Scaling the equations: dealing with astronomical numbers

$$M_\odot/m_n \approx 10^{57}$$

$$\frac{dP}{dr} = -\frac{GM(r)\rho(r)}{r^2} = -\frac{GM(r)\mathcal{E}(r)}{c^2r^2}$$

$$\frac{dp}{dx} = -\cancelto{1}{\left(\frac{2GM_0}{c^2R_0}\right)}\left[\frac{m(x)\varepsilon(x)}{2x^2}\right]$$

$$\frac{dM}{dr} = 4\pi r^2\rho(r) = 4\pi r^2\frac{\mathcal{E}(r)}{c^2}$$

$$\frac{dm}{dx} = \cancelto{1}{\left(\left[\frac{4\pi R_0^3}{3M_0}\right]\left[\frac{\mathcal{E}_0}{c^2}\right]\right)}3x^2\varepsilon(x)$$

Relativistic free Fermi Gas

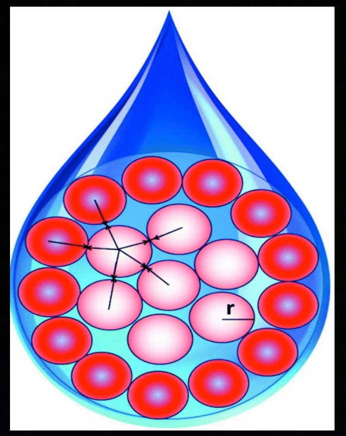
$$\mathcal{E} = \mathcal{E}_0\left[x_{\rm F}\,y_{\rm F}\left(x_{\rm F}^2+y_{\rm F}^2\right) - \ln(x_{\rm F}+y_{\rm F})\right]$$

$$\boxed{\begin{aligned} r = R_0\,x;\ \ M &= M_0\,m \\ P = \mathcal{E}_0\,p;\ \ \mathcal{E} &= \mathcal{E}_0\,\varepsilon \end{aligned}}$$

$$\boxed{\begin{aligned} \mathcal{E}_0 &= \frac{1}{8\pi^2}\frac{(mc^2)^4}{(\hbar c)^3} = 1.285\ \mathrm{GeV/fm^3} \\ M_0 &= 2.837\,M_\odot\ ;\ R_0 = 8.378\ \mathrm{km} \end{aligned}}$$

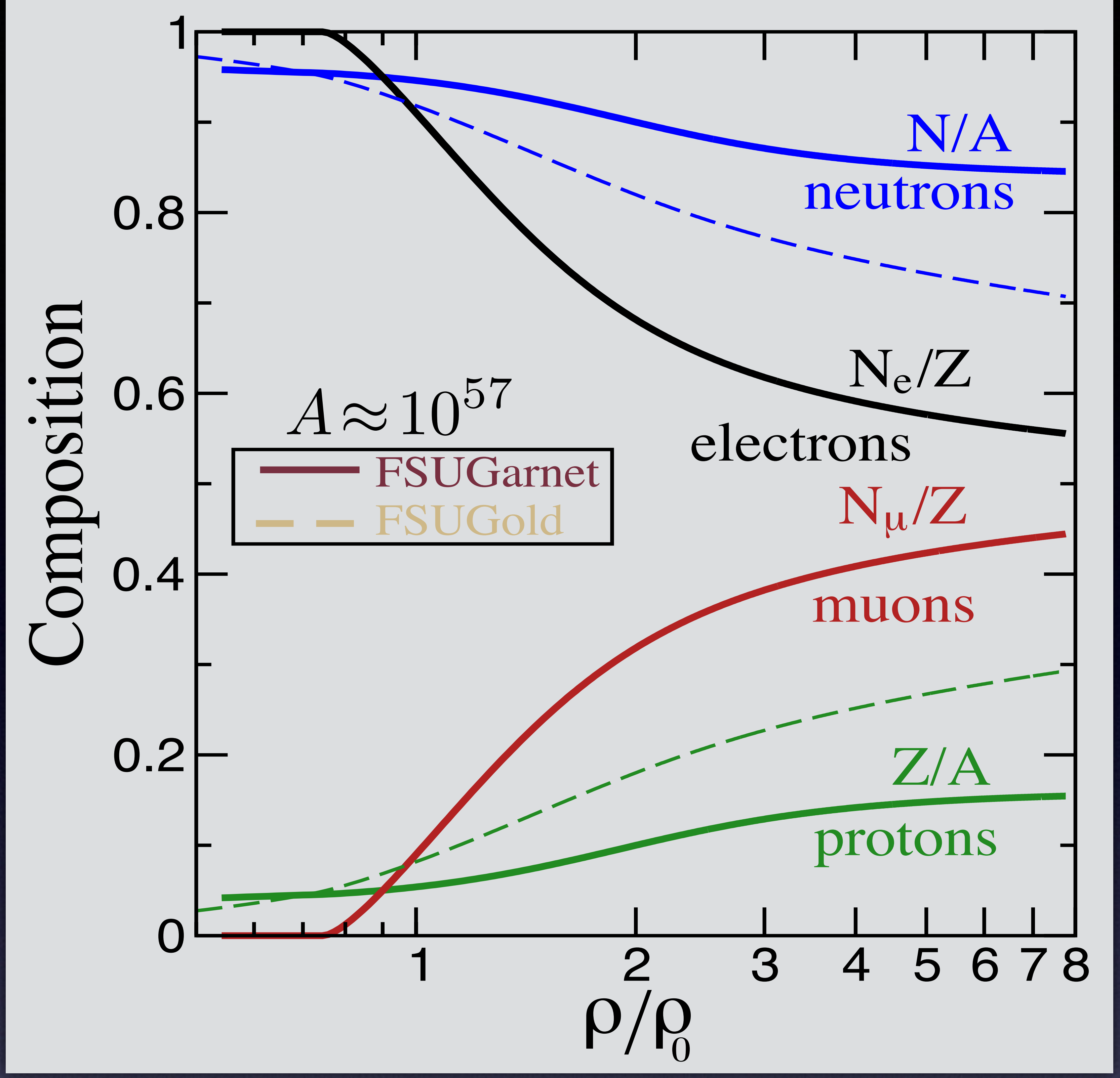
Nuclear Physics 101:The Liquid Drop Model

Bethe-Weizsäcker Mass Formula *(circa 1935-36)*



$R = r_0 A^{1/3}$

- Nuclear forces saturate ⟶ equilibrium density
- Nuclei penalized for developing a surface
- Nuclei penalized by Coulomb repulsion
- Nuclei penalized for isospin imbalance (N≠Z)

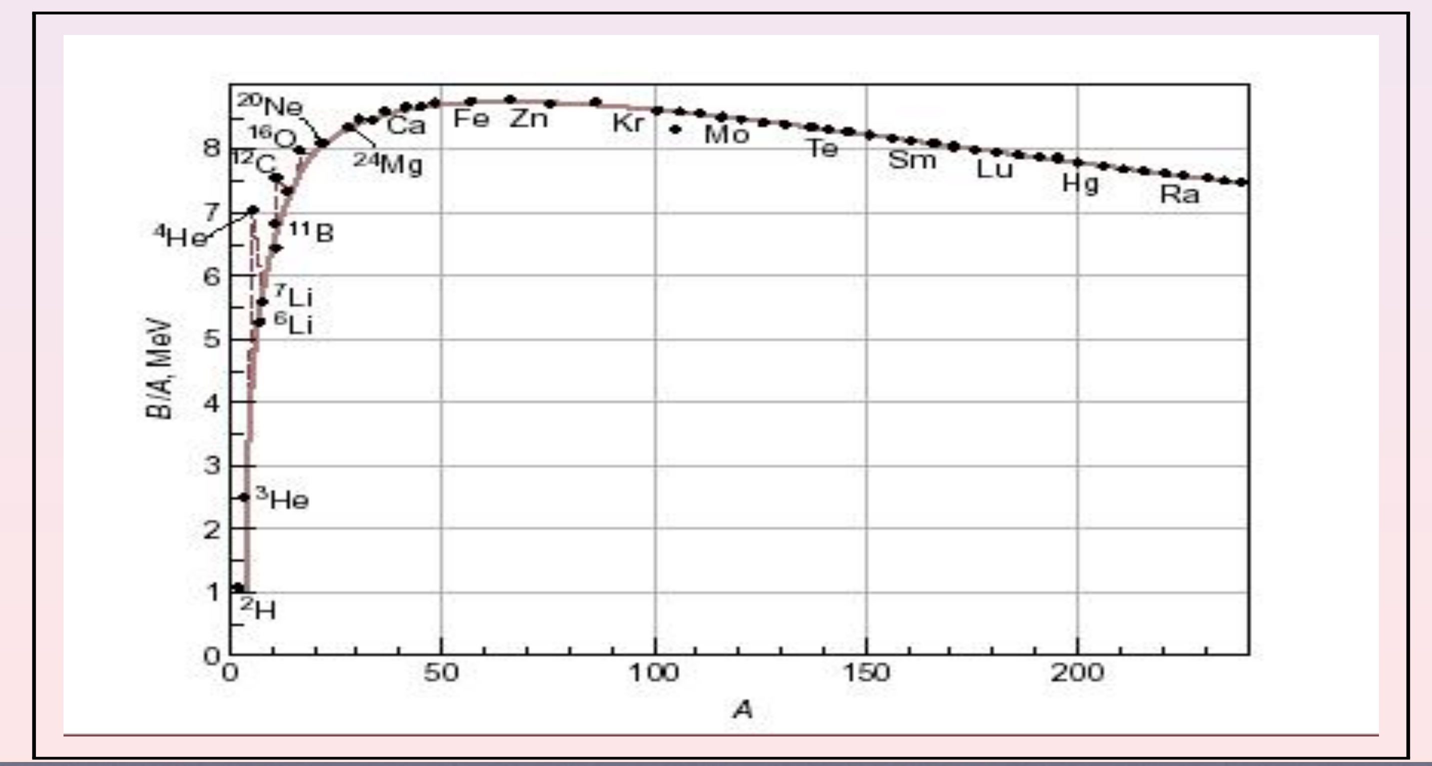


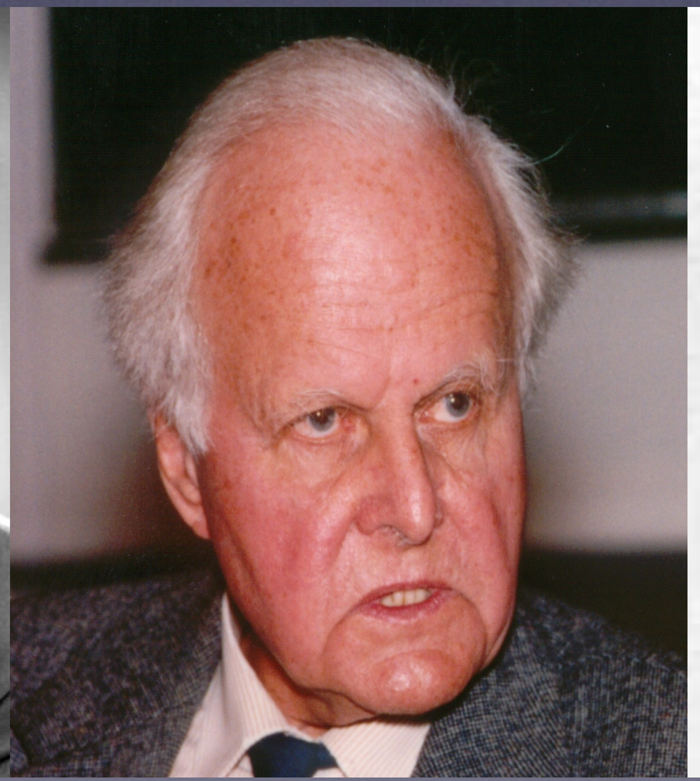
- $B(Z,N) = -a_v A + a_s A^{2/3} + a_c Z^2/A^{1/3} + a_a (N-Z)^2/A + \ldots$
  $+$ **shell corrections** $(2, 8, 20, 28, 50, 82, 126, \ldots)$

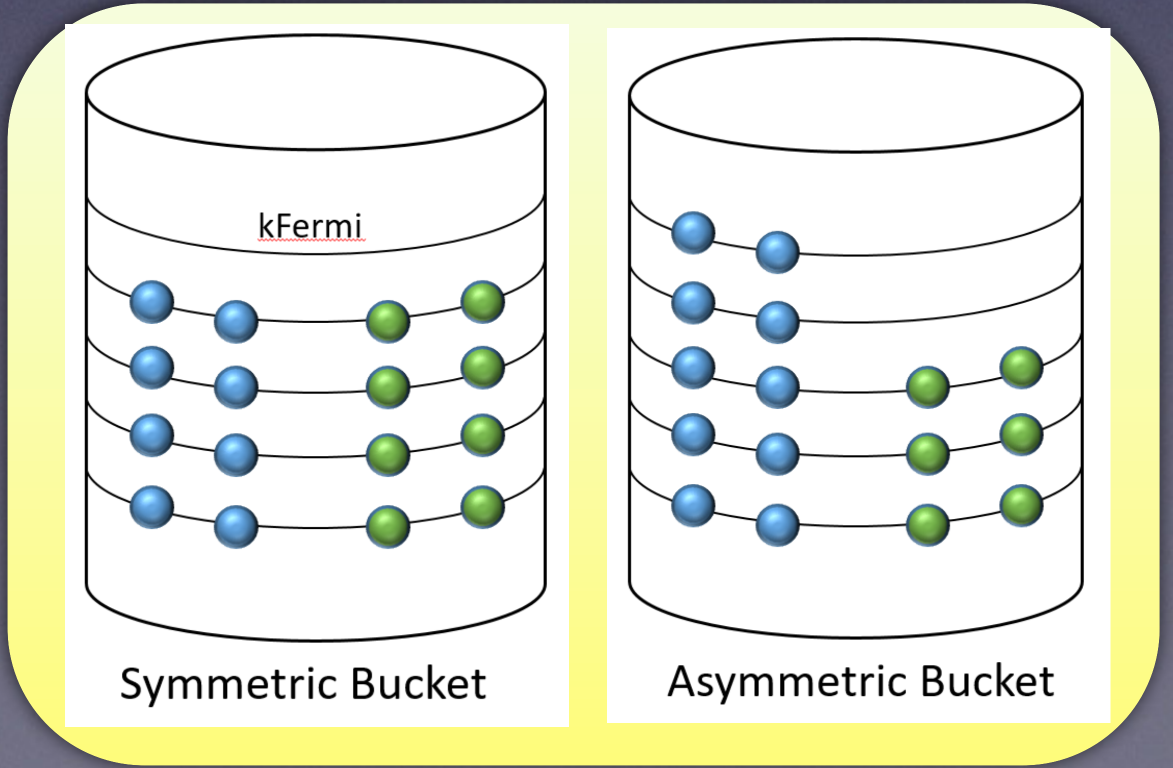
$$\boxed{a_v \simeq 16.0,\ a_s \simeq 17.2,\ a_c \simeq 0.7,\ a_a \simeq 23.3 \text{ (in MeV)}}$$

Neutron stars are gravitationally bound!

