# Pion and Kaon Structure through calculation of Current-Current Matrix Elements

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with

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*Mapping Parton Distribution Amplitudes and Functions*

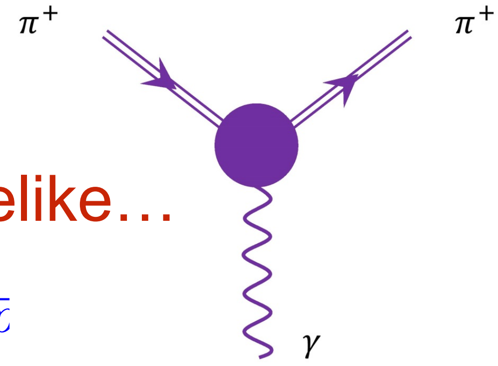
Trento 2018

# Outline

- Pion and Kaon Form Factors
- Light-Cone Distributions from Euclidean-space Lattice QCD
- Lattice Cross Sections
- Why the Pion and Kaon?
- Preliminary Results for PDFs
- Summary and Future Plans

# Pion Electro-Magnetic form factors

## Paradigm for LQCD Calculations of matrix elements




$$\langle \pi(\vec{p}_f) \mid V_\mu(0) \mid \pi(\vec{p}_i) \rangle = (p_i + p_f)_\mu F(Q^2)$$

where

$$V_\mu = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d$$

$$-Q^2 = [E_\pi(\vec{p}_f) - E_\pi(\vec{p}_i)]^2 - (\vec{p}_f - \vec{p}$$

Spacelike…

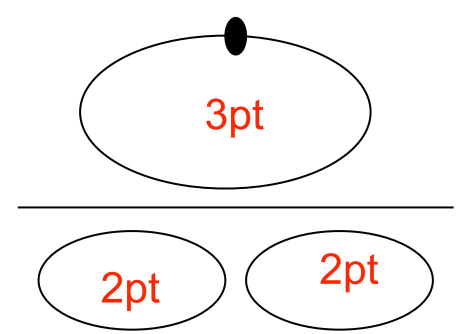
$$\Gamma_{\pi^+ \mu \pi^+}(t_f, t; \vec{p}, \vec{q}) = \sum_{\vec{x},\vec{y}} \langle 0 | \phi(\vec{x}, t_f) V_\mu(\vec{y}, t) \phi^\dagger(\vec{0}, 0) | 0 \rangle e^{-i\vec{p}\cdot\vec{x}} e^{-i\vec{q}\cdot\vec{y}},$$

Resolution of unity – insert states

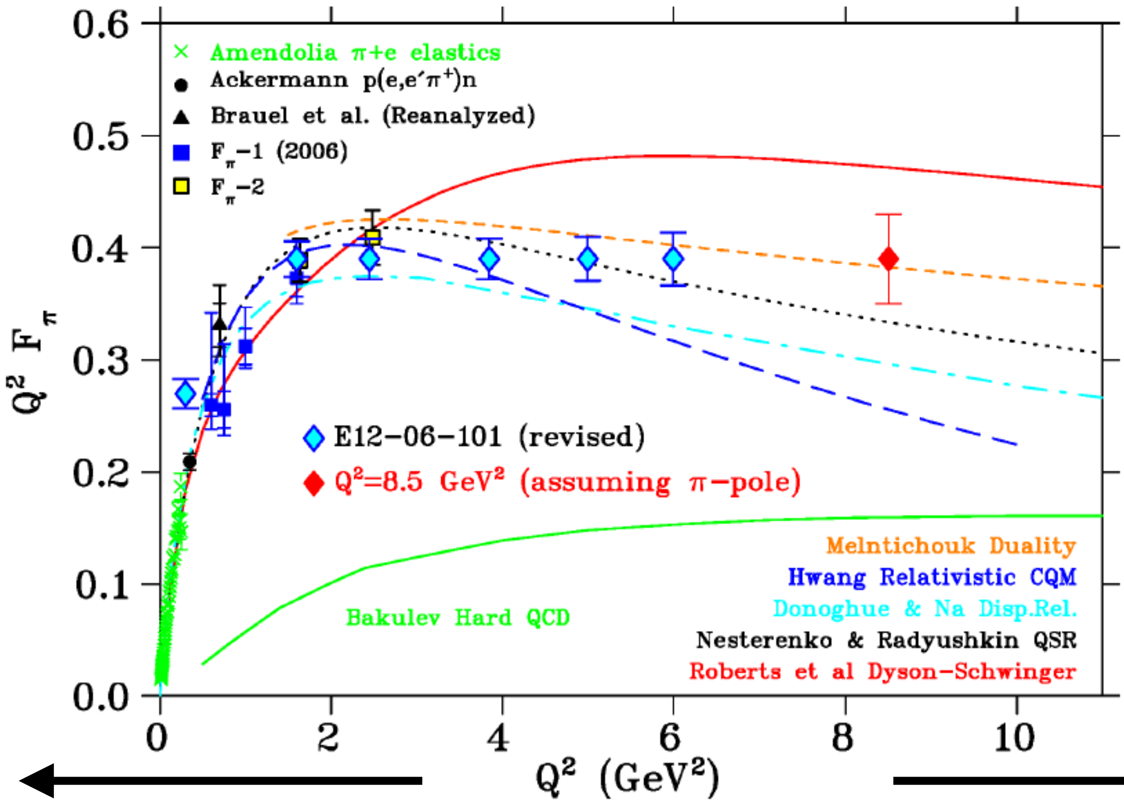
$$\langle 0 \mid \phi(0) \mid \pi, \vec{p} + \vec{q} \rangle \langle \pi, \vec{p} + \vec{q} \mid V_\mu(0) \mid \pi, \vec{p} \rangle \langle \pi, \vec{p} \mid \phi^\dagger \mid 0 \rangle e^{-E(\vec{p}(t - t_i)} e^{-E(\vec{p}+\vec{q})(t_f - t)}$$

