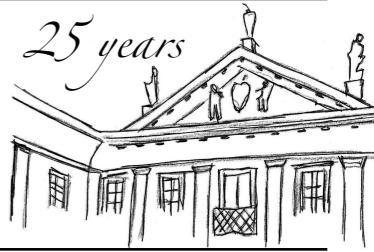




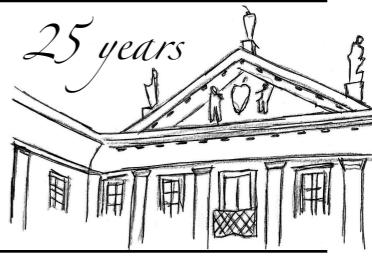
European Centre for Theoretical Studies
in Nuclear Physics and Related Areas



Parton Distribution Amplitudes for heavy-light pseudoscalar mesons

Daniele Binosi
ECT* - Fondazione Bruno Kessler, Italy

Mapping Parton Distribution Amplitudes and Functions
ECT*, Trento, Italy
September 13, 2018

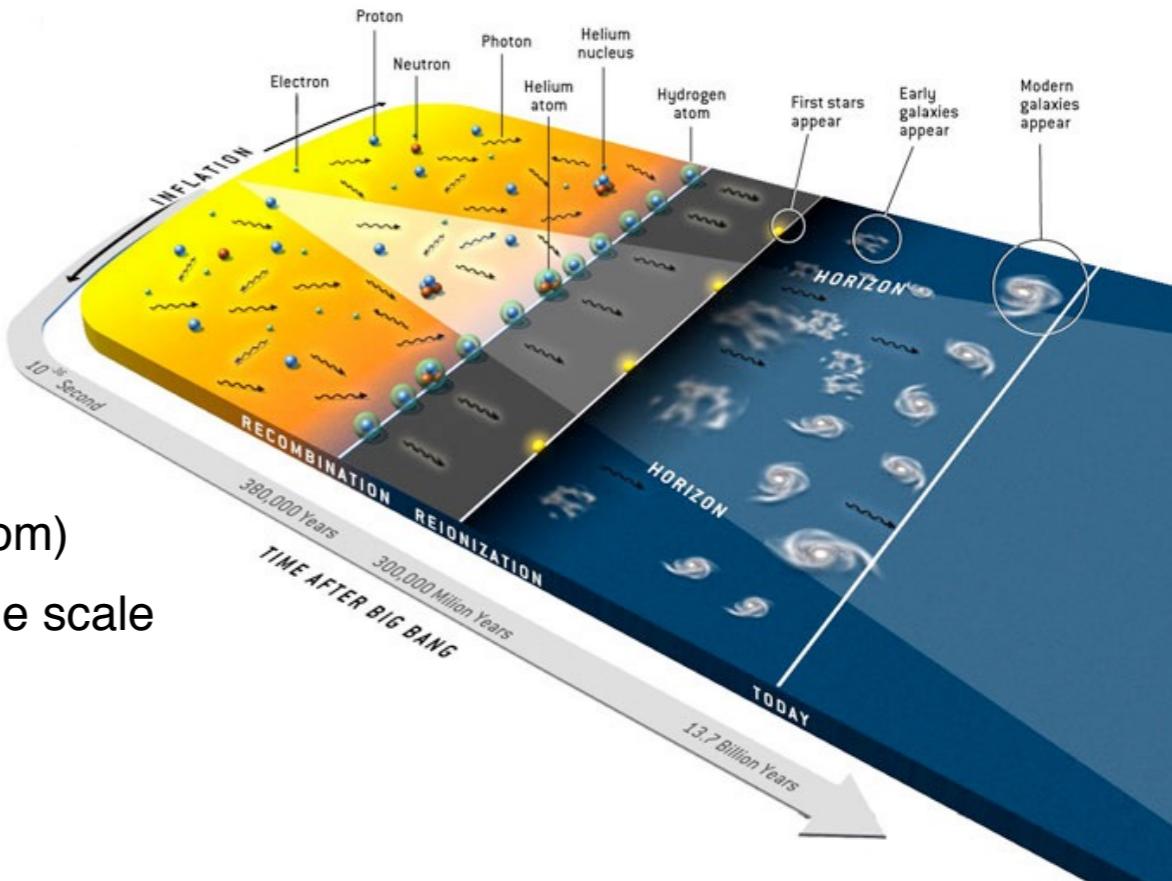


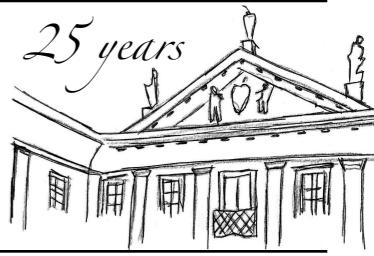
Quantum Chromo Dynamics

Strong

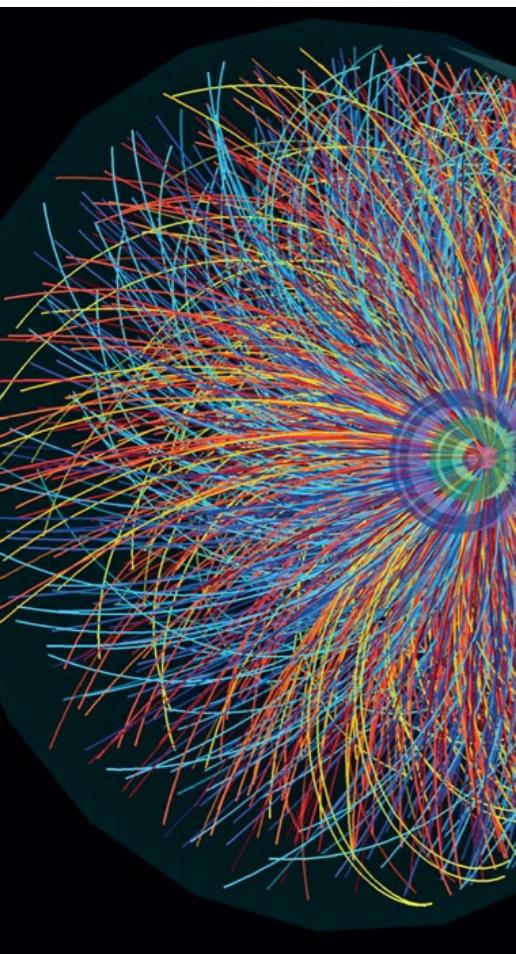
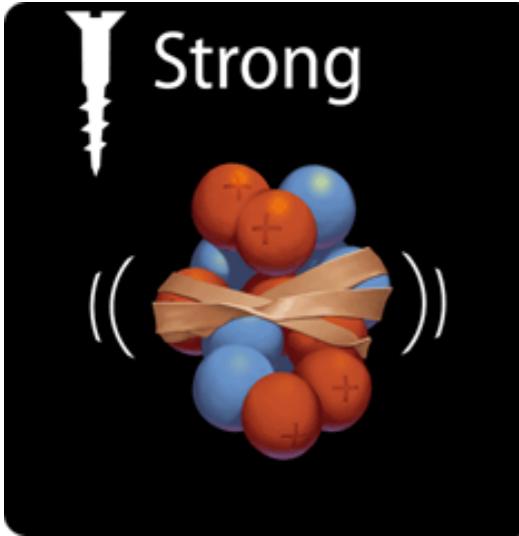


- QCD started 10^{-6} secs. after the Big-Bang:
it is the glue that binds us all, and understanding its dynamics has profound implications
 - Explain how massless gluons and light quarks are confined and bind together to form hadrons...
 - ...thereby explain the origin of ~98% of the mass in the visible Universe
- QCD is *likely* a perfect theory
nothing needs to be added or changed
 - Validated over an incredible energy range: $0 \lesssim E \leq 8000$ GeV
 - Unlikely to break down at any energy scale (asymptotic freedom)
 - No intrinsic parameters, just need one observable to define the scale $\Lambda_{\text{QCD}} \simeq 200 - 300$ MeV QCD's "standard kilogram"
- QCD is a *theory* not an effective theory
- However, it is innately nonperturbative
a priori no idea what such theory can produce





QCD: degrees of freedom



- **QCD basic degrees of freedom:**
matter (quarks); gauge (gluons)

$$q_f^i \begin{cases} \text{color} & i = 1, 2, 3 \\ \text{flavor} & f = u, d, s, c, b, t \end{cases}$$

$$A_\mu^a \begin{cases} \text{color} & a = 1, \dots, 8 \\ \text{spin} & \varepsilon_\mu^{\pm, 0} \end{cases}$$

- **QCD action:**
encodes all the dynamics

$$S_{\text{QCD}} = \int d^4x (\mathcal{L}_I + \mathcal{L}_{\text{GF+FPG}})$$

$$\mathcal{L}_I = \bar{q}_f^i (i\gamma^\mu D_\mu - m)_{ij} q_f^j - \frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu}$$

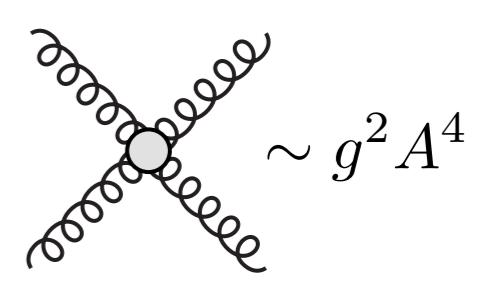
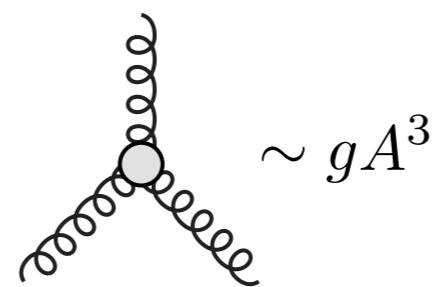
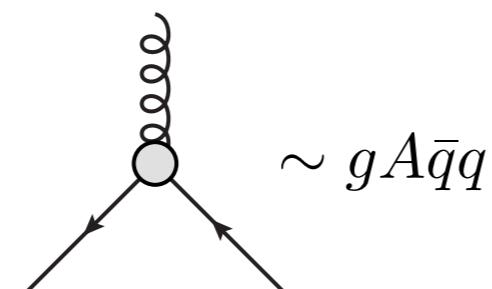
$$D_\mu = \partial_\mu - ig A_\mu^a T^a$$

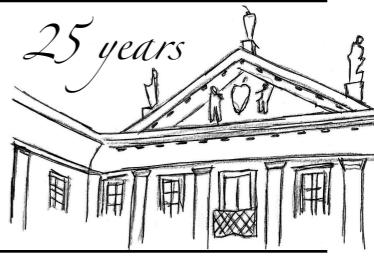
$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c$$

$$\mathcal{L}_{\text{GF+FPG}} = s(\bar{c}^a \mathcal{F}^a - \xi/2 \bar{c}^a b^a)$$

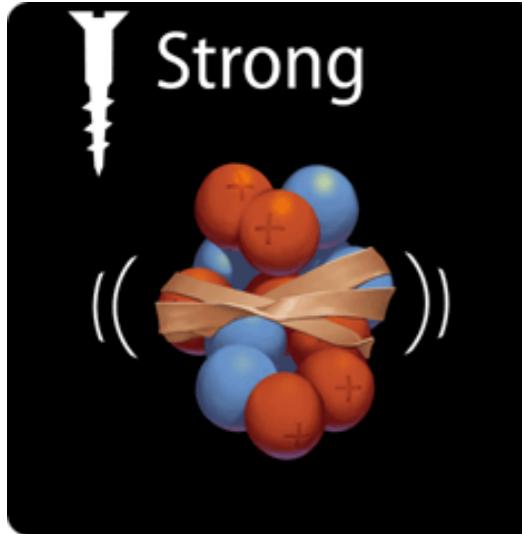
- **Gauge fixing + FP ghost:**
BRST exact, does not appear in spectrum

