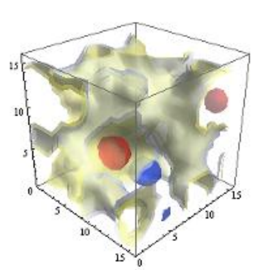
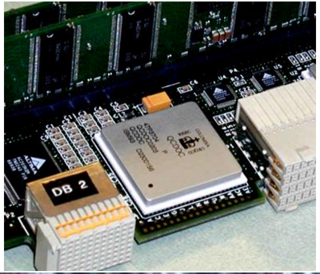
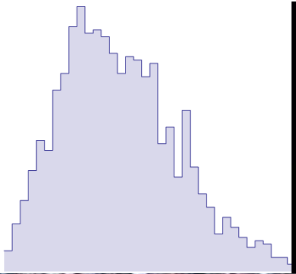
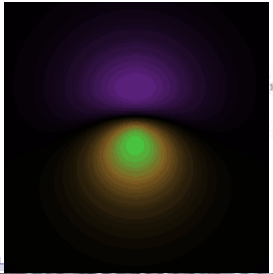
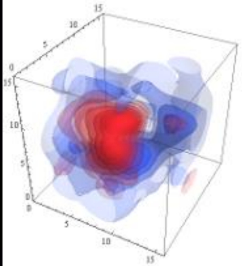
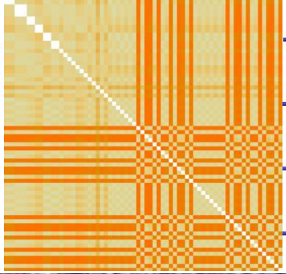
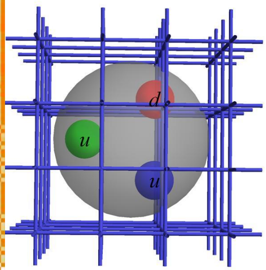
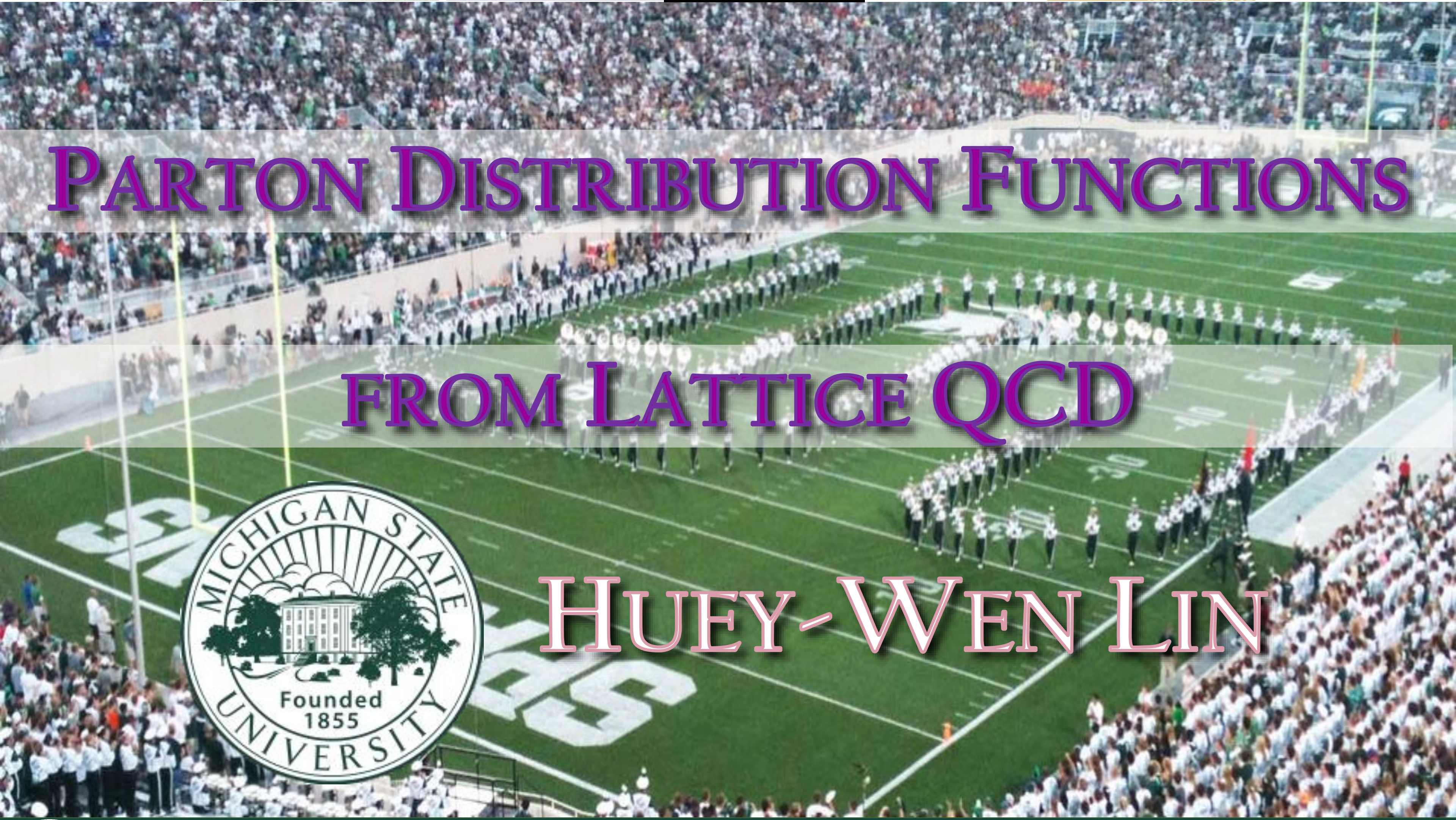








Parton Distribution Functions from Lattice QCD



Huey-Wen Lin

MICHIGAN STATE UNIVERSITY

What can we do on the lattice?

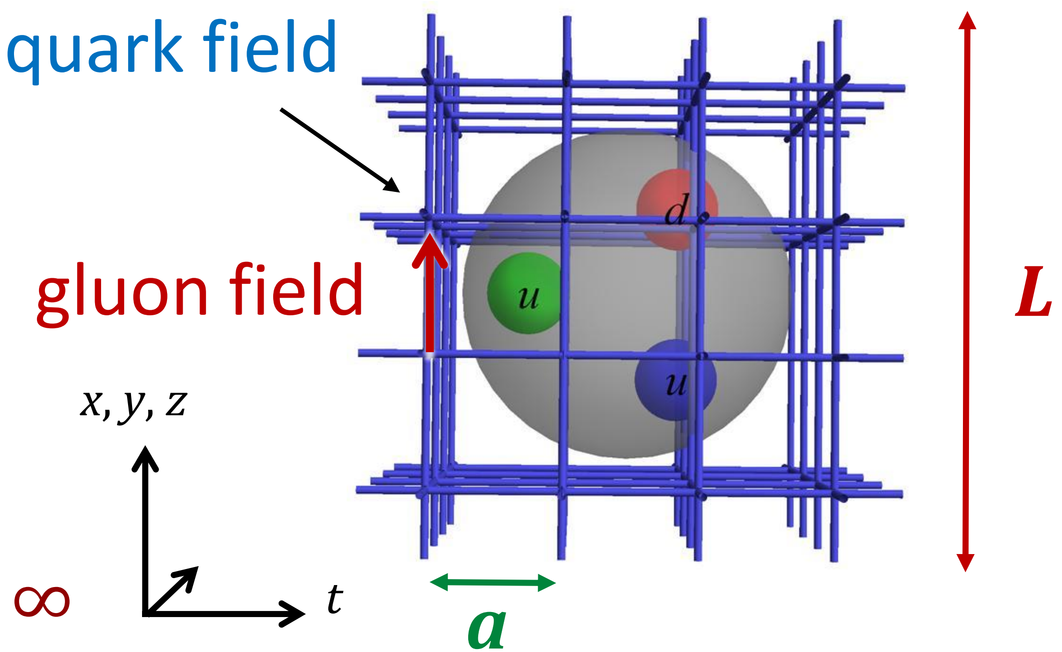
Lattice QCD 101

- Lattice QCD is an ideal theoretical tool for investigating strong-coupling regime of quantum field theories
- Physical observables are calculated from the path integral

$$\langle 0 | O(\bar{\psi}, \psi, A) | 0 \rangle = \frac{1}{Z} \int \mathcal{D}A\, \mathcal{D}\bar{\psi}\, \mathcal{D}\psi\, e^{iS(\bar{\psi},\psi,A)} O(\bar{\psi}, \psi, A)$$

in **Euclidian** space

- Quark mass parameter (described by m_π)
- Impose a UV cutoff
  discretize spacetime
- Impose an infrared cutoff
  finite volume



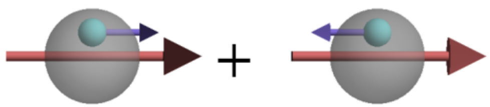
- Recover physical limit

$$m_\pi \to m_\pi^{\text{phys}},\ a \to 0,\ L \to \infty$$

PDFs on the Lattice

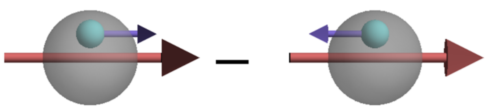
§ Lattice calculations rely on operator product expansion, only provide moments

most well known



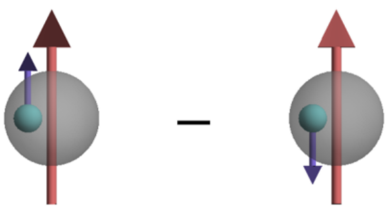
Quark density/unpolarized

$$\langle x^n \rangle_q = \int_{-1}^{1} dx\, x^n q(x)$$



Helicity
longitudinally polarized

$$\langle x^n \rangle_{\Delta q} = \int_{-1}^{1} dx\, x^n \Delta q(x)$$



Transversity
transversely polarized

$$\langle x^n \rangle_{\delta q} = \int_{-1}^{1} dx\, x^n \delta q(x)$$

very poorly known

§ True distribution can only be recovered with all moments

Problem with Moments

§ For higher moments, ops mix with lower-dimension ops

- Renormalization is difficult too

§ Relative e

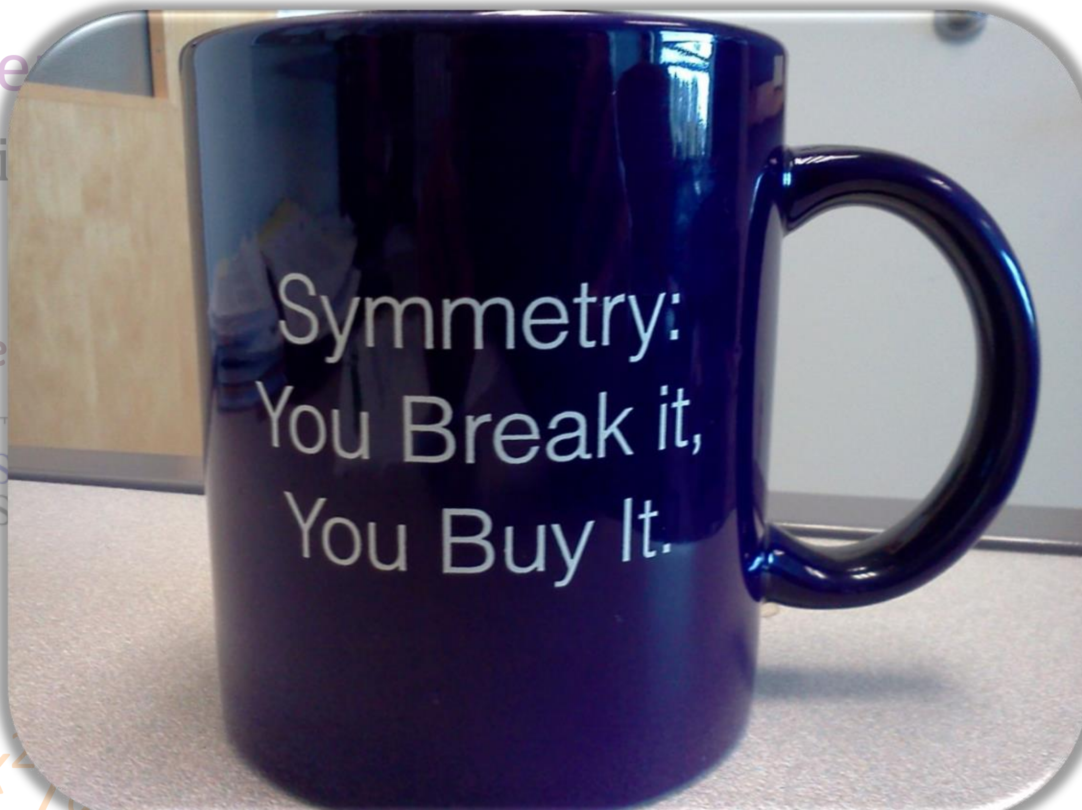
- Calculati

Dolgov et

Göckeler e

SAM):