$$\Gamma_{\pi^+\pi^+}(t, 0; \vec{p}) = \sum_{\vec{x}} \langle 0 \mid \phi(\vec{x}, t_f) \phi^\dagger(0) \mid 0 \rangle e^{-i\vec{p}\cdot\vec{x}}$$

$$\propto |\langle 0 | \phi(0) | \pi, \vec{p} \rangle|^2 e^{-E(\vec{p})t}$$



# Pion Experimental Summary



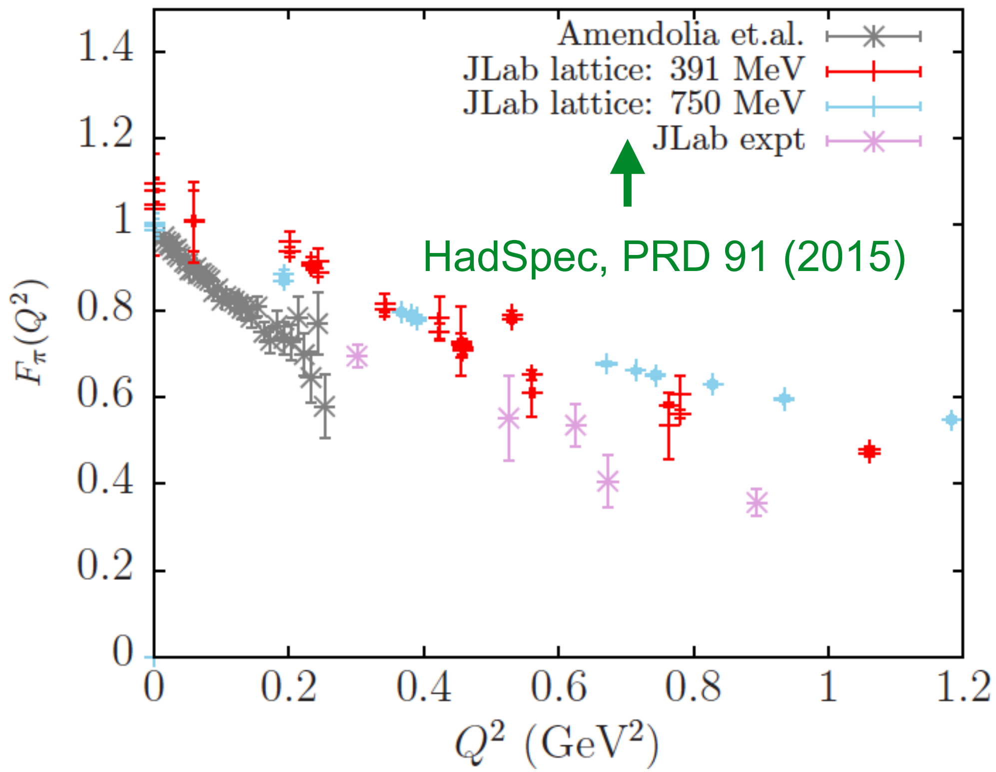

Thanks to
*Bipasha Chakraborty*

G. Huber, D.Gaskell, T. Horn
PR12-16-003

*Charge Radius* ←  → *"QCD"*

$$Q^2 \longrightarrow 30\,\mathrm{GeV}^2 \text{ at future EIC}$$

# Pion Form Factor - I




Briceno, Chakraborty, Edwards, Kusno, Orginos, DGR, Winter

## Anisotropic lattice

$$a_s \simeq 0.12\,\mathrm{fm}$$

$$a_t/a_s = 3.44$$

$$(L/a_s)^3 \times (T/a_t) = 20^3 \times 128$$

$$m_\pi = 750\,\mathrm{MeV}: \; 0.47(6)\,\mathrm{fm}$$

$$\langle r^2 \rangle = 6 \left. \frac{dF(q^2)}{dq^2} \right|_{q^2=0}$$

$$m_\pi = 391\,\mathrm{MeV}: \; 0.55(10)\,\mathrm{fm}$$

# Pion Form Factor - II

- Challenge to reach high momenta
  – discretization errors $p \leq 1/a$
- Signal-to-noise ratio

Need both spatial and temporal lattice spacing fine

$$C(t,\vec{p}) \equiv \sum_{\vec{x}} \langle 0 \mid \mathcal{O}(t,\vec{x})\mathcal{O}^\dagger(0,0) \mid 0 \rangle e^{-i\vec{p}\cdot\vec{x}} \rightarrow e^{-E(\vec{p})t}$$

$$C_{\sqrt{\sigma^2}}(t,\vec{p}) \longrightarrow e^{-m_\pi t}$$

**Boosted interpolating operators**

**Bali *et al*., Phys. Rev. D 93, 094515 (2016)**

Feynman-Hellmann method

*Probe correlators at small t*

Variational Method

# Variational Method

- Solve generalized eigenvalue equation

$$C(t)v^{(N)}(t,t_0) = \lambda_N(t,t_0)C(t_0)v^{(N)}(t,t_0).$$
$$\lambda_N(t,t_0) \longrightarrow e^{-E_N(t-t_0)},$$

- Find *optimal interpolating operator*, coupling to lowest state

$$\mathcal{O}_{N,\mathrm{proj}} = v_i^{(N)}\mathcal{O}_i$$

- Implement using *distillation* M.Peardon et al., arXiv:0905.2160

$$C_{3\mathrm{pt}} \rightarrow \langle 0 \mid \mathcal{O}_{\mathrm{proj}}(\vec{p}_f, t_f)V_\mu(\tau)\mathcal{O}_{\mathrm{proj}}(\vec{p}_i, t_i) \mid 0\rangle;\ \vec{q} = \vec{p}_f - \vec{p}_i$$

