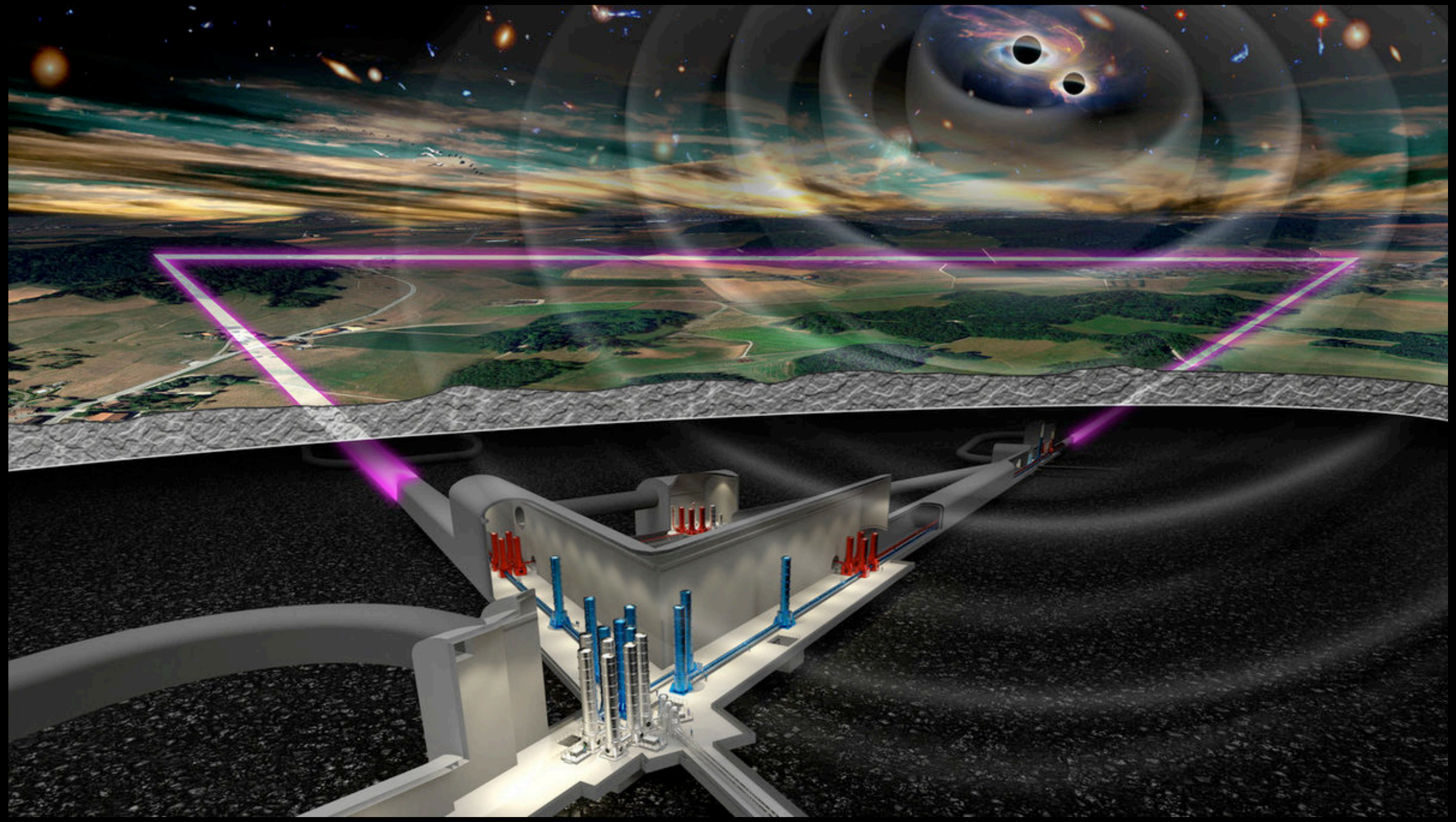
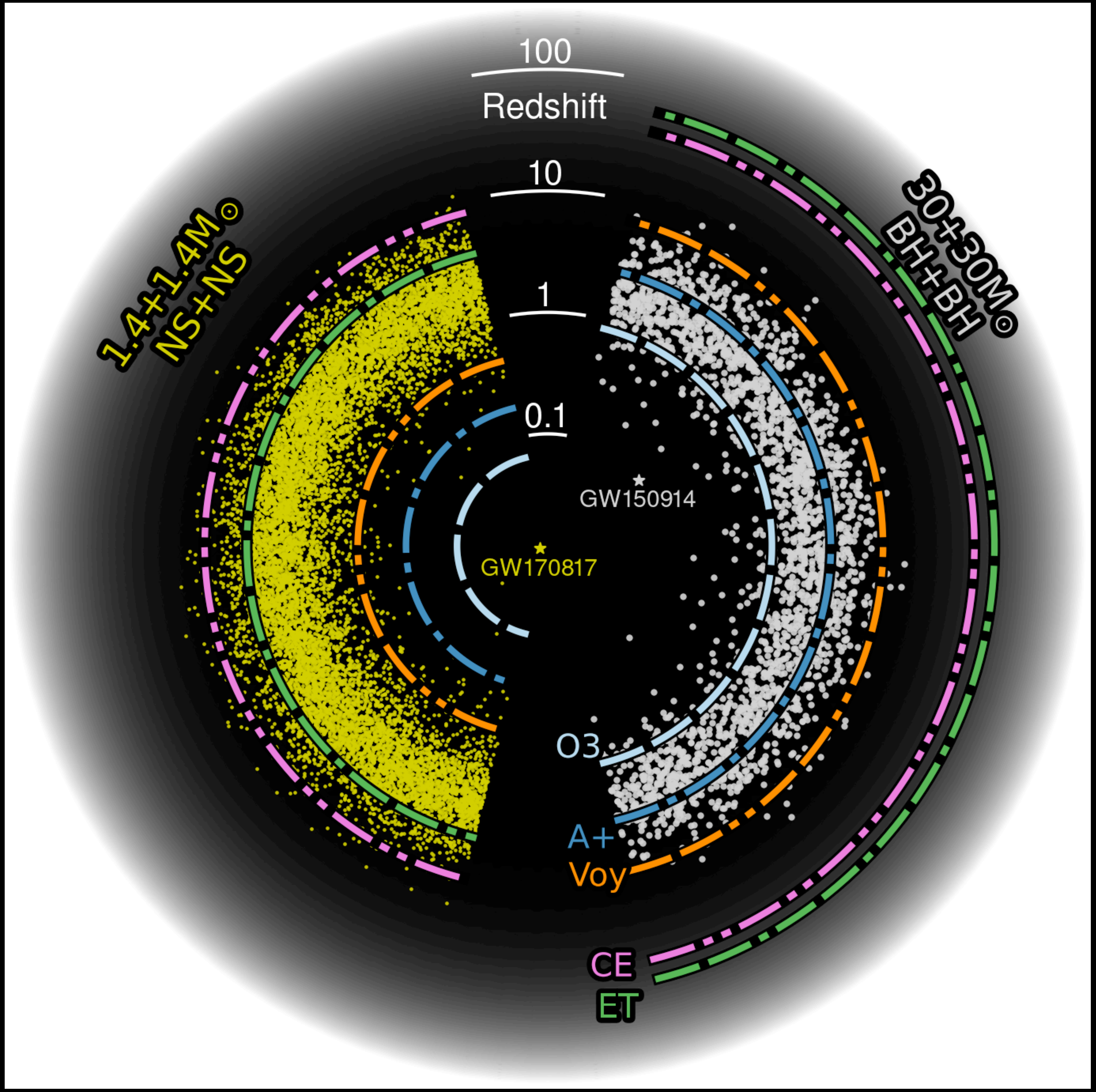
Gravitational Waves: Past, Present and Future

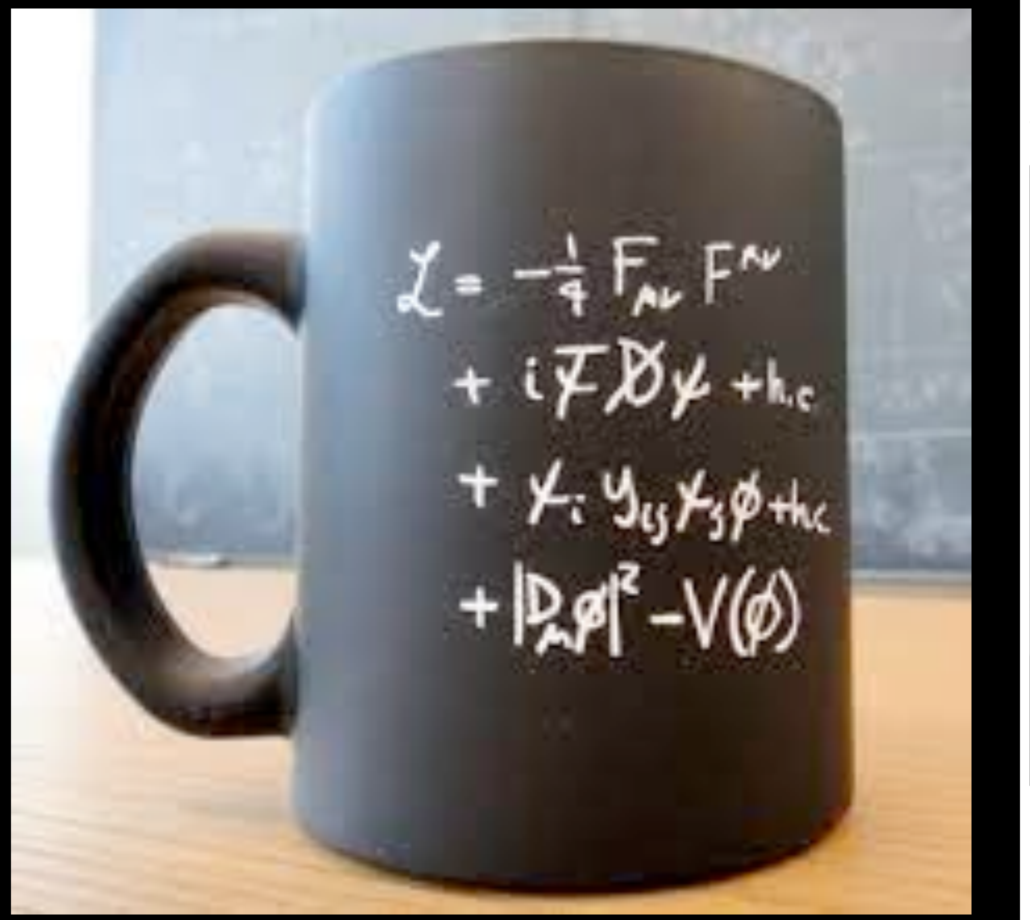


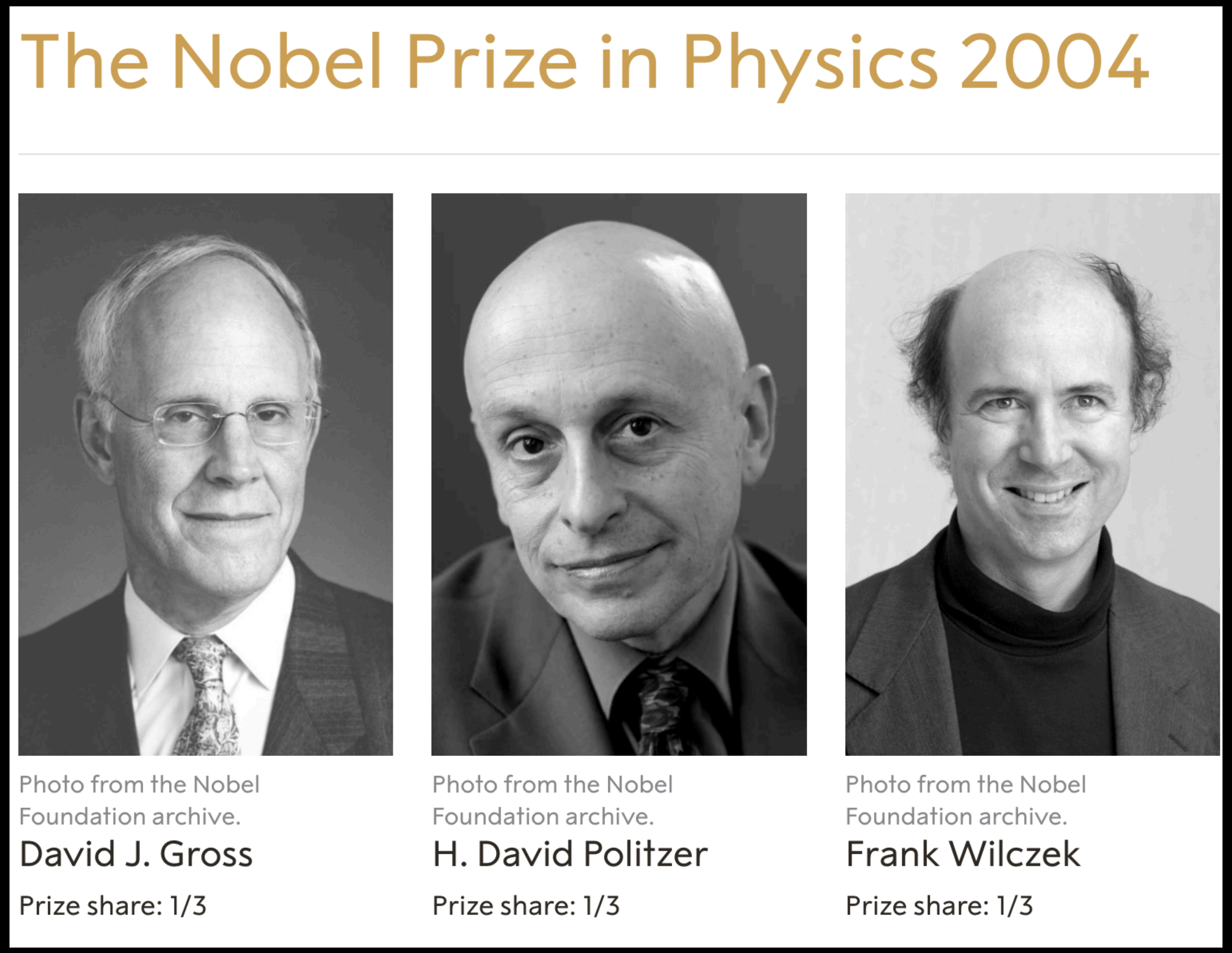


Thursday Afternoon Session: Anna Puecher

Neutron Stars meet Bayesian Inference

Model Building for the understanding of atomic nuclei and neutron stars





Walter Kohn
Nobel Laureate
Chemistry 1998

Quantum Chromodynamics (QCD) is the fundamental theory of the strong interactions

- Although the basic equations can be written in a coffee cup, their exact solution in the region of interest to atomic nuclei and neutron stars are unknown
- One must then resort to models that (hopefully!) embody the properties of QCD
- One such model is Density Functional Theory