- **QCD (self-)interactions**
dictate the theory's behavior

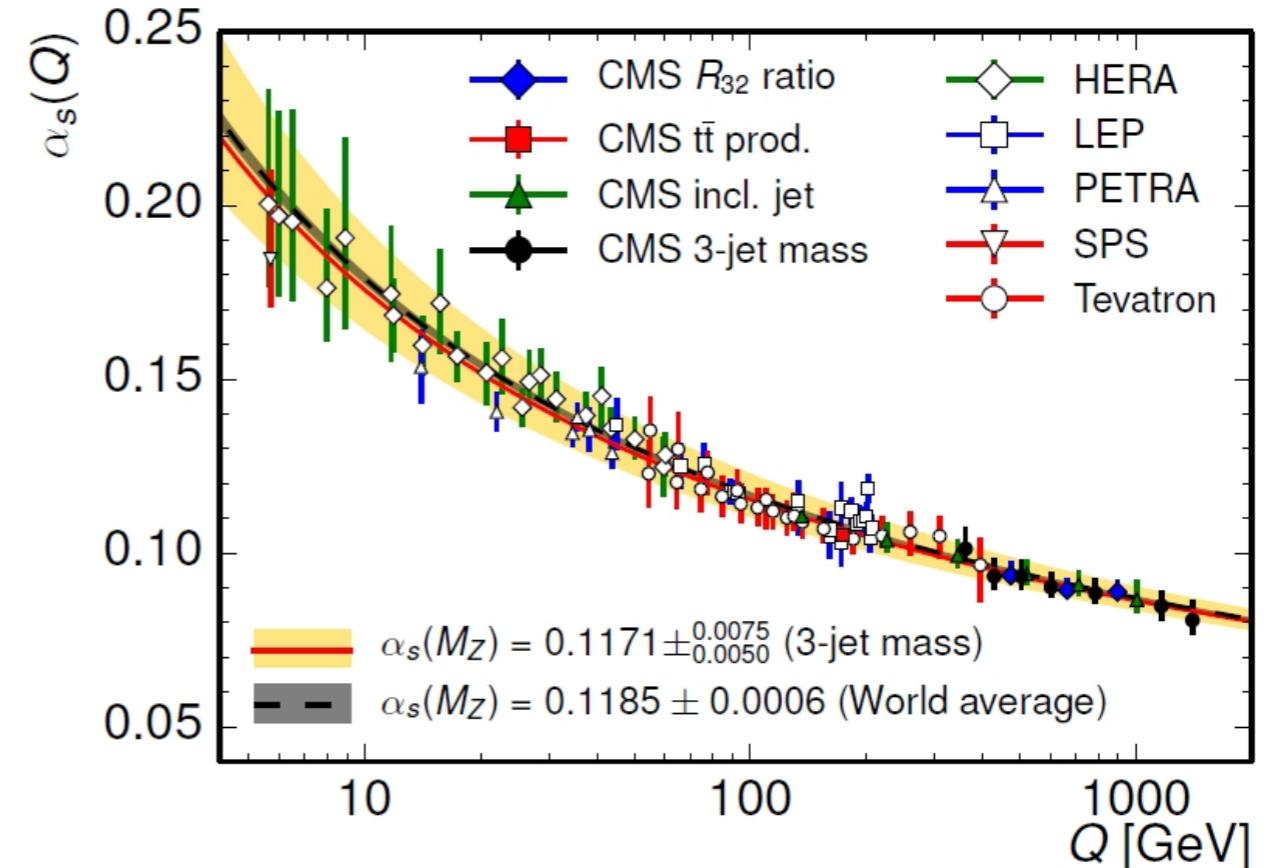
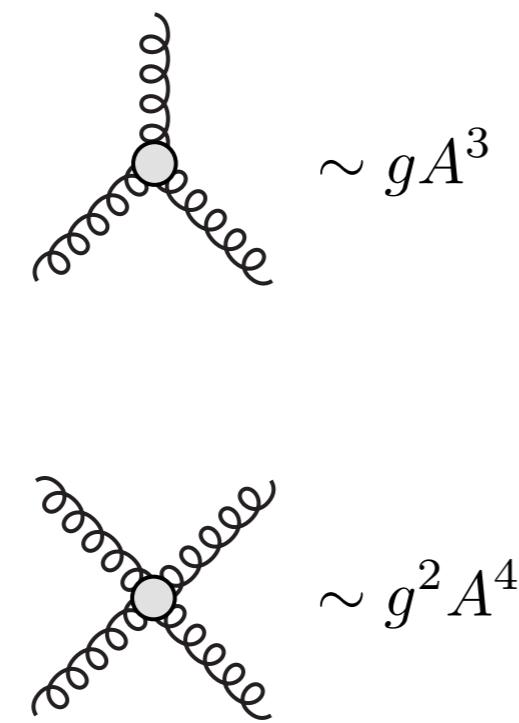




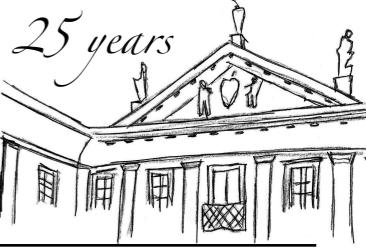
QCD: asymptotic freedom



- **Gluon self-interaction runs**
implies QCD is non perturbative and quark/gluons are confined

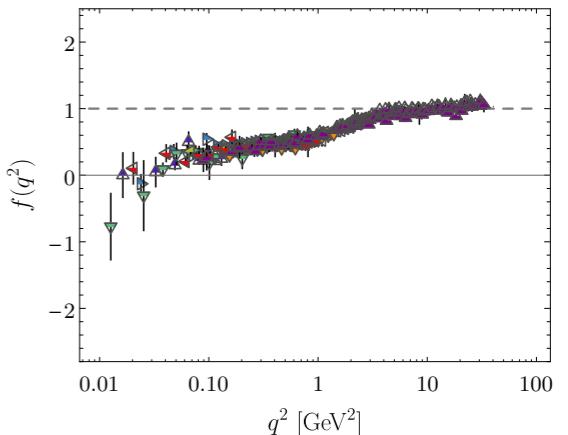
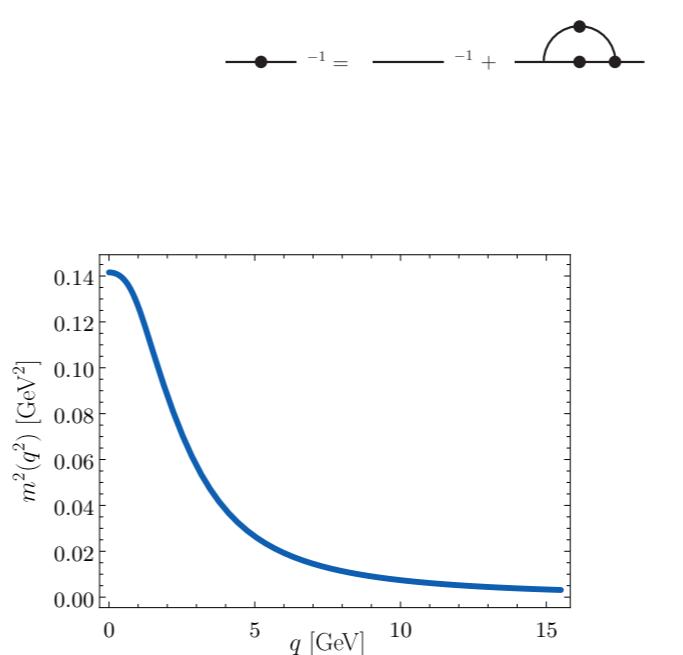
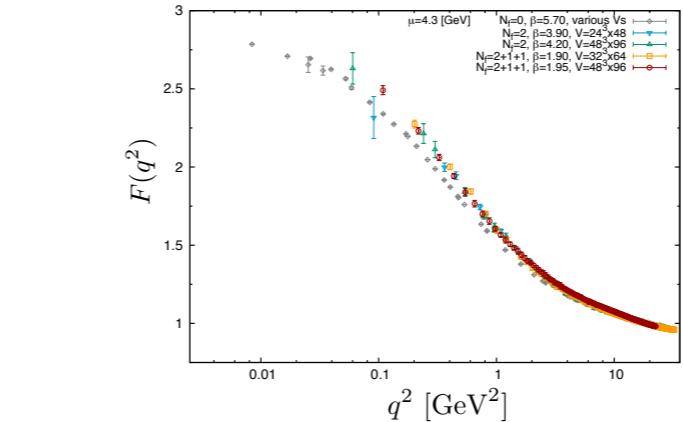
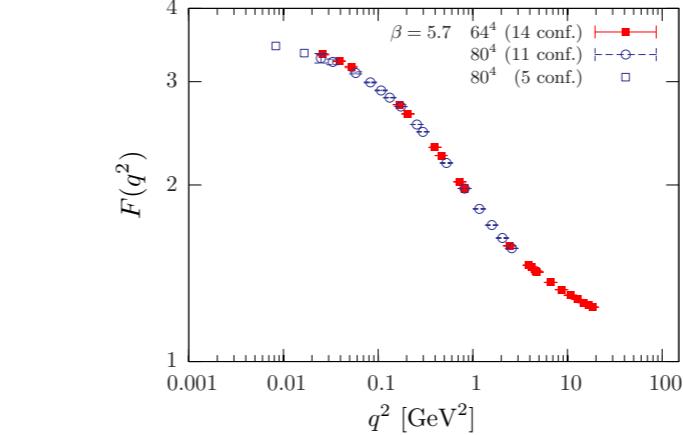
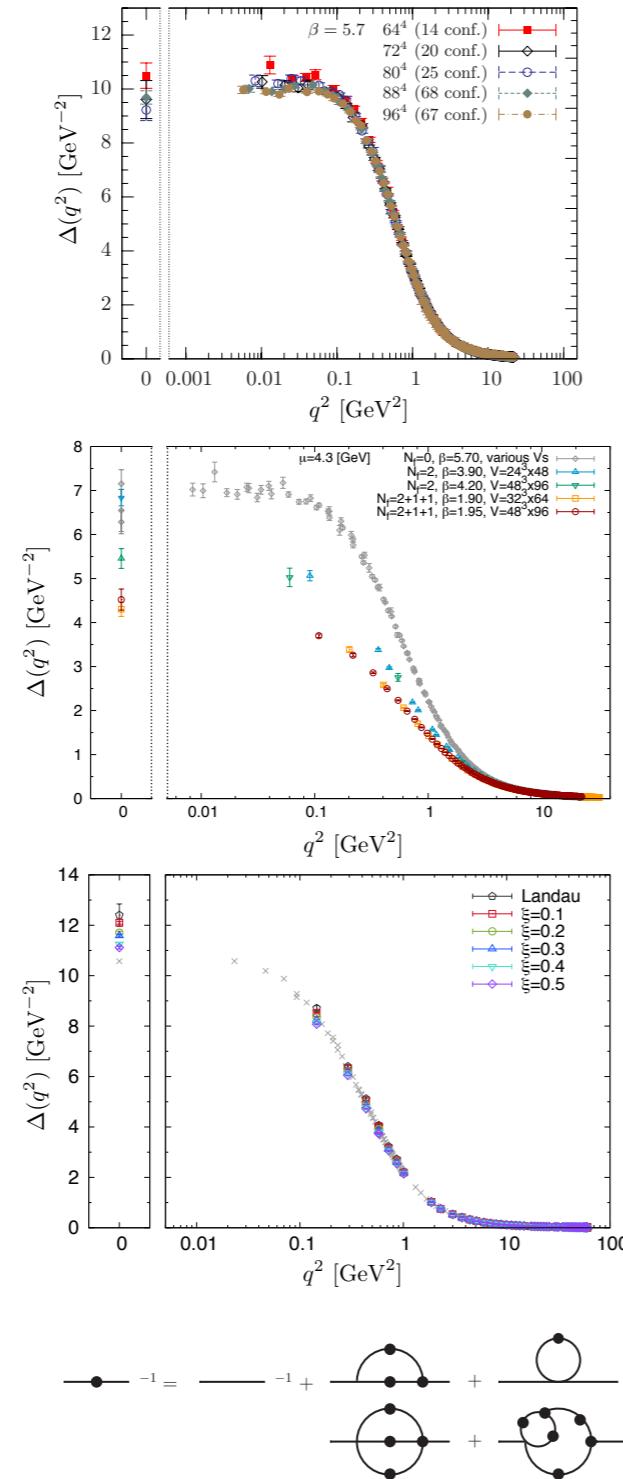
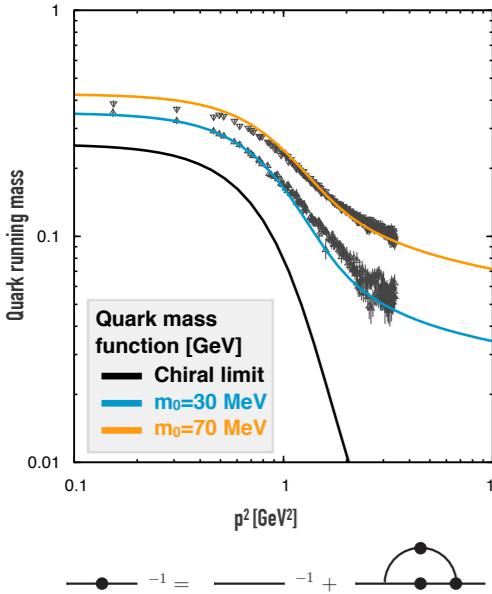