clover



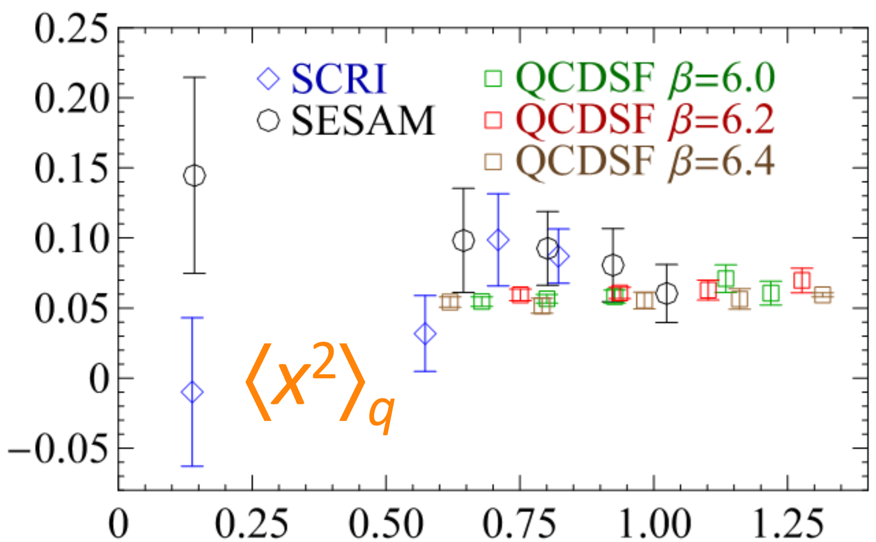
Problem with Moments

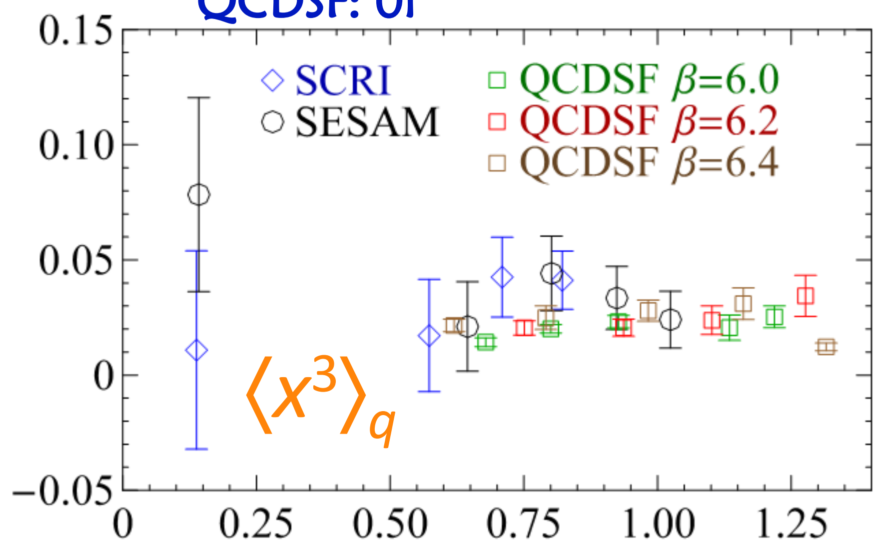
- § For higher moments, ops mix with lower-dimension ops
  - Renormalization is difficult too
- § Relative error grows in higher moments
  - Calculation would be costly and difficult

Dolgov et al. PRD66, 034506 (2002)
Göckeler et al. PRD71, 114511 (2005)

LHPC (SCRI, SESAM):
2f, Wilson and clover

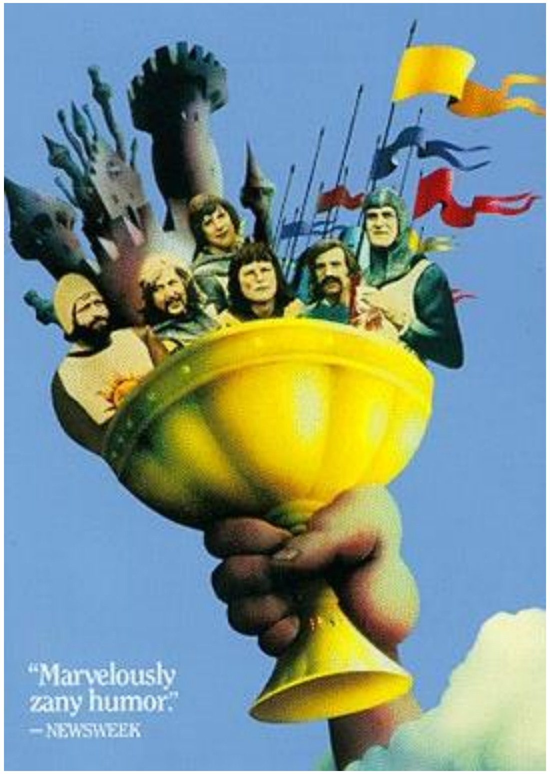
QCDSF: 0f





Beyond Traditional Moments?

- § Longstanding obstacle!
- § Holy grail of structure calculations
- § Applies to many structure quantities:
  - Generalized parton distributions (GPDs)
  - Transverse-momentum distributions (TMD)
  - Meson distribution amplitudes…
  - Wigner distribution



Beyond Traditional Moments?

§ Reaching for higher moments

- Fictitious heavy quarks (Detmold and Lin, hep-lat/0507007)
- Smeared lat. ops (Davoudi et al. 1204.4146)

§ Direct calculation of *x* dependence

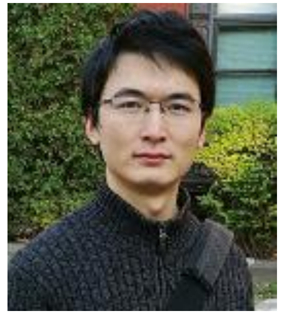
- Hadronic tensor currents (Liu et al., hep-ph/9806491, … 1603.07352)
- Inversion method/OPE without OPE (QCDSF, hep-lat/9809171, …1703.01153)
- Euclidean correlation functions (RQCD, 1709.04325)
- Lattice cross-section method (Y.-Q. Ma and J. Qiu, 2014, 2017)
- Large-momentum effective theory (LaMET) and variations
  - Original LaMET ("quasi-PDF" method) This talk
  - Pseudo-PDF method: differs in FT (A. Radyushkin, 2017)
  - Smeared quasi-PDF (C. Monahan and K. Orginos, 2017)

Lattice Parton Physics Project (LP^3)

https://www.pa.msu.edu/~hwlin/LP3/

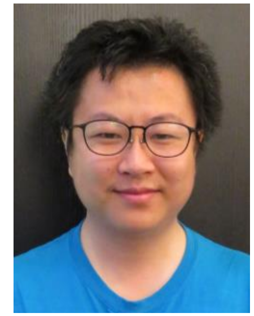
HL
(MSU)

Xiangdong Ji
(UMD)



Luchang Jin
(Conn)

Ruizi Li
(MSU)



Yi-Bo Yang
(MSU)

Yong Zhao
(MIT)

International collaborators

Jiunn-Wei Chen
(NTU)

Yu-Sheng Liu
(SJTU)

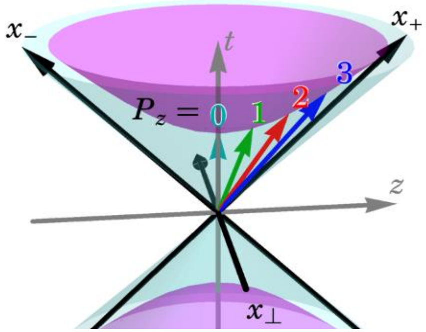
Andreas Schäfer
(Regensburg)

Jian-Hui Zhang
(Regensburg)

Large-Momentum Effective Theory (LaMET)

X. Ji, PRL. 111, 262002 (2013)

§ Calculate the parton distributions through the infinite-momentum frame Feynman, Phys. Rev. Lett. 23, 1415 (1969)



§ Finite-momentum quark distribution (quasi-distribution)

- Suggested operator:

$$\tilde{q}(x,\mu,P_z) = \int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \middle| \overline{\psi}(z)\gamma_z \exp\left(-ig\int_0^z dz'\, A_z(z')\right)\psi(0) \middle| P \right\rangle$$

$x = k_z/P_z$

Lattice z coordinate

hadron momentum $P_\mu = \{P_t, 0, 0, P_z\}$

Product of lattice gauge links

§ Take the infinite-P_z limit to recover lightcone functions

- Just another limit to take, like taking $a \to 0$ or $V \to \infty$

Progress in the theoretical development of LaMET

- **Renormalization:**

Ji and Zhang, 2015; Ishikawa et al., 2016, 2017; Chen, Ji and Zhang, 2016;

Xiong, Luu and Meiβner, 2017; Constantinou and Panagopoulos, 2017; Ji, Zhang, and Y.Z., 2017; J. Green et al., 2017; Ishikawa et al. (LP3), 2017; Wang, Zhao and Zhu, 2017; Spanoudes and Panagopoulos, 2018.

- **Factorization:**

Ma and Qiu, 2014, 2015, 2017; Izubuchi, Ji, Jin, Stewart and Y.Z., 2018.

- **One-loop matching:**

Xiong, Ji, Zhang and Y.Z., 2014; Ji, Schaefer, Xiong and Zhang, 2015; Xiong and Zhang, 2015; Constantinou and Panagopoulos, 2017; I. Stewart and Y. Z., 2017; Wang, Zhao and Zhu, 2017; Izubuchi, Ji, Jin, Stewart and Y.Z., 2018.

- **Power corrections:**

J.-W. Chen et al., 2016; A. Radyushkin, 2017.

- **Transvers momentum dependent parton distribution function:**

Ji, Xiong, Sun, Yuan, 2015; Ji, Jin, Yuan, Zhang and Y.Z., 2018; Ebert, Stewart and Y.Z., in progress.

