Joint Effect of Temperature and Magnetic Field in Neutron Star EoS

Luigi Scurto, Valéria Carvalho, Helena Pais, Constança Providência

Introduction

Neutron Stars

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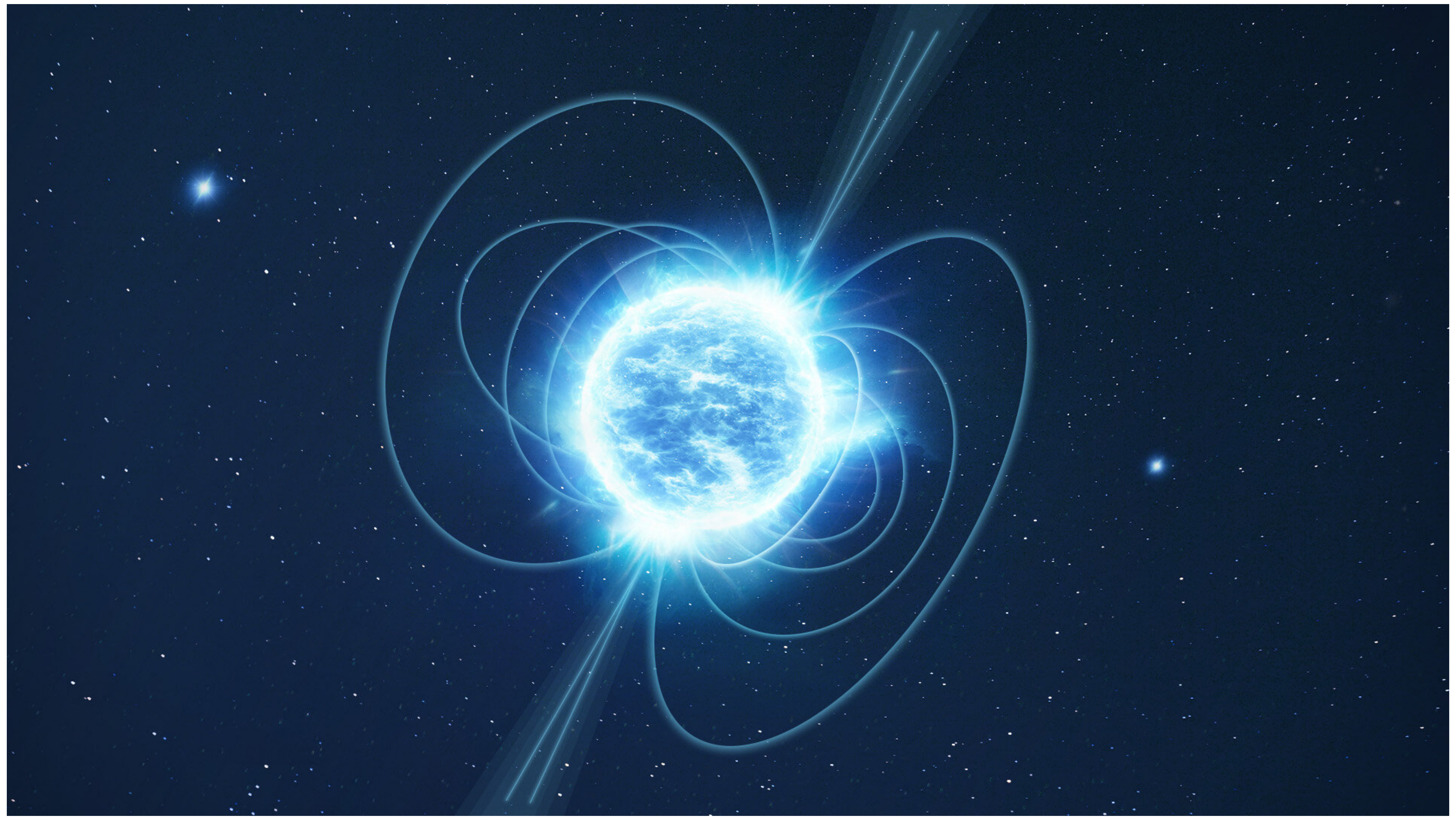
Compact objects of extreme interest in the multi messenger era of astrophysics



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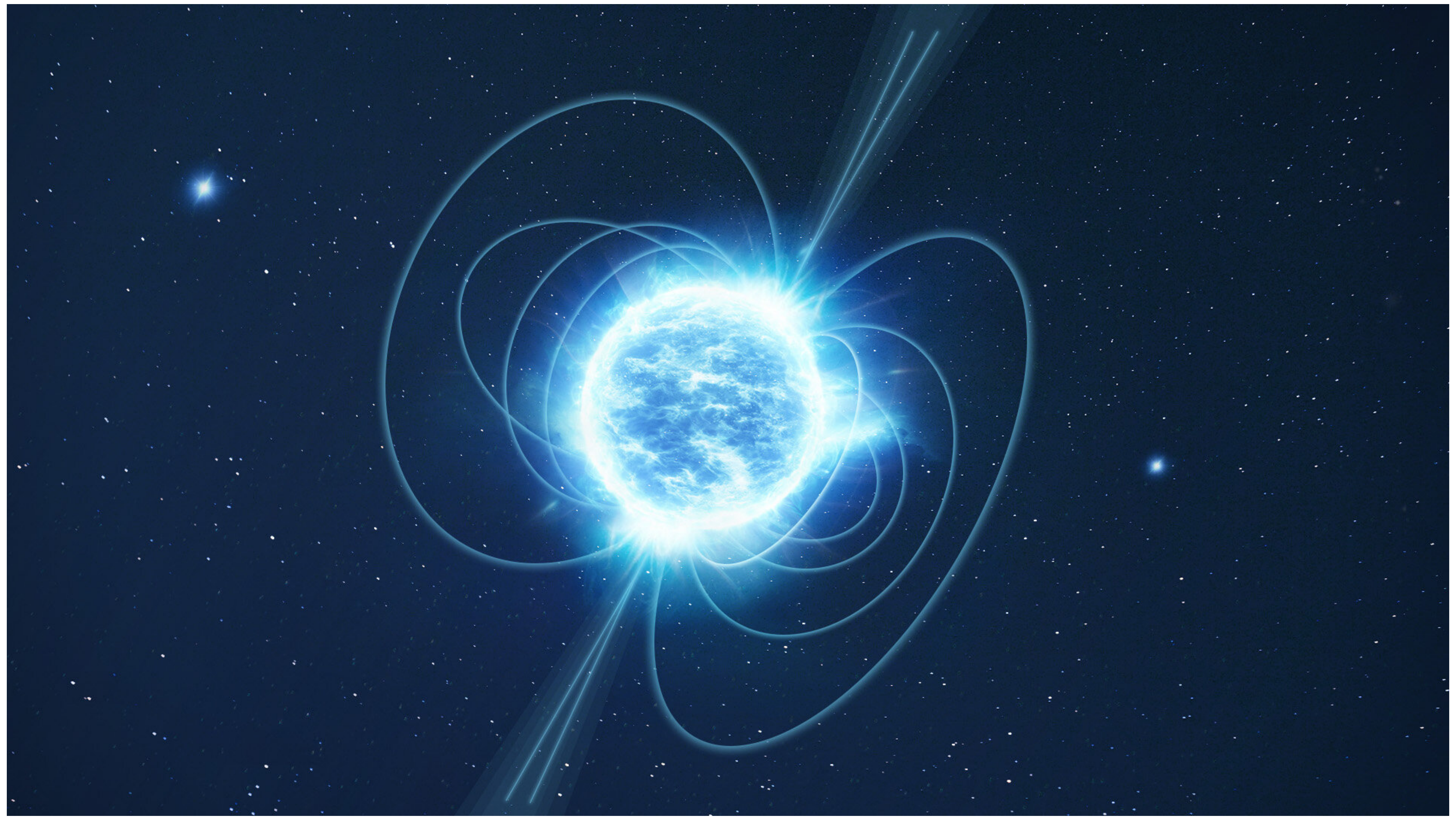

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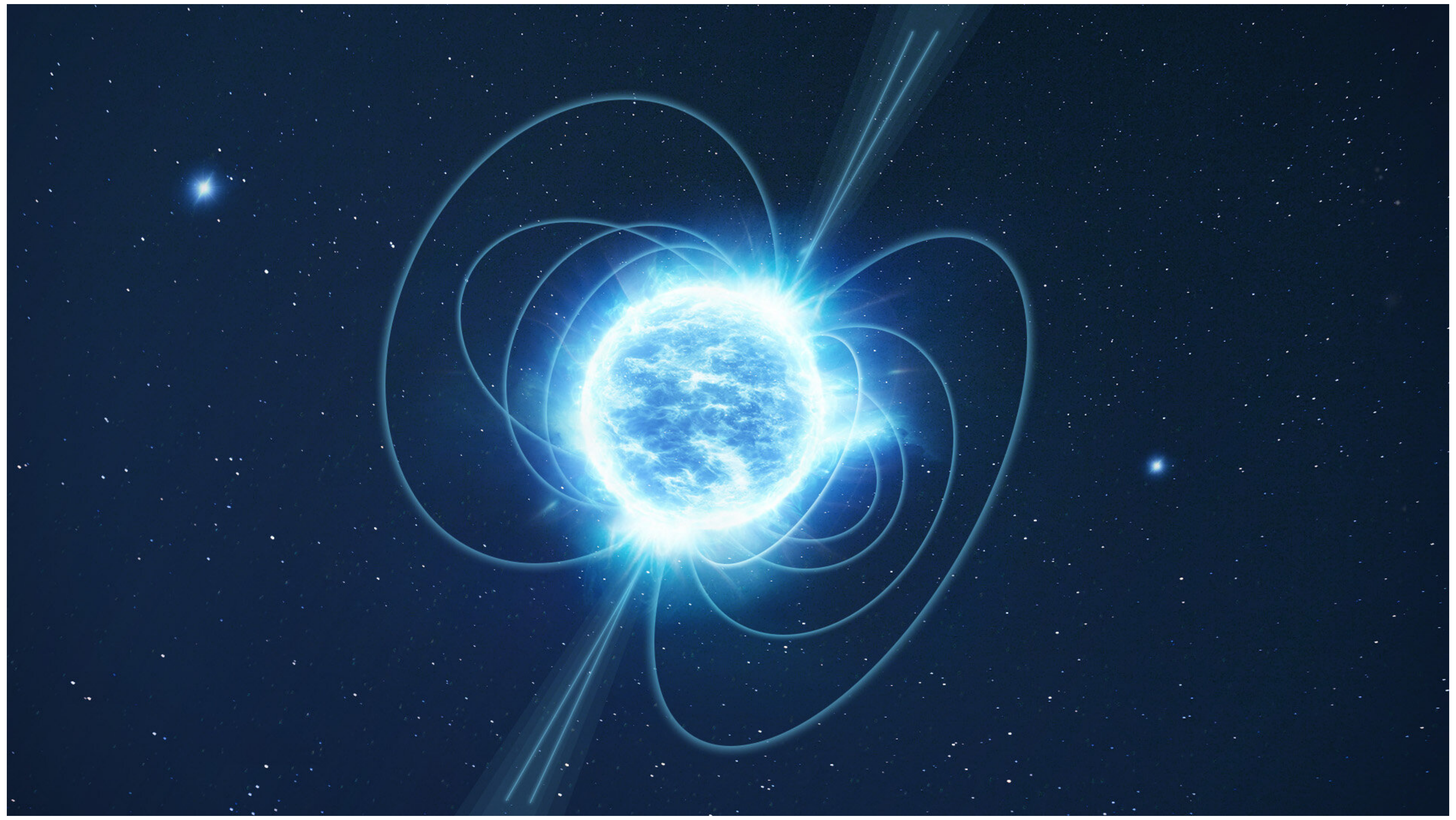