- **QCD interaction strength:**
grows with separation between gluons and quarks
- **Typical scale of hadron physics**
 $r \sim 2$ fm $\alpha \sim 0.5$
 - **Perturbation theory breaks down at this distance**
QCD is entirely nonperturbative across almost the entire proton's volume



QCD in the IR

- **Dynamical χ SB**
in the matter sector
- **Saturating gluon propagator**
dynamical gluon mass
- **IR massless ghost**
saturating dressing function
- **IR suppressed 3g vertex**
due to ghost-gluon dynamics





PDAs within a DS framework

- **Meson leading twist PDA**

projection of BS wave-function onto the light-cone

Chang, Cloet, Cobos-Martinez, Roberts, Schmidt, PRL 110 (2013)

$$f_{\text{PS}} \varphi(x, \zeta) = Z_2(\zeta, \Lambda) \text{Tr}_{\text{CD}} \int_{\text{d}q}^{\Lambda} \delta(n \cdot q_+ - xn \cdot P) \gamma_5(\gamma \cdot n) \chi_{\text{PS}}(q, P)$$

f_{PS}

- **Meson's decay constant**
(normalization of PDAs) $\int_0^1 \text{d}x \phi(x, \zeta) = 1$

$Z_2(\zeta, \Lambda)$

$$\int_{\text{d}q}^{\Lambda}$$

- **Quark wave-function renormalization**
evaluated at ζ (2 GeV)
- **Poincaré invariant regularization**
of 4-dimensional integral

$\delta(n \cdot q_+ - xn \cdot P)$

- **n light-cone vector; P meson momentum**
 $n \cdot P = -m_{\text{PS}}$ $P^2 = -m_{\text{PS}}^2$

$\chi_{\text{PS}}(q, P)$

- **BS wave-function:** $\chi_{\text{PS}}(q, P) = S_{f_1}(q_+) \Gamma_{\text{PS}}(q, P) S_{f_2}(q_-)$

$$\begin{aligned} q_+ &= q + \eta P \\ q_- &= q - (1 - \eta)P \end{aligned}$$

$\Gamma_{\text{PS}}(q, P)$

- **BS amplitude**
solution of the homogeneous BS equation

$$\Gamma_{\text{PS}}^{\alpha\beta}(k, P) = \int_q \mathcal{K}^{\alpha\alpha';\beta\beta'}(q_\pm, k_\pm) [S_{f_1}(q_+) \Gamma_{\text{PS}}^{\alpha\beta}(q, P) S_{f_2}(q_-)]_{\alpha'\beta'}$$

$S_{f_1}(q_+)$

- **Quark propagator**
solution of the gap equation

$$S_f^{-1}(k) = Z_2(-i\gamma \cdot k + m_f) + Z_1 \int_{\text{d}q}^{\Lambda} g^2 D^{\mu\nu}(k - q) \frac{\lambda^a}{2} \gamma_\mu S(q) \frac{\lambda^a}{2} \Gamma_\nu(q, k)$$



PDAs within a DS framework

- **Meson leading twist PDA**

projection of BS wave-function onto the light-cone

Chang, Cloet, Cobos-Martinez, Roberts, Schmidt, PRL 110 (2013)

$$f_{\text{PS}} \varphi(x, \zeta) = Z_2(\zeta, \Lambda) \text{Tr}_{\text{CD}} \int_{\text{d}q}^{\Lambda} \delta(n \cdot q_+ - xn \cdot P) \gamma_5(\gamma \cdot n) \chi_{\text{PS}}(q, P)$$

- **Not accessible** in Euclidean space

- **Mellin moments**

used for reconstructing the PDA

$$\langle x^m \rangle = \frac{1}{f_{\text{PS}}} Z_2(\zeta; \Lambda) \text{Tr}_{\text{CD}} \int_{\text{d}q}^{\Lambda} \frac{(n \cdot q_+)^m}{(n \cdot P)^{m+1}} \gamma_5(\gamma \cdot n) \chi_{\text{PS}}(q; P)$$

- **Highly oscillatory integral** in Euclidean space
- **Requires damping factor** and extrapolation

$$1/(1 + r^2 k^2)^m; \quad r^2 \rightarrow 0$$

- **Two possible representations**

for determining the full amplitude

- **Gegenbauer**

$$\varphi(x) = \mathcal{N} x^\alpha (1-x)^\beta$$

- **Gaussian**

$$\varphi(x) = \mathcal{N} [1 - (2x - 1)^2] e^{\alpha^2(1-(2x-1)^2) + \beta^2(2x-1)}$$

- **Main difference**

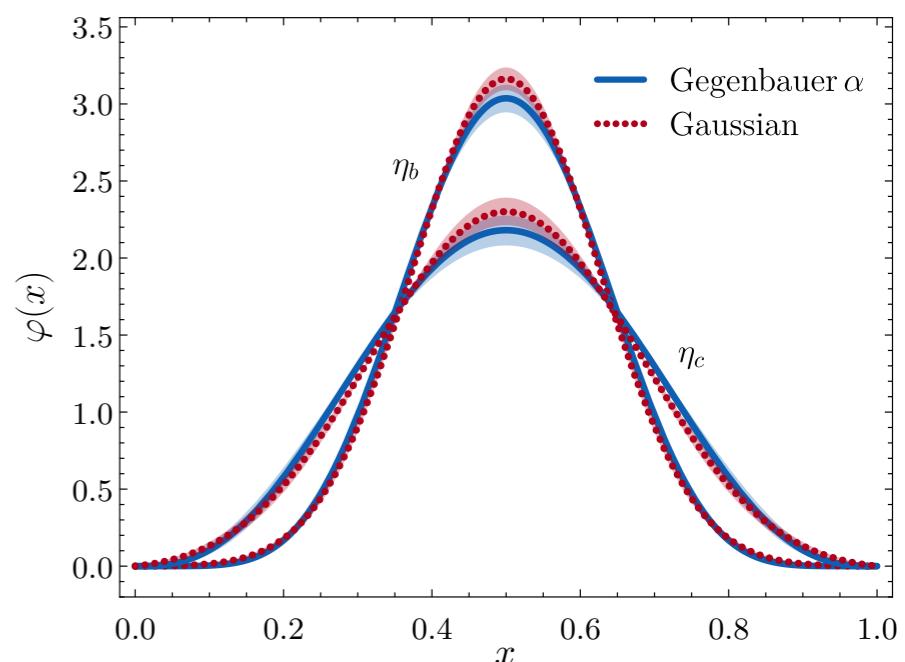
behavior as $x \rightarrow 1$

- **Affects the inverse moment** $\int_0^1 dx \phi(x)/(1-x)$

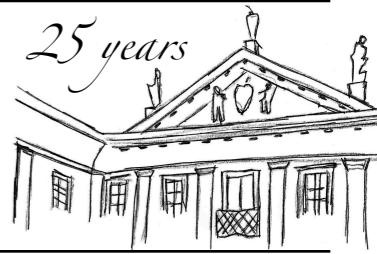
large Q^2 behavior of form factors

- **Heavy-heavy limit:** $\varphi(x) \rightarrow \delta(1-x/2)$

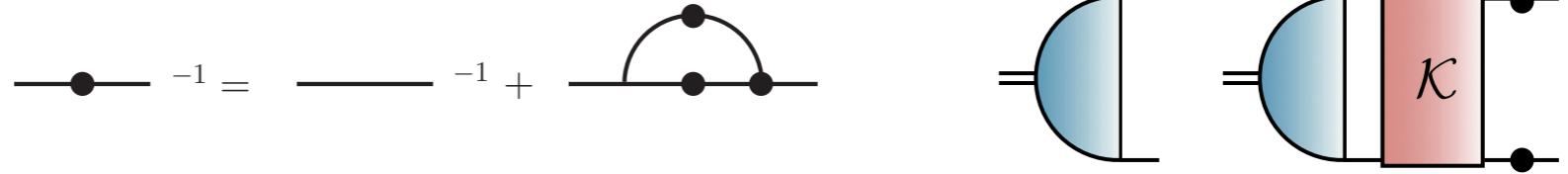
- **Heavy-light limit:** $\varphi(x) \rightarrow \delta(1-x)$



Propagators and BS amplitudes



- Use RL truncation for gap and BS equation



$$\Gamma_\nu = \gamma_\nu$$

$$\mathcal{K}[S\Gamma_{PS}S] = -P^{\mu\nu}\mathcal{G}\gamma_\mu[S\Gamma_{PS}S]\gamma_\nu$$

- For the interaction we set

Qin, Chang, Liu, Roberts and Wilson, PRC 84 (2011)

$$\mathcal{I}(k^2) = k^2 \frac{\mathcal{G}_{IR}(k^2) + \mathcal{G}_{UV}(k^2)}{4\pi}$$

$$\mathcal{G}_{IR}(k^2) = \frac{8\pi^2}{\omega^5} \varsigma^3 e^{-k^2/\omega^2}$$

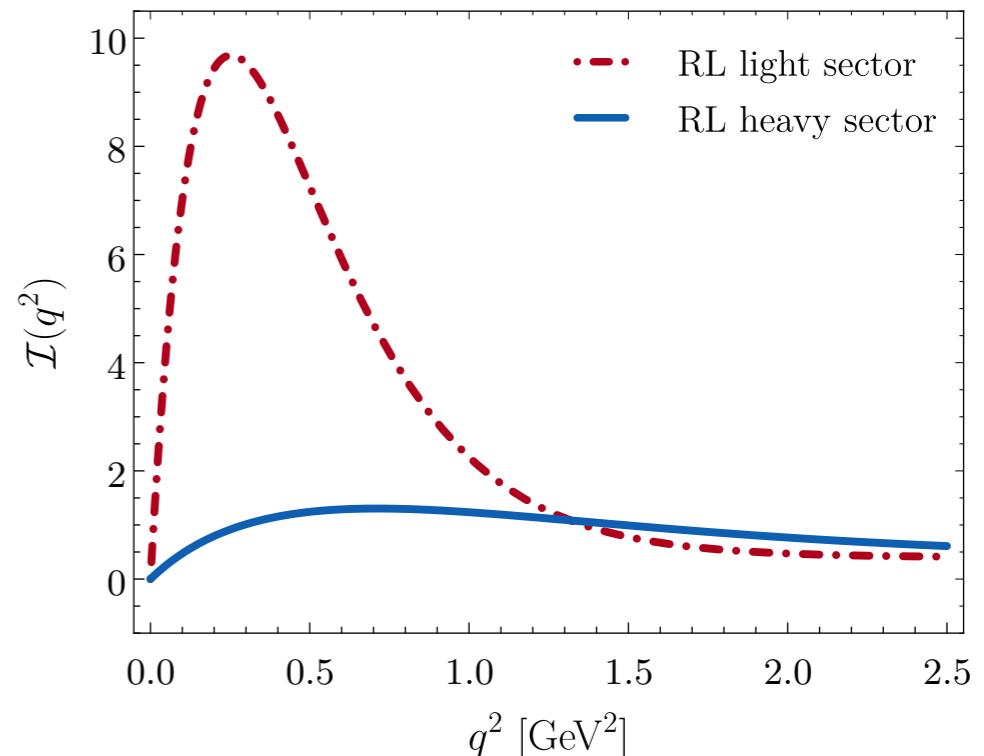
$$\mathcal{G}_{UV}(k^2) = \frac{96\pi^2}{25} \frac{1 - e^{-k^2/1[\text{GeV}^2]}}{k^2 \log[e^2 - 1 + (1 + k^2/\Lambda^2)^2]}$$