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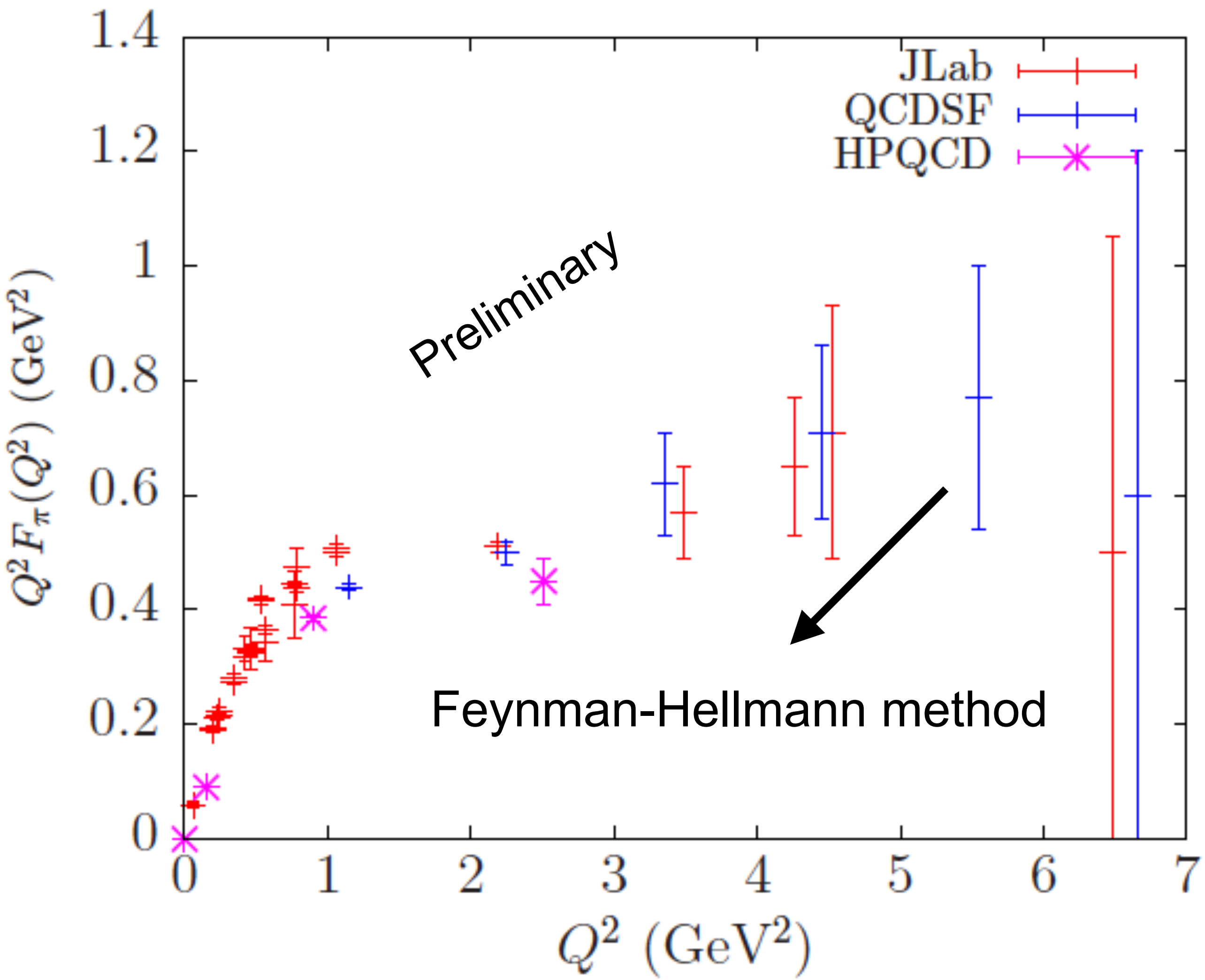
Feynman-Hellmann method

$$\begin{aligned} H &= H_0 + \lambda H_\lambda \\ \frac{\partial E_n}{\partial \lambda} &= \langle n \mid H_\lambda \mid n\rangle \end{aligned}$$

Reduces to calculation of energy-shift of two-point functions ***but*** repeat the calculation for each operator