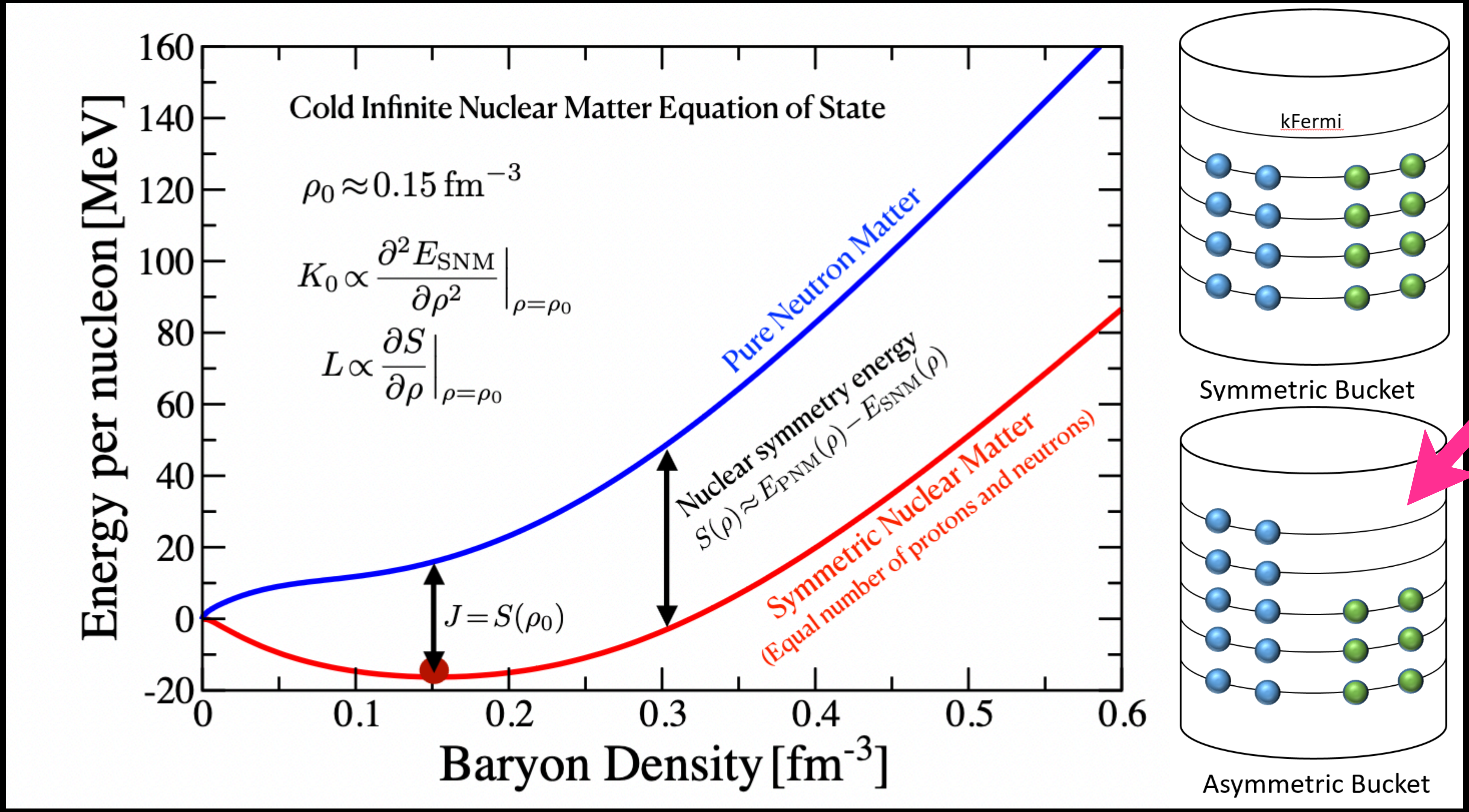
FSUGarnet ("Soft") ⟷ L small
FSUGold ("Stiff") ⟷ L large



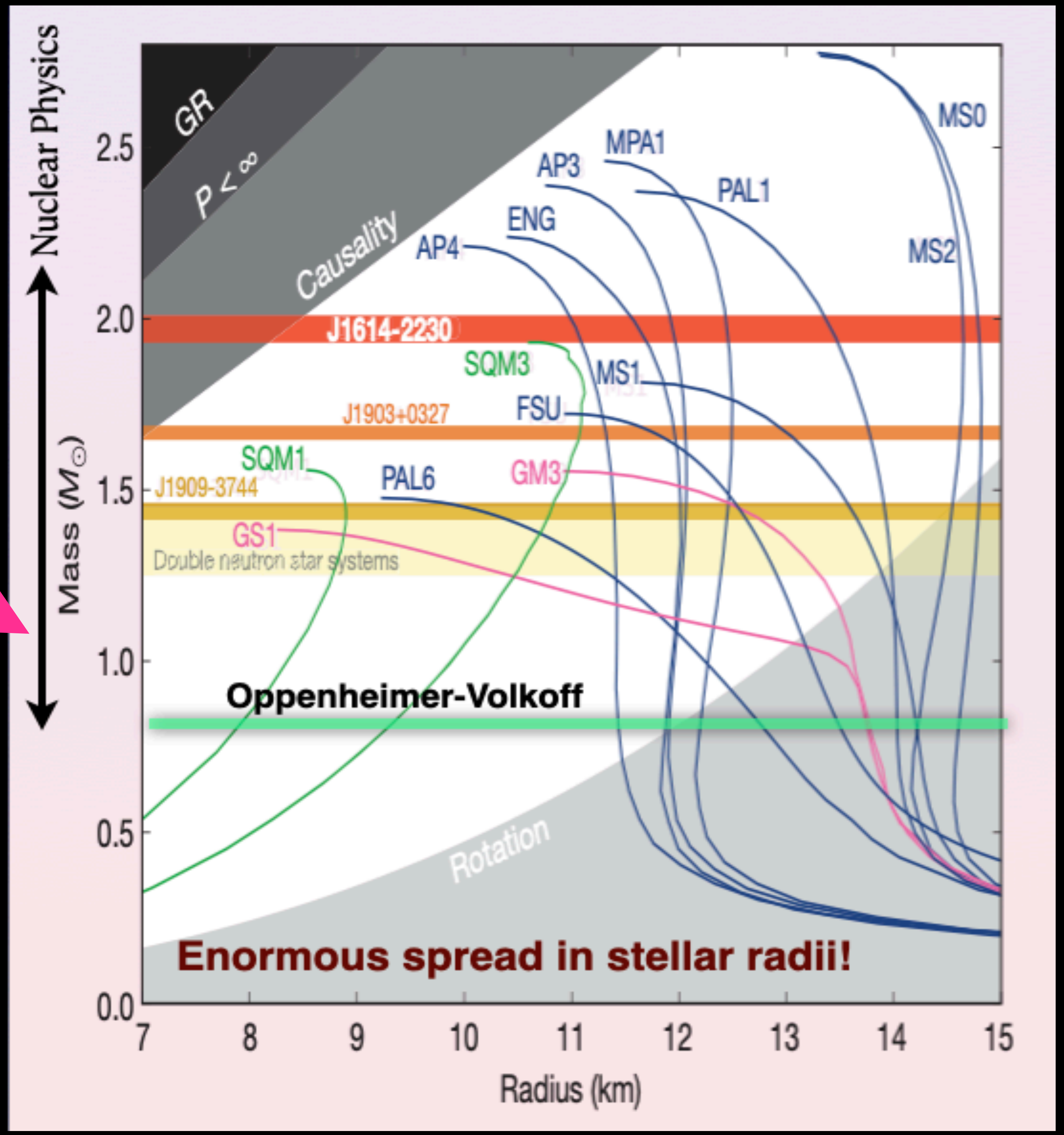




Neutron Stars and The Equation of State of Neutron-Rich Matter



Micro-Macro Connection



Only Physics that the TOV equation is sensitive to is the Equation of State

$$S(\rho_0) \approx \Big(E_{\mathrm{PNM}} - E_{\mathrm{SNM}}\Big)(\rho_0) = J$$

$$P_{\mathrm{PNM}} \approx \frac{1}{3} L \rho_0 \ \text{(Pressure of PNM)}$$

"Stiff" $\longrightarrow$ L large

"Soft" $\longrightarrow$ L small

PREX constrains L!

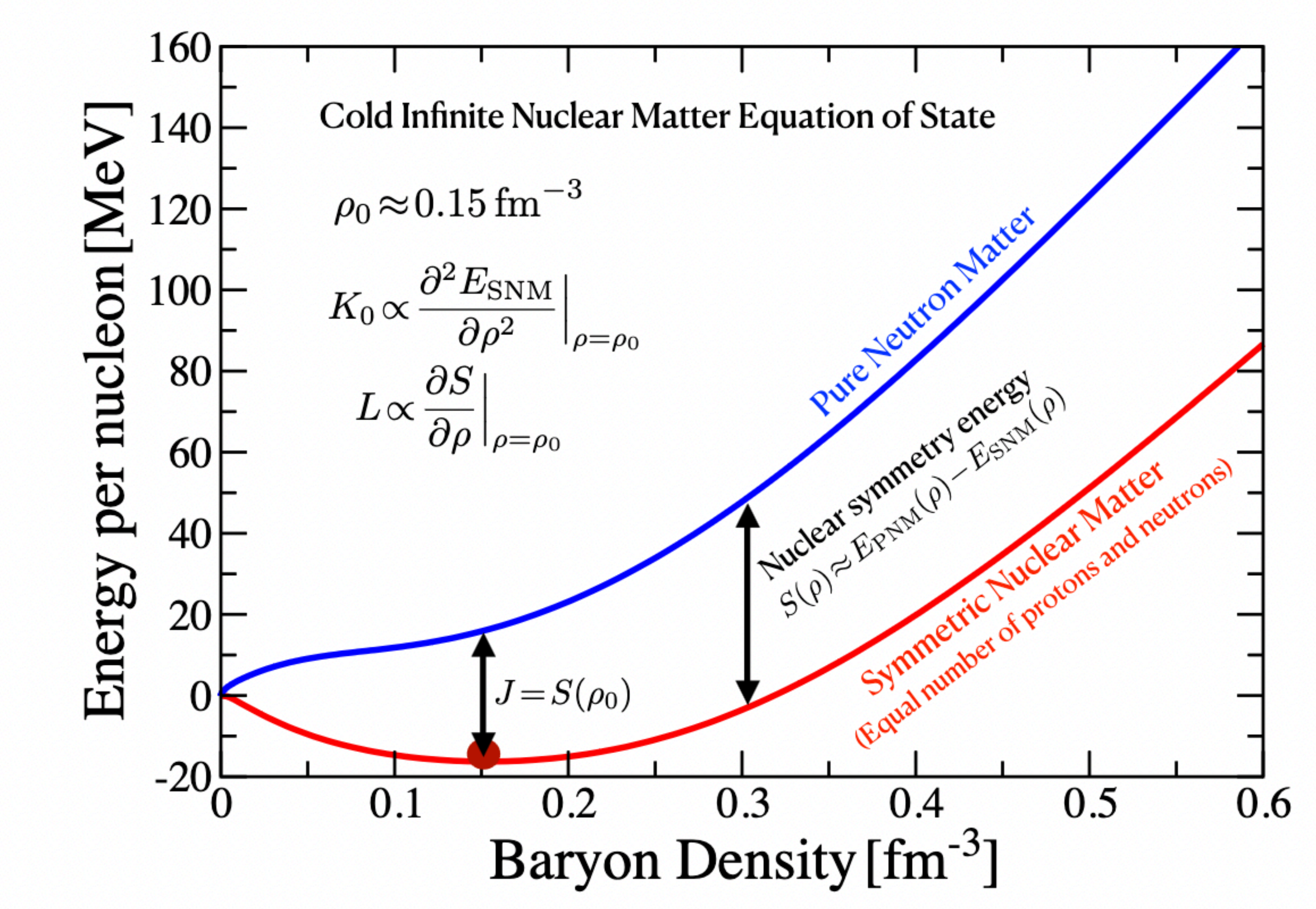
The Equation of State of Neutron-Rich Matter
… or more generally, asymmetric matter

Characterization in terms of a few bulk parameters of symmetric matter and the symmetry energy

$$T=0;\ \rho=\rho_n+\rho_p;\ \alpha=\frac{\rho_n-\rho_p}{\rho_n+\rho_p};\ x\equiv\frac{\rho-\rho_0}{3\rho_0}$$

$$\mathcal{E}(\rho,\alpha)\simeq\mathcal{E}_0(\rho)+\alpha^2\mathcal{S}(\rho)\simeq\left(\epsilon_0+\frac{1}{2}K_0x^2\right)+\left(J+Lx+\frac{1}{2}K_{\rm sym}x^2\right)\alpha^2$$

Bulk Property	Inferred Value	Observable
Saturation density	$\rho_0\approx 0.15\,{\rm fm}^{-3}$	Interior density ★★★★
Binding energy per nucleon	$\varepsilon_0\approx -16\,{\rm MeV}$	Nuclear masses ★★★★
Nuclear Incompressibility	$K_0\approx 230\,{\rm MeV}$	Nuclear breathing mode ★★★
Symmetry energy	$J\approx 32\,{\rm MeV}$	Masses of neutron-rich nuclei ★★★
Symmetry slope	$L\approx 100\,{\rm MeV}$	Neutron skin of neutron-rich nuclei ★★



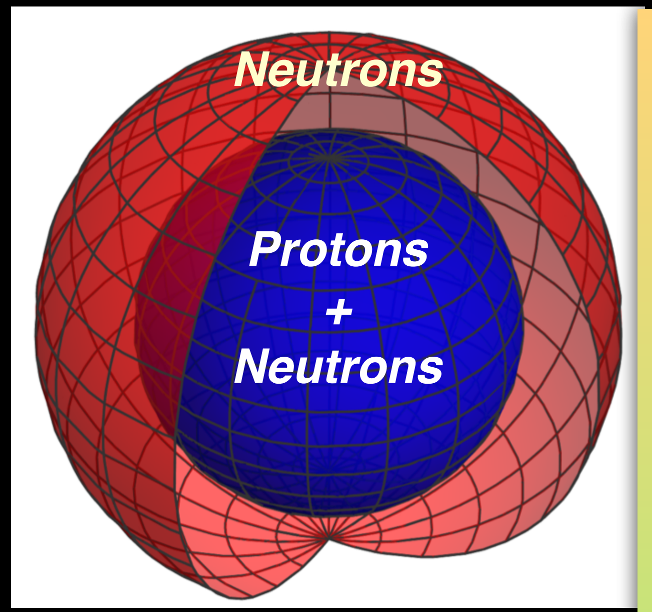
$$\mathcal{E}(\rho,\alpha\!=\!1)\equiv\mathcal{E}_{\rm PNM}(\rho)\simeq\mathcal{E}_0(\rho)+\mathcal{S}(\rho)$$

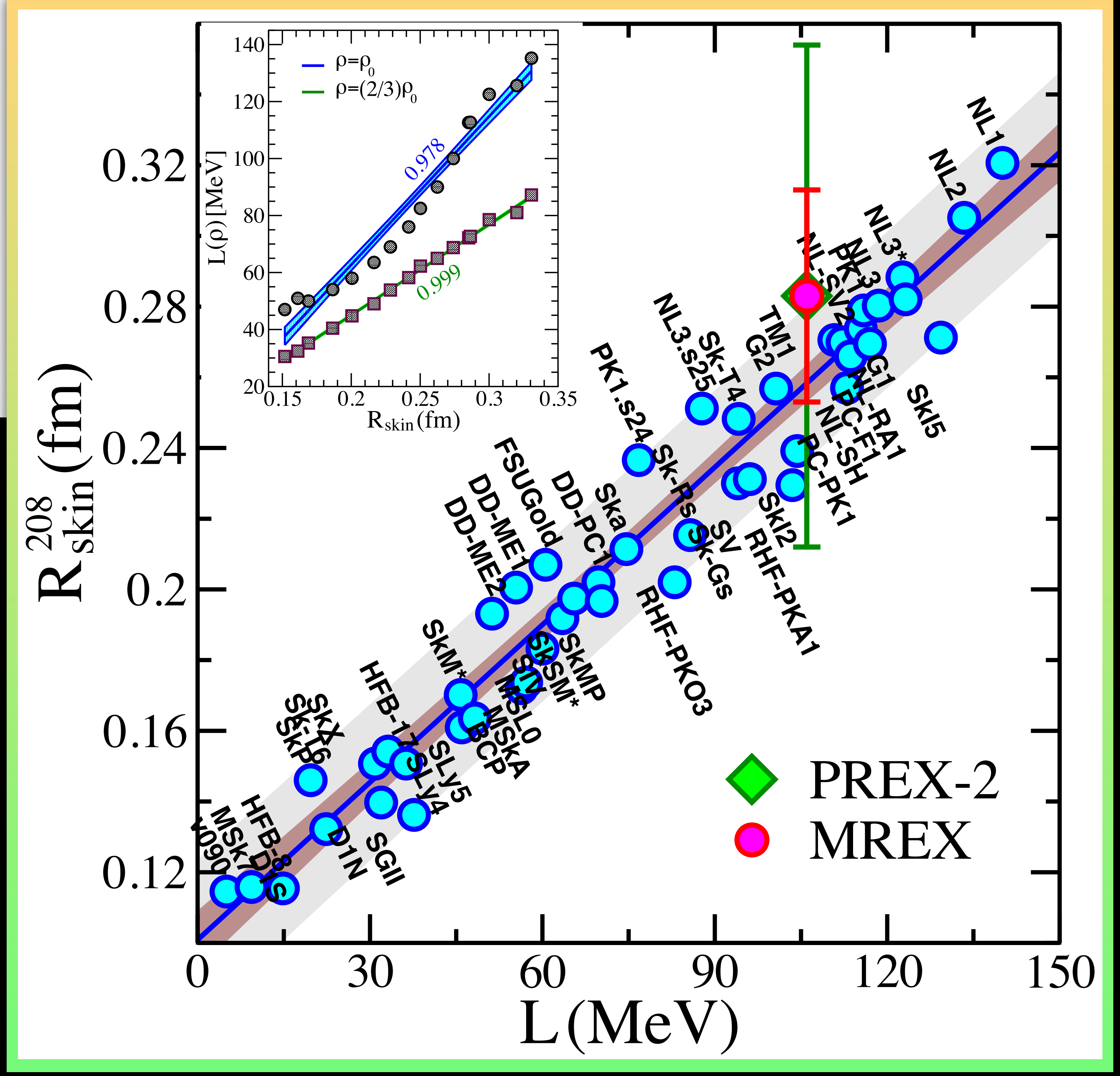
$$P_{\rm PNM}(\rho_0)\simeq P_{\rm SNM}(\rho_0)+\frac{1}{3}\rho_0L=\frac{1}{3}\rho_0L$$

The slope of the symmetry energy "L" is closely related to the pressure of pure neutron matter at saturation density