- Light quark sector (GeV)

$$\varsigma = 0.8; \quad \omega = 0.5$$

- Heavy quark sector (GeV)

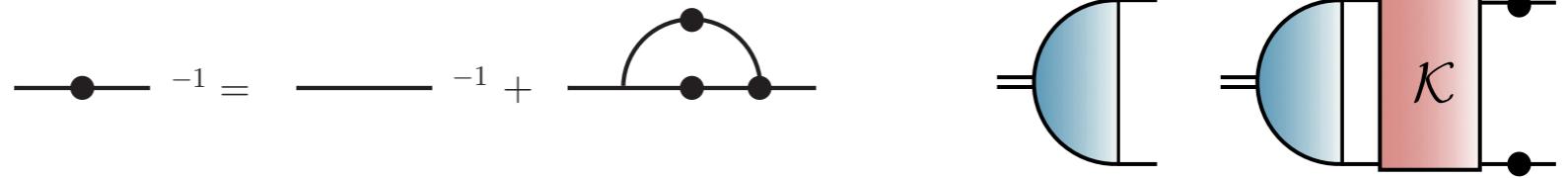
$$\varsigma = 0.6; \quad \omega = 0.8$$



Propagators and BS amplitudes



- Use RL truncation for gap and BS equation



$$\Gamma_\nu = \gamma_\nu$$

$$\mathcal{K}[S\Gamma_{PS}S] = -P^{\mu\nu}\mathcal{G}\gamma_\mu[S\Gamma_{PS}S]\gamma_\nu$$

- For the interaction we set

Qin, Chang, Liu, Roberts and Wilson, PRC 84 (2011)

$$\mathcal{I}(k^2) = k^2 \frac{\mathcal{G}_{IR}(k^2) + \mathcal{G}_{UV}(k^2)}{4\pi}$$

$$\mathcal{G}_{IR}(k^2) = \frac{8\pi^2}{\omega^5} \varsigma^3 e^{-k^2/\omega^2}$$

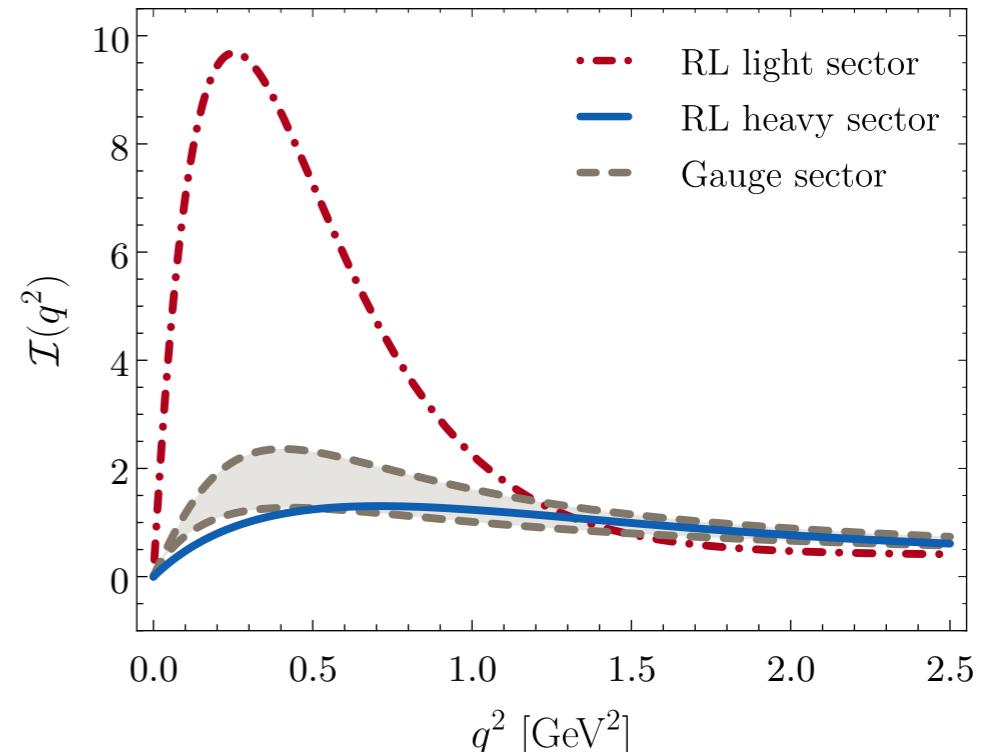
$$\mathcal{G}_{UV}(k^2) = \frac{96\pi^2}{25} \frac{1 - e^{-k^2/1[\text{GeV}^2]}}{k^2 \log[e^2 - 1 + (1 + k^2/\Lambda^2)^2]}$$

- Light quark sector (GeV)

$$\varsigma = 0.8; \quad \omega = 0.5$$

- Heavy quark sector (GeV)

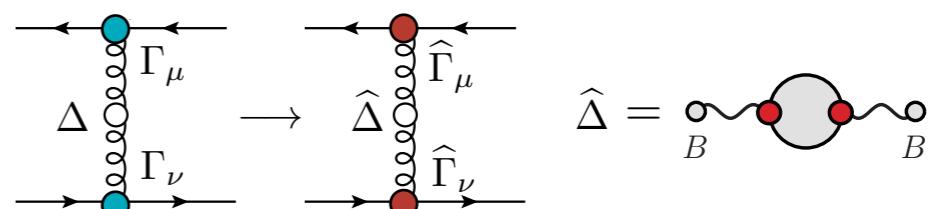
$$\varsigma = 0.6; \quad \omega = 0.8$$



- Heavy sector interaction

overlaps with gauge sector kernel determination

DB, Chang, Papavassiliou, Roberts, PLB 742 (2015)





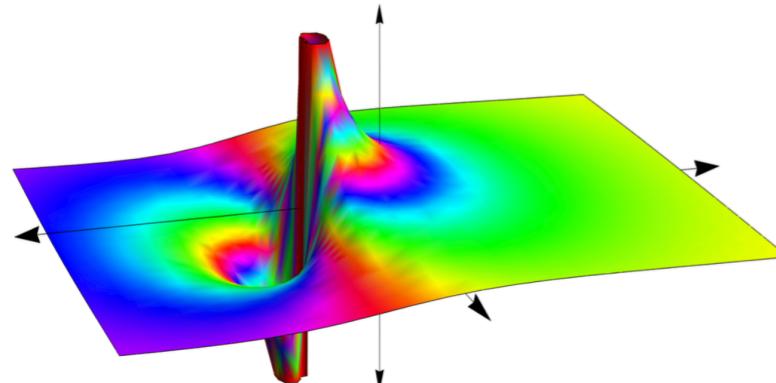
Propagators and BS amplitudes

- **SDE solutions**

dramatic change in the analytic structure

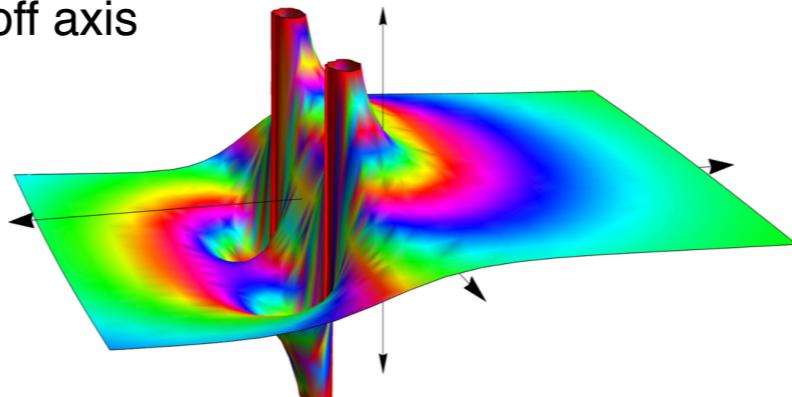
- **Normal particle**

mass pole on the real axis



- **SDE solutions**

interactions move the mass pole off axis

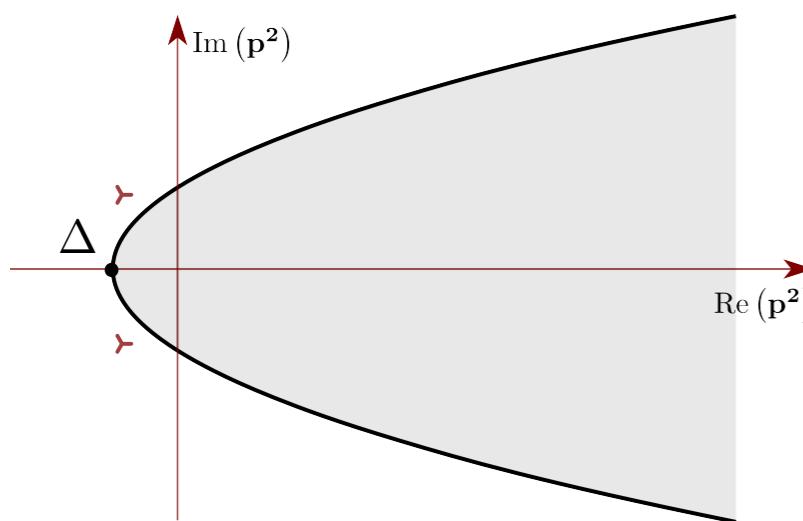


- **SDE singularities**

impacts our ability to find BS solutions

- **Quark propagators**

need to be known on a parabolic region



- **Cauchy th. not valid**

if singularities are inside the contour

- **Maximize parabola apices**

probe bound states up to $M = \Delta_h + \Delta_l$ ($\Delta_h \gg \Delta_l$)

- **Use freedom to unbalance legs**

in BS equation

$$q_+ = q + \eta P$$

$$q_- = q - (1 - \eta)P$$

$$\eta \in [\Delta_h/M, 1 - \Delta_l/M]$$

- **Still, if gap is too big**

no BS solution exists anyway

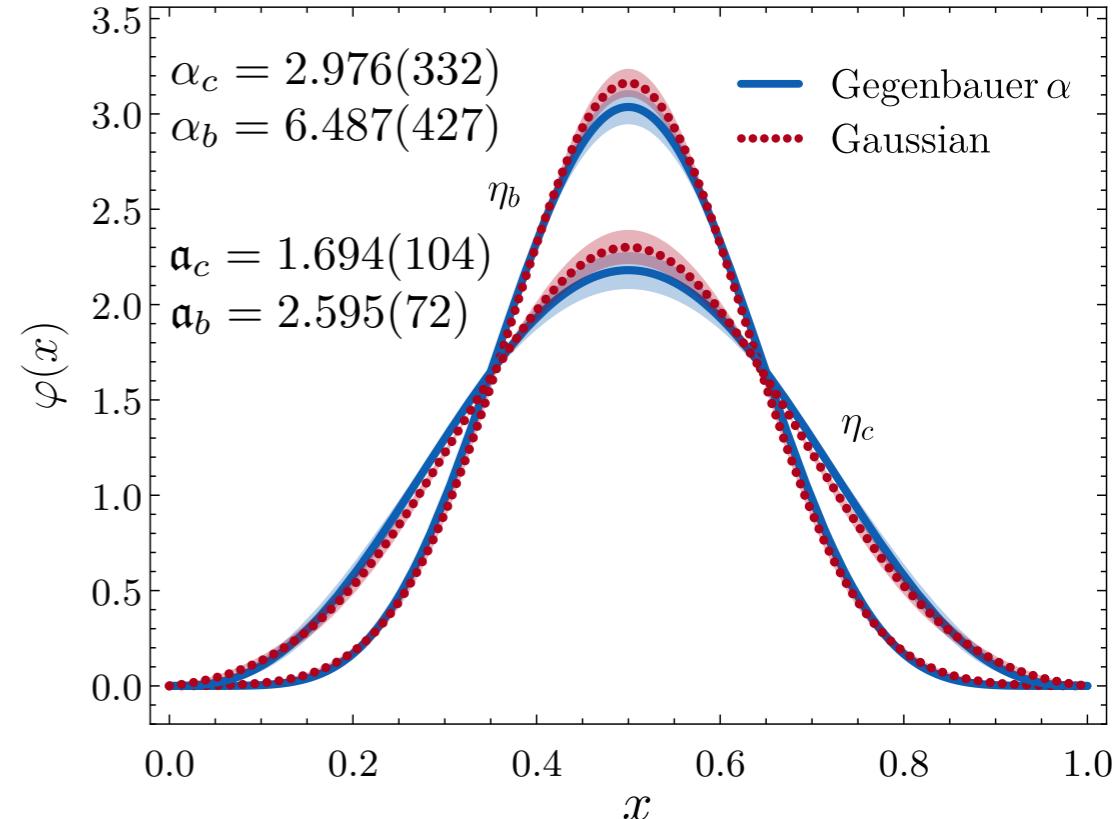
Heavy-light systems in RL



- **Choose current masses**
for heavy quarks (in GeV): $m_c = 1.25$, $m_b = 4.35$
- **Fix interaction parameters**
to heavy sector values: $\varsigma = 0.6$; $\omega = 0.8$

