Power corrections and renormalons in parton quasi-distributions

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based on: VB, Alexei Vladimirov, Jian-Hui Zhang, *in preparation*

ECT* workshop: Mapping parton distribution amplitudes and functions

Parton distributions from Euclidean observables

- PDFs from Euclidean correlation functions at large momenta — VB, D.Müller '08; X.Ji '13
- Non-standard observables
- Power corrections ?

Concept

M. Beneke, Phys.Rept. 317 (1999)
M. Beneke, V. Braun, hep-ph/0010208

- Leading twist calculation "knows" about the necessity to add a power correction

  Example:

$$F_2(x,Q^2) = 2x\int_x^1 \frac{dy}{y} C(y,Q^2/\mu^2) q(\tfrac{x}{y},\mu^2) + \frac{1}{Q^2} D_2(x)$$

$$C(y) = \delta(1-y) + \sum_{n=0}^{\infty} c_n \alpha_s^{n+1}, \qquad \alpha_s = \alpha_s(\mu)$$

Cut-off scheme

Imagine the separation between CFs and MEs is done using exlicit cutoff at $|k| = \mu$.
CFs will be modified compared to usual calculation by terms $\sim \mu^2/Q^2$

$$C(y)|^{\rm cut} = \delta(1-y) + \sum_{n=0}^{\infty} c_n \alpha_s^{n+1} - \frac{\mu^2}{Q^2} d(x) + \mathcal{O}\left(\frac{\mu^4}{Q^4}\right)$$

The dependence on μ must cancel:

- Logarithmic terms $\ln Q^2/\mu^2$ in CFs against μ-dependence in PDFs
- Power-terms μ^2/Q^2 against the higher-twist contributions

This means that $D_2(x)$ *in the cutoff scheme* must have the form

$$D_2(x) = \mu^2 2x \int_x^1 \frac{dy}{y} d_2(x) Q(\frac{x}{y}) + \delta D_2(x)$$

— related to quadratic UV divergences in matrix elements of twist-4 operators (in this scheme!)

Dimensional regularization

- In dim.reg. power-like terms in the CFs do not appear. Instead, the coefficients c_k (e.g., in $\overline{\text{MS}}$) diverge factorially with the order k

— The factorial divergence implies that the sum of the pert. series is only defined to a power accuracy and this ambiguity (renormalon ambiguity) must be compensated by adding a non-perturbative higher–twist correction

— Detailed analysis [Beneke:2000kc]: the asymptotic large-order behavior of the coefficients (the renormalons) is in one-to-one correspondence with the sensitivity to extreme (small or large) loop momenta

— Infrared renormalons in the l.t. CF are compensated by ultraviolet renormalons in the MEs of twist-four operators. At the end the same picture re-appears: only the details depend on the factorization method

Deep inelastic scattering

Quadratic term in μ is spurious since its sole purpose is to cancel the similar contribution to the CF $\Rightarrow$ does not contribute to any physical observable.

— Assume that the "true" twist four term is of the same order, get a **renormalon model**

$$D_2(x) = \varkappa\Lambda^2_{\rm QCD} 2x \int_x^1 \frac{dy}{y} d_2(x) Q(\frac{x}{y}) \,, \qquad \varkappa = \mathcal{O}(1)$$

One-loop result:

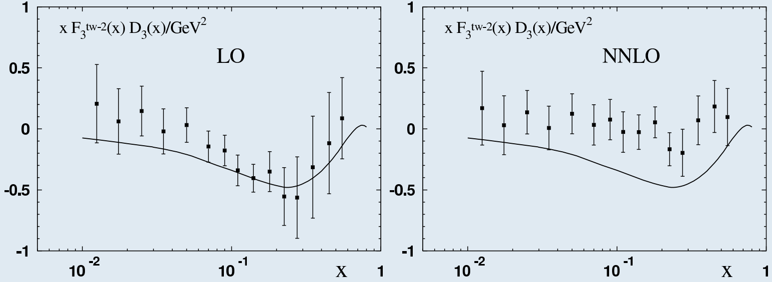
$$d_2^{(q)} = -\frac{4}{[1-x]_+} + 4 + 2x + 12x^2 - 9\delta(1-x) - \delta'(1-x)$$
$$d_L^{(q)} = 8x^2 - 4\delta(1-x)$$

Main conclusions:

- ratio twist-4/twist-2 is target-independent (if assumption correct)
- enhancement at $x \to 1$:
$$\left[\frac{\Lambda^2}{Q^2(1-x)}\right]^n$$

Is $\varkappa$ universal? E.g. the same for F_L and F_2? Dokshitzer, Webber: $\varkappa \to \bar{\alpha}_0$ universal nonperturbative coupling