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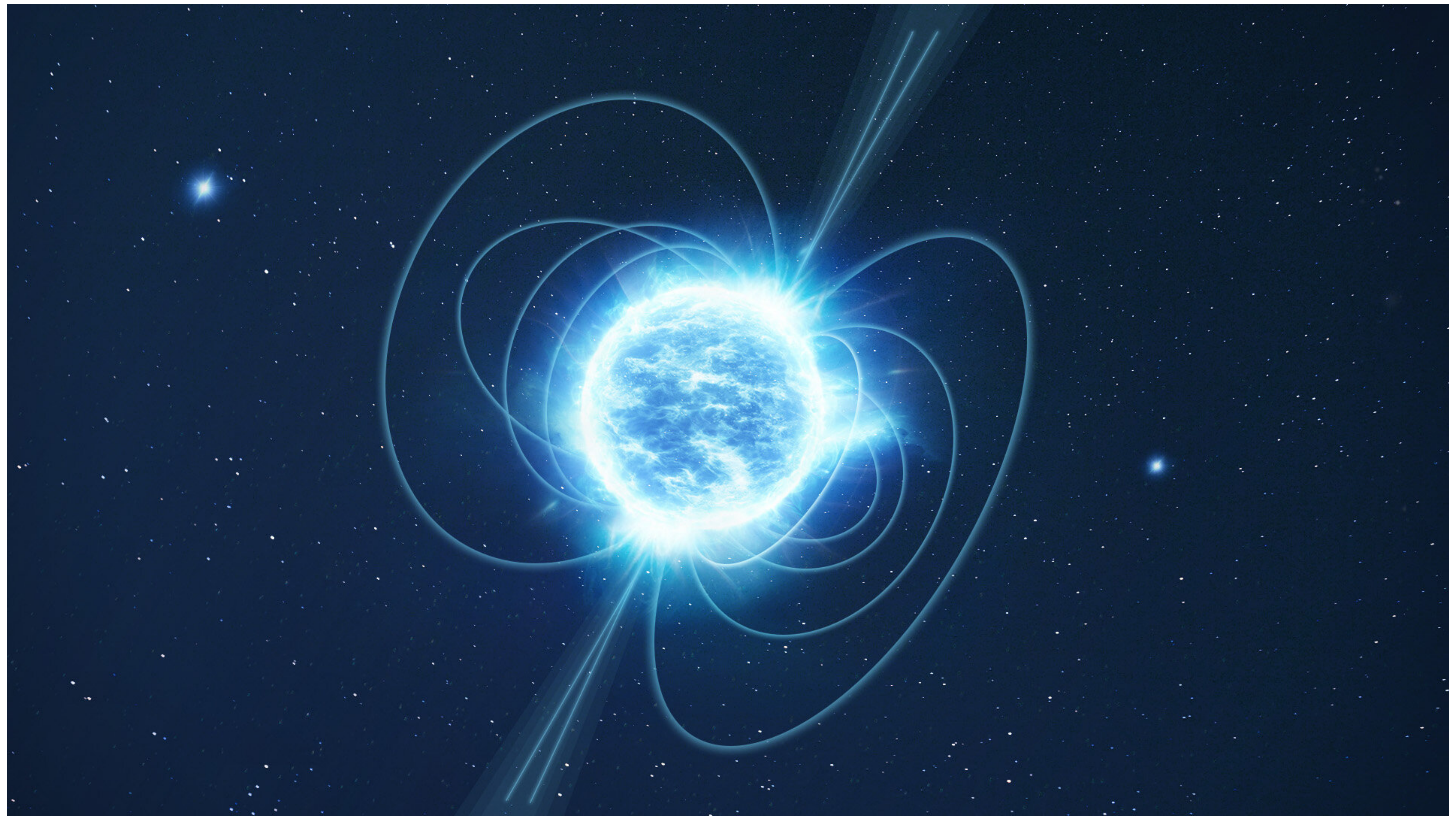

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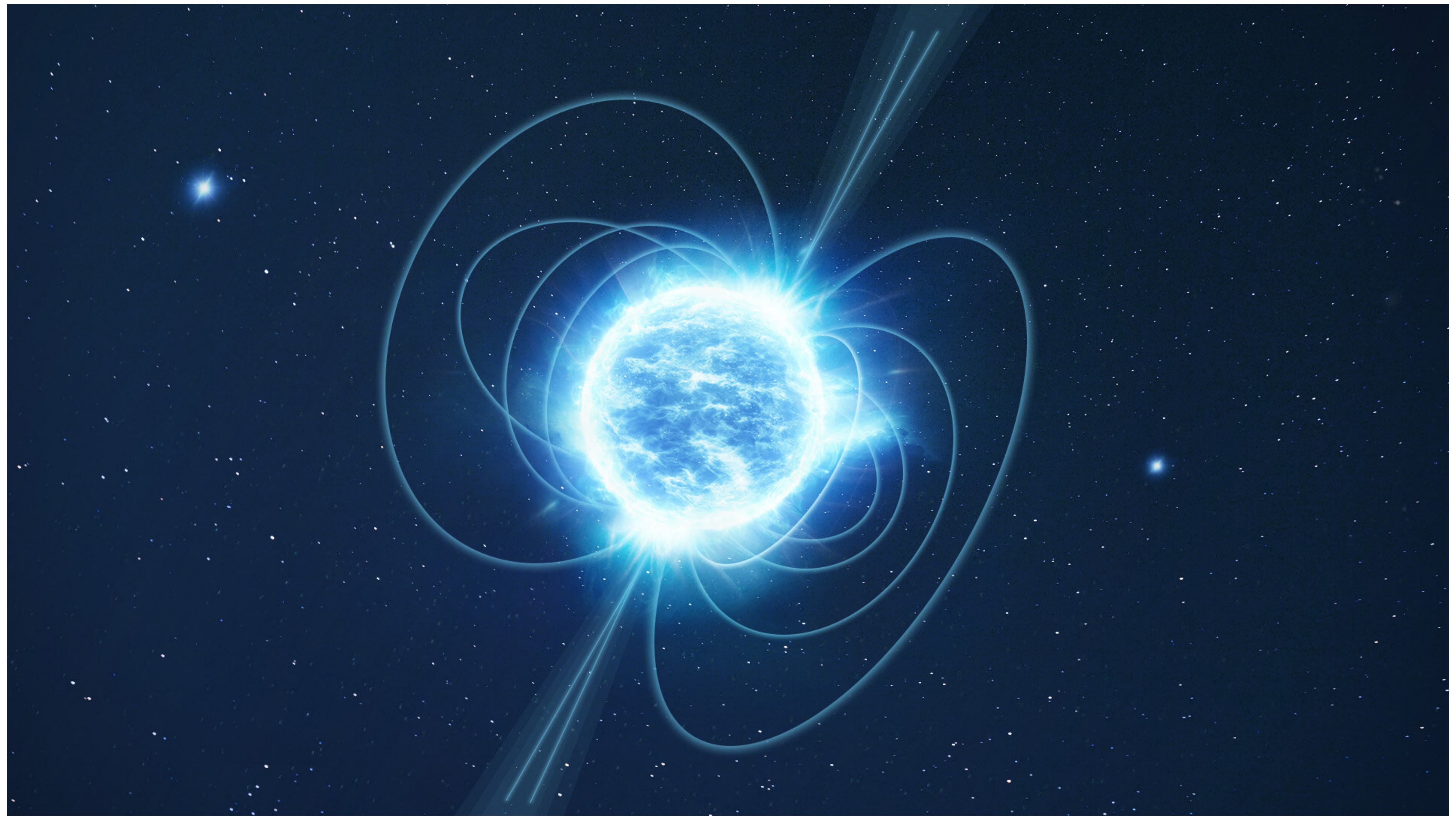

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Their interior composition is still very poorly constrained !

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Neutron Star	Sun

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Sun

$$M = 1 M_\odot$$

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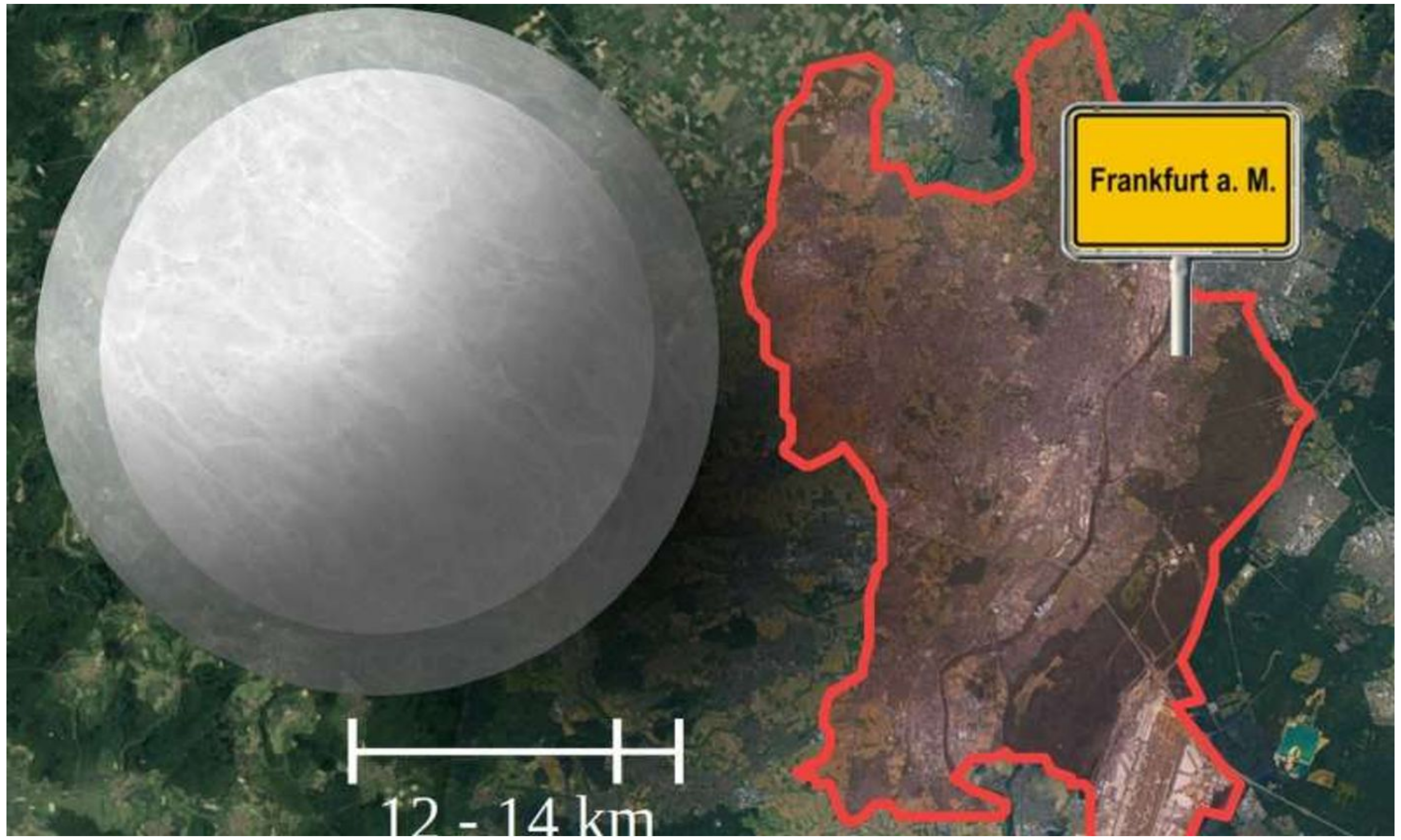

Credit: Lukas Weih, Goethe University

Roughly a hundred trillion times denser than the sun !

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→ **Is this a good approximation ?**

Framework

Relativistic Mean Field Approximation

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Mesons

$$\mathscr{L}_\sigma = \frac{1}{2}\left(\partial_\mu\phi\partial^\mu\phi - m_\sigma^2\phi^2\right)$$

$$\mathscr{L}_\rho = -\frac{1}{4}\mathbf{B}_{\mu\nu}\cdot\mathbf{B}^{\mu\nu} + \frac{1}{2}m_\rho^2\mathbf{b}_\mu\cdot\mathbf{b}^\mu$$

$$\mathscr{L}_\omega = -\frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} + \frac{1}{2}m_\omega^2 V_\mu V^\mu$$

The mesons are then replaced with their ground state expectation value

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$$\mathscr{L}_{int} = -\frac{1}{6}\kappa\phi^3 - \frac{1}{24}\lambda\phi^4 + \Lambda_{\omega\rho}g_\omega^2 g_\rho^2 V_\mu V^\mu \mathbf{b}_\mu \cdot \mathbf{b}^\mu + \frac{\xi}{4!}g_\omega^4(V_\mu V^\mu)^2$$

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$$g_i = a_i + (b_i + d_i x^3) e^{-c_i x} \quad \text{with} \quad x = \rho/\rho_0$$

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[L. Scurto, H. Pais, F. Gulminelli, Phys. Rev. D 109, 103015 (2024)]

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The main effect of the field is the Landau Quantization for charged particles

$$\rho_i = \frac{k_i^{F^3}}{3\pi^2} \longrightarrow \rho_i = \frac{|q|B}{2\pi^2}\sum_{\nu=0}^{\nu_{\max}^i} g_\nu k_{i,\nu}^F$$