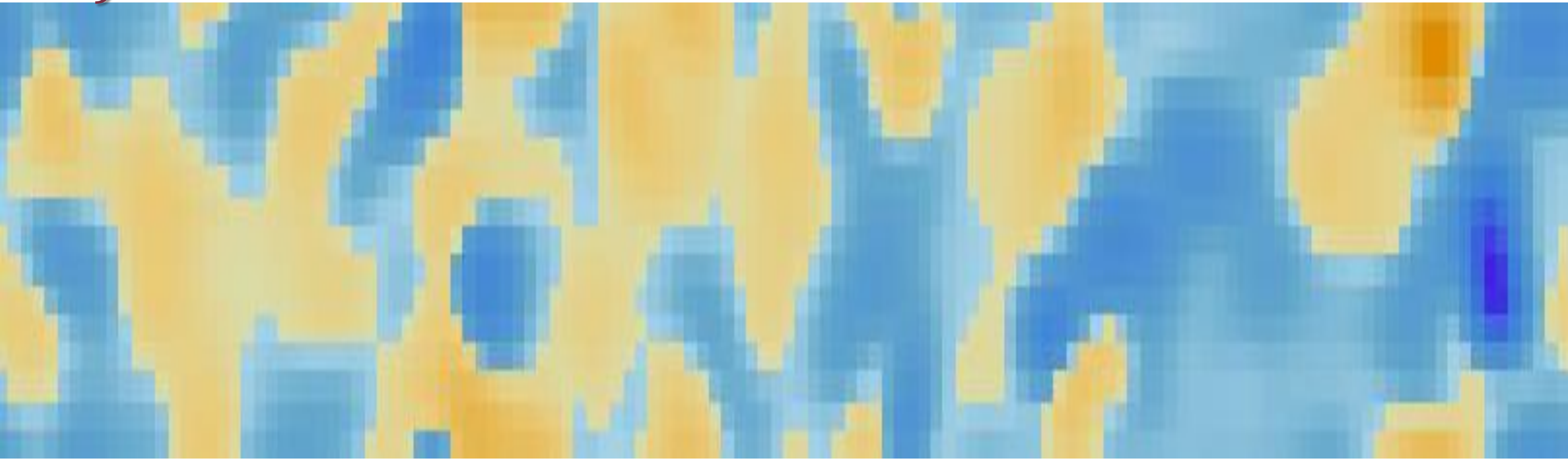
Slide credit: Yong Zhao, CIPANP 2018 Plenary talk

LaMET: Step-by-Step

Large-Momentum Effective Theory for PDFs

X. Ji, PRL. 111, 262002 (2013)

1) Calculate nucleon matrix elements on the lattice



Thanks to MILC collaboration for sharing their 2+1+1 HISQ lattices

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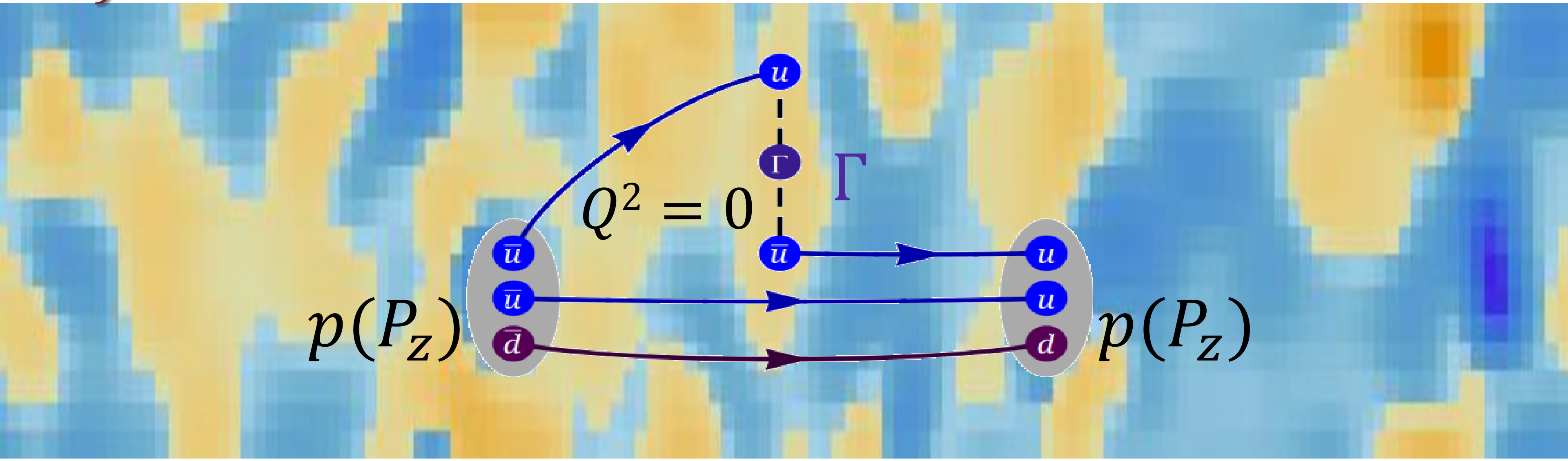

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LaMET: Step-by-Step

Large-Momentum Effective Theory for PDFs

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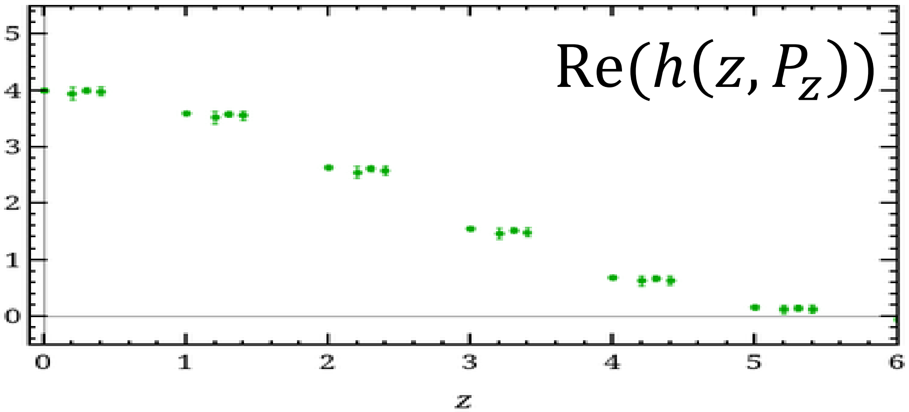
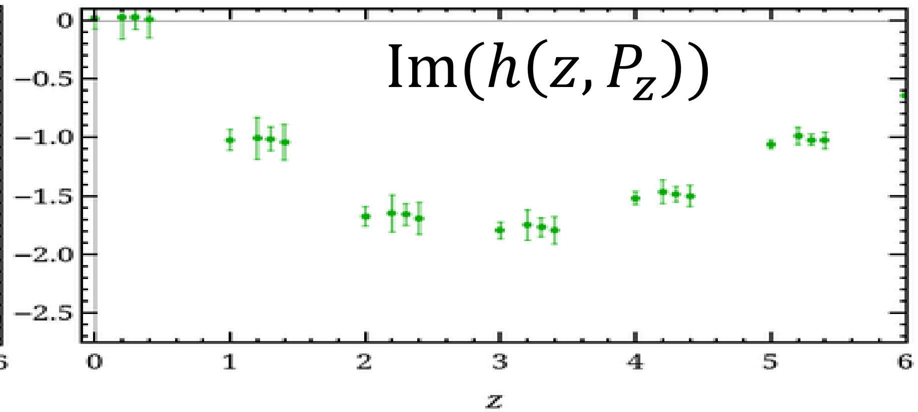
$$h(z, Pz) = \left\langle P \left| \bar{\psi}(z)\, \gamma_z \exp\left(-ig \int_0^z dz'\, A_z(z') \right) \psi(0) \right| P \right\rangle$$





$P_z =$ 2.6 GeV

$M_\pi \approx$ **135 MeV**, $a \approx 0.09$ fm

LP[3] 1804.01483

Ruizi Li

Blinded 3-state fits produced consistent results

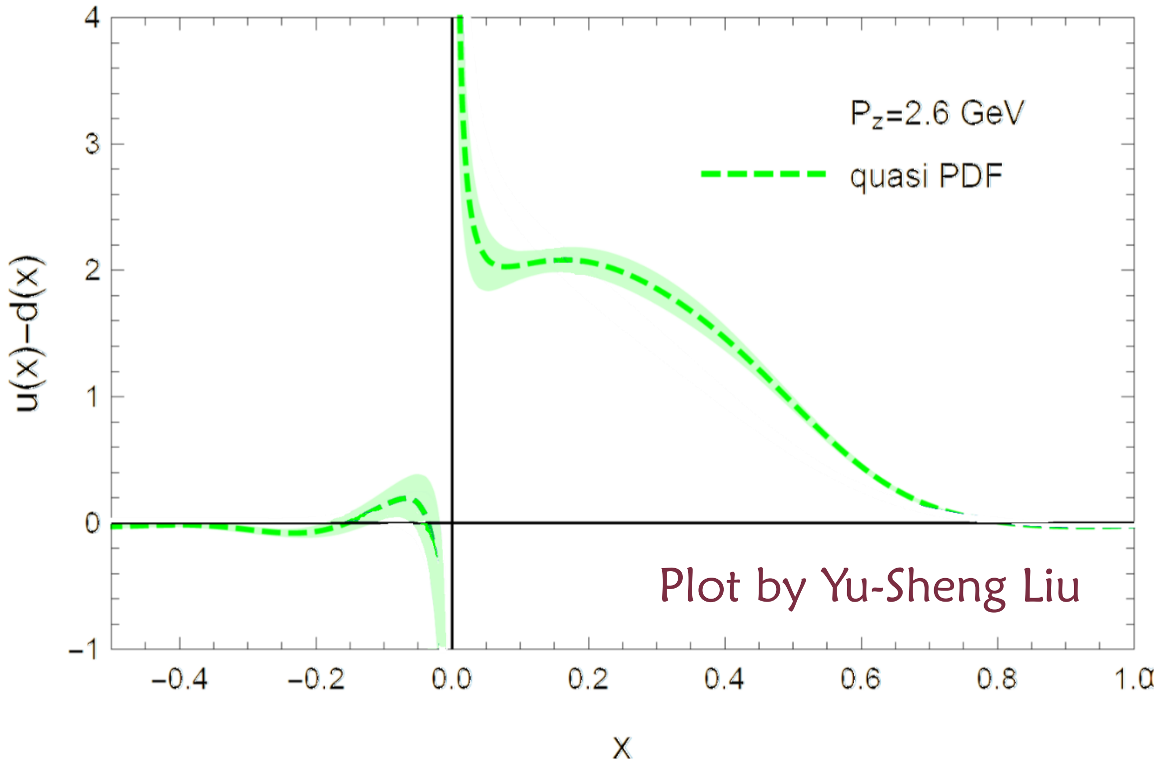
LaMET: Step-by-Step

Large-Momentum Effective Theory for PDFs

X. Ji, PRL. 111, 262002 (2013)

2) Compute "quasi-distribution" via

$$\tilde{q}(x,\mu,P_z) = \int \frac{dz}{4\pi} e^{-i\,x\,zP_z}\, h(z,\mu,P_z)$$



Yu-Sheng Liu

LaMET: Step-by-Step

Large-Momentum Effective Theory for PDFs

X. Ji, PRL. 111, 262002 (2013)

3) Recover true distribution (take $P_z \to \infty$ limit)

$$\tilde{q}(x,\mu,P_z) = \int_{-\infty}^{\infty} \frac{dy}{|y|} Z\left(\frac{x}{y}, \frac{\mu}{P_z}\right) \boldsymbol{q}(\boldsymbol{y},\boldsymbol{\mu}) + \mathcal{O}\left(M_N^2/P_z^2\right) + \mathcal{O}\left(\Lambda_{\mathrm{QCD}}^2/P_z^2\right)$$