- **Evaluate properties of η mesons**

	Exp.		lQCD	
	m_i	f_i	m_i	f_i
η_c	2.98	0.272	2.98	0.238
η_b	9.38	0.501	9.39	/



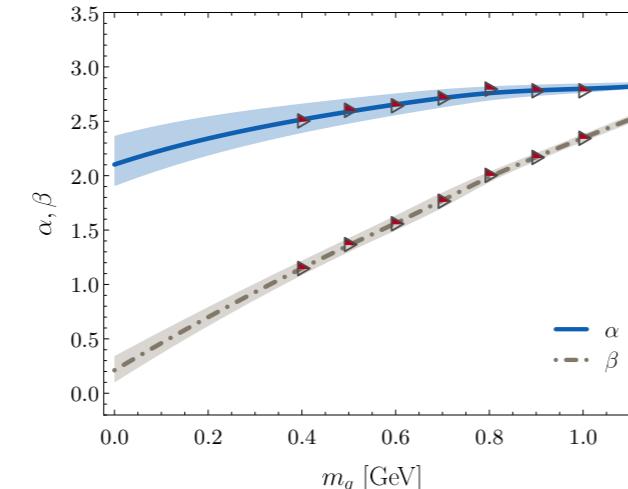
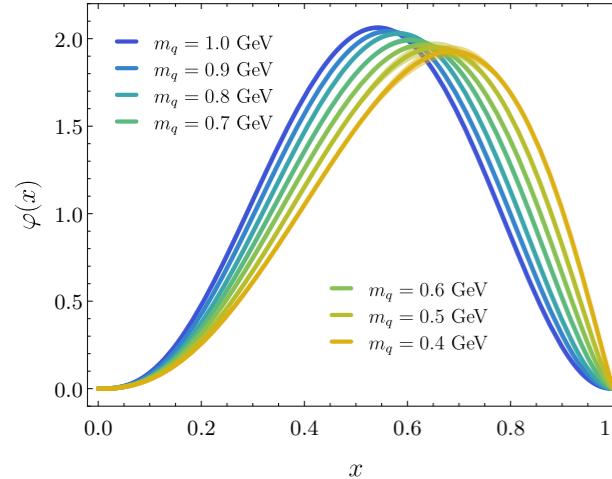
- **Construct an array of fictitious $h\bar{q}$ pseudoscalar mesons**
down to the lowest m_q RL can accommodate
 - **Charm sector:** $m_q \sim 0.4$
 - **Bottom sector:** $m_q \sim 1.3$
- **Estimate PDAs parameters**
use them to (SPM) extrapolate to lower m_q masses

Charm sector

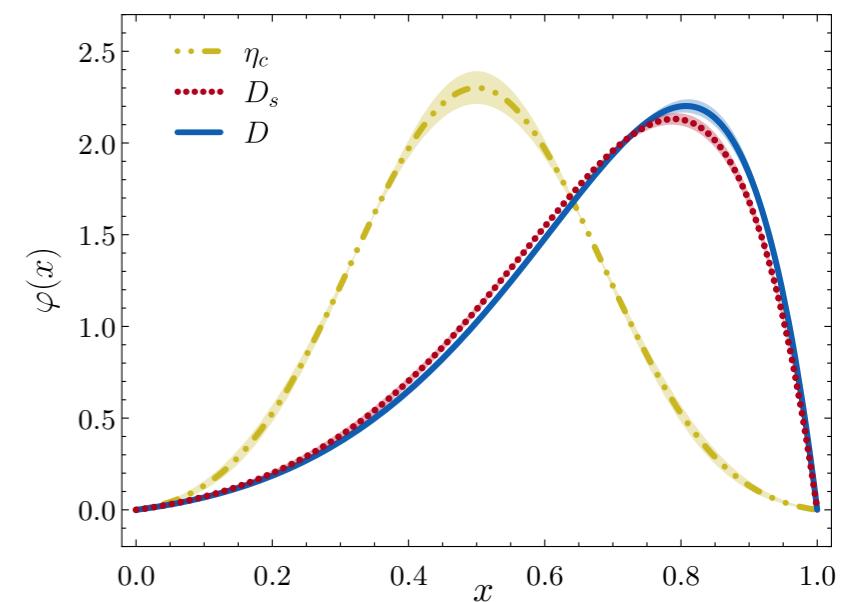
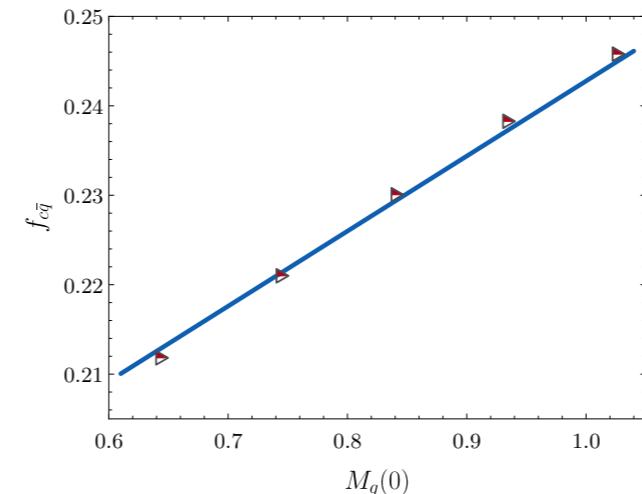
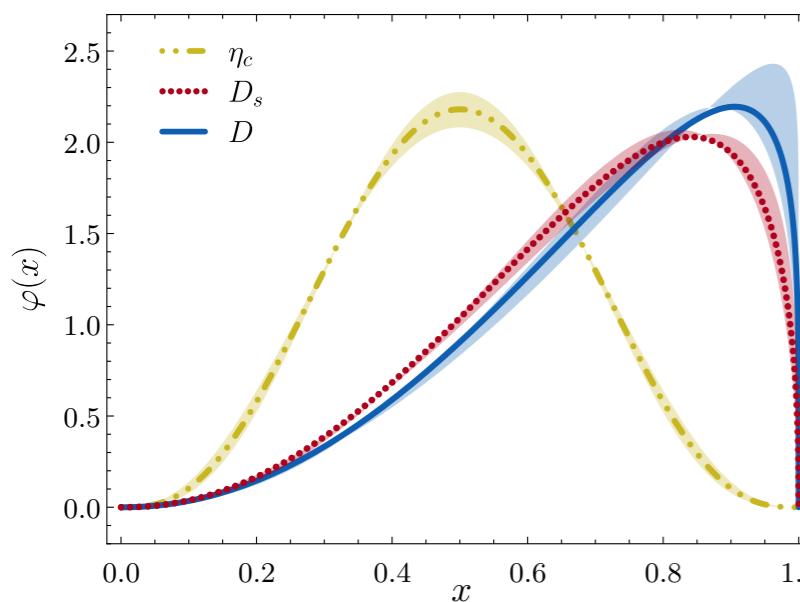
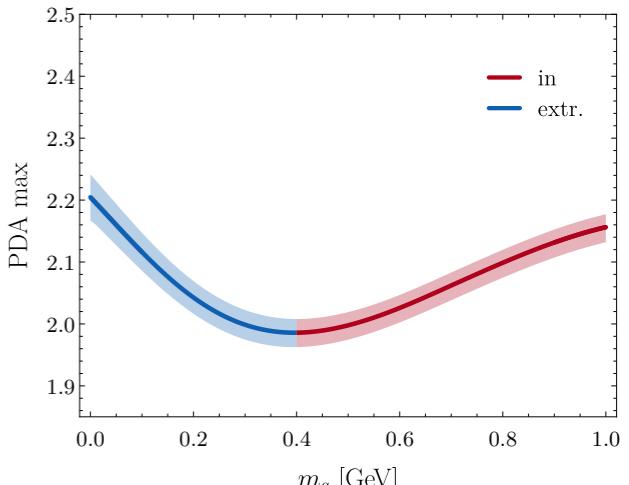
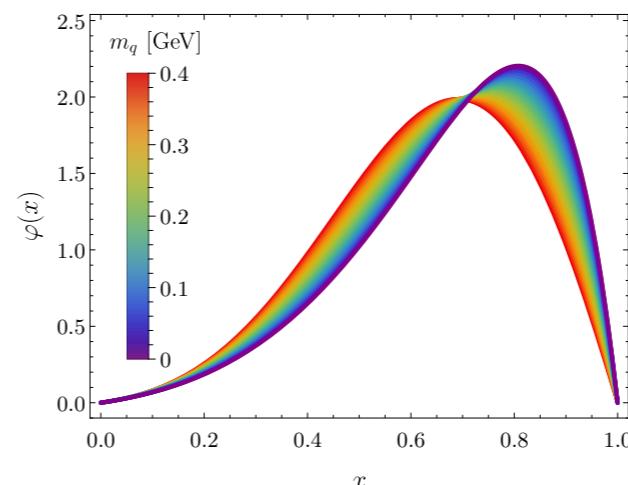
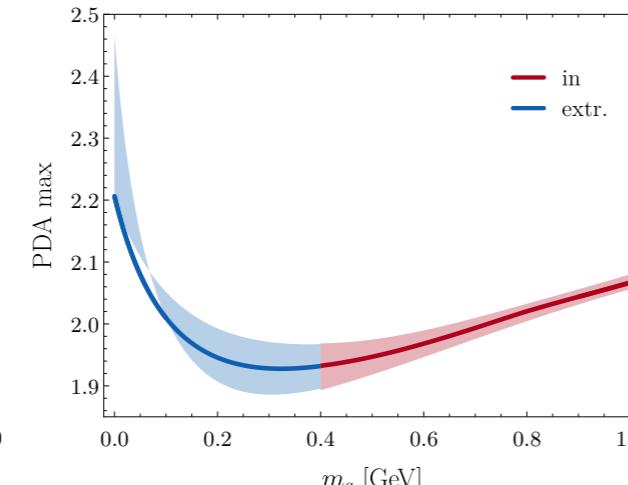
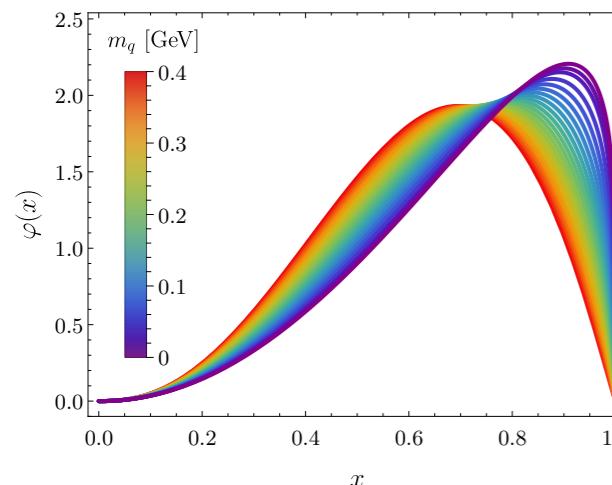
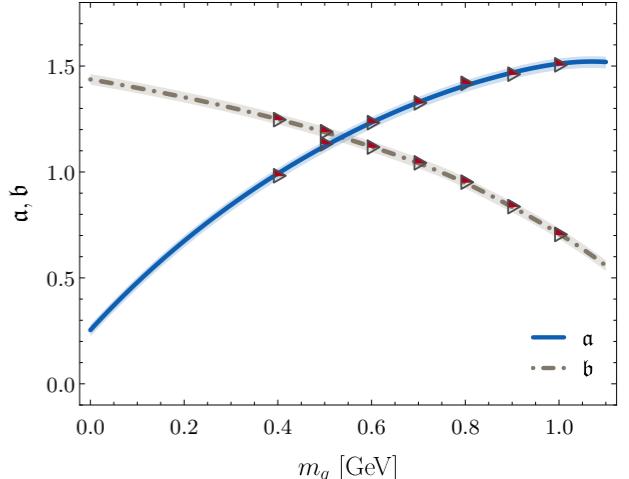
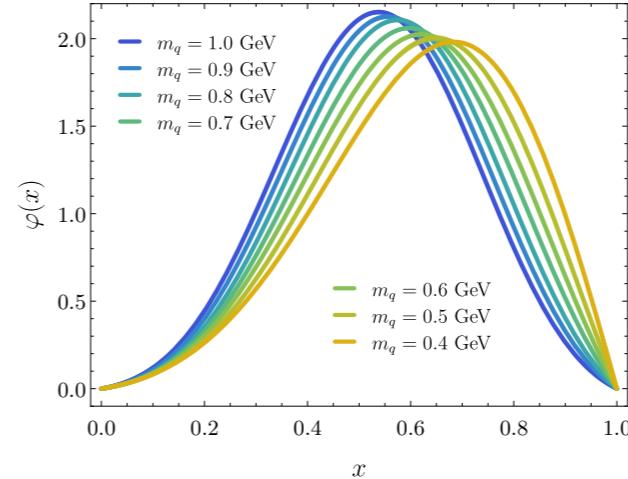
m_q	α	β	\mathfrak{a}	\mathfrak{b}
m_s	$2.212^{0.228}_{0.184}$	$0.418^{0.102}_{0.123}$	$0.442^{0.025}_{0.030}$	$1.404^{0.025}_{0.025}$
m_u	$2.107^{0.254}_{0.168}$	$0.220^{0.107}_{0.143}$	$0.262^{0.025}_{0.030}$	$1.436^{0.025}_{0.025}$



