# Valence-quark parton distribution function of nucleon

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September 13, 2018

# Outline

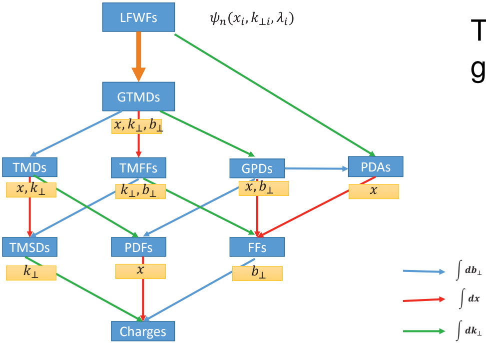
# Light Front

*Benefits of the light-front on studying hadron structure:*

- Quantum mechanics-like wave functions can be defined;
- Quantum-mechanics-like expectation values can be defined and evaluated;
- Parton distributions are correlation functions at equal LF-time $x_+$; namely, within the initial surface $x_+ = 0$ and can thus be expressed directly in terms of ground state LF wavefunctions.

# Light Vector Meson PDAs

The light front quantities can be generally sorted into two parts:




The light front quantities can be generally sorted into two parts:

- Parton in hadron state: $< 0|J_\mu|\Psi >$ — better understanding on the theory
- Parton in scattering process: $< \Psi|J_\mu|\Psi >$— closer to experiments.

## DSE framework on light front

*The computation in light front is very natural in the framework of Dyson-Schwinger equations, since the light front wave function is just the light front projection of bound states' Lorentz covariant wavefunction:*

- Delivered the first QCD-connected unification of the parton distribution amplitudes of light-light and heavy-heavy mesons
- The first QCD-connected prediction of pion and kaon form factors on the entire domain of spacelike $Q^2$
- QCD-connected computation of valence-quark distribution functions in the kaon and pion

## PDAs

For parton distribution amplitude, the definition can be easily expressed with quark propagator and Beth-Salpeter amplitude:

$$
\begin{aligned}
& f_{PS}\phi_{PS}(x,\zeta) \\
=& tr_{CD} Z_2(\zeta,\Lambda) \int_{dq}^{\Lambda} \delta(n\cdot q_+ - xn\cdot P)\gamma_5\gamma\cdot n\chi_5(q;P)\,, \quad (1a) \\
& f_V^{\perp}\phi_{V\perp}(x,\zeta) m_V^2 \\
=& n\cdot P tr_{CD} Z_T(\zeta,\Lambda) \int_{dq}^{\Lambda} \delta(n\cdot q_+ - xn\cdot P)\sigma_{\mu\lambda}P_\mu\chi_\lambda(q;P)\,, \quad (1b) \\
& f_V n\cdot P\phi_{V\parallel}(x,\zeta) \\
=& m_V tr_{CD} Z_2(\zeta,\Lambda) \int_{dq}^{\Lambda} \delta(n\cdot q_+ - xn\cdot P) n\cdot\gamma n_\lambda\chi_\lambda(q;P)\,, \quad (1c)
\end{aligned}
$$

## An analytic Example

Consider the quark propagator and BS amplitude with the following form ($\Delta_M^\nu(z) = 1/(z+M^2)^\nu$):

$$
\begin{aligned}
S(p) &= [-i\gamma \cdot p + M]\Delta_M(p^2)\,, \\
\rho_\nu(z) &= \frac{1}{\sqrt{\pi}}\frac{\Gamma(v+3/2)}{\Gamma(\nu+1)}(1-z^2)^\nu\,, \\
\Gamma_\lambda(q;P) &= i\gamma_\lambda \frac{M^3}{f_\rho}\int_{-1}^{1} dz\rho_\nu(z)\Delta_M^\nu(q^2_{+z})\,,
\end{aligned}
$$

After Feynmann parametrization, we could get the analytic form of distribution amplitude.