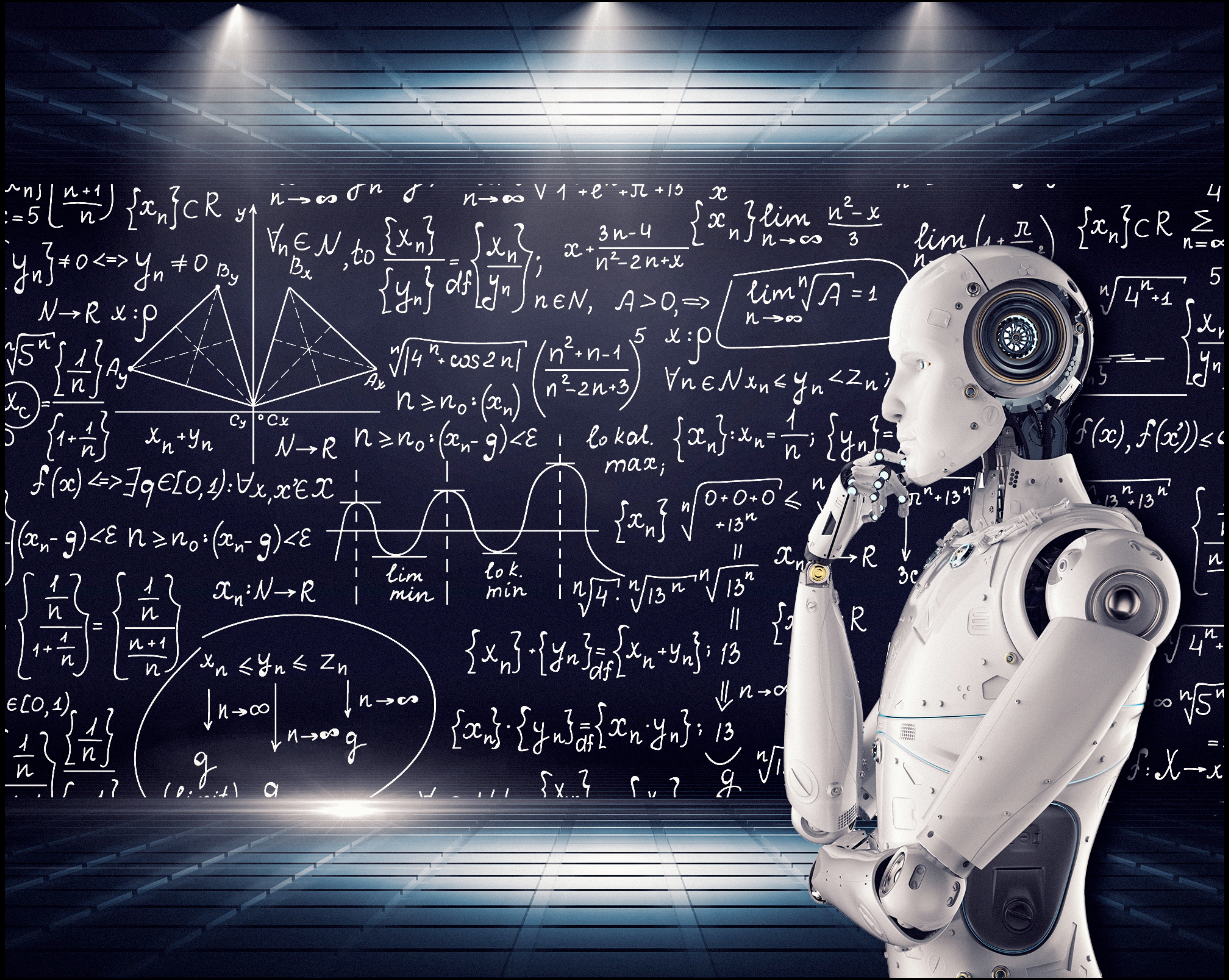
Covariant Density Functional Theory

- Empirical parameters calibrated to physical observables
- Ground state properties emerge from functional minimization

$$\mathcal{L}_{\rm Yukawa} = \bar{\psi}\left[g_{\rm s}\phi - \left(g_{\rm v}V_\mu + \frac{g_\rho}{2}\tau\cdot\mathbf{b}_\mu + \frac{e}{2}(1+\tau_3)A_\mu\right)\gamma^\mu\right]\psi$$

$$\mathcal{L}_{\rm self} = \frac{\kappa}{3!}(g_{\rm s}\phi)^3 - \frac{\lambda}{4!}(g_{\rm s}\phi)^4 + \frac{\zeta}{4!}g_{\rm v}^4(V_\mu V^\mu)^2 + \Lambda_{\rm v}\left(g_\rho^2\,\mathbf{b}_\mu\cdot\mathbf{b}^\mu\right)\left(g_{\rm v}^2 V_\nu V^\nu\right)$$

A Brief Introduction to Machine Learning

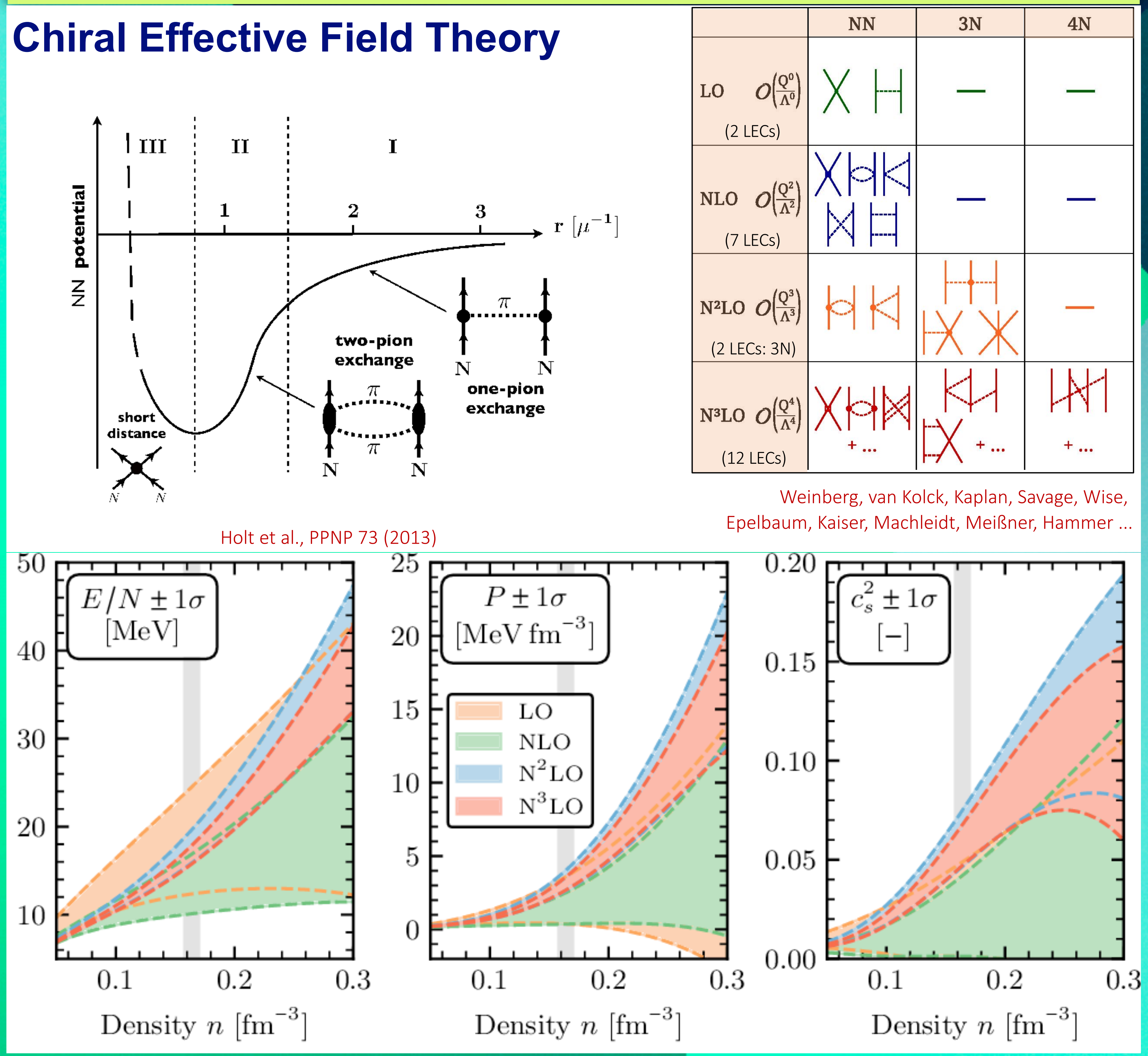


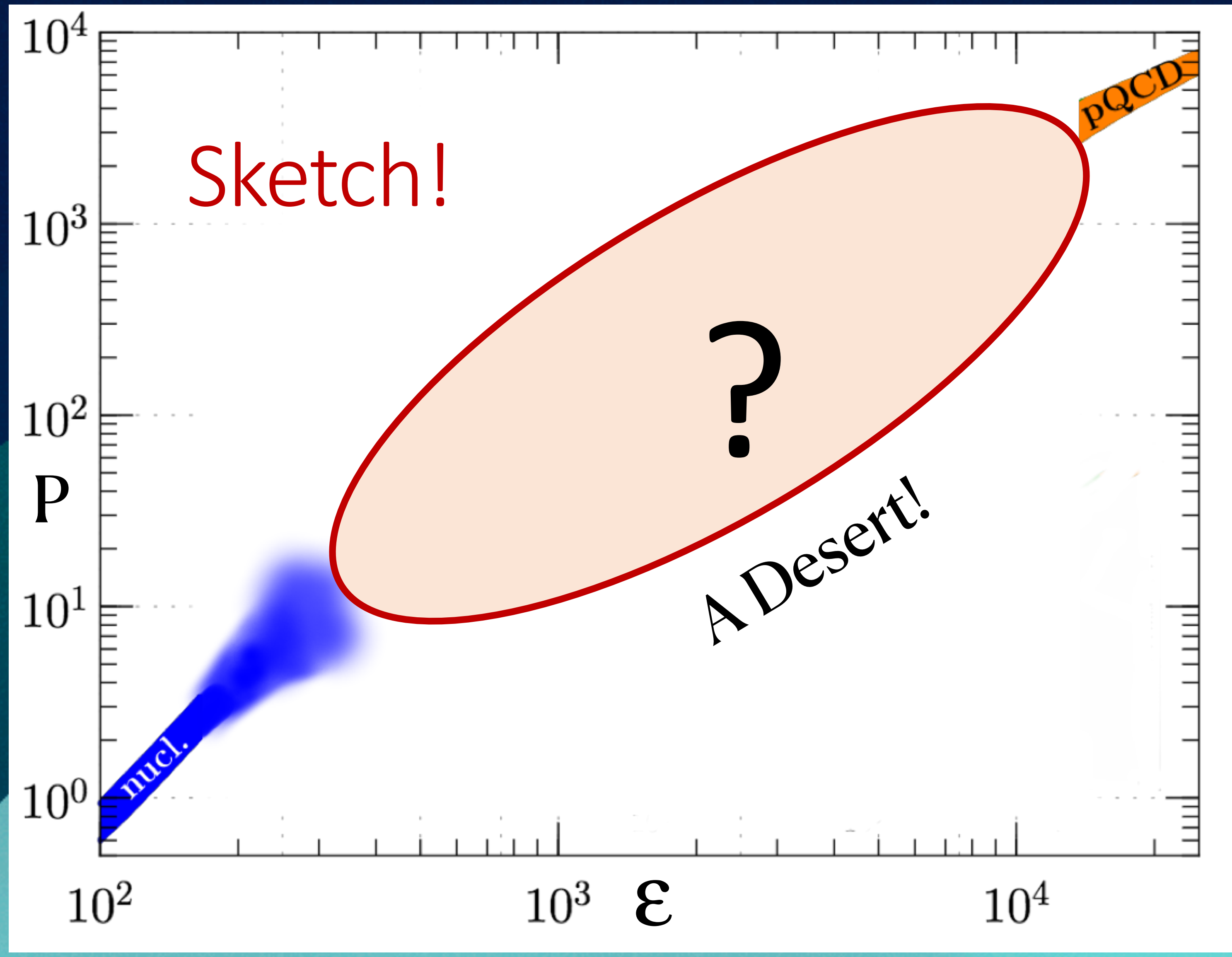
... it has to be brief since I am just learning it myself ...

The Tools of the Trade

Chiral Effective Field Theory

- A theory of nucleons, pions, and unresolved contact interactions
- Systematic, Improvable, and quantifiable
- Breaks down at ~1.5 normal nuclear density





QCD Made Simple

Quantum chromodynamics is conceptually simple. Its realization in nature, however, is usually very complex. But not always.

Frank Wilczek

Quantum chromodynamics, familiarly called QCD, is the modern theory of the strong interaction.[1] Historically its roots are in nuclear physics and the description of ordinary matter—understanding what protons and neutrons are and how they interact. Nowadays QCD is used to describe most of what goes on at high-energy accelerators.

to the presence or motion of color charge, very similar to the way photons respond to electric charge.

Quarks and gluons

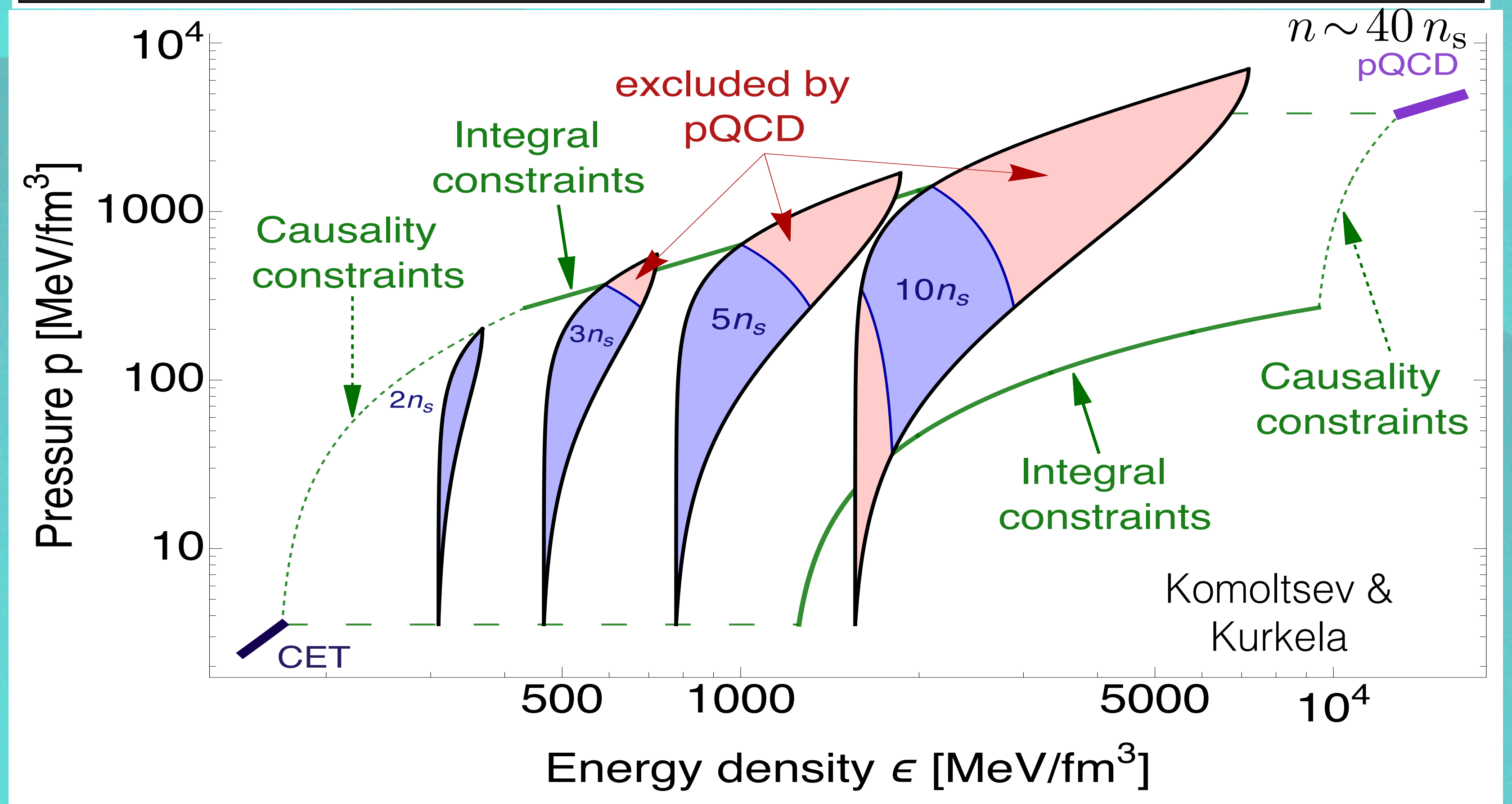
One class of particles that carry color charge are the quarks. We know of six different kinds, or "flavors," of quarks—denoted u, d, s, c, b, and t, for: up, down,

$$\mathcal{L} = \frac{1}{4g^2} G^a_{\mu\nu} G^a_{\mu\nu} + \sum_j \bar{q}_j (i\gamma^\mu D_\mu + m_j) q_j$$

where $G^a_{\mu\nu} \equiv \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + i f^a_{bc} A^b_\mu A^c_\nu$

and $D_\mu \equiv \partial_\mu + i t^a A^a_\mu$

That's it!