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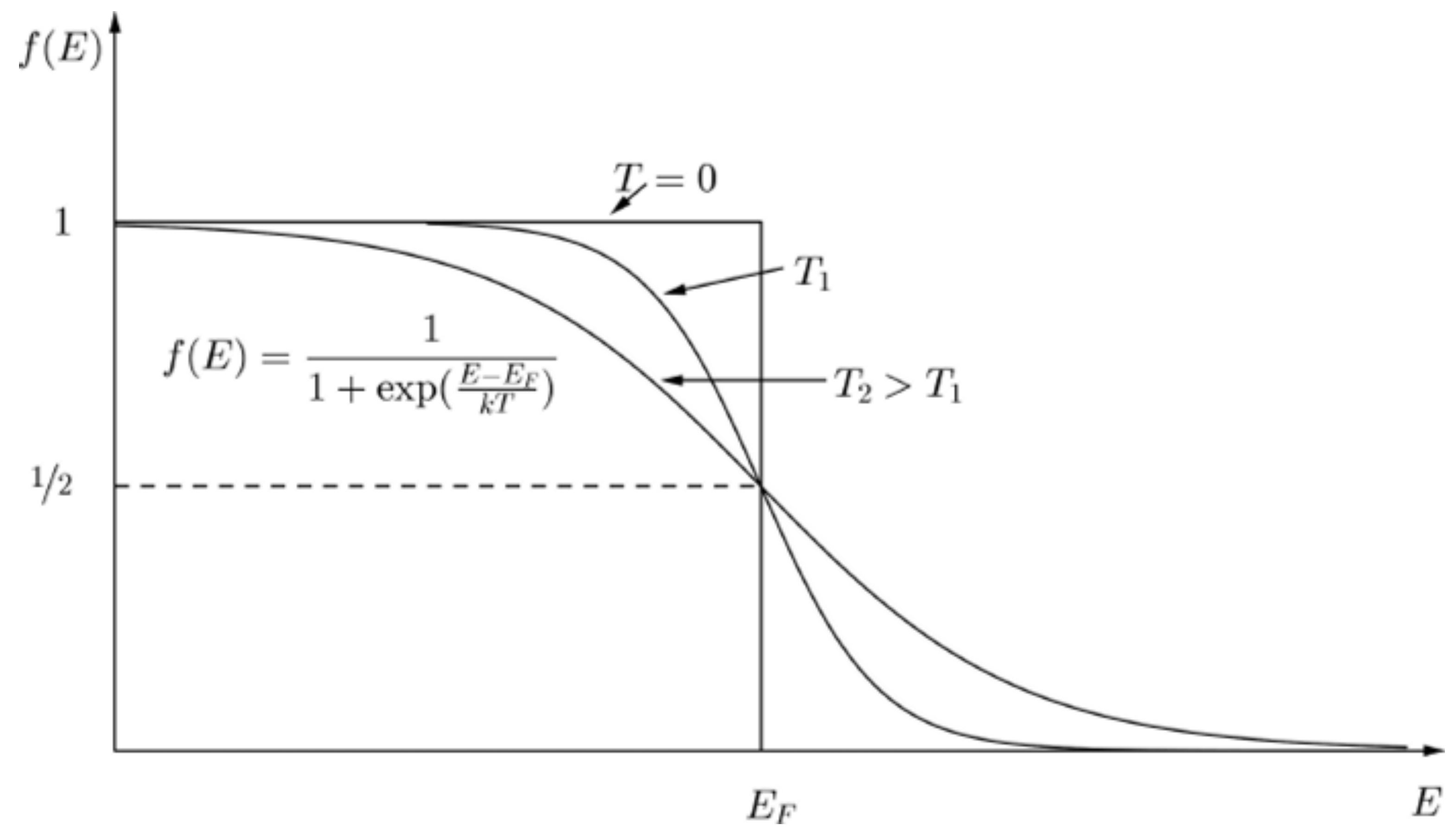

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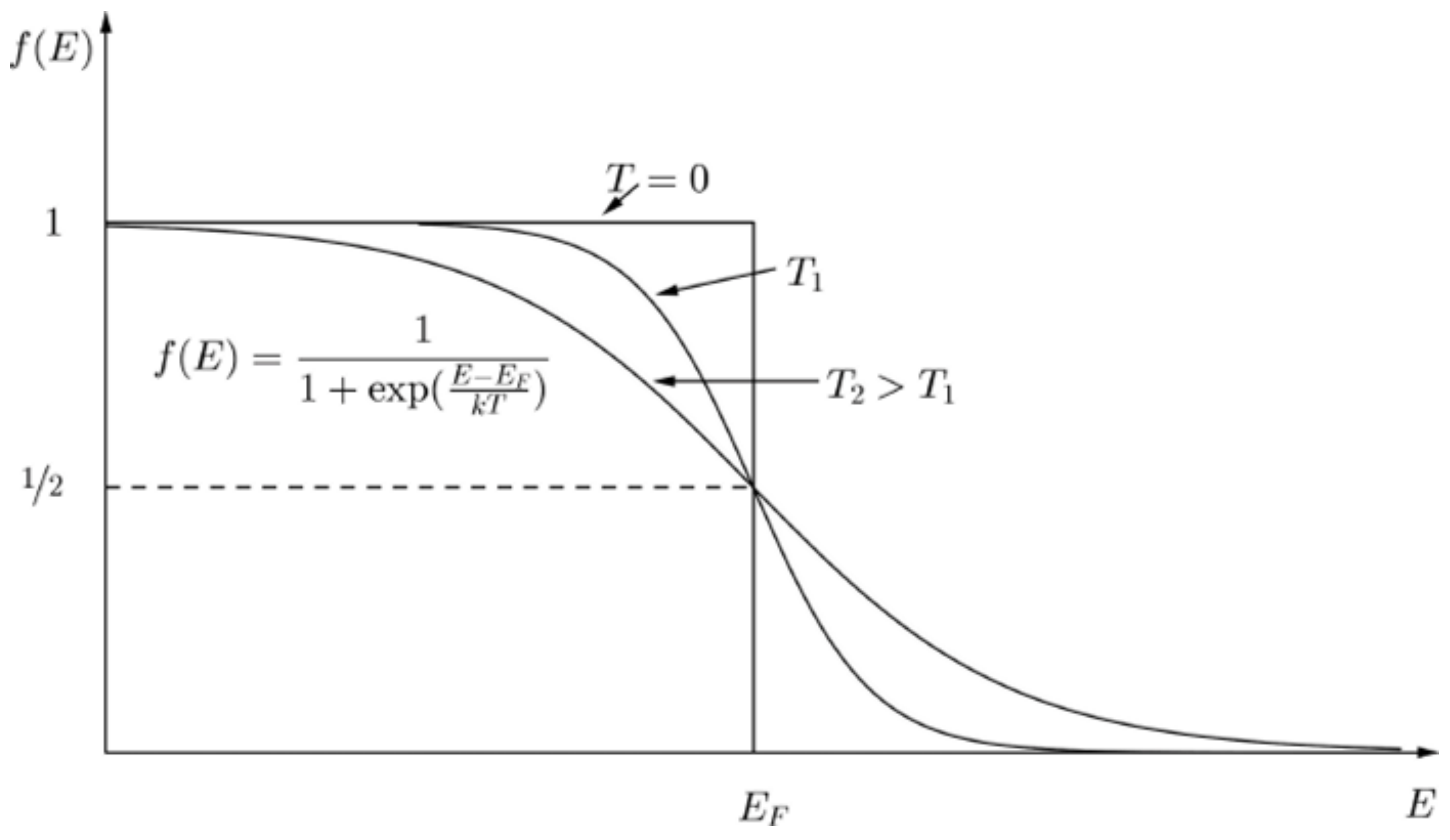

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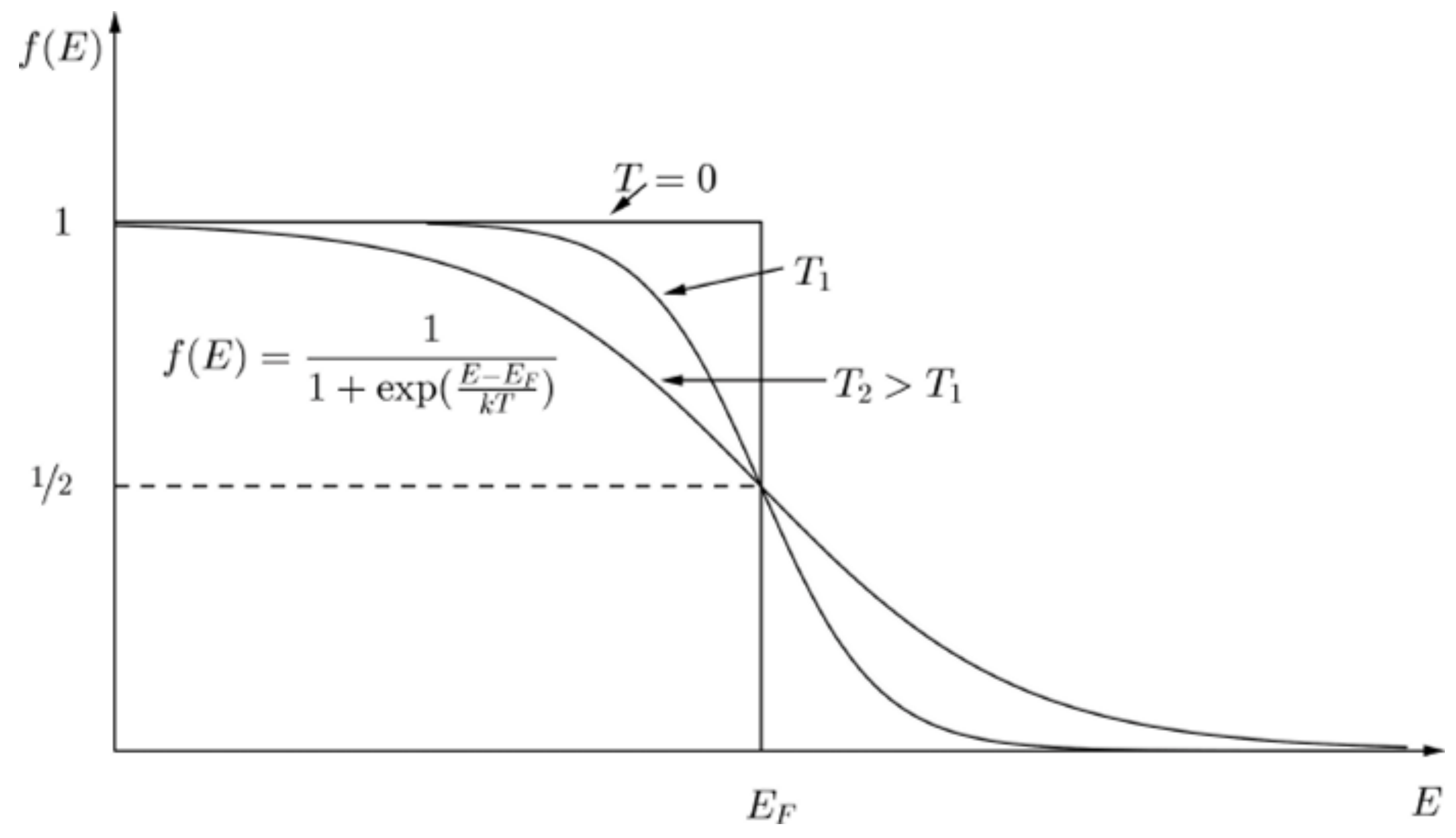

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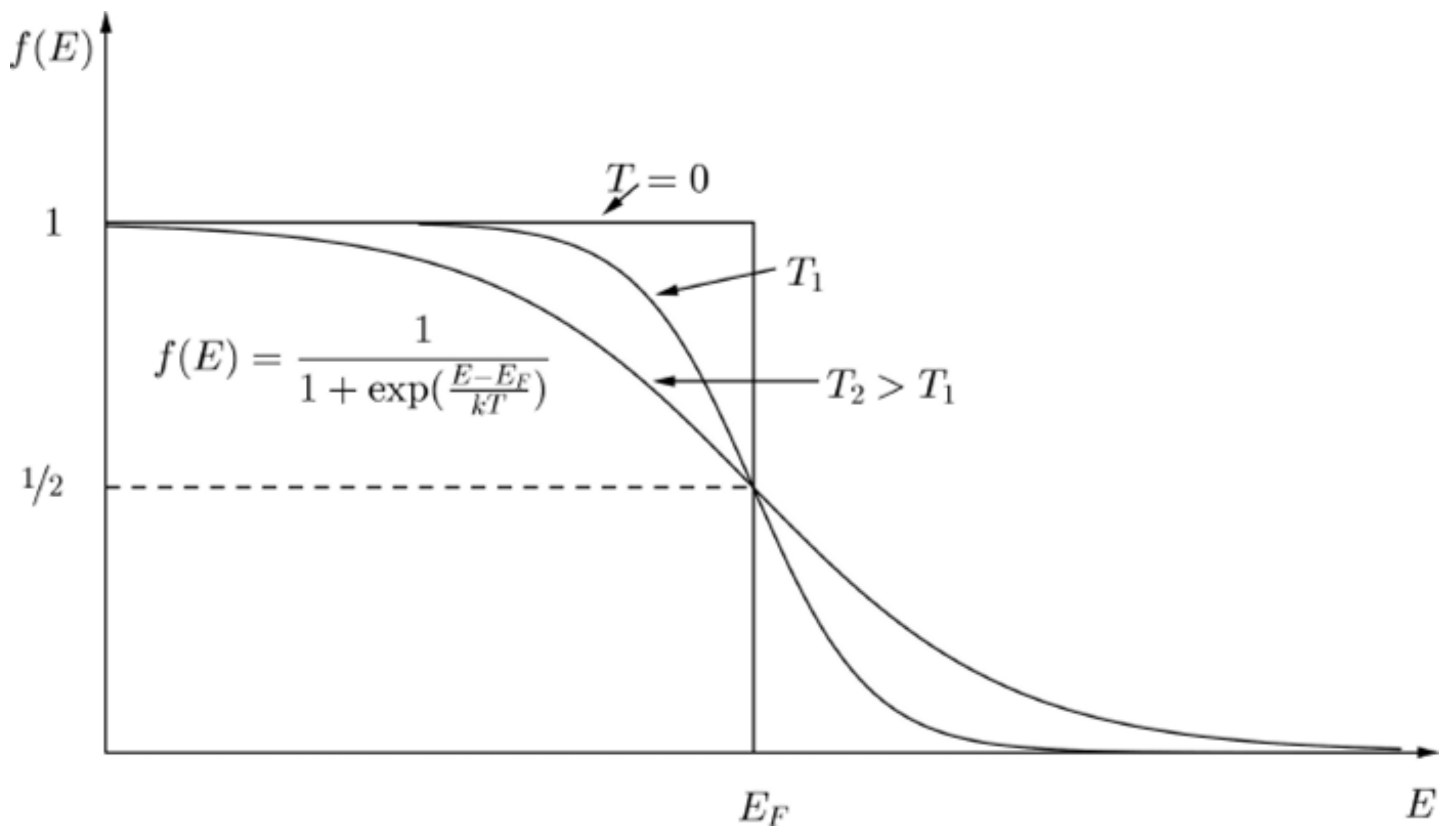