- *If $\nu = 1$, the ultraviolet behaviors of BS amplitudes act as $1/q^2$ which is the asymptotic behavior of QCD and we found the PDA go back to the asymptotic form 6x(1-x)*
- *If $\nu = 0$, it's a point-like structure and people obtain a constant amplitude*

## Realistic case

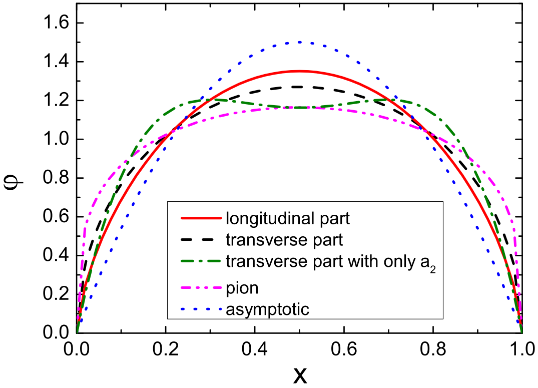
- Compute the moments with the above definition
- Reconstruct PDAs with Gegenbauer polynomials of order $\alpha$:

$$\phi(x) \approx \phi_m(x) = N_\alpha [x(1-x)]^{\alpha-1/2}[1 + \sum_{j=2,4,\ldots}^{j_{max}} a_j^\alpha C_j^\alpha(2x-1)]$$

Noticing that, $\alpha = 3/2$-basis is the conformal expansion, and people have found that

> *the fixed $\alpha = 3/2$-basis converges slower than with Gegenbauer polynomials of order $\alpha$*

# $\pi$ and $\rho$ Distribution Amplitudes

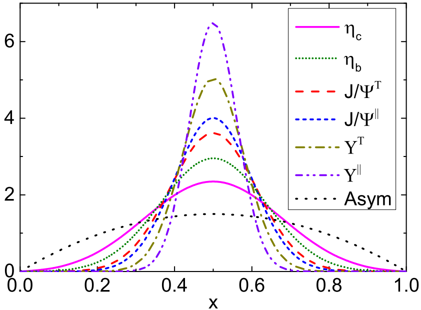



- Light mesons' PDAs are all broader than asymptotic form
- Double-humped shape caused by the incomplete expansion with $\alpha = 3/2$ basis

[1] F. Gao, L. Chang, Y. X. Liu, C. D. Roberts, et al. Phys. Rev. D 90 014011(2014).

# PDAs for quarkonium

We also obtain the PDAs for quarkonium.




- For the same quark mass, the different polarization affect meson's amplitudes
- As the quark mass goes larger, the PDAs tend to be $\delta$ function

$$\phi_{\Upsilon_{\|}} <_N \phi_{\Upsilon_{\perp}} <_N \phi_{J/\Psi_{\|}} <_N \phi_{J/\Psi_{\perp}} <_N \phi_{\eta_b} <_N \phi_{\eta_c} <_N \phi_{asymptotic}$$

## Critical Mass Scale

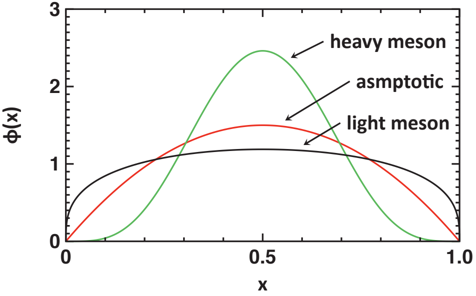
*PDAs for light meson:*

- Broader than the asymptotic form $\phi^{asy}(x) = 6x(1-x)$;
- The broadest shape of PDA, $\phi(x) = constant$, means the meson is point-like.

*PDAs for heavy meson:*

- Narrower than the asymptotic form $\phi^{asy}(x) = 6x(1-x)$;
- The narrowest shape of PDA, $\phi(x) = \delta(x - 1/2)$, means the meson is like a two-static-particle system.

# Critical Mass Scale




There must exist a critical mass at which $\phi(x) = \phi^{asy}(x)$

| | $\varphi_{PS}$ | $\varphi_{V,\perp}$ | $\varphi_{V,\parallel}$ |
|---|---|---|---|
| $m_{cri}$ | 0.15 GeV | 0.13 GeV | 0.12 GeV |

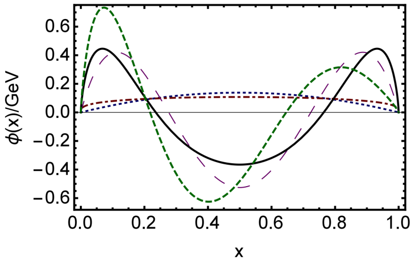
*The critical mass typically lies just above the s-quark mass.*

# PDAs of Excited States

- In the chiral limit, the decay constant of excited states becomes 0 owing to the relation:

$$f_{PS}\, M_{PS}^2 = 2 m_q\, \rho_{PS}$$

  The zeroth moment of its PDA becomes 0, PDAs become negative.
- The PDAs for excited states $\pi_1$ (dark solid) and $K_1$ (green dashed) compared with those for ground states:



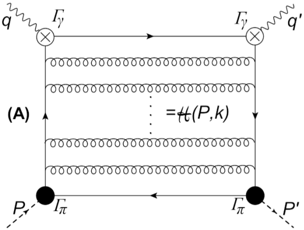

Indicates that PDAs of n-th radial excited states contains 2n zeros, which is similar to the radial wave function in the quantum mechanics.

