Graduated Visitor Future postdoc

FROM BETHE-SALPETER WAVEFUNCTIONS TO GPDS

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Mapping PDAs and PDFs
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Motivation

- Understanding strong interactions is still being a challenge for physicists, even decades after the formulation of the fundamental theory of quarks and gluons, namely, **Quantum Chromodynamics** (**QCD**).

- QCD is characterized by two emergent phenomena: **confinement** and **dynamical chiral symmetry breaking** (**DCSB**), which have far reaching consequences in the hadron spectrum and their properties.

- Due to the non perturbative nature of QCD, unraveling the hadron structure, from the fundamental degrees of freedom, is an outstanding problem.

- **Dyson-Schwinger equations** (**DSEs**) combine the IR and UV behavior of the theory at once. Therefore, DSEs are an ideal platform to study quarks, gluons and hadrons.

Outline

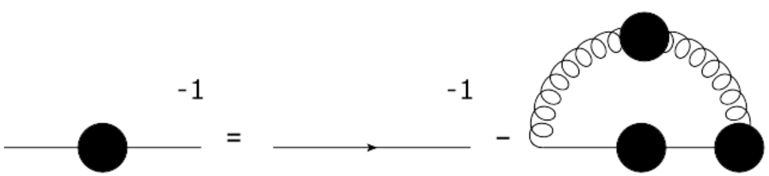
1. **The basics (DSEs)**
   - **Quark propagator and Bethe-Salpeter equation**
   - Perturbation theory integral representations
2. PDAs and TFFs [revisited]
3. η-η' PDAs and TFFs
4. LFWFs, GPDs, PDFs and EFFs
5. Conclusions and remarks

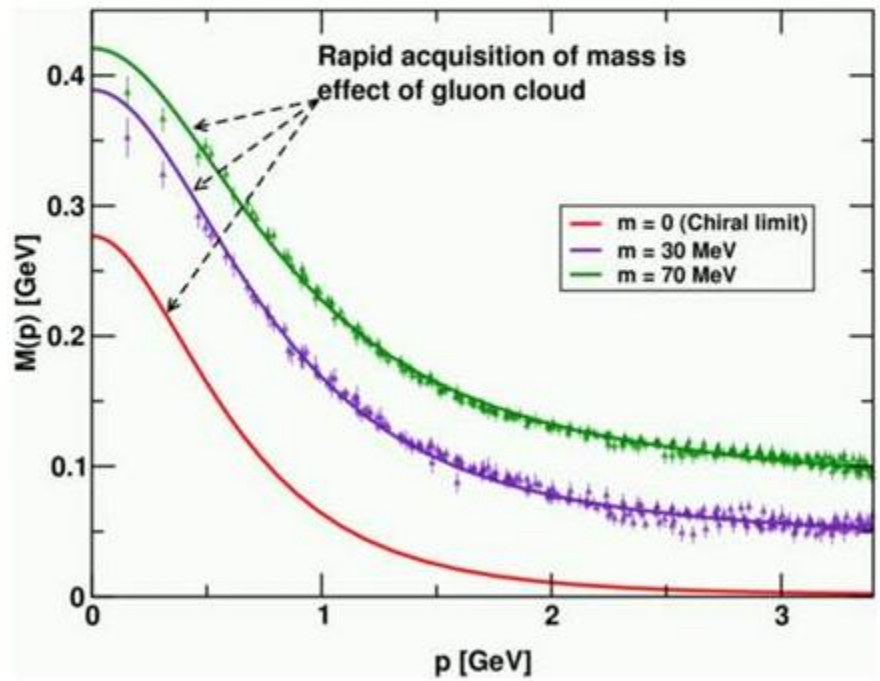
The basics: Dyson-Schwinger equations

- DCSB is a critical emergent phenomenom in QCD. It is the most important mass generating mechanism for visible matter in the universe (~98% of proton mass).

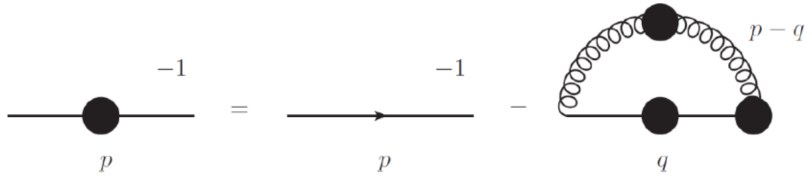
- Quark propagator encodes this phenomenom in the mass function, M(p^2).

$$S(p;\zeta) = \frac{Z(p^2;\zeta)}{p^2 + M(p^2)}$$



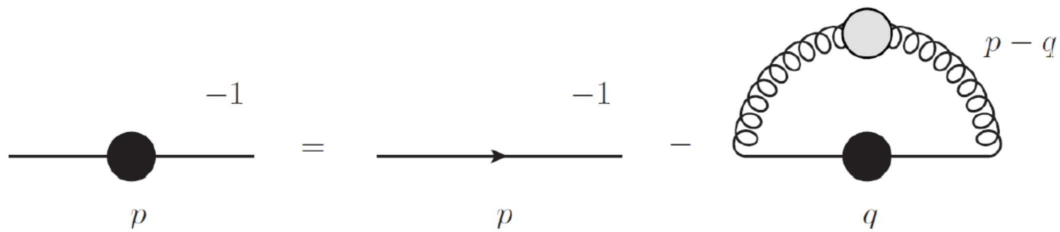


Quark propagator



- DSEs are an infinite tower of coupled equations. To extract the encoded physics, one must systematically truncate.

- The simplest symmetry preserving truncation, which gives a correct description of pseudoscalar and light vector meson phenomenology, is the **rainbow** truncation:



$$S^{-1}(p,\zeta) = [Z_{2F} S_0^{-1}(p)] + \int_q^{\Lambda} G((p-q)^2) D^0_{\mu\nu}(p-q,\zeta) \frac{\lambda^a}{2}\gamma_\mu S(q) \frac{\lambda^a}{2}\gamma_\nu,$$

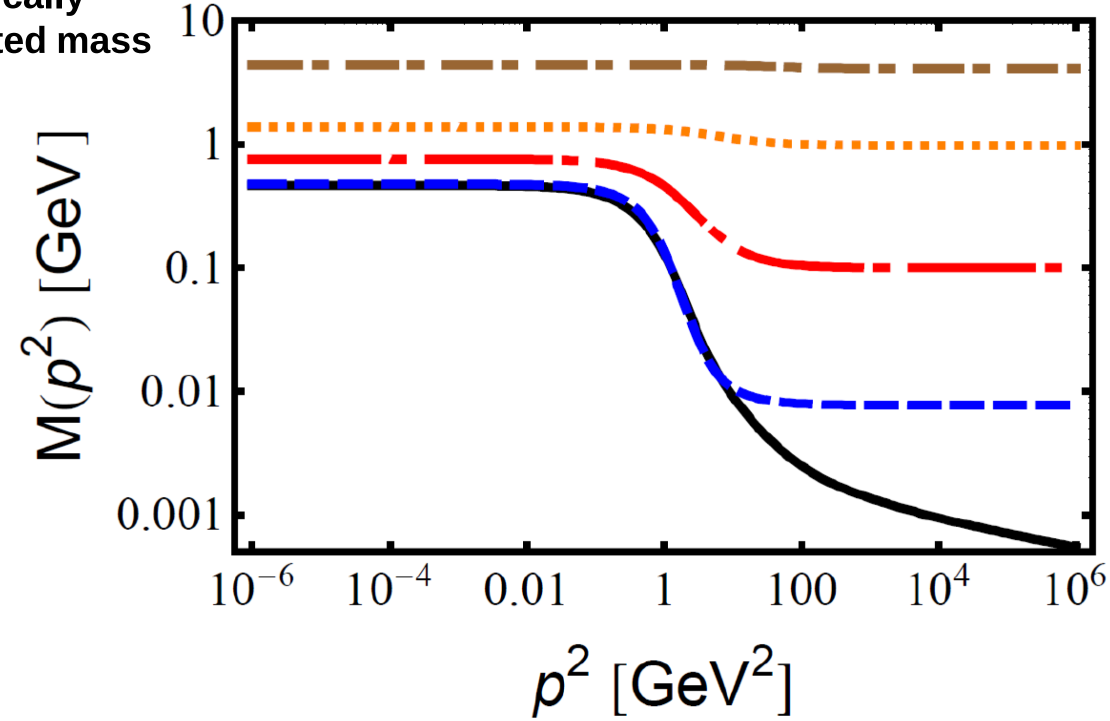
$$S(p,\zeta) = Z(p^2;\zeta^2)(i\gamma\cdot p + M(p^2))^{-1} = (i\gamma\cdot p\, A(p^2;\zeta^2) + B(p^2;\zeta^2))^{-1}$$

$G(k^2)$: Phys.Rev. C84, 042202(R) (2011).

Mass function

- **Mass function** for different current-quark masses: The **lighter** the **quark** is, the **stronger** the effect of **DCSB** is. Even when m=0, a dynamically generated mass appears (this is DCSB).

Dynamically generated mass



Current-quark mass

m_b = 4.19 GeV

m_c = 1.21 GeV

m_s = 106 MeV

$m_{u/d}$ = 4.7 MeV

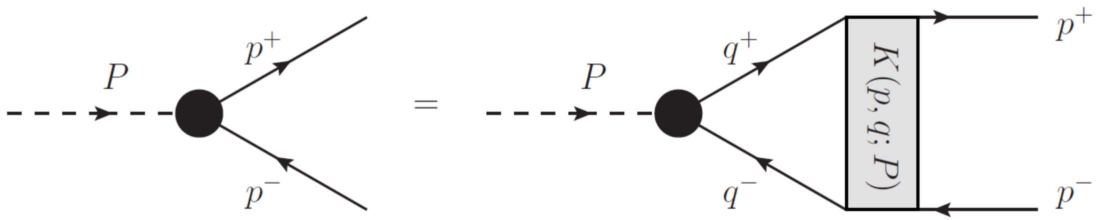
m = 0

Bethe-Salpeter equation

- Quarks and gluons are not found free in nature, they form hadrons. Baryons are colorless bound states of three quarks and mesons are colorless bound states of quark-antiquark pair.

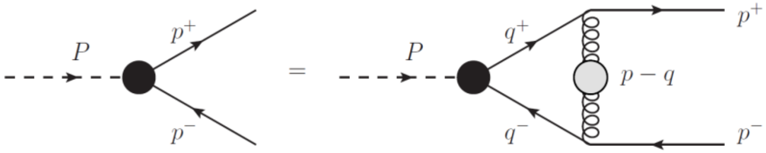
- The **Bethe-Salpeter equation** (**BSE**) is the equation in quantum field theory that describes meson-like systems:

$$\Gamma_M(p;P) = \int_q K(q,p;P)[S(q^+)\Gamma_M(q;P)S(q^-)] \, , \; q^\pm = q \pm P/2 \, .$$



- $\Gamma(p;P)$ is the **Bethe-Salpeter amplitude** (**BSA**). $\mathrm{K}(q,p;P)$ is the scattering kernel, which should be determined and is related to the truncation of the gap equation.