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When both are present, we have both the integration over momenta and the sum of Landau Levels

Results

Effect on the EoS

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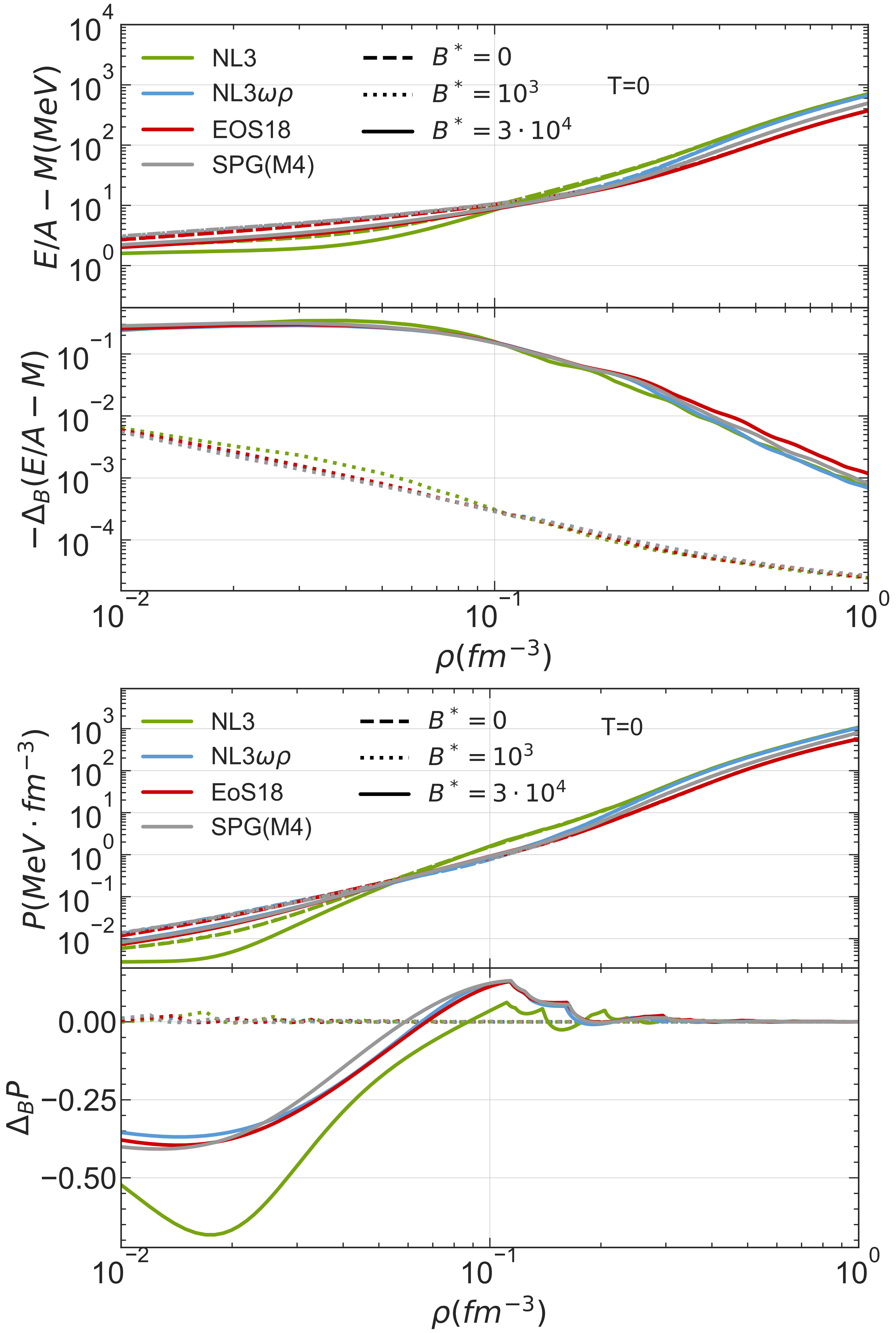

The effect on the EoS of the magnetic field appears to be very small and decreases at higher densities

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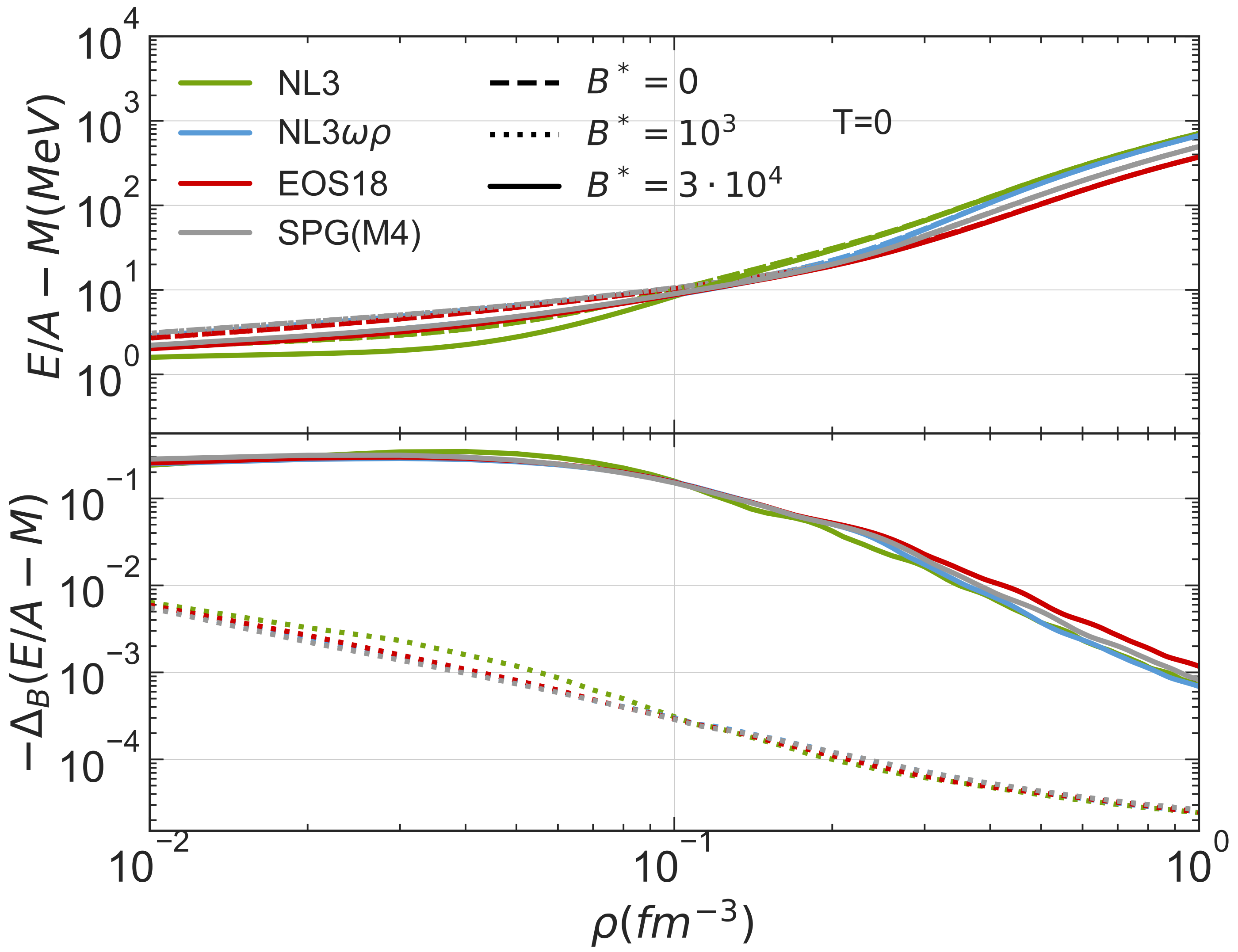