# Form Factor at high $Q^2$

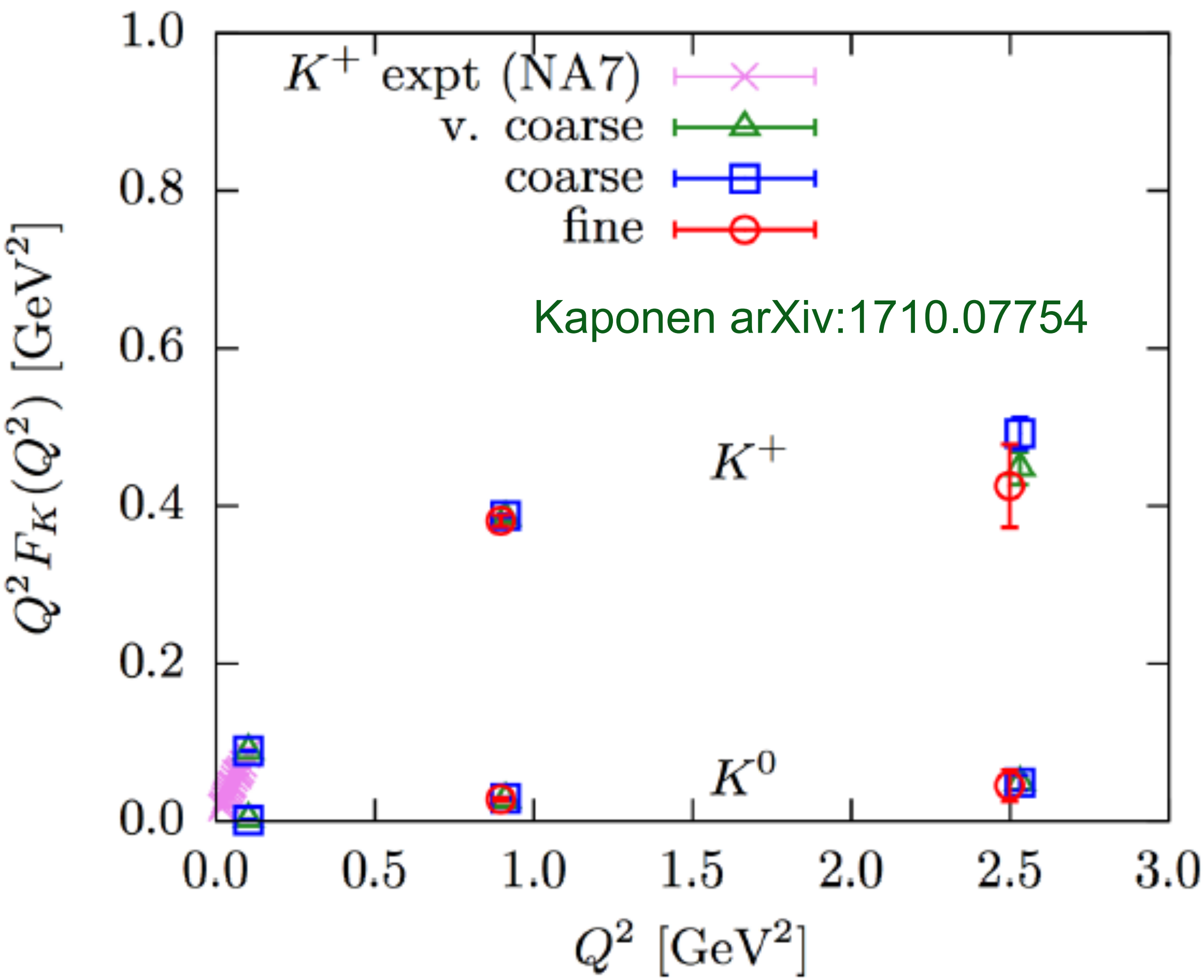



arXiv:1702.01513
arXiv:1710.07554

Form factor at high momenta achievable

Apply methods to fine isotropic lattice

# Kaon Form Factor




$m_\pi \simeq 310\,\mathrm{MeV}$

# Lattice PDFs Introduction - I

- Euclidean lattice precludes the calculation of light-cone correlation functions
  - So… …Use *Operator-Product-Expansion* to formulate in terms of *Mellin Moments*

$$q(x,\mu) = \int \frac{d\xi^-}{4\pi} e^{-ix\xi^- P^+} \langle P \mid \bar{\psi}(\xi^-)\gamma^+ e^{-ig\int_0^{\xi^-} d\eta^- A^+(\eta^-)}\psi(0) \mid P\rangle$$

$$\downarrow$$

$$\langle P \mid \bar{\psi}\gamma_{\mu_1}(\gamma_5)D_{\mu_2}\dots D_{\mu_n}\psi \mid P\rangle \to P_{\mu_1}\dots P_{\mu_n} a^{(n)}$$

- Moment Methods
  - Extended operators: Z.Davoudi and M. Savage, PRD 86,054505 (2012)
  - Valence heavy quark: W.Detmold and W.Lin, PRD73, 014501 (2006)

KF Liu, SJ Dong, PRL72, 1790 (1994)

- Hadronic Tensor (HT) $W_{\mu\nu} = \frac{1}{4\pi}\int d^4z\, e^{iq.z}\langle p \mid J_\mu(z)^\dagger J_\nu(0) \mid p\rangle$

$$C_4(\vec{p},\vec{q},\tau) = \sum_{\vec{x}_f} e^{-i\vec{p}.\vec{x}_f} \sum_{\vec{x}_2,\vec{x}_1} e^{-i\vec{q}.(\vec{x}_2-\vec{x}_1)}\langle N(\vec{x}_f,t_f)J_\mu(\vec{x}_2,t_2)J_\nu(\vec{x}_1,t_1)\bar{N}(\vec{0},t_0)\rangle$$

This is a ***four-point*** function - *time-dependent operator.*

# Introduction - II

- Quasi-PDF (qPDF) interpreted in LaMET (Large Momentum Effective Theory) was proposed by X.Ji **X. Ji, Phys. Rev. Lett. 110 (2013) 262002**

$$q(x,\mu^2,P^z) = \int \frac{dz}{4\pi} e^{izk^z} \langle P \mid \bar{\psi}(z)\gamma^z e^{-ig\int_0^z dz' A^z(z')} \psi(0) \mid P > + \mathcal{O}((\Lambda^2/(P^z)^2), M^2/(P^z)^2))$$

*Quasi distributions* approach light-cone distributions in limit of large *P*$^z$

$$q(x,\mu^2,P^z) = \int_x^1 \frac{dy}{y} Z\left(\frac{x}{y}, \frac{\mu}{P^z}\right) q(y,\mu^2) + \mathcal{O}(\Lambda^2/(P^z)^2, M^2/(P^z)^2)$$

- Pseudo-PDF (pPDF) recognizing generalization of PDFs in terms of *Ioffe Time.* $\nu = p \cdot z$ ⟶ V.Braun