Bethe-Salpeter equation



- The Interaction kernel, K(q,p;P), is related to the truncation of the gap equation via the axial vector Ward-Takahashi identity (**Phys.Lett. B733 (2014) 202-208**, Qin et al.):

$$[\Sigma(p^+)\gamma_5 + \gamma_5\Sigma(p^-)] = \int_q K(p,q;P)[\gamma_5 S(q^-) + S(q^+)\gamma_5] \; .$$

- A kernel sufficient for many needs:

$$K(p,q;P) = -k^2 G(k^2) D^0_{\mu\nu}(k) \left[\frac{\lambda^c}{2}\gamma_\mu\right] \left[\frac{\lambda^c}{2}\gamma_\nu\right] \; , \; k = p - q.$$

- Together with the rainbow approximation, it is called **rainbow-ladder** truncation (**RL**).

$$\Gamma_M(p;P) = -\int_q k^2 G(k^2) D^0_{\mu\nu}(k) \frac{\lambda^c}{2}\gamma_\mu [S(q^+)\Gamma_M(q;P)S(q^-)] \frac{\lambda^c}{2}\gamma_\nu \; .$$

Bethe-Salpeter equation

- For a pseudoscalar meson, the BSA is characterized by 4 tensor structures:

$$\Gamma_M(p;P) = \gamma_5(iE_M(p;P) + \gamma\cdot P F_M(p;P) + \gamma\cdot p\, p\cdot P G_M(p;P) + p_\alpha \sigma_{\alpha\beta} P_\beta H_M(q;P))$$

- In the case of the pion, the axial vector Ward-Takahashi identity relates the dominant BSA, E_π(p;P), with the quark propagator as follows:

$$f_\pi E_\pi(p^2) = B(p^2)$$ **"Goldstone's theorem"**

- The relationship above is **exact in the chiral limit**, and it implies that the **two-body** problem is **solved** (almost) completely, once solution of **one body** problem **is known**.

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Perturbation theory integral representations

- The quark propagator may be expressed as:

$$S(p;\zeta) = -i\gamma \cdot p\, \sigma_v(p^2;\zeta) + \sigma_s(p^2;\zeta)$$

- Numerical solutions are parametrized in terms of N pairs of complex conjugate poles:

$$\sigma_v(q) = \sum_{k=1}^{N} \left(\frac{z_k}{q^2+m_k^2} + \frac{z_k^*}{q^2+m_k^{*2}} \right) \ , \ \sigma_s(q) = \sum_{k=1}^{N} \left(\frac{z_k m_k}{q^2+m_k^2} + \frac{z_k^* m_k^*}{q^2+m_k^{*2}} \right) \ .$$

- These are constrained to the UV conditions of the free quark propagator form. For our computations, N=2 is adequate.

Phys.Rev. D67 (2003) 054019.

M. S. Bhagwat, M. A. Pichowsky, and P. C. Tandy

Perturbation theory integral representations

- BSAs are written in a Nakanishi-like representation (**Phys. Rev. 130, 1230–1235 (1963).** N. Nakanishi):

$$\mathcal{F}^k(p;P) = \int_{-1}^{1} dz\ \rho(z) \int_0^{\infty} d\Lambda\ \delta(\Lambda - \Lambda_c) \frac{1}{(p^2 + z\ p\cdot P + \Lambda^2)^n}\ .$$

- All BSAs can be written in such simple form.
- More than fifty years old. **Forgotten** for most of that time.
- **Rediscovered** 5 years ago: pion PDA and elastic form factor (Phys.Rev.Lett. 111 (2013) no.14, 141802; Phys.Rev.Lett. 110 (2013) no.13, 132001).
- **Now put to great use**: PDAs, form factors, PDFs, etc.; mesons and baryons; light and heavy.

Salmè's talk
Ding's talk

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Parton distribution amplitudes

- The PDA is a projection of the system's Bethe-Salpeter wave-function onto the light-front. It is therefore process independent and hence plays a crucial role in explaining and understanding a wide range of a given meson's properties.

- Given a pseudoscalar meson with total momentum P, a resolution scale ζ and a light-cone four-vector n ($n^2 = 0, n.P = -m_M$), the PDA reads as:

$$f_M \phi_M(x;\zeta) = Z_2 \int_q \delta(n \cdot q^+ - x n \cdot P) \gamma_5 \gamma \cdot n [S(q^+) \Gamma_M(q;P) S(q^-)] \ .$$

- The moments of the distribution are given by:

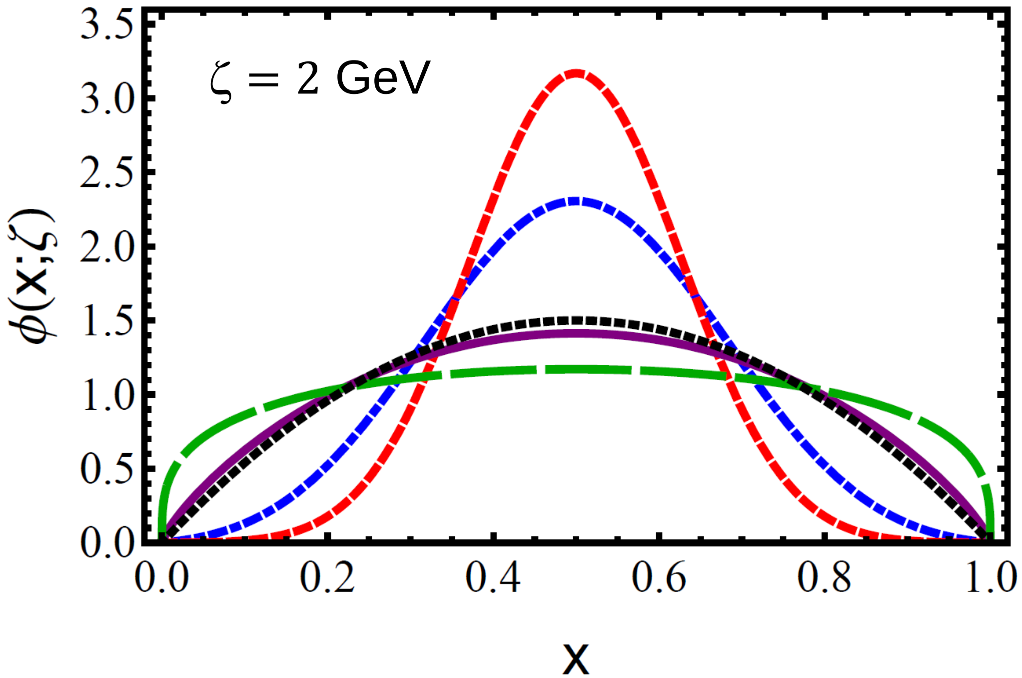
$$f_M (n \cdot P)^{m+1} < x^m >= \mathrm{tr}_{CD} Z_2 \int_q (n \cdot q^+)^m \gamma_5 \gamma \cdot n [S(q^+) \Gamma_M(q;P) S(q^-)] \ ,$$

$$< x^m >= \int_0^1 dx \, x^m \phi_M(x;\zeta) \ .$$

Ding's talk

PDAs from DSEs

Phys.Rev.Lett. 110 (2013) no.13, 132001
Phys.Lett. B753 (2016) 330-335



❖[Red]: $\phi_{\eta_b}(x)$.

❖[Blue]: $\phi_{\eta_c}(x)$.

❖[Purple]: $\phi_{\eta_s}(x)$.

❖[Green]: $\phi_{\pi}(x)$.

❖[Black]: $\phi_{CL}(x)$.

- Dilation of pion PDA is a manifestation of DCSB; for the heavier systems PDAs are narrow.
- s-quark is the boundary between strong and weak mass generation being dominant.
- The last five years have shown us what the PDAs for mesons are, with **complete certainty**.

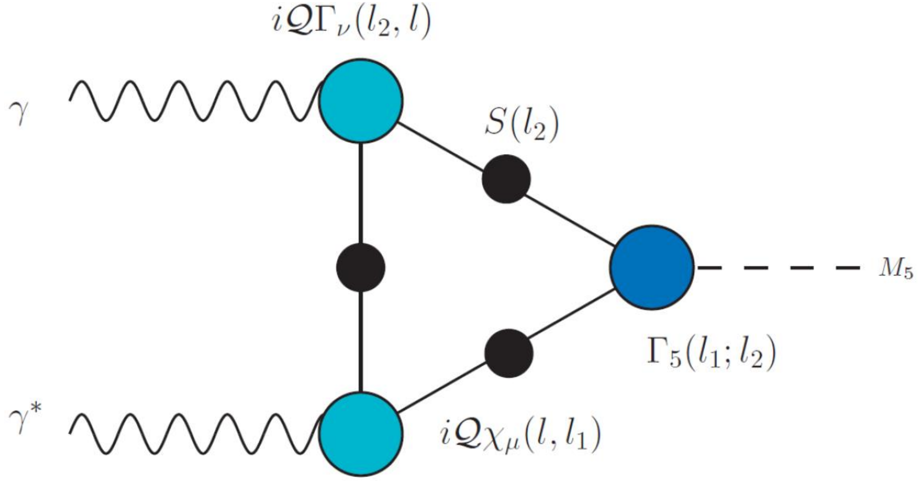
$\gamma\gamma^*$ transition form factors

- For a pseudoscalar meson M_5, the $\gamma\gamma^*\to M_5$ transition is written as:

$$T_{\mu\nu}(k_1,k_2) = \epsilon_{\mu\nu\alpha\beta}k_{1\alpha}k_{2\beta}G_{M_5}(k_1^2, k_1\cdot k_2, k_2^2)\ ,$$

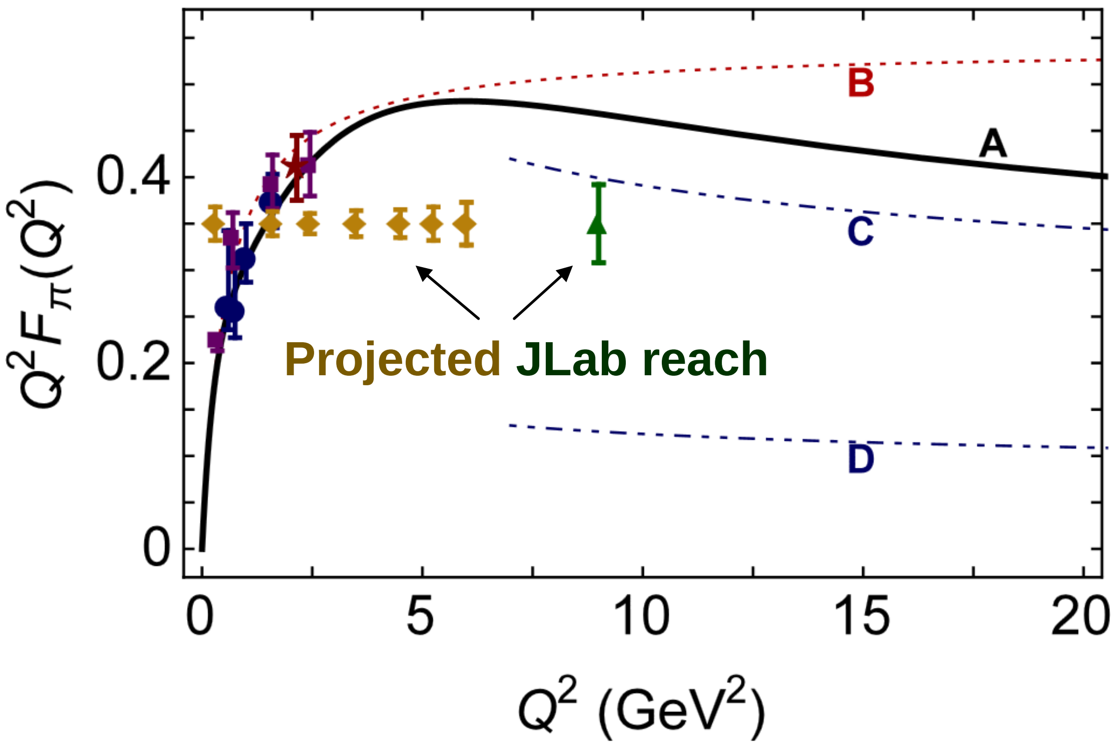
$$T_{\mu\nu}(k_1,k_2) = \mathrm{tr}\int \frac{d^4l}{(2\pi)^4} i\mathcal{Q}\chi_\mu(l,l_1)\Gamma_{M_5}(l_1,l_2)S(l_2)i\mathcal{Q}\Gamma_\nu(l_2,l)$$

- Quark propagator and BSA are known from their PTIRs, **quark-photon vertex** can be written in terms of quark propagator dressing functions.



Pion elastic form factor

Phys.Rev.Lett. 111 (2013) no.14, 141802. Chang et al.