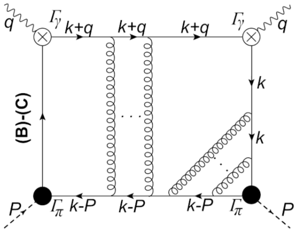
## *Our results for PDAs and their implications:*

- First QCD-connected computation of the pointwise behavior of PDAs for both light and heavy mesons.
- First to put this in print and compute the value of critical mass.
  - Lies in the neighborhood of the s-quark current-mass.
  - Indication that no expansions in the s-quark mass can be reliable (e.g., ChPT) because it defines a transition boundary for internal hadron dynamics.
- Connection between PDAs and the radial wave function in the quantum mechanics
  - PDAs of n-th radial excited states contains 2n zeros.

# Diagrams for PDF

*A corrected leading-order expression of parton distribution function includes two diagrams:*



The first diagram can be described in terms of the derivative of propagator based on ward identity:

$$\Gamma_n = n_\mu \frac{\partial S(k \pm P)}{\partial k_\mu} \tag{2}$$

The Second diagram can be similarly described by the derivative of the vertex:

$$\tilde{\Gamma}_n = n_\mu \frac{\partial \Gamma(k; P)}{\partial k_\mu} \tag{3}$$

*Only by considering both diagrams, people can obtain the correct momentum sum rule:*

- without meson-cloud corrections and dressed-gluon distribution , $< x >^\pi_q = \frac{1}{2}$

# PDF of pion

Employing the same representation for meson as in the computation of PDAs:

$$\begin{aligned} S(p) &= [-i\gamma \cdot p + M]\Delta_M(p^2)\,, \\ \rho_\nu(z) &= \frac{1}{\sqrt{\pi}}\frac{\Gamma(v+3/2)}{\Gamma(\nu+1)}(1-z^2)^\nu\,, \\ \Gamma_\lambda(q;P) &= i\gamma_\lambda \frac{M^3}{f_\rho}\int_{-1}^{1} dz\rho_\nu(z)\Delta_M^\nu(q^2_{+z})\,, \end{aligned}$$

With $\nu = 1$, the BS wavefunction owns asymptotic behaviour of QCD,

- $q(x \to 1) \propto (1-x)^2$.

# Quark-diquark model for nucleon

Considering the quark-diquark model for nucleon. Giving the algebraic model for quark propagator $S(k)$, diquark propagator $D^{s,av}(k)$ and the quark-diquark amplitude $\Gamma^{s,av}(k;P)$as followings:

$$
\begin{aligned}
S^{-1}(k) &= i\not{k} + M, \\
D^{s}(k) &= \frac{1}{k^2 + M_s^2} \\
D^{av}(k) &= \left(\delta_{\mu\nu} + \frac{k_\mu k_\nu}{k^2}\right)\frac{1}{k^2 + M_{av}^2}, \\
\Gamma^{s}(k;P) &= \int_{-1}^{1} dz\rho(z)\left(F(k_z)^3 + i\alpha(\not{k} - \frac{\not{P}k\cdot P}{P^2})F(k_z)^{5/2}\right), \\
\Gamma^{av}_{\mu}(k;P) &= \beta\frac{1}{\sqrt{3}}(\gamma_\mu - \frac{P_\mu k\cdot P}{P^2})\int_{-1}^{1} dz\rho(z)F(k_z)^{5/2},
\end{aligned} \tag{4}
$$

There will exist [$ud$]-scalar diquark, [$ud$]-axial vector diquark and [$uu$]-axial vector diquark.

