Equation of state and gravitational waves

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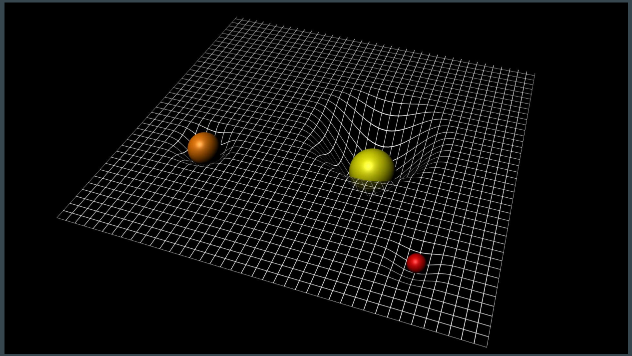
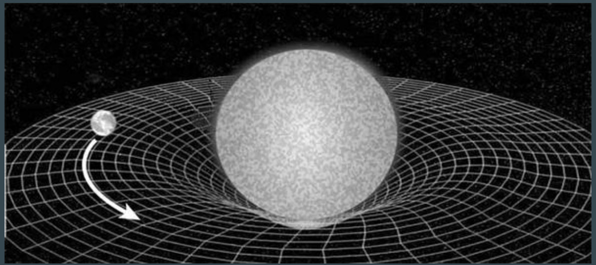
Tim Dietrich , Henrique Gieg, Vsevolod Nedora, Adrian Abac, Edoardo Giangrandi, Hauke Koehn, Nina Kunert, Ivan Markin, Anna Neuweiler, Henrik Rose, Federico Schianchi,Matthew Beaudoin, Ramon Jaeger, Sahil Jhavar, Hannes Kroke, Ranjith Mudimadugula, Karol Pawel Peters, Ashwin Shirke, Sophia Ve Dahm

General Relativity and gravitational waves



Credit:ESA–C.Carreau

General Relativity: masses deform the spacetime geometry producing curvature, and this geometry is what determines how massive objects move



$g_{\alpha\beta}$: spacetime metric

Einstein Field Equations

$$R_{\alpha\beta} - \frac{1}{2} R g_{\alpha\beta} = \frac{8\pi G}{c^4} T_{\alpha\beta}$$

curvature

source of the curvature (mass and energy)

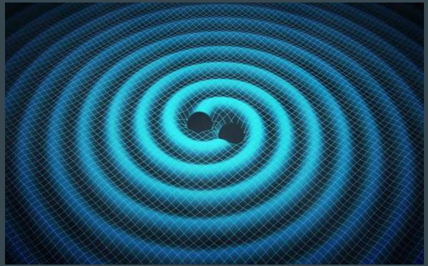
General Relativity and gravitational waves

Weak field limit $g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta}$ $|h_{\alpha\beta}| \ll 1$

↳ linearized theory of gravity (expansion of Einstein's equation around flat metric to linear order in $h_{\alpha\beta}$)

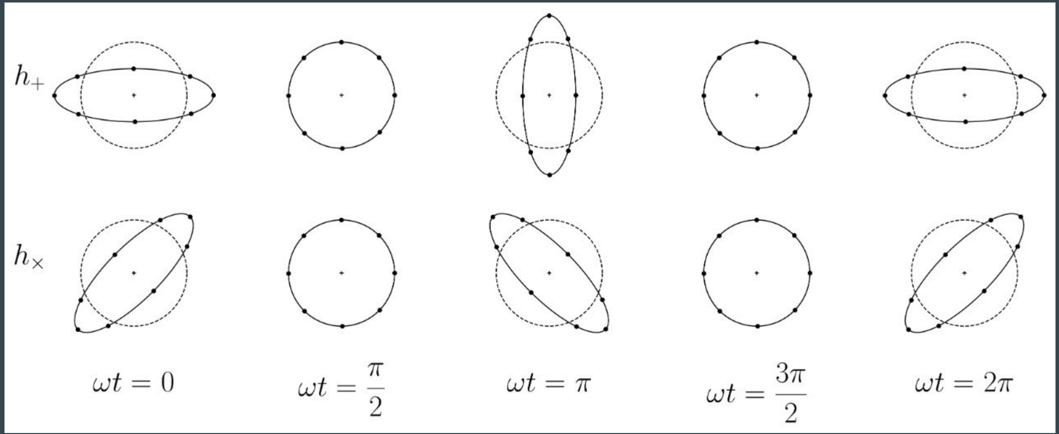
Linearized Einstein equations: $\Box\, \bar{h}_{\alpha\beta} = -\frac{16\pi G}{c^4} T_{\alpha\beta}$

in vacuum $\Box\, \bar{h}_{\alpha\beta} = 0$ $\Longrightarrow$ wave- equation!



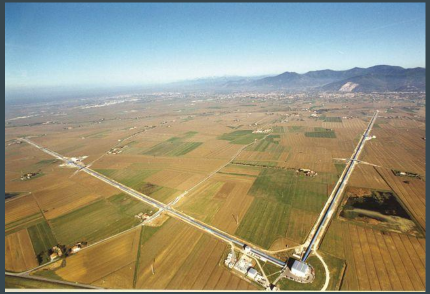
Perturbations of spacetime propagate on the flat spacetime background as plane waves $\Longrightarrow$ ***gravitational waves.***

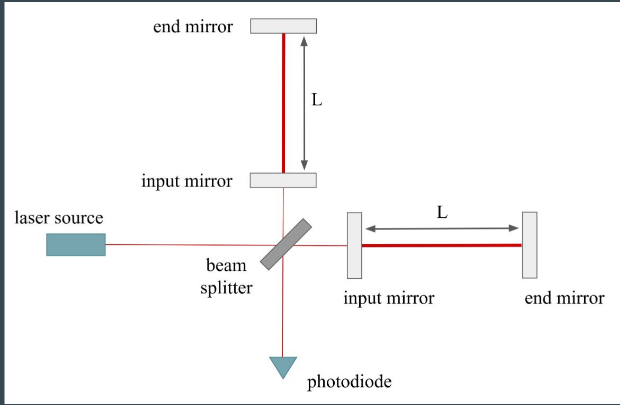
Gravitational waves detection



Gravitational waves: stretch and compress spacetime

Laser interferometers

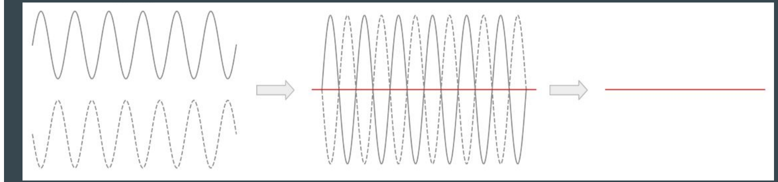




destructive interference

laser 1

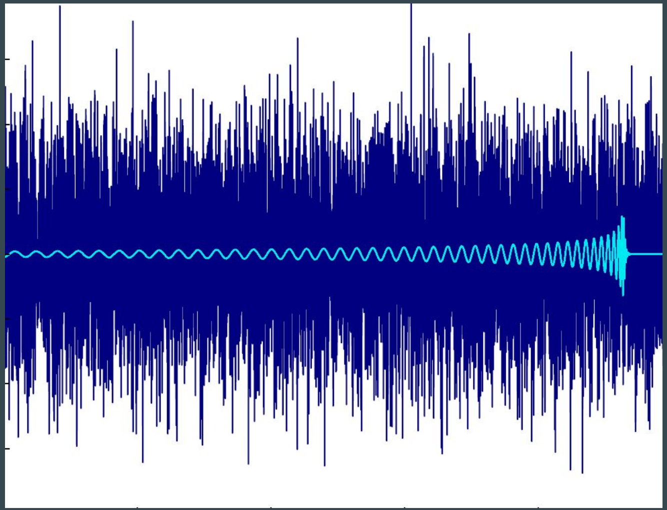
output



laser 2

$$h(t) = \frac{\delta L}{L} \sim \mathcal{O}(10^{-20})$$

Identifying signals

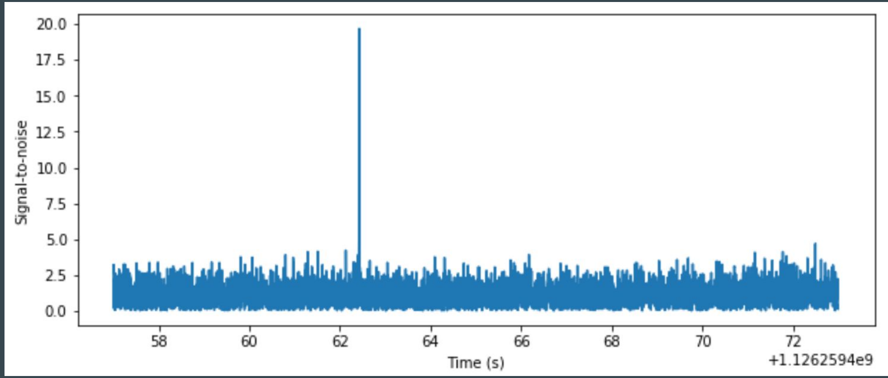


Matched filtering

Compare data with *template banks* (only main parameters)

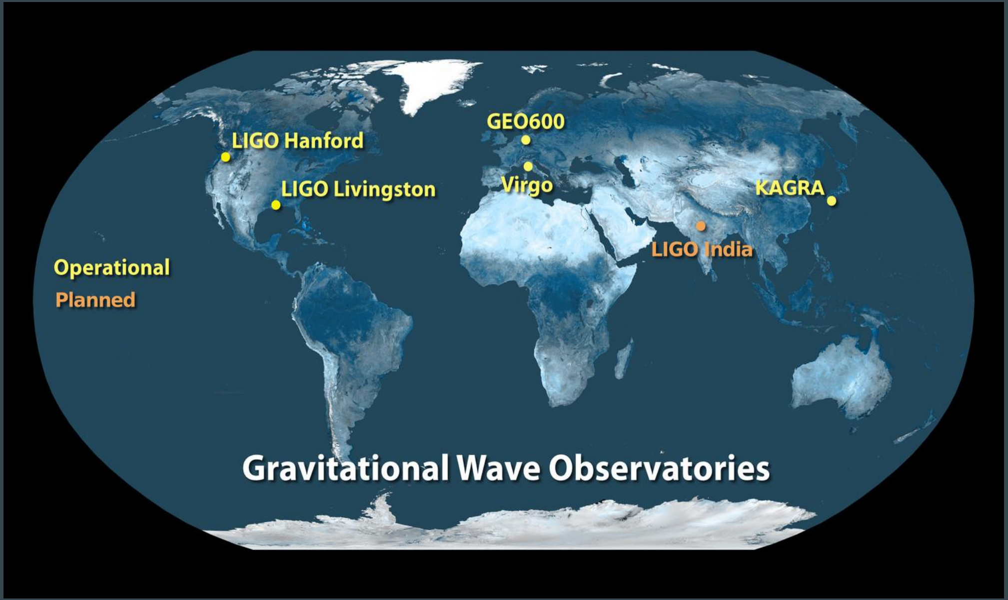
⇩

Find ***trigger time***



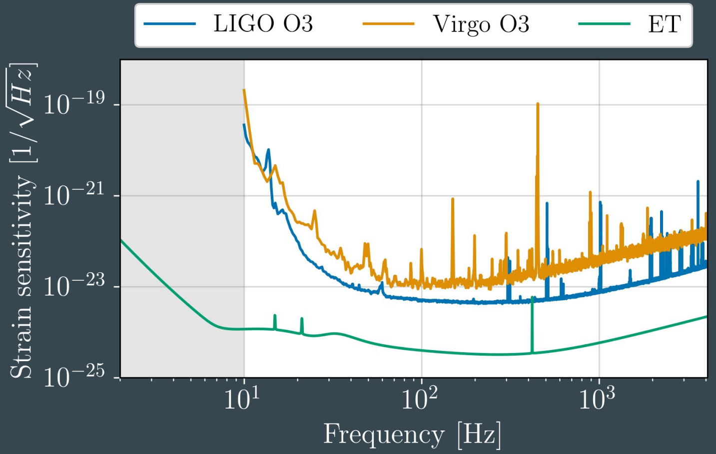
If you want to know more: tutorial here

Gravitational waves detectors

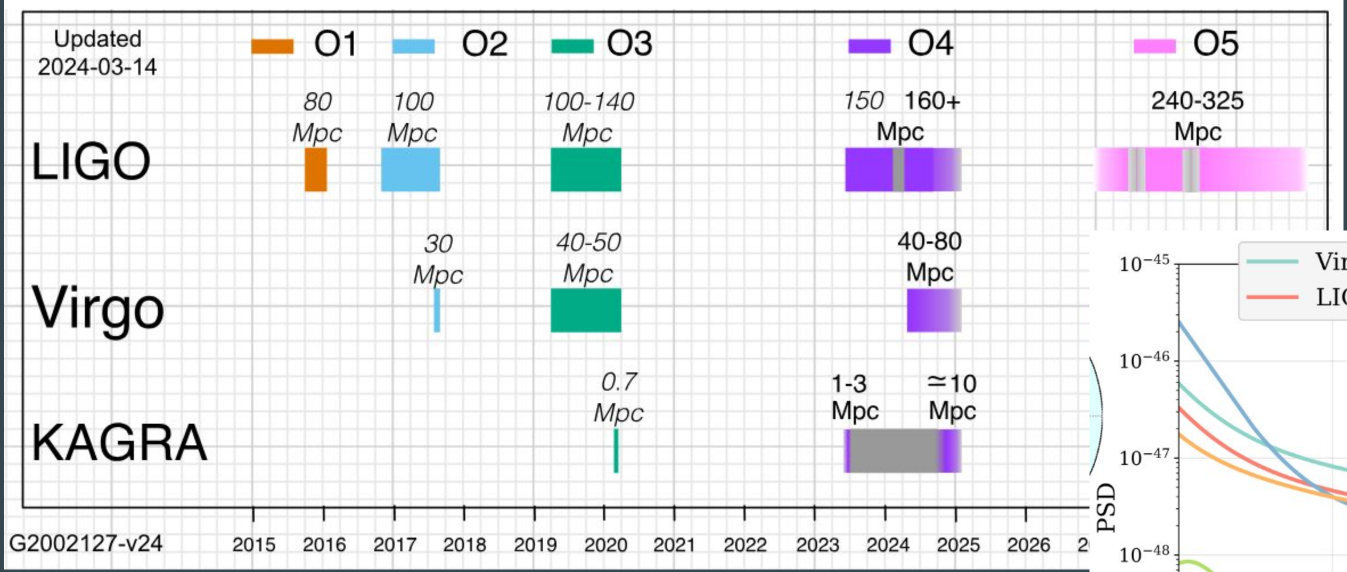


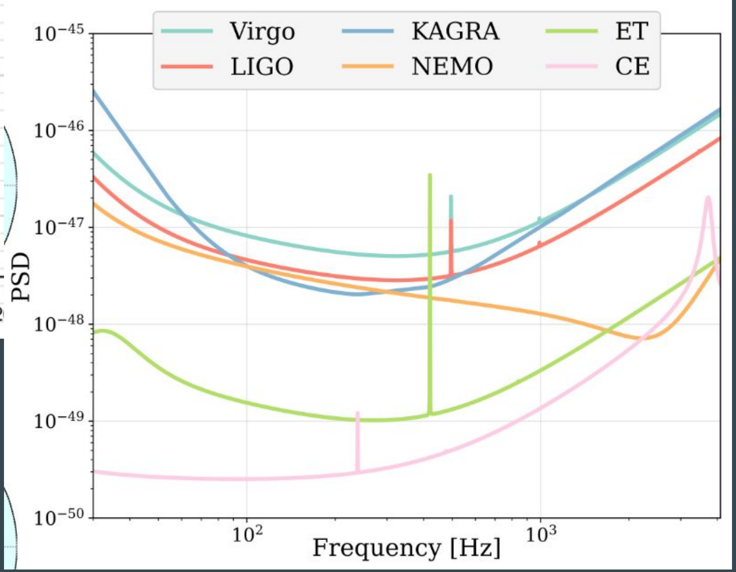
Next-generation detectors:

- Einstein Telescope (ET) in Europe
- Cosmic Explorer (CE) in the US

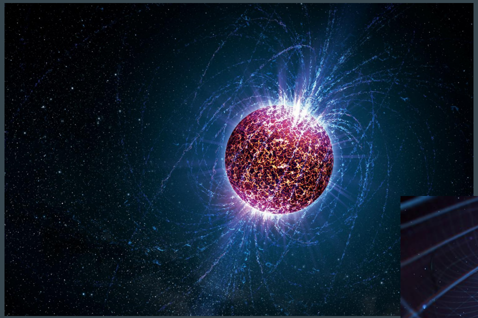
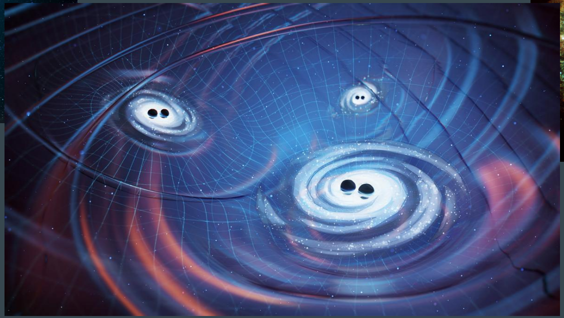


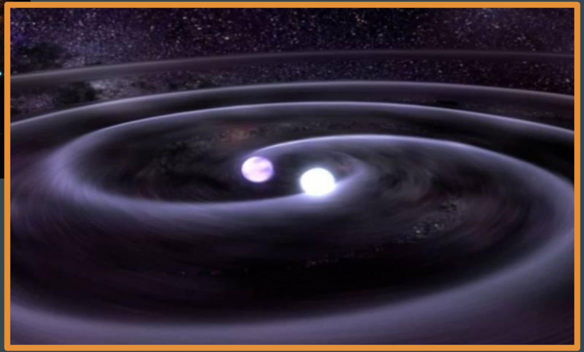
Gravitational waves detectors





Gravitational waves sources



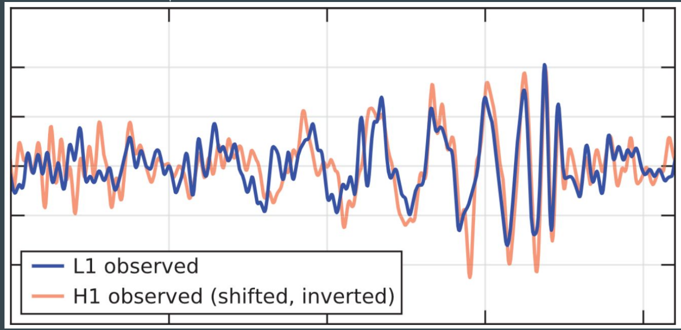
Leading order: *mass quadrupole moment*

What we have

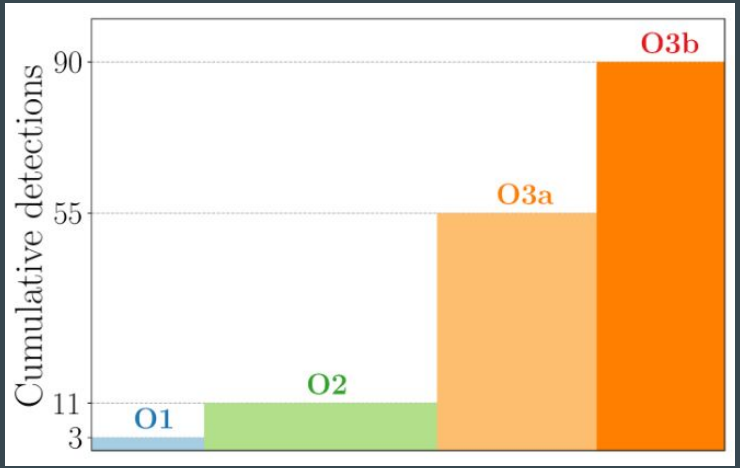
Sept '15 - Jan '16 | Nov '16 - Aug '17 | Apr '19 - Mar '20 | May '23 - ongoing

O1 | O2 | O3a | O3b | O4

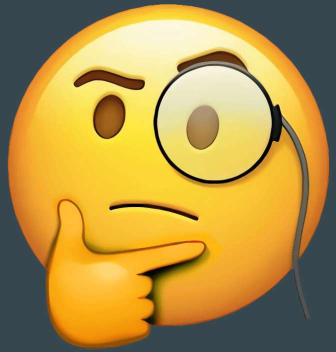
GW150914



GW170817

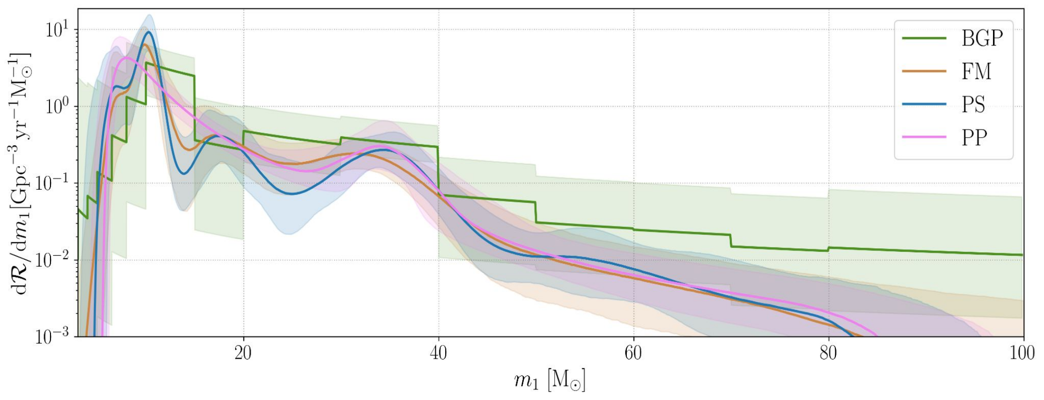


What science can we do with gravitational waves?



Population properties

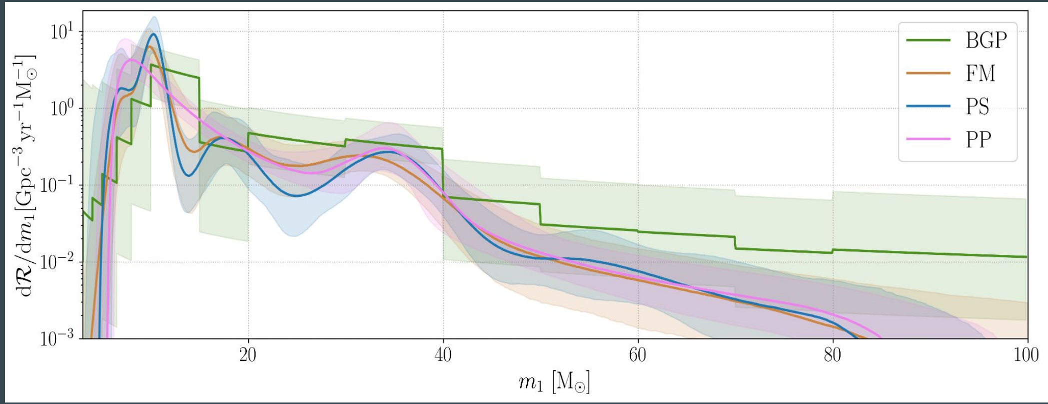
Masses



LVK, *Phys.Rev.X* 13 (2023) 1, 011048

Population properties

MASSES



LVK, *Phys.Rev.X* 13 (2023) 1, 011048

SPINS

Binaries formation channels:

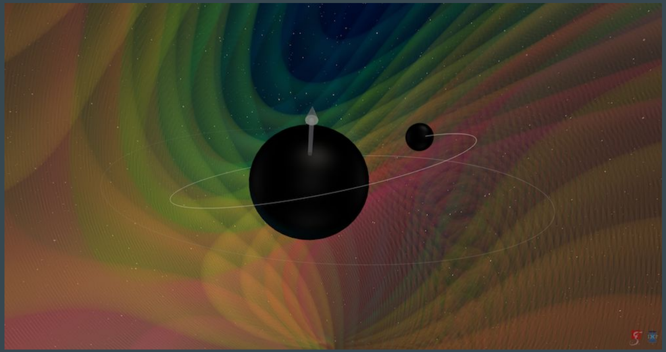
- Isolated binary evolution: aligned spins
- Dynamical interaction: precession, misaligned spins

Spin properties $\Longleftrightarrow$ Formation channels

Investigate and model different mechanisms

Special events

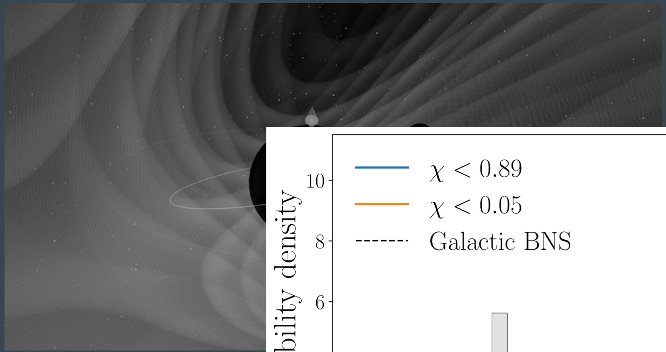
- GW190412: unequal masses

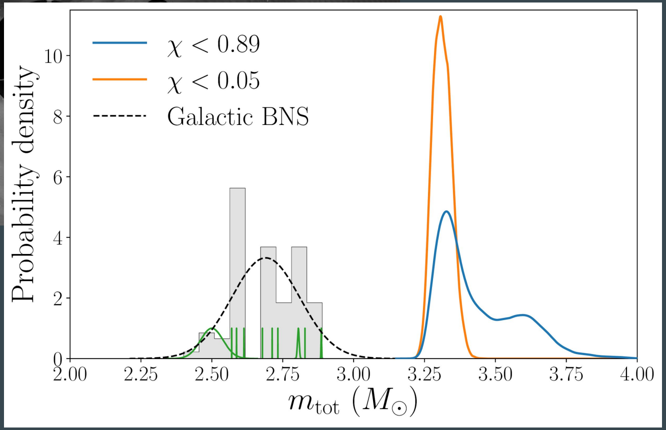


Special events

- GW190412: unequal masses

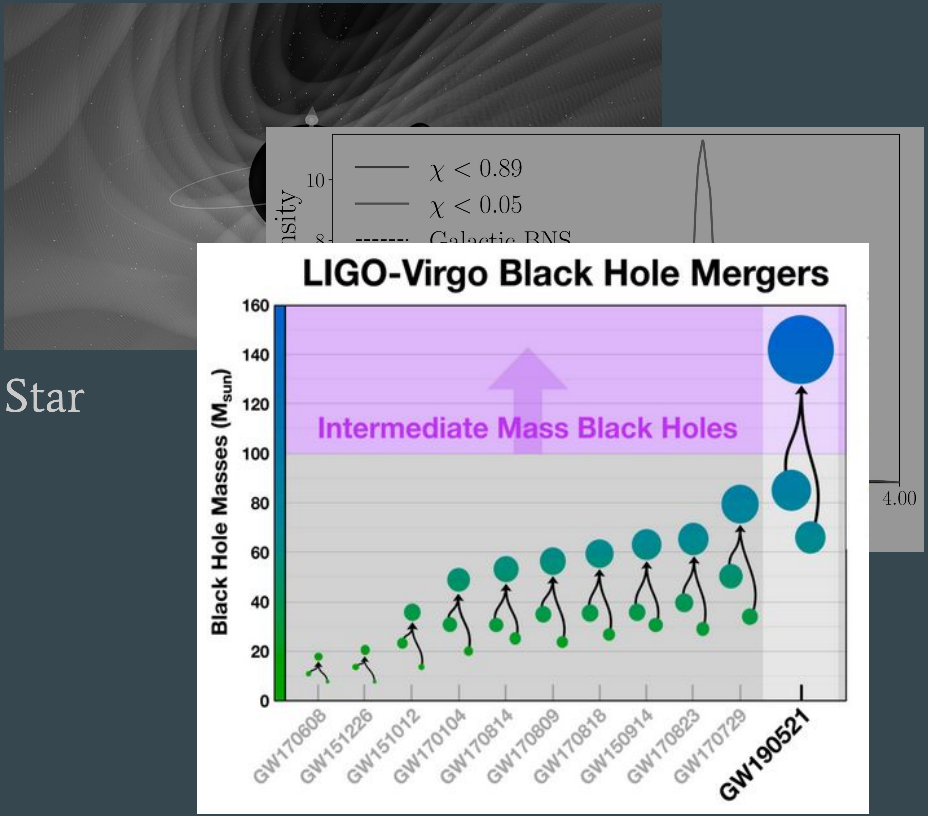
- GW190425: heavy Binary Neutron Star





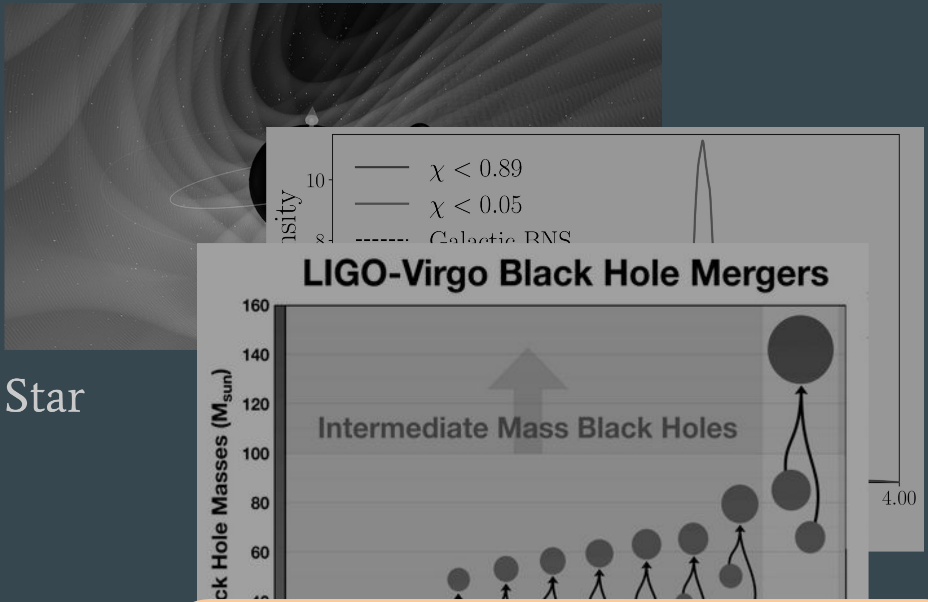
Special events

- GW190412: unequal masses
- GW190425: heavy Binary Neutron Star
- GW190521: the most massive