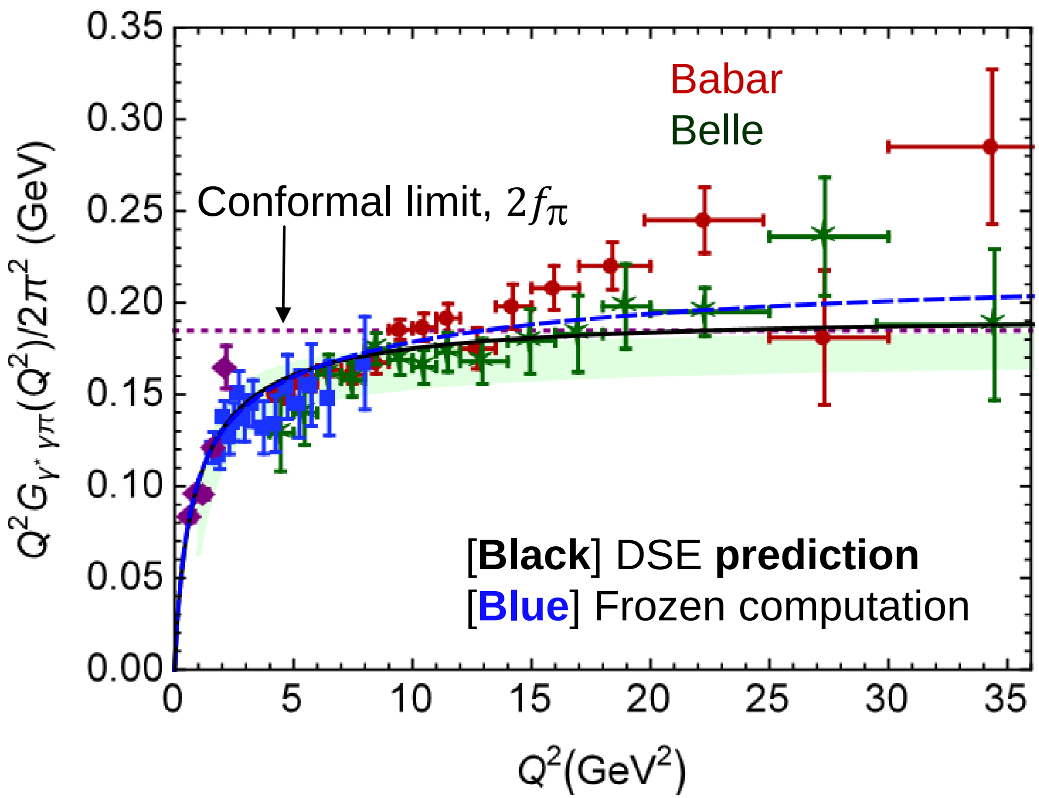
DSE **prediction** ($\zeta = 2\ GeV, r = 0.66\ fm$)

Result computed with $\phi(x; \zeta = 2\ GeV)$

Asymptotic QCD: $\phi^{cl}(x) = 6x(1-x)$

- **DSE prediction** of charged pion form factor **motivated a re-evaluation** of the reach **of JLab12 program**, with the aim of testing such prediction (so that green point was added).

Pion transition form factor

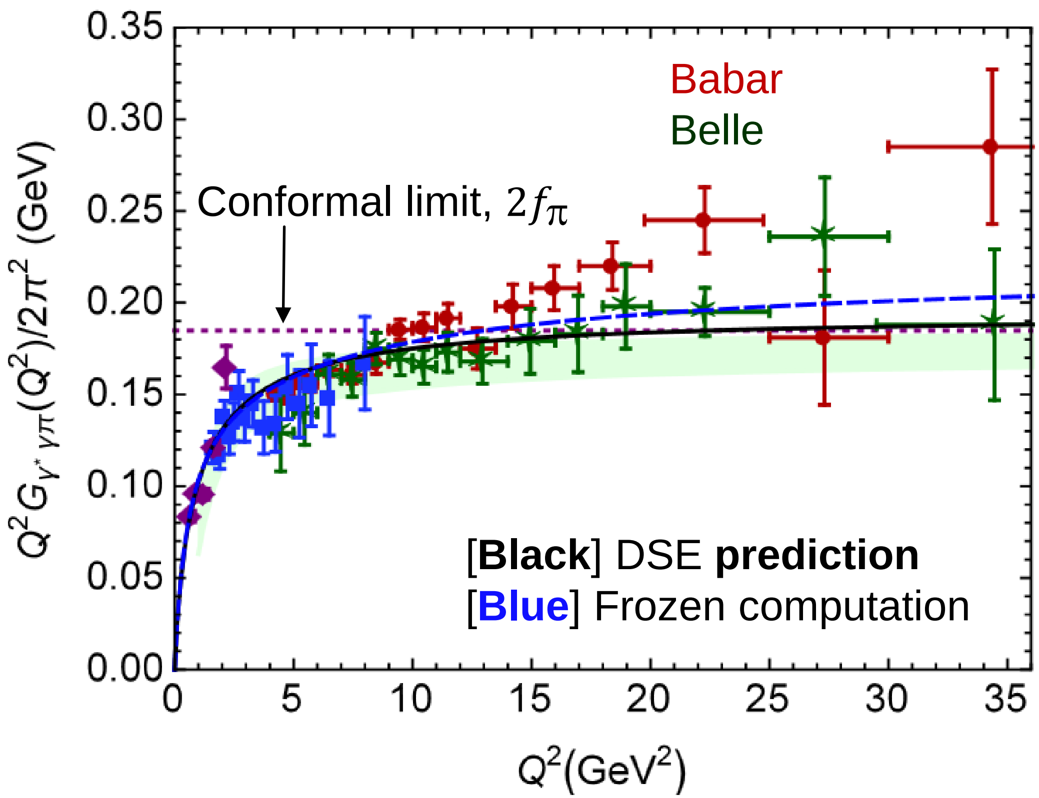


- $F_\pi(Q^2)$ evolves as $1/Q^2$ with scaling violations (~ ln Q^2).
- For the transition form factor, the **conformal limit is a constant**, independent of logarithms.
- But, **the way you reach that constant is completely determined by the logarithms**. We improved DSE methods to include this effect.
- Nothing depends depends on arbitrary scale, but everything depends on Λ_{QCD} which is the RGI scale that defines QCD.

Phys.Rev. D93 (2016) no.7, 074017.

KR, L. Chang, A. Bashir, J.J. Cobos-Martinez, L.X. Gutiérrez-Guerrero, C.D. Roberts, P.C. Tandy

Pion transition form factor



DSE prediction does not conciliate with Babar:

- A pion **PDA** that is a **broad, concave** function at the hadronic scale, **explains both** $F_\pi(Q^2)$, $G_\pi(Q^2)$ and their hard photon limits.
- Babar data, however favors a *flat-top* PDA, which yields a correct power-law, but produces an erroneous value of the anomalous dimension.
- Belle data strongly supports our conclusions.

Phys.Rev. D93 (2016) no.7, 074017.

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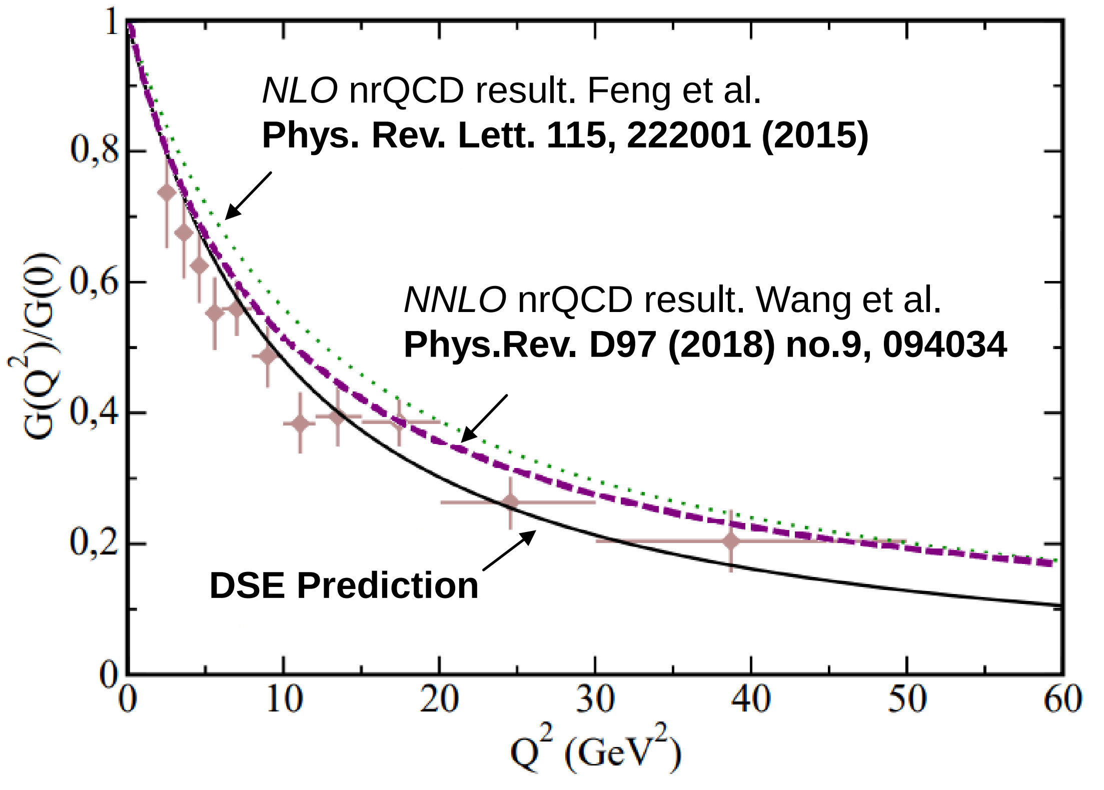
η_c, η_b transition form factors



Phys.Rev. D95 (2017) no.7, 074014.

"Partonic structure of neutral pseudoscalars via two photon transition form factors"

KR, M. Ding, A. Bashir, L. Chang, C.D. Roberts

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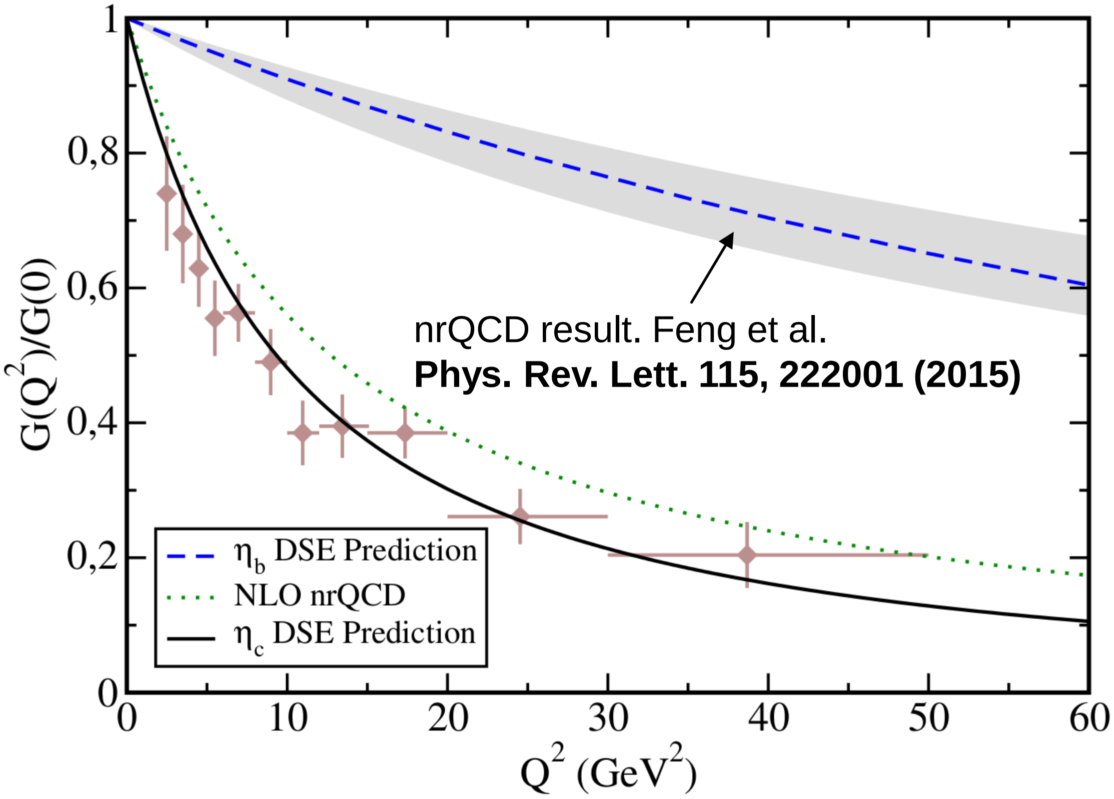