Finite $P_z \leftrightarrow \infty$ perturbative matching

$$Z(x,\mu/P_z) = C\delta(x-1) - \frac{\alpha_s}{2\pi} Z^{(1)}(x,\mu/P_z)$$

Non-singlet case only

LP3, 1807.06566

Yu-Sheng Liu (SJTU)

Yong Zhao (MIT)

LaMET: Step-by-Step

Large-Momentum Effective Theory for PDFs

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Dominant correction
(for nucleon);
known scaling form

HWL et al. 1402.1462

J.-W. Chen et al, 1603.06664

LaMET: Step-by-Step

Large-Momentum Effective Theory for PDFs

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complicated higher-twist operator;
smaller P_z correction for nucleon
J.-W. Chen et al, 1603.06664 and references within
(extrapolate it away)

§ Some similarity to more broadly-studied HQET…

$$O\left(\frac{m_b}{\Lambda}\right) = Z\left(\frac{m_b}{\Lambda}, \frac{\Lambda}{\mu}\right) o(\mu) + \mathcal{O}\left(\frac{1}{m_b}\right) + \cdots$$

LaMET: Step-by-Step

Large-Momentum Effective Theory for PDFs

X. Ji, PRL. 111, 262002 (2013)

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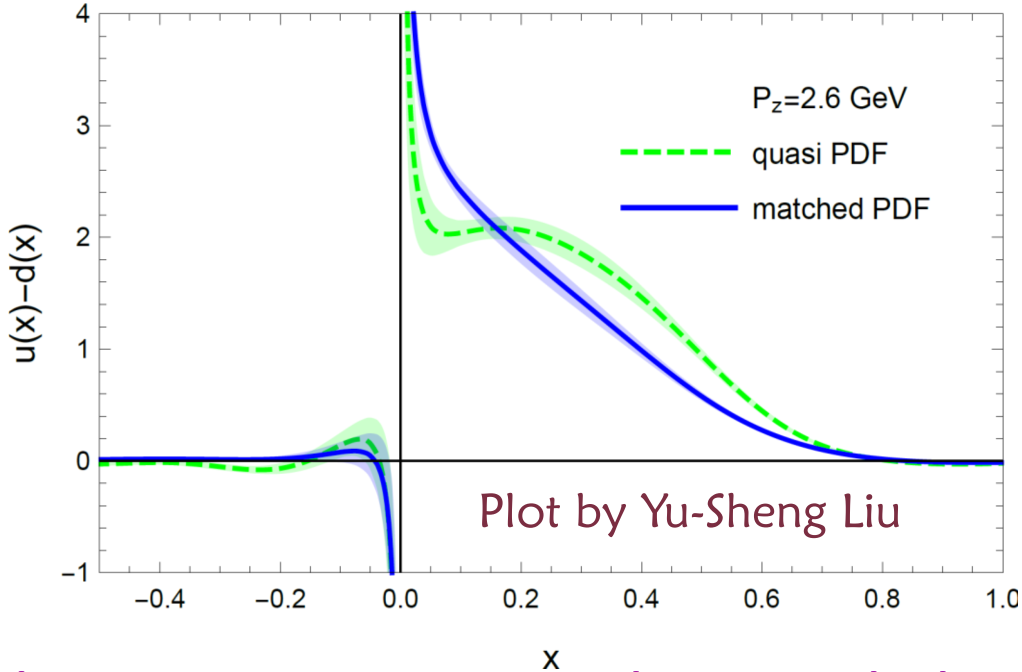
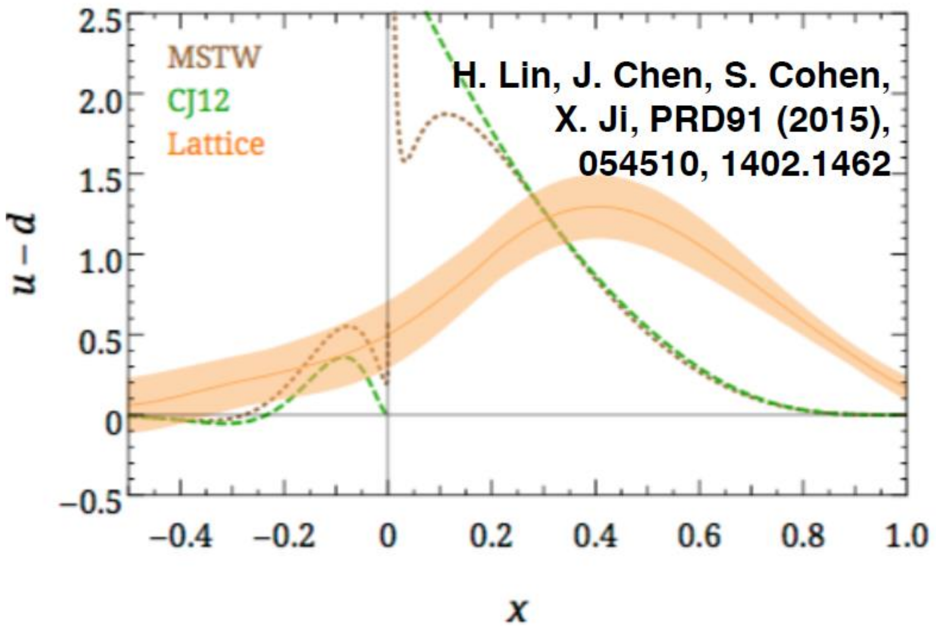
$a \approx 0.09$ fm
LP³ 1804.01483



- Matching is a crucial step in recovering the true lightcone distribution

Nucleon Unpolarized PDF

§ From 2014 to 2018

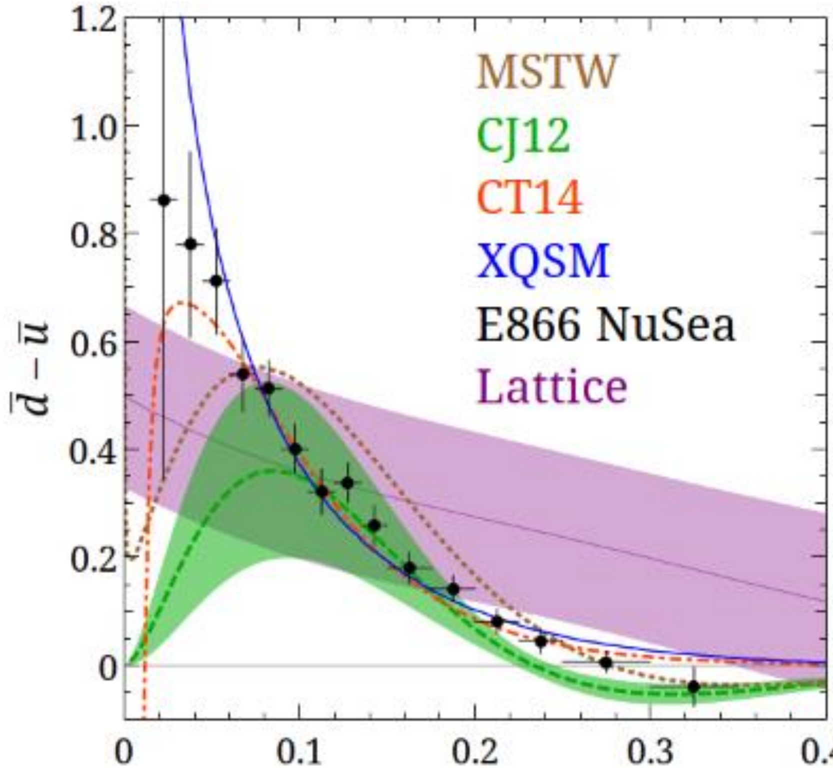


First result in 2014!

- $M_\pi \approx$ **310 MeV**, $a \approx 0.12$ fm ($M_\pi L \approx 4.5$)
- Largest $P_z \approx 1.3$ GeV
- 1-loop $\overline{\text{MS}}$ matching + target-mass correction

Nucleon Unpolarized PDF

§ From 2014 to 2018



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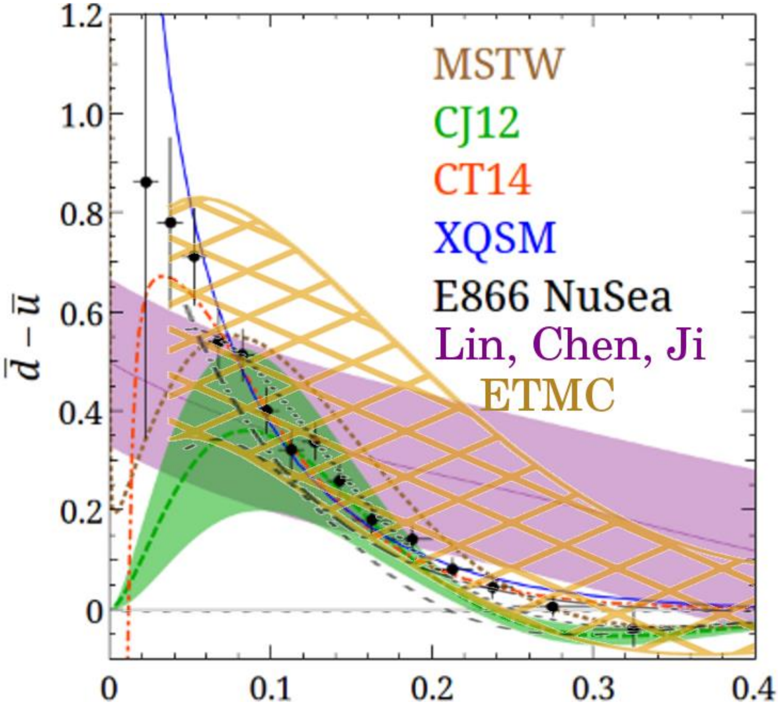
- 1-loop $\overline{\text{MS}}$ matching + target-mass correction

First sea flavor asymmetry ever calculated!

Overcomes
decades of obstacles in
LQCD structure calculations

Nucleon Unpolarized PDF

§ From 2014 to 2018



Similar results repeated by ETMC,
at $M_\pi \approx 373$ MeV, a$\approx 0.8fm$

ETMC, 1504.07455

Nucleon Unpolarized PDF

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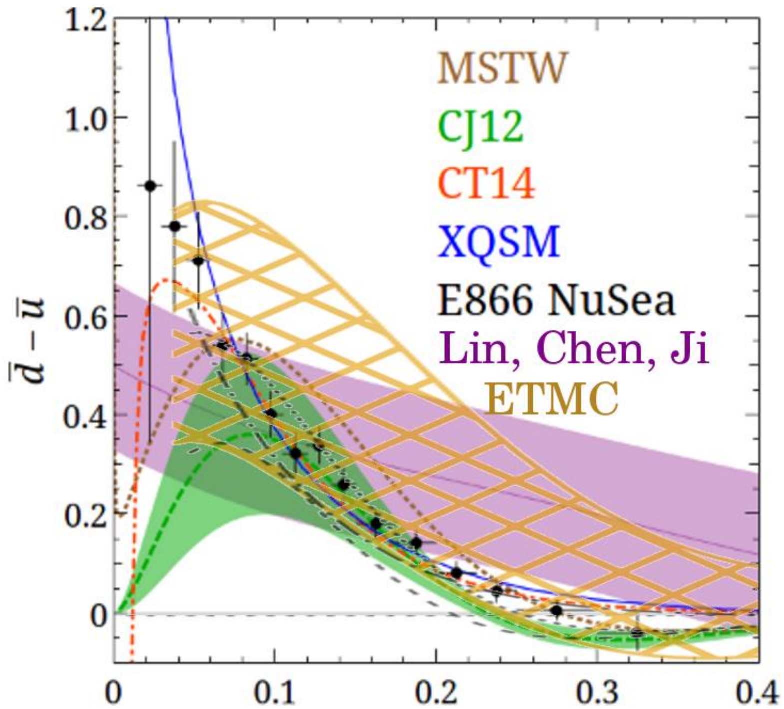