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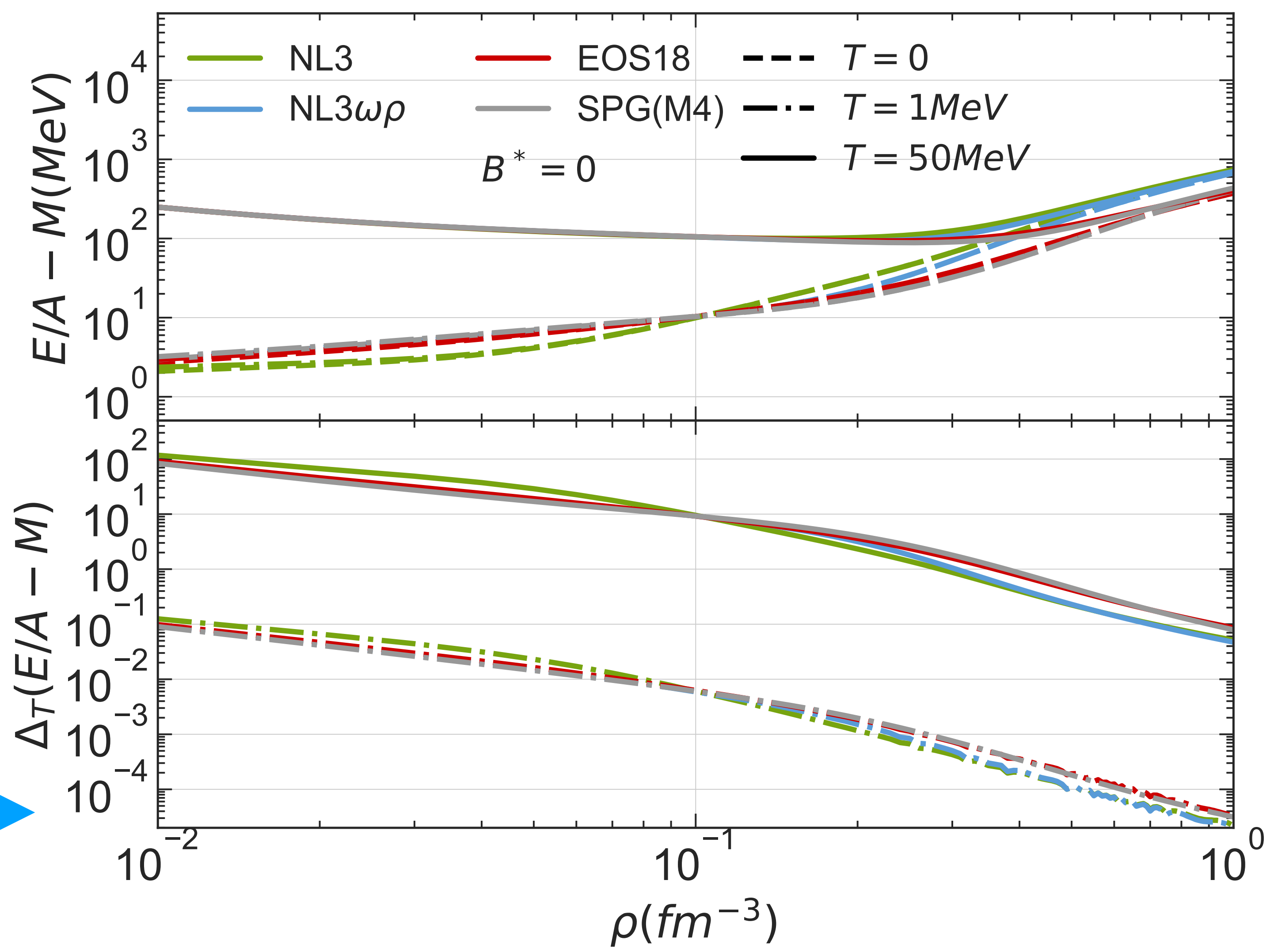
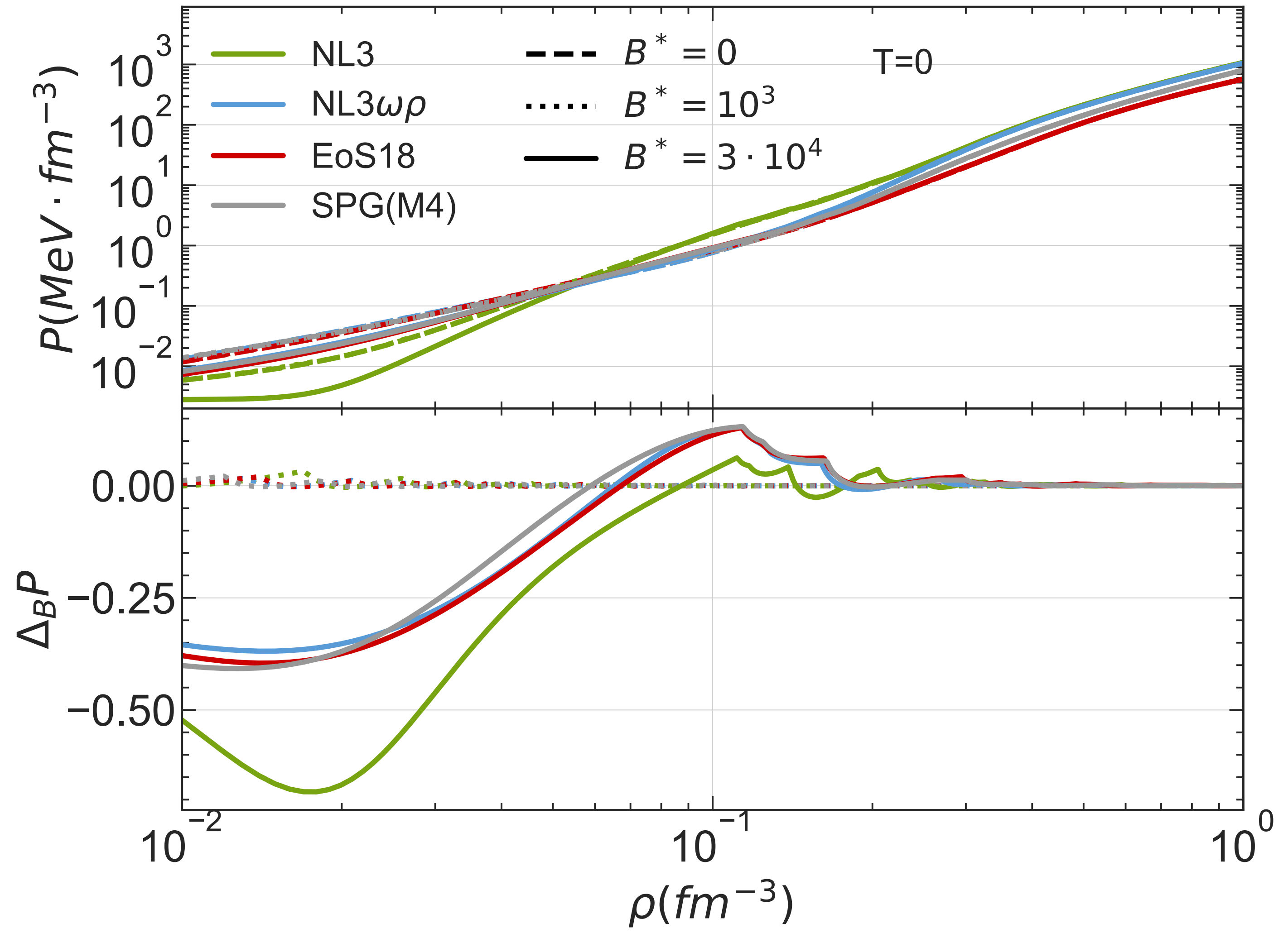
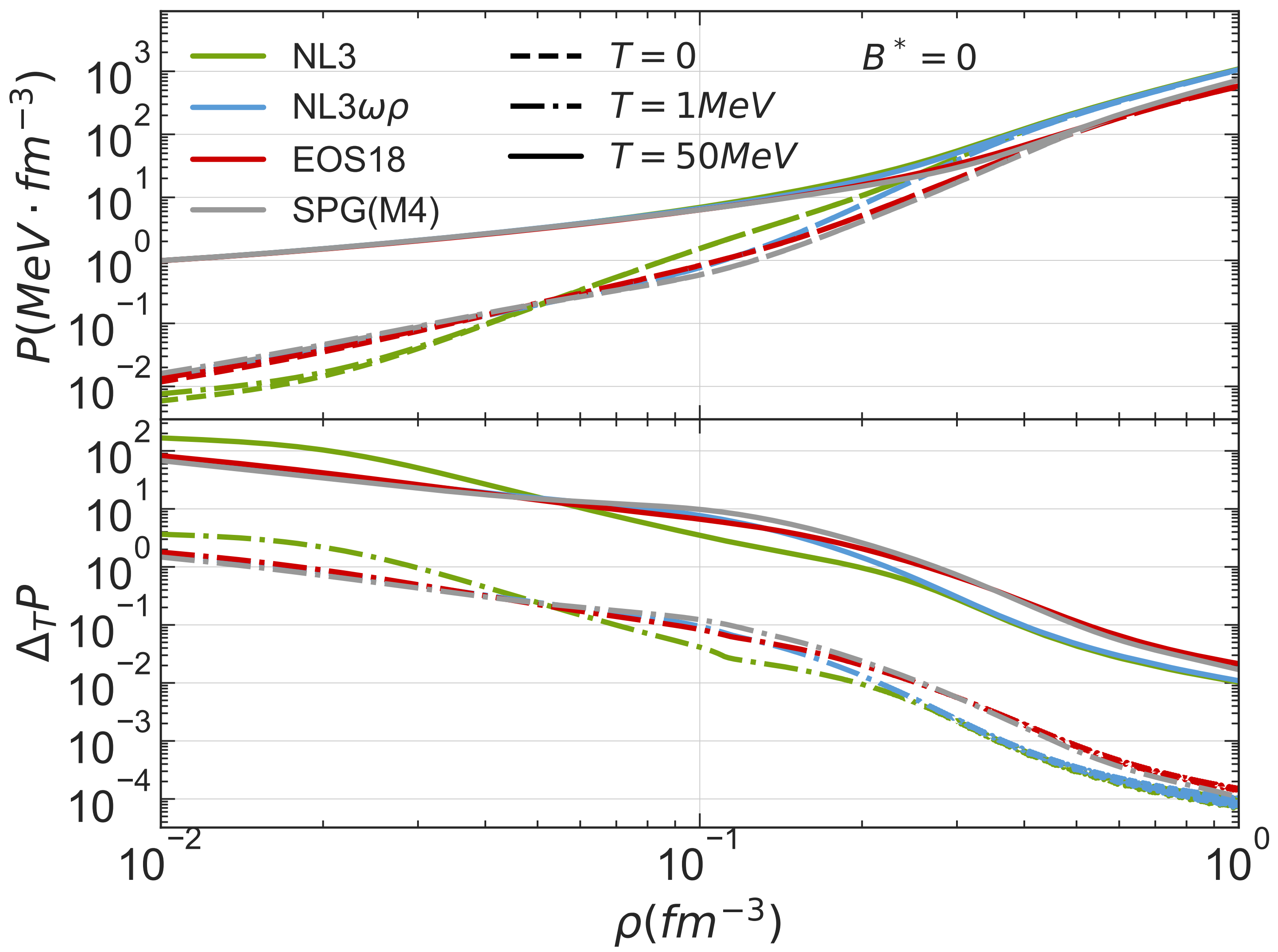


On the other hand, the effect of temperature appears to be opposite to the one of the magnetic field, but much stronger





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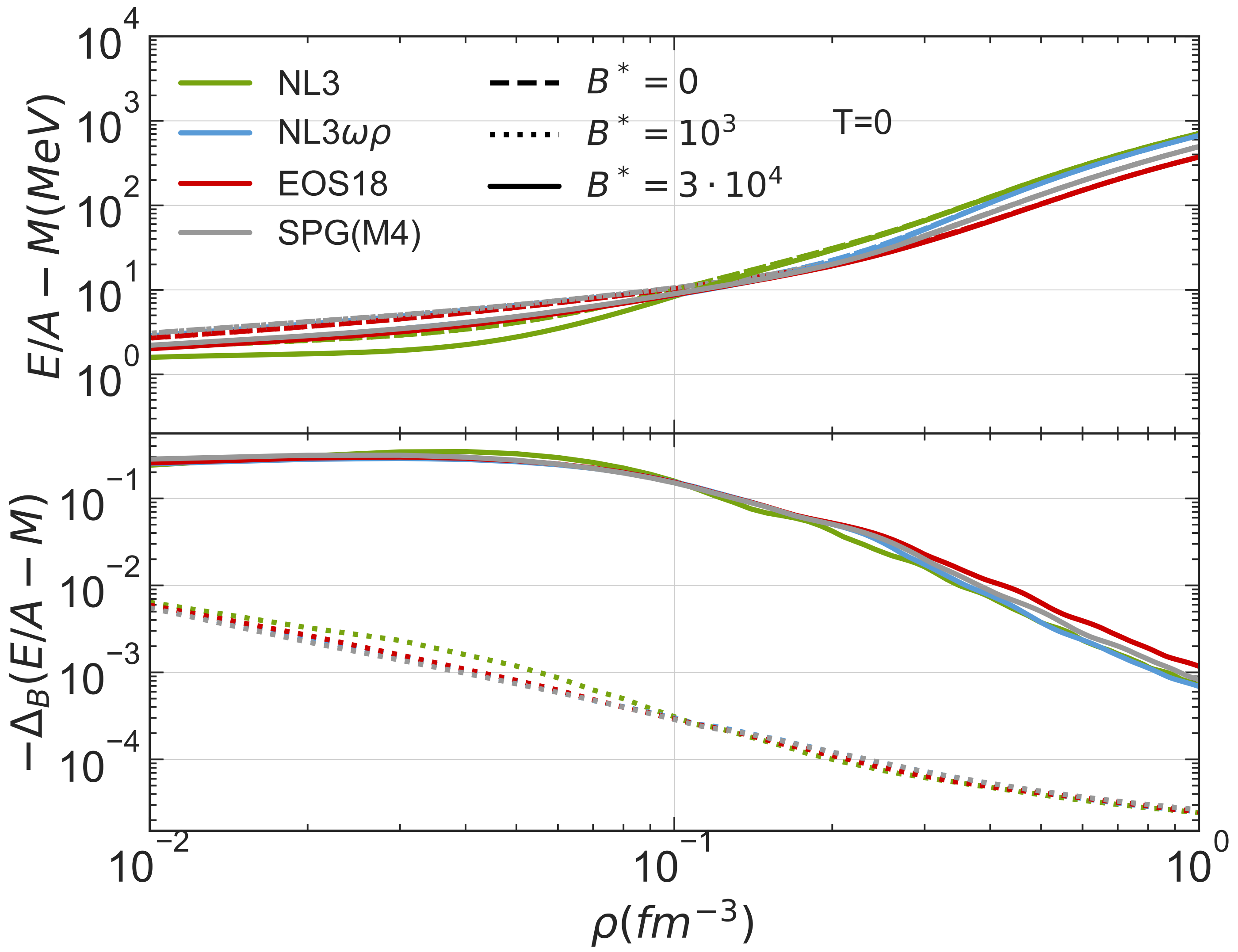