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η- η′ mesons

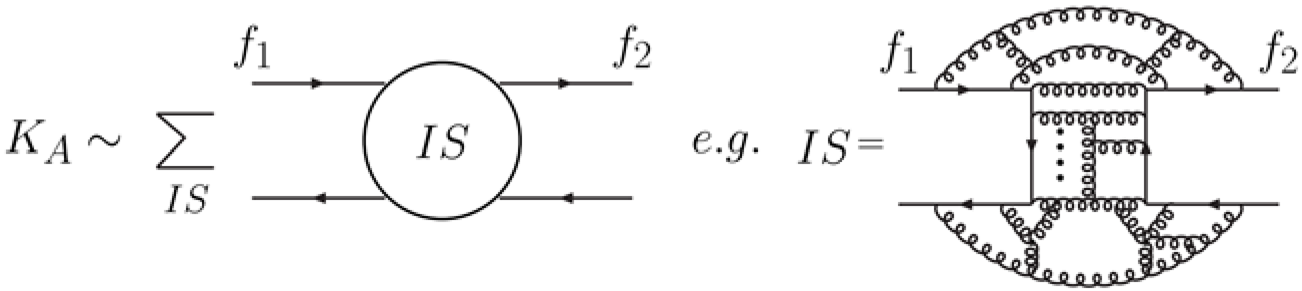
- To produce flavor mixing, ladder kernel is insufficient, non Abelian anomaly must be included:

$$\Gamma(p;P) = \int_q (K_L + K_A)^{rs}_{tu}\ (\mathcal{S}\ \Gamma(q;P)\ \mathcal{S})_{sr}\ ,\ \mathcal{S} = \mathrm{diag}(S^q, S^q, S^s)\ .$$

Ladder **Anomaly**

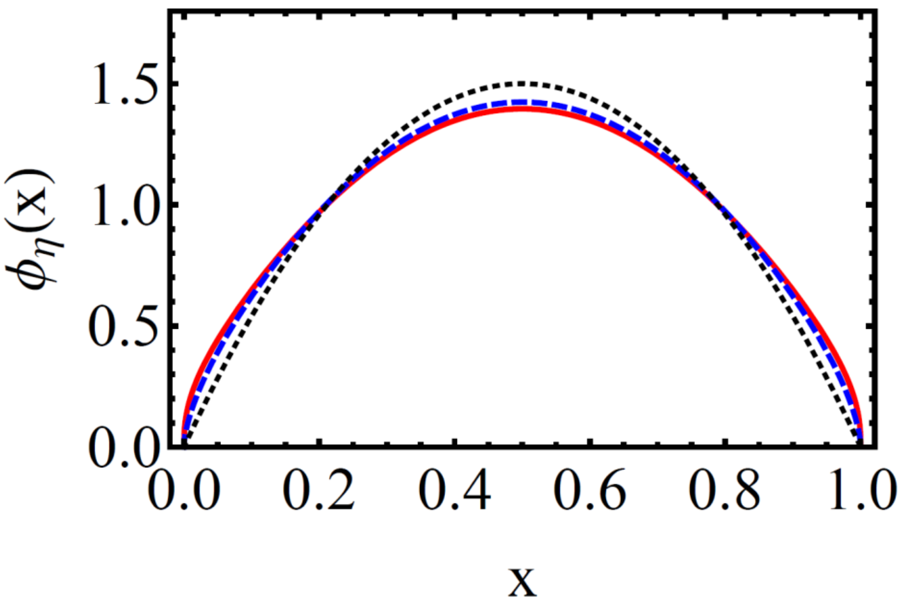
- Anomaly kernel is based upon Mandar et al. work (**Phys. Rev. C76, 045203 (2007)**):

$$(K_A)^{rs}_{tu}(q,k,P) = -\xi(k-q)\{\cos\theta_\xi^2[\zeta\gamma_5]_{rs}[\zeta\gamma_5]_{tu} - \sin\theta_\xi^2[\zeta\gamma\cdot P\gamma_5]_{rs}[\zeta\gamma\cdot P\gamma_5]_{tu}$$

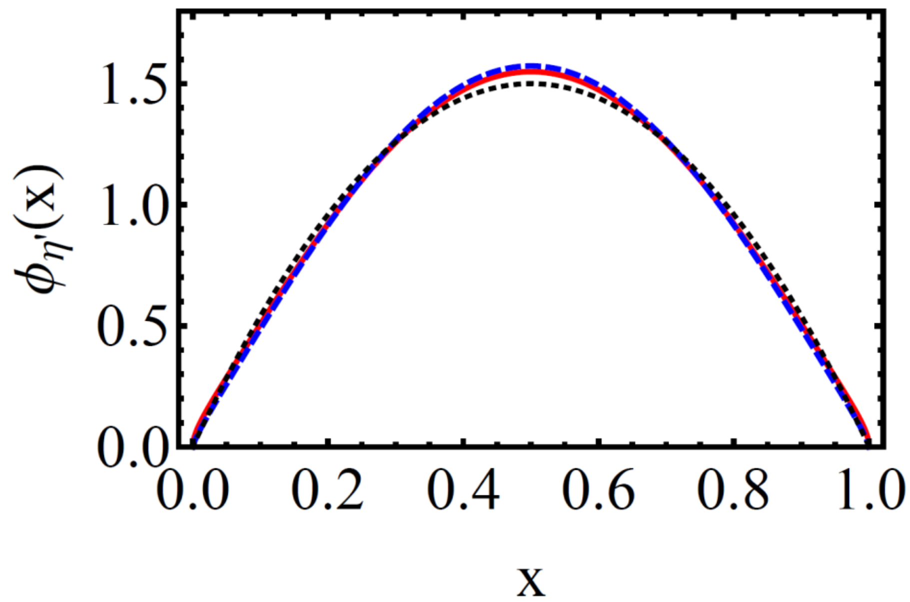


η- η′ PDAs

η-η′ PDAs lie close to the conformal distribution.



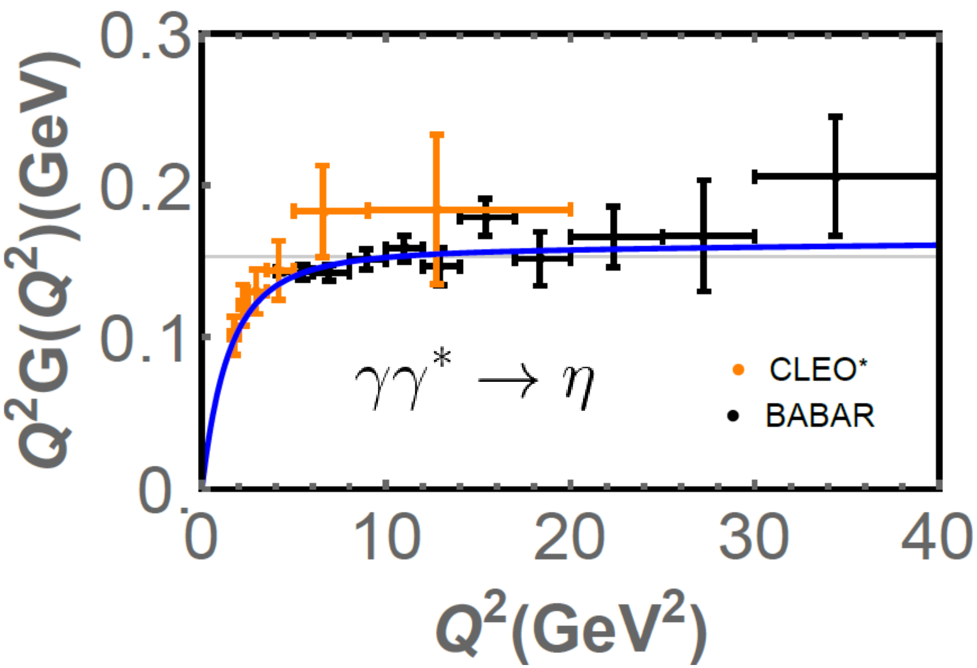
[Blue] $\phi_\eta^s(x;\zeta = 2\ \text{GeV})$.
[Red] $\phi_\eta^q(x;\zeta = 2\ \text{GeV})$.
[Black] $\phi(x) = 6x(1-x)$.

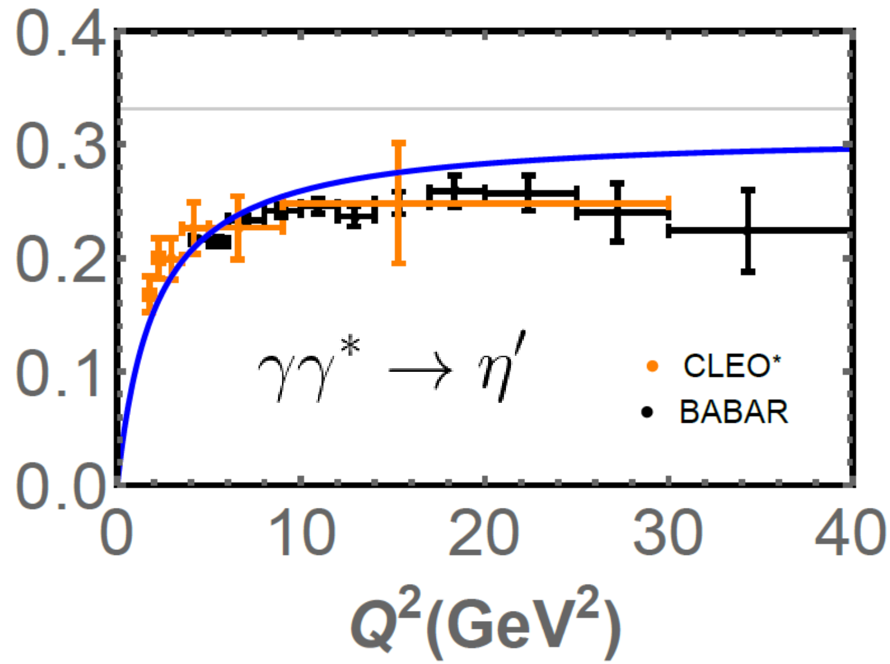


[Blue] $\phi_{\eta'}^s(x;\zeta = 2\ \text{GeV})$.
[Red] $\phi_{\eta'}^q(x;\zeta = 2\ \text{GeV})$.
[Black] $\phi(x) = 6x(1-x)$.

η- η′ TFFs

- The **good agreement** of our preliminary computation **with experimental data** is encouraging.