Example: CCFR data on $F_3(x,Q^2)$:

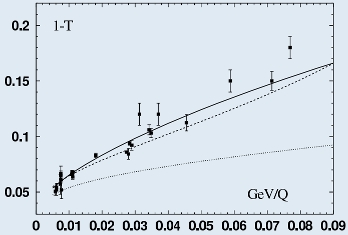


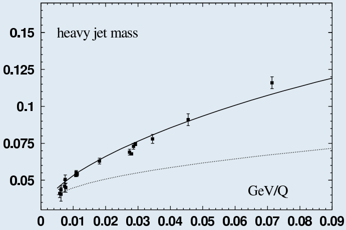
Hadronic event shape variables in e^+e^- annihilation

$$\langle S \rangle = \int d\mathsf{PS}[p_i] |\mathcal{M}_{q\bar{q}g}|^2 S(p_i)$$

e.g. thrust $S = 1 - T$

$$\langle 1 - T \rangle = t_1 \alpha_s(\mu) + \left[t_2^{(0)} + t_2^{(1)} \ln \frac{\mu^2}{Q^2} \right] \alpha_s^2(\mu) + \varkappa_T \frac{\Lambda_{\rm T}}{Q} + \mathcal{O}(1/Q^2)$$



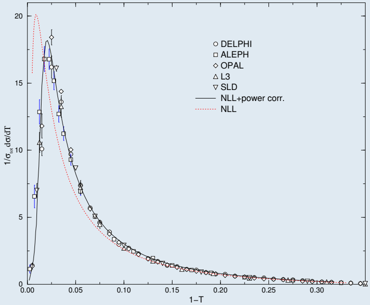


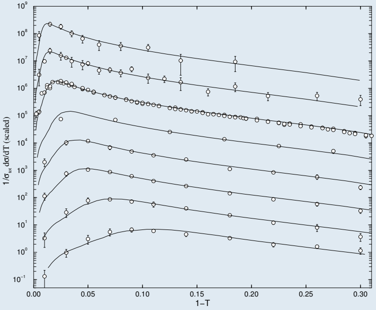
Dotted lines: NLO, solid: with $\frac{\Lambda}{Q}$, dashed: NLO with $\mu = 0.07Q$.
Universality $\Lambda_{\rm thrust} \simeq \Lambda_{\rm jet\, mass}$ works to 20%.

Event shape distributions

$$\frac{d\sigma}{dt}(t) \;\mapsto\; \frac{d\sigma}{dt}(t-\Lambda/Q) + \mathcal{O}\left(1/(tQ)^2\right)$$

If $t \sim \Lambda/Q$ all terms in $1/(tQ)^k$ have to be resummed — "shape function"





— fitted for $Q = 91.2$ GeV, used for $Q = 14, 22, 35, 44, 55, 91, 133, 161$ GeV:

Quasi-distributions (1)

- Ioffe-time quasi-distributions

$$\langle N(p)|\bar{q}(z)[z,0]\not{z}q(0)|N(p)\rangle = 2(pz)\mathcal{I}^{\parallel}(z^2,pz)$$
$$\langle N(p)|\bar{q}(z)[z,0]\not{\epsilon}q(0)|N(p)\rangle = 2(p\epsilon)\mathcal{I}^{\perp}(z^2,pz)\,, \qquad (\epsilon\cdot z)=0$$

QCD factorization (light-ray OPE)

$$\mathcal{I}^{\parallel(\perp)}(z^2,pz) = C^{\parallel(\perp)}(\mu_F)\otimes\int_{-1}^{1}dx\,e^{ixpz}q(x,\mu_F) \quad\longleftarrow\quad \text{Ioffe-time distribution}$$
$$+\,\mathcal{O}(z^2)$$

Quasi-distributions (2)

Let $z^\mu \mapsto z v^\mu, \quad z \in \mathbb{R}$

- parton quasi-distributions [Ji:2013dva]

$$\mathcal{Q}^{\parallel(\perp)}(x,p) = (pv)\int_{-\infty}^{\infty} \frac{dz}{2\pi} e^{-ixz(pv)}\, \mathcal{I}^{\parallel(\perp)}(z^2v^2, pvz)$$

- parton pseudo-distributions [Radyushkin:2017cyf]

$$\mathcal{P}(x,z) = z\int_{-\infty}^{\infty} \frac{d(pv)}{2\pi} e^{-ixz(pv)}\, \mathcal{I}^{\perp}(z^2v^2, pvz)$$