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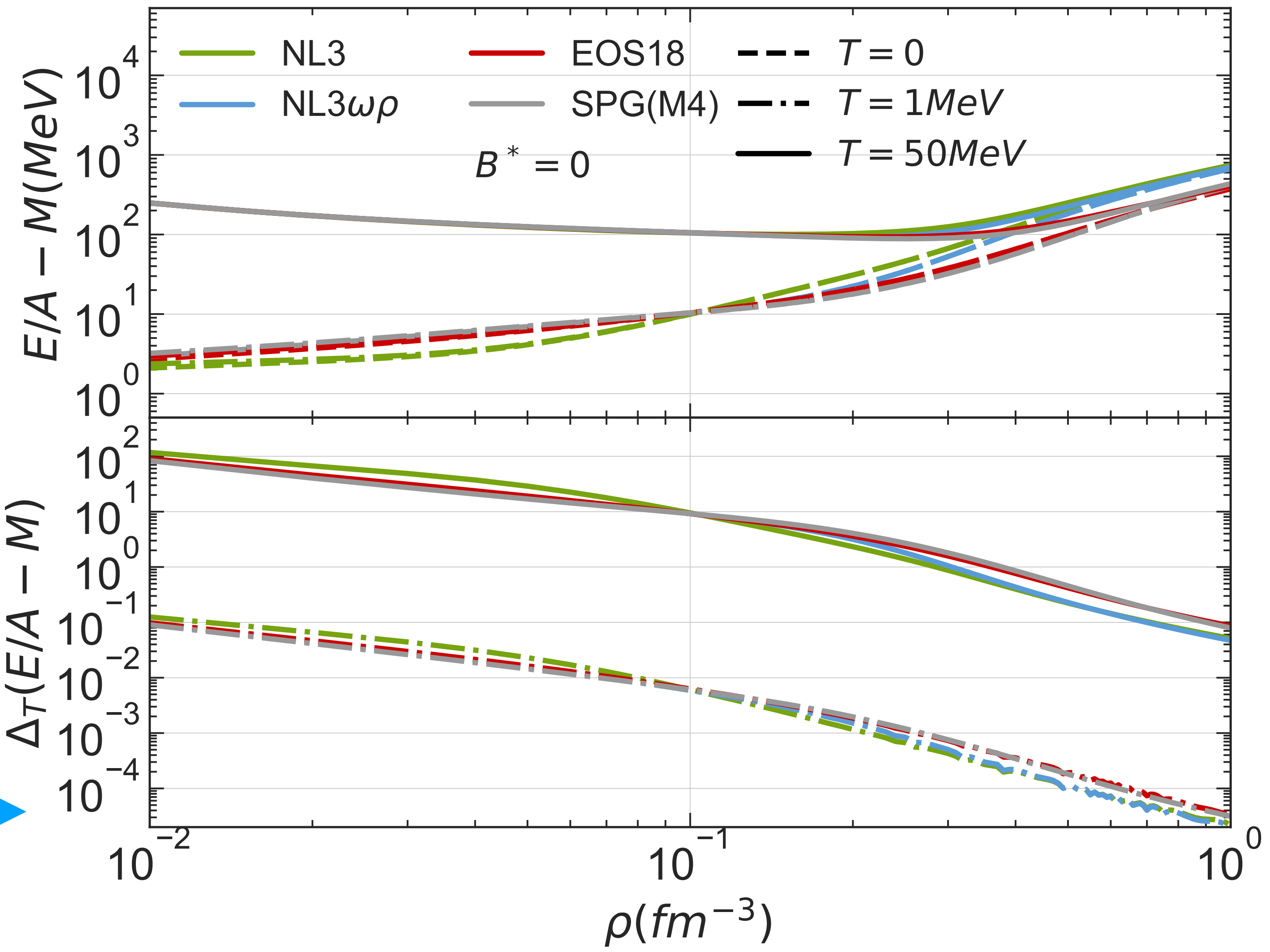
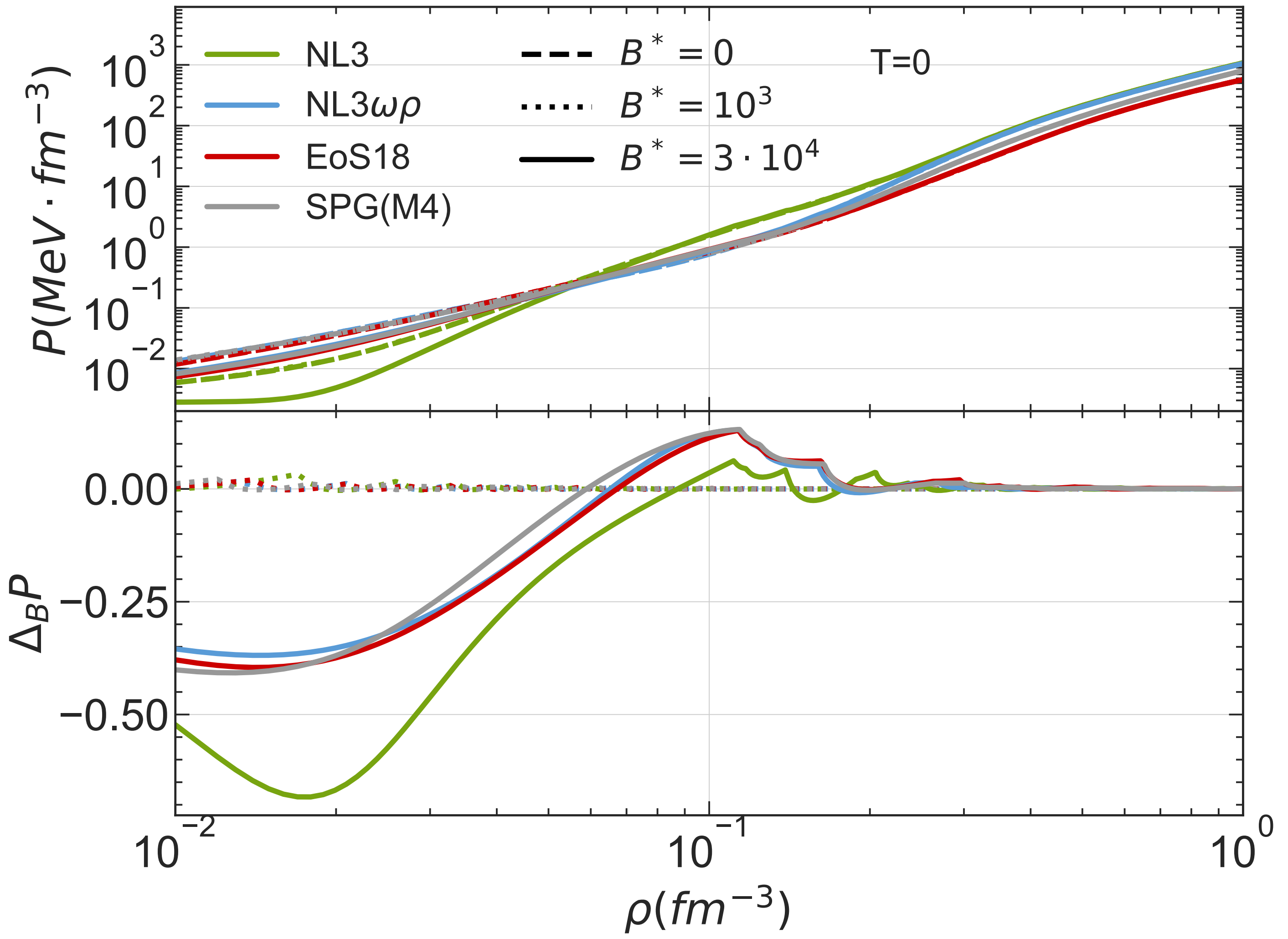
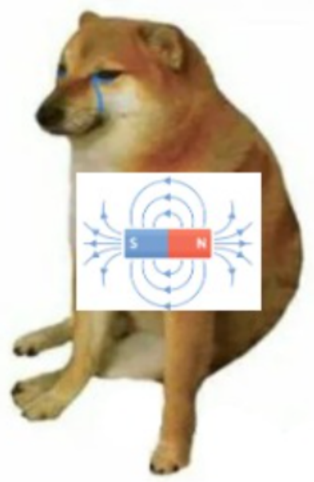
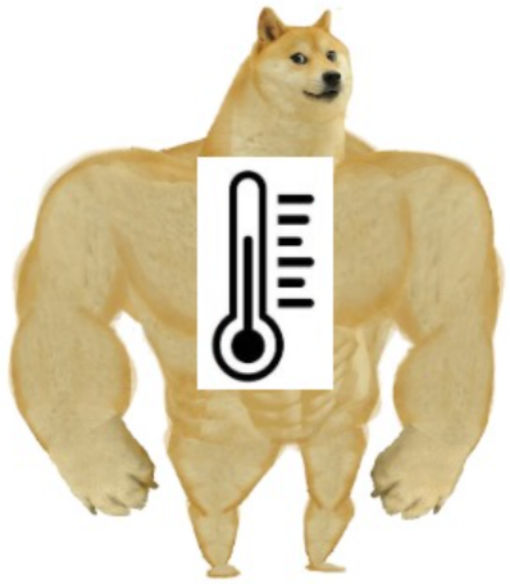
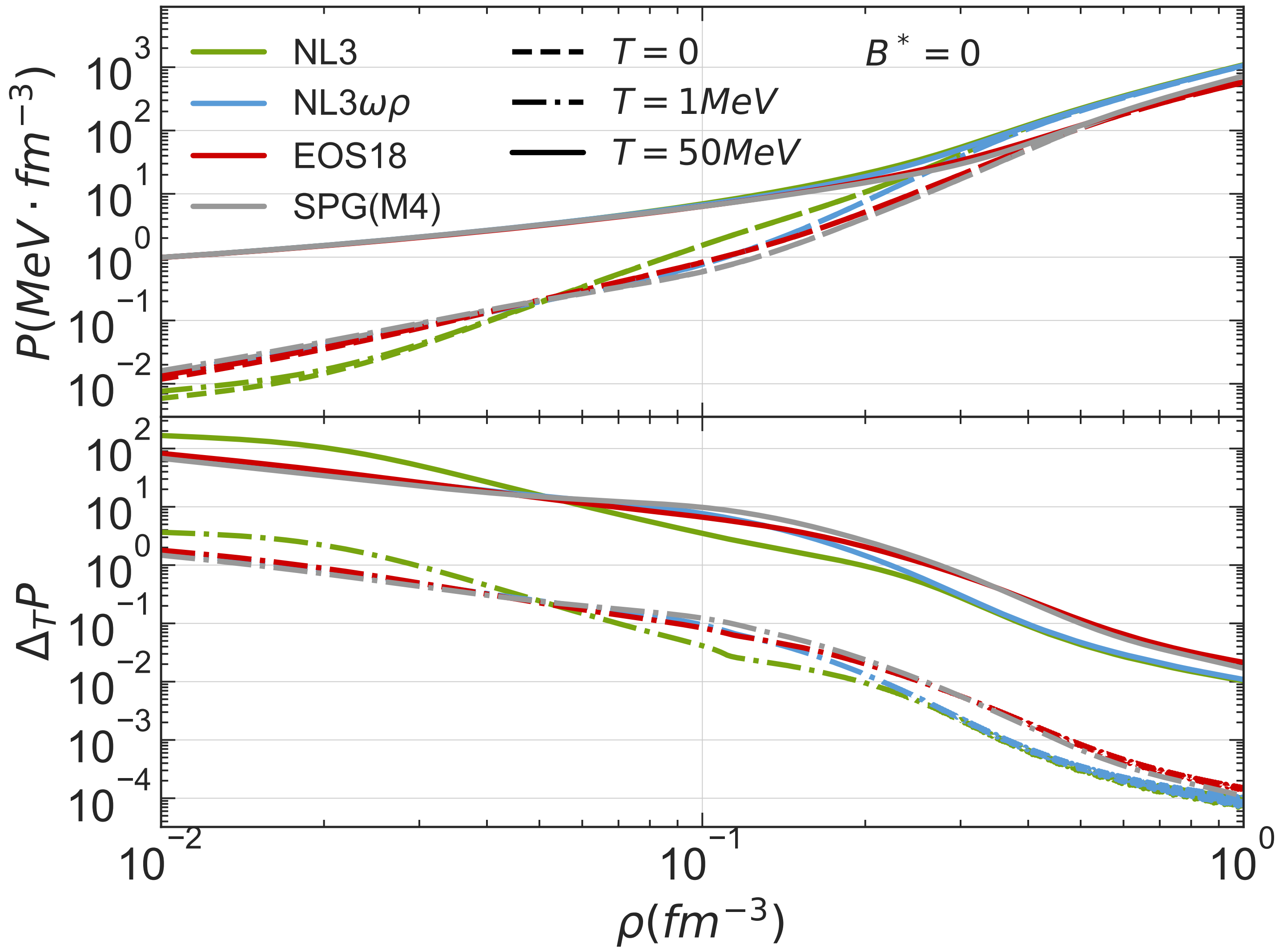