Special events

- GW190412: unequal masses
- GW190425: heavy Binary Neutron Star
- GW190521: the most massive
- GW190814: a mysterious compact object

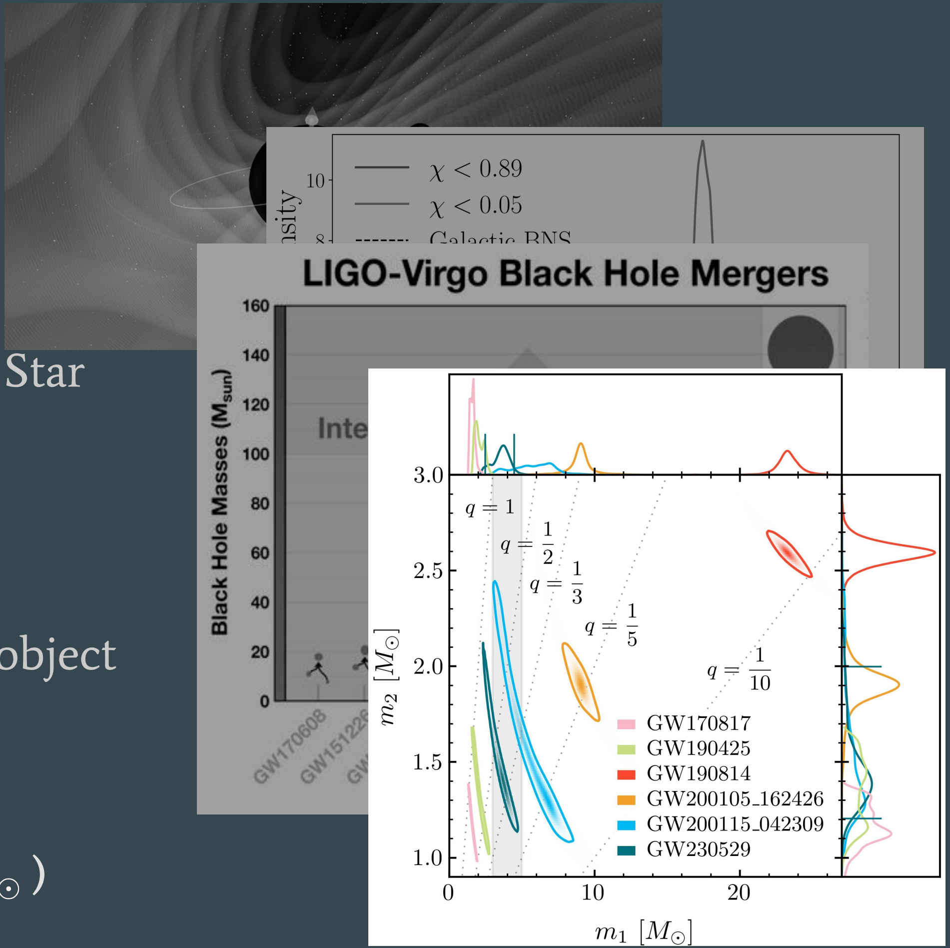


$m_1 \sim 23\ M_\odot$ => Black Hole

$m_2 : 2.50 - 2.67\ M_\odot$ => ???

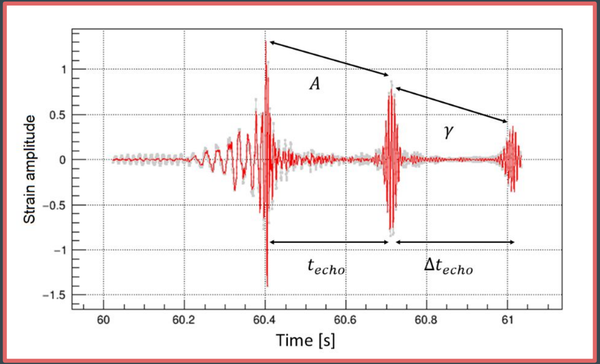
Special events

- GW190412: unequal masses
- GW190425: heavy Binary Neutron Star
- GW190521: the most massive
- GW190814: a mysterious compact object
- GW230529: mass-gap event
  Primary object $m_1 = 3.6 M_\odot$
  BH in mass lower mass gap (3-5 $M_\odot$)



Testing general relativity

- ➢ Dynamical, strong-field regime
- ➢ Comparing data with general relativity predictions



Phys.Rev.D 108 (2023) 6, 064018

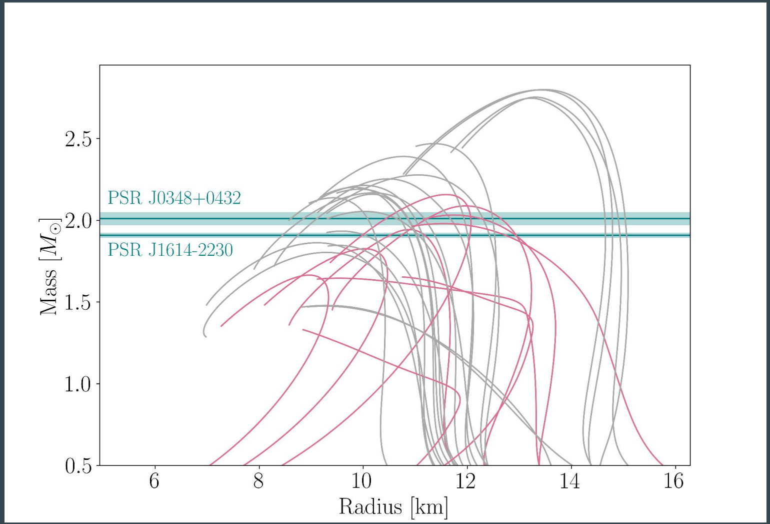
- Parametrized deviations from the phase evolution
- Test the nature of compact objects: echoes (no horizon)
- Propagation of gravitational waves

• • •

Most recent LVK results:
arXiv:2112.06861

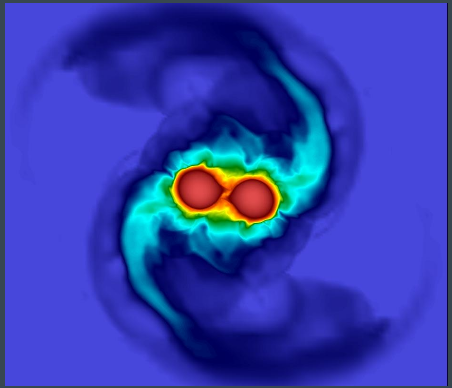
No violations of general relativity found until now!

Nuclear matter properties



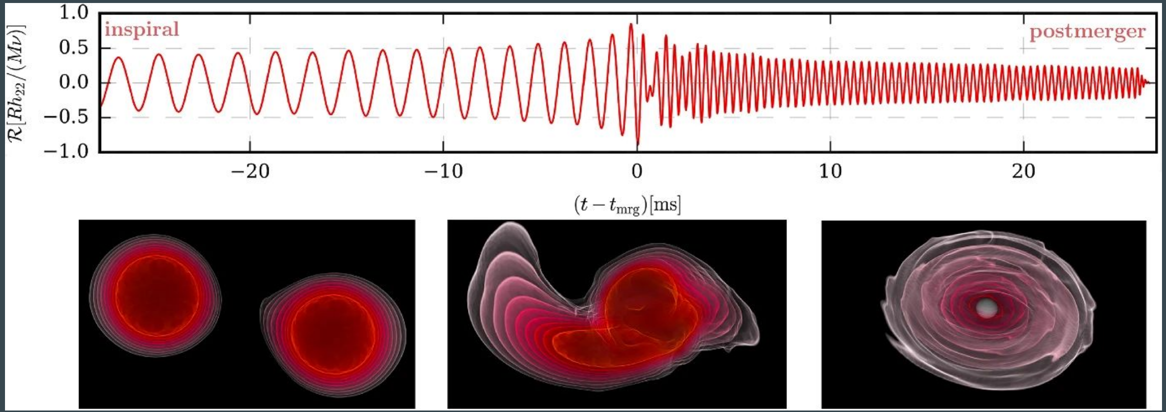
Neutron stars: supranuclear-dense matter

Equation of state :
relation between pressure and density
⟺ mass - radius
⟺ mass - tidal deformability parameter



University of Birmingham

Nuclear matter properties

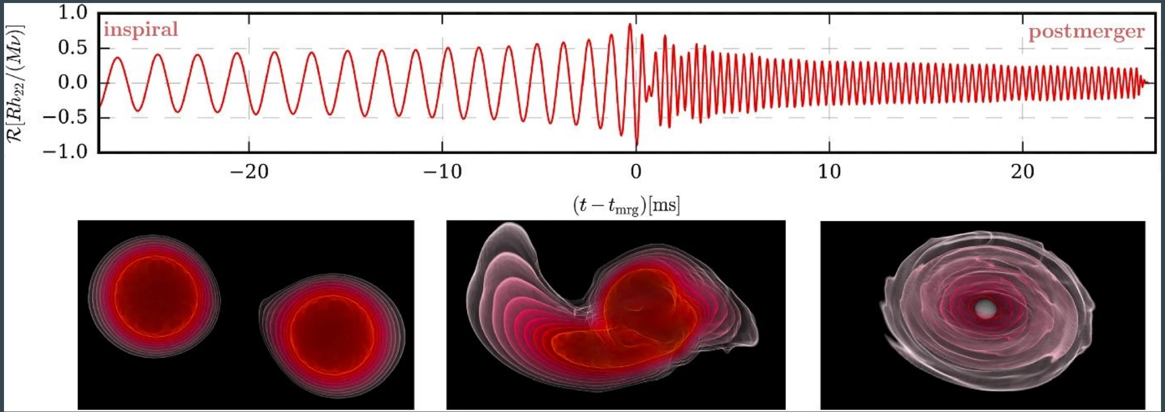


Gen. Rel.Grav. 53, 27 (2021)

- measure the system parameters (masses and tidal deformabilities)
- study the postmerger (future)

- multimessenger astrophysics NMMA
- combine with nuclear information [Nature 606, 276 (2022)]

Nuclear matter properties



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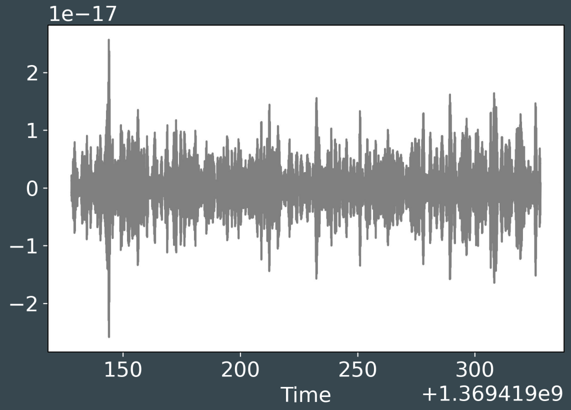
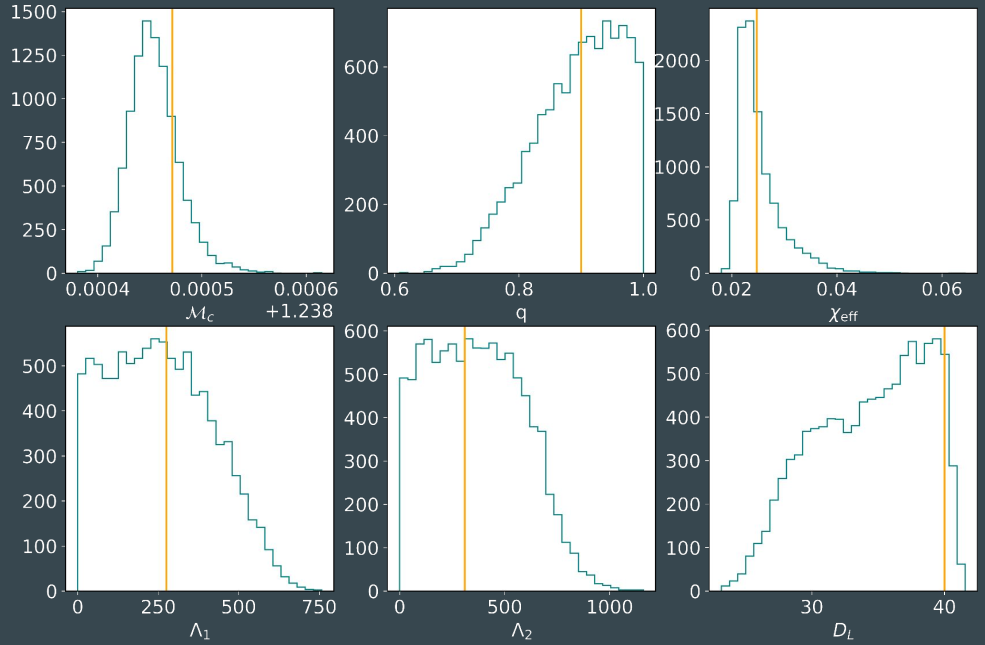
How do we estimate the source parameters?