Normalized (scale-invariant) quasi-distributions

Quasidistributions are scale-dependent (Z_q in axial gauge) and in renormalization schemes not based on dim. reg. also suffer from a linear UV divergence of the Wilson line. Problem can be avoided by considering a scale-independent ratio:

- diving out the value at zero proton momentum [Orginos:2017kos]

$$\mathbf{I}(z, pv) = \mathcal{I}(z, pv, \mu)/\mathcal{I}(z, 0, \mu)$$

- or, alternatively, normalizing the qITD to the vacuum correlator

$$\widehat{\mathbf{I}}(z, pv) = \mathcal{I}(z, pv, \mu)/\mathcal{N}(z, \mu)\,, \qquad \mathcal{N}(z, \mu) = \left(\frac{2iN_c}{\pi^2 z^2}\right)^{-1} \langle 0|\bar{q}(z)\not{z}[z, 0]q(0)|0\rangle$$

In this way one can define the scale-independent qPDF/pPDF

$$\mathbf{Q}(x, p) = (pv)\int_{-\infty}^{\infty} \frac{dz}{2\pi} e^{-ixz(pv)}\, \mathbf{I}(z, pv)$$

$$\mathbf{P}(x, z) = z\int_{-\infty}^{\infty} \frac{d(pv)}{2\pi} e^{-ixz(pv)}\, \mathbf{I}(z, pv)$$

and similarly for $\widehat{\mathbf{Q}}(x, p)$ and $\widehat{\mathbf{P}}(x, z)$

- normalization to the vacuum correlator does not affect leading $\mathcal{O}(z^2)$ power corrections, whereas the normalization to zero momentum, as we will see, has a nontrivial effect.

Borel transform and renormalons

- light-ray OPE

$$\bar{q}(zv)\not{v}[zv,0]q(0) = \int_0^1 d\alpha\, H^{\|}(z,\alpha,\mu,\mu_F)\Pi^{\mu_F}_{\text{l.t.}}[\bar{q}(\alpha z v)\not{z}q(0)] + \dots$$

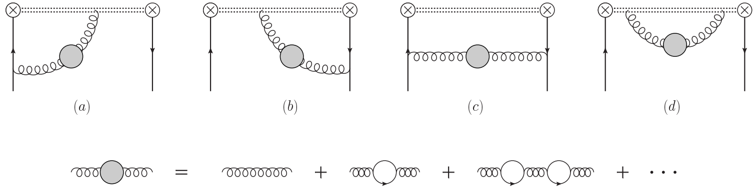
$$H = \delta(1-\alpha) + \sum_{k=0}^{\infty} h_k a_s^{k+1}\,, \qquad a_s = \frac{\alpha_s(\mu)}{4\pi}\,, \qquad h_k \propto k!$$

- A convenient way to handle such a series is to consider the Borel transform

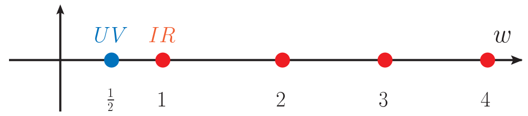
$$B[H](w) = \sum_{k=0}^{\infty} \frac{h_k}{k!}\left(\frac{w}{\beta_0}\right)^k \qquad H = \delta(1-\alpha) + \frac{1}{\beta_0}\int_0^\infty dw\, e^{-w/(\beta_0 a_s)} B[H](w)$$

However, integration is obscured by singularites on the integration path — renormalons

- Bubble-chain approximation 't Hooft, '77



Singularities of the Borel transform



- Ultraviolet renormalon at $w = 1/2$ M. Beneke, VB, '94

$$B[H^{\parallel(\perp)}] \overset{w\to 1/2}{=} \frac{4C_F}{w-1/2}\left(-\frac{v^2z^2\mu^2e^{5/3}}{4e^{-2\gamma_E}}\right)^{1/2}$$

— removed by normalization or by explicit subtraction

- Infrared renormalon at $w = 1$

$$B[H^{\parallel}](w) \overset{w\to 1}{=} \frac{-4C_F}{1-w}\left[\alpha + \bar\alpha\ln\bar\alpha\right]\left(-\frac{v^2z^2\mu^2e^{5/3}}{4e^{-2\gamma_E}}\right)$$

$$B[H^{\perp}](w) \overset{w\to 1}{=} \frac{-4C_F}{1-w}\left[\alpha + \bar\alpha\ln\bar\alpha + \alpha\bar\alpha\right]\left(-\frac{v^2z^2\mu^2e^{5/3}}{4e^{-2\gamma_E}}\right)$$