The effect of temperature tends to wash away the effect of the magnetic field





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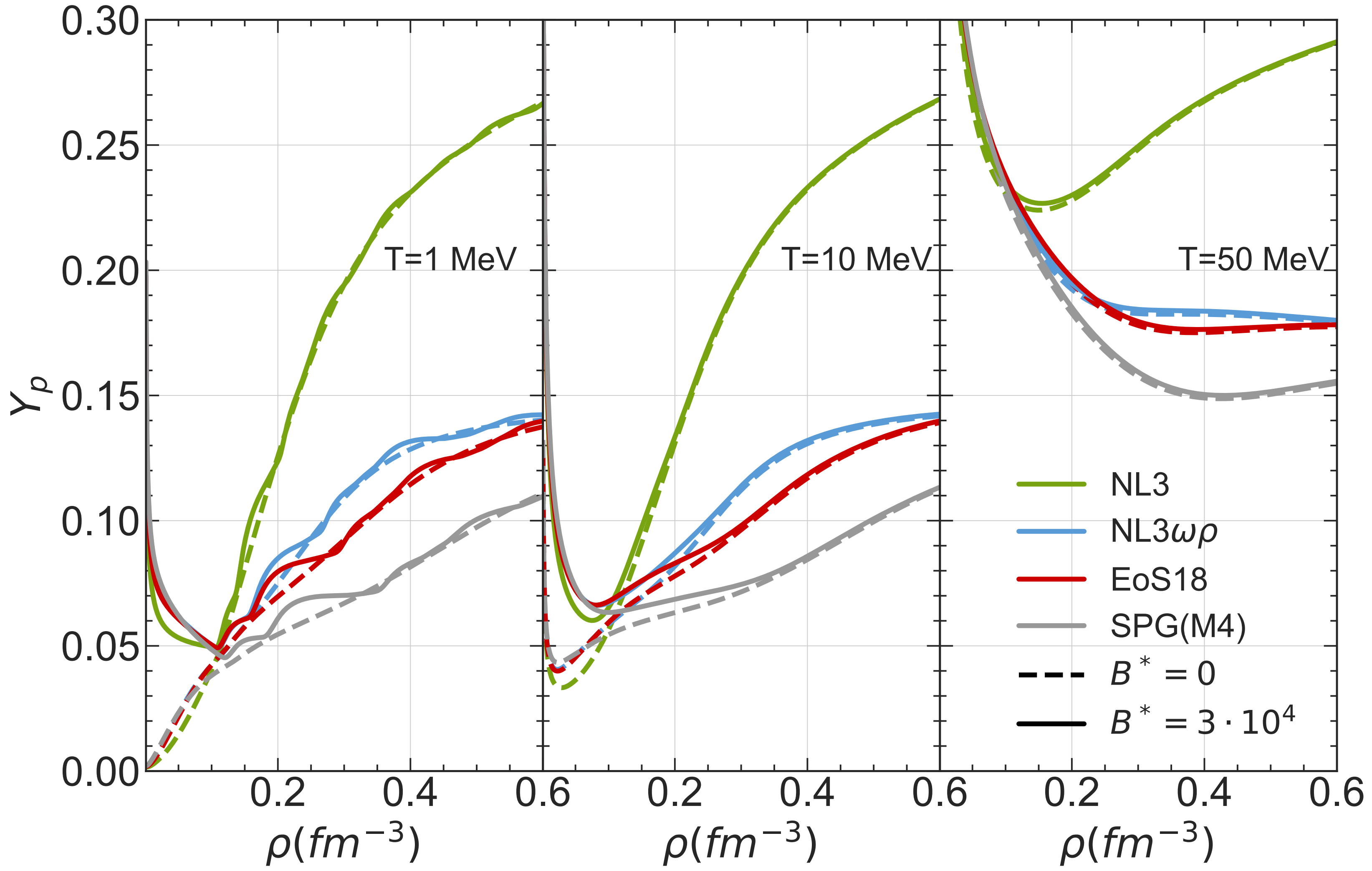

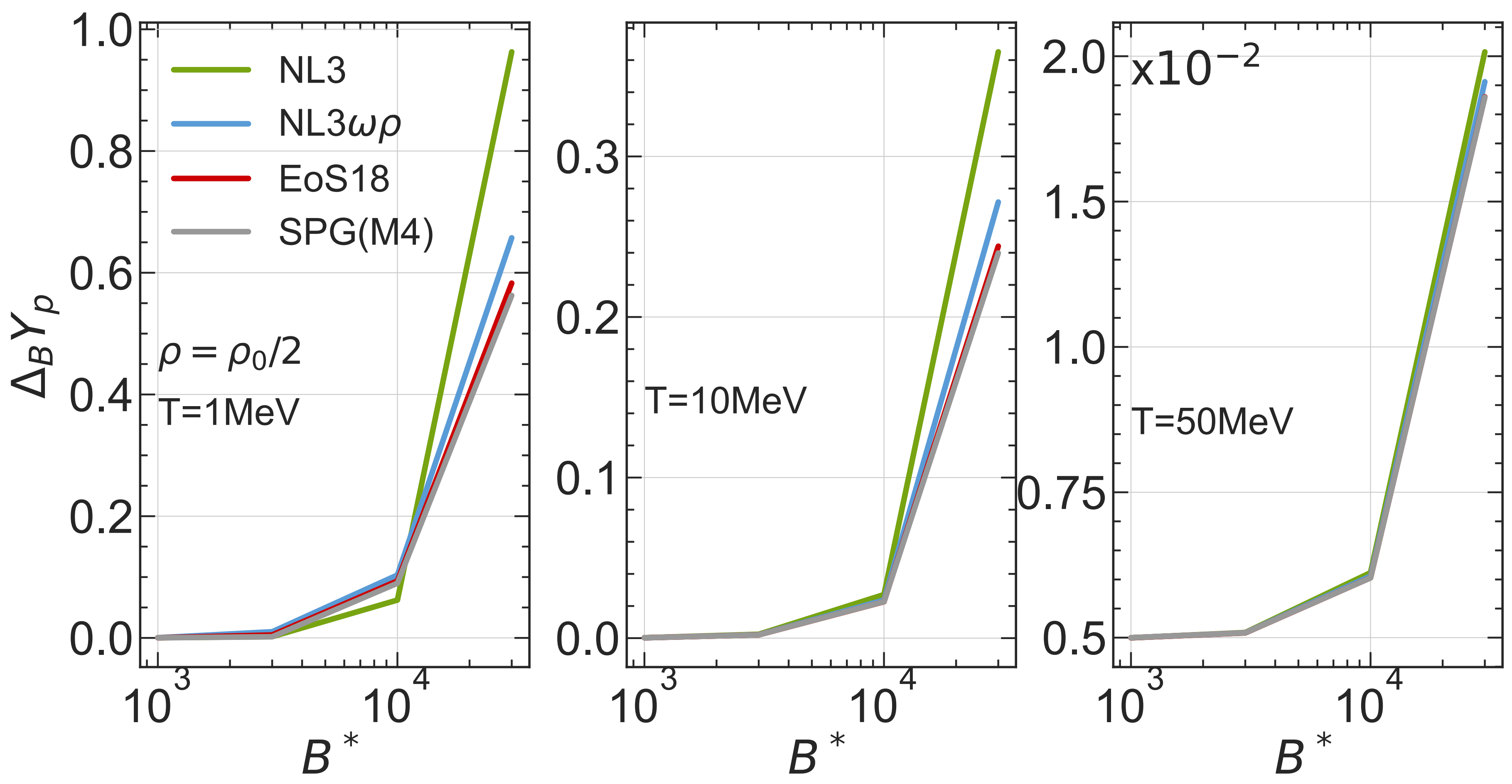

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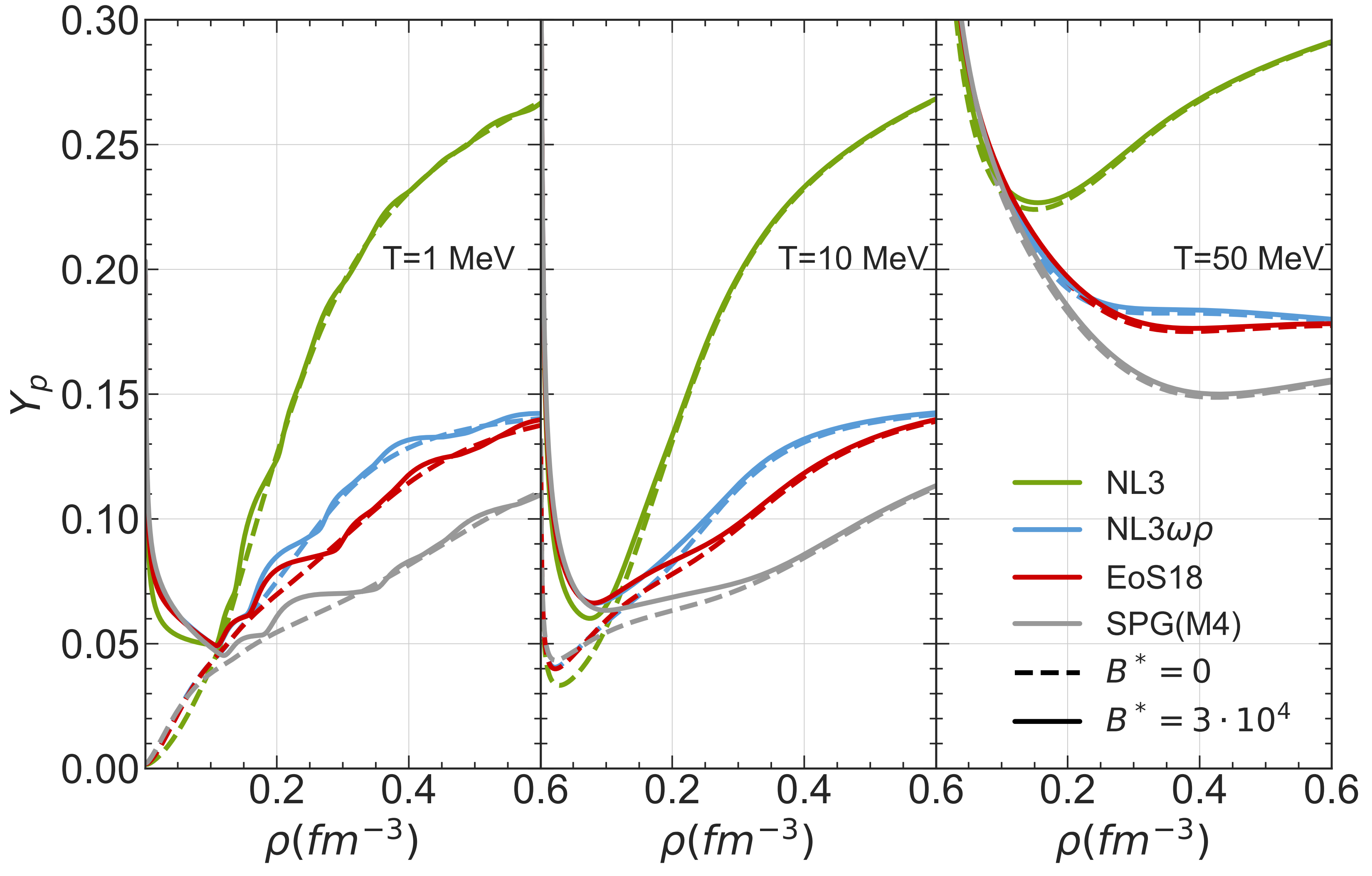