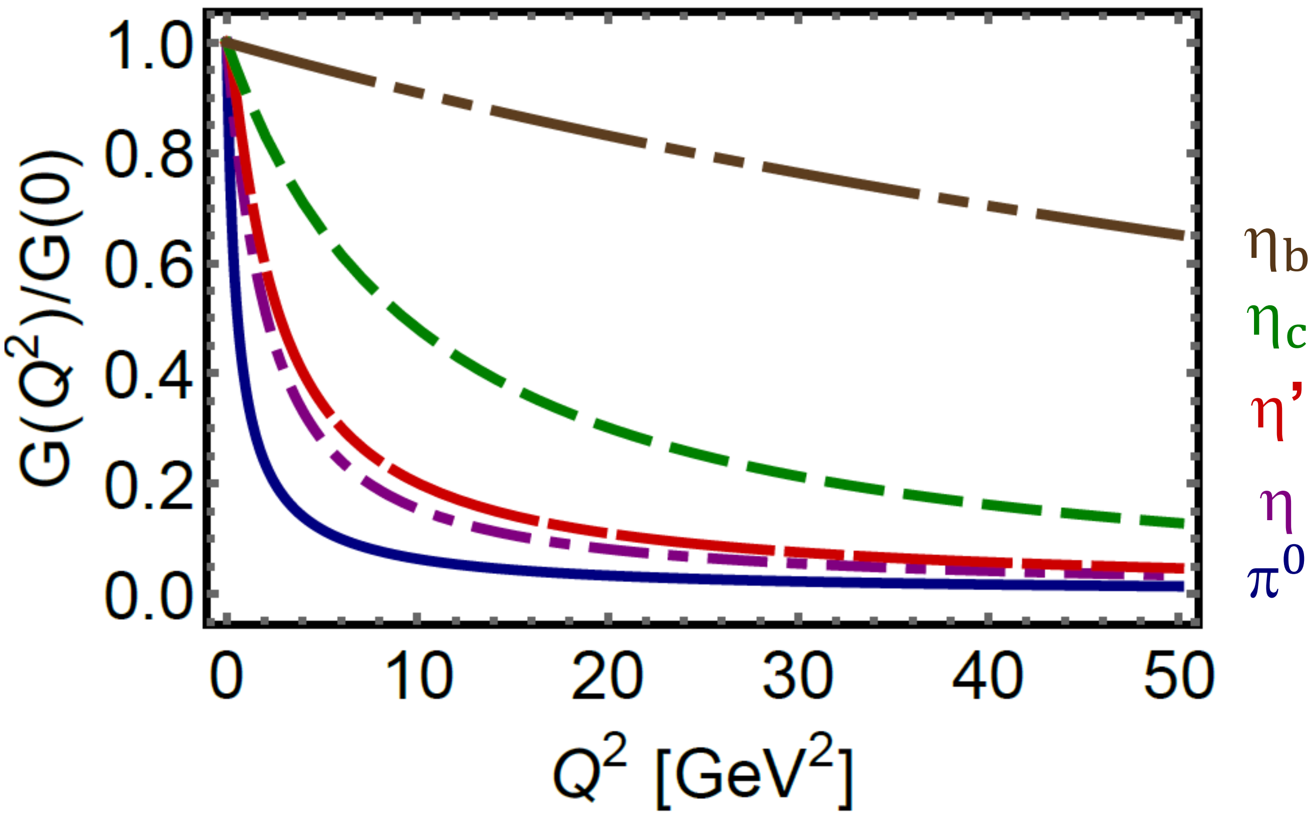




- Subtleties, however, must be taken into consideration: c-cbar contributions, **scale dependence of** the flavor mixed **decay constants**, for example.

See **Phys.Rev. D90 (2014) no.7, 074019**

$\gamma\gamma^*$ transition form factors



Phys.Rev. D93 (2016) no.7, 074017. K. Raya et al.

Phys.Rev. D95 (2017) no.7, 074014. K. Raya, M. Ding et al.

η- η' in preparation… M. Ding, K. Raya et al.

$\gamma\gamma^*$ TFFs

Phys.Rev. D95 (2017) no.7, 074014. KR et al.
Phys.Rev. D93 (2016) no.7, 074017. KR et al.

- We described a computation of $\gamma\gamma^* \to$ neutral pseudoscalar transition form factors, in which all elements employed are determined by solutions of QCD's Dyson-Schwinger equations.

- The novel analysis techniques we employed made it possible to compute $F_\pi(Q^2), G(Q^2)$, on the entire domain of space-like momenta, for the first time in a framework with a **direct connection to QCD**.

- Our QCD based theoretical computation demonstrates that the results of asymptotic QCD are faithfully reproduced, while also successfully agreeing with *"all"* data. It strongly suggests that **Belle**, not Babar, **is correct** on neutral pion.

- Starting from quarks and gluons, the picture of $\gamma\gamma^* \to$neutral pseudoscalar **has been completed**.

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LFWF: pion and Kaon

- Following **Phys.Rev. D97 (2018) no.9, 094014**, we model Kaon Bethe-Salpeter wave function as:

$$n_K \chi_K^{(2)}(k_-^K; P_K) = \underbrace{\mathcal{M}(k; P_K)}_{\mathbf{1}} \int_{-1}^{1} d\omega \, \underbrace{\rho_K(\omega)}_{\mathbf{2}} \underbrace{\mathcal{D}(k; P_K)}_{\mathbf{3}} \, ,$$

1: Leading twist contribution to PDA (only γ_5 BSA):

$$\mathcal{M}(k; P_K) = -\gamma_5 [\gamma \cdot P_K M_u + \gamma \cdot k (M_u - M_s) + \sigma_{\mu\nu} k_\mu P_{K\nu}] \, ,$$

2: Sprectral weight: To be chosen later.

3: Product of 3 quadratic forms in the denominator:

$$\mathcal{D}(k; P_K) = \Delta(k^2, M_u^2) \Delta((k - P_K)^2, M_s^2) \hat{\Delta}(k_{\omega-1}^2, \Lambda_K^2) \, ,$$

$$\text{where: } \Delta(s,t) = [s+t]^{-1}, \; \hat{\Delta}(s,t) = t\Delta(s,t) \, .$$

LFWF: pion and Kaon

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- Combining denominators through Feynman parametrization:

$$\chi_K^{(2)}(k_-^K; P_K) = \mathcal{M}(k; P_K) \int_0^1 d\alpha \; 2\chi_K(\alpha; \sigma^3(\alpha)) \; , \; \sigma = (k - \alpha P_K)^2 + \Omega_K^2 \; ,$$

where Ω_K^2 depends on the model and Feynman parameters, and:

$$\chi_K(\alpha; \sigma^3) = \left[\int_{-1}^{1-2\alpha} d\omega \int_{1+\frac{2\alpha}{\omega-1}}^{1} dv + \int_{1-2\alpha}^{1} d\omega \int_{\frac{\omega-1+2\alpha}{\omega+1}}^{1} dv \right] \frac{\rho_K(\omega)}{n_K} \frac{\Lambda_K^2}{\sigma^3} \; .$$

- Pion case is recovered when $M_s \to M_d$.

LFWF: pion and kaon

Chang's talk

- The pseudoscalar LFWF can be written:

$$f_K \psi_K^{\uparrow\downarrow}(x, k_\perp^2) = \mathrm{tr}_{CD} \int_{dk_{||}} \delta(n\cdot k - x n \cdot P_K)\gamma_5 \gamma \cdot n \chi_K^{(2)}(k_-^K; P_K) \ .$$

- The moments of the distribution are given by:

$$< x^m >_{\psi_K^{\uparrow\downarrow}} = \int_0^1 dx x^m \psi_K^{\uparrow\downarrow}(x, k_\perp^2) = \frac{1}{f_K n \cdot P} \int_{dk_{||}} \left[\frac{n\cdot k}{n\cdot P}\right]^m \gamma_5 \gamma \cdot n \chi_K^{(2)}(k_-^K; P_K)$$

$$= \frac{12}{f_K} \int_0^1 d\alpha \alpha^m \mathcal{Y}_K(\alpha; \sigma^2) \Rightarrow \psi_K^{\uparrow\downarrow}(x, k_\perp^2) = \frac{12}{f_K} \mathcal{Y}_K(x; \sigma_\perp^2) \ .$$

$$\mathcal{Y}_K(\alpha; \sigma^2) = [M_u(1-\alpha) + M_s \alpha]\mathcal{X}(\alpha; \sigma_\perp^2) \ , \ \sigma_\perp = k_\perp^2 + \omega_K^2 \ .$$

- Notably, from the definiton of the moments, one can identify the LFWF. Compactness of these results is one merit of the algebraic model.

LFWF: pion and kaon

$$\varphi_K(x) = \frac{1}{16\pi^3}\int d^2k_\perp \psi_K^{\uparrow\downarrow}(x,k_\perp^2)$$

- Spectral density is chosen as:

$$u_G\rho_G(\omega) = \frac{1}{2b_0^G}\left[\text{sech}^2\left(\frac{\omega-\omega_0^G}{2b_0^G}\right)+\text{sech}^2\left(\frac{\omega+\omega_0^G}{2b_0^G}\right)\right][1+\omega\, v_G]\,,$$

Λ_π	b_0^π	w_0^π	v_π	Λ_K	b_0^K	w_0^K	v_K
M_u	0.1	0.73	0	$2\Lambda_\pi$	b_0^π	0.95	0.16

$$M_u = 0.31\ \text{GeV},\ M_s = 1.2M_u,\ m_\pi = 0.140\ \text{GeV},\ m_K = 0.49$$

- In order to produce broad, concave PDAs such that:

$$\langle(2x-1)^2\rangle_{\varphi_\pi} := \int_0^1 dx\,(2x-1)^2\varphi_\pi(x) \approx 0.25\,,$$
$$\langle 2x-1\rangle_{\varphi_K} \approx -0.04\,,\ \langle(2x-1)^2\rangle_{\varphi_K} \approx 0.25\,.$$

$$\frac{<1/x>_{AM}}{<1/x>_{CL}} \approx 1.15\,.$$

Nakanishi weight

Side note

- **Note**: If spectral density is chosen as: $\rho_G(\omega;\nu) \sim (1-z^2)^\nu$,

one obtains closed algebraic forms of PDAs and PDFs:

$$\phi(x;\nu) \sim [x(1-x)]^\nu \, , \quad u_\pi(x;\nu) \sim [x(1-x)]^{2\nu} \, .$$

- In particular, asymptotic PDA corresponds to ν=1.

Sketching the pion's valence-quark generalised parton distribution

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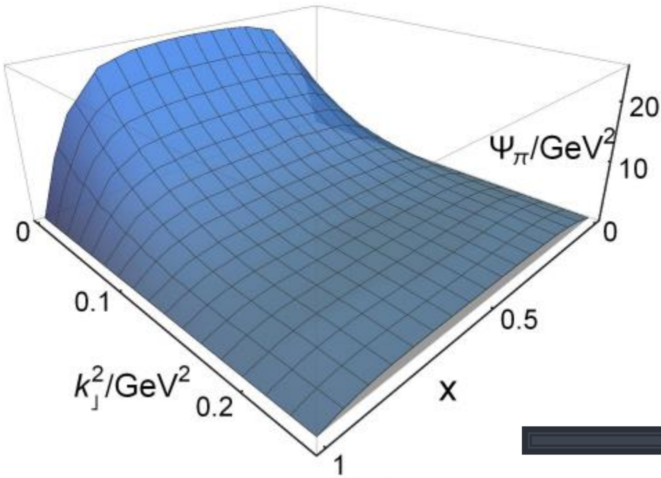
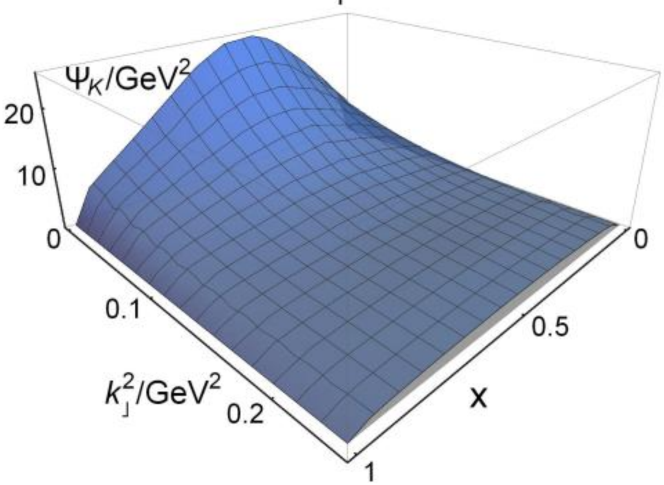
[d]*Departamento de Física Aplicada, Facultad de Ciencias Experimentales, Universidad de Huelva, Huelva E-21071, Spain*
[e]*Institute for Advanced Simulation, Forschungszentrum Jülich and JARA, D-52425 Jülich, Germany*