Similar results repeated by ETMC,
at $M_\pi \approx 373$ MeV, a$\approx 0.8fm$
ETMC, 1504.07455

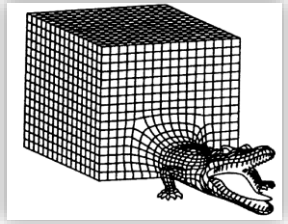
§ Updated results in 2017/18

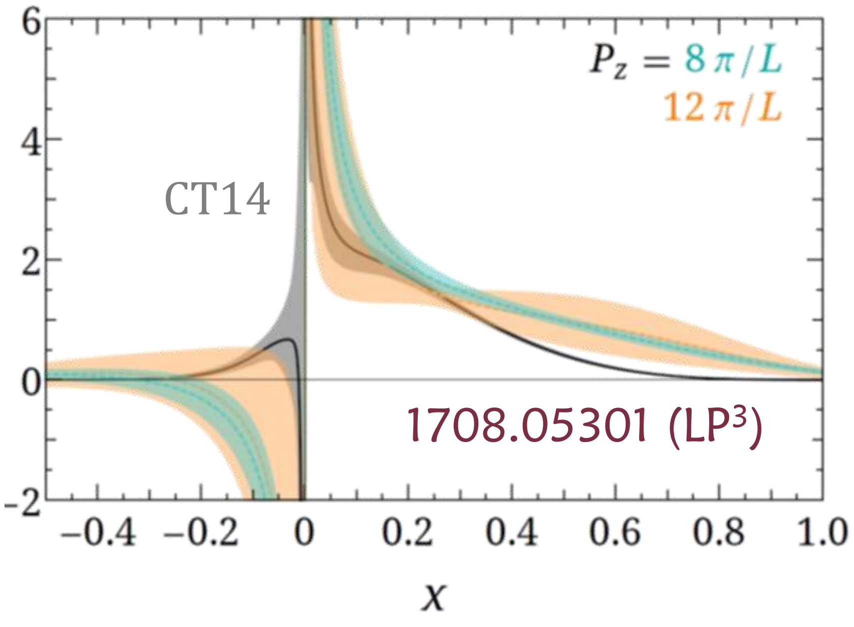
- RI/MOM nonperturbative renormalization and corresponding matching to lightcone distribution
- Improved quasi-distribution definition

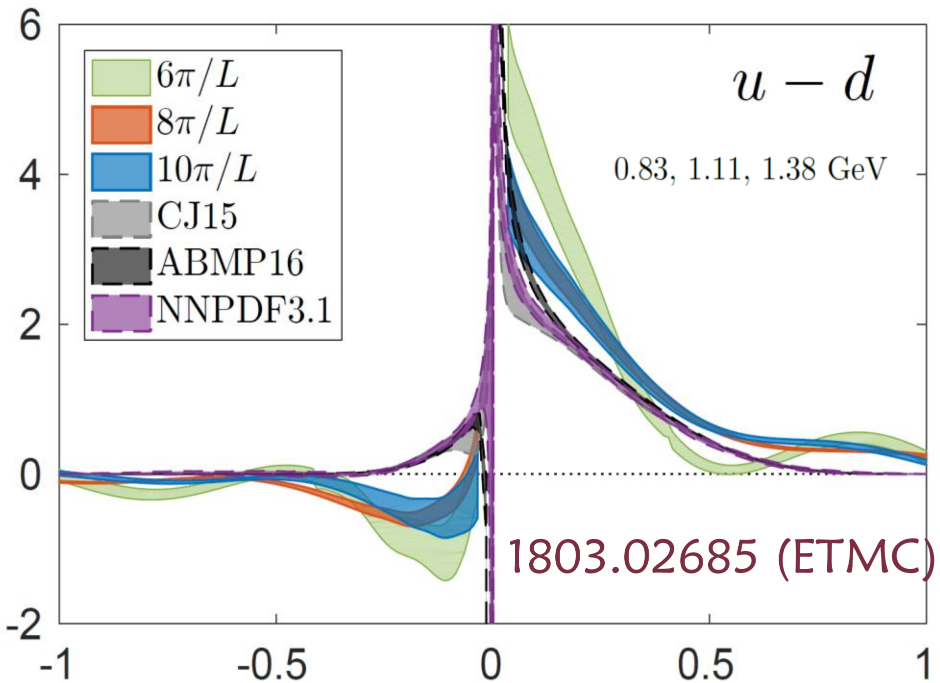
Physical Pion Mass Results

§ Exciting! Two collaborations' results at physical pion mass

- Boost momenta $P_z \leq 1.4$ GeV
- Study of systematics still needed





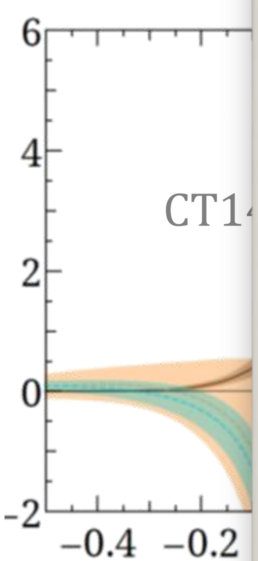


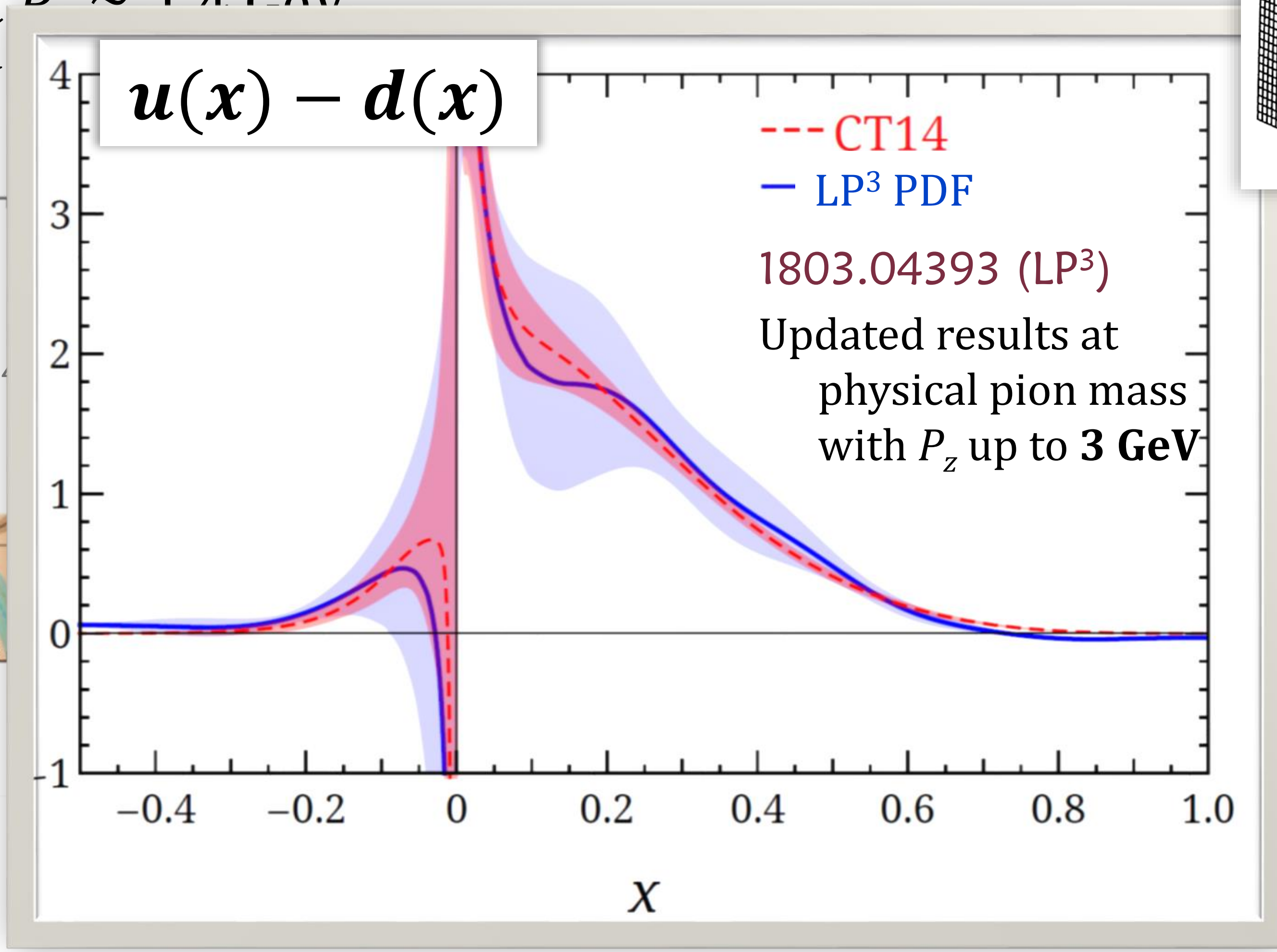
Not use any parametrization form like $xf(x,\mu_0) = a_0 x^{a_1}(1-x)^{a_2}P(x)$

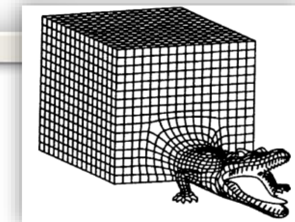
Physical Pion Mass Results

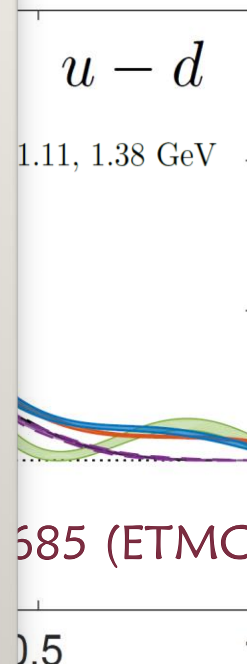
§ Exciting! Two collaborations' results at physical pion mass