Heaven and Earth

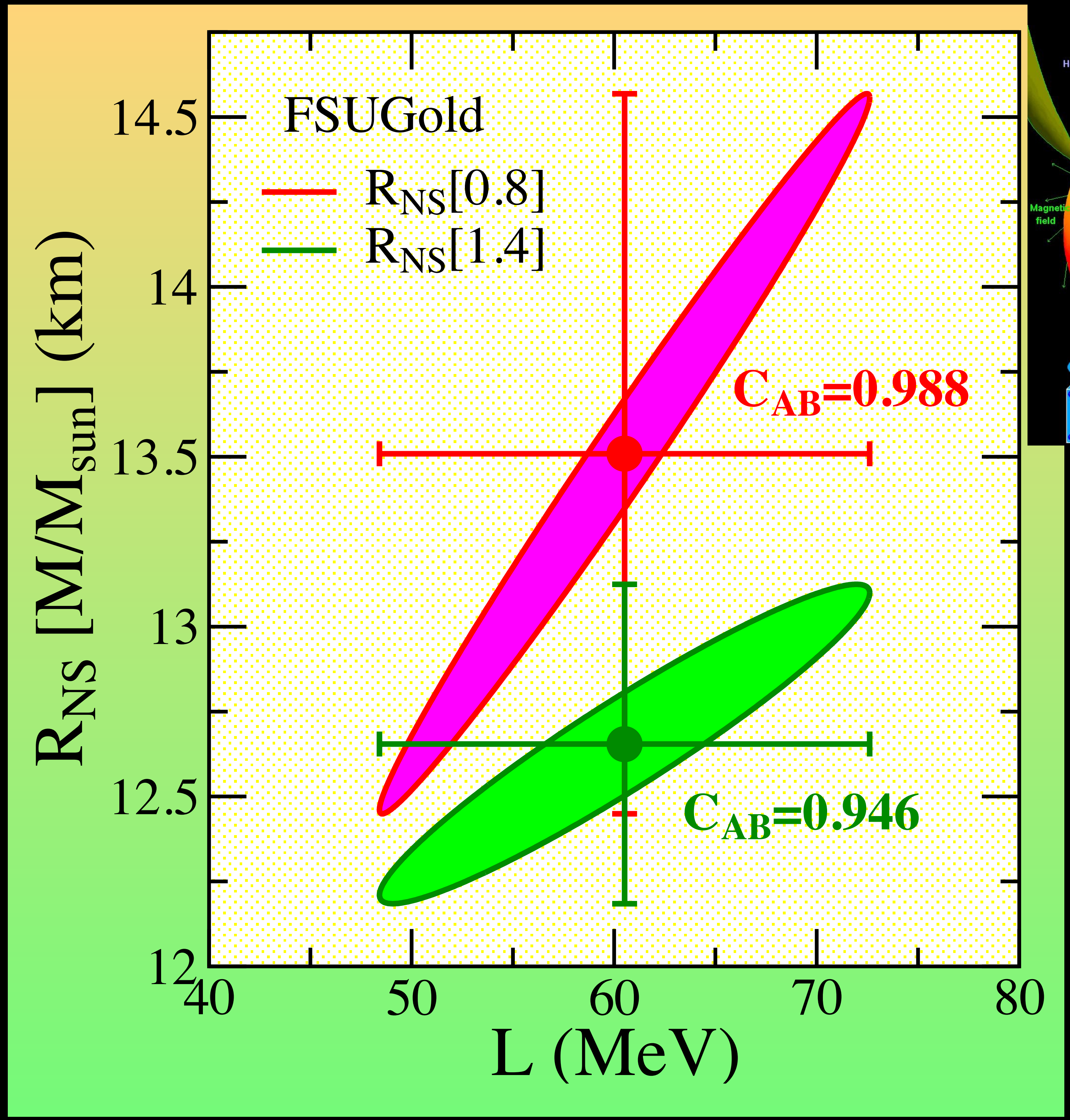
Laboratory Constraints on the EOS

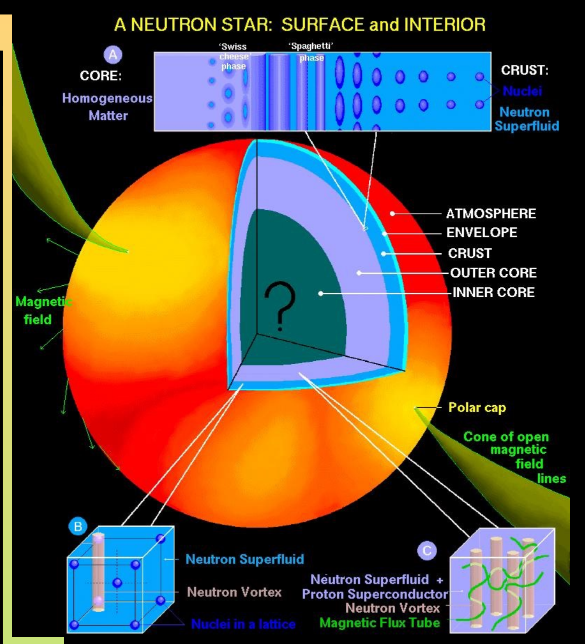




18 orders

magnitude



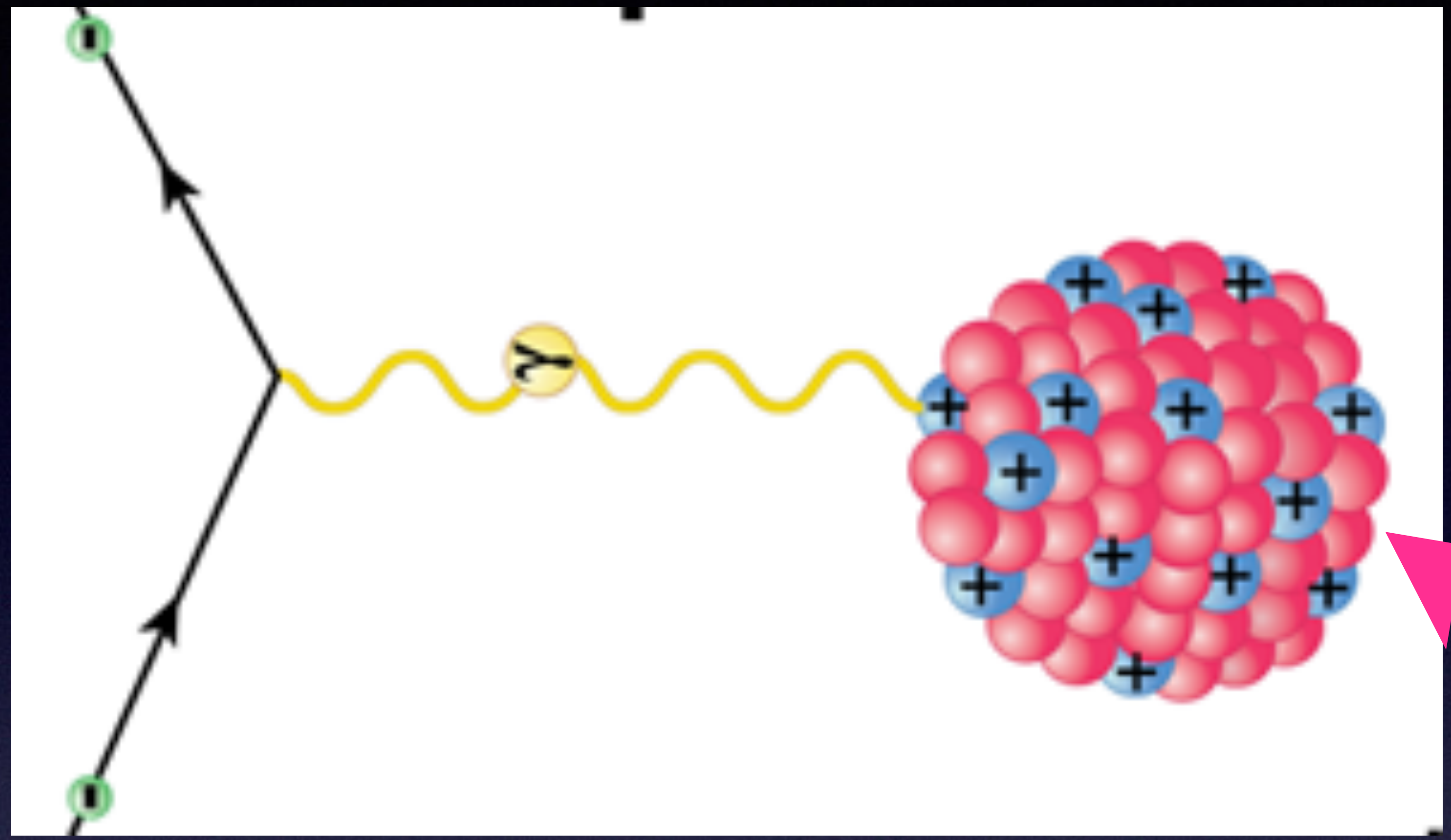


Nuclear interaction is responsible for describing finite nuclei and neutron stars!

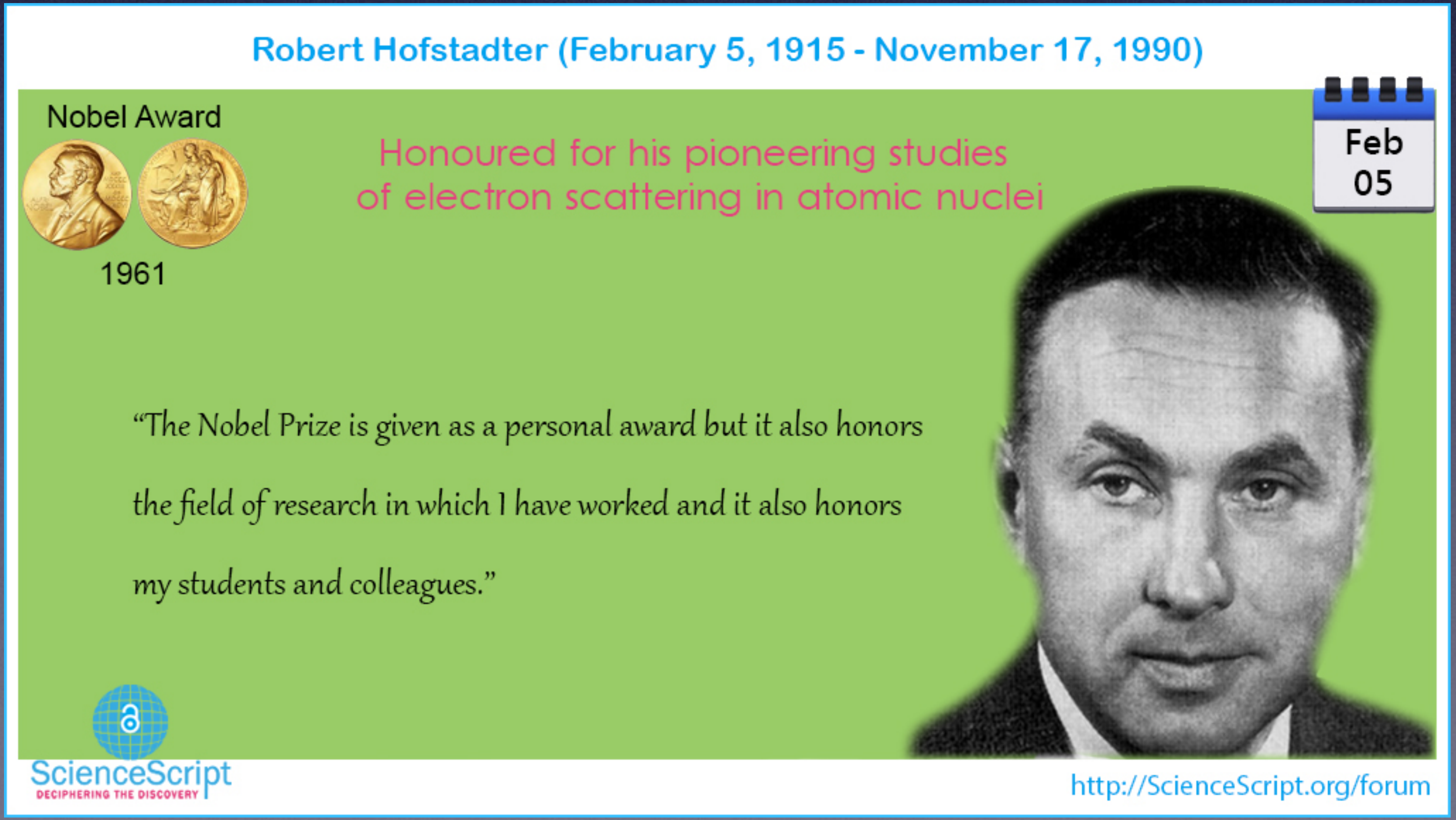
The slope of the symmetry energy L controls both the neutron skin of heavy nuclei as well as the radius of (low mass) neutron stars — objects that differ in size by 18 orders of magnitude!

Parity Conserving e-Nucleus Scattering

Searching for an accurate picture of the proton distribution



$$\left(\frac{d\sigma}{d\Omega}\right)_{J=0} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \left|F_{\text{ch}}(q)\right|^2$$



Nuclear information is contained in one single form factor, whose Fourier transform gives the spatial distribution

Diffraction – Hofstadter, Nobel (1961)

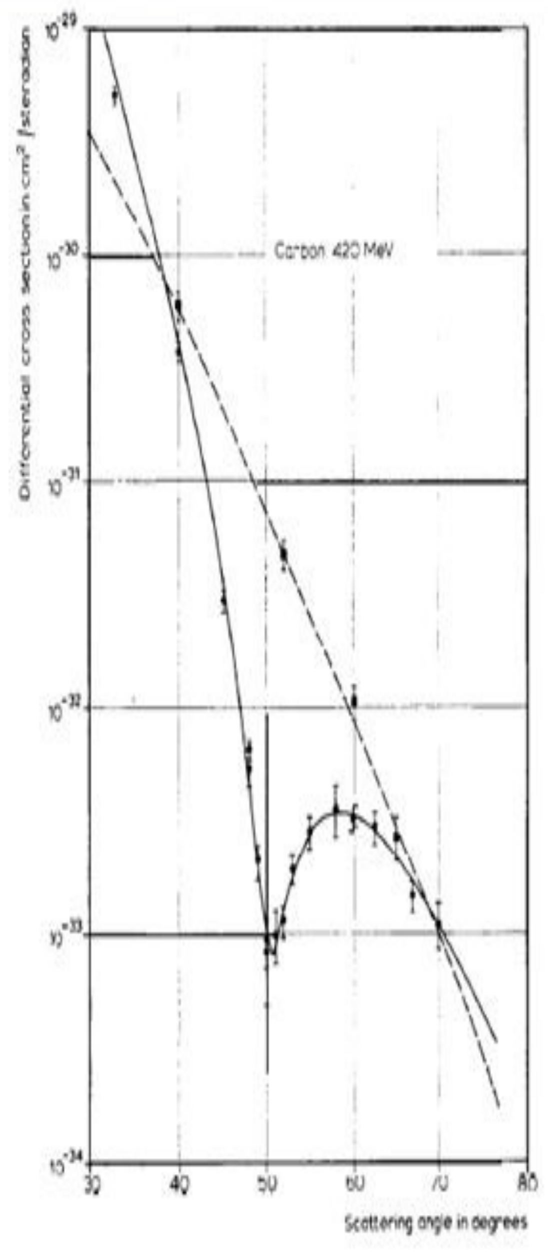
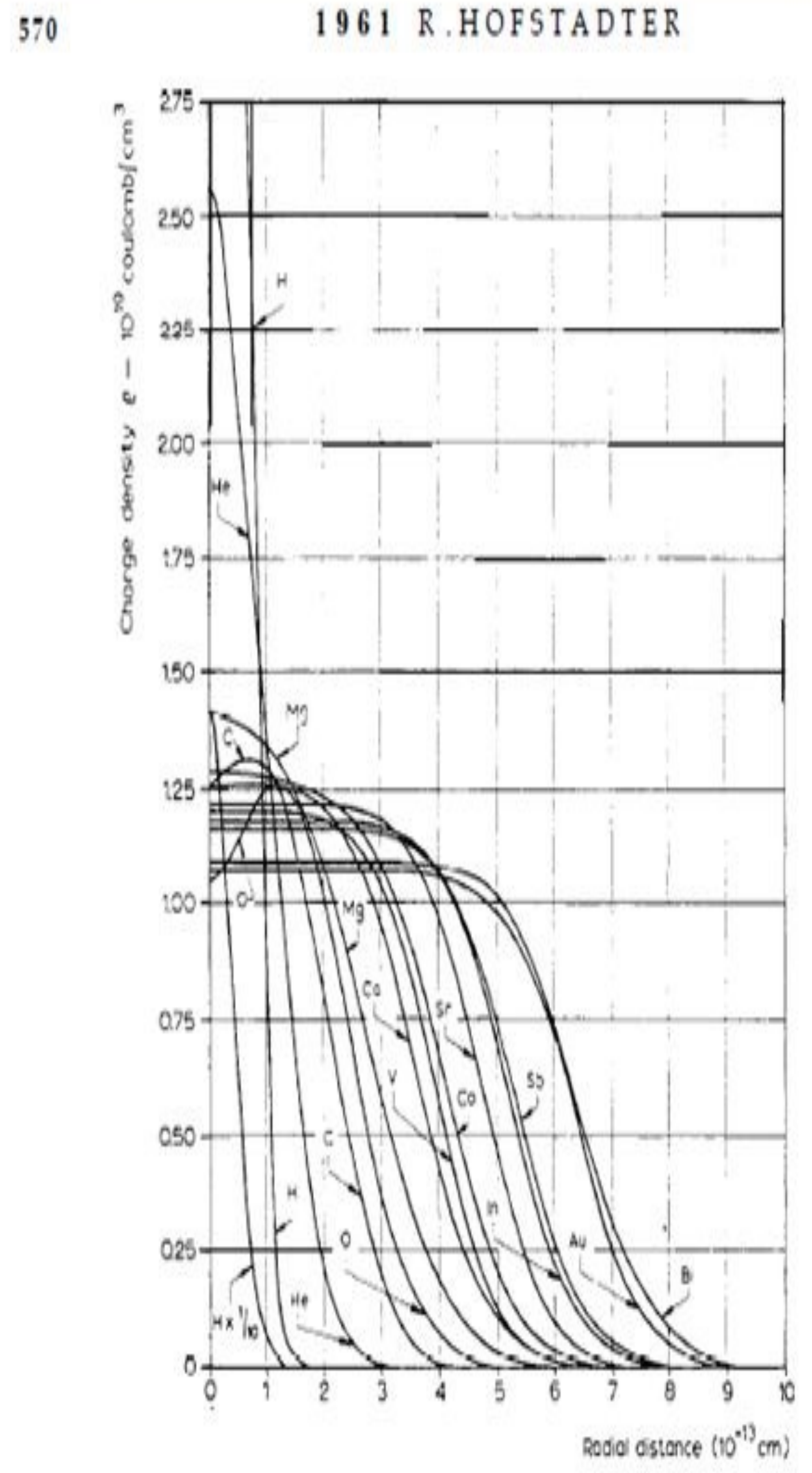


Fig. 5. This figure shows the elastic and inelastic curves corresponding to the scattering of 420-MeV electrons by ^{12}C. The *solid circles*, representing experimental points, show the elastic-scattering behavior while the *solid squares* show the inelastic-scattering curve for the 4.43-MeV level in carbon. The *solid line* through the elastic data shows the type of fit that can be calculated by phase-shift theory for the model of carbon shown in Fig. 8.