**A. Radyushkin, PLB767 (2017)**

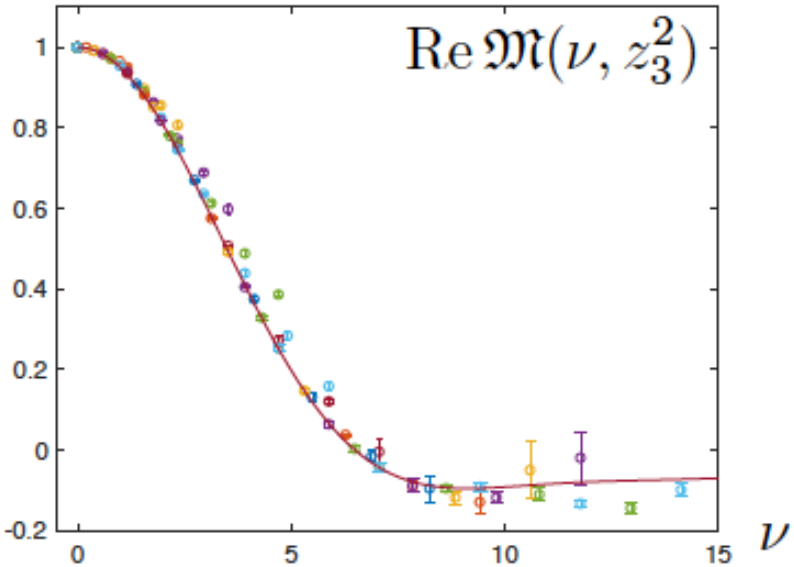
$$\mathcal{M}^\alpha(z,p) = \langle p \mid \bar{\psi}(z)\gamma^\alpha \exp\left(-ig\int_0^z dz' A^z(z')\right) \psi(0) \mid p\rangle$$

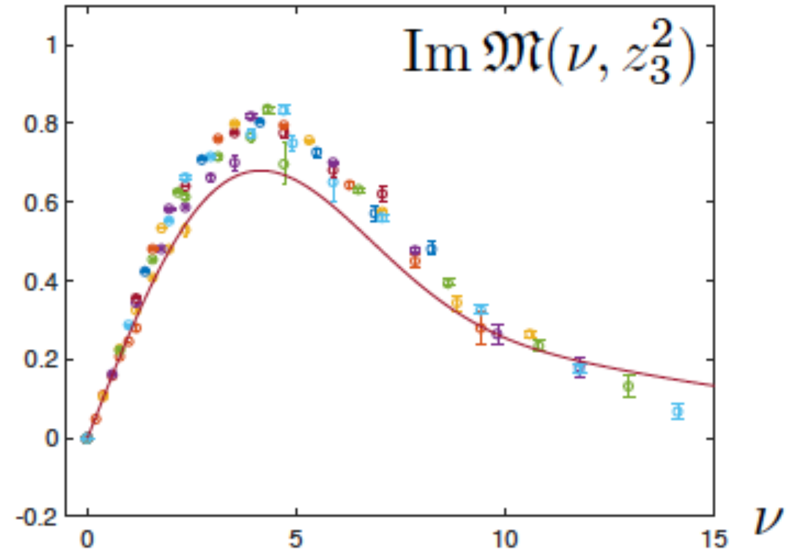
# Quasi- and Pseudo-PDFs - I

Ioffe-time PDFs related to regular PDFs through: $M(\nu, z_3^2) = \int_{-1}^{1} f(x, 1/z_3^2) e^{ix\nu}$

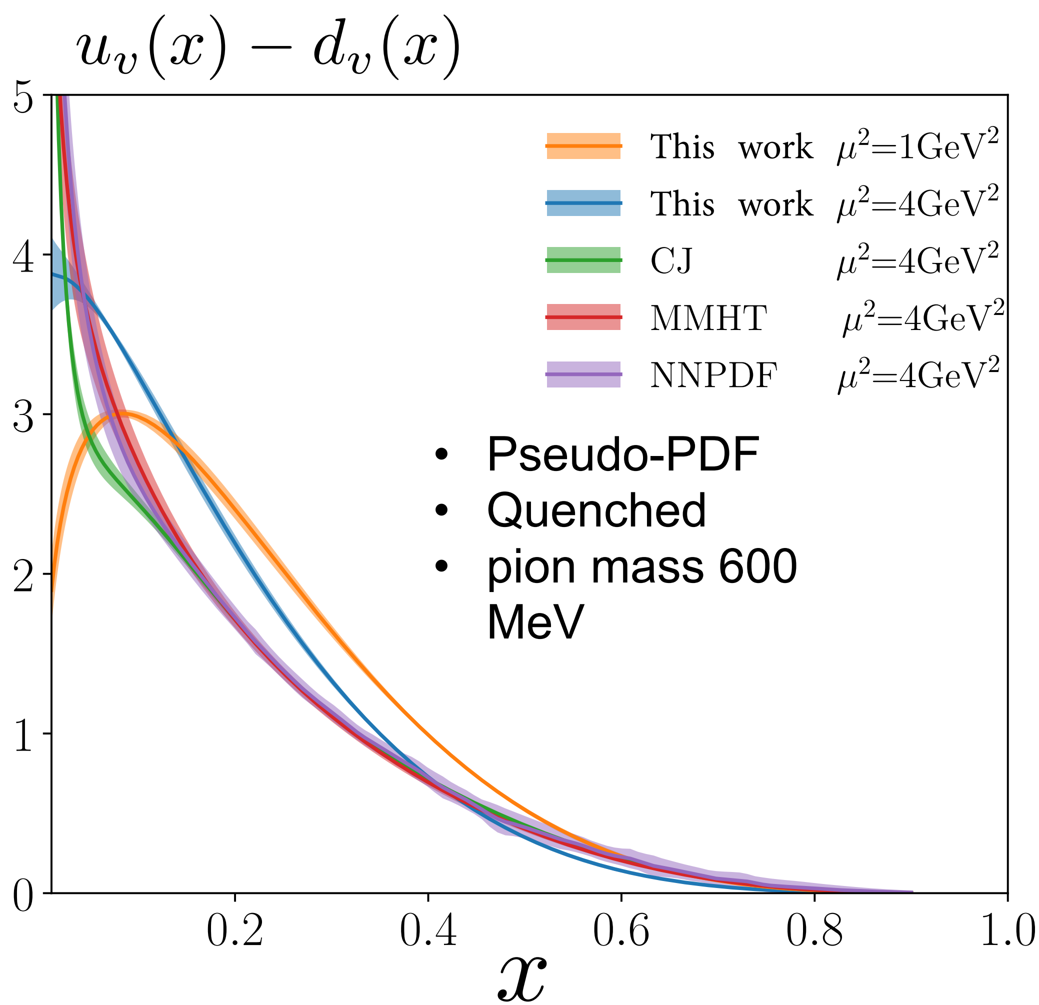
Introduce reduced matrix element with well-defined continuum limit: $\mathscr{M}(\nu, z_3^2) = \dfrac{M(\nu, z_3^2)}{M(0, z_3^2)}$