## quark distribution

After this, the PDF from the quark (as in left diagram) can be defined as:

$$\Lambda_+(P)\not{n}u_1(x)\Lambda_+(P) = \int d^4k(2\pi)^4\delta(n\cdot k - xn\cdot P)\Lambda_+(P)(O^s + O^{av})\Lambda_+(P), \quad (5)$$

$$\Lambda_+(P)\not{n}d_1(x)\Lambda_+(P) = 2\int d^4k(2\pi)^4\delta(n\cdot k - xn\cdot P)\Lambda_+(P)(O^{av})\Lambda_+(P), \quad (6)$$

with

$$O^s = \bar{\Gamma}^s(k - P/2; -P)n_\sigma \frac{\partial S(k)}{\partial k_\sigma}\Gamma^s(k - P/2; P)D^s(k - P)$$

and

$$O^{av} = \bar{\Gamma}^s(k - P/2; -P)n_\sigma \frac{\partial S(k)}{\partial k_\sigma}\Gamma^{av}(k - P/2; P)D^{av}_{\mu\nu}(k - P)$$

.

## diquark distribution

the diquark distribution as in right diagram can be expressed as:

$$\Lambda_+(P)\not{n} f^{s,av}(x)\Lambda_+(P) = \int d^4k(2\pi)^4\delta(n\cdot k - xn\cdot P)\Lambda_+(P)\tilde{O}^{s,av}\Lambda_+(P), \quad (7)$$

with

$$\tilde{O}^s = -O^s + n_\delta \frac{d(-\bar{\Gamma}^s(k-P/2;-P)S(k)\Gamma^s(k-P/2;P)D^s(k-P))}{dk_\delta}$$

and

$$\tilde{O}^{av} = -O^{av} + n_\delta \frac{d(-\bar{\Gamma}^{av}_\mu(k-P/2;-P)S(k)\Gamma^{av}_\nu(k-P/2;P)D^{av}_{\mu\nu}(k-P))}{dk_\delta}$$

## quark distribution in diquark

The quark distribution in diquark can be taken into account by:

$$\tilde{u}_{s,av} = \int_x^1 \frac{1}{y} f^{s,av}(y) f_{q/s,av}(x/y). \tag{8}$$

where $f_{q/s,av}$ is chosen as:$f_{q/s,av}(z) = 30z^2(1-z)^2$. The final parton distribution functions of $u$ and $d$ quark are:

$$\begin{aligned} u(x) &= f^u(x) + \tilde{u}^s(x) + 5\tilde{u}^{av}(x) \\ d(x) &= f^d(x) + \tilde{d}^s(x) + \tilde{d}^{av}(x) \end{aligned} \tag{9}$$

## $x \to 1$

*Behaviour at $x \to 1$:*

The denominator are
$(k_- P_+(x + (z-1)/2) + k_\perp^2 + M^2)^a (k_- P_+(x + (z'-1)/2) + k_\perp^2 + M^2)^a (k_- P_+ x + k_\perp^2 + M^2)^2 (k_- P_+(x-1) + k_\perp^2 + M^2)$.
For the higher order singularities:

- Can be rewritten with Cauchy formula as $\int dk_- f(k_-)/(k_- - k'_-)^n = d^{n-1} f(k'_-)/d{k'_-}^{n-1}$.
- More derivatives mean more quark-photon vertex and thus are suppressed, here we focus on the contribution from the first order singularity, mass pole, that is, from $(k_- P_+(x-1) + k_\perp^2 + M^2)$.

## $x \to 1$

Within the residue theorem, if employing $\rho(z) = (1 - z^2)$ which leads to the asymptotic behaviour of QCD, the denominator always goes to $(1 - x)^5$.
After then we can study the behaviour of the numerator, the leading order of $(1 - x)$ will also come from the pole $1/((k - P)^2 + M^2)$ at which $k_- \sim \frac{k_\perp^2 + M^2}{1-x}$, and thus,

- The quark pdf includes $L = 0$ of scalar diquark contributes $(1 - x)^0$ and thus, totally, it will be $(1 - x)^5$.
- The quark pdf from $L = 1$ and also the axial vector diquark is $(1 - x)^5 \times \frac{k_\perp^2}{(1-x)^2} \sim (1 - x)^3$.

## $x \to 1$

*For the diquark pdf, the direct computation gives:*

From $L = 0$ of scalar diquark is $(1 - x)^6$ and from the others are $(1 - x)^3$.

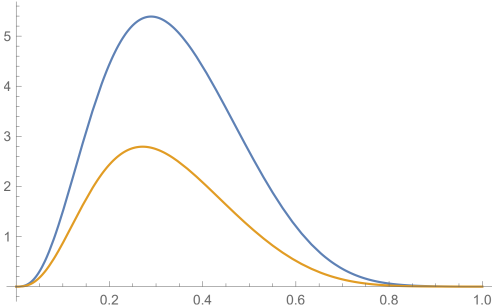
*After including the quark distribution in diquark:*

From $L = 0$ of scalar diquark is $(1 - x)^9$ and from the others are $(1 - x)^6$.