Diffractive electron scattering on nuclei and the resulting charge density distributions, images of spherical nuclei

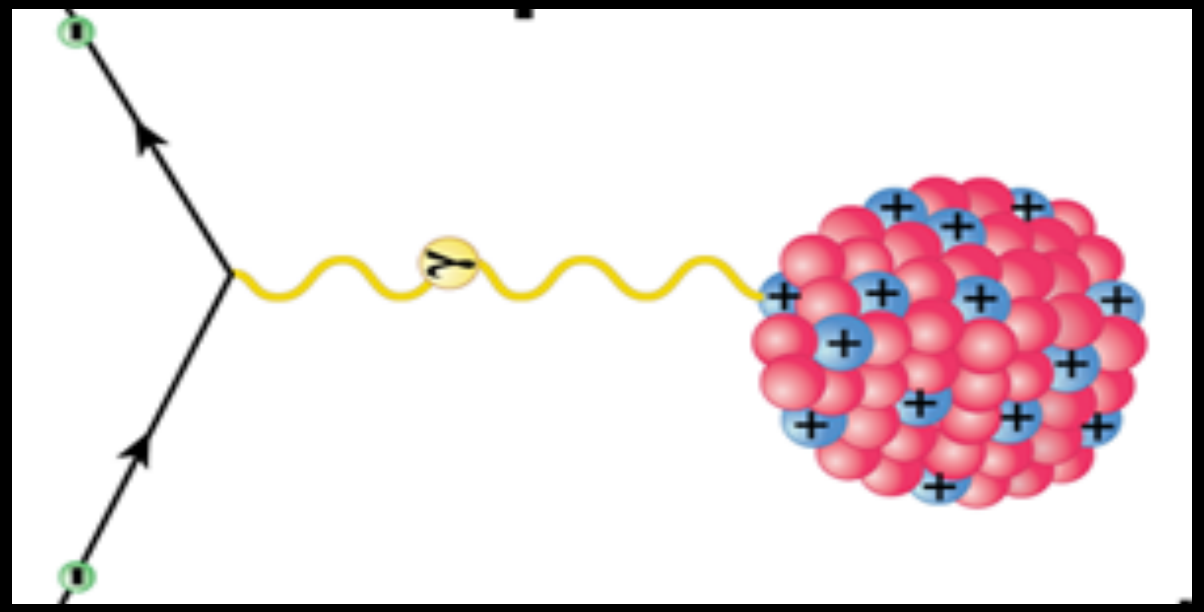
The Charge Form Factor

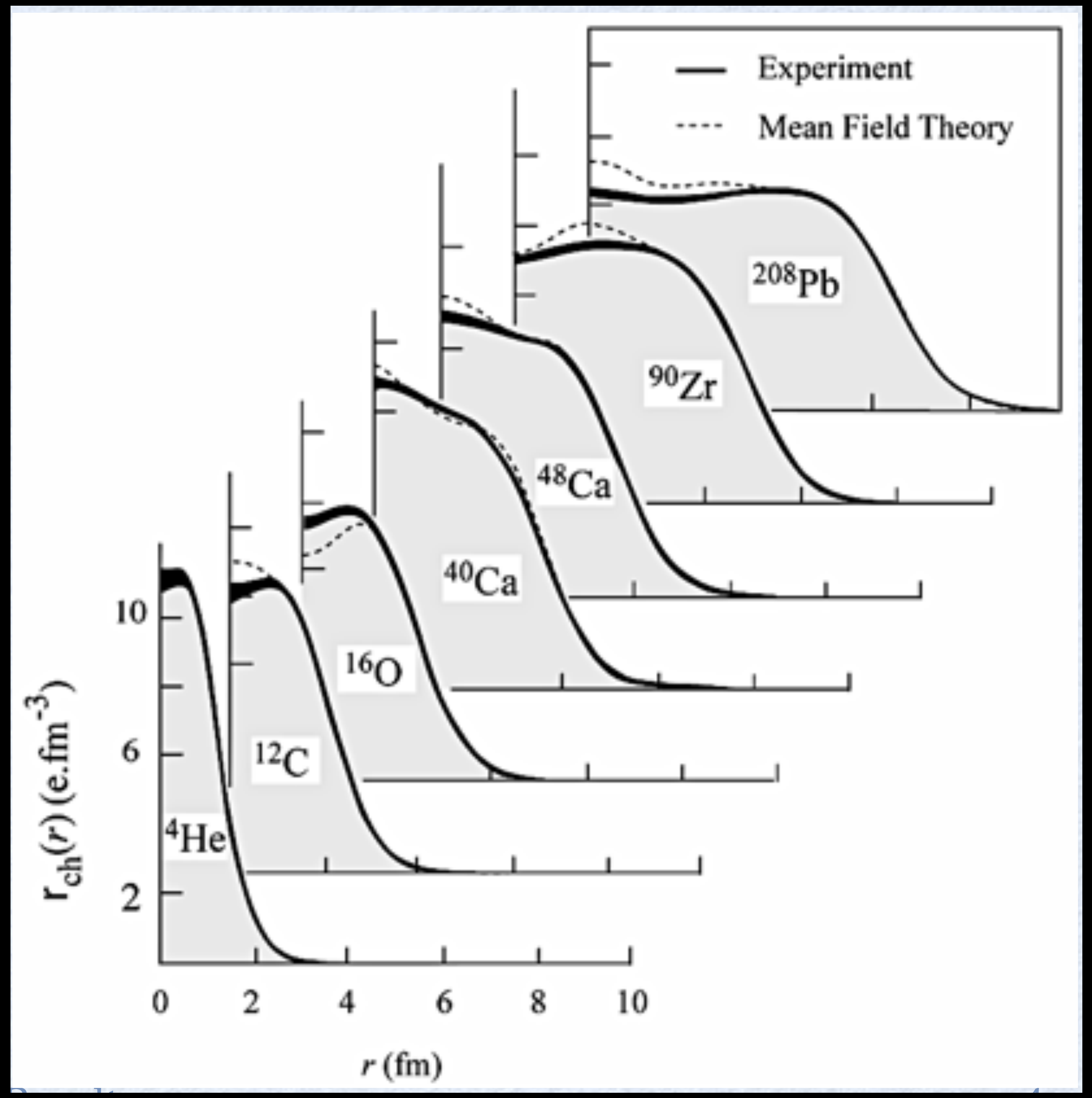
- Ultrarelativistic Mott Scattering:

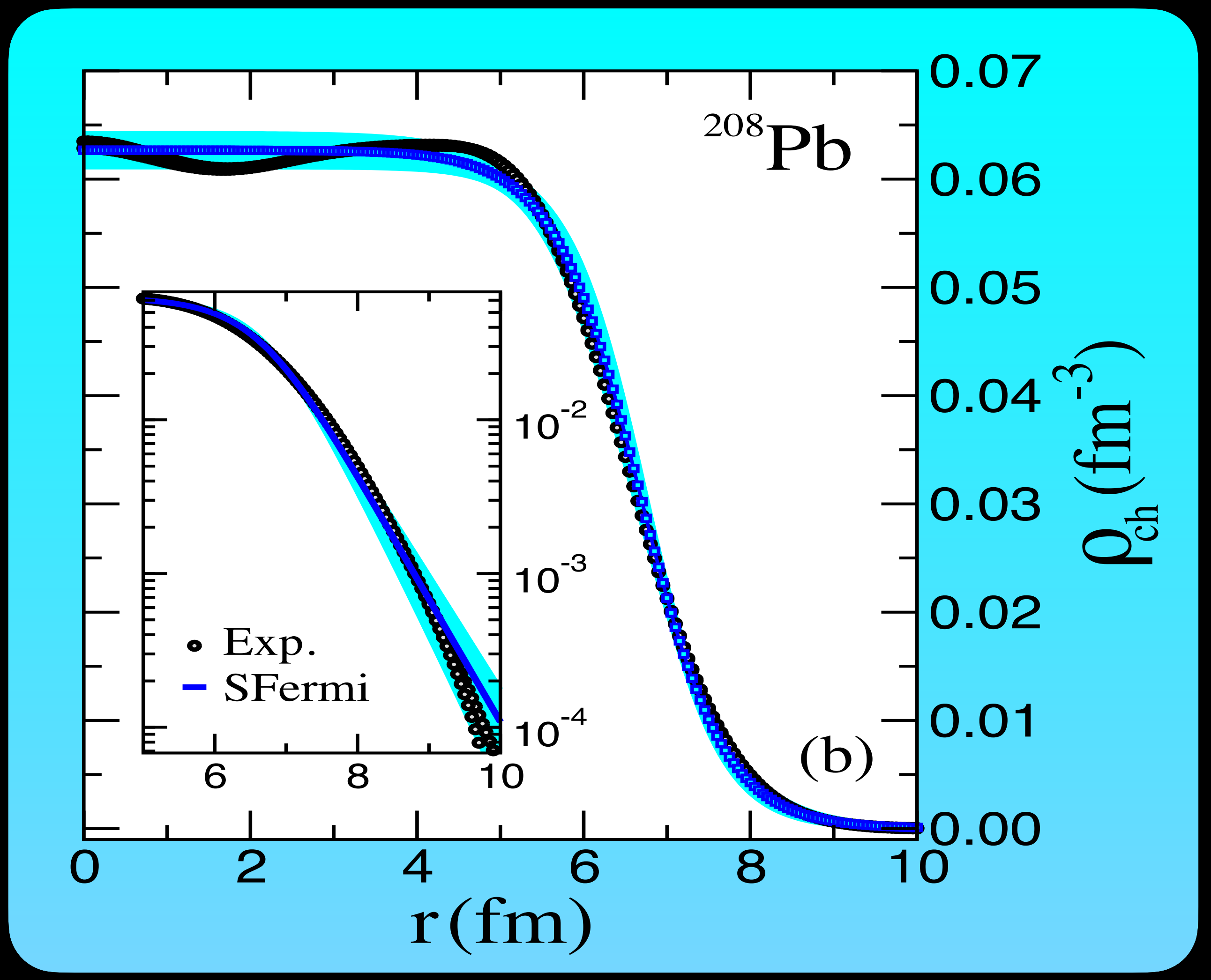
$$\left(\frac{d\sigma}{d\Omega}\right)_{\rm M} = \frac{Z^2\alpha^2\cos^2(\theta/2)}{4E^2\sin^4(\theta/2)}$$

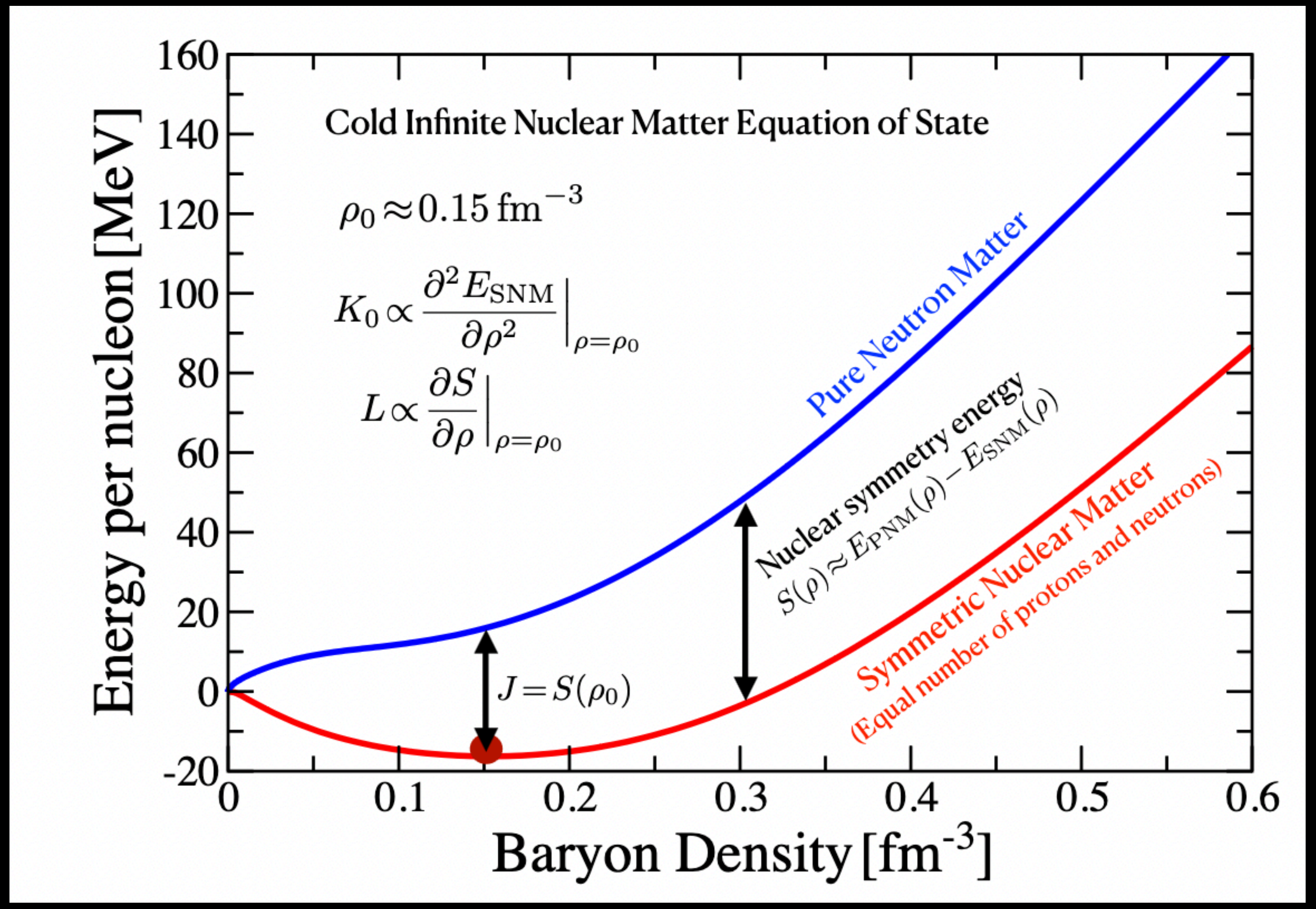
- e-Scattering from J=0 spherical nuclei:

$$\left(\frac{d\sigma}{d\Omega}\right)_{J=0} = \left(\frac{d\sigma}{d\Omega}\right)_{\rm Mott}\left|F_{\rm ch}(q)\right|^2$$









Uniform interior is a clear manifestation of nuclear saturation: the existence of an equilibrium density