Parameter estimation

Source parameters $\Longleftrightarrow$ GW signal



Parameter estimation



We assume that from match filtering we know there is a signal

Parameter estimation

We need:

- The measured GW data
- A model H to describe our data, whose parameters we want to estimate (ex: the masses)
- An idea of the range in which the parameters lie (ex: for a NS, $m_1, m_2 \in [1, 3]M_\odot$)

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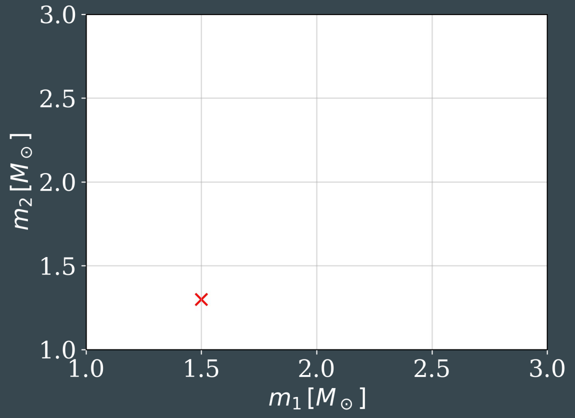
1. Pick some values for the parameters



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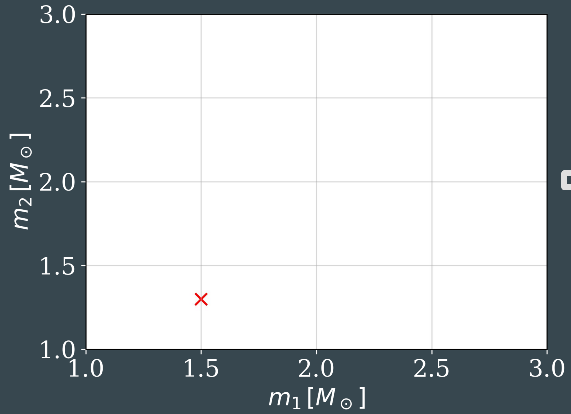
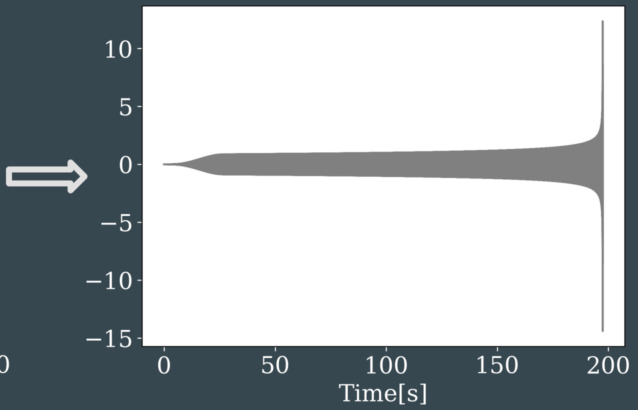


2. Use the model to compute the GW waveform for those parameters values



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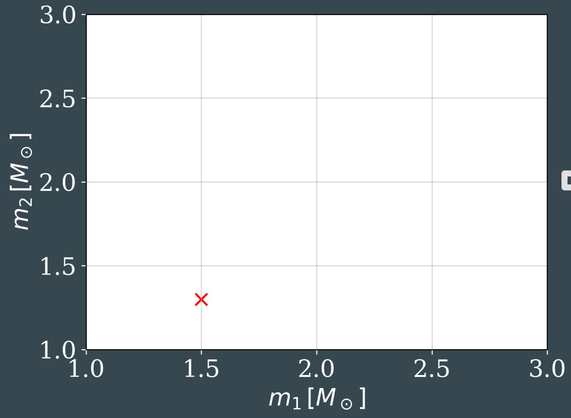

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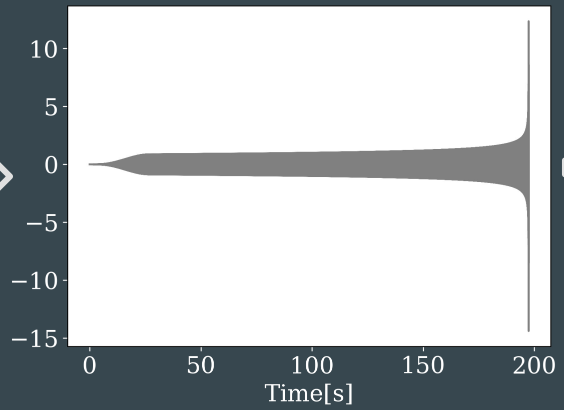
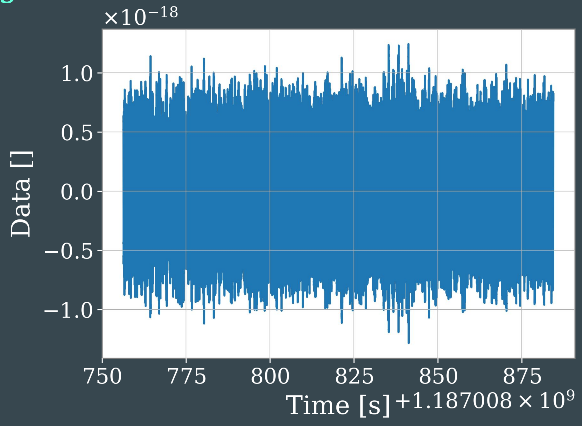


3. Compare to GW data



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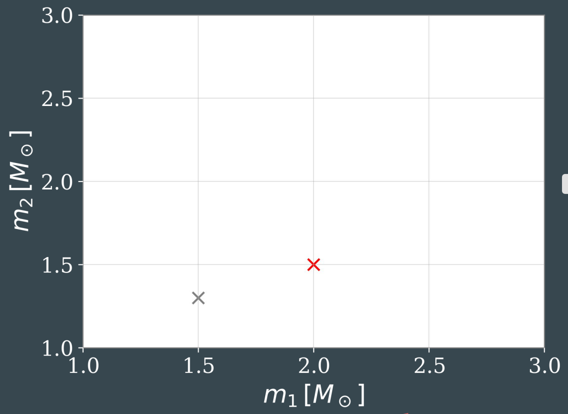

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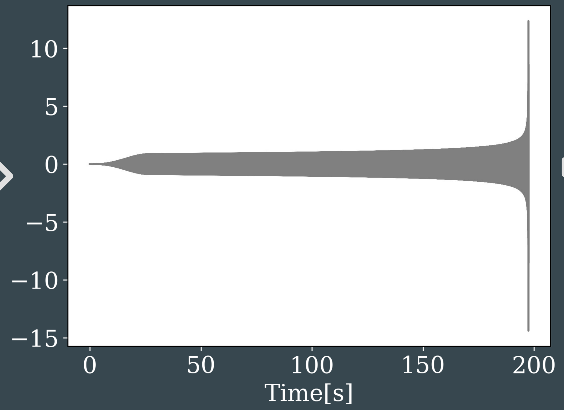

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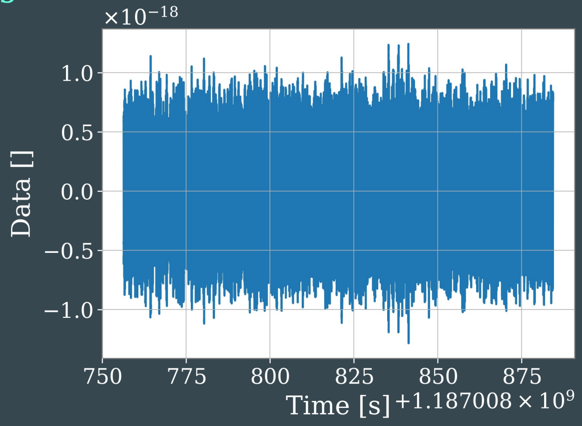
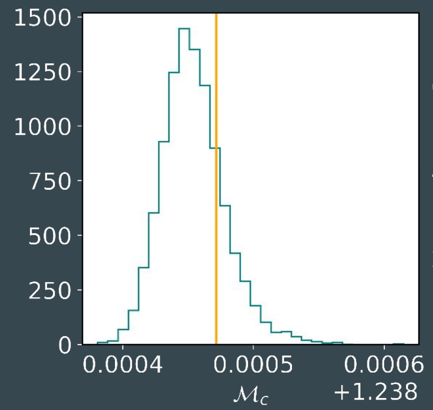


Parameter estimation - Bayesian inference



Bayes theorem:

$$p(d|\vec{\theta},\mathcal{H}_s) \propto \exp\left[-\frac{1}{2}\left\langle d-h(\vec{\theta})|d-h(\vec{\theta})\right\rangle\right]$$

$$p(\vec{\theta}|\mathcal{H}_s,d) = \frac{p(d|\vec{\theta},\mathcal{H}_s)\,p(\vec{\theta}|\mathcal{H}_s)}{p(d|\mathcal{H}_s)}$$

likelihoood

prior

Posterior probability density

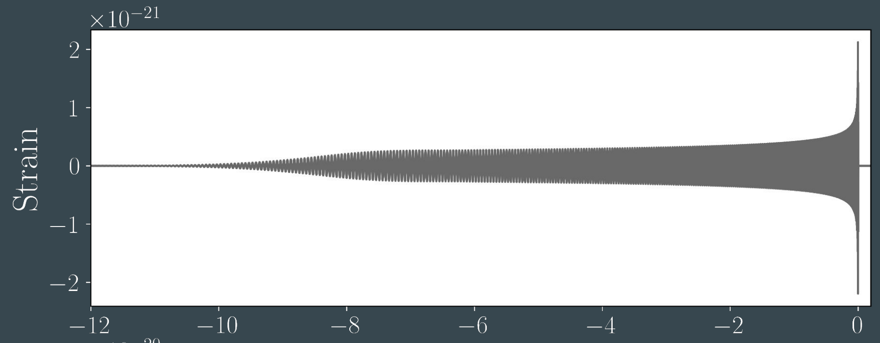
evidence

Evidence → Model comparison

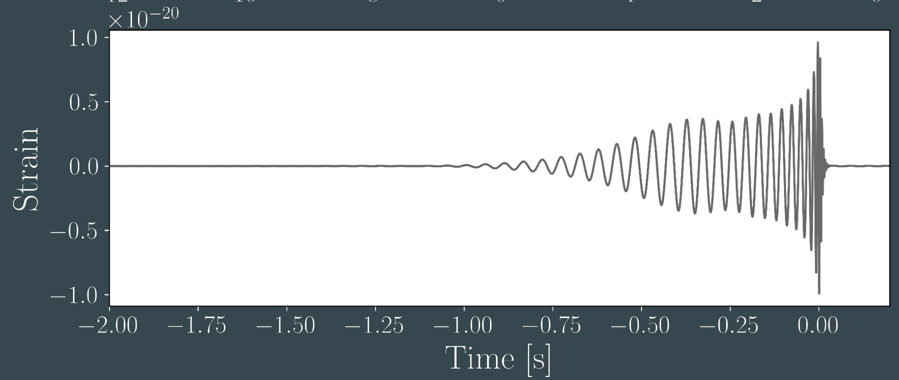
$$\mathcal{B}_A^B = \frac{p(d|H_B)}{p(d|H_A)} = \frac{Z_B}{Z_A}$$

Parameter estimation - waveform models

$$m_1 = 10\, M_\odot$$
$$m_2 = 8\, M_\odot$$



$$m_1 = 45\, M_\odot$$
$$m_2 = 36\, M_\odot$$



Parameters: masses, distance, spins, inclination, sky location...

Accurate models needed: include tidal effects, spins, precession, higher-order modes, eccentricity...

Waveform models

GW signal $\Longrightarrow$ solve Einstein Field Equations: $R_{\alpha\beta} - \frac{1}{2}Rg_{\alpha\beta} = \frac{8\pi G}{c^4}T_{\alpha\beta}$

Numerical Relativity $\Longrightarrow$ Computationally expensive

NRsurrogate *models:* directly interpolated on a set of pre-computed NR waveforms

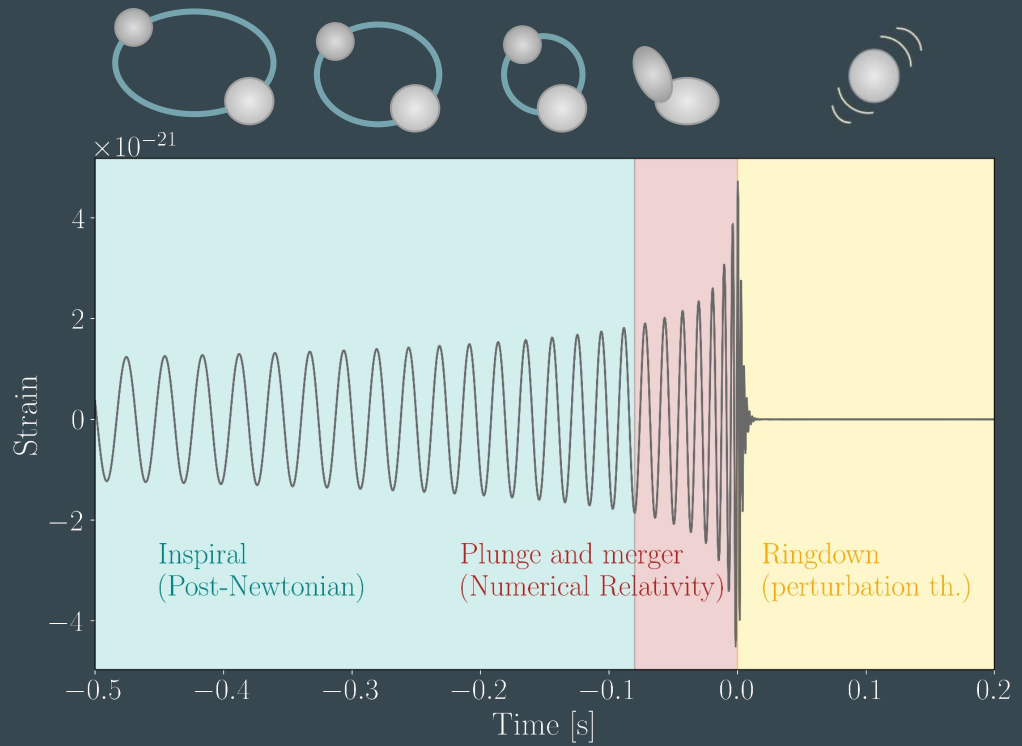
✓ very accurate
X limited parameters space

→ ***Approximations***

Waveform models

GW signal $\Rightarrow$ solve Einstein Field Equations: $R_{\alpha\beta} - \frac{1}{2}Rg_{\alpha\beta} = \frac{8\pi G}{c^4}T_{\alpha\beta}$

Numerical Relativity $\Longrightarrow$ Computationally expensive $\Longrightarrow$ ***Approximations***



NSs postmerger?