- Boost P ~ 1.4 GeV
- Study









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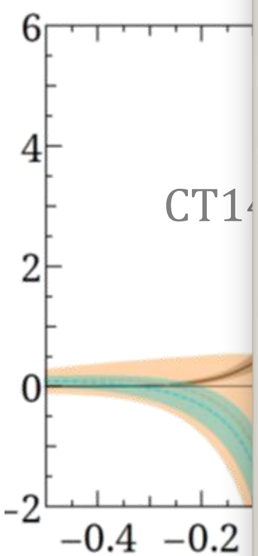
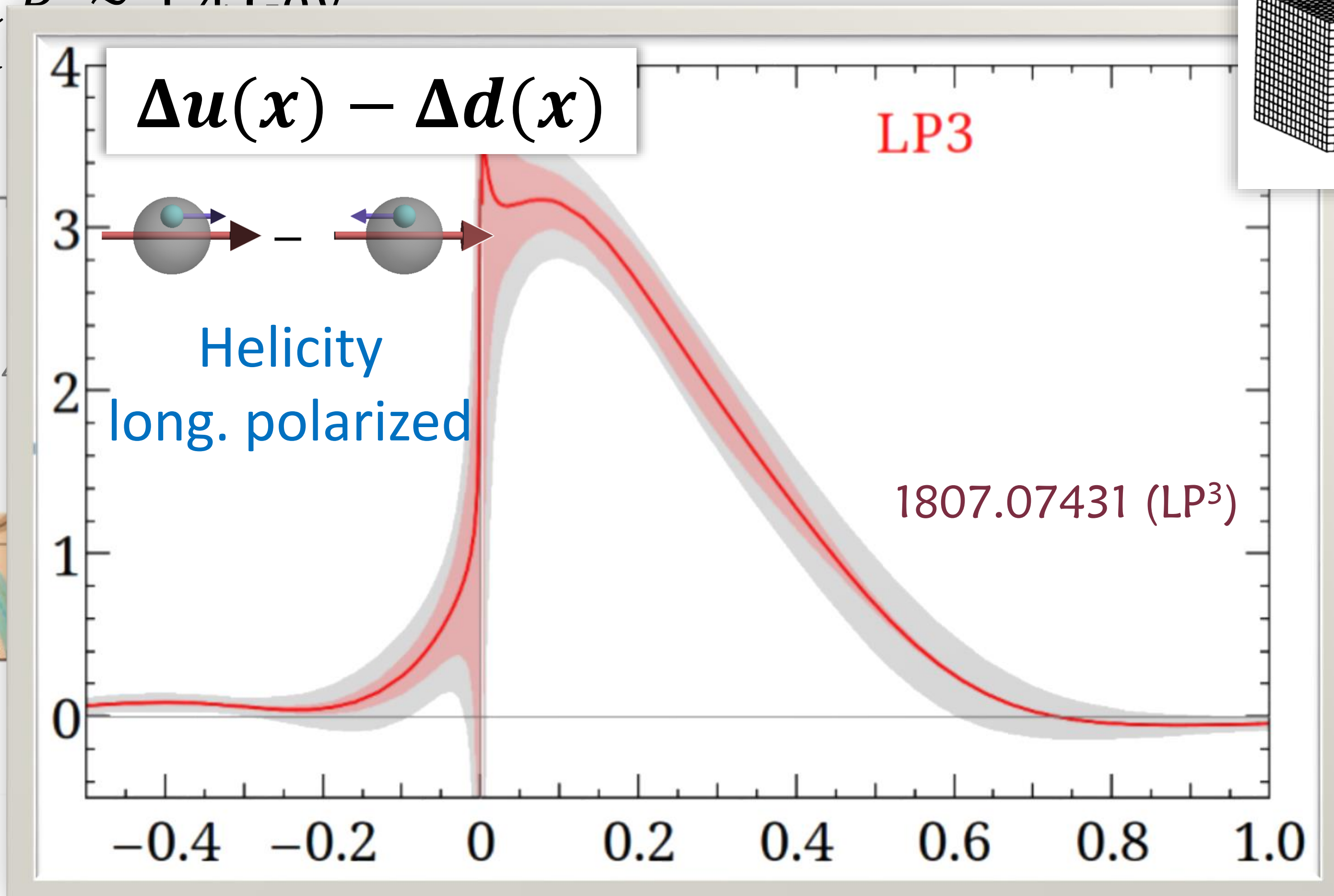
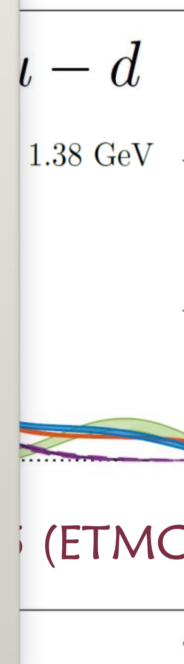
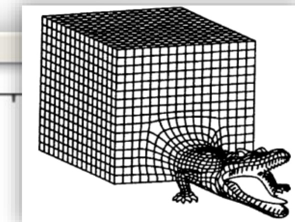