Form Factor is the Fourier transform of the Density

A Two-Parameter "Symmetrized Fermi" Form Factor

- Conventional "Fermi" function
  ["cusp" at the origin (r=0)]

$$\rho_{\mathrm{F}}(r) = \frac{\rho_0}{1+e^{(r-c)/a}}$$

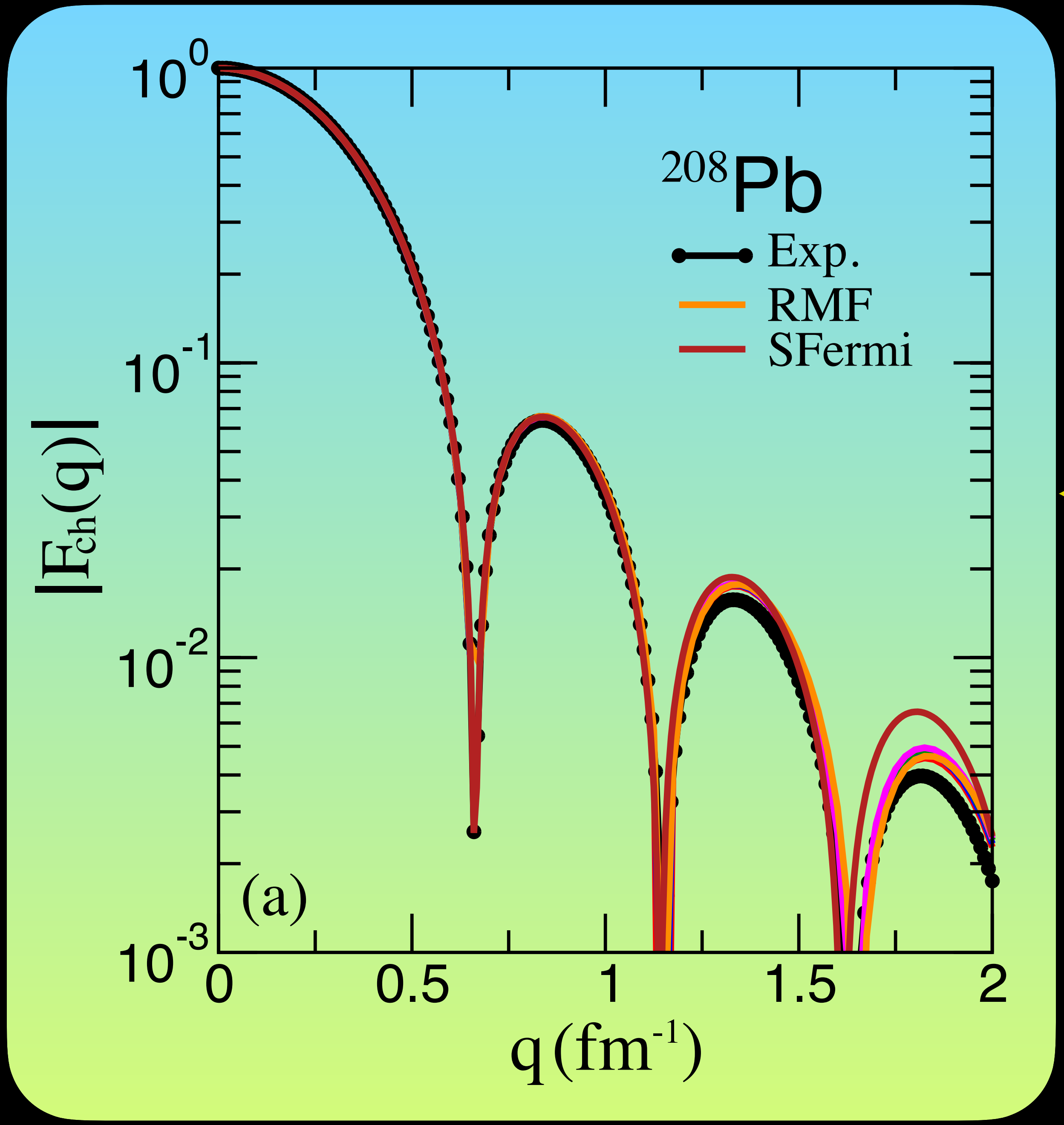
- "Symmetrized Fermi" function

$$\rho_{\mathrm{SF}}(r) = \rho_{\mathrm{F}}(r) + \left[\rho_{\mathrm{F}}(-r) - \rho_0\right]$$

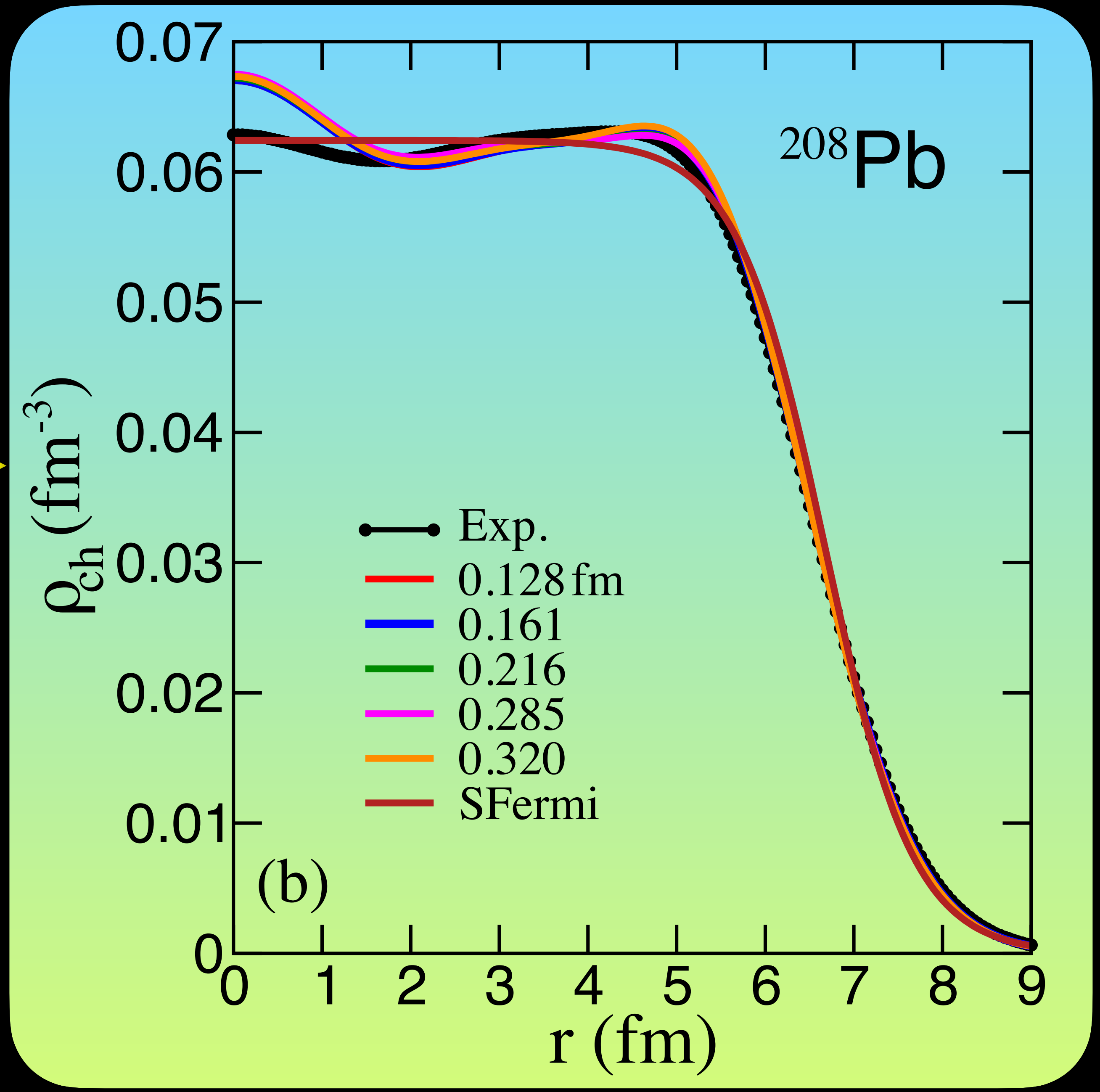
$$F_{\mathrm{SF}}(q) = \frac{3}{qc\Big((qc)^2+(\pi qa)^2\Big)}\left(\frac{\pi qa}{\sinh(\pi qa)}\right)\left[\frac{\pi qa}{\tanh(\pi qa)}\sin(qc) - qc\cos(qc)\right]$$

$$F_{\mathrm{SF}}(q) \to \frac{\cos(qc+\delta)}{qc}e^{-\pi qa}$$

Diffractive Oscillations ("c")
modulated by an exponential falloff ("πa")



F.T.



$$R^2 = \langle r^2\rangle = \frac{3}{5}c^2 + \frac{7}{5}(\pi a)^2$$

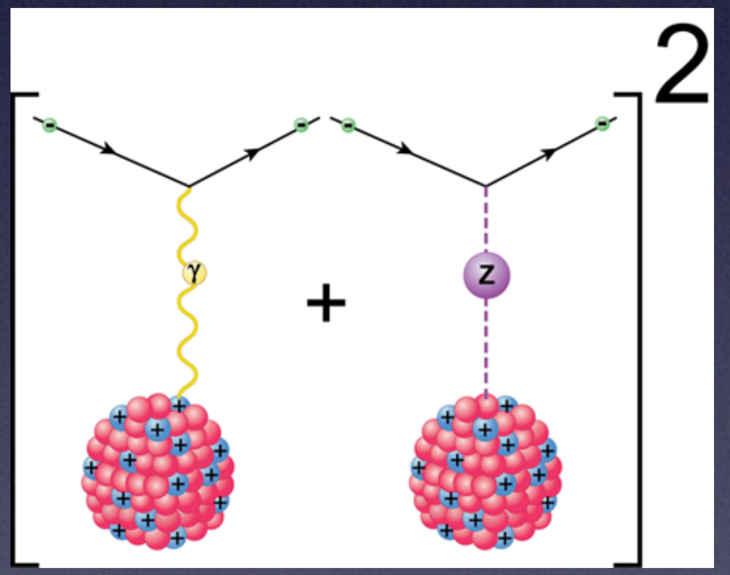
$$R^4 = \langle r^4\rangle = \frac{3}{7}c^4 + \frac{18}{7}(\pi a)^2c^2 + \frac{31}{7}(\pi a)^4$$

Form Factor ("q") and Spatial Density ("r") are related by a Fourier transform

Parity Violating e-Nucleus Scattering

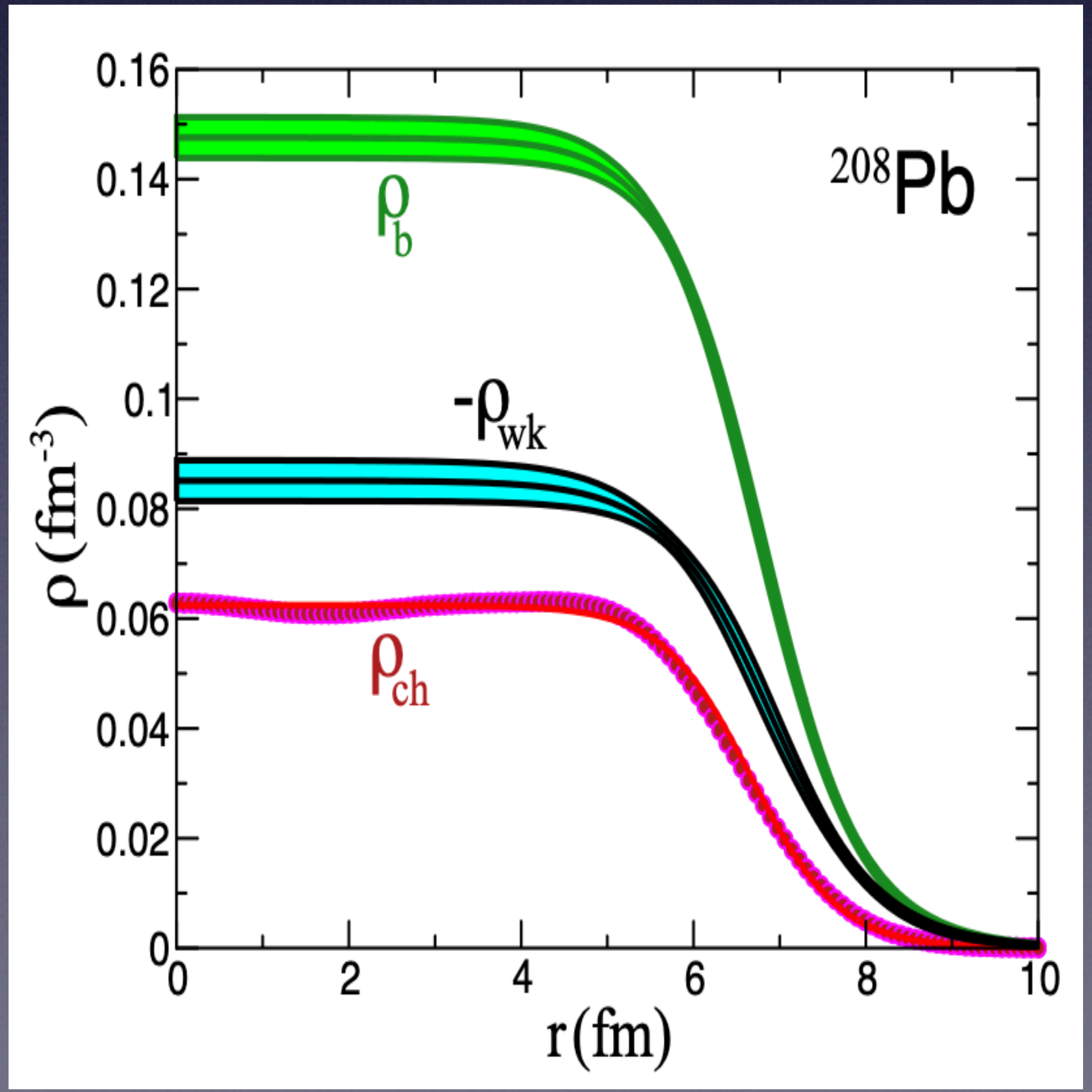
Searching for an accurate picture of the neutron distribution

- Charge (proton) density known with enormous precision
  - Probed via parity-conserving elastic e-scattering
- Weak-charge (neutron) density known very poorly known
  - Probed via parity-violating elastic e-scattering



$$A_{\rm PV} \equiv \left[\frac{\left(\frac{d\sigma}{d\Omega}\right)_R - \left(\frac{d\sigma}{d\Omega}\right)_L}{\left(\frac{d\sigma}{d\Omega}\right)_R + \left(\frac{d\sigma}{d\Omega}\right)_L}\right] = \left(\frac{G_{\rm F}Q^2}{4\pi\alpha\sqrt{2}}\right)\frac{F_{wk}(Q^2)}{F_{ch}(Q^2)} \simeq 10^{-6}$$

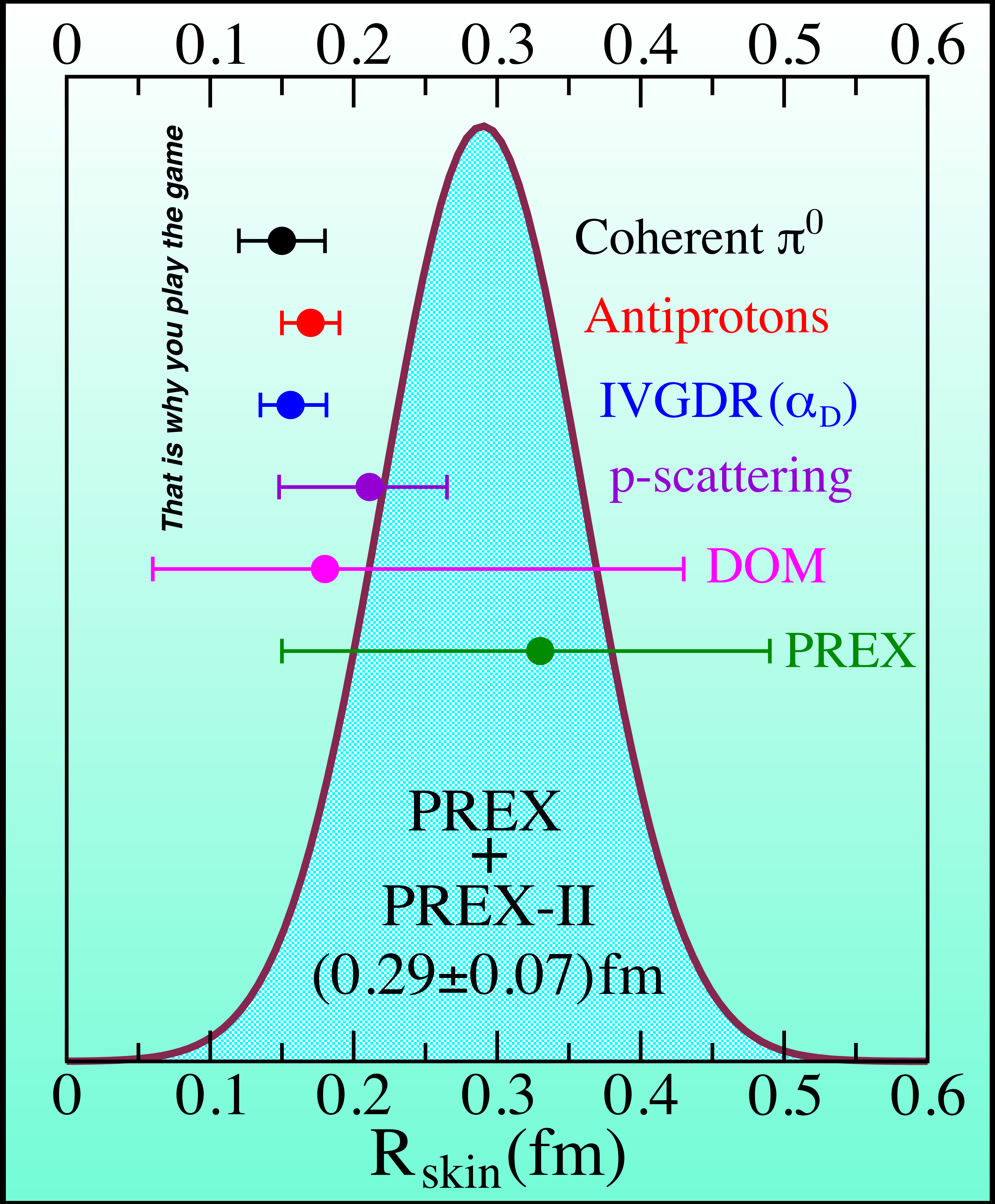
- *Electric-charge density dominated by protons*
- *Weak-charge density dominated by neutrons*



	up-quark	down-quark	proton	neutron
γ-coupling	$+2/3$	$-1/3$	$+1$	0
Z_0-coupling	$\approx +1/3$	$\approx -2/3$	≈ 0	-1