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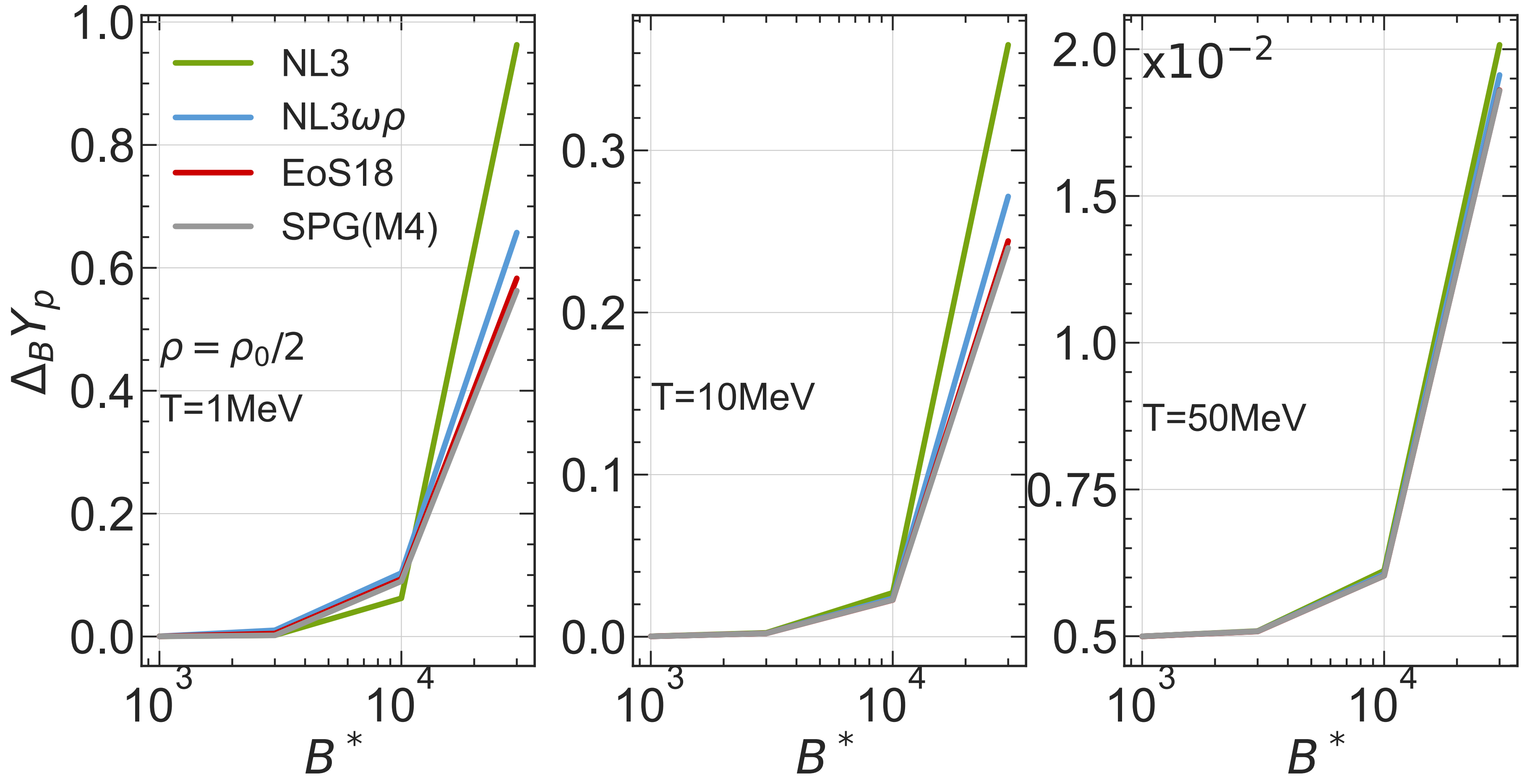
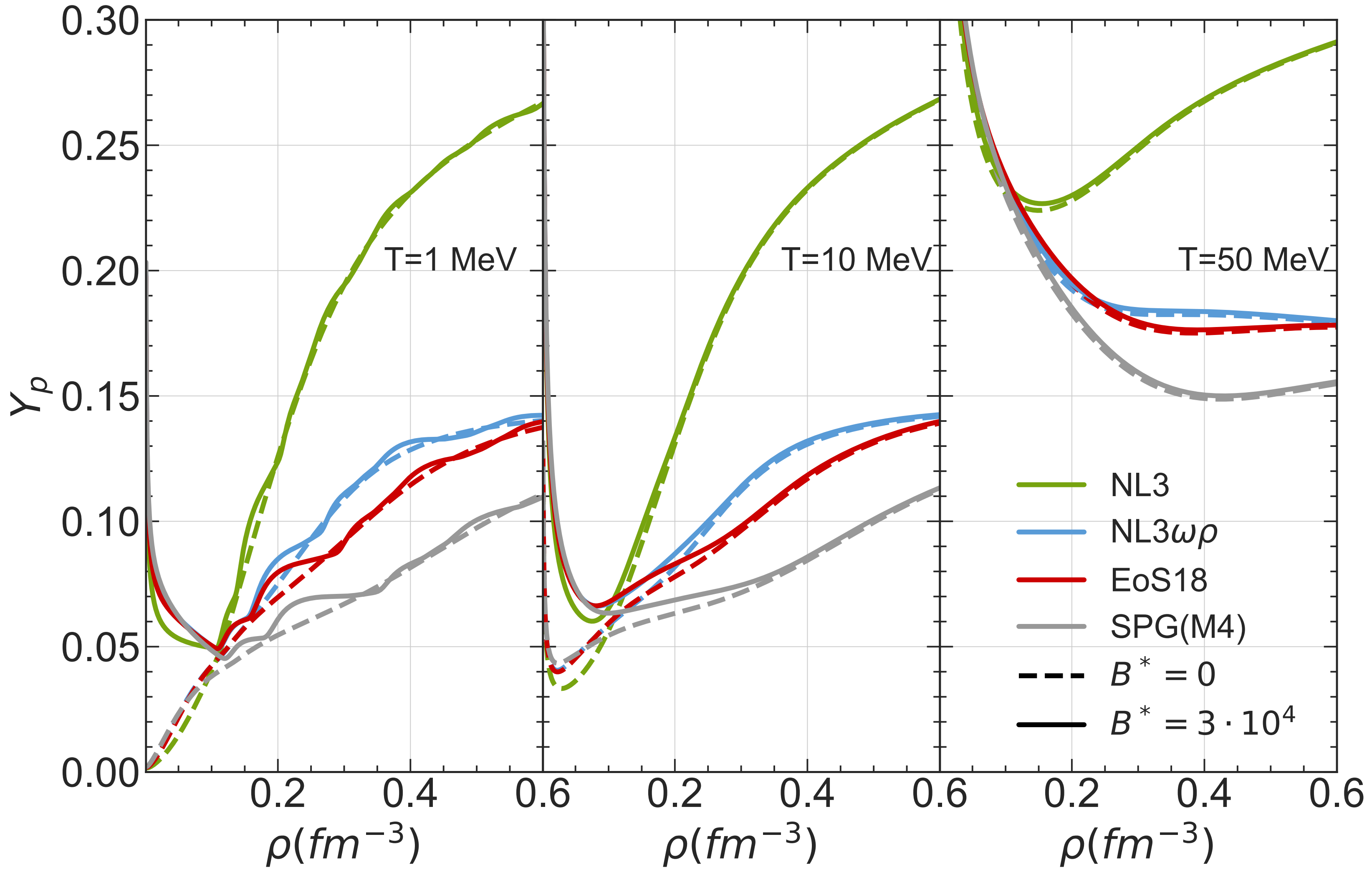


The effect is more relevant at low densities and, for very strong magnetic fields, is relevant up to temperatures around 10 MeV

Effect on the composition



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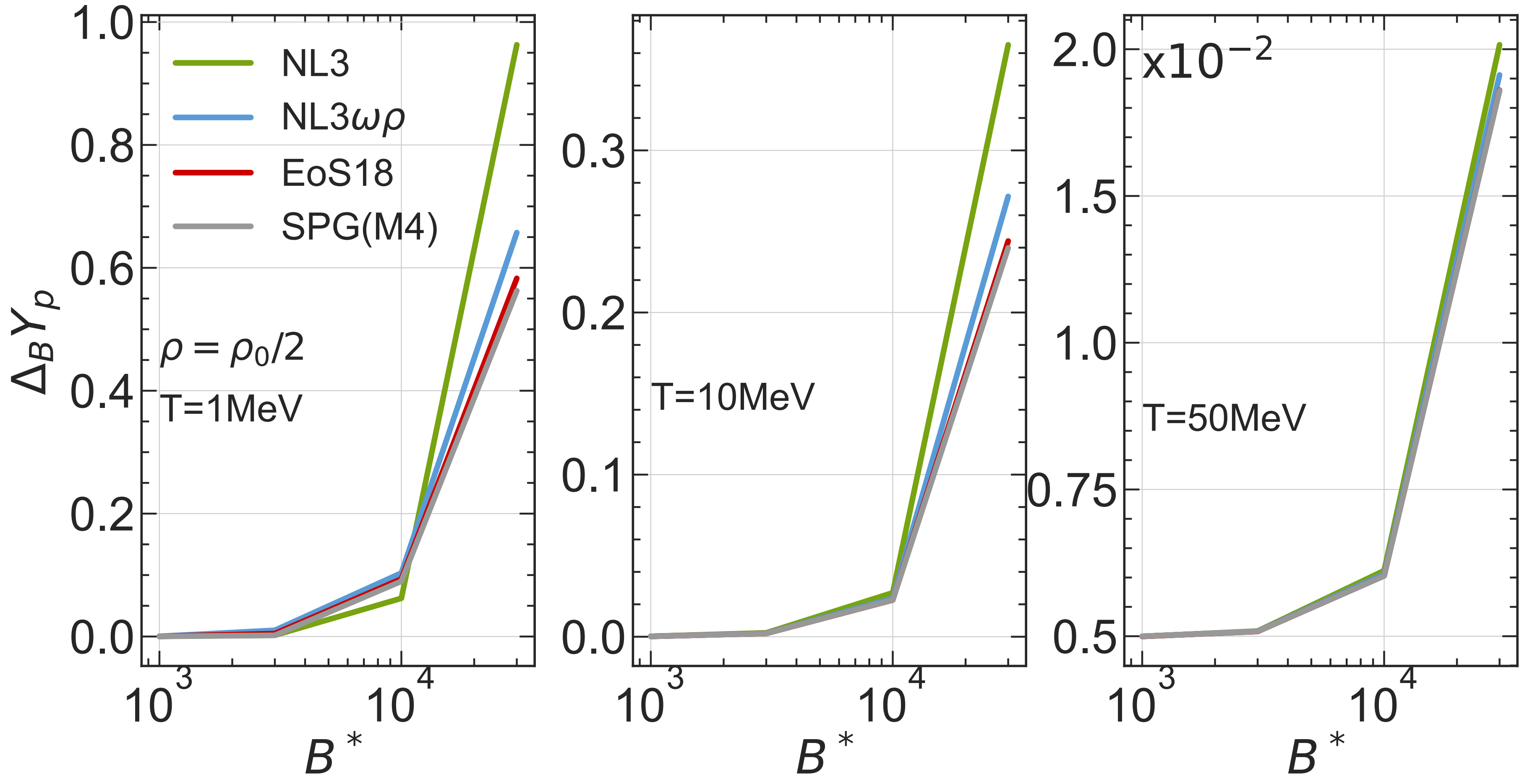
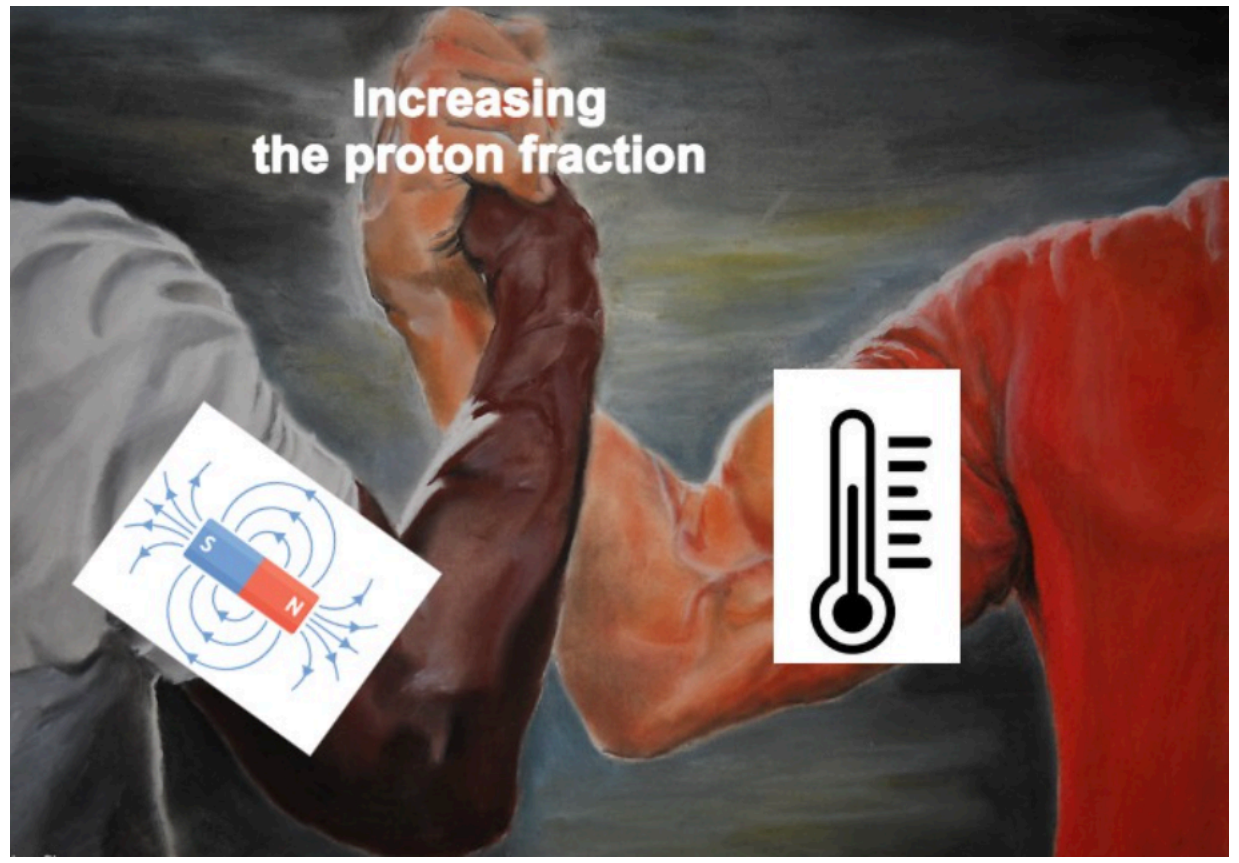


This is due to the fact that magnetic fields and finite temperatures both have the effect of increasing the proton fraction



Conclusions

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Is this a good approximation ?

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Yes for static properties ← **Is this a good approximation ?**

- In the case of the EoS, the effect of the magnetic field is orders of magnitude bigger than the one of the magnetic field and acts in the opposite sense

Conclusions

Yes for static properties ← Is this a good approximation ? → Partially for dynamical properties

- In the case of the EoS, the effect of the magnetic field is orders of magnitude bigger than the one of the magnetic field and acts in the opposite sense

- In the case of the composition, magnetic field and temperature both tend to increase the proton fraction, so the joint effect should be taken into account in the case of strong magnetic field and intermediate temperatures

Thank you !