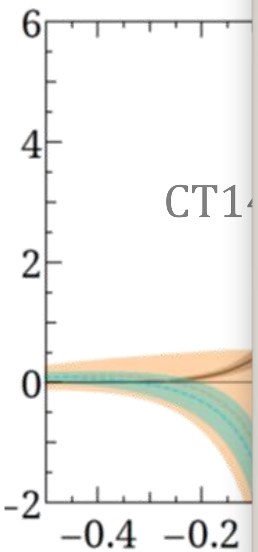


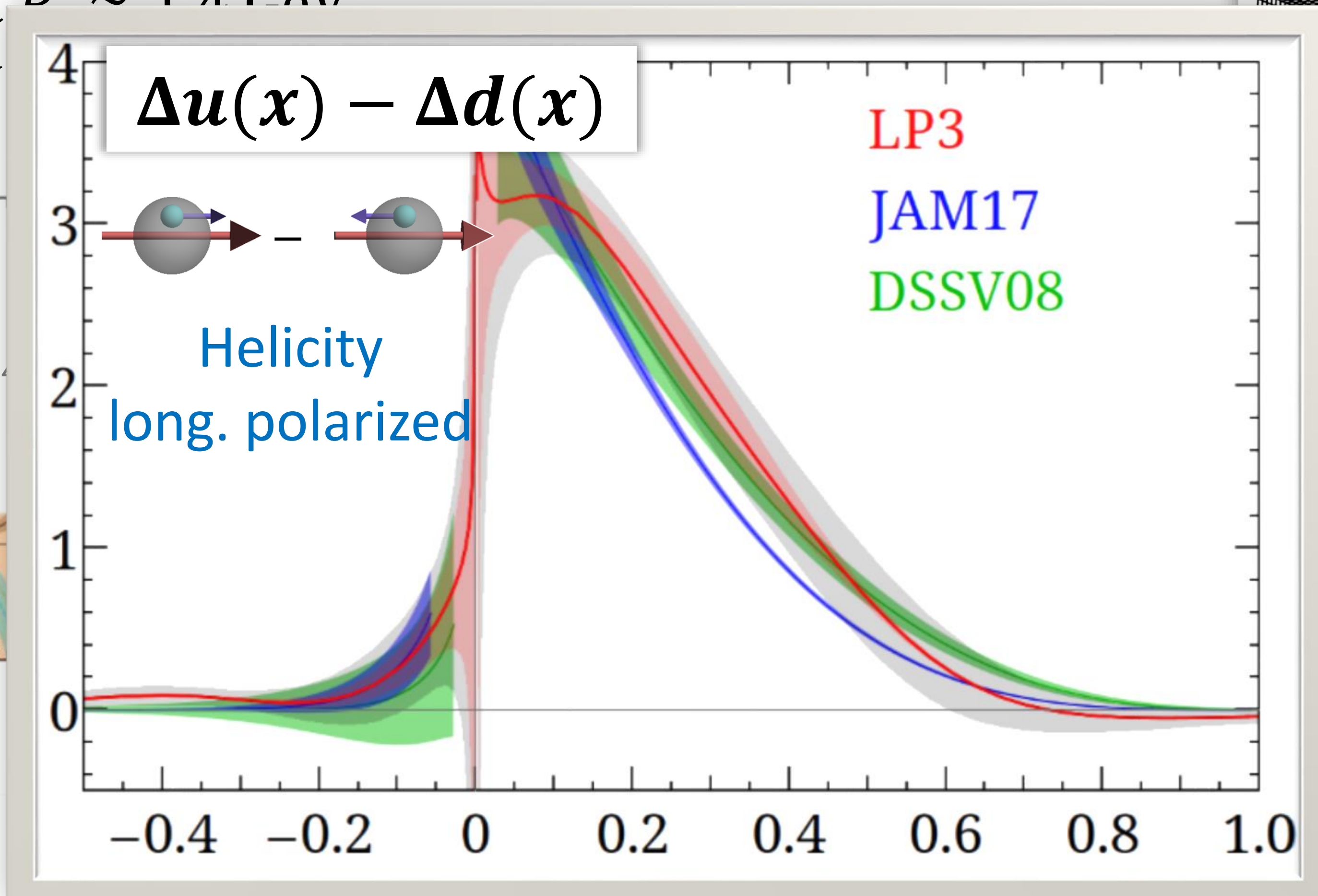


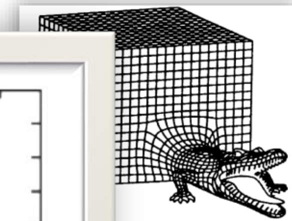
Physical Pion Mass Results

§ Exciting! Two collaborations' results at physical pion mass

- Boost $P \sim 1.4$ GeV
- Study







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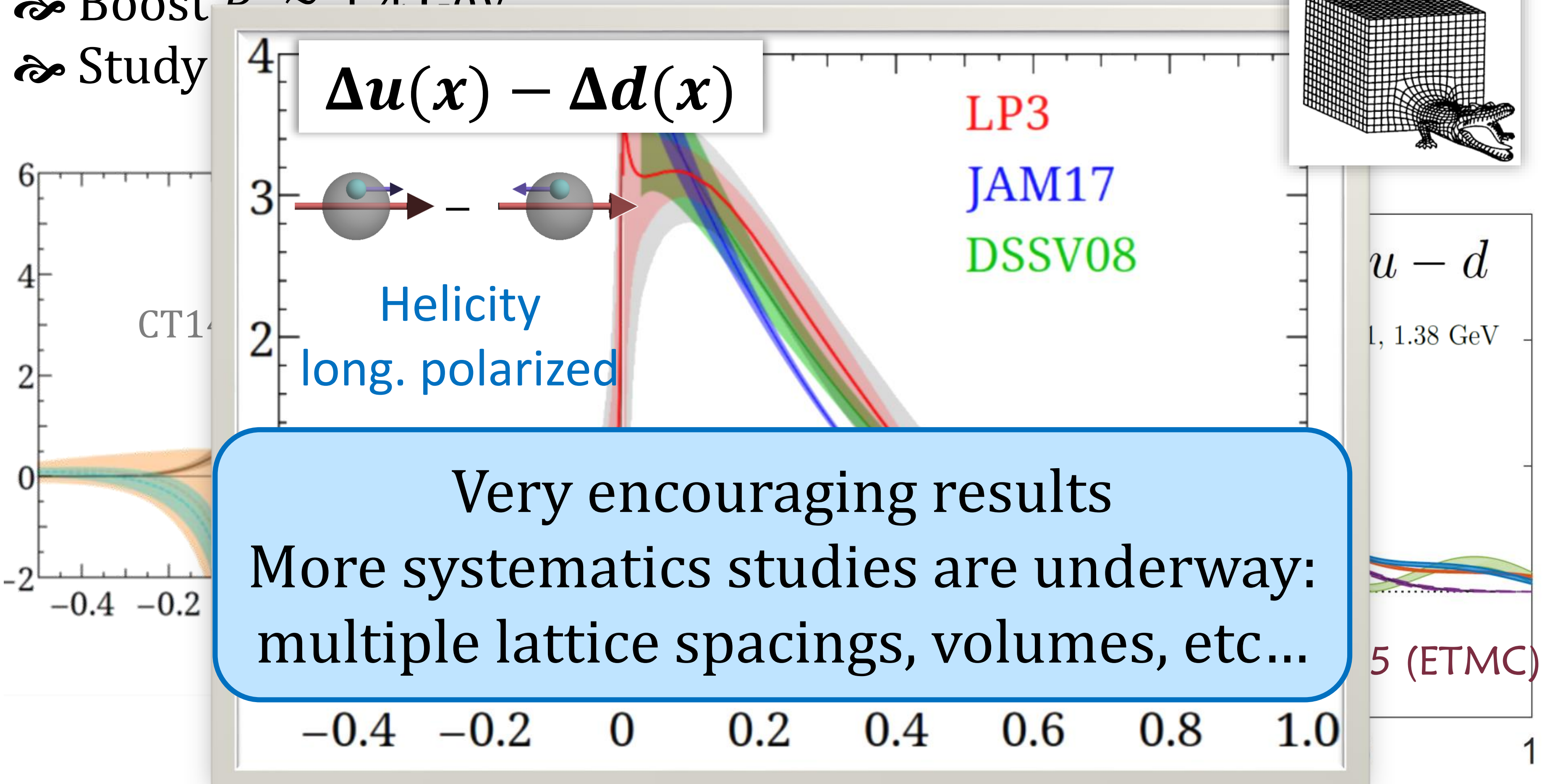
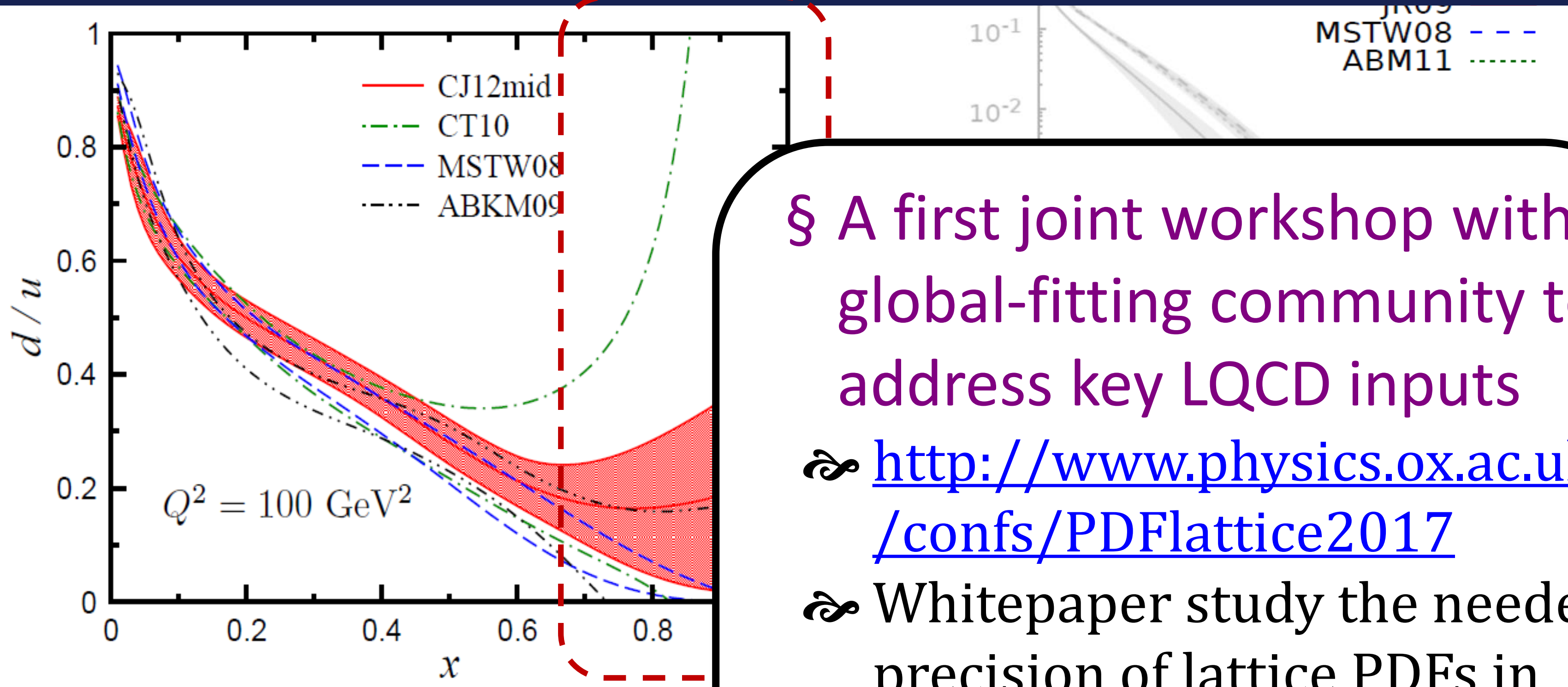
- Boost $P \sim 1.4$ GeV
- Study



Very encouraging results
More systematics studies are underway:
multiple lattice spacings, volumes, etc…

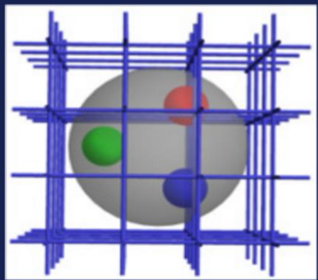
Parton Distributions and Lattice Calculations in the LHC era (PDFLattice 2017)

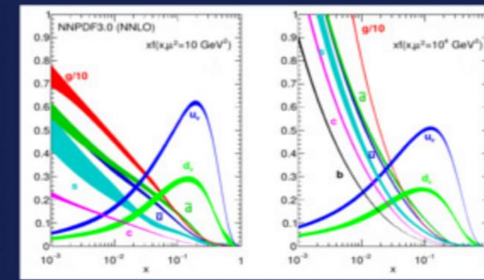
22-24 March 2017, Oxford, UK



Jimenez-Delgado, Melnitchouk, O
J.Phys. G40 (2013) 09310

- A first joint workshop with global-fitting community to address key LQCD inputs
  - http://www.physics.ox.ac.uk/confs/PDFlattice2017
  - Whitepaper study the needed precision of lattice PDFs in the large-x region





Parton Distributions and Lattice Calculations in the LHC era
(PDFLattice 2017)

22-24 March 2017, Oxford, UK

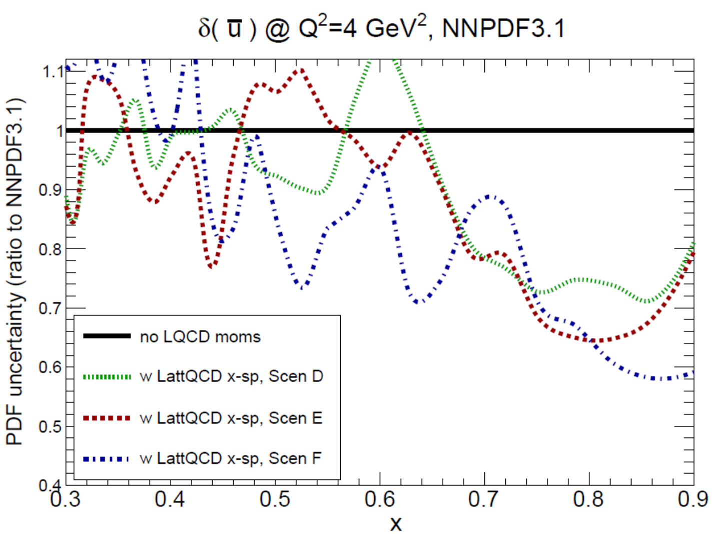
§ Implementing the pseudo-data from LQCD with $x = 0.7$–0.9

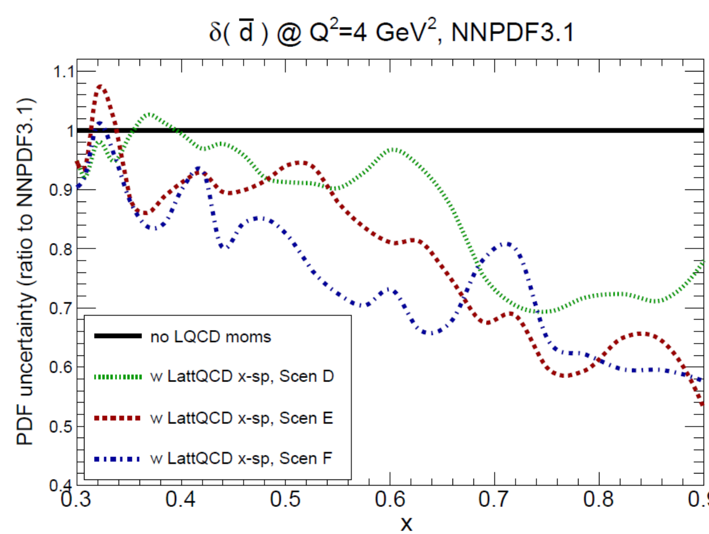
$$u(x_i, Q^2) - d(x_i, Q^2) \text{ and } \bar{u}(x_i, Q^2) - \bar{d}(x_i, Q^2)$$

D: 12%

E: 6%

F: 3%



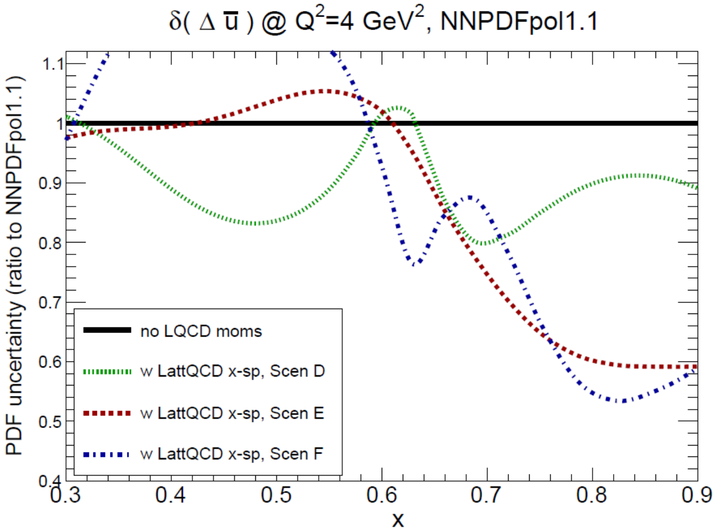


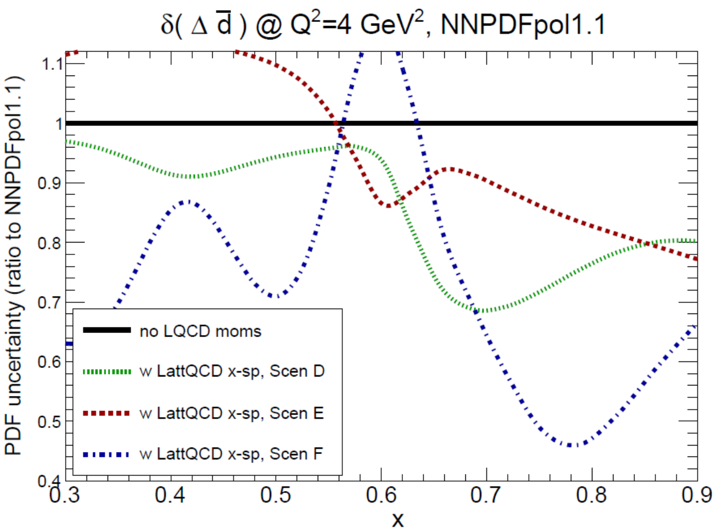
Lin et al, Progress in Particle and Nuclear Physics 100, 106 (2018)

§ Implementing the pseudo-data from LQCD with $x = 0.7–0.9$

$$\Delta u(x_i, Q^2) - \Delta d(x_i, Q^2) \text{ and } \Delta\bar{u}(x_i, Q^2) - \Delta\bar{d}(x_i, Q^2)$$

D: 12%
E: 6%
F: 3%





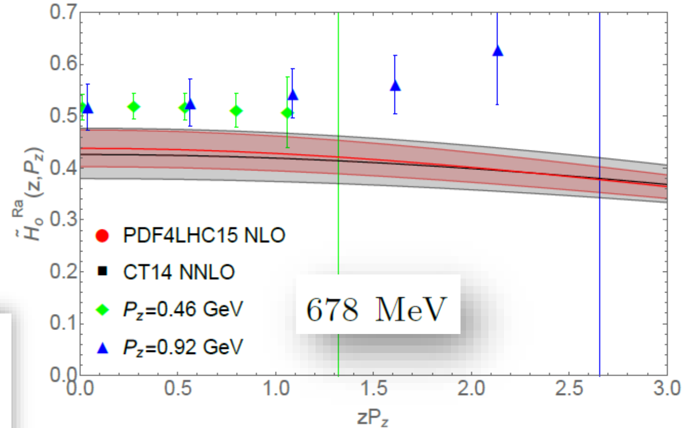
Lin et al, Progress in Particle and Nuclear Physics 100, 106 (2018)