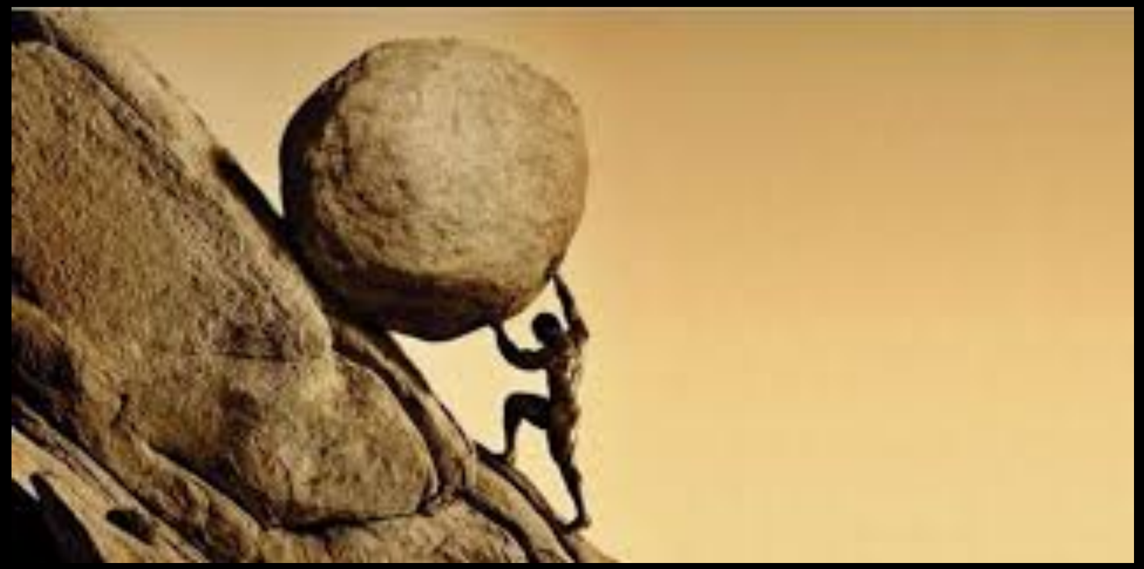
$$g_{\rm v} = 2t_z - 4Q\sin^2\theta_{\rm W} \approx 2t_z - Q$$

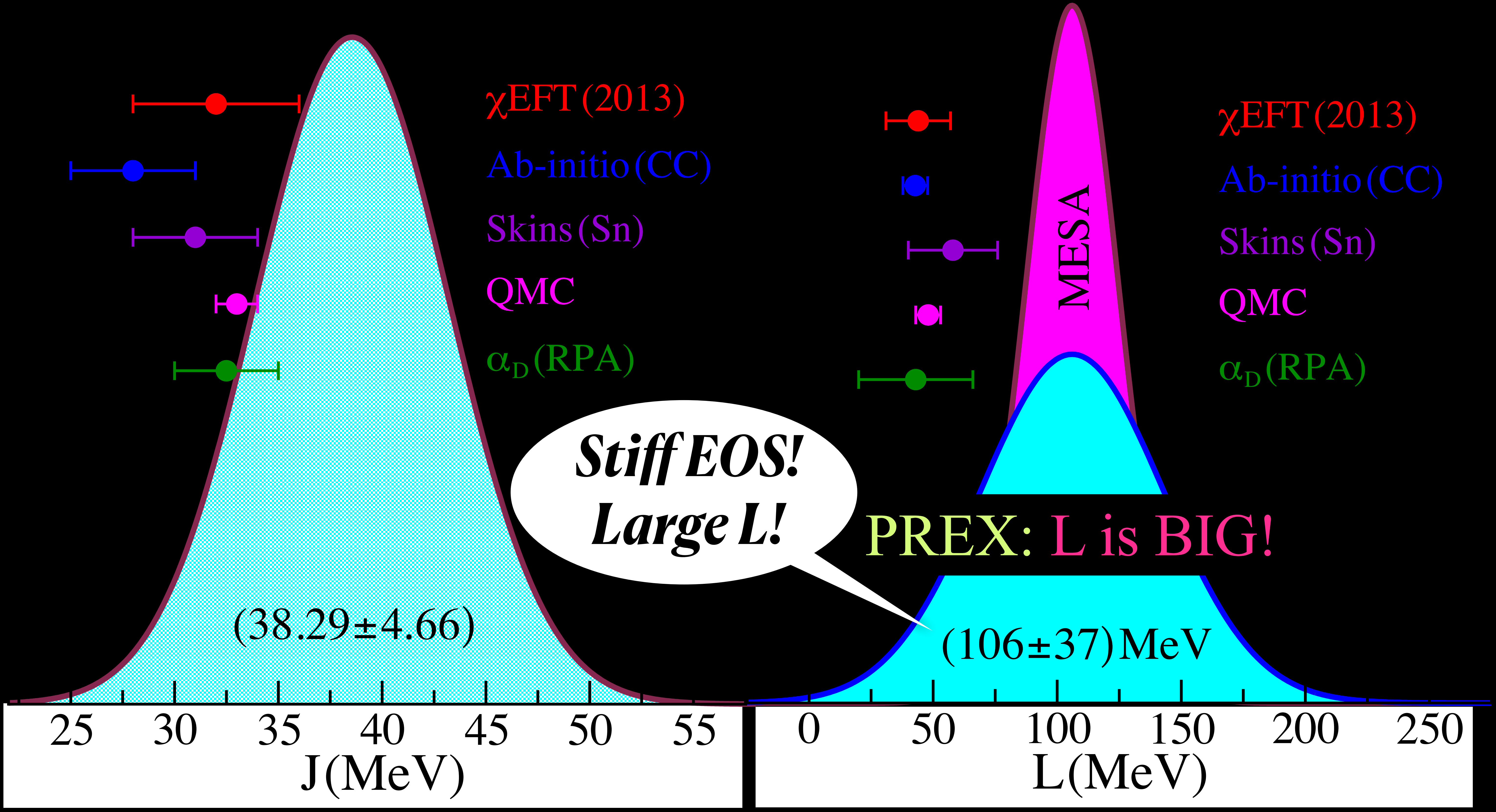
PREX-2 (Oct 29, 2020)
Ciprian Gal - DNP Meeting
Adhikari et al., PRL 126, 172502 (2021)



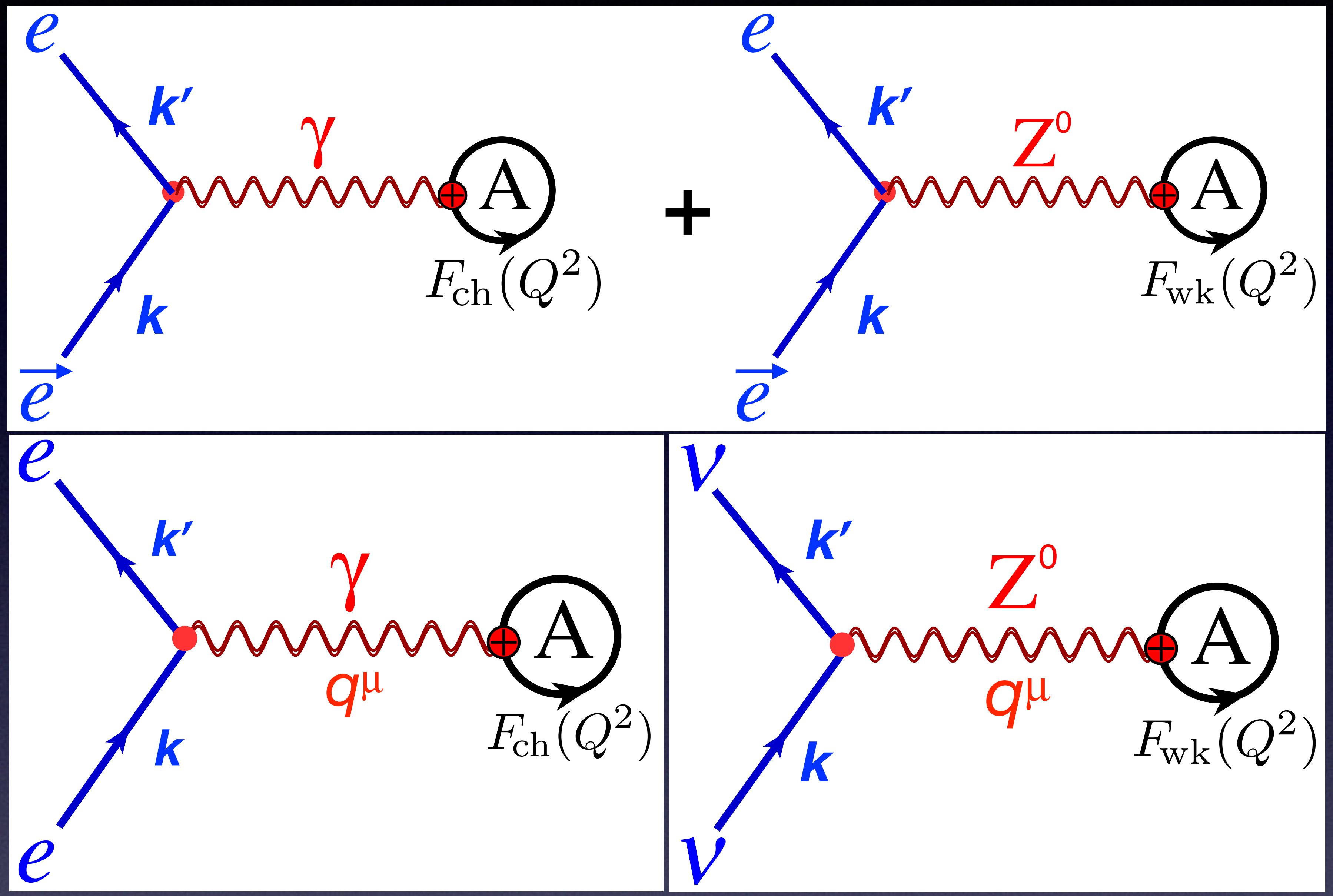
Conservation of difficulty: PVES provides the cleanest constraint on the EOS of neutron-rich matter in the vicinity of saturation density

Heroic effort from our experimental colleagues





Electroweak Probes of Nuclear Densities

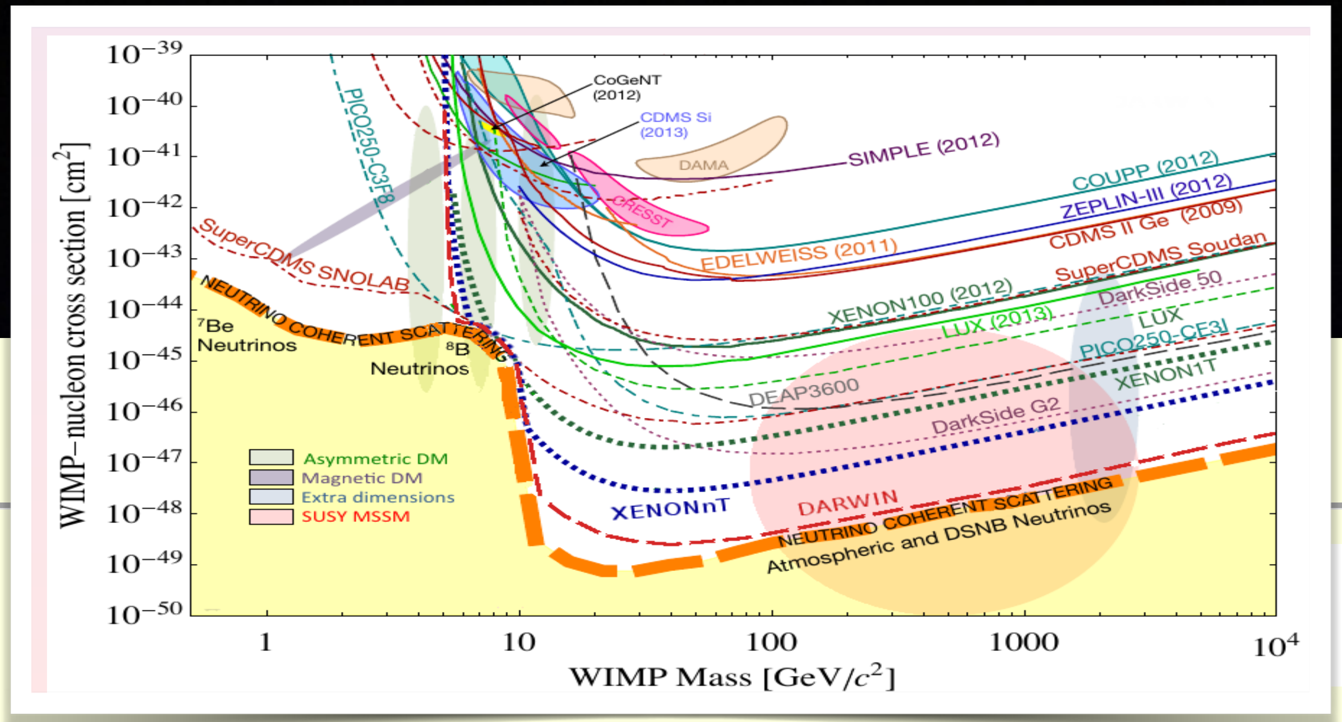


$$A_{PV}(Q^2) = \frac{G_F Q^2}{4\pi\alpha\sqrt{2}} \frac{Q_{wk} F_{wk}(Q^2)}{Z F_{ch}(Q^2)}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\rm EM} = \left[\frac{\alpha^2\cos^2(\theta/2)}{4E^2\sin^4(\theta/2)}\left(\frac{E'}{E}\right)\right] Z^2 F^2_{\rm ch}(Q^2)$$

$$\left(\frac{d\sigma}{dT}\right)_{\rm NC} = \frac{G_F^2}{8\pi} M\left[2-2\frac{T}{E}-\frac{MT}{E^2}\right] Q^2_{\rm wk} F^2_{\rm wk}(Q^2)$$

CEvNS



Science

REPORTS

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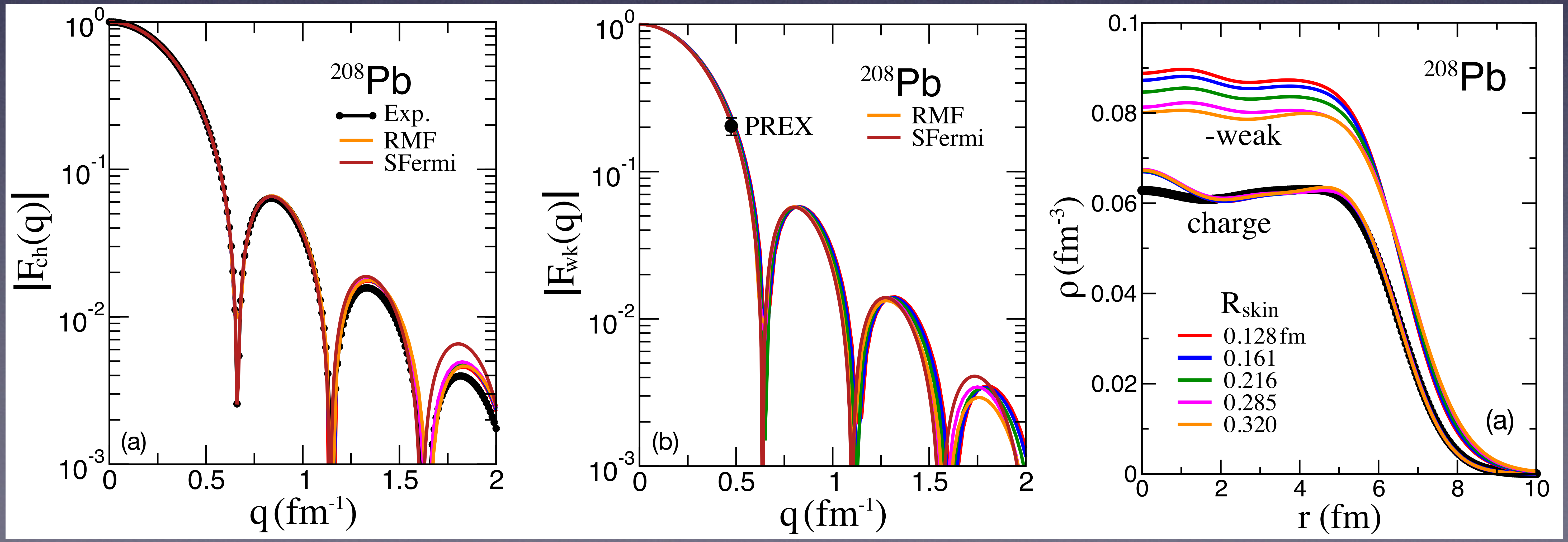
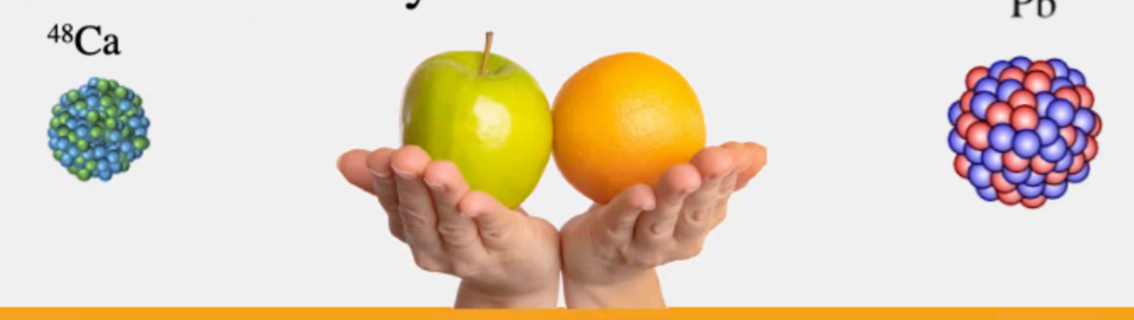
Observation of coherent elastic neutrino-nucleus scattering