How to link χEFT to pQCD

Lattice QCD at finite density ($\mu/T \ll 1$)

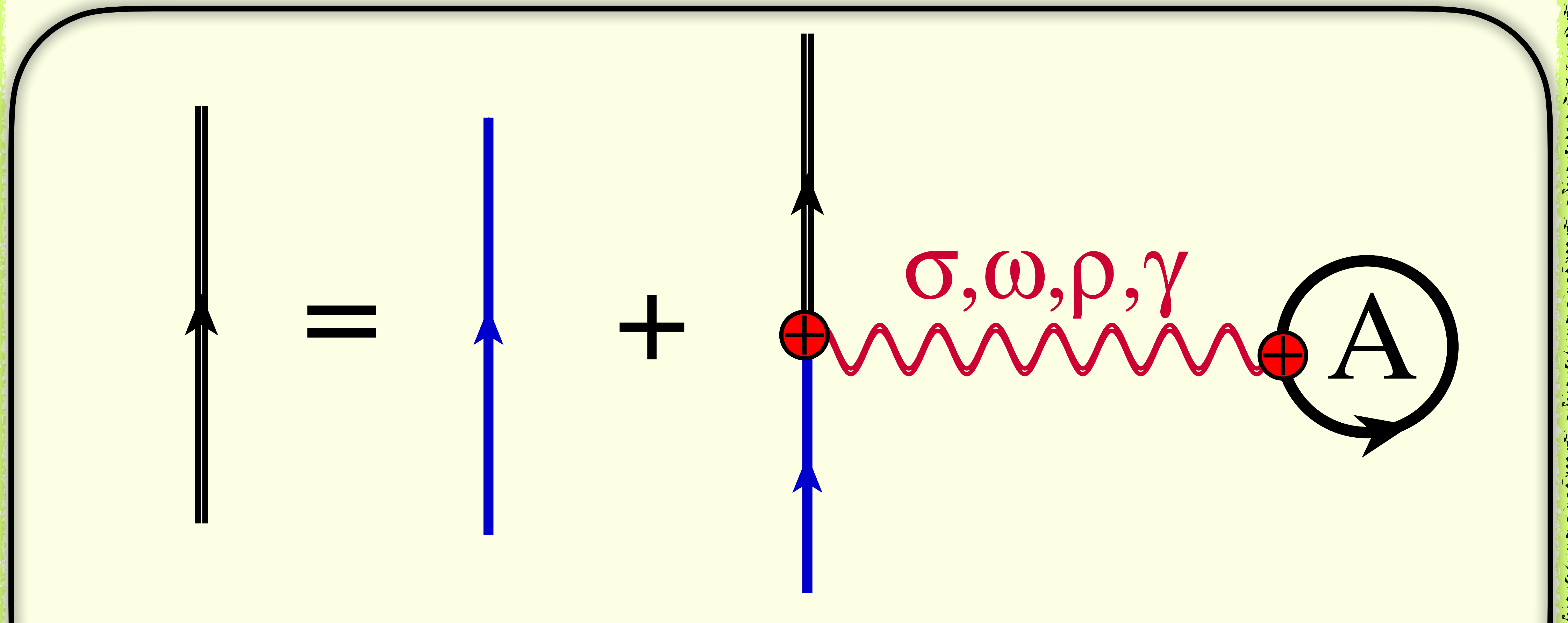
Covariant Density Functional Theory (Relativistic MFT with a slight twist)

Covariant Density Functional Theory

$$\mathcal{L}_{\rm Yukawa} = \bar{\psi}\left[g_{\rm s}\phi - \left(g_{\rm v}V_\mu + \frac{g_\rho}{2}\tau\cdot\mathbf{b}_\mu + \frac{e}{2}(1+\tau_3)A_\mu\right)\gamma^\mu\right]\psi$$

$$\mathcal{L}_{\rm self} = \frac{\kappa}{3!}(g_{\rm s}\phi)^3 - \frac{\lambda}{4!}(g_{\rm s}\phi)^4 + \frac{\zeta}{4!}g_{\rm v}^4(V_\mu V^\mu)^2 + \Lambda_{\rm v}\left(g_\rho^2\,\mathbf{b}_\mu\cdot\mathbf{b}^\mu\right)\left(g_{\rm v}^2 V_\nu V^\nu\right)$$

- σ: intermediate range scalar attraction (2π exchange)
- ω: short-range vector repulsion (contact term in χEFT)
- ρ: isospin (flavor) dependent short-range interaction
- γ: long-range Coulomb repulsion between protons



Self-consistent DFT ground state

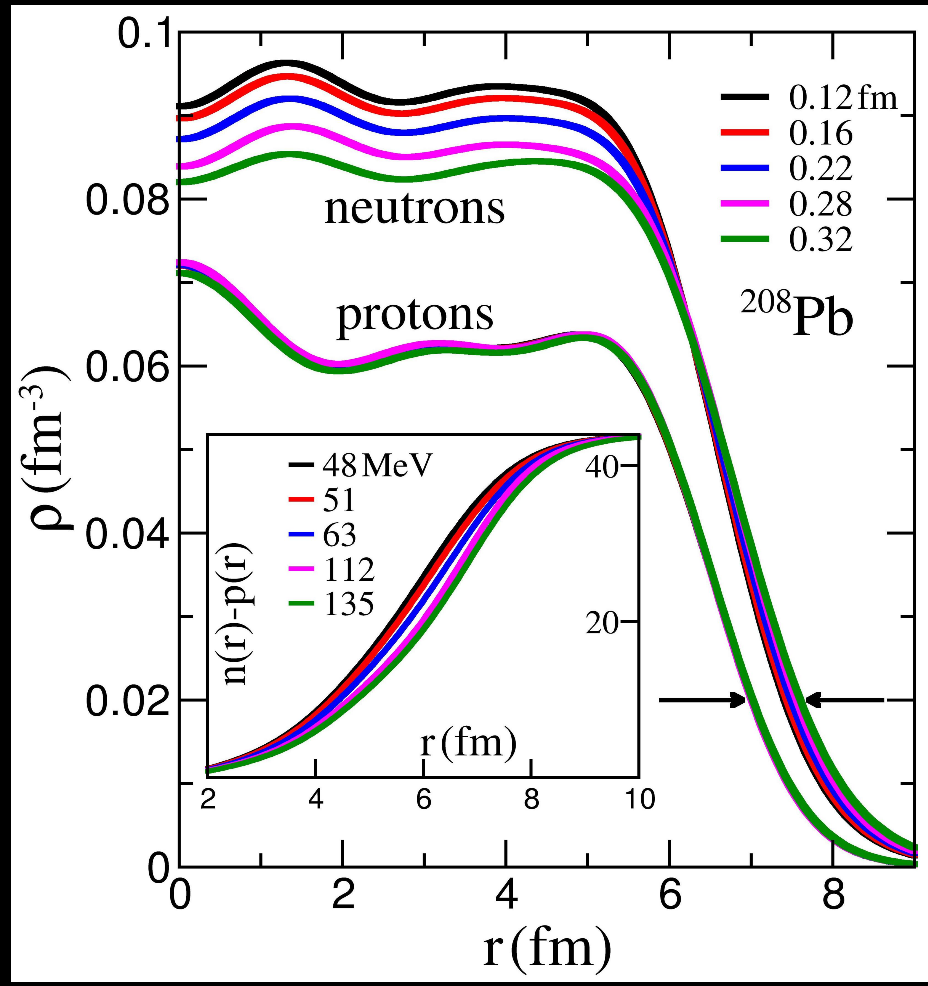
Walter Kohn
Nobel Laureate
Chemistry 1998

Anatomy of a self-consistent Covariant DFT calculation

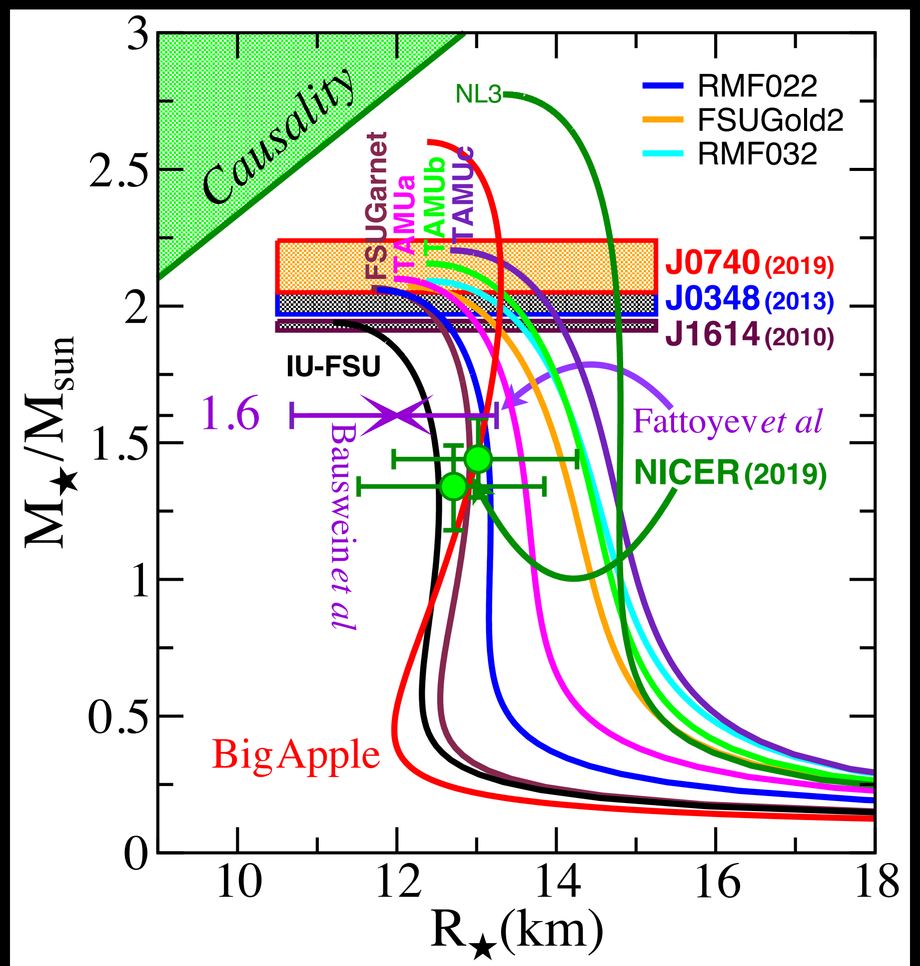
The Hohenberg-Kohn Theorem: The ground state energy can be obtained variationally: the density that minimizes the total energy is the exact ground state density

- Empirical parameters calibrated to physical observables
- Ground state properties emerge from functional minimization

From finite nuclei to neutron stars!



18 orders
magnitude



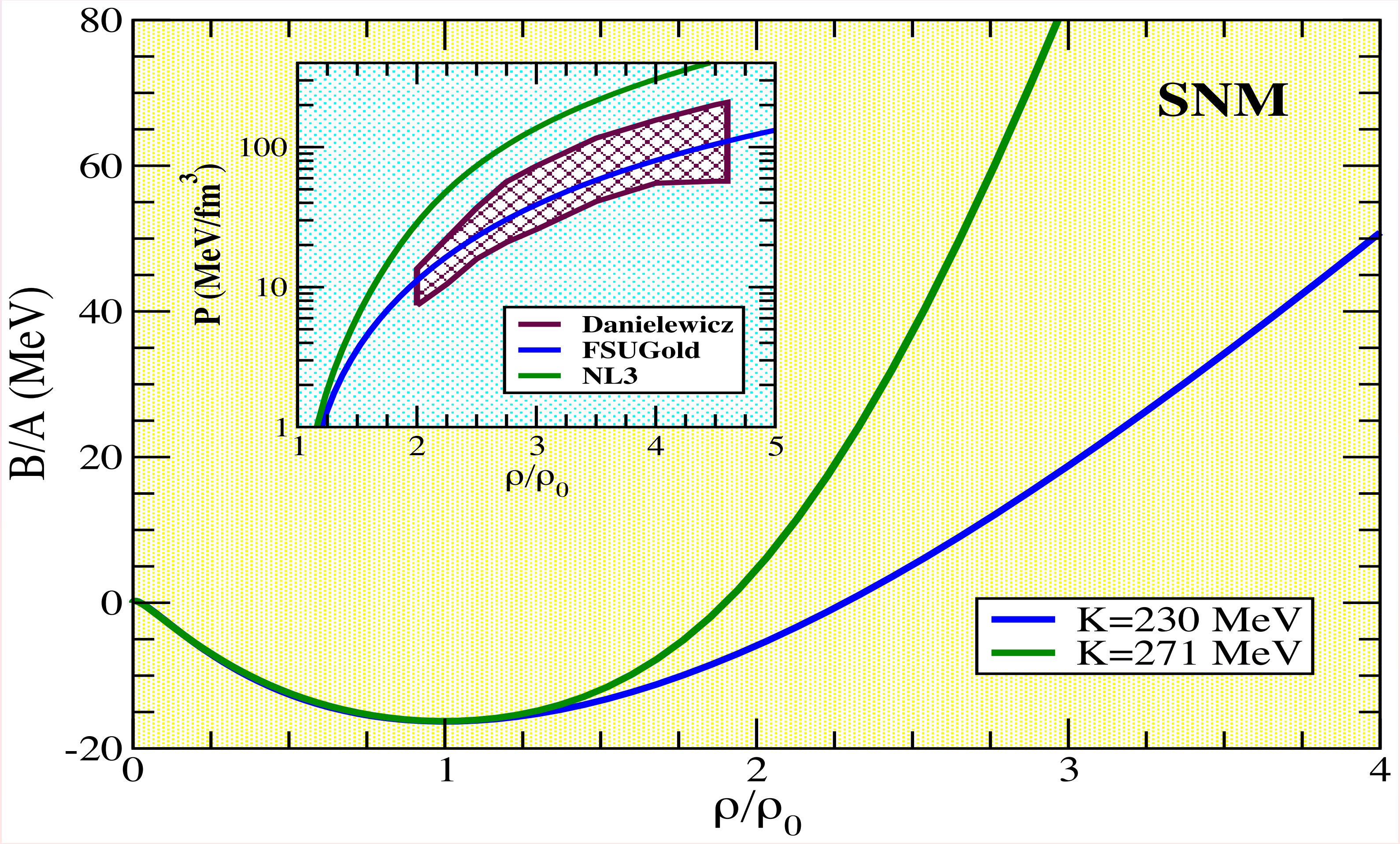
Covariant Density Functional Theory

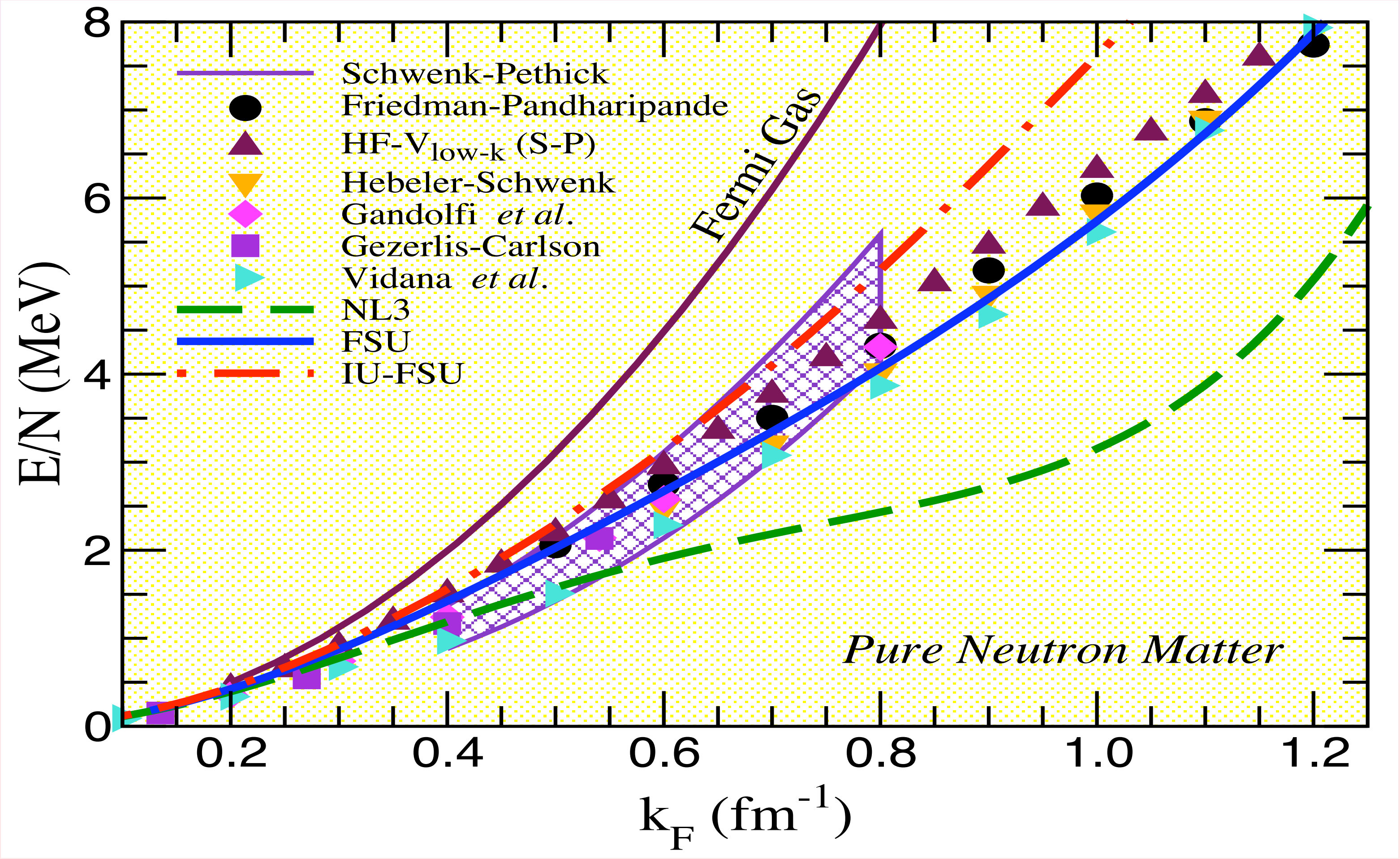
Relativistic Density Functional: The Effective Lagrangian Density

$$\mathscr{L}_{\rm int} = g_{\rm s}\bar{\psi}\psi\phi - g_{\rm v}\bar{\psi}\gamma^{\mu}\psi V_{\mu} - \frac{g_{\rho}}{2}\bar{\psi}\gamma^{\mu}\boldsymbol{\tau}\cdot\mathbf{b}_{\mu}\psi - e\bar{\psi}\gamma^{\mu}\tau_{p}\psi A_{\mu}$$
$$-\frac{\kappa}{3!}(g_{\rm s}\phi)^3 - \frac{\lambda}{4!}(g_{\rm s}\phi)^4 + \Lambda_{\rm v}(g_{\rm v}^2 V^{\mu}V_{\mu})(g_{\rho}^2 b^{\mu}b_{\mu}) + \frac{\zeta}{4!}g_{\rm v}^4(V_{\mu}V^{\mu})^2$$