Waveform models - Post Newtonian expansion

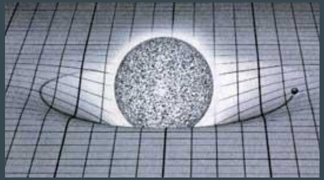
- GWs carry energy -> remove energy from the system -> orbit shrinks

Energy balance: $\mathcal{F} = -\frac{dE}{dt}$

GW luminosity (flux)

Orbital energy loss

Waveform models - Post Newtonian expansion



- <u>Curvature induced by the source</u>

Inspiral: Assume binary components at large distance and with small velocities

Energy balance: $\mathcal{F}(v) = -\dot{E}(v)$

expand both sides in power series of (v/c): ***post-Newtonian expansion***

Post-Newtonian (PN) expansion = *expansion in (v/c)* n-th PN order $\iff \mathcal{O}\left(\frac{v^{2n}}{c^{2n}}\right)$

compute energy and flux up to needed order

$\Longrightarrow$ compute the GW phase $h(t) = B(t)e^{i\Phi(t)}$

$$\dot{\phi}(t) = v^3/M_{\text{tot}}$$
$$\dot{v} = -\mathcal{F}(v)/E'$$

Taylor family models

$(E' = dE/dv)$

Waveform models - Post Newtonian expansion

$$\mathcal{E}_4(v) = -\frac{1}{2}\eta v^2 \left\{1 - \left(\frac{3}{4} + \frac{1}{12}\eta\right) v^2 - \left(\frac{27}{8} - \frac{19}{8}\eta + \frac{1}{24}\eta^2\right) v^4 + \left[-\frac{675}{64} + \left(\frac{34445}{576} - \frac{205}{96}\pi^2\right)\eta - \frac{155}{96}\eta^2 - \frac{35}{5184}\eta^3\right] v^6 + \left[-\frac{3969}{128} + \left(-\frac{123671}{5760} + \frac{9037}{1536}\pi^2 + \frac{896}{15}\gamma_{\mathrm{E}} + \frac{448}{15}\ln(16v^2)\right)\eta + \left(-\frac{498449}{3456} + \frac{3157}{576}\pi^2\right)\eta^2 + \frac{301}{1728}\eta^3 + \frac{77}{31104}\eta^4\right] v^8 + \mathcal{O}(v^{10})\right\}$$

Currently: energy up to 4 PN, flux up to 4.5 PN

$$\mathcal{F}_{4.5}(v) = \frac{32}{5}\eta^2 v^{10} \left\{1 - \left(\frac{1247}{336} + \frac{35}{12}\eta\right) v^2 + 4\pi v^3 - \left(\frac{44711}{9072} - \frac{9271}{504}\eta - \frac{65}{18}\eta^2\right) v^4 - \left(\frac{8191}{672} + \frac{583}{24}\eta\right)\pi v^5 + \left[\frac{6643739519}{69854400} + \frac{16}{3}\pi^2 - \frac{1712}{105}\gamma_{\mathrm{E}} + \left(\frac{41}{48}\pi^2 - \frac{134543}{7776}\right)\eta - \frac{94403}{3024}\eta^2 - \frac{775}{324}\eta^3 - \frac{856}{105}\ln\left(16v^2\right)\right] v^6 - \left(\frac{16285}{504} - \frac{214745}{1728}\eta - \frac{193385}{3024}\eta^2\right)\pi v^7 + \left[-\frac{323105549467}{3178375200} + \frac{232597}{4410}\gamma_{\mathrm{E}} - \frac{1369}{126}\pi^2 + \frac{39931}{294}\ln 2 - \frac{47385}{1568}\ln 3 + \frac{232597}{8820}\ln\left(v^2\right) + \left(-\frac{1452202403629}{1466942400} + \frac{41478}{245}\gamma_{\mathrm{E}} - \frac{267127}{4608}\pi^2 + \frac{479062}{2205}\ln 2 + \frac{47385}{392}\ln 3 + \frac{20739}{245}\ln\left(v^2\right)\right)\eta + \left(\frac{1607125}{6804} - \frac{3157}{384}\pi^2\right)\eta^2 + \frac{6875}{504}\eta^3 + \frac{5}{6}\eta^4\right] v^8 + \left[\frac{265978667519}{745113600} - \frac{6848}{105}\gamma_{\mathrm{E}} - \frac{3424}{105}\ln\left(16v^2\right) + \left(\frac{2062241}{22176} + \frac{41}{12}\pi^2\right)\eta - \frac{133112905}{290304}\eta^2 - \frac{3719141}{38016}\eta^3\right]\pi v^9 + \mathcal{O}\left(v^{10}\right)\right\},$$

Characteristic velocity of the binary

$$v = (\pi f_{\mathrm{gw}} M_{\mathrm{tot}})^{1/3}$$

Symmetric mass ratio

$$\eta = \frac{m_1 m_2}{(m_1+m_2)^2}$$

Post Newtonian expansion - GW phase

In frequency domain: $\tilde{h}(f) = A(f)e^{i\Psi(f)}$

GW phase: $$\Psi(f) = 2\pi f t_c - \phi_c - \frac{\pi}{4} + \frac{3}{128\eta} v^{-5} \left[\sum_{k=0}^{7} \phi_k(\eta)\, v^k\right]$$

$$\begin{cases} v = (\pi f_{\rm gw} M_{\rm tot})^{1/3} \\ \eta = \frac{m_1 m_2}{(m_1+m_2)^2} \end{cases}$$

- 0PN: $\phi_0 = 1$ $\qquad \Psi(f)_{\rm 0PN} = \frac{3}{128\eta} v^{-5} \propto (\mathcal{M}_c \pi f)^{5/3}$

  $\Longrightarrow$ phase evolution dominated by chirp mass

Post Newtonian expansion - GW phase

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- 0PN: $\phi_0 = 1$ $\quad \Psi(f)_{0\mathrm{PN}} = \frac{3}{128\eta} v^{-5} \propto (\mathcal{M}_c \pi f)^{5/3}$

$\Longrightarrow$ phase evolution dominated by chirp mass

$$\psi = \psi_{\mathrm{PP}} + \psi_{\mathrm{SO}} + \psi_{\mathrm{SS}} + \psi_{\mathrm{T}}$$

point-particle $\Psi(f)$

spin-orbit

spin-spin

tidal effects

Post Newtonian expansion - spins

- lowest spin correction at 1.5 PN (k=3):

$$\propto \underbrace{\frac{m_1\chi_1+m_2\chi_2}{M_{\rm tot}}}_{\chi_{\rm eff}} - \frac{76\,\eta\,(\chi_1+\chi_2)}{226}$$

χ_i = dimensionless spin aligned with orbital angular momentum

changing mass ratio can mimic spin effects -> degeneracy

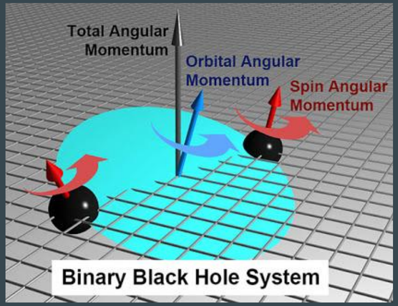
Post Newtonian expansion - spins

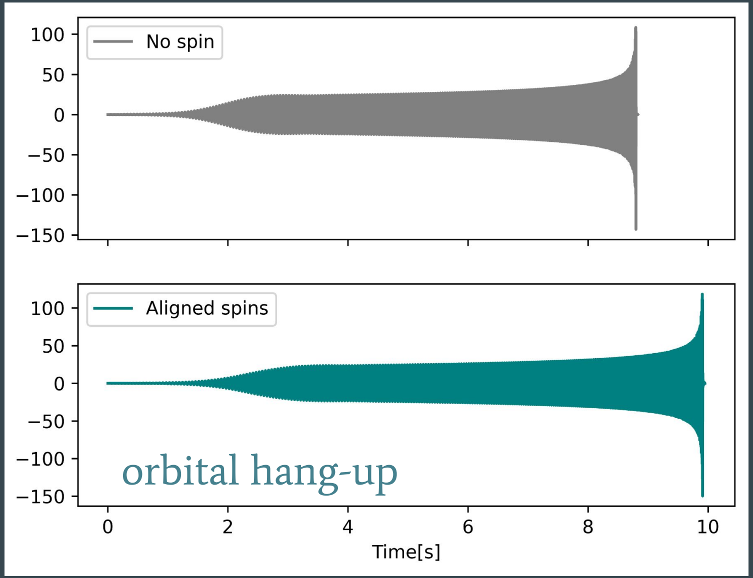
➢ lowest spin correction at 1.5 PN (k=3):

$$\propto \underbrace{\frac{m_1\chi_1 + m_2\chi_2}{M_{\text{tot}}}}_{\chi_{\text{eff}}} - \frac{76\,\eta\,(\chi_1+\chi_2)}{226}$$

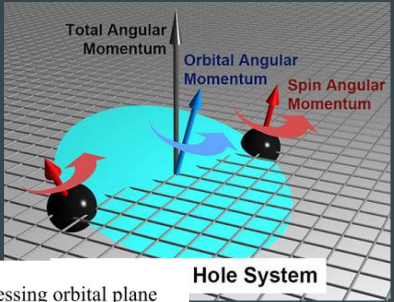
changing mass ratio can mimic spin effects -> degeneracy

χ_i = dimensionless spin aligned with orbital angular momentum





Post Newtonian expansion - spins



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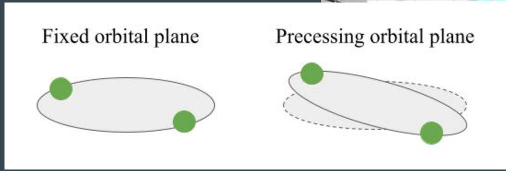
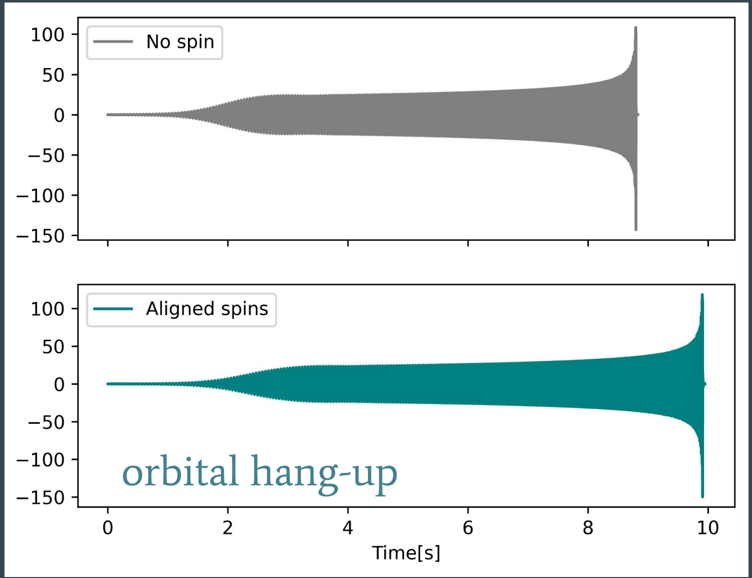
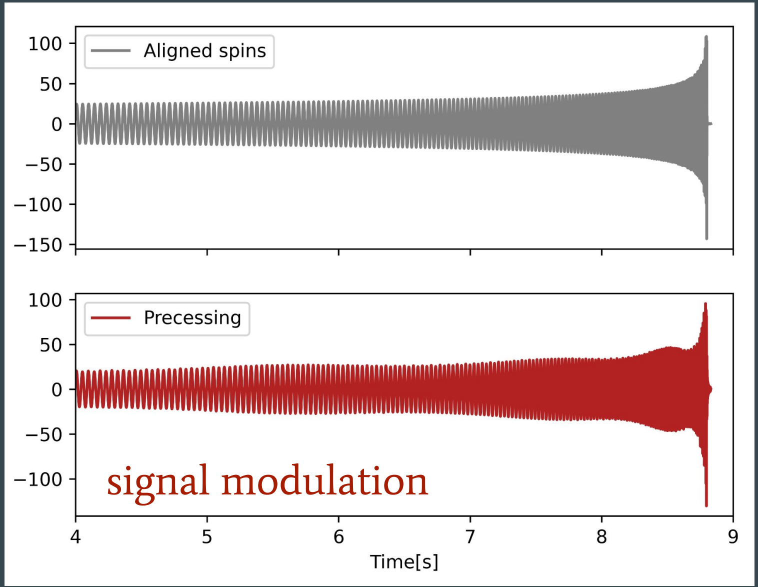
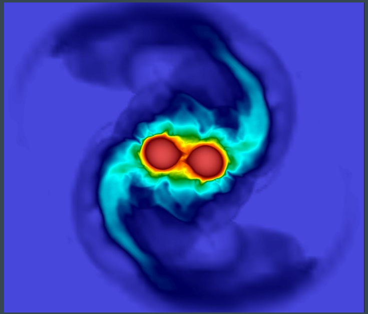
changing mass ratio can mimic spin effects -> degeneracy







Post Newtonian expansion - tidal corrections



University of Birmingham

$$Q_{ij} = -\Lambda m^5 \mathcal{E}_{ij}$$

Q_{ij} → quadrupole moment

$\mathcal{E}_{ij}$ → tidal field

Λ → tidal deformability parameter:

$$\Lambda = \frac{2}{3} k_2 \left(\frac{R}{m}\right)^5$$

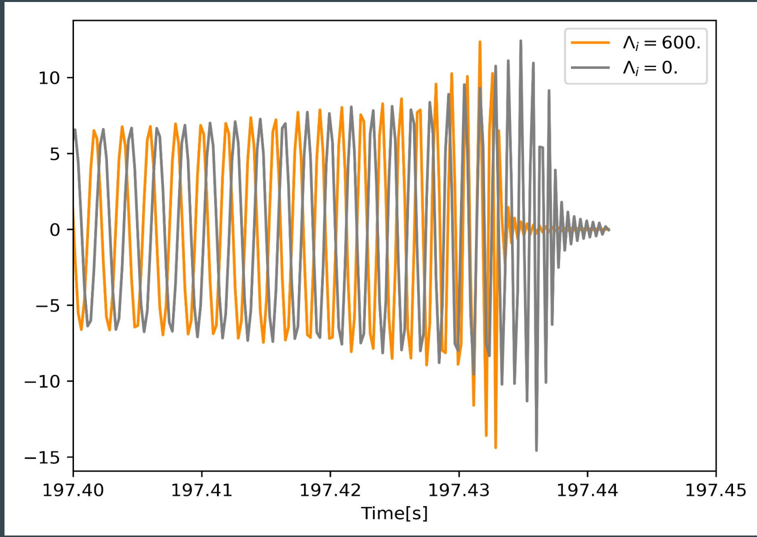
Tidal deformability
-> orbital energy loss
-> faster inspiral

$$\psi_(f) = \ldots - \frac{39}{2} \tilde{\Lambda} v^{10}$$

From 5PN order (~$\mathcal{O}(100)$ Hz)

$$\tilde{\Lambda} = \frac{16}{13}\left[\left(1 + \frac{12 m_2}{m_1}\right)\frac{\lambda_1^5}{M^5} + \left(1 + \frac{12 m_1}{m_2}\right)\frac{\lambda_2^5}{M^5}\right]$$

mass-weighted tidal deformability



6PN order $\propto \Delta\tilde{\Lambda}$

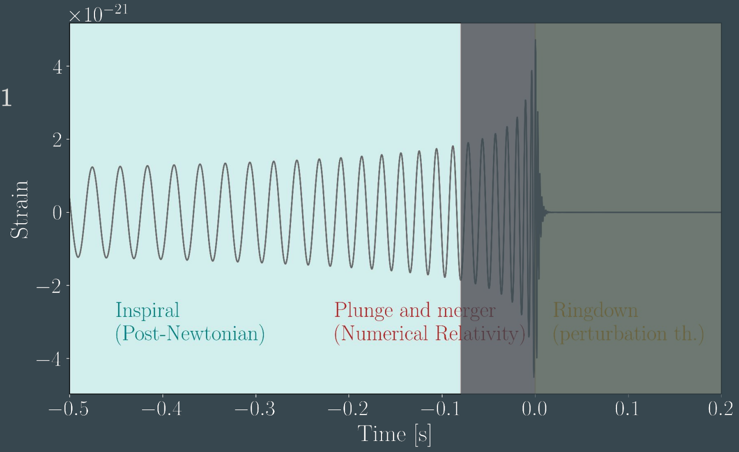
Waveform models - Post Newtonian

We measure chirp mass and mass-weighted tidal deformability, *not* radius!