Orginos, Radyushkin, Karpie and Zafeiropoulos, PRD96, 094503 (2017)



# Pseudo-PDFs - II




$$M(\nu, z_3^2) = \int_{-1}^{1} f(x, 1/z_3^2) e^{ix\nu}$$

Lattice "data" — PDF — Known kernel

Fit to

$$q_v(x) = N(a,b) x^a (1-x)^b$$

**Quasi-PDFs**

↓

Huey-Wen Lin (Friday)

# Pseudo- and Quasi-PDFs

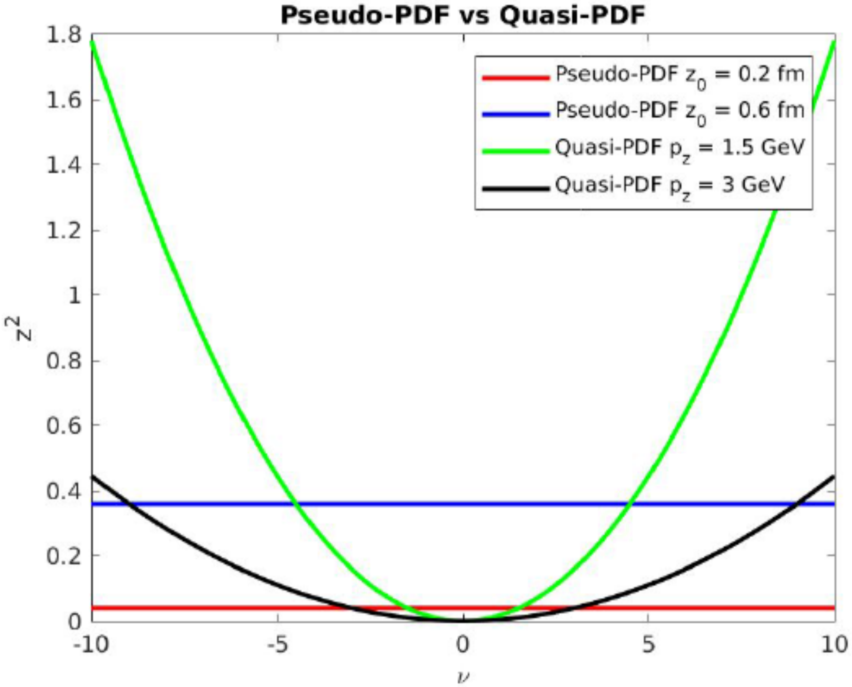
Relation between qPDF and pPDF approaches

Slide from **Joe Karpie**

- Both integrals of Ioffe-Time Distribution Function
- Should yield same PDF after matching and systematic controls

$$P(x, z_0^2) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\nu \, e^{-i\nu x} M(\nu, z_0^2)$$

$$Q(x, p_z^2) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\nu \, e^{-i\nu x} M(\nu, \frac{\nu^2}{p_z^2})$$



# "Lattice Cross Sections"

- Good "Lattice Cross Sections" (LCS) Ma and Qiu, Phys. Rev. Lett. 120 022003

$$\sigma_n(\omega, \xi^2, P^2) = \langle P \mid T\{\mathcal{O}_n(\xi)\} \mid P \rangle$$

*Expressed in coordinate space*

Short distance scale

where

$$\sigma_n(\omega, \xi^2, P^2) = \sum_a \int_{-1}^{1} \frac{dx}{x} f_a(x, \mu^2) K_n^a(x\omega, \xi^2, x^2P^2, \mu^2) + \mathcal{O}(\xi^2 \Lambda_{\rm QCD}^2)$$

*Calculated in LQCD*

Structure function

*Calculated in perturbation theory ("process dependent")*

Factorize in $\omega = P \cdot \xi,\ \xi^2 P^2$ providing $\xi << \frac{1}{\Lambda_{\rm QCD}}$

$P \to \sqrt{s}$ *Collision energy*

$\xi \to \frac{1}{Q}$ *Hard Probe*

Momentum space $\tilde{\sigma}(\tilde{\omega}, q^2P^2) \equiv \int \frac{d^4\xi}{\xi^4} \sigma(P \cdot \xi, \xi^2, P^2)$ $\tilde{\omega} = 1/x_B$

# Lattice Cross Sections - II

- Quasi- and Pseudo-distributions particular case

$$\mathcal{O}(\xi) = \bar{\psi}(0)\Gamma W(0, 0+\xi)\psi(\xi)$$

Wilson Line

Current-current correlators

$$\mathcal{O}_{j_1 j_2}(\xi) \equiv \xi^{d_{j_1}+d_{j_2}-2} Z_{j_1}^{-1} Z_{j_2}^{-1} j_1(\xi)\, j_2(0)$$