The Encoding:

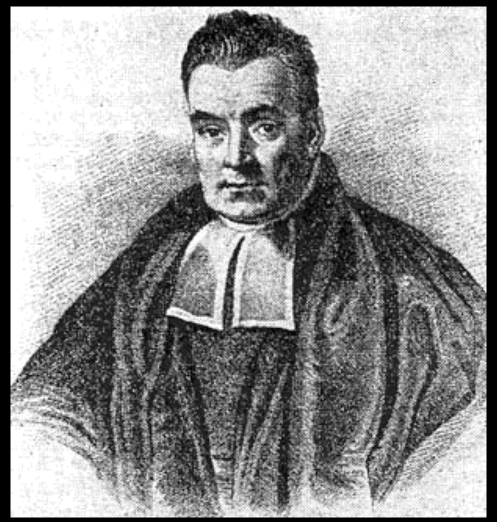
- $g_{\rm s}$ and $g_{\rm v}$: saturation properties (ρ_0 , ε_0 $\longrightarrow$ masses, charge radii)
- g_{ρ}: symmetry energy ($J \equiv a_4 \longrightarrow$ masses, charge radii)
- κ and λ: nuclear compressibility ($K_0 \longrightarrow$ ISGMR)
- $\Lambda_{\rm v}$: slope symmetry energy ($L \longrightarrow$ neutron skins, neutron-star radii)
- ζ: high-density component of EOS (limiting neutron-star mass)





Neutron Stars meet Bayesian Inference

Model Building for the understanding of atomic nuclei and neutron stars



Thomas Bayes
(1701-1761)

$$P(M|D) = \frac{P(D|M)P(M)}{P(D)}$$

Posterior ← $P(M|D)$; Likelihood ← $P(D|M)$; $P(M)$ → Prior; $P(D)$ → Marginal Likelihood

$$\mathcal{L}_{\rm Yukawa} = \bar{\psi}\left[g_{\rm s}\phi - \left(g_{\rm v}V_\mu + \frac{g_\rho}{2}\tau\cdot\mathbf{b}_\mu + \frac{e}{2}(1+\tau_3)A_\mu\right)\gamma^\mu\right]\psi$$

$$\mathcal{L}_{\rm self} = \frac{\kappa}{3!}(g_{\rm s}\phi)^3 - \frac{\lambda}{4!}(g_{\rm s}\phi)^4 + \frac{\zeta}{4!}g_{\rm v}^4(V_\mu V^\mu)^2 + \Lambda_{\rm v}\left(g_\rho^2\,\mathbf{b}_\mu\cdot\mathbf{b}^\mu\right)\left(g_{\rm v}^2 V_\nu V^\nu\right)$$

M: A theoretical MODEL with parameters and biases

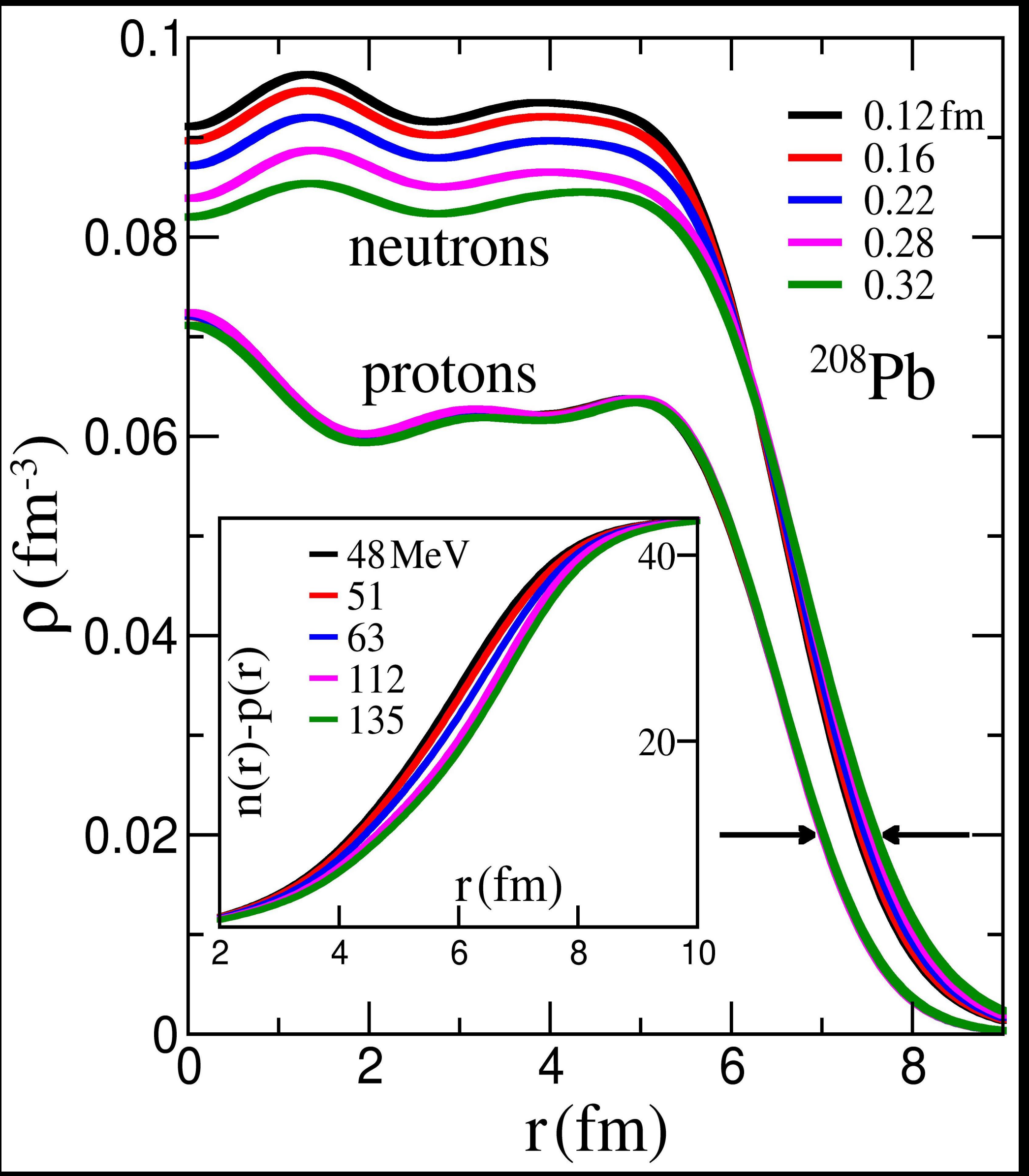
D: A collection of experimental and observational DATA

$$(g_{\rm s}, g_{\rm v}, g_\rho, \kappa, \lambda, \Lambda_{\rm v}) \Longleftrightarrow (\rho_0, \epsilon_0, M^*, K, J, L)$$

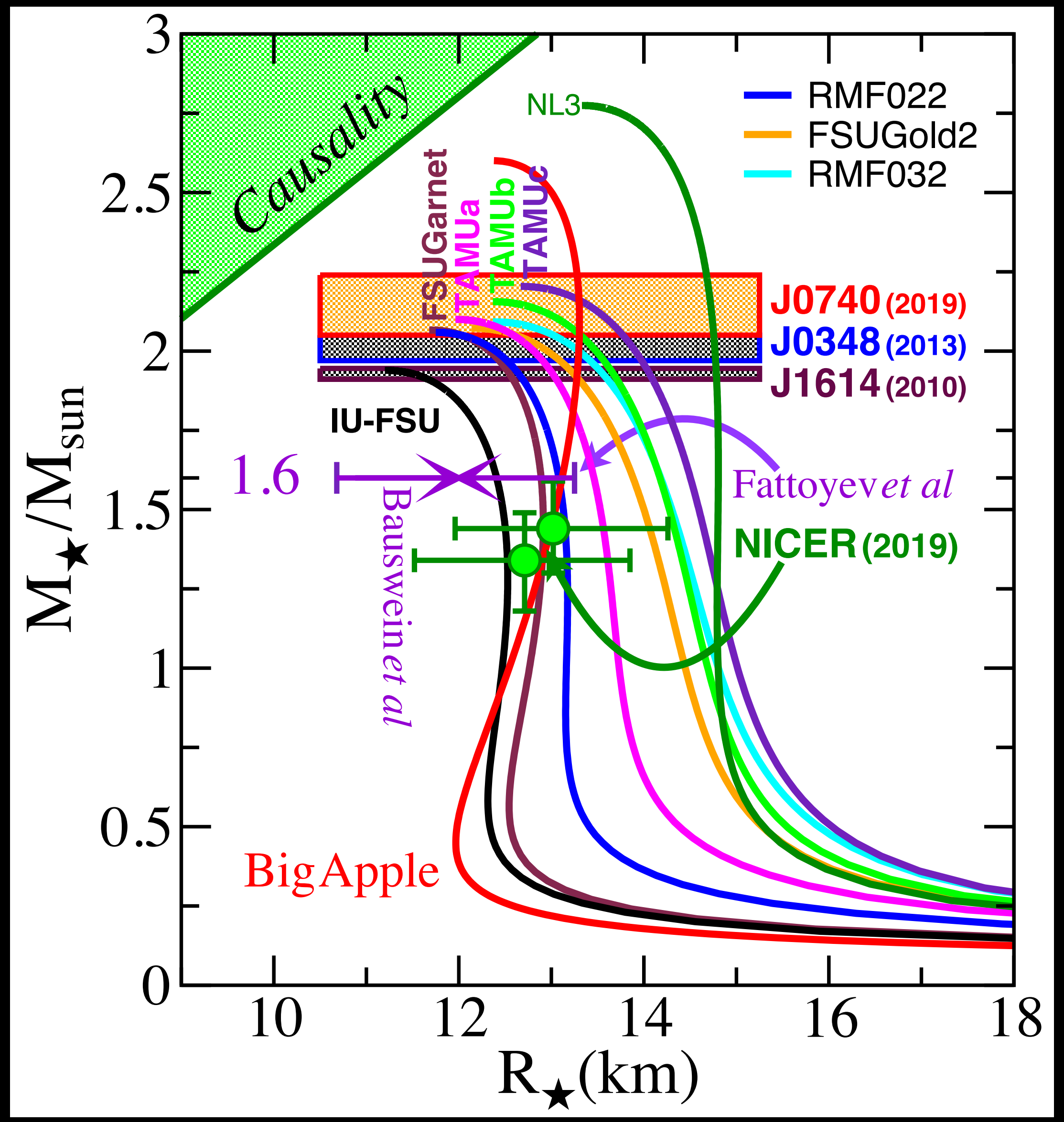
From finite nuclei to neutron stars!

The Prior *P(M)*: An insightful transformation in DFT

The Likelihood provides new evidence to update *P(M)*



18 orders
magnitude



$$P(D/M) \simeq \exp(-\chi^2/2)$$

$$\chi^2(D,M) = \sum_{n=1}^{N} \frac{\left(O_n^{(\rm th)}(M) - O_n^{(\rm exp)}(D)\right)^2}{\Delta O_n^2}$$

The marginal likelihood (or evidence) is an overall normalization factor in Monte Carlo simulations

Covariant Density Functional Theory: From Finite Nuclei to Neutron Stars

> HOW THE SAUSAGE IS MADE is the practical *and often unpleasant or messy aspects of a process that are usually not made public*.

$$\mathcal{L}_{\rm Yukawa} = \bar{\psi}\left[g_{\rm s}\phi - \left(g_{\rm v}V_{\mu} + \frac{g_{\rho}}{2}\tau\cdot\mathbf{b}_{\mu} + \frac{e}{2}(1+\tau_{3})A_{\mu}\right)\gamma^{\mu}\right]\psi$$

$$\mathcal{L}_{\rm self} = \frac{\kappa}{3!}(g_{\rm s}\phi)^{3} - \frac{\lambda}{4!}(g_{\rm s}\phi)^{4} + \frac{\zeta}{4!}g_{\rm v}^{4}(V_{\mu}V^{\mu})^{2} + \Lambda_{\rm v}\left(g_{\rho}^{2}\,\mathbf{b}_{\mu}\cdot\mathbf{b}^{\mu}\right)\left(g_{\rm v}^{2}V_{\nu}V^{\nu}\right)$$

The Kohn-Sham (Mean-field like) Equations

In the particular case of the Lagrangian density given in Equation 1, the classical meson fields satisfy Klein–Gordon equations containing both nonlinear meson interactions and ground-state baryon densities as source terms. That is,

$$\left(\nabla^{2} - m_{\rm s}^{2}\right)\phi_{0}(r) - \frac{\partial U_{\rm eff}}{\partial\phi_{0}} = -g_{\rm s}\rho_{\rm s}(r), \qquad 5a.$$

$$\left(\nabla^{2} - m_{\rm v}^{2}\right)V_{0}(r) + \frac{\partial U_{\rm eff}}{\partial V_{0}} = -g_{\rm v}\rho_{\rm v}(r), \qquad 5b.$$

$$\left(\nabla^{2} - m_{\rho}^{2}\right)b_{0}(r) + \frac{\partial U_{\rm eff}}{\partial b_{0}} = -\frac{g_{\rho}}{2}\rho_{3}(r). \qquad 5c.$$

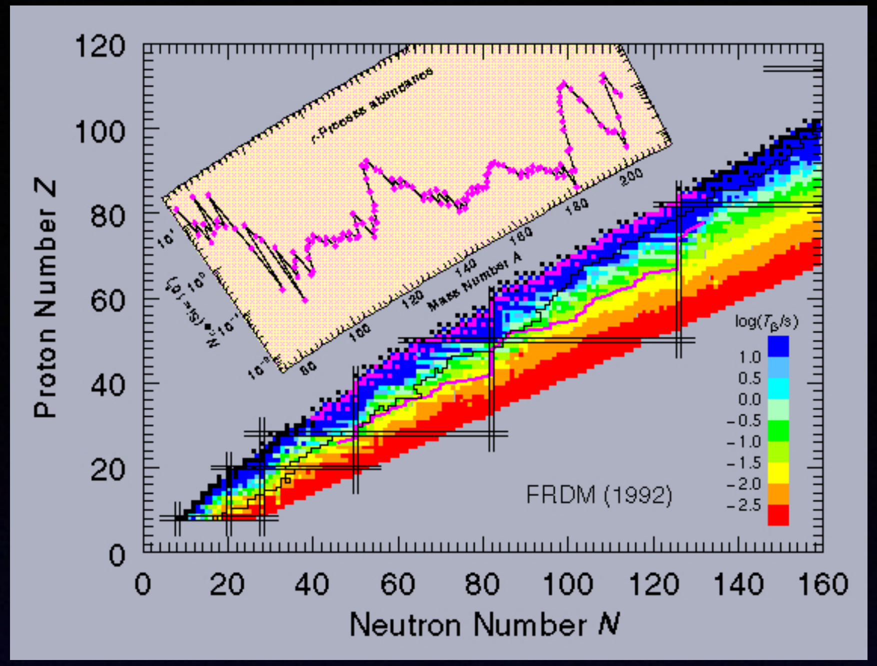
In turn, the Coulomb field satisfies the much simpler Poisson's equation,

$$\nabla^{2}A_{0} = -e\rho_{p}, \qquad 6.$$

$$\left[-i\boldsymbol{\alpha}\cdot\nabla + g_{\rm v}V_{0}(r) + \frac{g_{\rho}}{2}\tau_{3}b_{0}(r) + e\tau_{p}A_{0}(r) + \beta\left(M - g_{\rm s}\phi_{0}(r)\right)\right]\psi(\mathbf{r}) = E\psi(\mathbf{r}). \qquad 7.$$