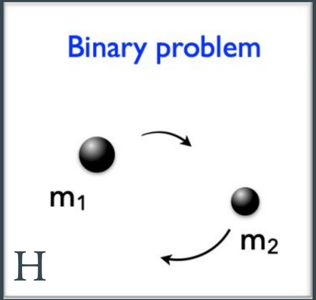
- PN approximation: expansion in (v/c)
- It explains why observe certain effects
- We can derive models directly from it (*TaylorF2*), analytical and fast

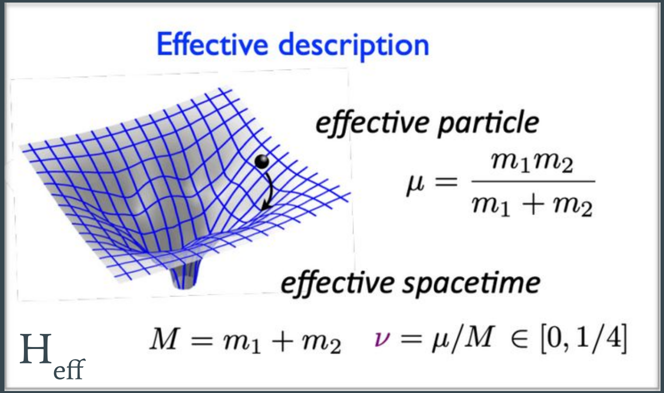
but

- PN expansion valid until ISCO $f_{\rm ISCO} = \left(6^{3/2}\pi M_{tot}\right)^{-1}$
- Taylor models for inspiral only
- Not extremely accurate (velocity increases)



Waveform models - Effective one body





Define a map between the Hamiltonian of the binary problem and the one of effective description:

$H_{eff} \Longrightarrow$ Concise resummation of PN results

$\Downarrow$

Add terms corresponding to not-yet-computed PN orders and calibrate to NR simulations

Equations of motion

$H \Longrightarrow$

$$\frac{dx^i}{dt} = \frac{\partial H}{\partial p_i}, \qquad \frac{dp_i}{dt} = -\frac{\partial H}{\partial x^i} + \mathcal{F}_i$$

$\Longrightarrow$ Gravitational wave signal

Waveform models - Effective one body

- Inspiral-merger-ringdown (IMR) model:

  EOB dynamics until lighting + quasinormal ringdown modes

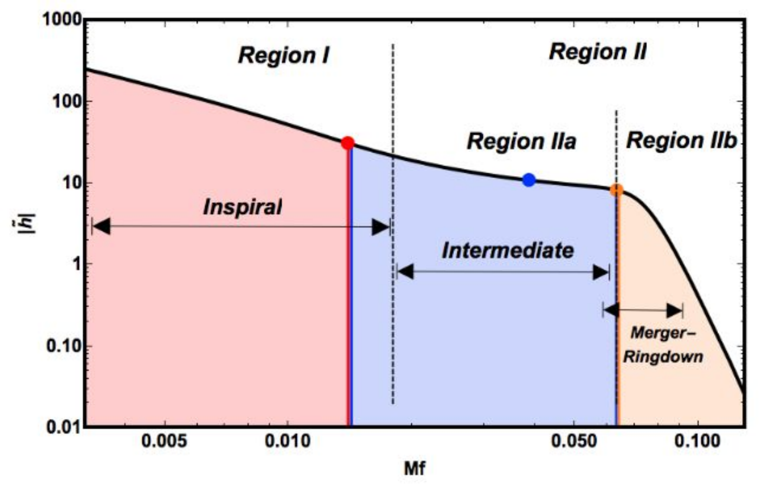
- Extend region of validity by calibrating to NR waveforms
- Examples: *SEOBNRv4PHM, SEOBNRv5, TEOBResumS*

✓ Very accurate
X Slow

Waveform models - Phenom

- Phenomenological ansatz with coefficients calibrated to NR waveforms
- Three different regions
- $\tilde{h}(f) = A(f)e^{i\Phi(f)}$

  Different phase and amplitude ansatz for each region



[Phys. Rev. D 93, 044007]

Example: phase *IMRPhenomD*

$$\Phi_{\rm Ins} = \Phi^{F2}_{\rm spin}(f;\Xi) + \frac{1}{\eta}\left(\sigma_0 + \sigma_1 f + \frac{3}{4}\sigma_2 f^{4/3} + \frac{3}{5}\sigma_3 f^{5/3} + \frac{1}{2}\sigma_4 f^2\right)$$

$$\Phi_{\rm Int} = \frac{1}{\eta}\left(\beta_0 + \beta_1 f + \beta_2 \ln(f) - \frac{\beta_3}{3} f^{-3}\right)$$

$$\Phi_{\rm MR} = \frac{1}{\eta}\left[\alpha_0 + \alpha_1 f - \alpha_2 f^{-1} + \frac{4}{3}\alpha_3 f^{3/4} + \alpha_4 \tan^{-1}\left(\frac{f-\alpha_5 f_{\rm RD}}{f_{\rm damp}}\right)\right]$$

Full IMR waveform: require continuity

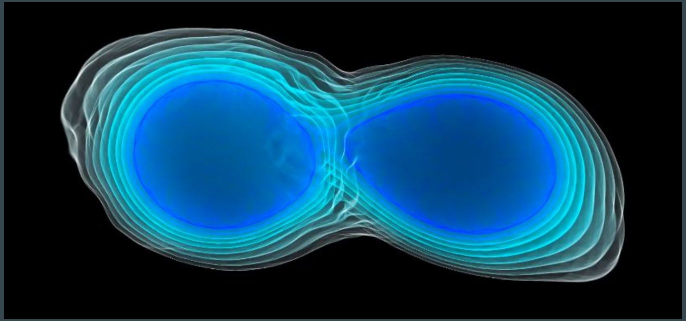
$\{\sigma_i, \beta_i, \alpha_i\}$ calibrated to hybrid waveforms

✓ Fast

Waveform models - effects of matter

$\psi = \psi_{\rm PP} + \psi_{\rm SO} + \psi_{\rm SS} + \psi_{\rm T}$ → Tidal deformability

NRTidal: closed-form expression to describe tidal effects, calibrated to NR

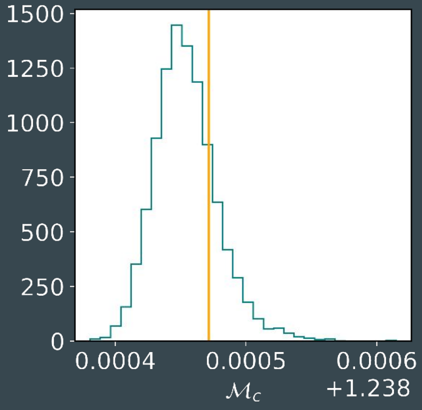


Most recent: NRTidalv3 [arXiv:2311.07456]

- different EoS
- also not-equal mass

Add phase corrections to existing BBH models
IMRPhenomD_NRTidalv3, IMRPhenomXAS_NRTidalv3, IMRPhenomXP_NRTidalv3, SEOBNRv5_ROM_NRTidalv3

Parameter estimation - Bayesian inference



Bayes theorem:

$$p(\vec{\theta}|\mathcal{H}_s, d) = \frac{p(d|\vec{\theta},\mathcal{H}_s)\,p(\vec{\theta}|\mathcal{H}_s)}{p(d|\mathcal{H}_s)}$$

likelihoood

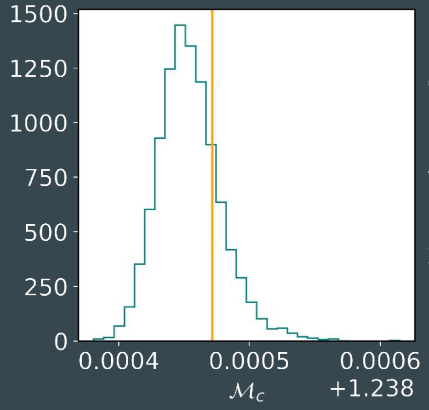
prior

Posterior probability density

evidence

Likelihood and evidence are multidimensional integrals on the parameters space:
15 parameters for BBHs,
17 for BNSs

Parameter estimation - Bayesian inference



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likelihoood

prior

Posterior probability density

evidence

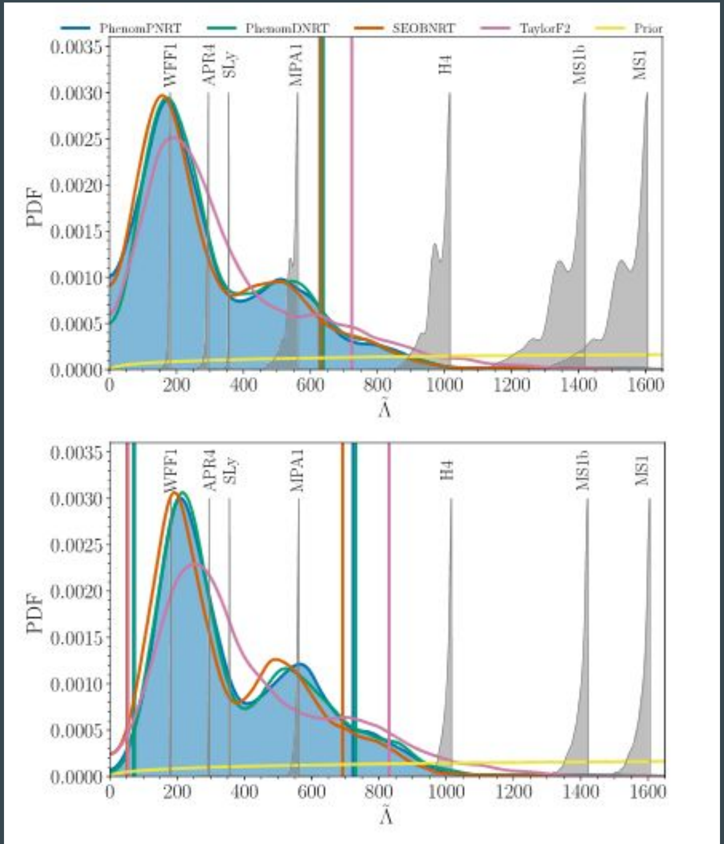
Likelihood and evidence are multidimensional integrals on the parameters space:
15 parameters for BBHs,
17 for BNSs

Stochastic sampling

- ➔ nested sampling (evidence)
- ➔ Markov Chain Monte Carlo (MCMC)

(see backup slides for details)

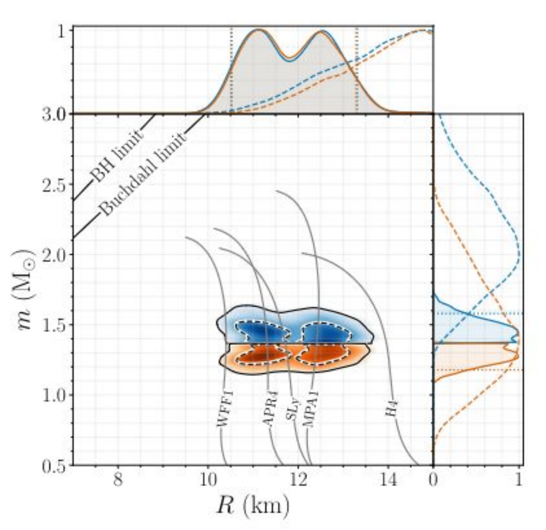
Neutron stars EOS with current detectors



Phys.Rev.X 9 (2019) 1, 011001

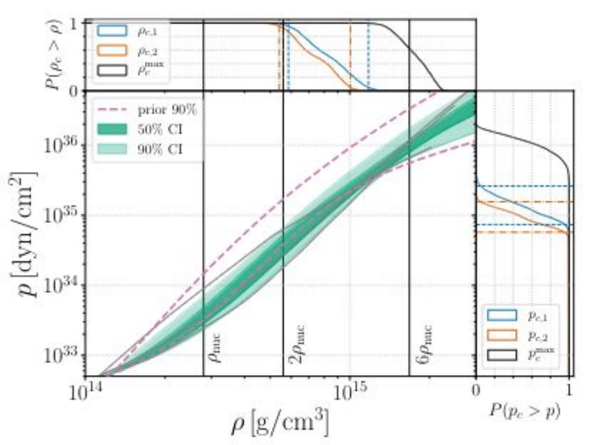
Two BNSs events:

- GW170817
  (close, loud, multimessenger)
- GW190425 (far, no $\tilde{\Lambda}$ information)



Phys.Rev.Lett. 121 (2018) 16, 161101

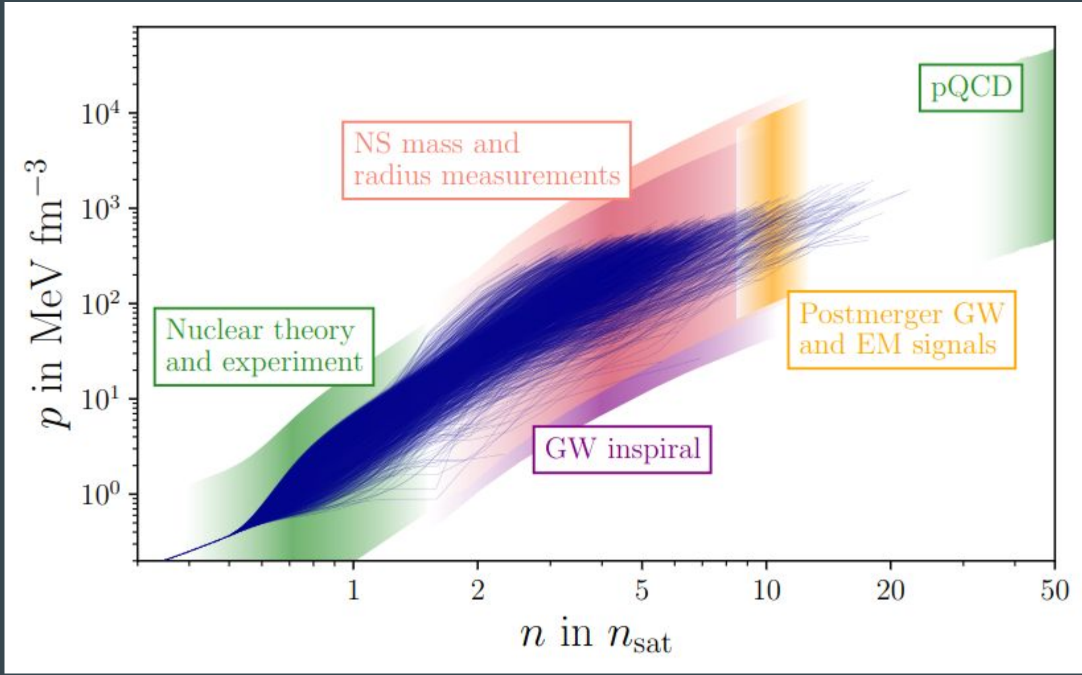
Neutron stars EOS with current detectors