D. Akimov,[1,2] J. B. Albert,[3] P. An,[4] C. Awe,[4,5] P. S. Barbeau,[4,5] B. Becker,[6] V. Belov,[1,2] A. Brown,[4,7] A. Bolozdynya,[2] B. Cabrera-Palmer,[8] M. Cervantes,[5] J. I. Collar,[9*] R. J. Cooper,[10] R. L. Cooper,[11,12] C. Cuesta,[13†] D. J. Dean,[14] J. A. Detwiler,[13] A. Eberhardt,[13] Y. Efremenko,[6,14] S. R. Elliott,[12] E. M. Erkela,[13] L. Fabris,[14] M. Febbraro,[14] N. E. Fields,[9‡] W. Fox,[3] Z. Fu,[13] A. Galindo-Uribarri,[14] M. P. Green,[4,14,15] M. Hai,[9§] M. R. Heath,[3] S. Hedges,[4,5] D. Hornback,[14] T. W. Hossbach,[16] E. B. Iverson,[14] L. J. Kaufman,[3||] S. Ki,[4,5] S. R. Klein,[10] A. Khromov,[2] A. Konovalov,[1,2,17] M. Kremer,[4] A. Kumpan,[2] C. Leadbetter,[4] L. Li,[4,5] W. Lu,[14] K. Mann,[4,15] D. M. Markoff,[4,7] K. Miller,[4,5] H. Moreno,[11] P. E. Mueller,[14] J. Newby,[14] J. L. Orrell,[16] C. T. Overman,[16] D. S. Parno,[13¶] S. Penttila,[14] G. Perumpilly,[9] H. Ray,[18] J. Raybern,[5] D. Reyna,[8] G. C. Rich,[4,14,19] D. Rimal,[18] D. Rudik,[1,2] K. Scholberg,[5] B. J. Scholz,[9] G. Sinev,[5] W. M. Snow,[3] V. Sosnovtsev,[2] A. Shakirov,[2] S. Suchyta,[10] B. Suh,[4,5,14] R. Tayloe,[3] R. T. Thornton,[3] I. Tolstukhin,[3] J. Vanderwerp,[3] R. L. Varner,[14] C. J. Virtue,[20] Z. Wan,[4] J. Yoo,[21] C.-H. Yu,[14] A. Zawada,[4] J. Zettlemoyer,[3] A. M. Zderic,[13] COHERENT Collaboration#

FRIB



Among the Open Questions: Who Ordered That?

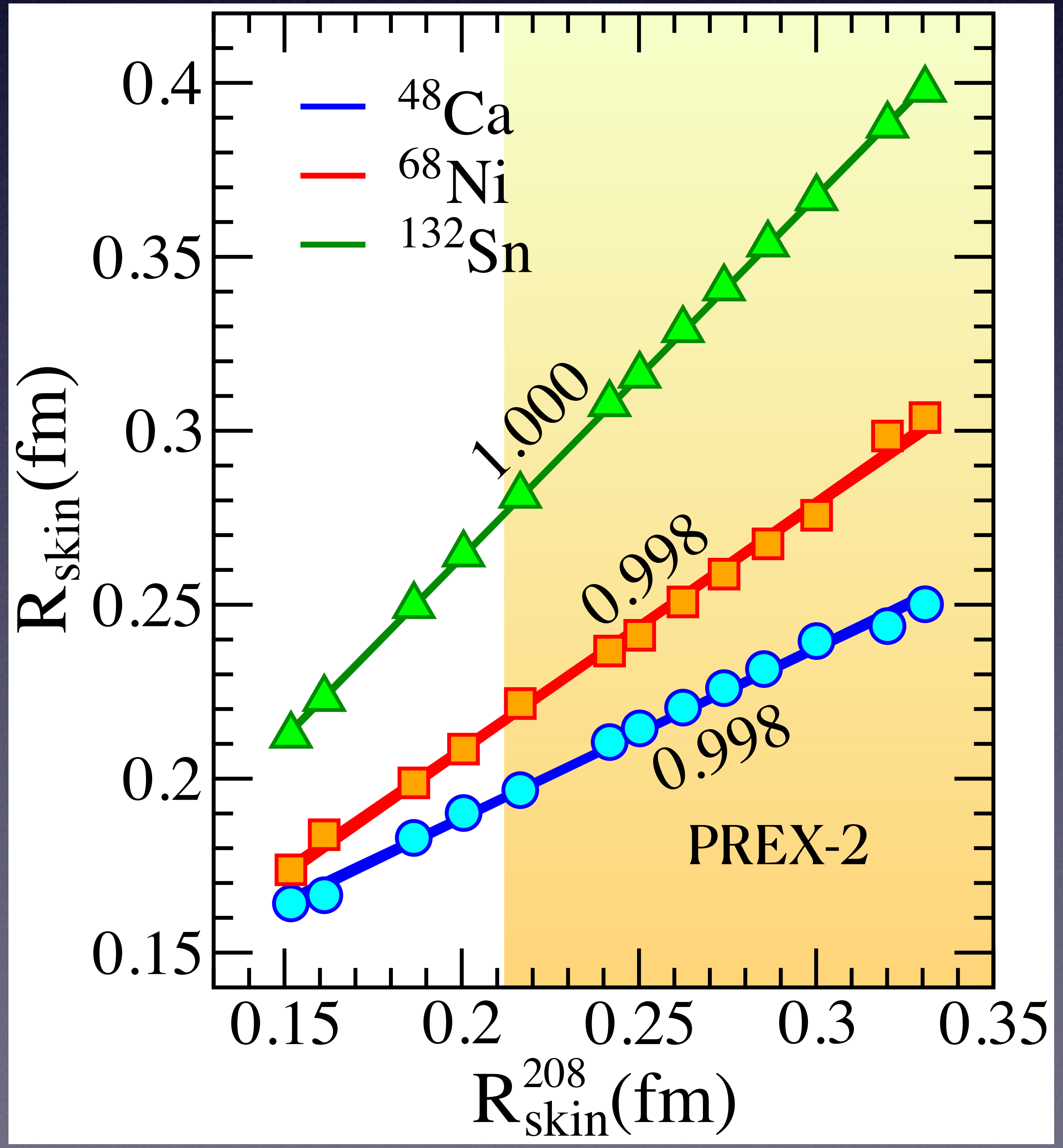


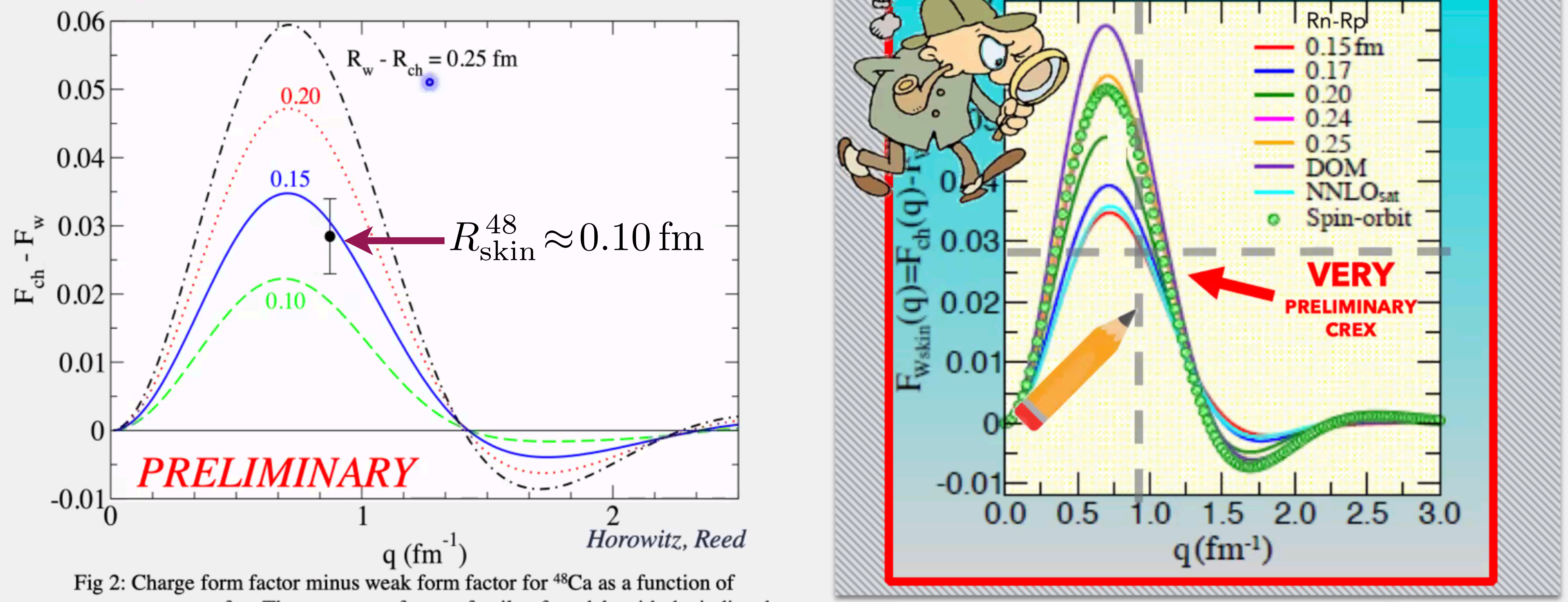
Preliminary Observations:

- CREX result is consistent with a thin neutron skin prediction (e.g. coupled cluster calculations) and is strongly inconsistent with predictions of a very thick skin
- At this point it appears potentially challenging for DFT models to reproduce both the CREX result of a thin skin in ^{48}Ca and the PREX result of a relatively thick skin in ^{208}Pb.

No theoretical model that I know of can reproduce both!

Isidor Isaac Rabi





Comparing to Theory

Observation:

- CREX result is consistent with a thin neutron skin prediction (e.g. coupled cluster calculations) and is strongly inconsistent with predictions of a very thick skin

Fig 2: Charge form factor minus weak form factor for ^{48}Ca as a function of momentum transfer. The curves are for one family of models with the indicted R_{Wskin} = weak minus charge rms radii. The error bar shows the CREX result.

Figure taken from J.Mammei CevNS 2019 talk (Jorge Piekarewicz plot), shows various curves for a family of R_{nskin} = Rn-Rp values. Also DOM and NNLO (coupled cluster). Warning: theories shown may (or may not) require further SO correction.

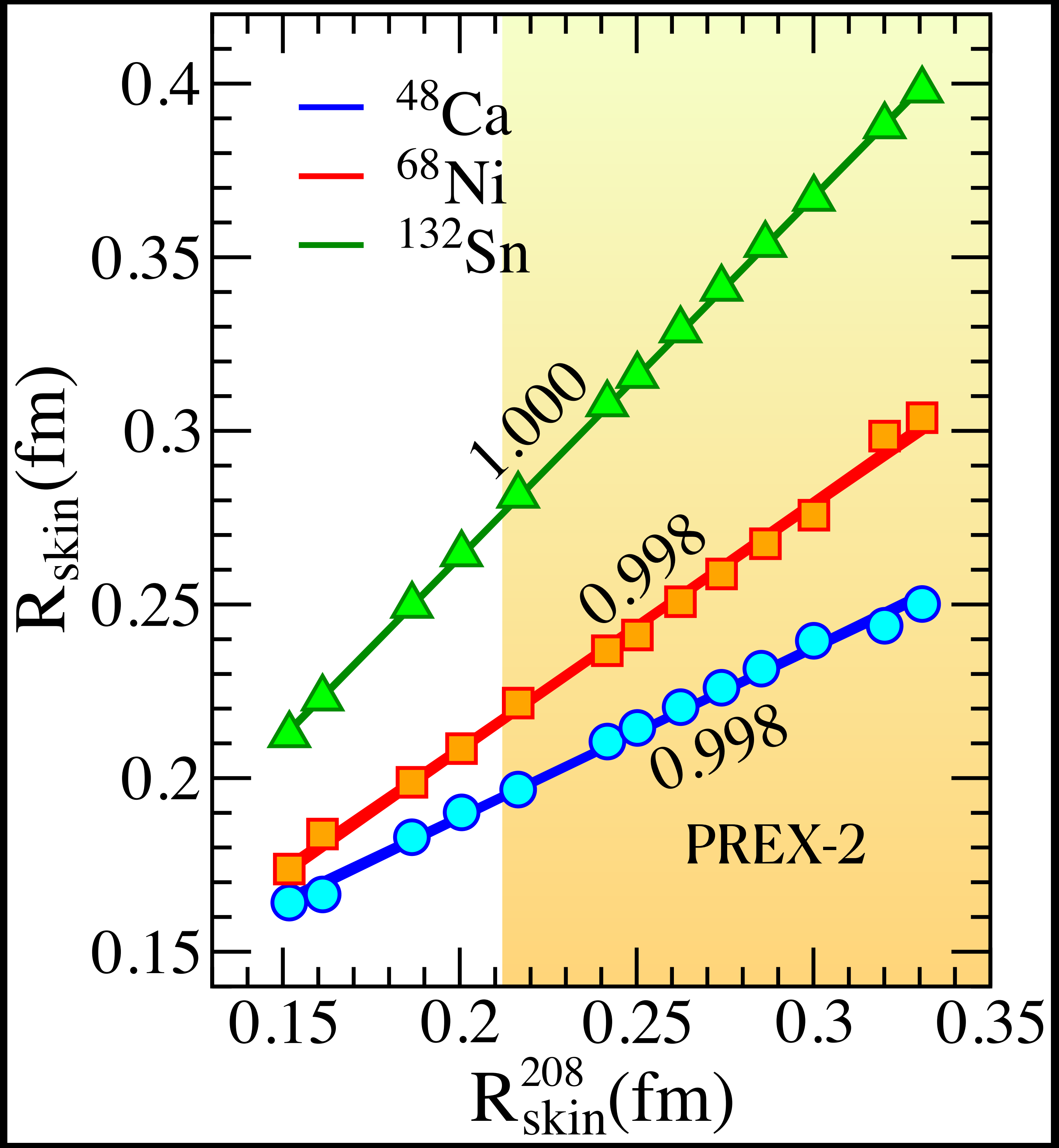
The PREX-CREX Dilemma

(No theoretical model can reproduce both!)