The above set of equations—Equations 5–7—represents the effective KS equations for the nuclear many-body problem. As such, this set of mean-field equations must be solved self-consistently. That is, the single-particle orbitals satisfying the Dirac equation are generated from the various meson fields, which in turn satisfy Klein–Gordon equations with the appropriate ground-state densities as the source terms. This process demands an iterative procedure in which mean-field potentials of the Wood–Saxon form are initially provided to solve the Dirac equation for the occupied nucleon orbitals, which are then combined to generate the appropriate densities for the meson field. The Klein–Gordon equations are then solved with the resulting meson fields providing

Model Building: From Finite Nuclei to Neutron Stars



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Building relativistic mean field models for finite nuclei and neutron stars

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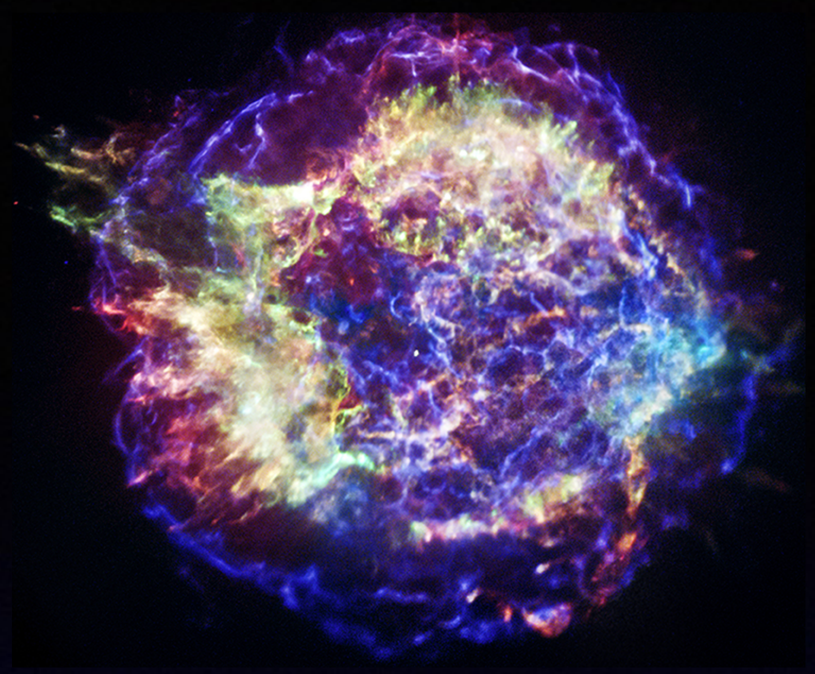


$$\mathcal{L}_{\rm Yukawa} = \bar{\psi}\left[g_{\rm s}\phi - \left(g_{\rm v}V_\mu + \frac{g_\rho}{2}\tau\cdot\mathbf{b}_\mu + \frac{e}{2}(1+\tau_3)A_\mu\right)\gamma^\mu\right]\psi$$

$$\mathcal{L}_{\rm self} = \frac{\kappa}{3!}(g_{\rm s}\phi)^3 - \frac{\lambda}{4!}(g_{\rm s}\phi)^4 + \frac{\zeta}{4!}g_{\rm v}^4(V_\mu V^\mu)^2 + \Lambda_{\rm v}\left(g_\rho^2\,\mathbf{b}_\mu\cdot\mathbf{b}^\mu\right)\left(g_{\rm v}^2 V_\nu V^\nu\right)$$

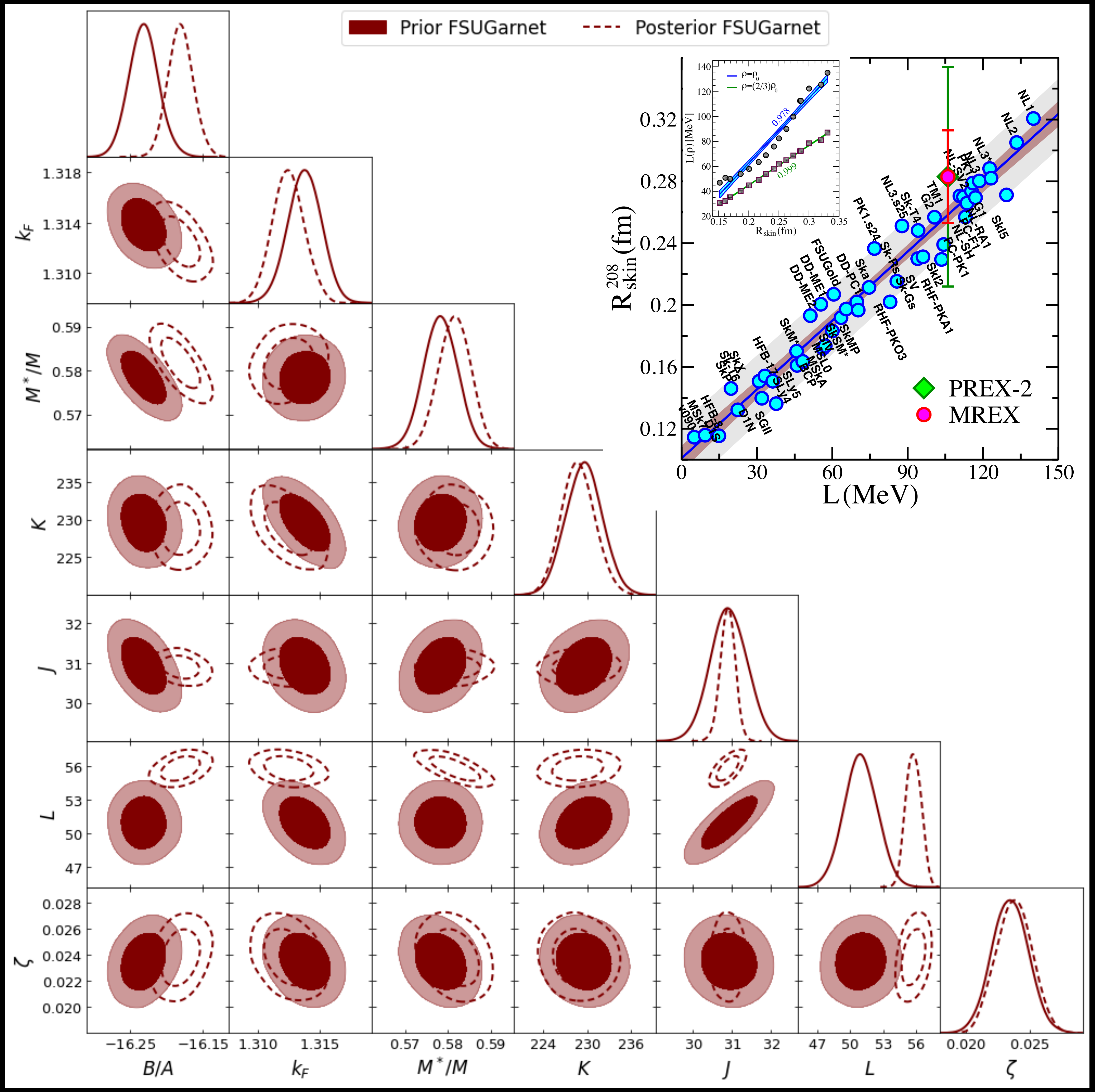
Nuclear Density Functional Theory (DFT)

- ***Ab-initio calculations of heavy nuclei remains daunting task***
- ***Search for energy functional valid over a large physics domain***
  "From finite nuclei all the way to neutron stars"
- ***Incorporate physics insights into the construction of the functional***
- ***Accurately calibrated to various properties of finite nuclei***
  masses, charge radii, and giant monopole resonances
- ***Empirical constants encode physics beyond mean field***
- ***Empirical constants obtained from the optimization of a quality measure***

Nucleus	Observable	Experiment	NL3	FSU	FSU2
^{16}O	B/A	7.98	8.06	7.98	8.00
	$R_{\rm ch}$	2.70	2.75	2.71	2.73
^{40}Ca	B/A	8.55	8.56	8.54	8.54
	$R_{\rm ch}$	3.48	3.49	3.45	3.47
^{48}Ca	B/A	8.67	8.66	8.58	8.63
	$R_{\rm ch}$	3.48	3.49	3.48	3.47
^{68}Ni	B/A	8.68	8.71	8.66	8.69
	$R_{\rm ch}$	—	3.88	3.88	3.86
^{90}Zr	B/A	8.71	8.70	8.68	8.69
	$R_{\rm ch}$	4.27	4.28	4.27	4.26
^{100}Sn	B/A	8.25	8.30	8.24	8.28
	$R_{\rm ch}$	—	4.48	4.48	4.47
^{116}Sn	B/A	8.52	8.50	8.50	8.49
	$R_{\rm ch}$	4.63	4.63	4.63	4.61
^{132}Sn	B/A	8.36	8.38	8.34	8.36
	$R_{\rm ch}$	4.71	4.72	4.74	4.71
^{144}Sm	B/A	8.30	8.32	8.32	8.31
	$R_{\rm ch}$	4.95	4.96	4.96	4.94
^{208}Pb	B/A	7.87	7.90	7.89	7.88
	$R_{\rm ch}$	5.50	5.53	5.54	5.51

Nucleus	TAMU	RCNP	NL3	FSU	FSU2
^{90}Zr	17.81 ± 0.35	—	18.76	17.86	17.93 ± 0.09
^{116}Sn	15.90 ± 0.07	15.70 ± 0.10	17.19	16.39	16.47 ± 0.08
^{144}Sm	15.25 ± 0.11	15.77 ± 0.17	16.29	15.55	15.59 ± 0.09
^{208}Pb	14.18 ± 0.11	13.50 ± 0.10	14.32	13.72	13.76 ± 0.08

Bayesian Inference for Uncertainty Quantification: Model building for the understanding of atomic nuclei, neutron stars, and unveiling correlations



$$\mathcal{L}_{\rm Yukawa} = \bar{\psi}\left[g_{\rm s}\phi - \left(g_{\rm v}V_\mu + \frac{g_\rho}{2}\tau\cdot\mathbf{b}_\mu + \frac{e}{2}(1+\tau_3)A_\mu\right)\gamma^\mu\right]\psi$$

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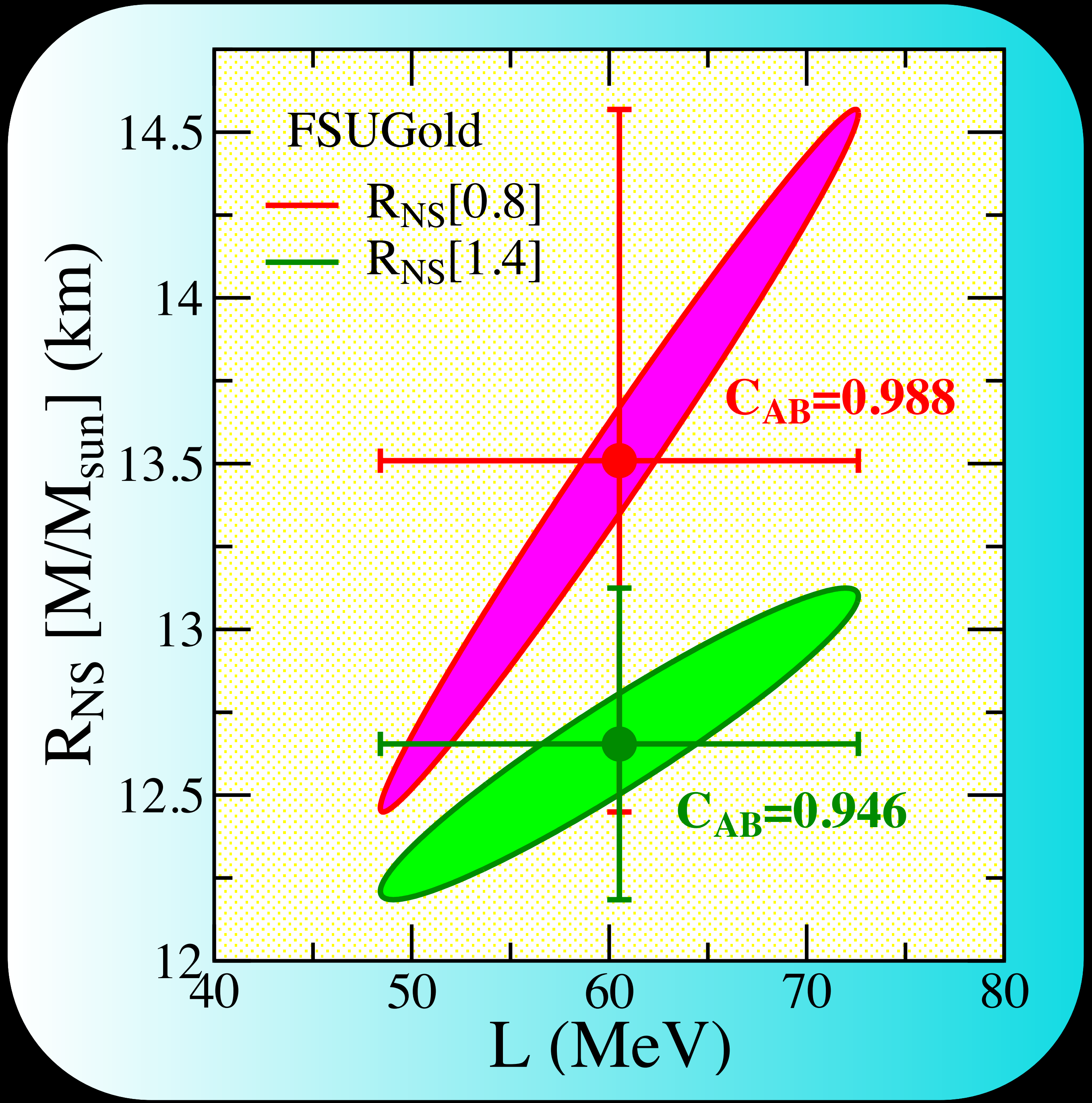
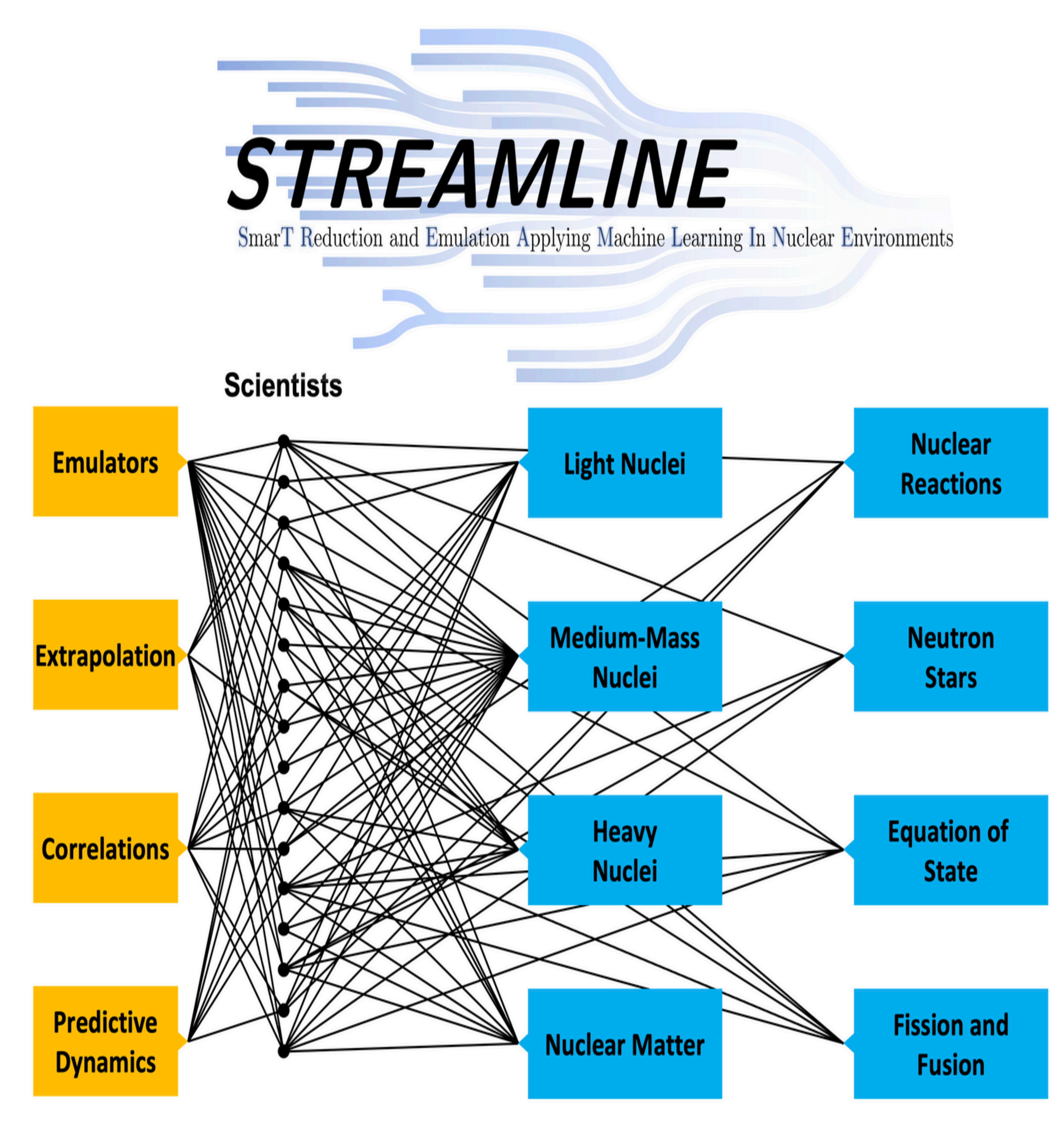
$$(g_{\rm s}, g_{\rm v}, g_\rho, \kappa, \lambda, \Lambda_{\rm v}) \Longleftrightarrow (\rho_0, \epsilon_0, M^*, K, J, L)$$



Neutron skins and stellar radii (quantities that differ by 18 orders of magnitude) are both sensitive to L!