- Gegenbauer

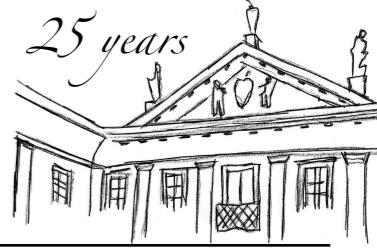


- Gaussian

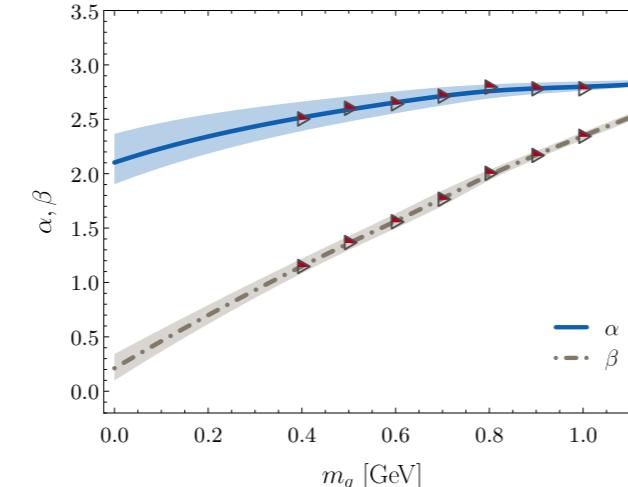
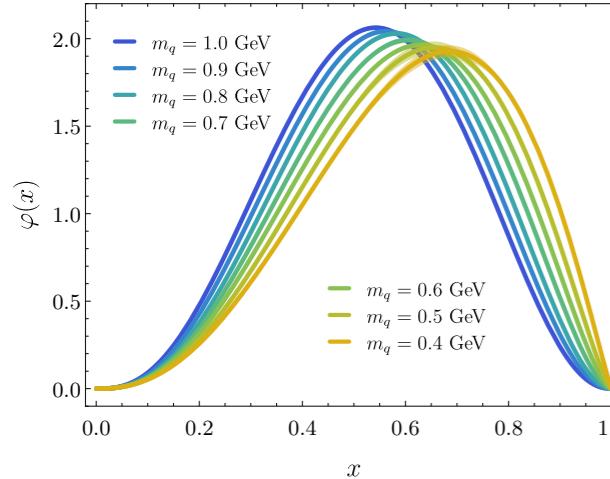


Charm sector

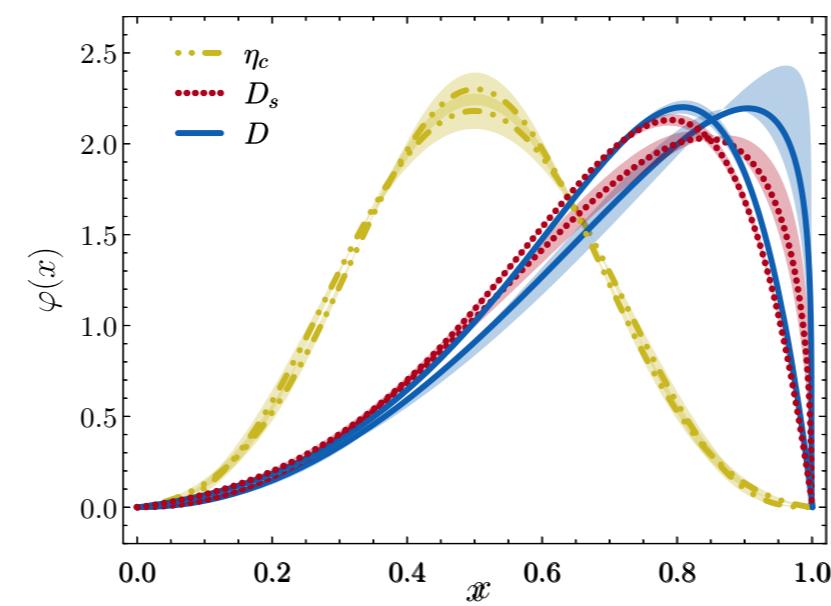
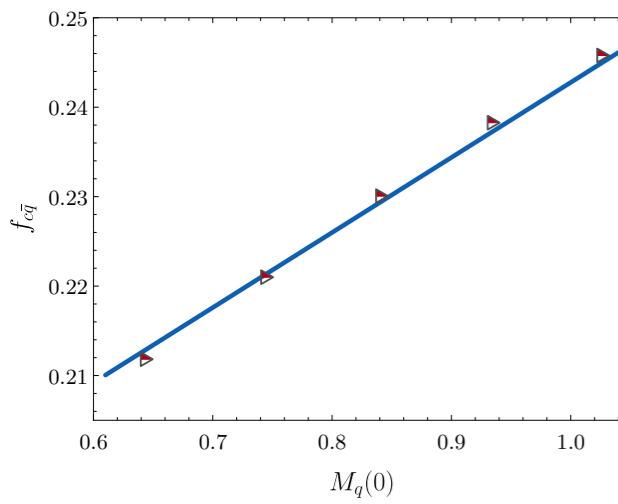
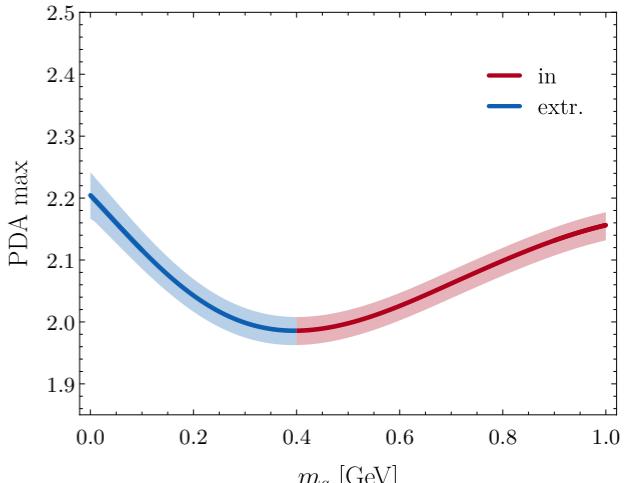
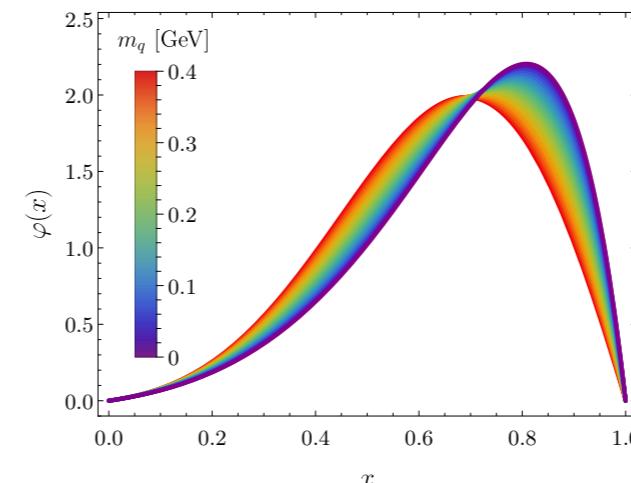
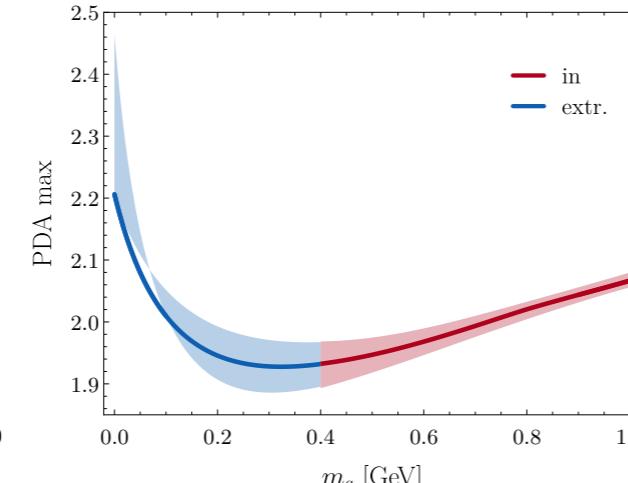
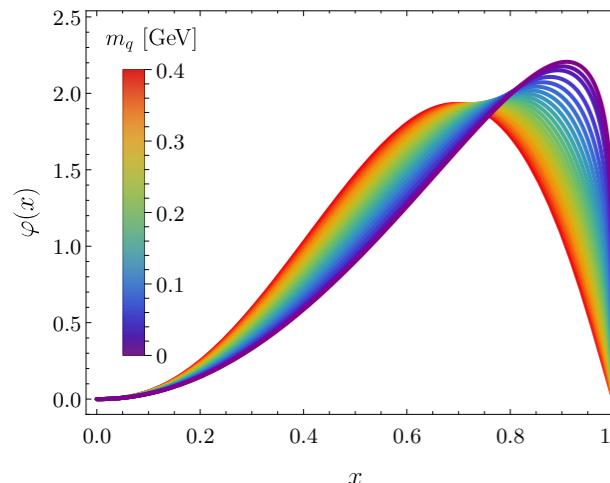
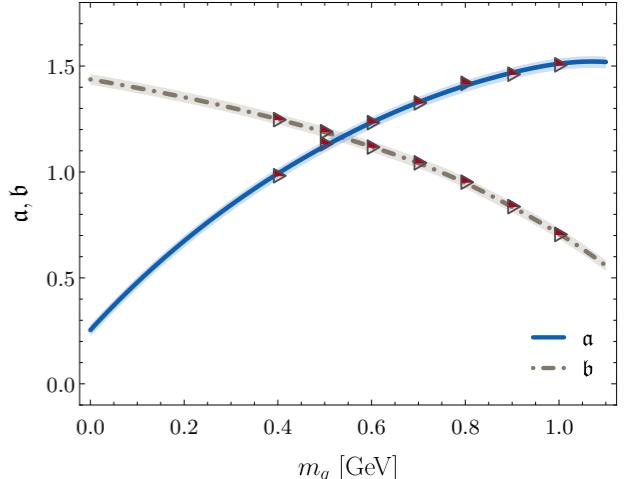
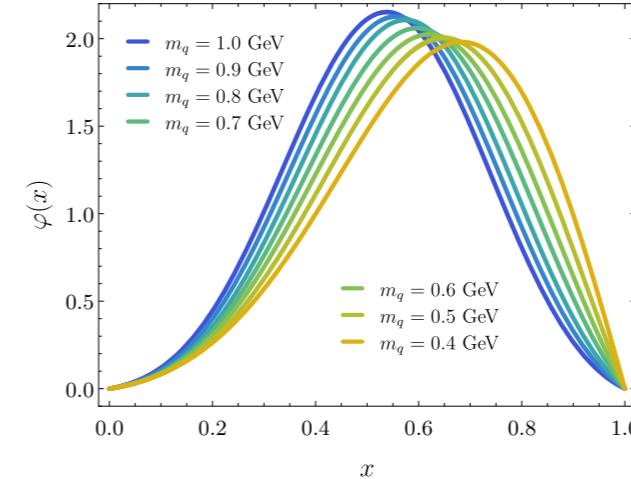
m_q	α	β	α	β
m_s	$2.212^{0.228}_{0.184}$	$0.418^{0.102}_{0.123}$	$0.442^{0.025}_{0.030}$	$1.404^{0.025}_{0.025}$
m_u	$2.107^{0.254}_{0.168}$	$0.220^{0.107}_{0.143}$	$0.262^{0.025}_{0.030}$	$1.436^{0.025}_{0.025}$