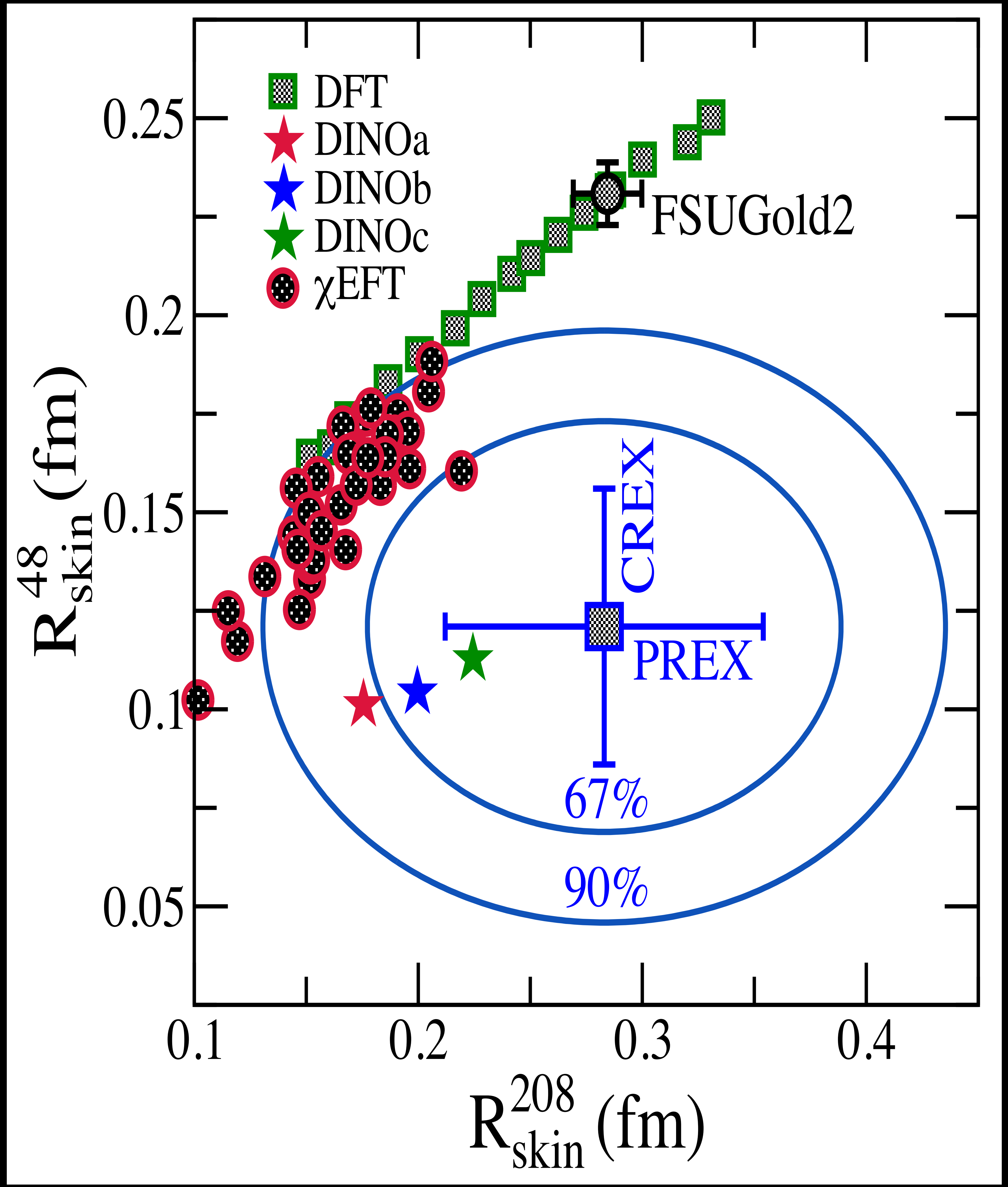
Combined Theoretical Analysis of the Parity-Violating Asymmetry for ^{48}Ca and ^{208}Pb

Paul-Gerhard Reinhard[1,*] Xavier Roca-Maza[2,†] and Witold Nazarewicz[3,‡]

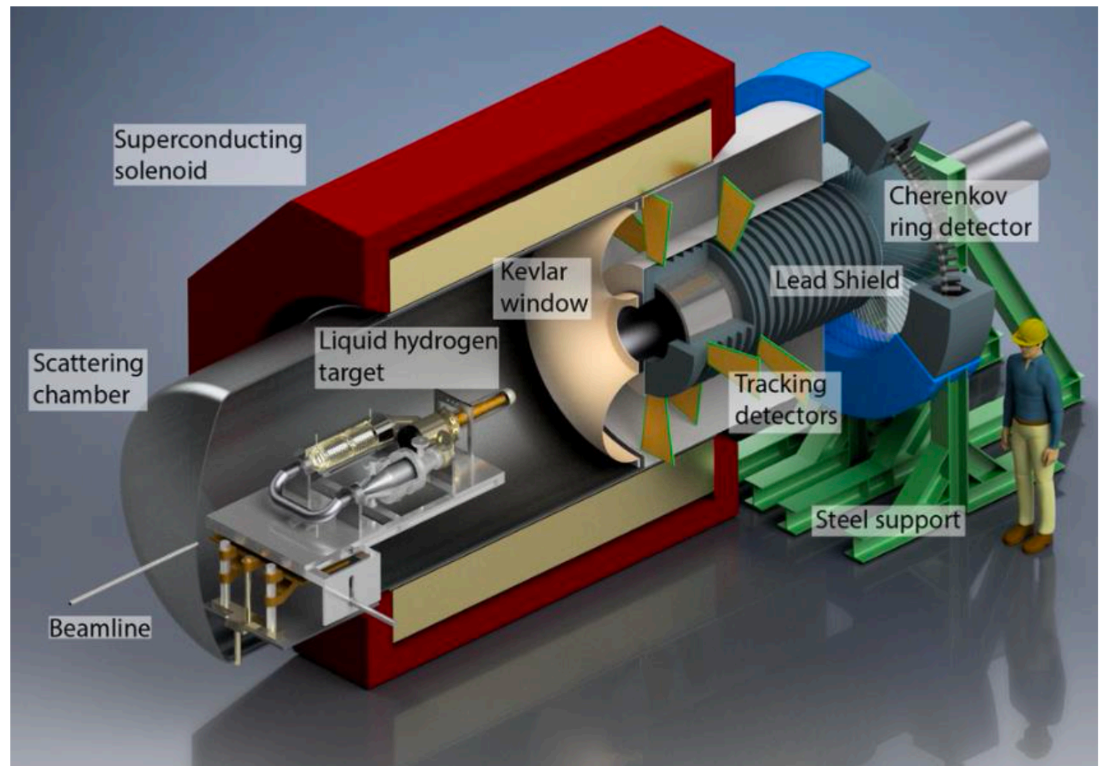
"We conclude that the simultaneous accurate description of the PV asymmetry in calcium and lead cannot be achieved by our models that accommodate a pool of global nuclear properties …"



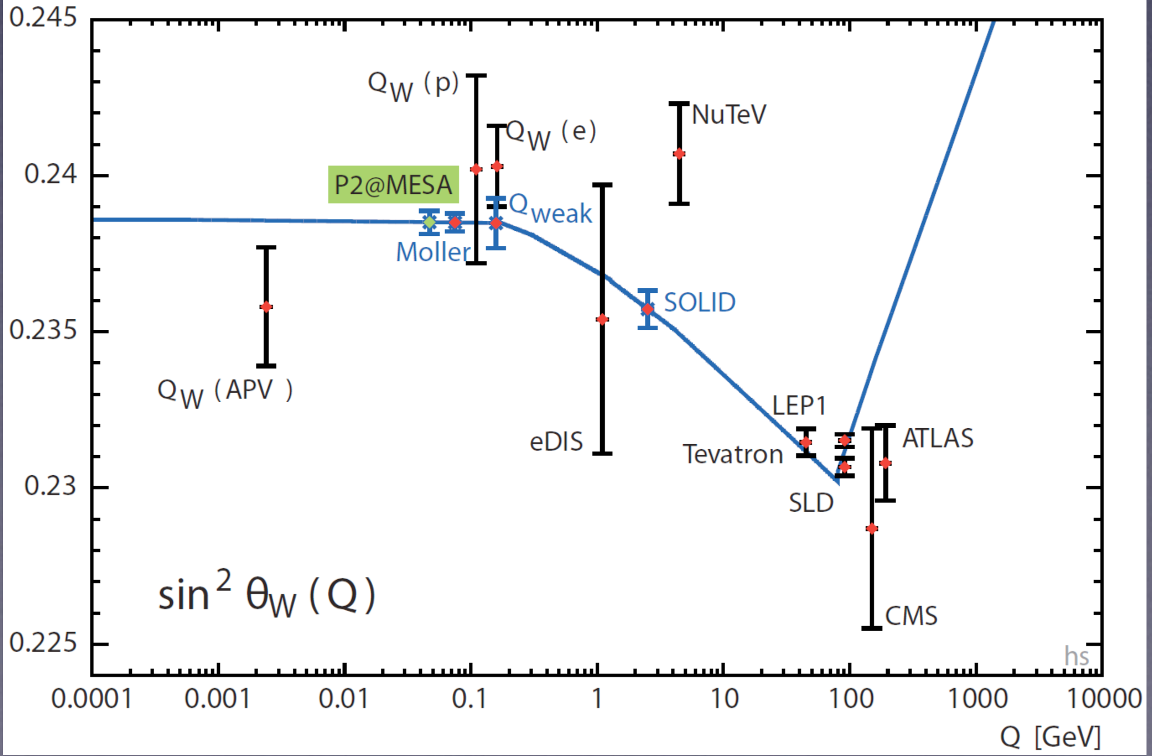
- Density Functional Theory in all its flavors predicts a strong correlation
- 34 "non-implausible" chiral interactions also display a similar correlation
- Modifications to existent DFT models can "break" the strong correlation — but at the expense of generating unphysical behavior in other observables



The P2 experiment

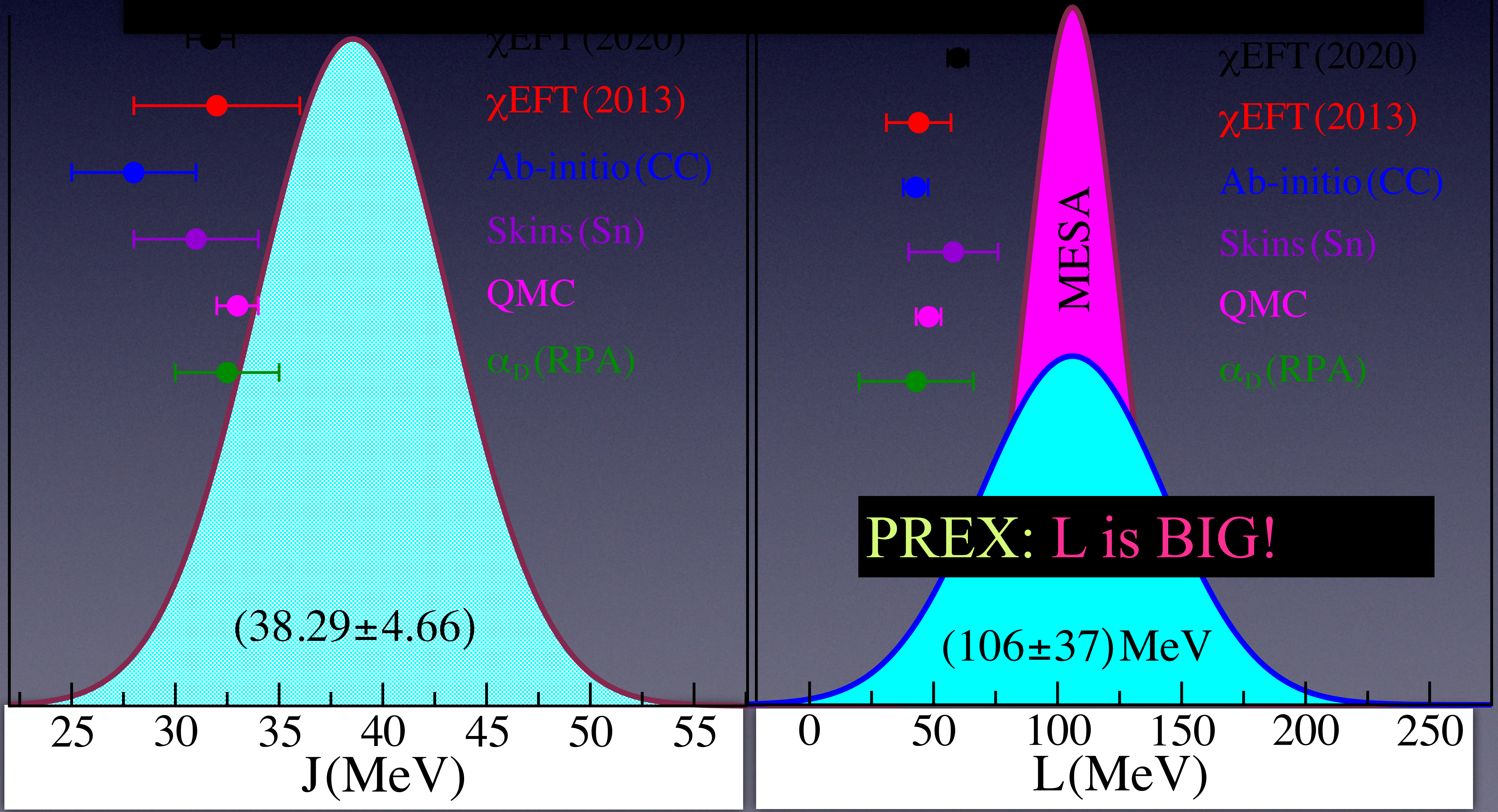


- Aimed to measure weak mixing angle $\sin^2\theta_W$ through parity-violating elastic electron scattering on hydrogen
- Uses solenoid spectrometer with tracking detectors and Cherenkov detector
- The same setup but with ^{208}Pb target can be used for neutron skin measurement to confirm/confront PREX results



Was PREX a Statistical Fluke?

The MESA Facility in Mainz will provide the most precise electroweak measurement of the neutron skin thickness of 208Pb (+/- 0.03 fm)



MREX@MESA

The P2 experiment

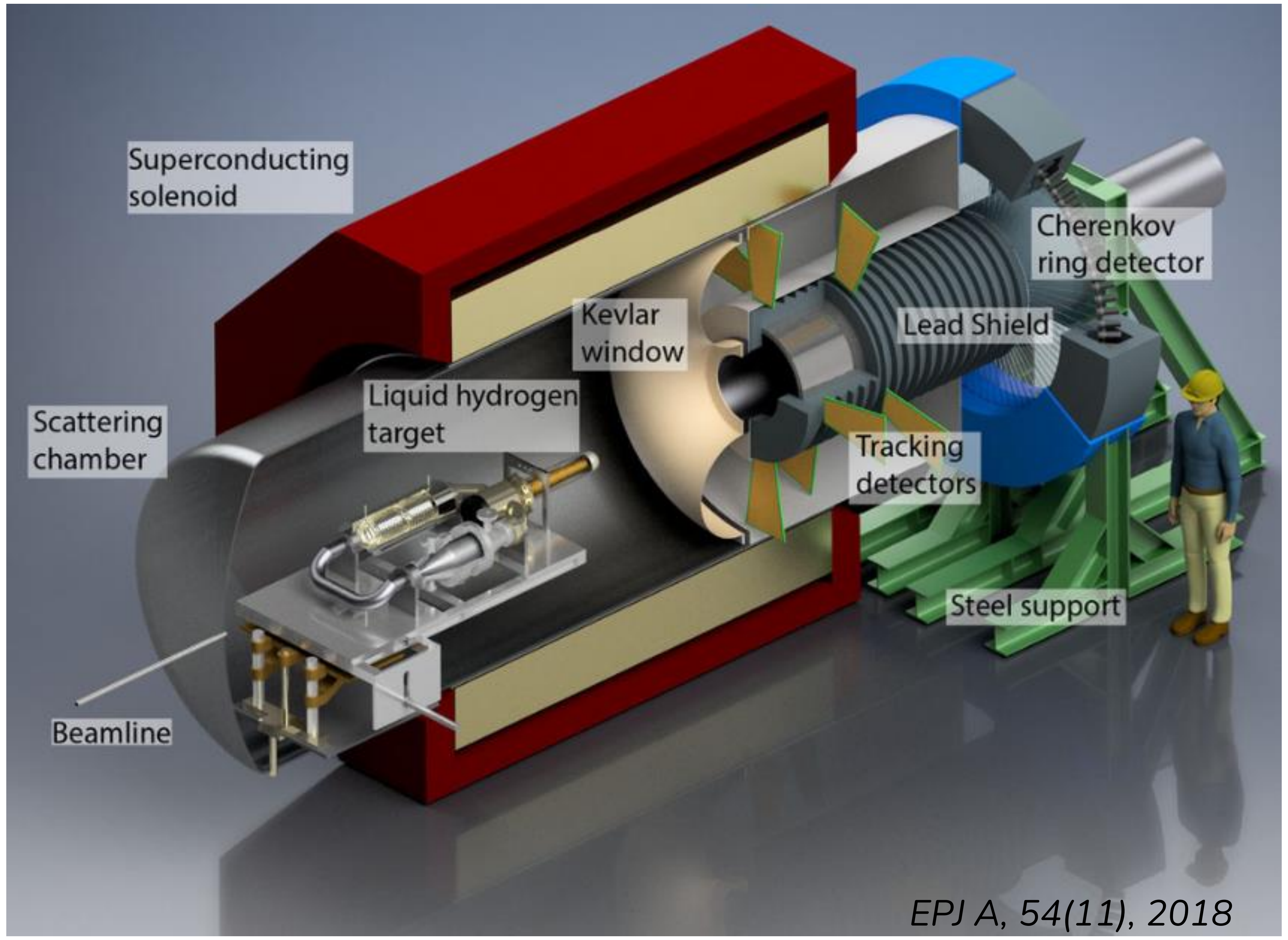


Fig.12 CAD drawing of the P2 detector

- Aimed to measure weak mixing angle $\sin^2\theta_W$ through parity-violating elastic electron scattering on hydrogen
- Uses solenoid spectrometer with tracking detectors and Cherenkov detector
- The same setup but with ^{208}Pb target can be used for neutron skin measurement to confirm/confront PREX results

Wednesday Afternoon Session: Concettina ("tt") Sfienti