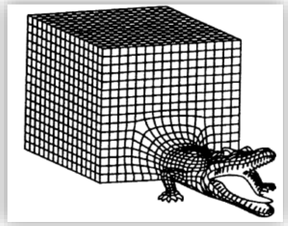
Gluon PDF

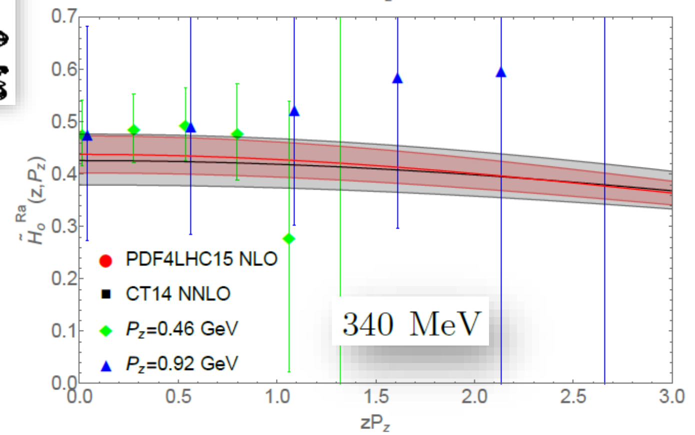
§ Pioneering first glimpse into gluon PDF using LaMET

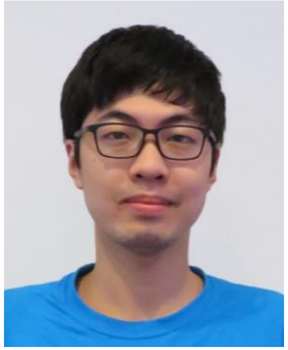
- Lattice details: overlap/2+1DWF, 0.16fm, 340-MeV sea pion mass
- Study strange/light-quark
- Promising results using coordinate-space comparison, but signal does not go far in *z*
- Hard numerical problem to be solved

Fan. et al, 1808.02077

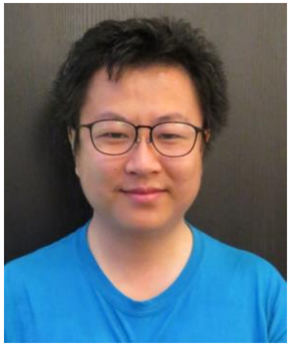








Zhouyou Fan



Yi-Bo Yang

A NEW HOPE

It is a period of war and economic uncertainty.

Turmoil has engulfed the galactic republics.

Basic truths at foundation of the human civilization are disputed by the dark forces of the evil empire.

A small group of QCD Knights from United Federation of Physicists has gathered in a remote location on the third planet of a star called Sol on the inner edge of the Orion–Cygnus arm of the galaxy.

The QCD Knights are the only ones who can tame the power of the Strong Force, responsible for holding atomic nuclei together, for giving mass and shape to matter in the Universe.

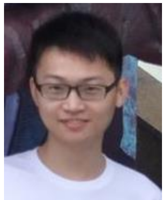
They carry secret plans to build the most powerful

Summary & Outlook

Exciting time for studying structure on the lattice

- **Overcoming longstanding obstacle to full *x*-distribution**
  - Progress made in first lattice pion PDF (1804.01483) & meson distribution amplitudes (1702.0008,1712.10025)

J. Zhang



R. Zhang

- **Exciting/promising physical pion mass results**
- **Further systematics study needed**
  - Need further work on larger momentum boost with finer lattice spacing, higher statistics, higher-order matching, …

- **LQCD impacts for current PDFs in the next few years**
  - Large-*x* isovector PDFs (examples shown)
  - More work need to be done in small-*x* region

The work of HL is sponsored by NSF CAREER Award under grant PHY 1653405

Backup Slides

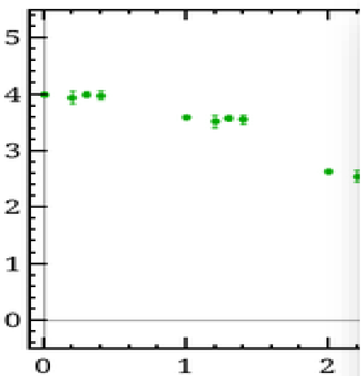
LaMET: Step-by-Step

Large-Momentum Effective Theory for PDFs

X. Ji, PRL. 111, 262002 (2013)

1) Calculate nucleon matrix elements on the lattice

$$h(z,Pz) = \left\langle P \middle| \bar{\psi}(z)\,\gamma_z \exp\!\left(-ig\int_0^z dz'\, A_z(z')\right)\psi(0) \middle| P \right\rangle$$



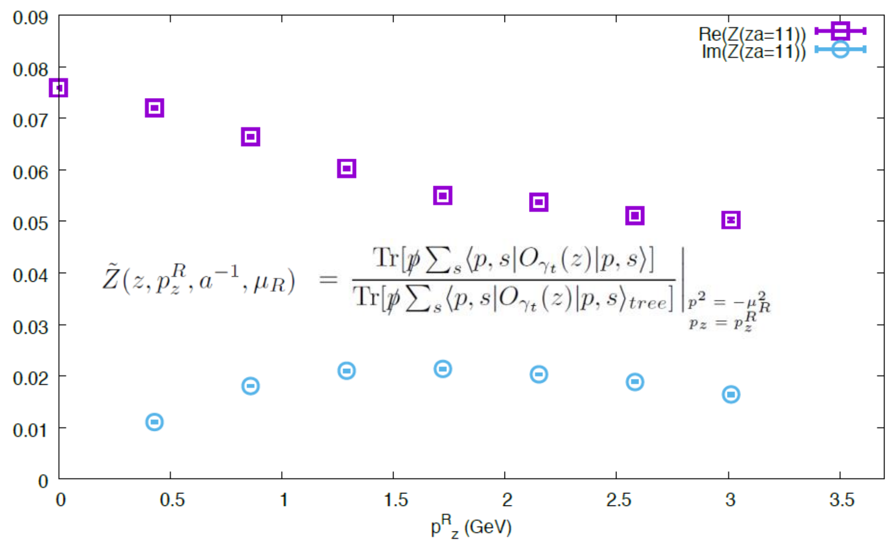
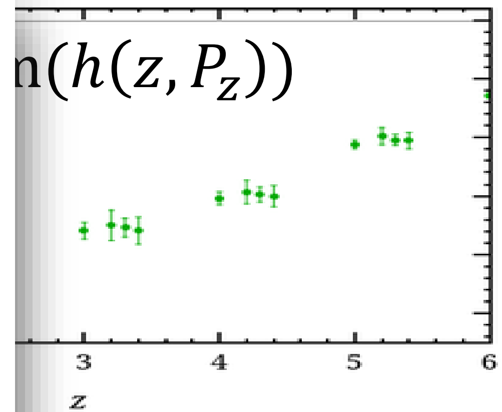


FIG. 1. The values of $Z(z)$ (the inverse of the renormalization constant) at $\mu_R = 3.7$ GeV and $z = 11a \approx 1.0$ fm as a function of p_z^R. Note that $Z(z)$ becomes stable at large p_z^R.



$P_z =$ 2.6

$M_\pi \approx$ **135 Me**

LP[3] 1804.01483

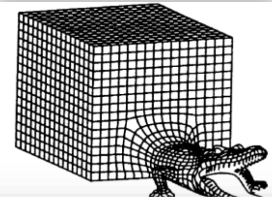
Blinded 3-state fits produced consistent results

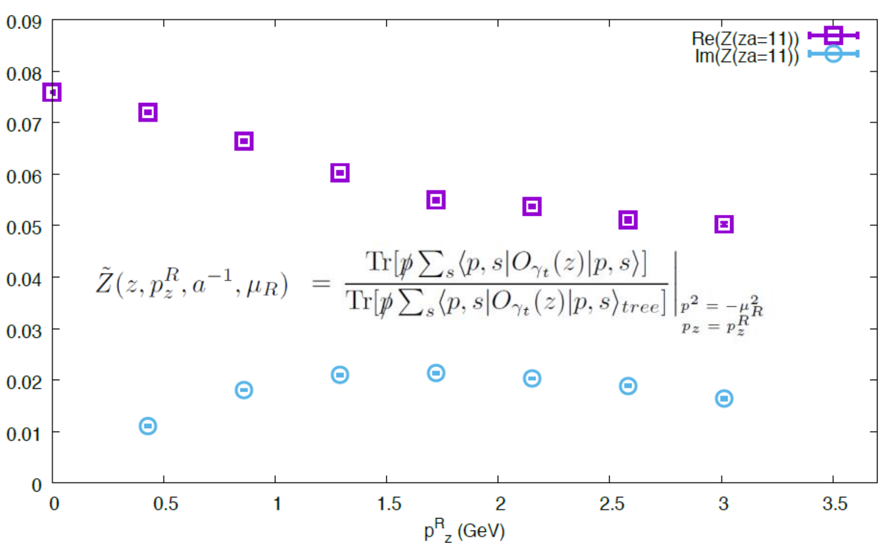
Ruizi Li

Physical Pion Mass Results

§ Exciting! Two collaborations' results at physical pion mass

- Study of systematics still needed





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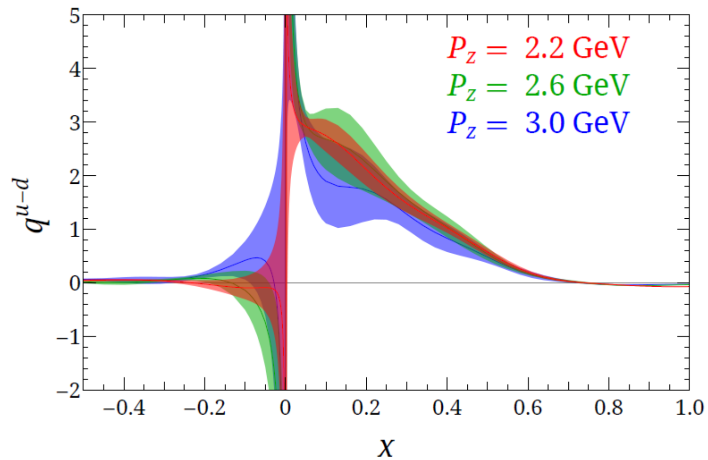

FIG. 3. Nucleon boost momentum dependence of the matched unpolarized isovector PDFs. The parameters of the central value of matched PDF is $(\mu_R, p_z^R) = (3.7, 2.2)$ GeV. For quark asymmetry, the shape is consistent throughout most x regions. However, in the antiquark region, there is a significant change in distribution as momentum increases.

Pseudo-PDF

- § A variation of LaMET: A. Radyushki, 1705.01488
  - $\mathcal{P}(x, z^2, \mu, \epsilon) = \int dz(p_z/2\pi)e^{ix\cdot v}h(v, z^2, \mu, \epsilon)$
- § Versus quasi-PDF Ji, Zhang, Zhao 1706.07416
  - $\tilde{q}(x, p_z, \mu, \epsilon) = \int (dz/2\pi)e^{ix\cdot z\, p_z}h(zp_z, z^2, \mu, \epsilon)$
- § Similarity and issues: $h(v, z^2)/h(0, z^2) = M(v, z^2)$
- § One of the numerical attractions
  - Similar matrix elements; same problems we have
  - Extension to other structures is not clear yet