Leading power corrections to quasi-PDFs

$$\mathcal{Q}^{\parallel}(x,p) = q(x) - \frac{v^2\Lambda^2}{x^2(pv)^2}\int_{|x|}^{1}\frac{dy}{y}\left[\frac{y^2}{[1-y]_+} + 2\delta(\bar{y}) + \delta'(\bar{y})\right]q(\tfrac{x}{y})$$

$$\mathcal{Q}^{\perp}(x,p) = q(x) - \frac{v^2\Lambda^2}{x^2(pv)^2}\int_{|x|}^{1}\frac{dy}{y}\left[\frac{y^2}{[1-y]_+} + 3\delta(\bar{y}) + \delta'(\bar{y}) - 2y^2\right]q(\tfrac{x}{y})$$

where $\Lambda = \mathcal{O}(\Lambda_{\mathrm{QCD}})$

For a numerical study, present the result in the form

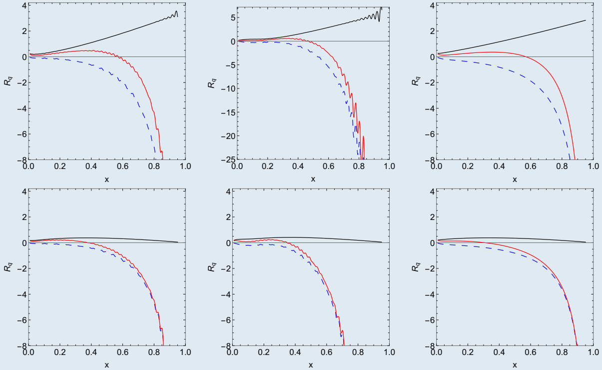
$$\mathcal{Q}^{\parallel(\perp)}(x,p) = q(x)\left\{1 - \frac{v^2\Lambda^2}{x^2(1-x)(pv)^2}\mathcal{R}_{\mathcal{Q}}^{\parallel(\perp)}(x)\right\}$$

Leading power corrections to quasi-PDFs

u-quark | d-quark | $q(x) = x^{-1/2}(1-x)^3$

$\mathcal{R}^{\parallel}_{\mathcal{Q}}$

$\mathcal{R}^{\perp}_{\mathcal{Q}} - \mathcal{R}^{\parallel}_{\mathcal{Q}}$



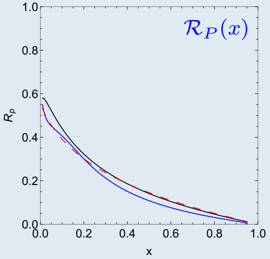
black: non-normalized qPDFs (original Ji's definition)
blue: subtraction terms from normalization at zero momentum
red: normalized qPDFs (after subtraction at zero momentum)

used: MSFW NLO valence quark PDFs at 2 GeV

Leading power corrections to pseudo-PDFs

$$\mathcal{P}(x,z,\mu) = q(x) + (v^2 z^2 \Lambda^2)\int_{|x|}^{1} \frac{dy}{y}\,(y + \bar{y}\ln\bar{y} + y\bar{y})q(\tfrac{x}{y}) \;\equiv\; q(x)\Big\{1 + (v^2 z^2 \Lambda^2)\mathcal{R}_P(x)\Big\}$$

$$\mathbf{P}(x,z,\mu) = q(x) + (v^2 z^2 \Lambda^2)\int_{|x|}^{1} \frac{dy}{y}\,[y + \bar{y}\ln\bar{y} + y\bar{y}]_+ q(\tfrac{x}{y}) \;\equiv\; q(x)\Big\{1 + (v^2 z^2 \Lambda^2)\mathbf{R}_P(x)\Big\}$$



MSFW u-quarks (black)
MSFW d-quarks (blue)
$q(x) = x^{-1/2}(1-x)^3$ (red)

$\mathcal{R}_P(x) \overset{x\to 1}{=} \mathcal{O}(1-x)\,,$ ☺

$\mathbf{R}_P(x) = \mathcal{R}_P(x) - \frac{5}{12}\,,$ ☹

Summary

- Power corrections for qPDFs have a generic behavior

$$\mathcal{Q}(x,p) = q(x)\left\{1 + \mathcal{O}\left(\frac{\Lambda^2}{p^2}\cdot\frac{1}{x^2(1-x)}\right)\right\}$$

Normalization to zero momentum considerably reduces the correction at $0.1 < x < 0.6$ at the cost of a strong enhancement at $x > 0.6$

- Power corrections for pPDFs have a generic behavior

$$\mathcal{P}(x,z) = q(x)\left\{1 + \mathcal{O}\left(z^2\Lambda^2(1-x)\right)\right\}$$

but the suppression at $x \to 1$ is lifted by the normalization to zero momentum

- Position space PDFs (qITDs) have flat power corrections at large Ioffe times