- *The behaviour of nucleon pdf at $x \to 1$ is $(1 - x)^3$*
- *The leading contribution comes from the quark distribution with $L = 1$ scalar diquark and also the axial vector diquark.*

## numerical results

The valence-quark pdf $u(x)$ and $d(x)$ at hadron scale $\zeta_H$:




The PDF satisfy the relation:

$$\int dx u(x) = 2 \int dx d(x) = 2 \tag{10}$$

$$\int dx x(u(x) + d(x)) = 1 \tag{11}$$

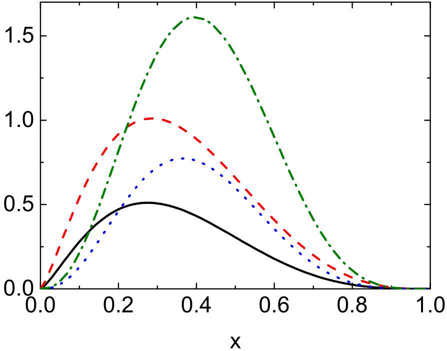
## evolution

The scale evolution can be obtained by:

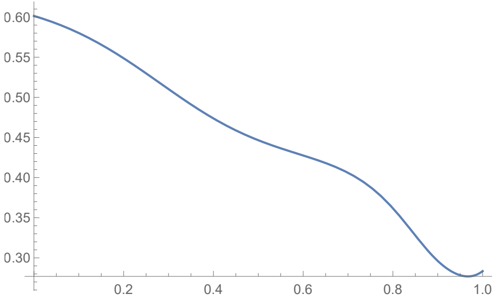
$$< x_\zeta^m > = < x_{\zeta_H}^m > [\frac{\alpha(\zeta^2)}{\alpha(\zeta_H^2)}]^{\gamma_0^m/\beta_0}, \alpha(\zeta^2) = \frac{4\pi}{\beta_0 ln(\zeta^2/\Lambda_{QCD}^2)}$$

$$\gamma_0^m = -\frac{4}{3}[3 + \frac{2}{(m+1)(m+2)} - 4\sum_{k=1}^{m+1}\frac{1}{k}]$$

$xu(x)$ and $xd(x)$ at hadron scale and at $\zeta = 2$ GeV:



The ratio of $d(x)/u(x)$:




Noticing that, if only with scalar diquark, the two diagrams lead to $d(x)/u(x) = 0$.

- The two-loop diagram will give a very small correction.

## gluon contribution

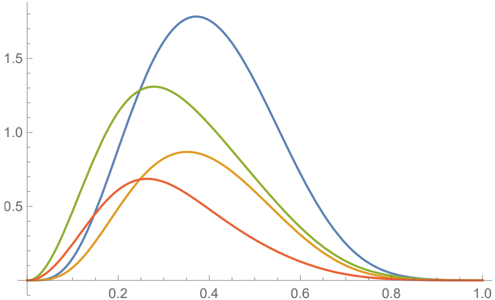
Considering the gluon contribution, the momentum of pion will be shifted into gluon distribution:

$$g(x) = \delta_q(x) = s_g x^{\alpha_g}(1-x)^{\beta_g} P_1^{\beta_g \alpha_g}(2x-1), \tag{12}$$

The gluon distribution is assigned as following:

$$\begin{aligned} u_f(x) &= u(x) - 2/3g(x) \\ d_f(x) &= d(x) - 1/3g(x) \end{aligned} \tag{13}$$

The valence-quark parton distribution function after adding the gluon contribution ($xu(x)$ and $xd(x)$):




At hadron scale:

$< x_u >$: 0.675 $\rightarrow$ 0.517
$< x_d >$: 0.324 $\rightarrow$ 0.245

*Results for nucleon PDF:*

A corrected leading-order expression of parton distribution function is employed here to compute the nucleon pdf in the quark-diquark picture

- The $x \to 1$ behaviour of nucleon pdf is $(1-x)^3$, which is contributed from the quark distribution with the $L=1$ component of scalar diquark and also axial quark;
- $d$ and $u$ valence-quark distribution have been obtained, also with gluon contribution as the shifting of the momentum distribution;
- Computation of the evolution of pdf and also the ratio of $d(x)$ and $u(x)$