- Gegenbauer



- Gaussian

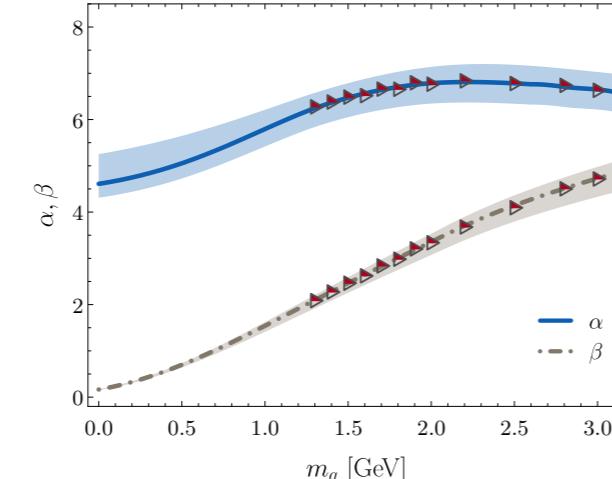
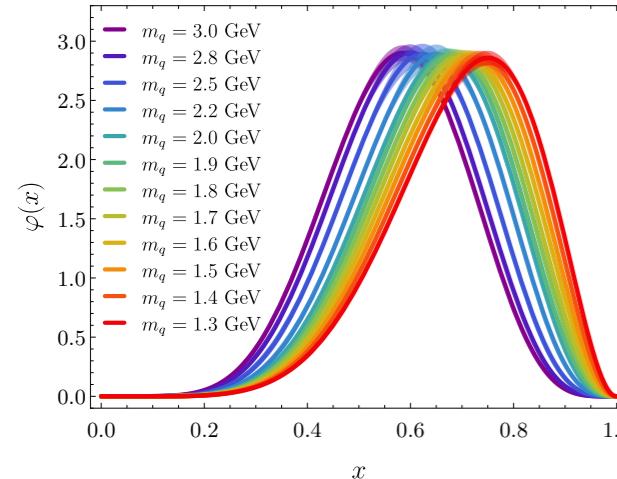


Bottom sector

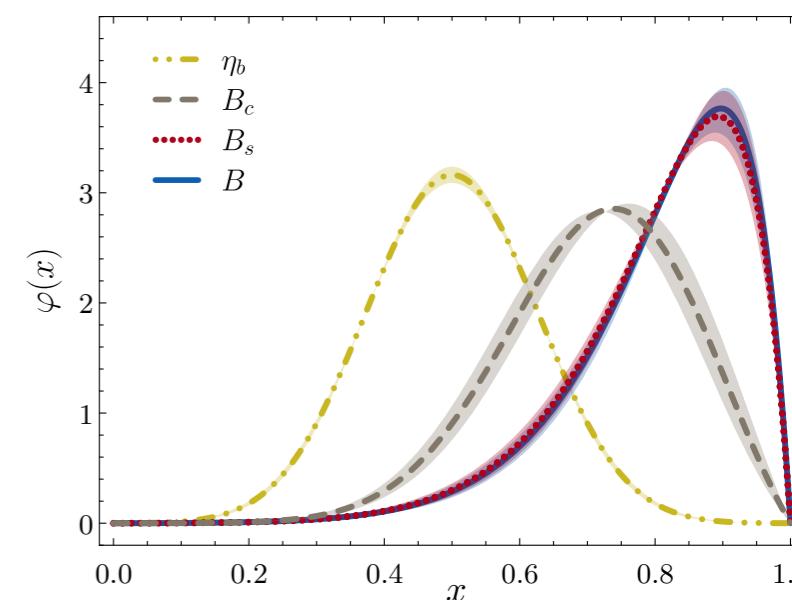
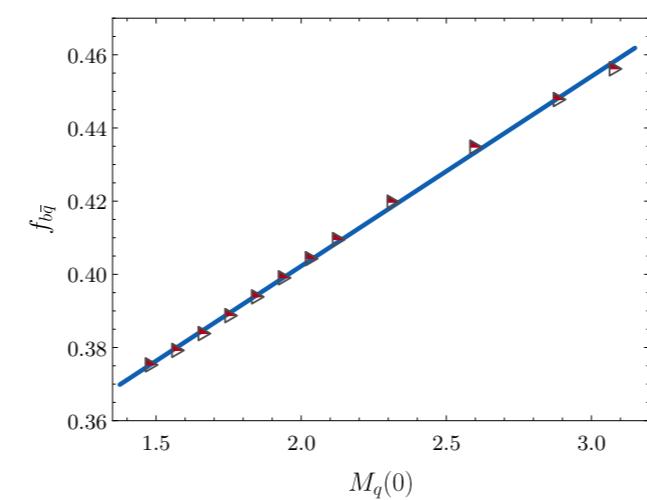
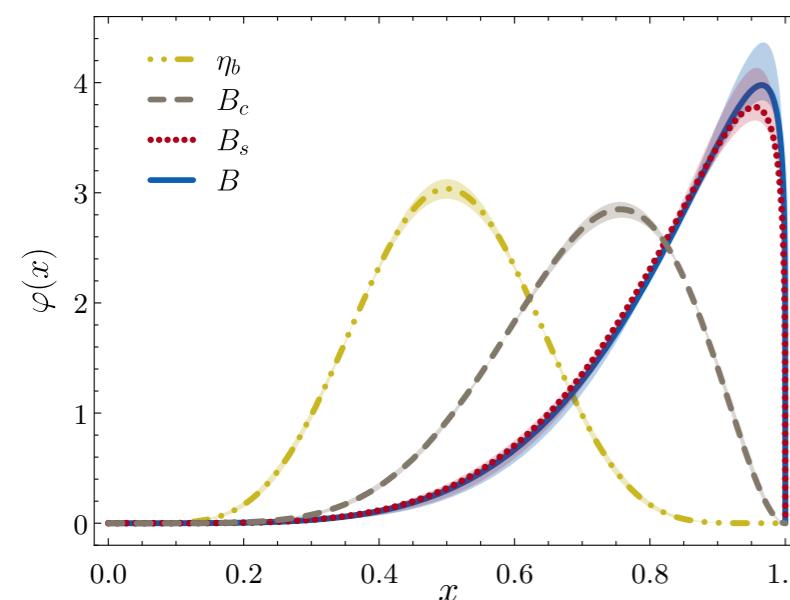
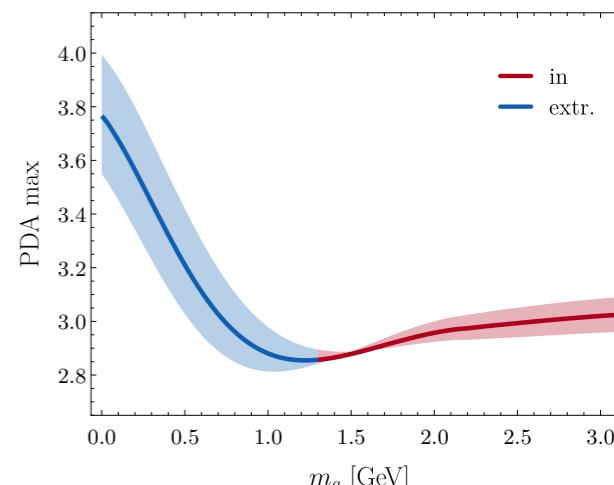
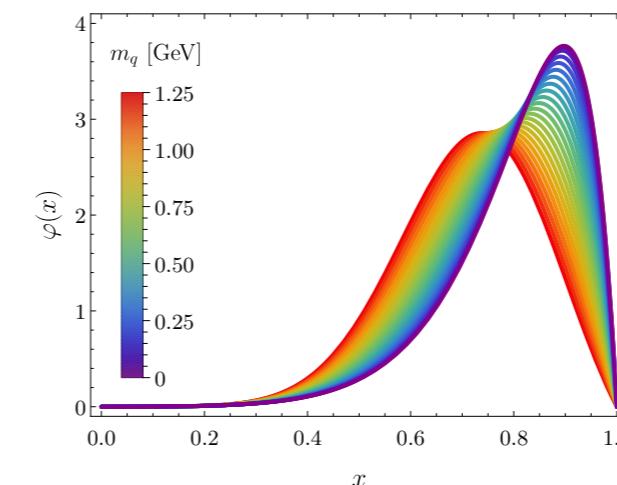
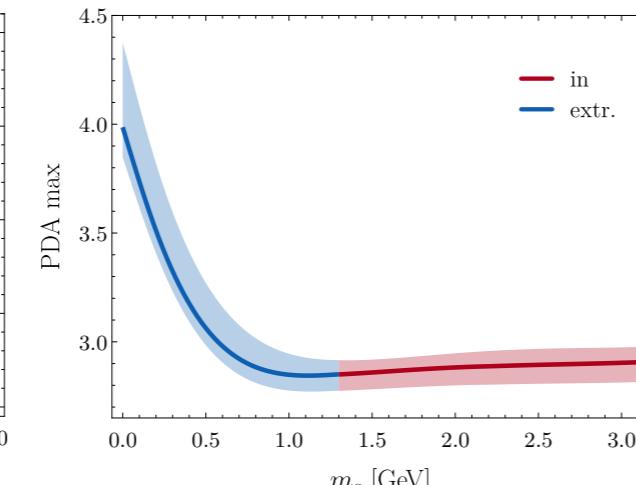
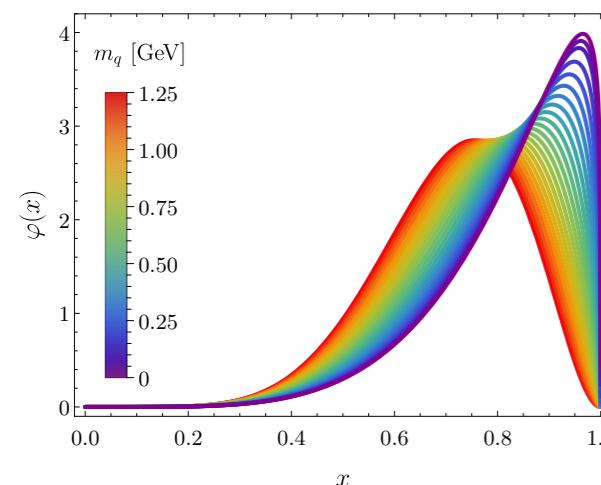
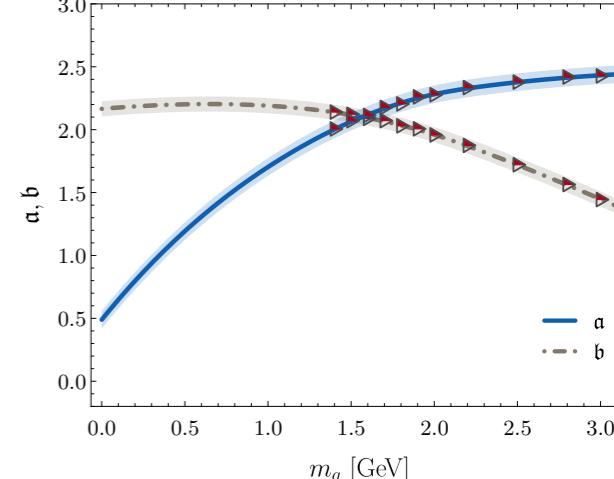
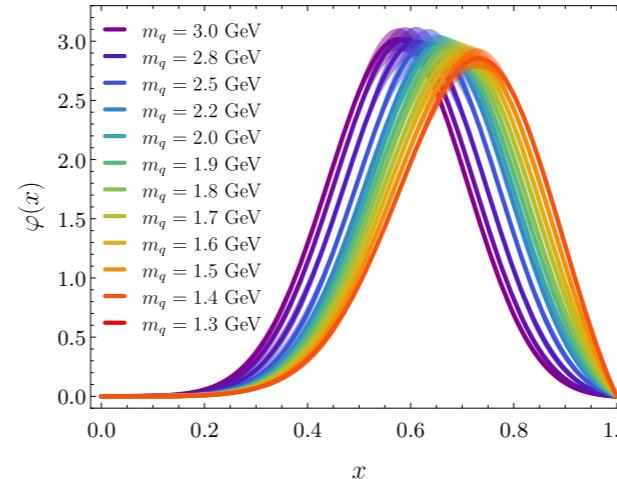
m_q	α	β	a	b
m_c	$6.179^{0.398}_{0.379}$	$1.994^{0.342}_{0.141}$	$1.901^{0.070}_{0.070}$	$2.163^{0.060}_{0.060}$
m_s	$4.660^{0.760}_{1.240}$	$0.230^{0.125}_{0.108}$	$0.621^{0.070}_{0.070}$	$2.174^{0.060}_{0.060}$
m_u	$4.612^{0.714}_{1.287}$	$0.144^{0.054}_{0.118}$	$0.495^{0.070}_{0.070}$	$2.166^{0.060}_{0.060}$