$d_j$ : Dimension of the current

$Z_j$ : Renormalization constant of the current

$Z_j$ already known for the lattice ensembles being used

Different choices of currents

$$j_S(\xi) = \xi^2 Z_S^{-1}[\overline{\psi}_q \psi_q](\xi), \qquad j_V(\xi) = \xi Z_V^{-1}[\overline{\psi}_q \gamma \cdot \xi \psi_q](\xi),$$

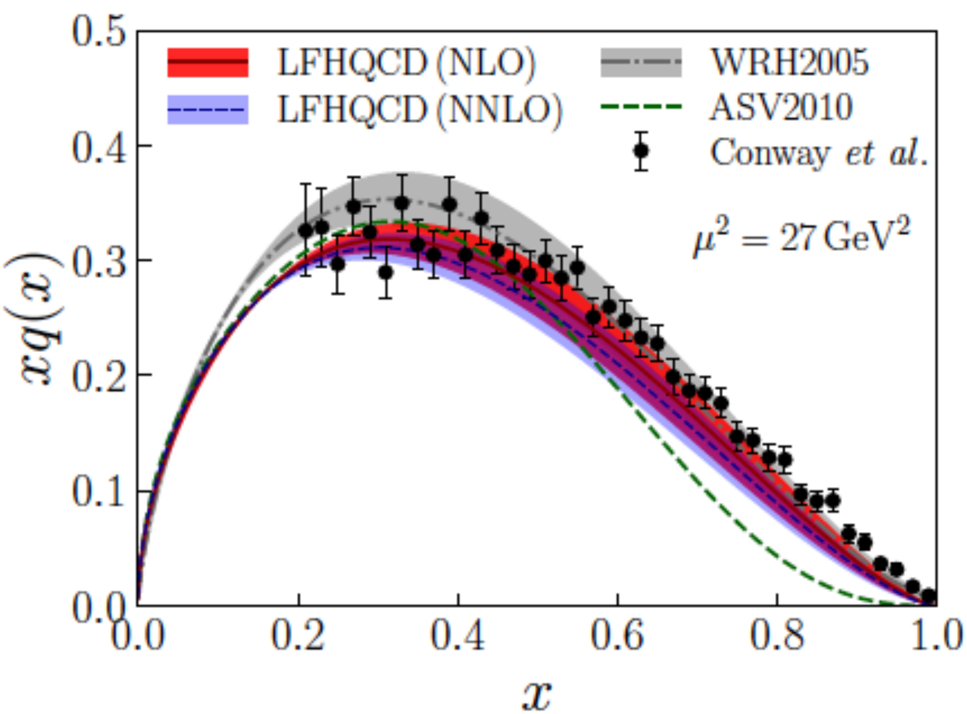
$$j_{V'}(\xi) = \xi Z_{V'}^{-1}[\overline{\psi}_q \gamma \cdot \xi \psi_{q'}](\xi), \qquad j_G(\xi) = \xi^3 Z_G^{-1}[-\frac{1}{4} F^c_{\mu\nu} F^c_{\mu\nu}](\xi), \ldots$$

flavor changing current

gluon distribution

# Why the Pion?

- u distribution of FNAL E615 to leading order
- C12-15-006 at Hall A will look at structure of pion
- C12-15-006A at Hall A will look at structure of Kaon
- No free pion target

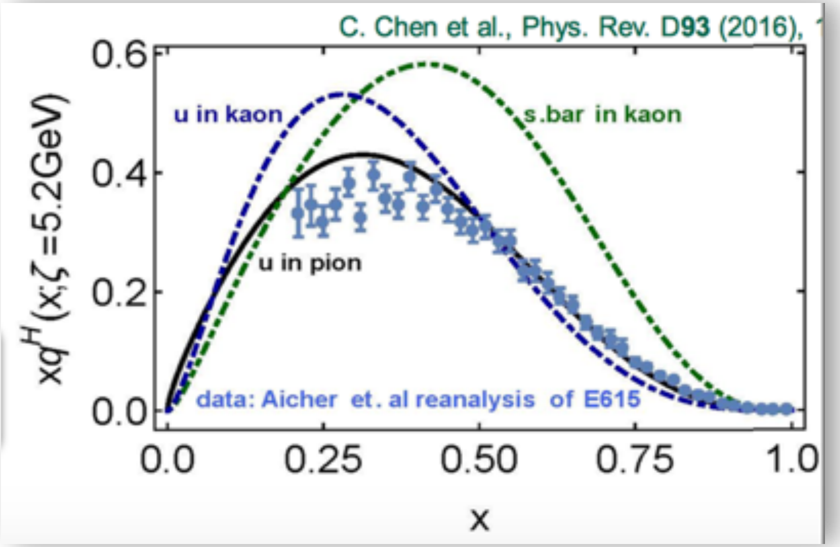


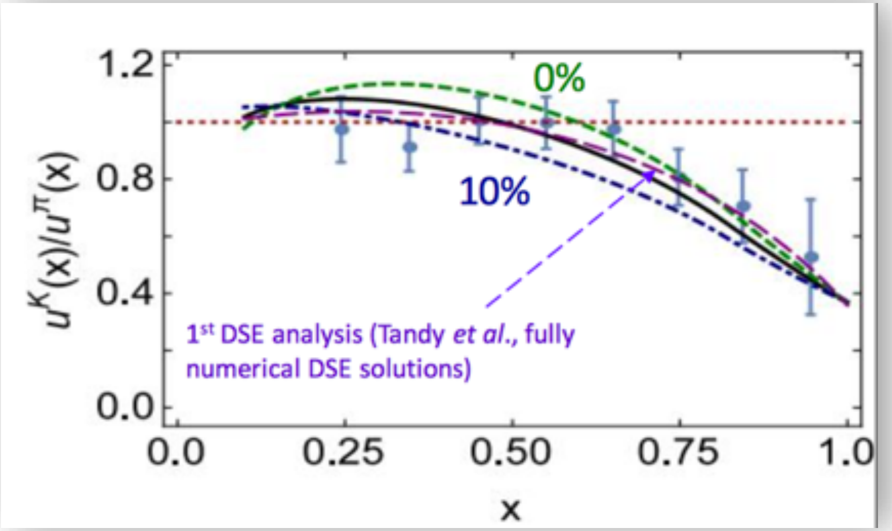

de Teramond, liu, Sufian, Dosch, Brodsky, Deur, PRL (2018)