Phys.Rev.Lett. 121 (2018) 16, 161101

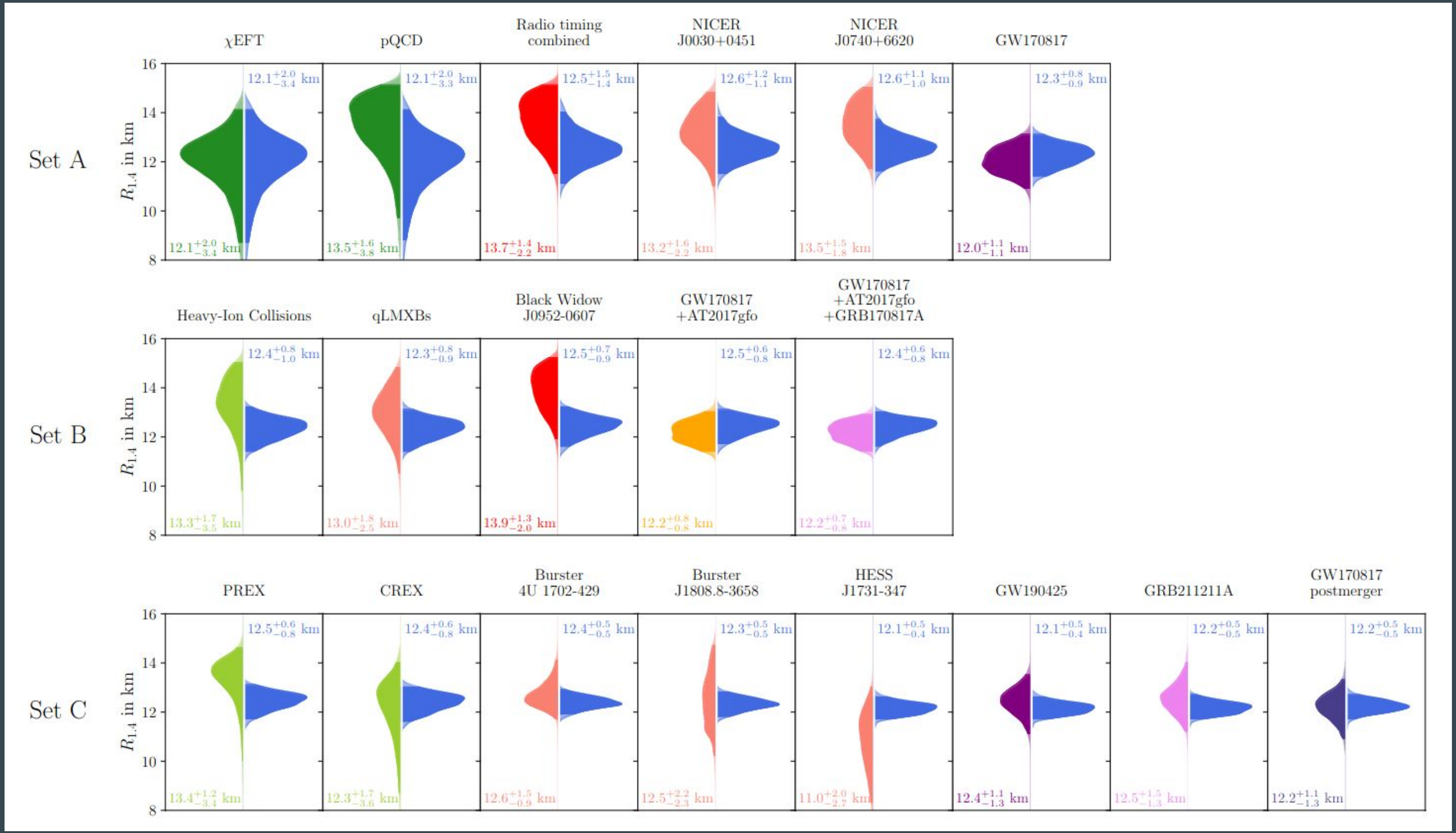
Combine with other sources of information!

Overview: https://arxiv.org/abs/2402.04172

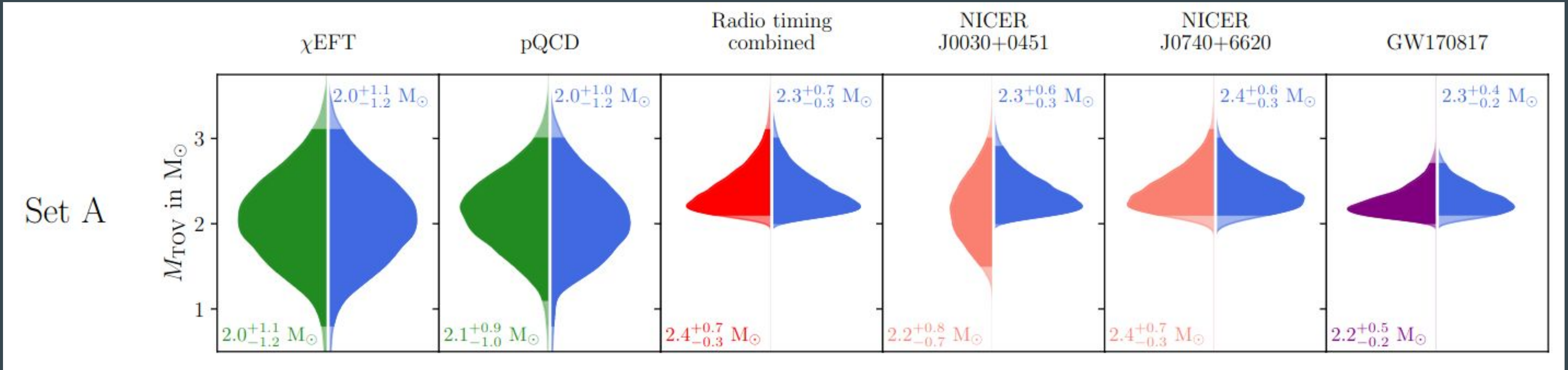


Sample on the EoS -> compute $\tilde{\Lambda}$ from mass and EoS -> use it in the model

Combining information



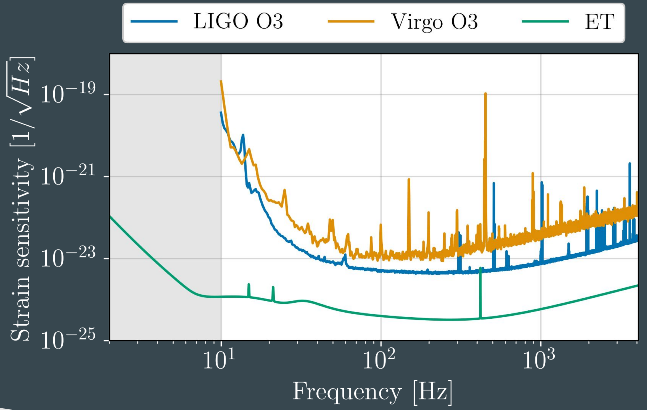
Combining information



Web interface: https://multi-messenger.physik.uni-potsdam.de/eos_constraints/

Future detectors

Extended frequency band

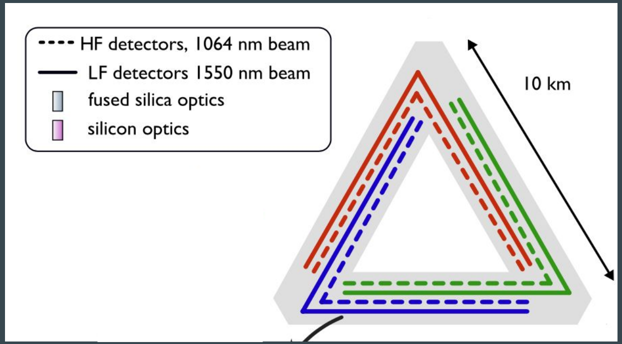


Problem: quantum noise

Radiation pressure noise at low frequencies → Decrease laser power

Shot noise at high frequencies → Increase laser power

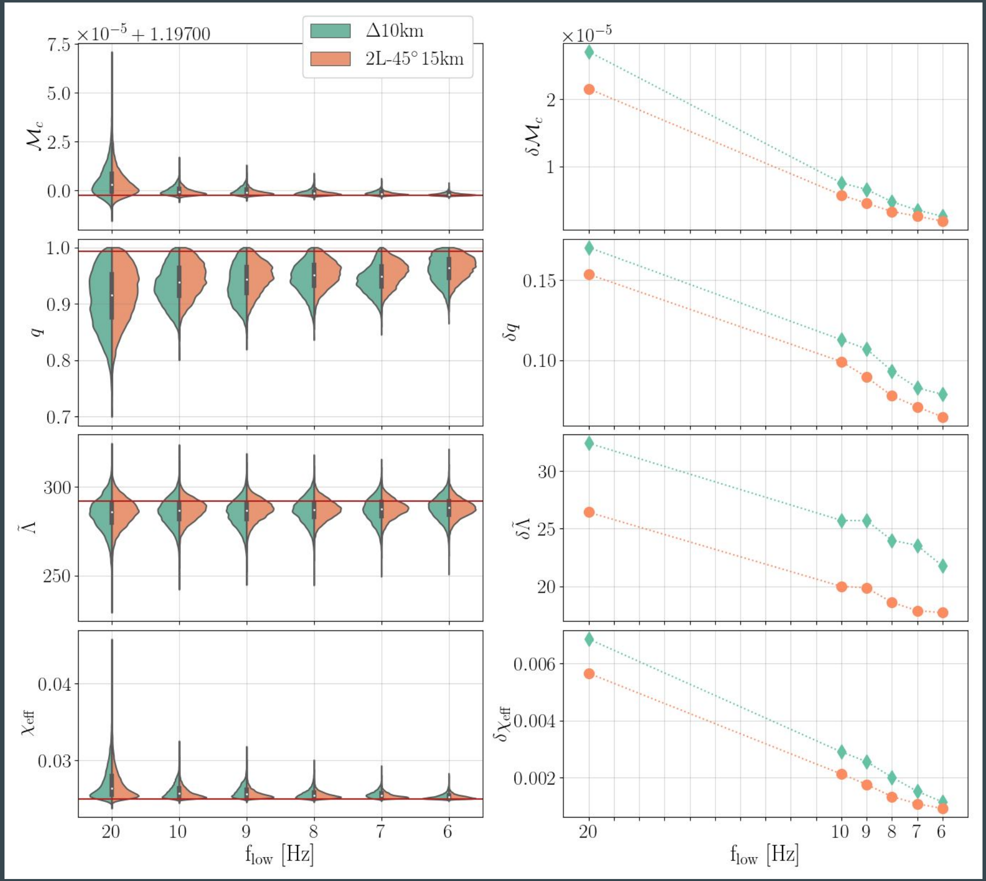
Einstein Telescope:
xylophone configuration



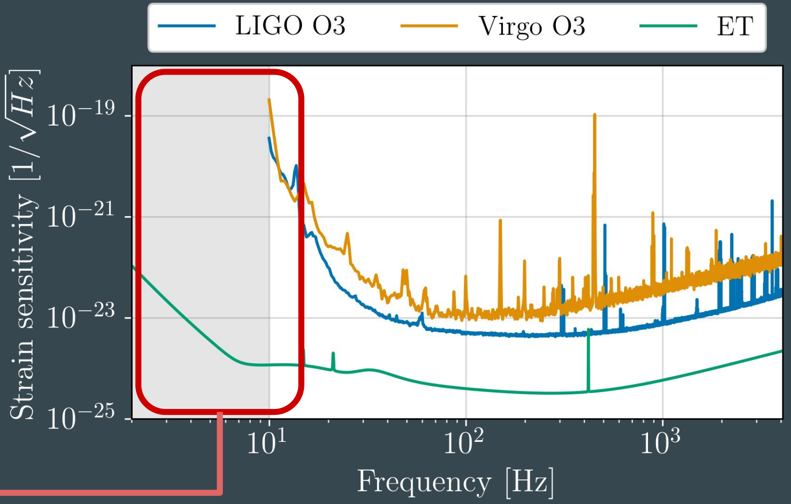
Plus (to improve low freqs):