Emulators and Reduced Order Models



Emulators are statistical ML models that faithfully reproduce the behavior of a complex physical system at a "tiny" fraction of the computational cost!

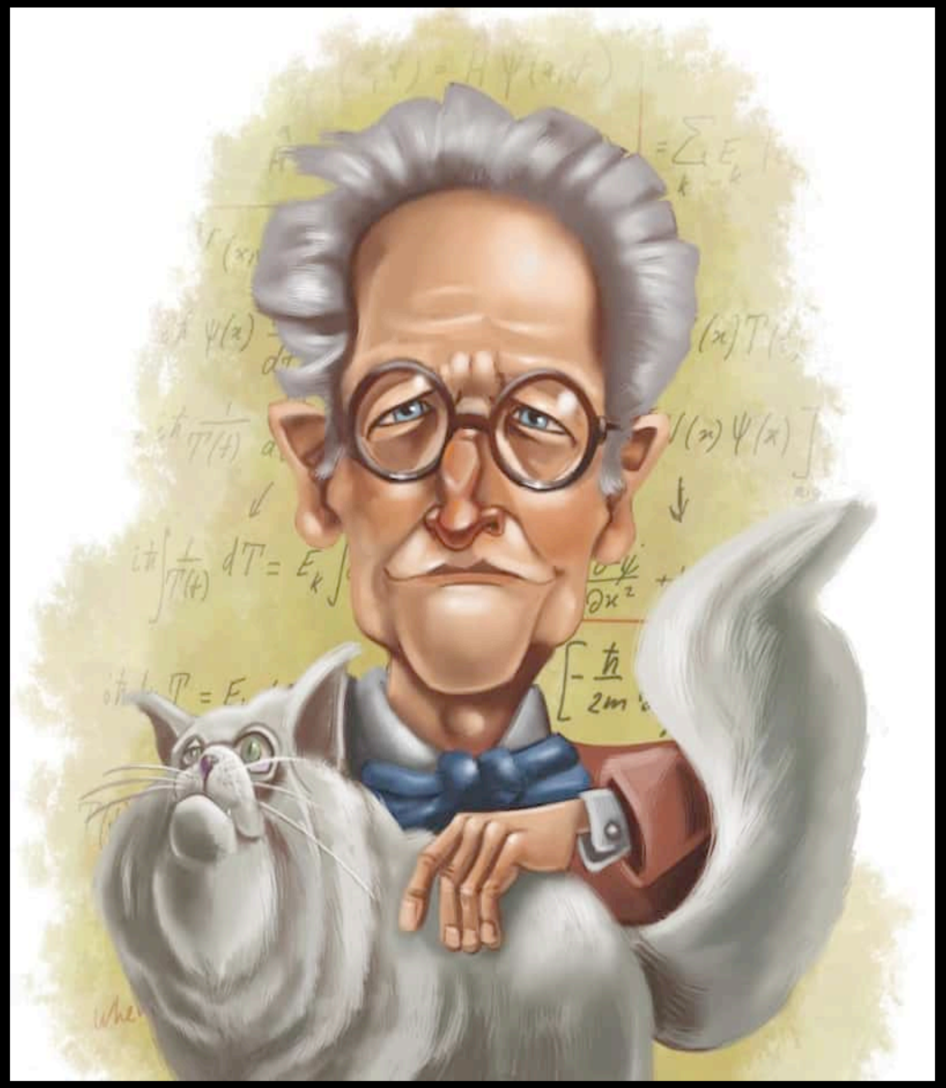
Neutron Stars meet Bayesian Emulators

Today's posterior is tomorrow's prior!

- The influx of experimental/observational data will challenge future model building.
- Given that updates to the model are computationally expensive, an emulator provides an inexpensive and (hopefully) accurate alternative to the original high-fidelity model.
- Reduced order models — although widely used in other fields — are just making their entrance in the nuclear science arena.

$$\left(-\frac{d^2}{dx^2}+V_\lambda(x)+\frac{\kappa(\kappa+1)}{x^2}\right)\phi_{n\kappa}(x)=\varepsilon\phi_{n\kappa}(x)$$

A large set of parameters



Neutron Stars meet Bayesian Emulators
Reduced Basis Methods

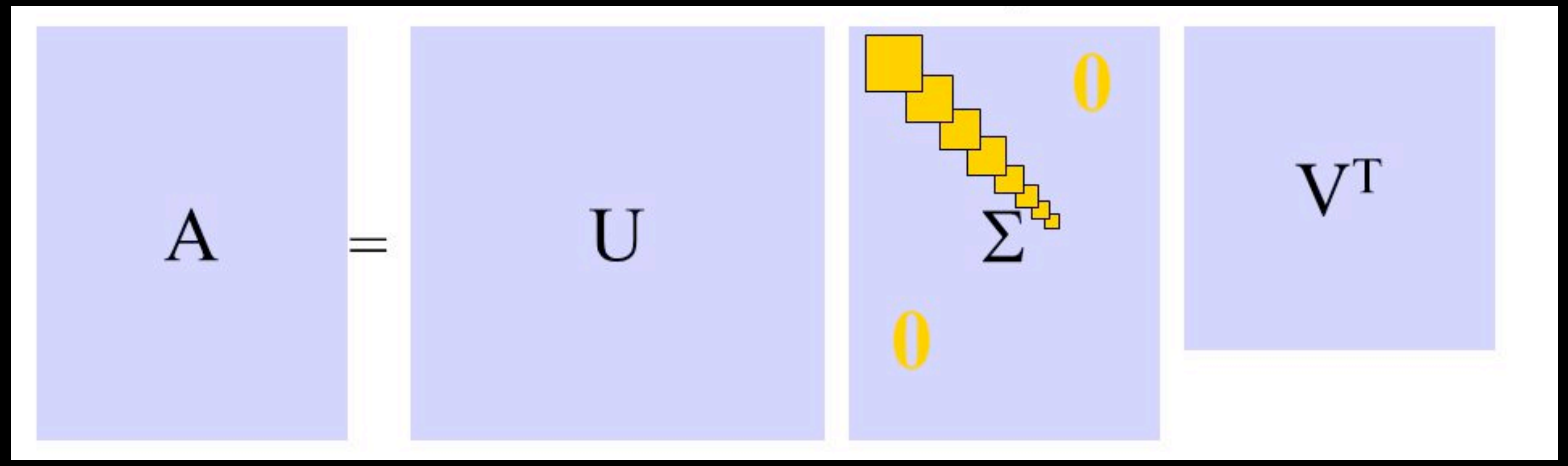
A "universal" reduced basis capable of accurately and efficiently reproduce the entire single-particle spectrum of a variety of nuclei at a small fraction in computational cost

1. Scaled Schrodinger equation as a function of some parameter(s) λ (for example, depth of the potential)

$$\left(-\frac{d^2}{dx^2} + V_\lambda(x) + \frac{\kappa(\kappa+1)}{x^2}\right)\phi_{n\kappa}(x) = \varepsilon\phi_{n\kappa}(x)$$

2. Solve the exact "high-fidelity" model for a few values of the model parameters λ and collect all the the resulting eigenfunctions $\phi_{n\kappa}(x, \lambda_i)$

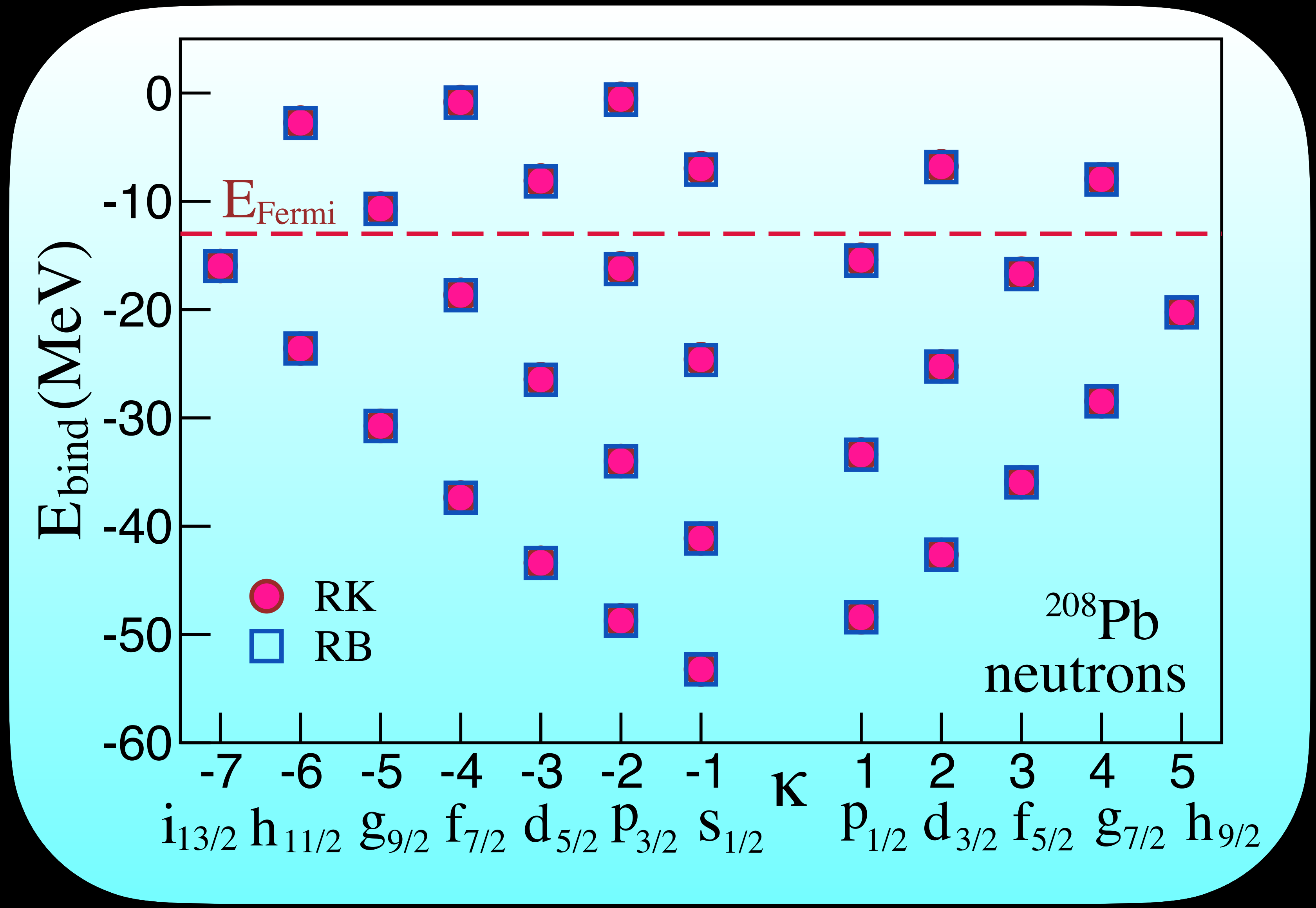
3. Implement a "singular value decomposition" (SVD) to construct the low-dimensional orthonormal and universal reduced basis:

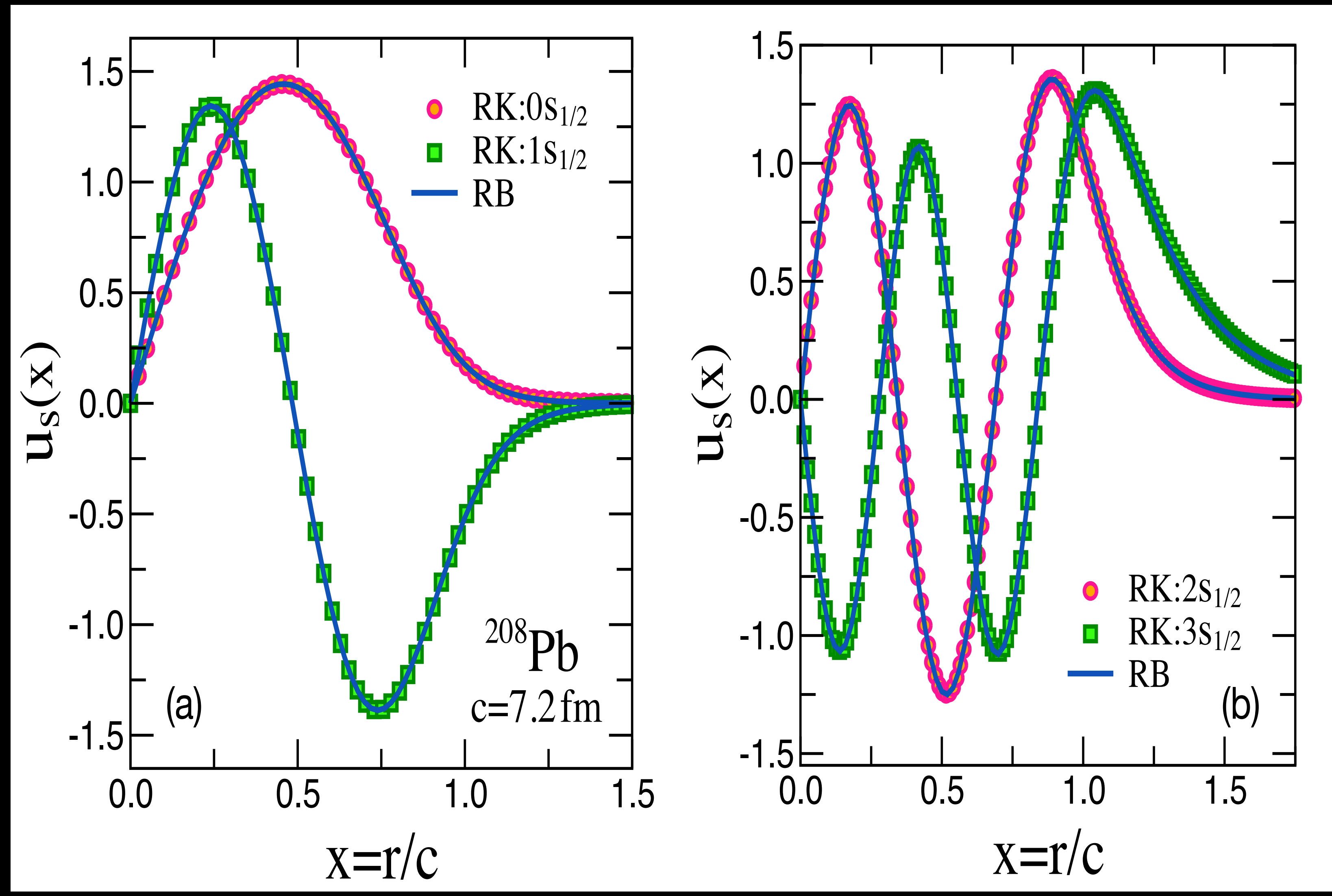


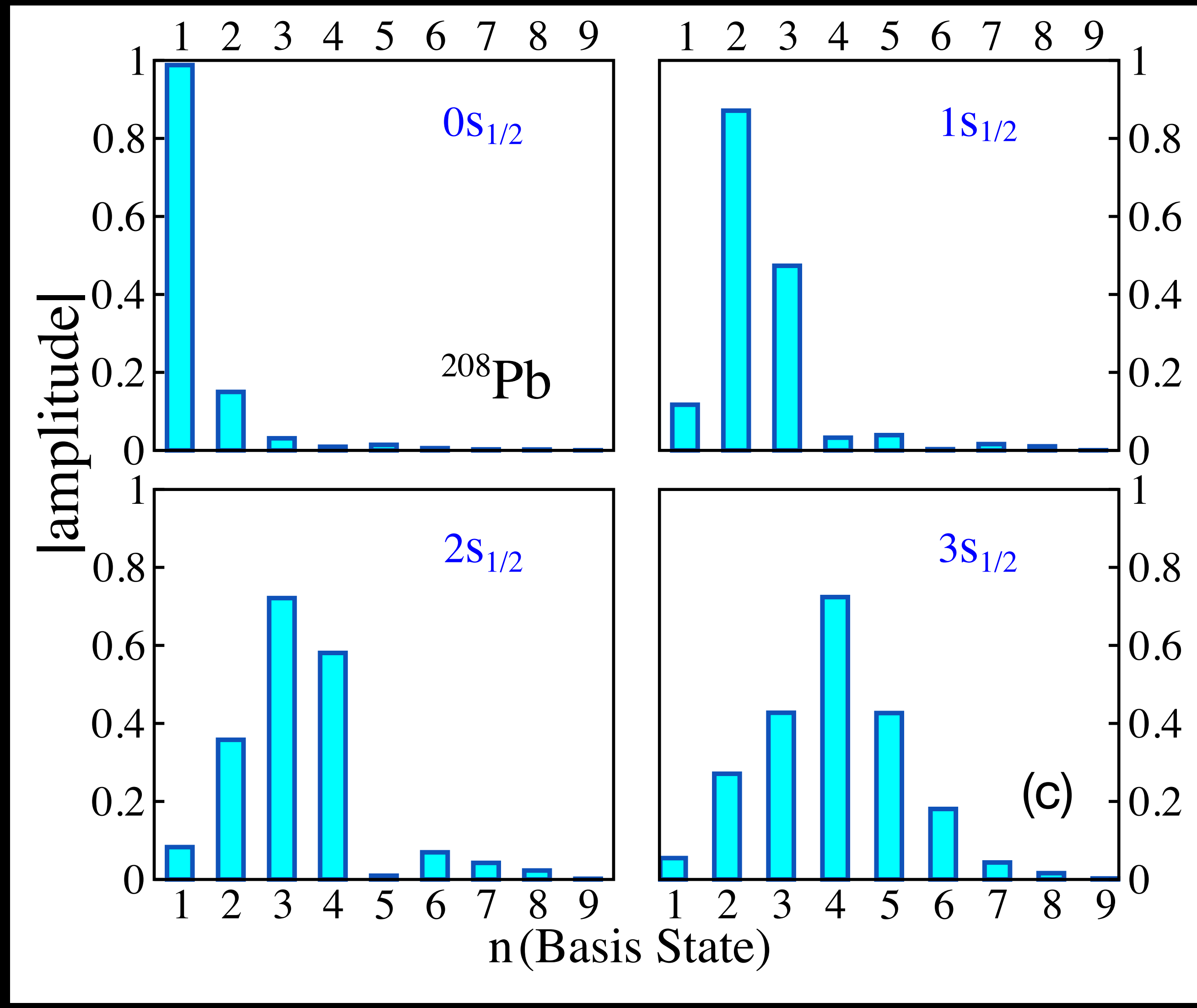
4. Compute the single-particle spectrum of all nuclei of interest at a small fraction in computational cost (three orders of magnitude in speedup!)

Neutron Stars meet Bayesian Emulators
Reduced Basis Methods

A “universal” reduced basis capable of accurately and efficiently reproduce the entire single-particle spectrum of a variety of nuclei at a small fraction in computational cost!





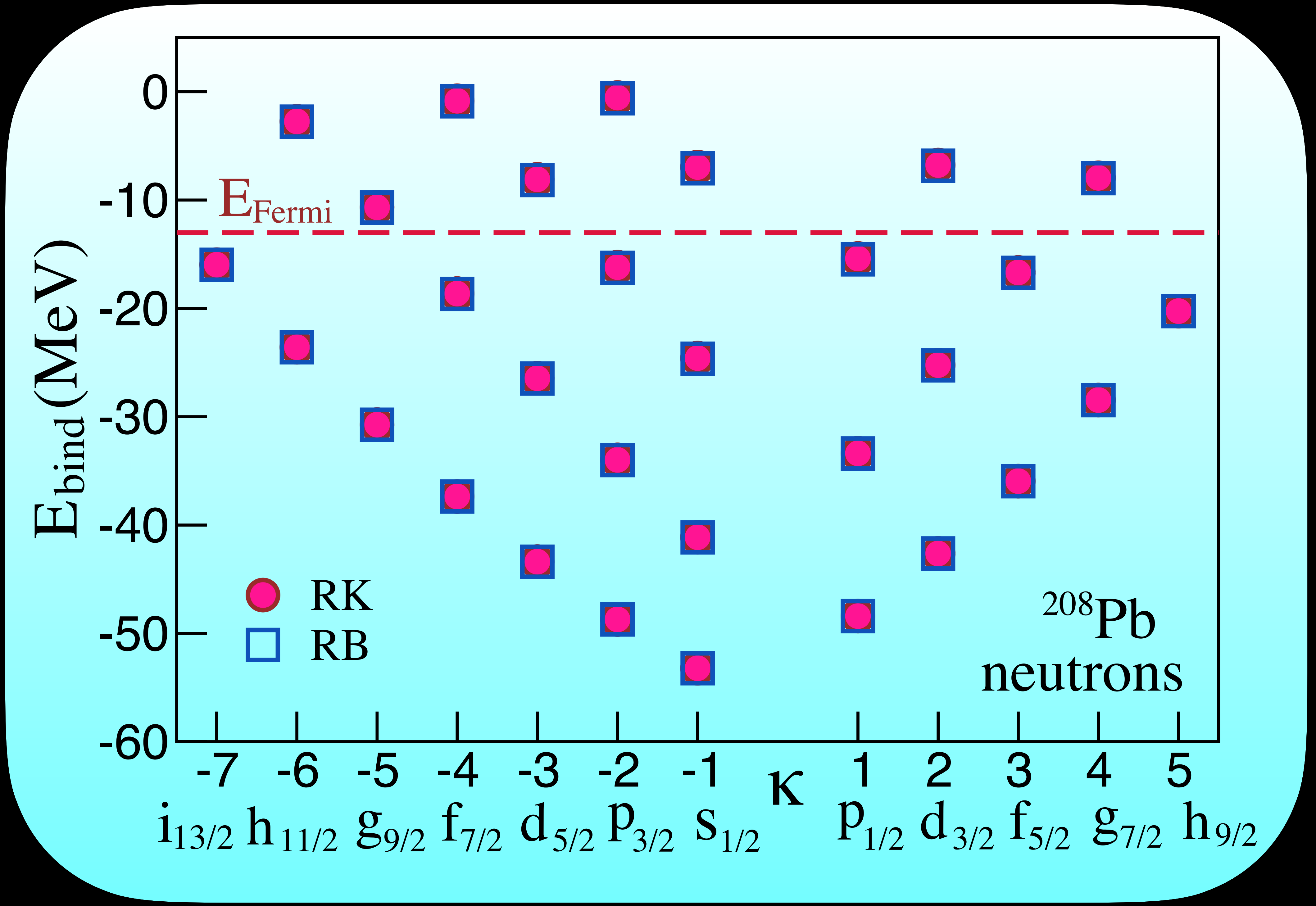


Neutron Stars meet Bayesian Emulators

Reduced Basis Methods

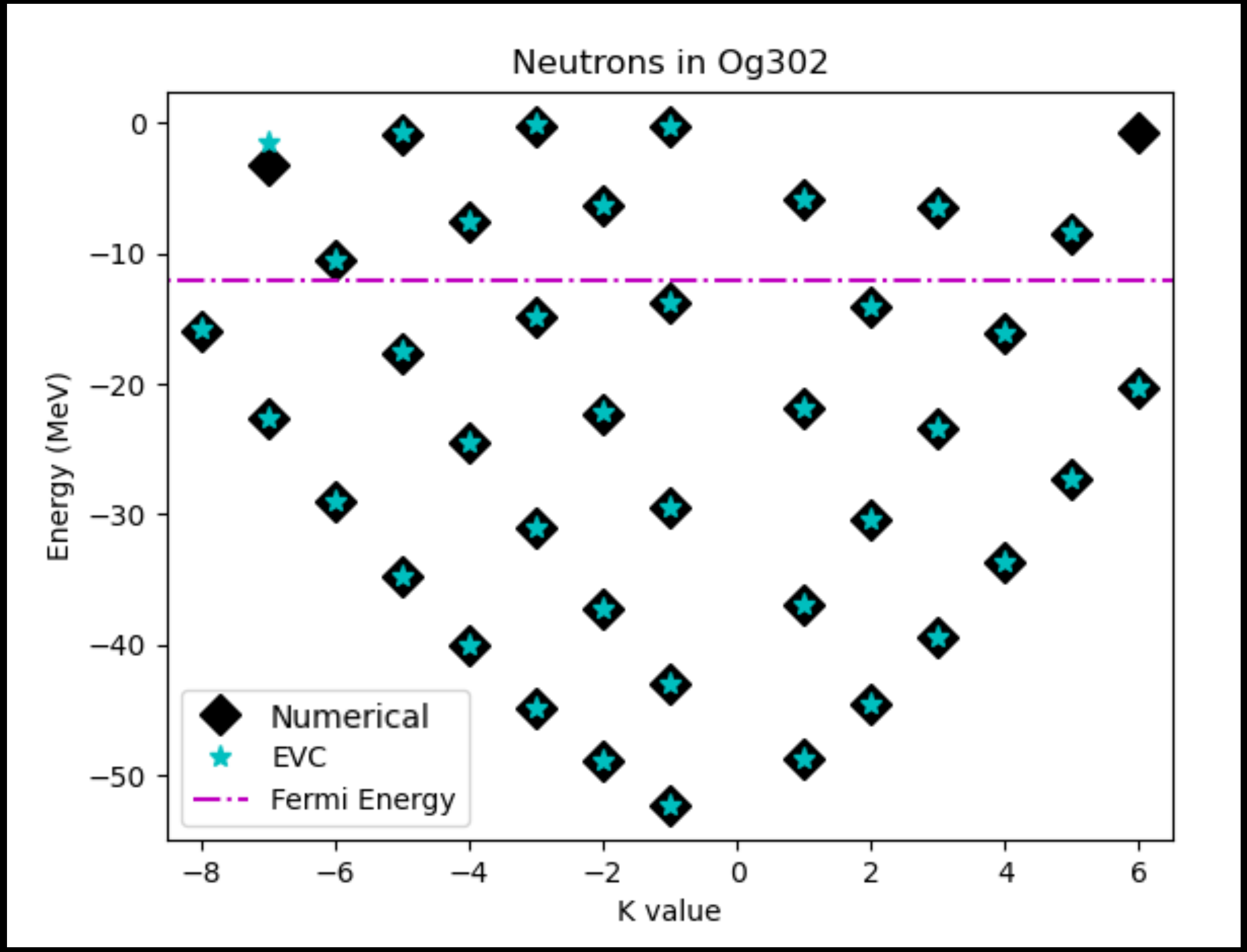
A "universal" reduced basis capable of accurately and efficiently reproduce the entire single-particle spectrum of a variety of nuclei at a small fraction in computational cost ... **and extrapolate!**