- Gegenbauer



- Gaussian

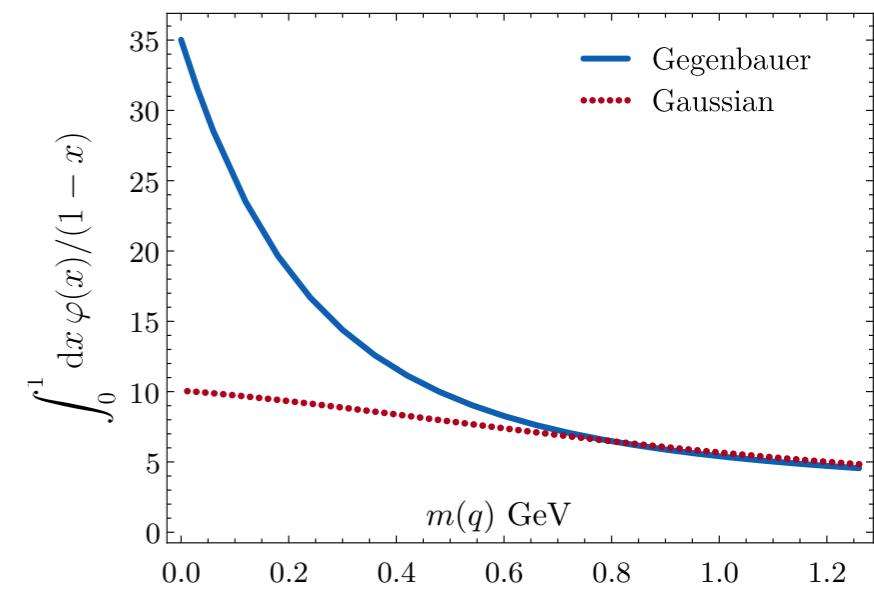
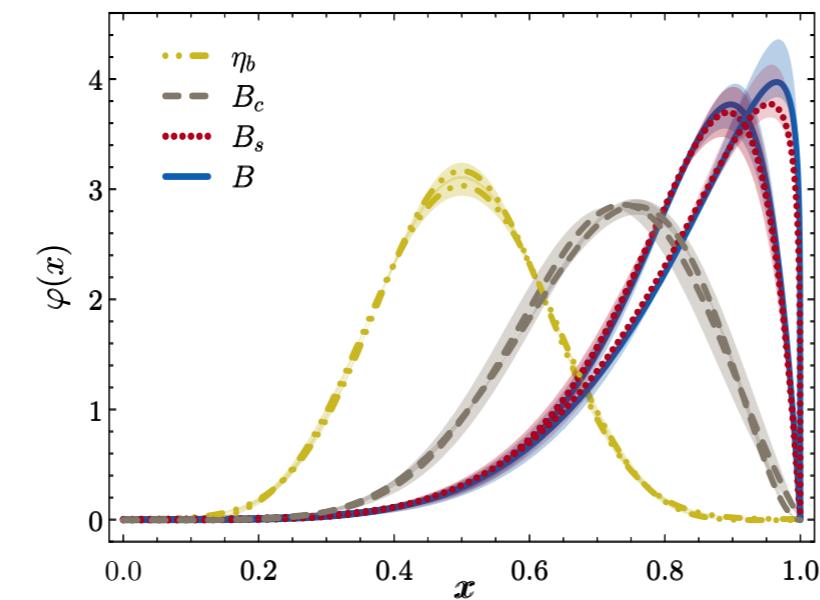
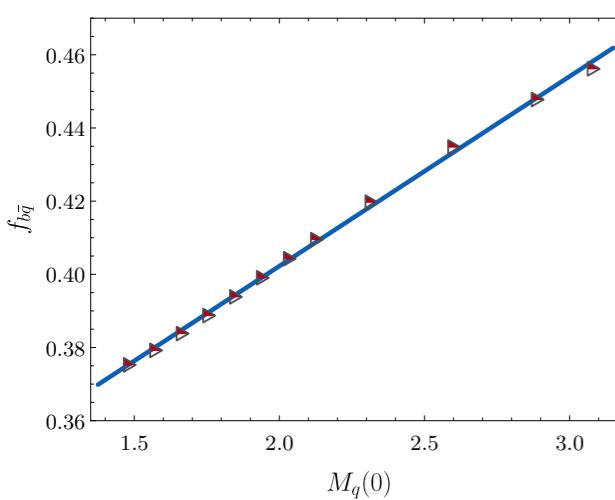
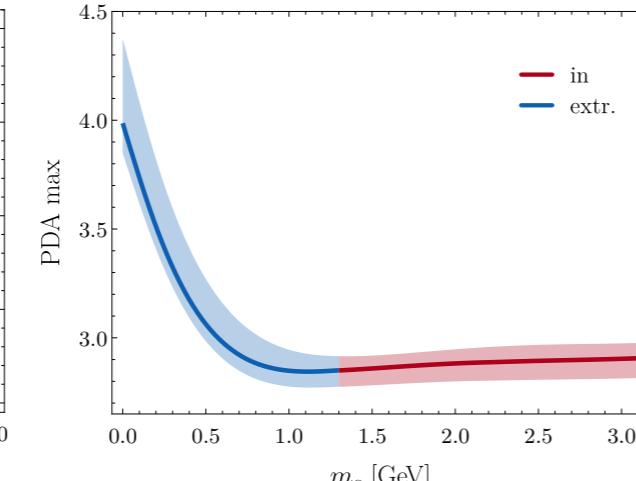
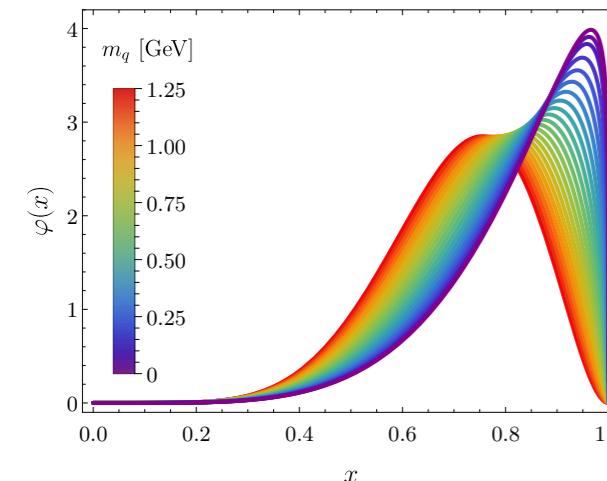
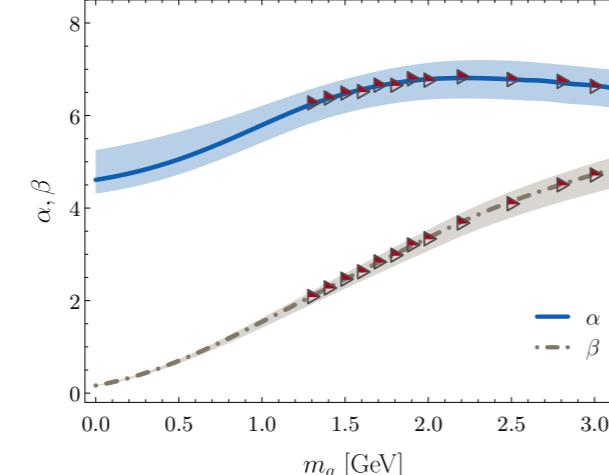
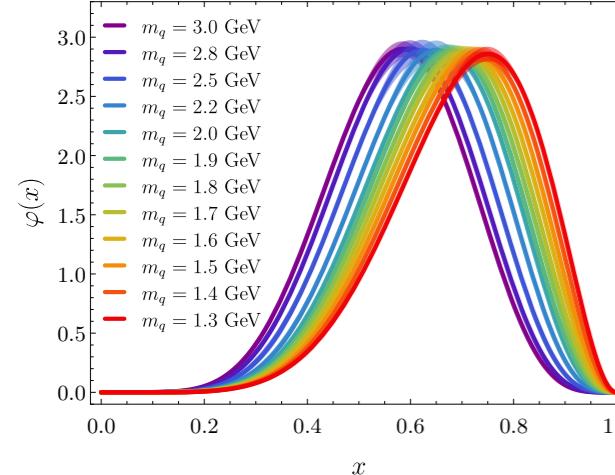


Bottom sector

m_q	α	β	a	b
m_c	$6.179^{0.398}_{0.379}$	$1.994^{0.342}_{0.141}$	$1.901^{0.070}_{0.070}$	$2.163^{0.060}_{0.060}$
m_s	$4.660^{0.760}_{1.240}$	$0.230^{0.125}_{0.108}$	$0.621^{0.070}_{0.070}$	$2.174^{0.060}_{0.060}$
m_u	$4.612^{0.714}_{1.287}$	$0.144^{0.054}_{0.118}$	$0.495^{0.070}_{0.070}$	$2.166^{0.060}_{0.060}$



- Gegenbauer





European Centre for Theoretical Studies
in Nuclear Physics and Related Areas



thankyou