Phys.Lett. B741 (2015) 190-196

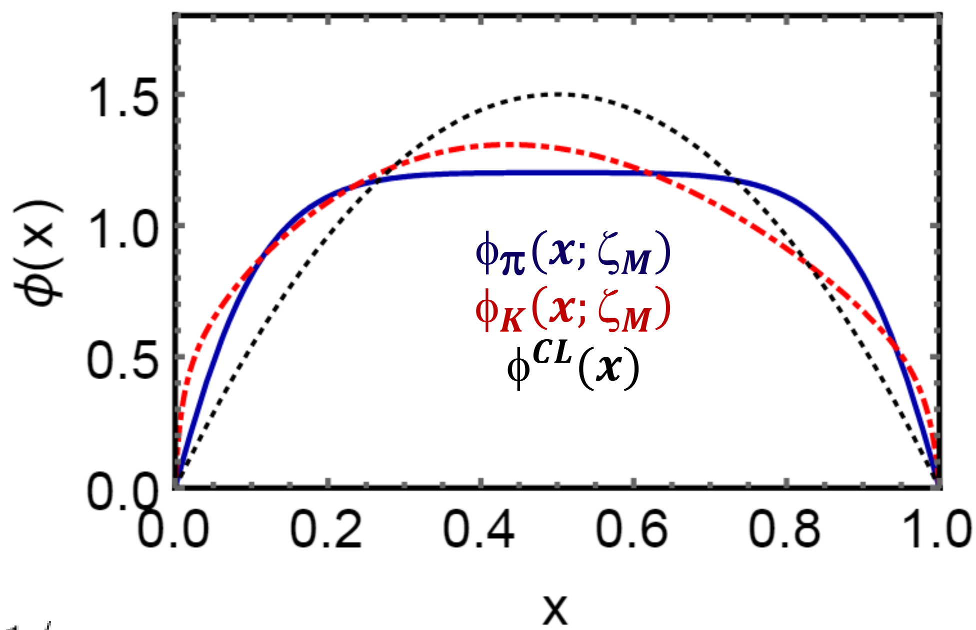
LFWF and PDA: pion and kaon





Phys.Rev. D97 (2018) no.9, 094014

$$\varphi_K(x) = \frac{1}{16\pi^3}\int d^2k_\perp \psi_K^{\uparrow\downarrow}(x, k_\perp^2)$$

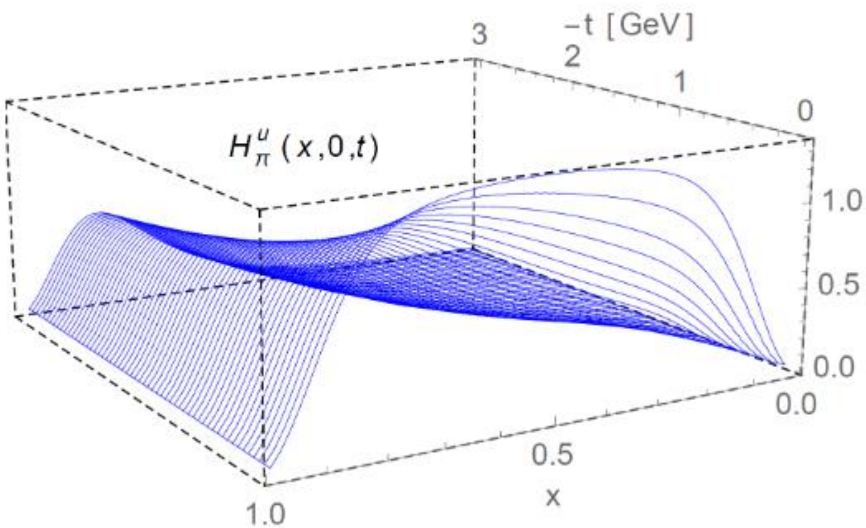


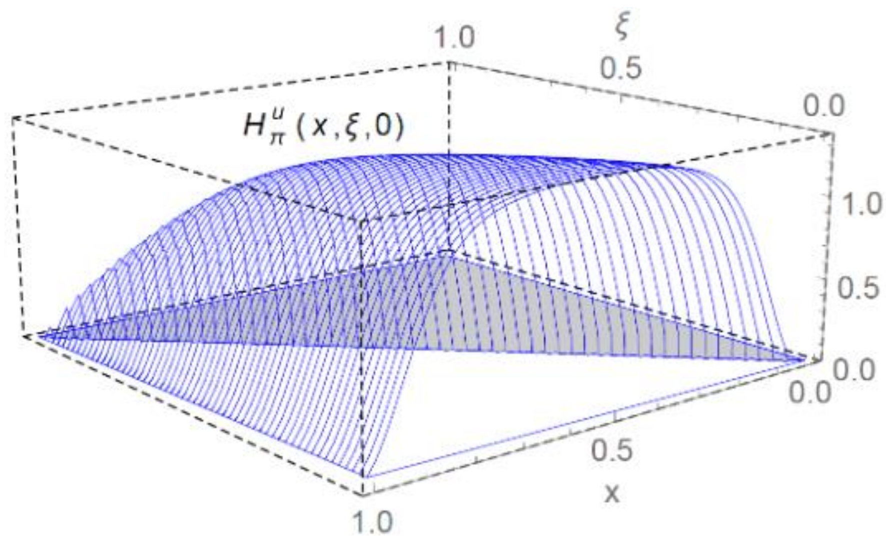
$$\frac{<1/x>_{AM}}{<1/x>_{CL}} \approx 1.15\,.$$

Overlap GPD representation: Pion

- A two-particle truncated expression for the pion and Kaon **GPDs**, in the **DGLAP** kinematic **domain**, is obtained from the **overlap of the LFWF**:

$$H_M^q(x,\xi,t) = \int \frac{\mathrm{d}^2\mathbf{k}_\perp}{16\,\pi^3} \Psi^*_{u\bar{f}}\left(\frac{x-\xi}{1-\xi}, \mathbf{k}_\perp + \frac{1-x}{1-\xi}\frac{\Delta_\perp}{2}\right) \Psi_{u\bar{f}}\left(\frac{x+\xi}{1+\xi}, \mathbf{k}_\perp - \frac{1-x}{1+\xi}\frac{\Delta_\perp}{2}\right) .$$

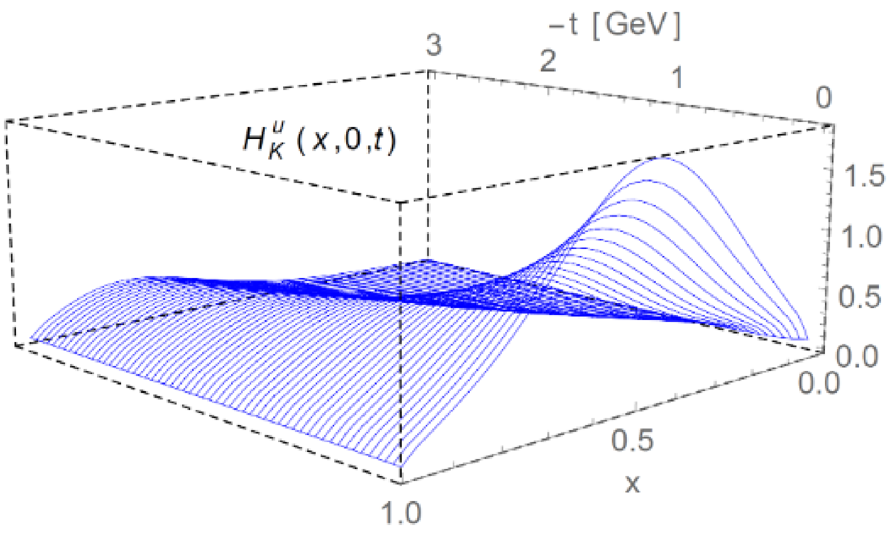


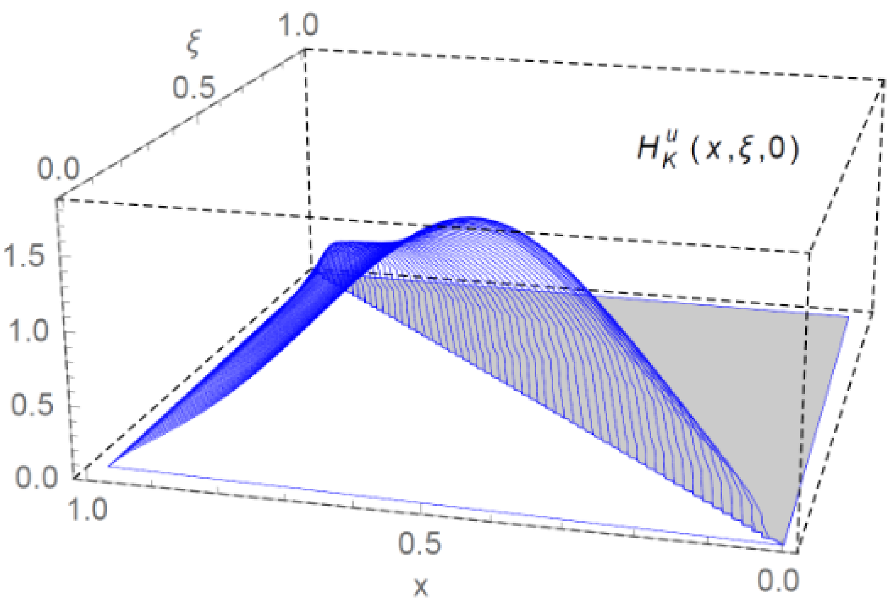


Overlap GPD representation: Kaon

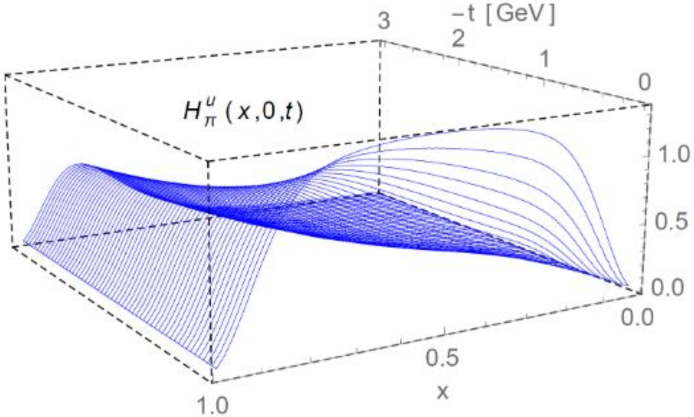
- A two-particle truncated expression for the pion and Kaon **GPDs**, in the **DGLAP** kinematic **domain**, is obtained from the **overlap of the LFWF**:

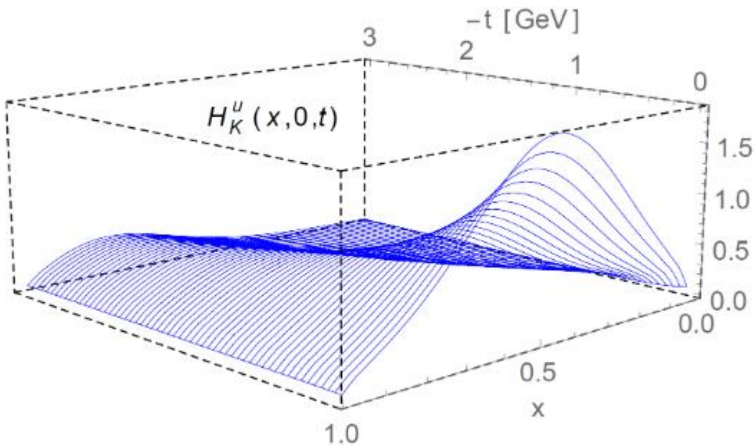
$$H_M^q\left(x,\xi,t\right) = \int \frac{\mathrm{d}^2\mathbf{k}_\perp}{16\,\pi^3}\Psi^*_{u\bar{f}}\left(\frac{x-\xi}{1-\xi},\mathbf{k}_\perp + \frac{1-x}{1-\xi}\frac{\Delta_\perp}{2}\right)\Psi_{u\bar{f}}\left(\frac{x+\xi}{1+\xi},\mathbf{k}_\perp - \frac{1-x}{1+\xi}\frac{\Delta_\perp}{2}\right)\,.$$



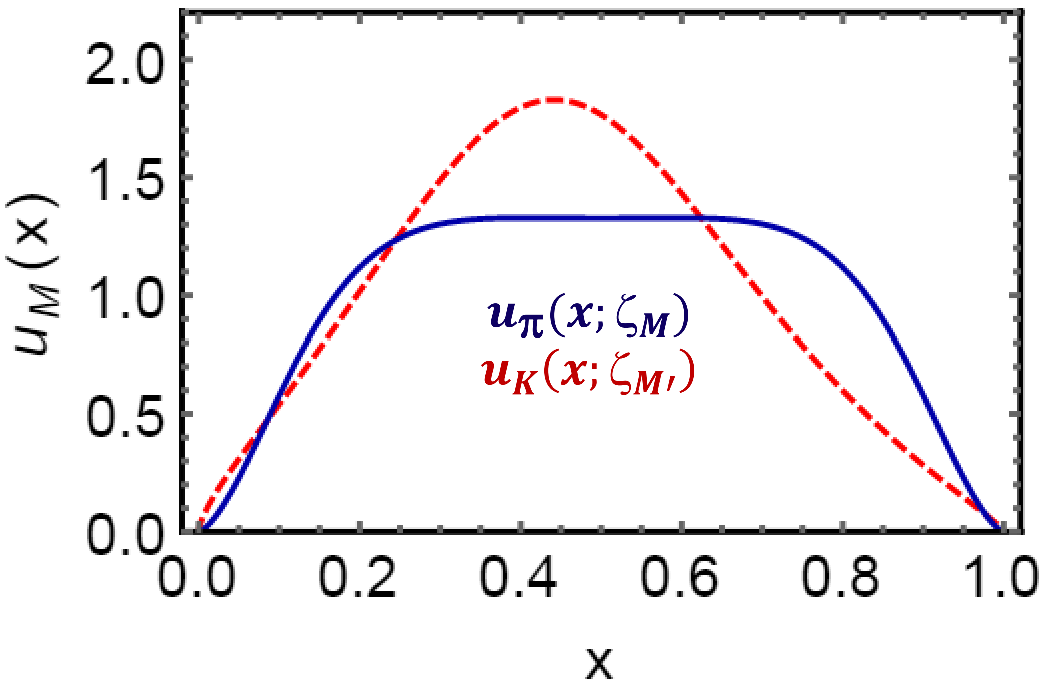


GPDs and PDFs: Pion and Kaon





$$H_M^u(x,0,0) = u_M(x)$$



$\zeta_{M,M'}$: **Intrinsic** model **scales**

Pion PDF

- To be able **to compare** the resulting pion **PDF** with experimental data, one **should obtain** the intrinsic **model scale**, **then** DGLAP **evolve to** the experiment scale, ζ=**5.2 GeV**.

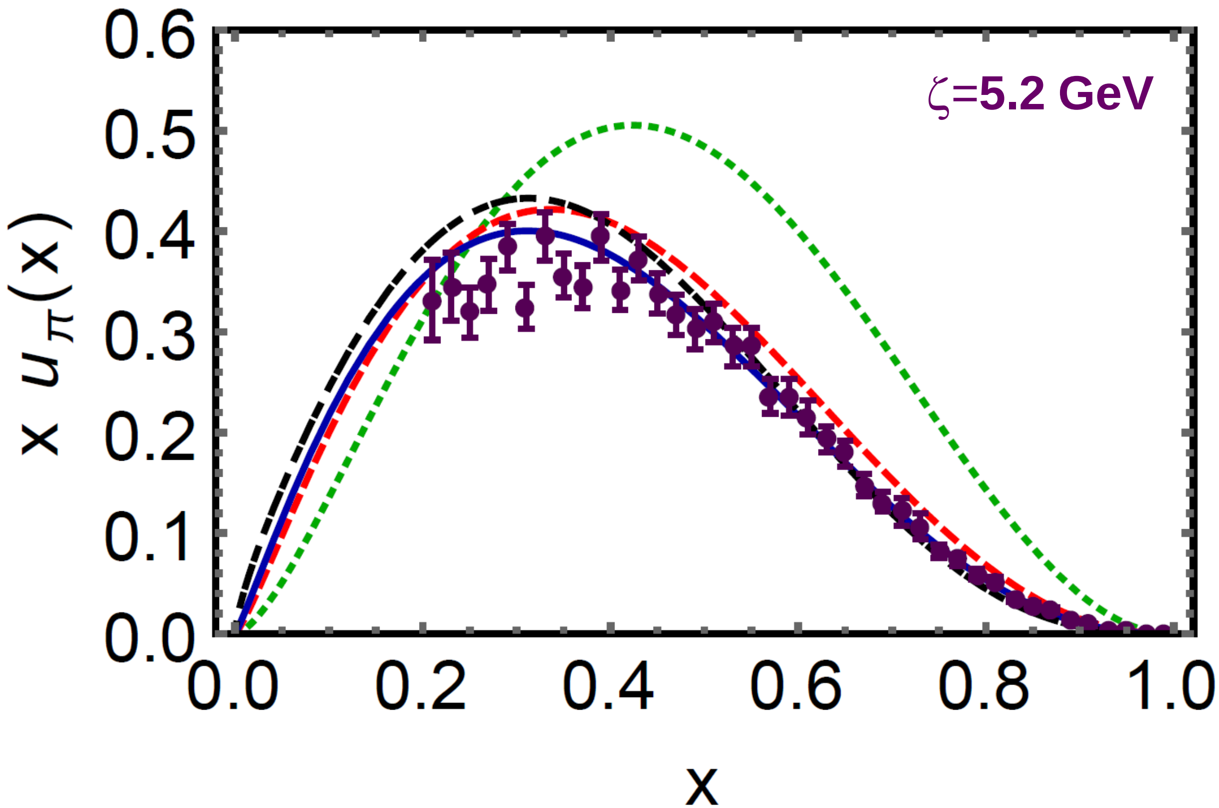
- The DGLAP evolution equations:

$$< x^m_{\zeta_H} >^u_M = \int_0^1 dx\, x^m u_M(x)\ ,\ < x^m_{\zeta} >^u_M = < x^m_{\zeta_H} >^u_M \left[\frac{\alpha(\zeta^2)}{\alpha(\zeta_H^2)}\right]^{\gamma_0^m/\beta_0}\ .$$

- We guess the best initial scale: such that, for example, when the PDF is evolved to 2 GeV, one obtains the average of lattice moments (Brommet et al., Best et al., Detmold et al.):

	$\langle x\rangle$	$\langle x^2\rangle$	$\langle x^3\rangle$
average	0.26(8)	0.11(4)	0.058(27)

Pion PDF



$< x > = 0.24$

$< x > = 0.26$

$< x > = 0.33$

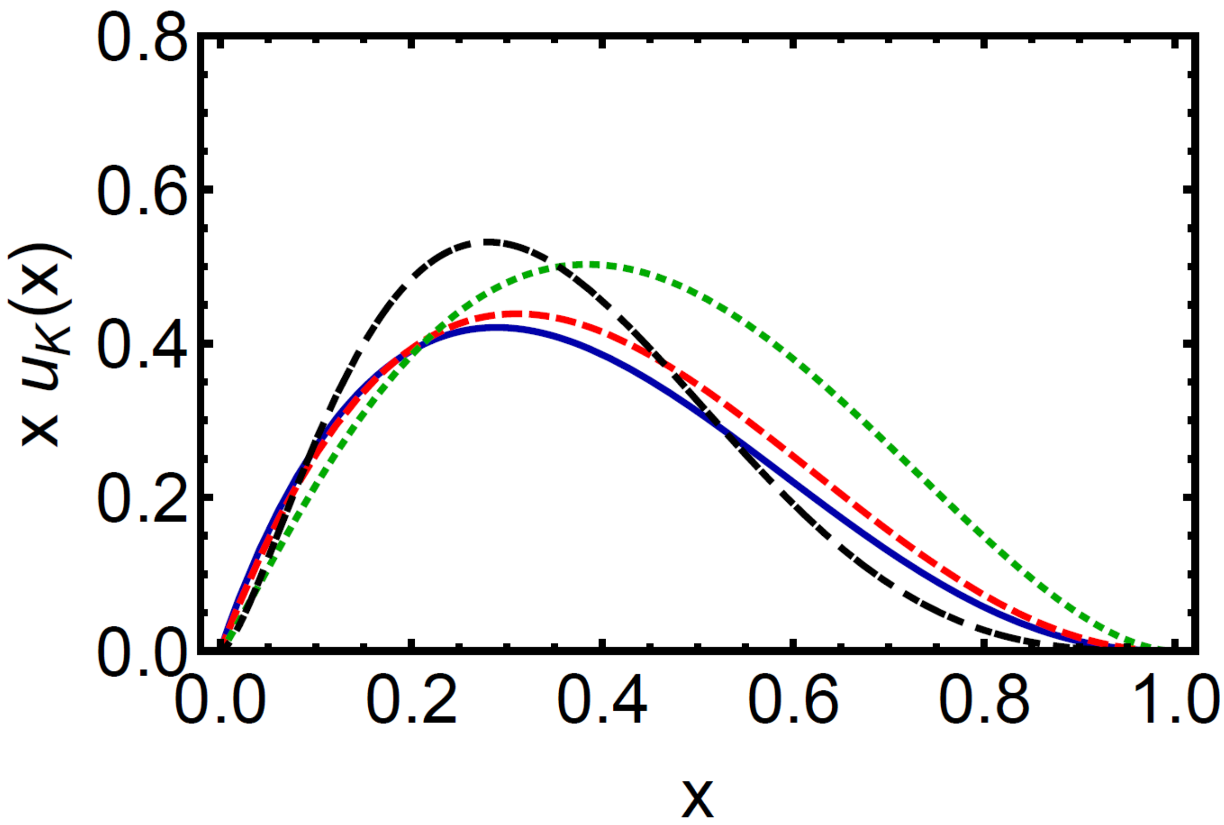
$< x > = 0.26$*

ζ_0 = 0.349 GeV: Obtained directly from the experimental data (π).

ζ_0 = 0.374 GeV: Obtained to best fit the lattice moments at 2 GeV (π).

ζ_0 = 0.510 GeV: Typical hadron scale. See for example: **Phys.Lett. B737 (2014) 23-29** and **Phys.Rev. D93 (2016) no.7, 074021***.

Kaon PDF



$< x > = 0.22$

$< x > = 0.24$

$< x > = 0.29$

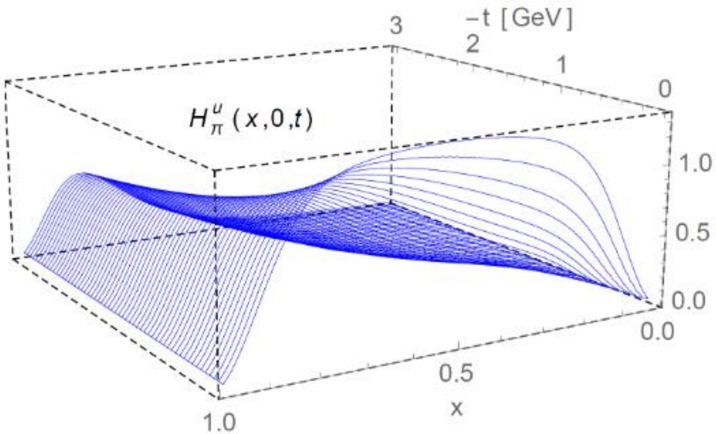
$< x > = 0.28$*

ζ_0 = 0.349 GeV: Obtained directly from the experimental data (π).