$^{208}\mathrm{Pb}(Z=82, N=126)$



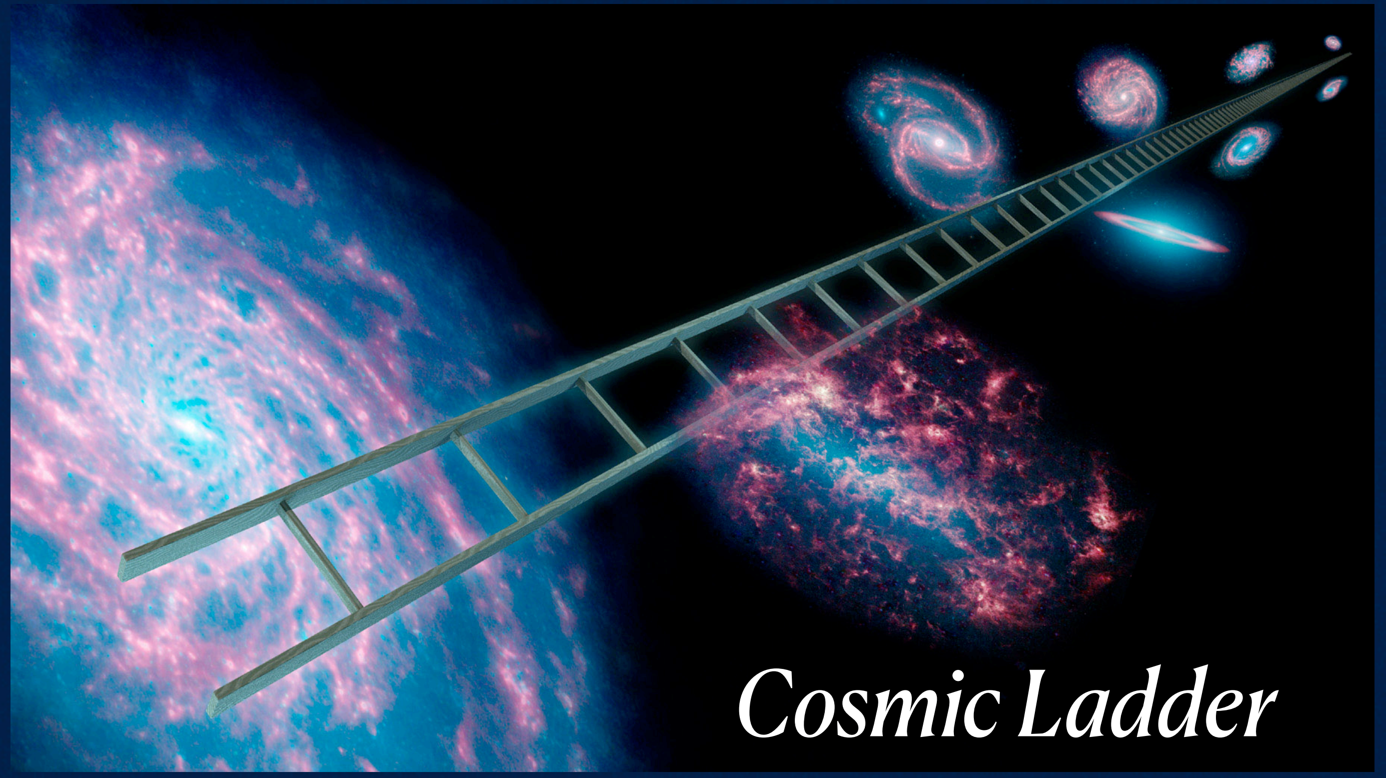
Genuine Extrapolation

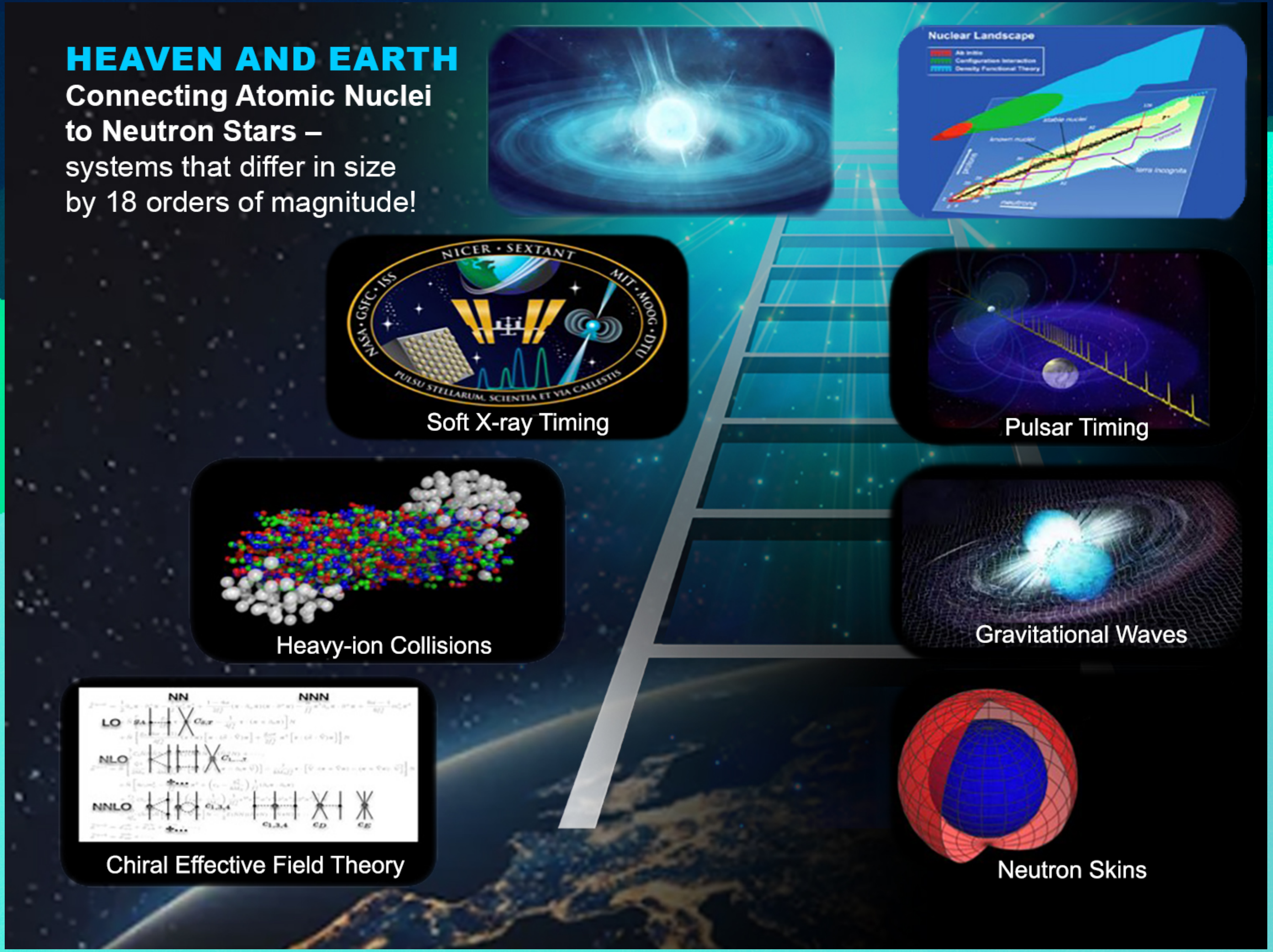
$^{302}\mathrm{Og}(Z=118, N=184)$



Heaven and Earth: Nuclear EOS Density Ladder

No single method can constrain the EOS over the entire density domain. Instead, each rung on the ladder provides information that can be used to determine the **EOS** at neighboring rungs

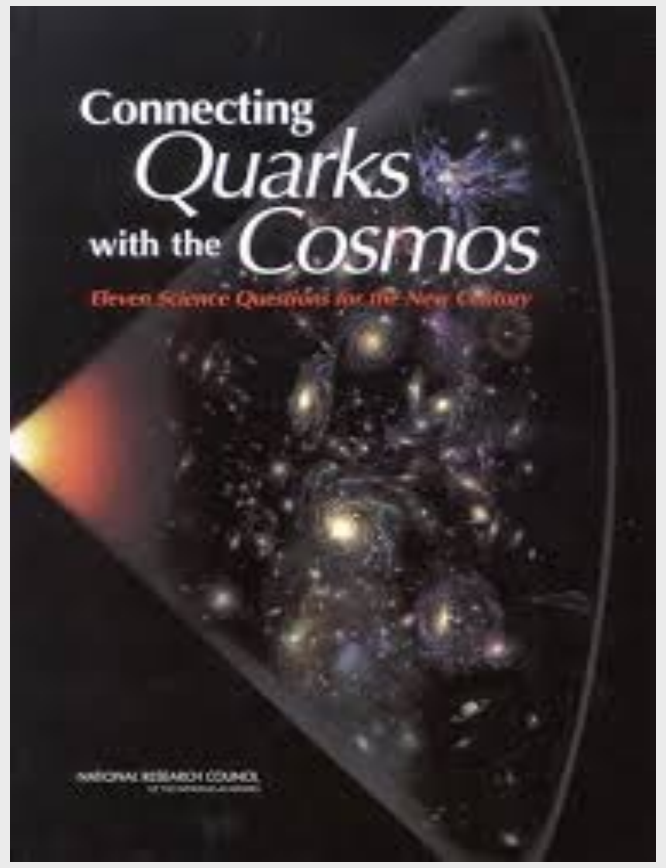




The Nuclear Physics of Neutron Stars

How were the heavy elements from iron to uranium made?

Are there new states of matter at ultrahigh temperatures and densities?



ECT*
EUROPEAN CENTRE
FOR THEORETICAL STUDIES
IN NUCLEAR PHYSICS AND RELATED AREAS

FONDAZIONE
BRUNO KESSLER

DTP/TALENT TRAINING SCHOOL

Trento, 15 July - 2 August 2024

Training in Advanced Low Energy Nuclear Theory

Nuclear theory for astrophysics

Organizers
Almudena Arcones (TU Darmstadt), Bruno Giacomazzo (Università degli Studi di Milano-Bicocca), Jorge Piekarewicz (Florida State University)

Student coordinator and Advisor
Bruno Giacomazzo

The school is truly multidisciplinary as it addresses fundamental questions in fields as diverse as astrophysics, gravitational physics, nuclear physics, and particle physics. Neutron stars are supported against gravitational collapse by nuclear interactions that become strongly repulsive at short densities leading, in turn, to an equation of state (EOS) capable of supporting neutron stars in excess of two solar masses. Neutron stars are unique cosmic laboratories that probe the strong interaction at the extremes of density and isospin asymmetry, and which may harbor exotic states of matter in their cores. Finally, the gravitational-wave and electromagnetic emission from the collision of binary neutron stars is starting to provide fundamental new insights into the astrophysical site for the r-process and on the nature of dense matter.
In this school we will discuss neutron stars and their EOS, core-collapse supernovae and neutron star mergers. These two high-energy events allow us to understand the extreme conditions in neutron stars as well as the origin of heavy elements in the universe.

Keynote Speakers and Lecturers
Almudena Arcones (TU Darmstadt), Andre da Silva Schneider (Universidade Federal de Santa Catarina), Bruno Giacomazzo (Università degli Studi di Milano-Bicocca), Alejandra Gonzalez (University of Jena), Martin Obergaulinger (University of Valencia), Albino Perego (University of Trento), Jorge Piekarewicz (Florida State University), Anna Puecher (University of Potsdam), Adriana Raduta (IFIN-HH Bucharest), Moritz Reichert (University of Valencia), Concettina Sfienti (Johannes Gutenberg-Universität), Om Sharan Salafia (INAF), Irene Tamborra (Niels Bohr Institute), Serena Vinciguerra (University of Amsterdam), Anna Watts (University of Amsterdam)

APPLICATIONS
Applications for the ECT* DTP/TALENT Training School 2024 should be made electronically through the ECT* web page. It should include: a curriculum vitae, a 1-page description of academic and scientific achievements, a short letter expressing the applicants' personal motivation for participating in the School. In addition, a reference letter from the candidate's supervisor should be sent to Barbara Gazzoli (gazzoli@ectstar.eu) for the attention of Professor Gert Aarts - Director of ECT*. For further details see www.ectstar.eu

Director of ECT*: Professor Gert Aarts

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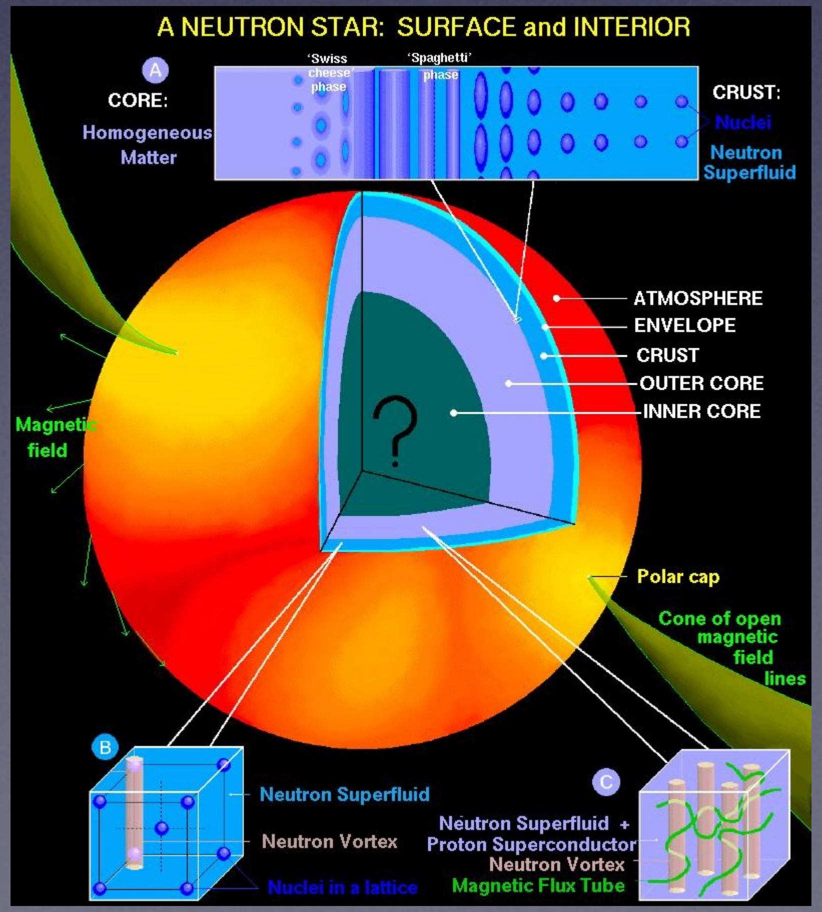
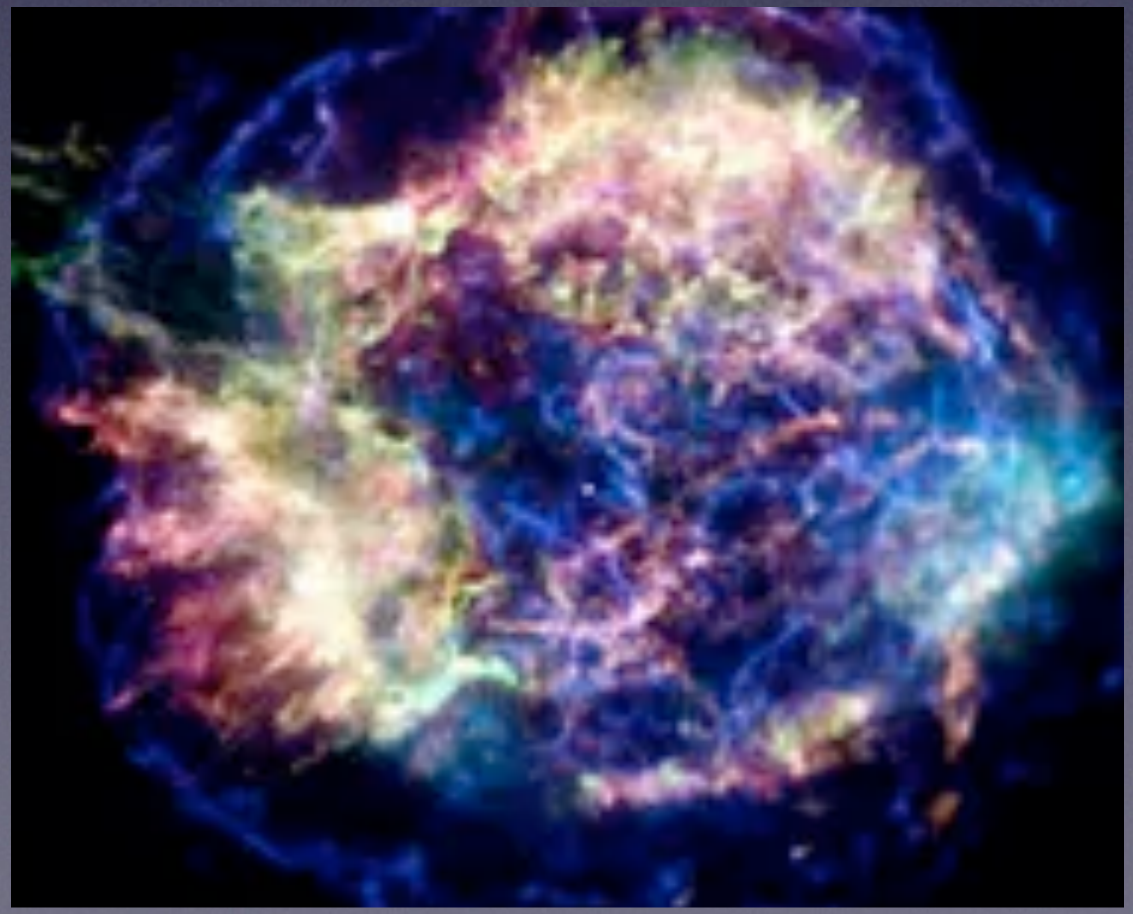
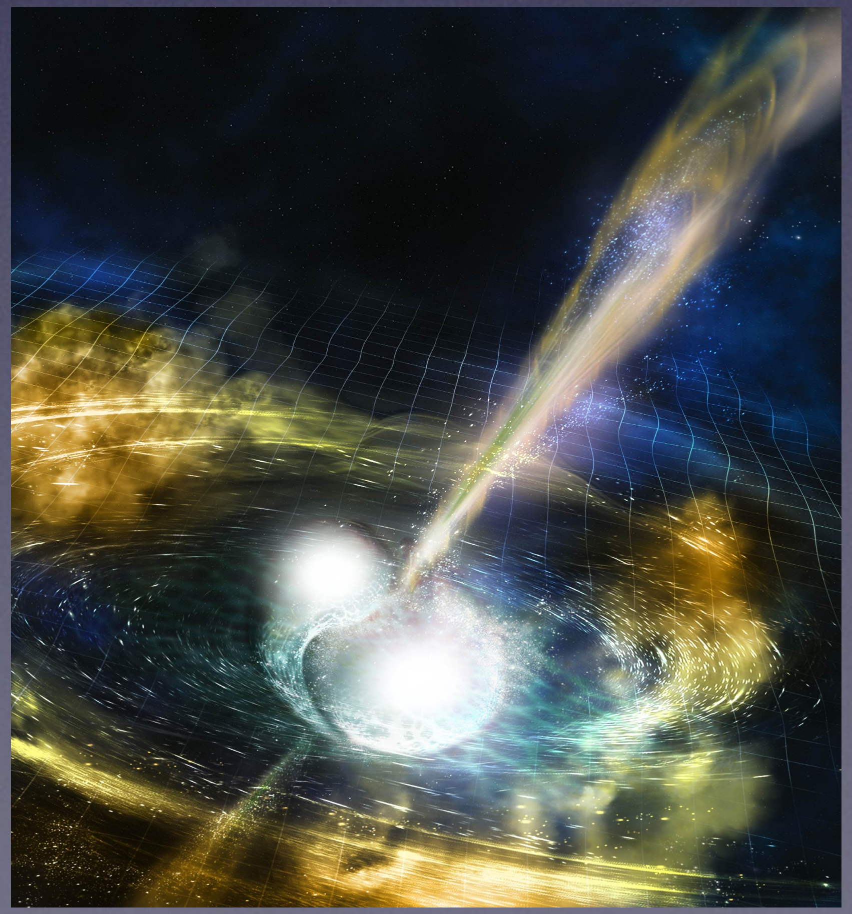
SUPPORTING INSTITUTIONS
INFN Istituto Nazionale di Fisica Nucleare
Theory Alliance FACILITY FOR RARE ISOTOPE BEAMS
EMMI

Nuclear Theory for Astrophysics

- ***First Week: Heaven and Earth — Informing the Equation of State of NS Matter***
  (Adriana, Anna, Concettina, Serena; Jorge)
- ***Second Week: Core Collapse Supernovae***
  (Andre, Francesco, Martin, Moritz; Almudena)
- ***Third Week: Neutron Star Mergers***
  (Albino, Alejandra, Om; Bruno)







We are all part of one big family with the common goal of decoding the wonderful and enigmatic character of neutron stars!