- cryogenic
- underground

Future detectors

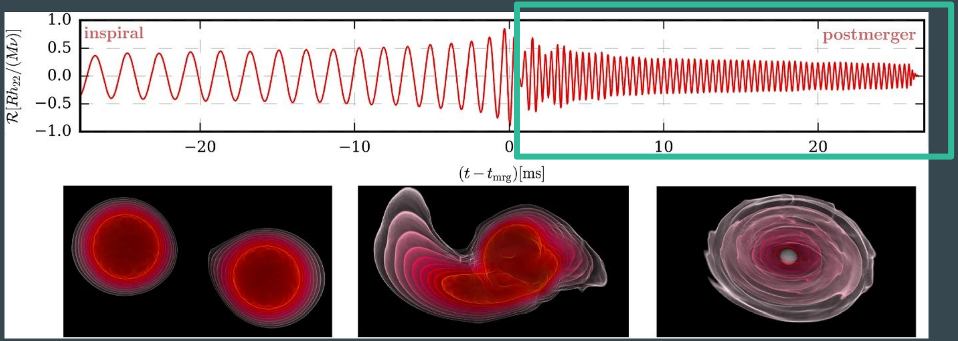


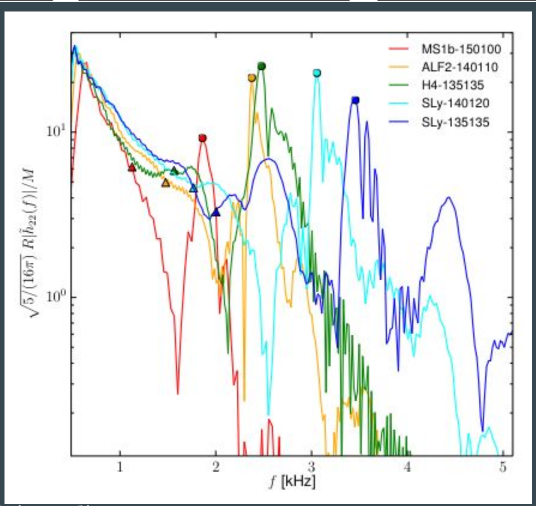
Extended frequency band



Phys.Rev.D 108 (2023) 2, 023018

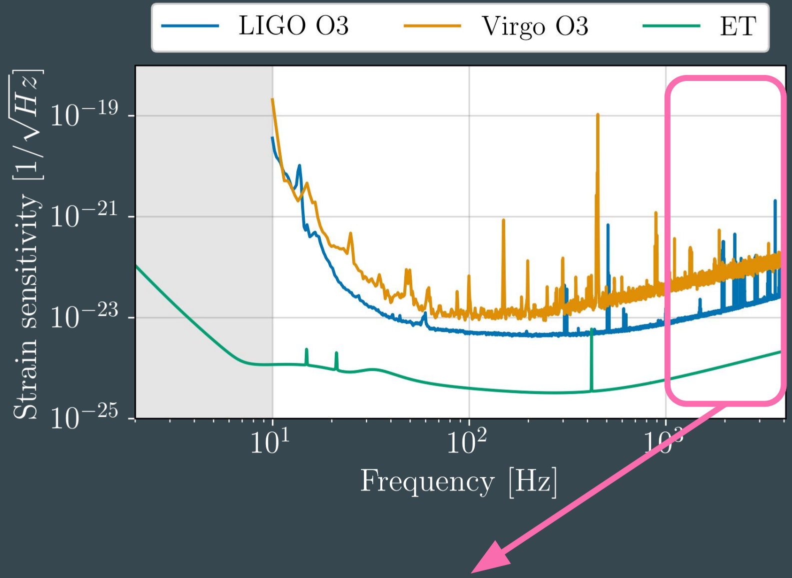
Future detectors





Phys.Rev.Lett. 115 (2015) 9, 091101

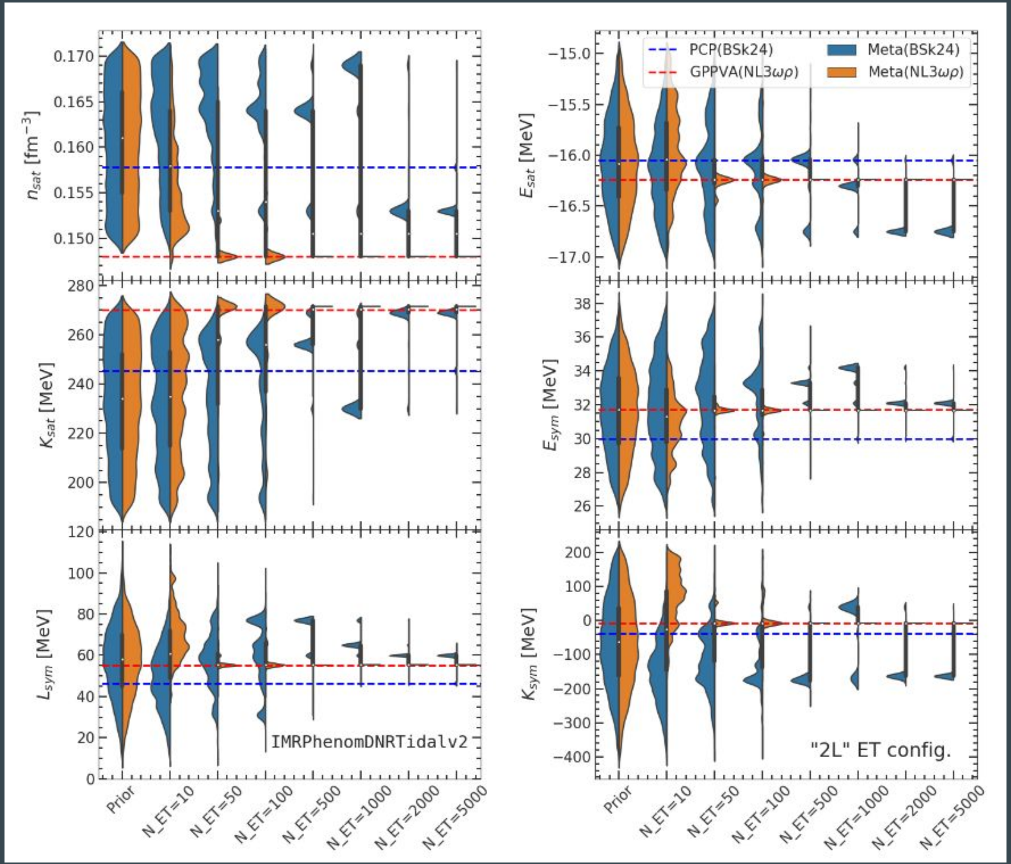
Extended frequency band



Postmerger

EoS in different temperature and density regime

Future detectors - nuclear parameters



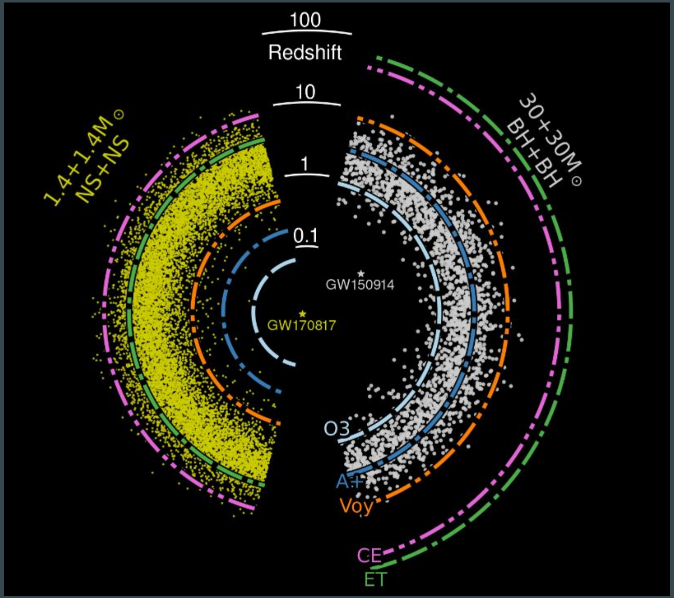
Posteriors for masses and lambdas + nuclear meta-model (expansion of energy per particle in density)

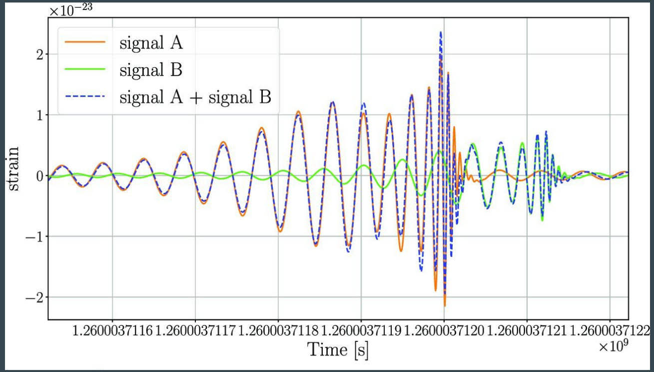
Phys.Rev.D 108 (2023) 12, 122006

Future detectors - challenges

More events, louder, more time in band

- ➢ Overlapping signals
- ➢ *Systematics* (ex: waveform models)
- ➢ Computational issues!



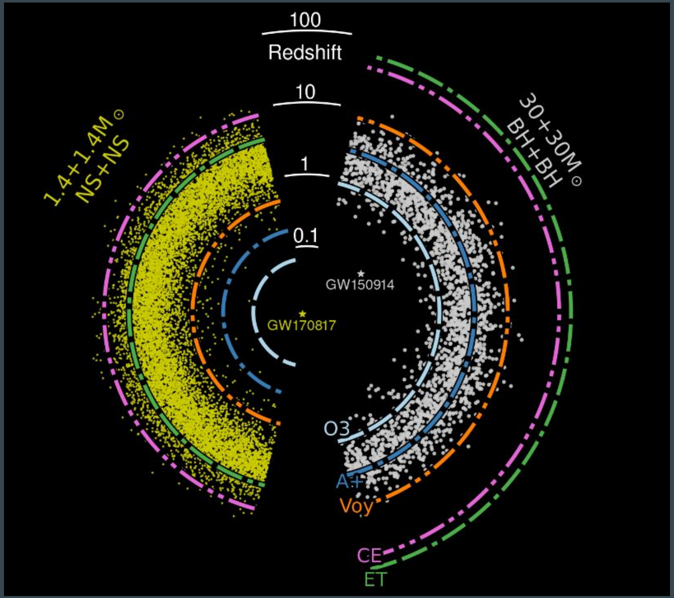


Phys.Rev.D 104 (2021) 4, 044003

Future detectors - challenges

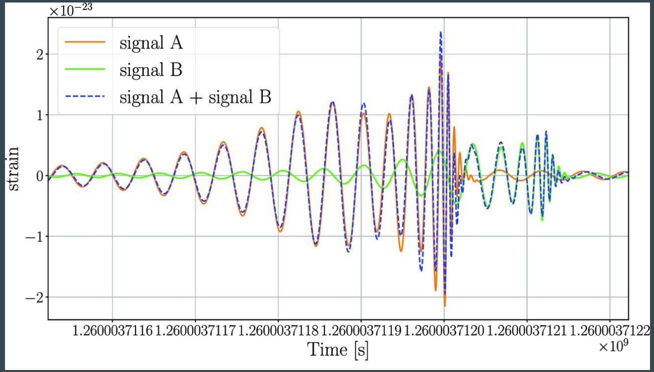
More events, louder, more time in band

- ➢ Overlapping signals
- ➢ *Systematics* (ex: waveform models)
- ➢ Computational issues!



BNS full parameter estimation with LIGO-Virgo takes weeks, with ET/CE months

- Faster waveform/samplers
- Machine learning to evaluate the posteriors
- Approximate likelihood (Reduced Order Quadratures, Relative Binning, Multibanding)



Phys.Rev.D 104 (2021) 4, 044003

Take-away message

- From GWs we can measure chirp mass and mass-weighted tidal deformability
- We can use this information (possibly combined with other data) to get information about the EoS
- To get chirp mass and $\tilde{\Lambda}$ we use Bayesian inference:
  - Computationally very expensive -> nested or mcmc sampling
  - We need waveform models -> accurate but fast ($\mathcal{O}(10^6)$ evaluations per analysis)
- Next generation detectors:
  - More precise measurements
  - Postmerger (different temperature and density, but very complicated)
  - Various challenges

Software

Parameter estimation : `bilby` (python)
documentation: https://lscsoft.docs.ligo.org/bilby/
[google ‘bilby software’]

- nested: `dynesty` [documentation: https://dynesty.readthedocs.io/en/latest/]
- mcmc: `bilby_mcmc`

`Bilby_pipe`, `parallel_bilby`: automation and parallelization

Waveforms: LALsuite library (C), documentation

Useful tools: pyCBC (python), documentation: https://pycbc.org/

Tutorials

Python notebook:

- *"toy_model_sampling.ipynb"* : how PE works, example on a toy model (no GW)
- *"waveforms_and_mismatch.ipynb"*: how to generate different GW waveforms for different parameters, mismatch computation between different waveforms
- *"gw_pe.ipynb"*: example of PE on GW signals (simplified sampler settings)

Nested sampling

$$\mathcal{P}(\vec{\theta}) = \frac{L(\vec{\theta})\pi(\vec{\theta})}{Z(\vec{\theta})} \implies$$ Estimate evidence, posterior as by-product

$$Z = \int L(\vec{\theta})\pi(\vec{\theta})d\vec{\theta}$$

Prior mass X(λ) := integral over the prior volume of all parameters with likelihood greater than a given value λ

$$X(\lambda) = \int_{\vec{\theta}:L(\vec{\theta})>\lambda} \pi(\vec{\theta})d\vec{\theta}$$

$$Z = \int_0^1 L(X)dX$$

discrete samples

$$Z \sim \sum_k g(L_k)\Delta X_k$$

Evidence → Model comparison

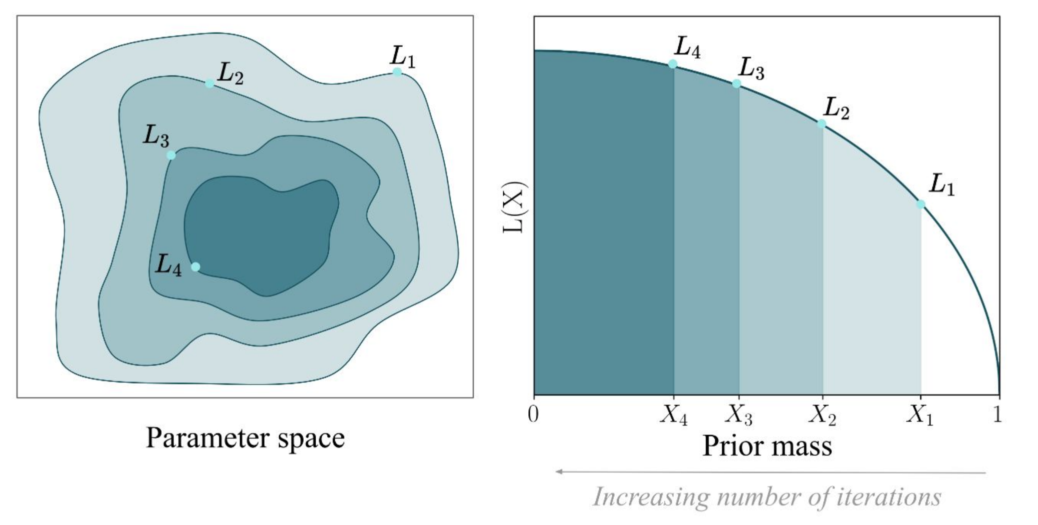
$$\mathcal{B}_A^B = \frac{p(d|H_B)}{p(d|H_A)} = \frac{Z_B}{Z_A}$$

Nested sampling

1. Sample N$_{\text{live}}$ points $\left\{\vec{\theta}_1, \vec{\theta}_2, \ldots, \vec{\theta}_{N_{live}}\right\}$ from the prior and compute their likelihood
2. For the point with the lowest likelihood L_k, estimate the corresponding prior mass X_k; save $(\vec{\theta}, L_k, X_k)$, and remove $\vec{\theta}_k$ from the set of live points
3. Increase evidence by $Z_k = f(L_k, X_k)$
4. Randomly draw $\vec{\theta}_j$ from the prior, with $L(\vec{\theta}_j) > L_k$
5. Repeat until termination condition

We obtain a series of $(\vec{\theta}, L_k, X_k)$ with increasing L_k

$$\Longrightarrow \qquad \mathcal{P}(\vec{\theta}_k) \simeq \frac{Z_k}{Z}$$



MCMC

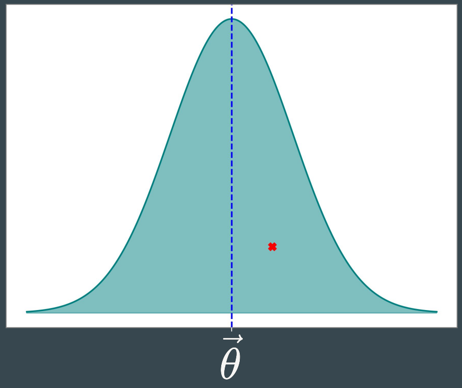
Sample from the posterior distribution

1. Sample $\vec{\theta}$ from the prior
2. Generate next point $\vec{\theta}'$ from a *proposal distribution* $Q(\vec{\theta}'|\vec{\theta})$
3. Accept the new point with probability

$$\alpha = \min\left\{1, \frac{L(\vec{\theta}')\pi(\vec{\theta}')Q(\vec{\theta}'|\vec{\theta})}{L(\vec{\theta})\pi(\vec{\theta})Q(\vec{\theta}|\vec{\theta}')}\right\}$$

accepted -> append $\vec{\theta}'$ to list of samples
rejected -> append $\vec{\theta}$ to list of samples

4. Repeat from the appended sample



Proposal distributions for GWs can be very complicated!