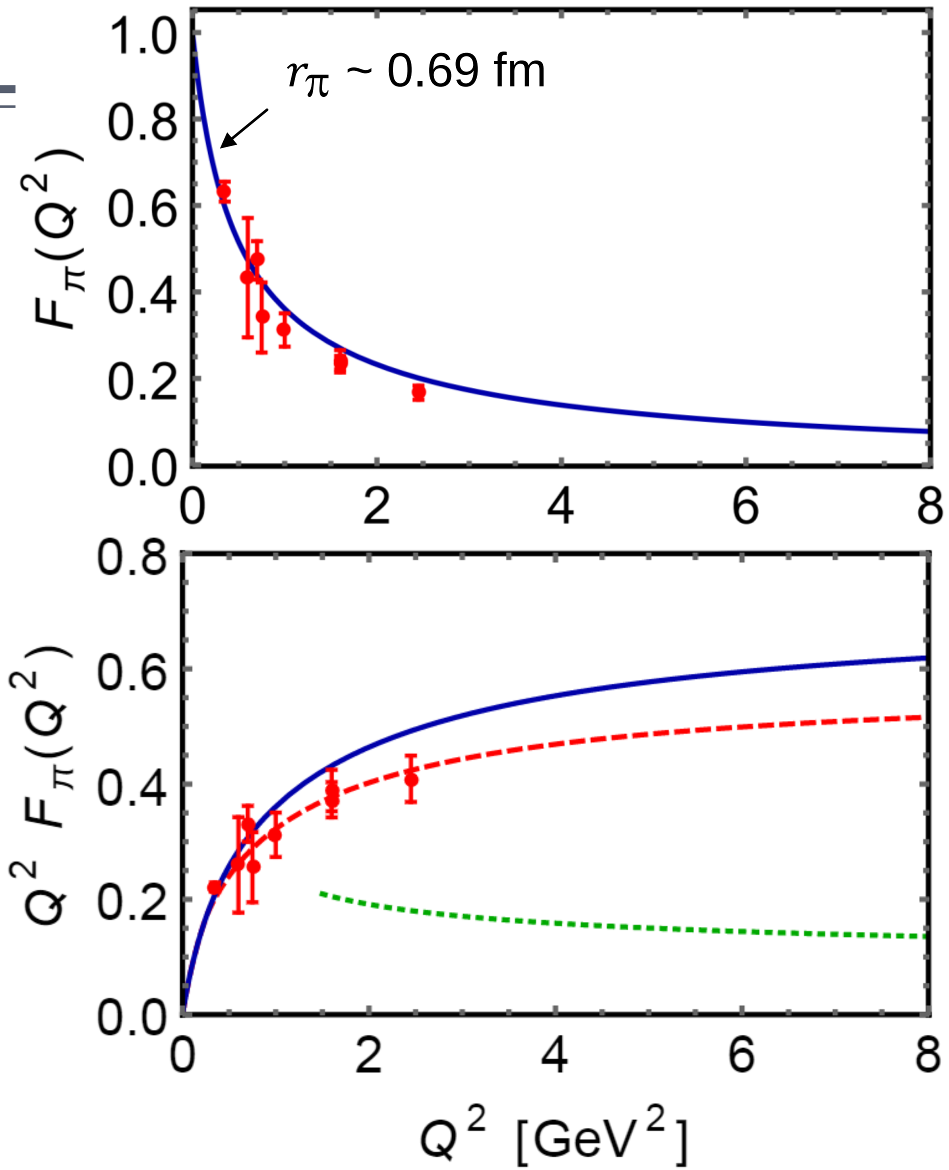
ζ_0 = 0.374 GeV: Obtained to best fit the lattice moments at 2 GeV (π).

ζ_0 = 0.510 GeV: Typical hadron scale. See for example: **Phys.Lett. B737 (2014) 23-29** and **Phys.Rev. D93 (2016) no.7, 074021***.

GPDs and EFF: Pion



$$F_M^q(-t = \Delta^2) = \int_{-1}^{1} dx\, H_M^q(x,\xi,t)$$

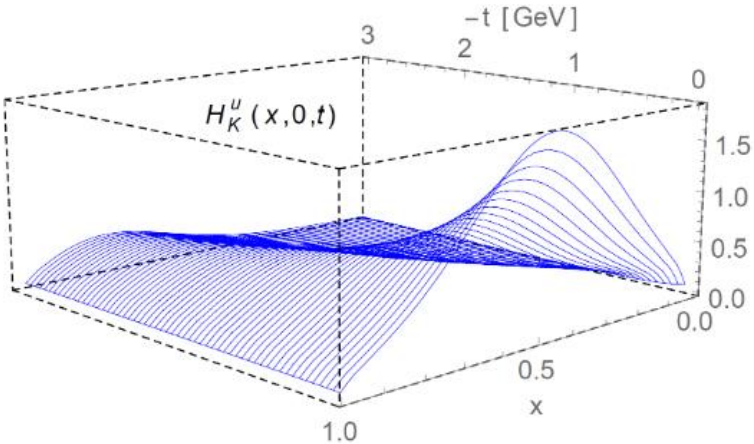


Blue: Computed from GPD

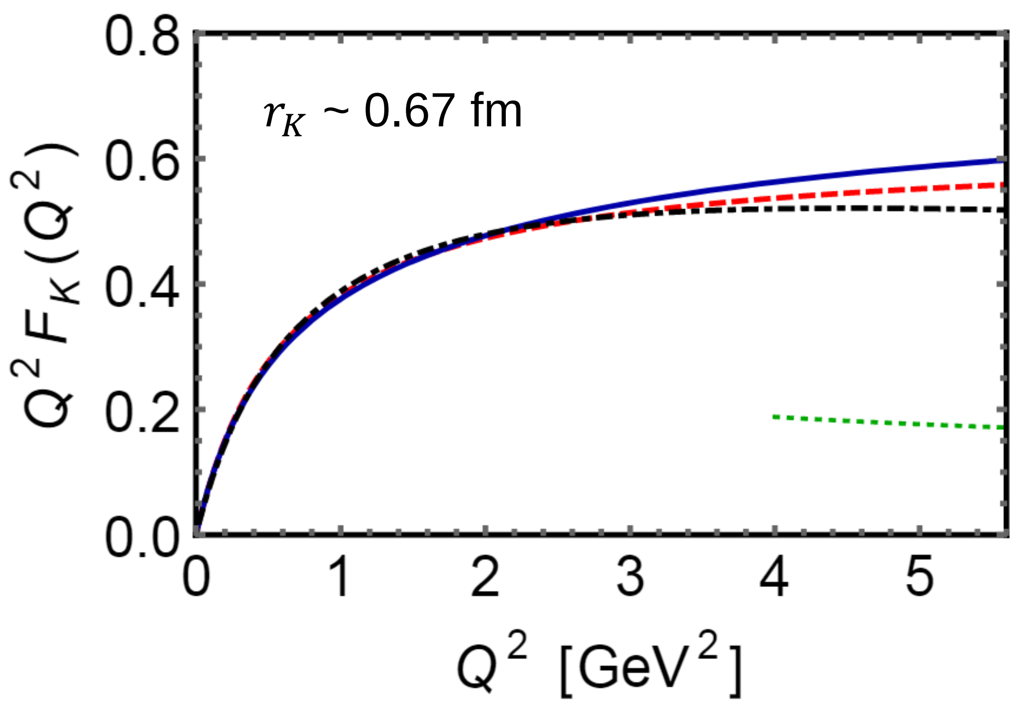
Green: Computed from HS formula

Red: ‘*Evolved*’ form factor

GPDs and PDFs: Kaon



$$F_M^q(-t = \Delta^2) = \int_{-1}^{1} dx\, H_M^q(x, \xi, t)$$



Blue: Computed from GPD

Green: Computed from HS formula

Black: DSE result, **Phys.Rev. D96 (2017) no.3, 034024**

GPDs, PDFs and EFFs

- Employing an AM formulated in the context of LFWF and quasidistributions, **Phys.Rev. D97 (2018) no.9, 094014,** we performed and exploratory study of:

  - Pion and kaon **valence quark GPDs**: DGLAP region, in the overlap representation (see, for example, **Phys.Lett. B780 (2018) 287-293**).
  - Pion and kaon **valence quark PDFs**.
  - Pion and kaon **elastic form factors**.

- **Qualitative results** of PDFs and EFFs seem to be **consistent**.

- **Quantitative results,** when a comparisson when realistic results is possible, are **compatible**.

In collaboration with J. Rodríguez-Quintero et al.

GPDs, PDFs and EFFs

- Short, medium and long term **goals**:

  ✓ Incorporation of missing ingredients: gluon content in kaon and pion (when studying PDFs), the rest of the Bethe-Salpeter amplitudes, for example.

  ✓ Improve our understanding how the parton distributions should **evolve with** their corresponding **evolution equations**.

  ✓ Following **Phys.Lett. B780 (2018) 287-293**, extend our GPD analysis to the ERBL región.

  Mountarde's talk

  ✓ Realistic computations, based upon the **real solutions** of the quark propagator **DSEs** and **BSEs**.

  Chang's talk

  ❖ Reduce model dependence and provide **trustable predictions**.

In collaboration with J. Rodríguez-Quintero et al.

Outline

1. The basics (**DSEs**)
   - Quark propagator and Bethe-Salpeter equation
   - Perturbation theory integral representations
2. PDAs and TFFs [revisited]
3. η-η' PDAs and TFFs
4. LFWFs, GPDs, PDFs and EFFs
5. **Conclusions and remarks**

Conclusions

Phys.Rev. D95 (2017) no.7, 074014. KR et al.
Phys.Rev. D93 (2016) no.7, 074017. KR et al.

- Within a single systematic and consistent approach, starting from quark propagator, **we unified** the description of $\gamma\gamma^*$ **transition form factors** with:
  - ✓ Valence quark **distribution amplitudes**
  - ✓ Charged pion elastic form factor
  - ✓ Masses, decay constants, etc.
- A sound understanding of the distribution of valence-quarks within mesons has been reached.
  - ➢ Smooth connection of **Goldstone modes** with systems containing the **heaviest valence quarks** that can today be studied experimentally.
- **Predictions** for all kinds **of parton distributions** for mesons (and baryons?) **are arising.** QCD based AM models provide excellent starting points for qualitative explorations.

Mezrag's talk

Gao's talk

Final remarks

Phys.Rev. D95 (2017) no.7, 074014. KR et al.
Phys.Rev. D93 (2016) no.7, 074017. KR et al.

- With several facilities at work all around the world, hadron physics is a very active field today: **it is the time to be interested in hadron physics**.

- Continuum QCD has evolved to the point where QCD connected predictions for elastic and transition form factors and parton distributions of **all types are within reach**:
  - **PDFs and GPDs:** Phys.Lett. B737 (2014) 23-29; Phys.Lett. B741 (2015) 190-196; Phys.Rev. D93 (2016) no.7, 074021
  - **PDAs and form factors:** Phys.Rev.Lett. 110 (2013) no.13, 132001; Phys.Rev.Lett. 111 (2013) no.14, 141802; Phys.Lett. B753 (2016) 330-335, Phys.Rev. D93 (2016) no.7, 074017; Phys.Lett. B783 (2018) 263-267.

- Lattice QCD and experiments provide crucial information to improve the theoretical predictions. Exist now an array of exciting predictions waiting for empirical validation.