Discrepancy in large-x behavior of pion distribution

# Why the pion *and* kaon

- Recent DSE calculations of pion and Kaon PDFs and ratios




First-principle lattice QCD calculation of needed –
First calculation of Kaon PDF on the lattice

# Why the Pion - II?

- Pion less computationally demanding that nucleon
  - *Larger signal-to-noise ratio*

$$C(t,\vec{p}) \equiv \sum_{\vec{x}} \langle 0 \mid \mathcal{O}(t,\vec{x})\mathcal{O}^\dagger(0,0) \mid 0\rangle e^{-i\vec{p}\cdot\vec{x}} \rightarrow e^{-E(\vec{p})t}$$

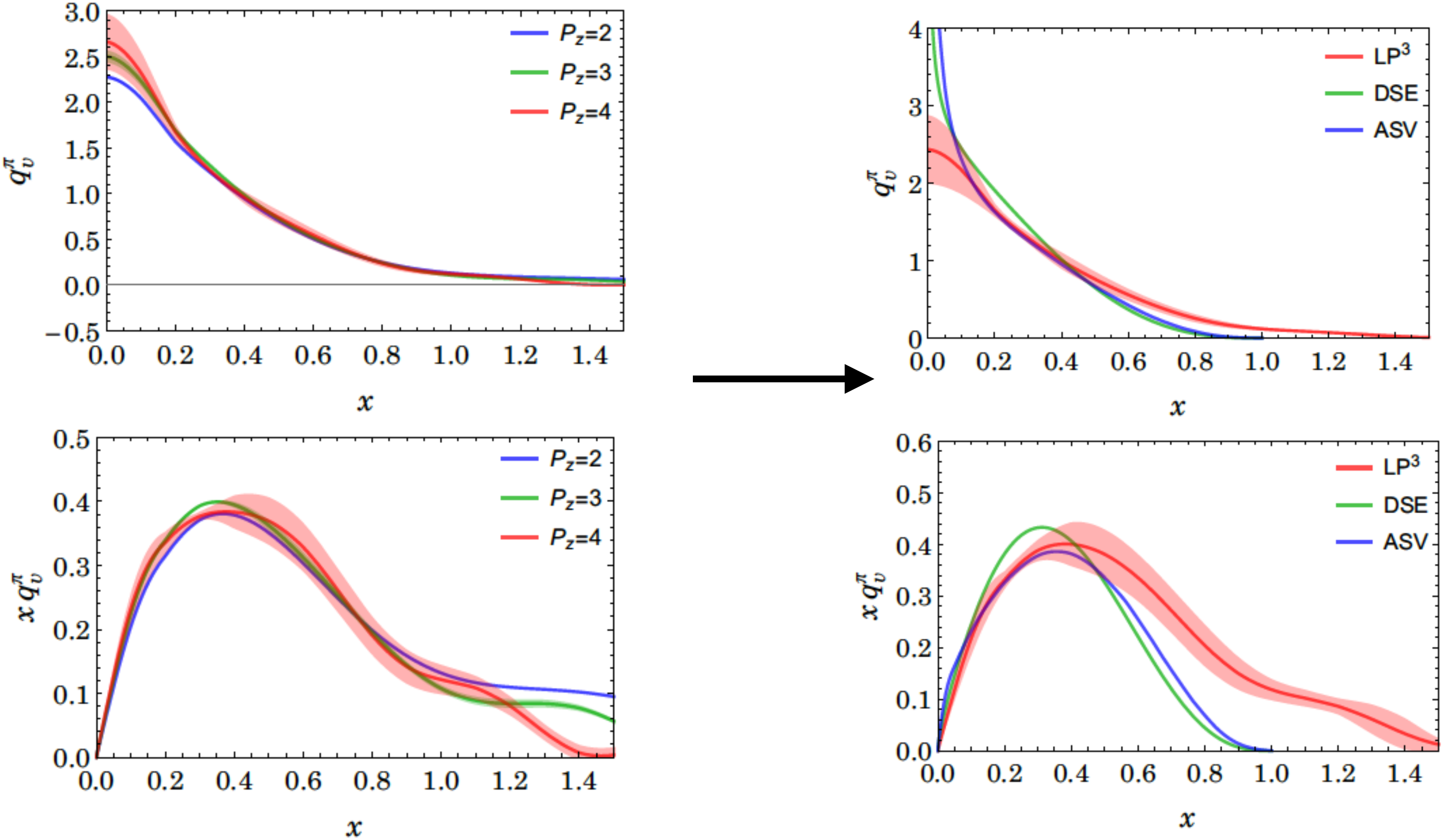
$$C_{\sqrt{\sigma^2}}(t,\vec{p}) \rightarrow \begin{cases} e^{-m_\pi t} & \text{Mesons} \\ e^{-(3m_\pi/2)t} & \text{Baryons} \end{cases}$$

- Important constraint on systematic uncertainty is understanding operator renormalization
  - *Operator renormalization "independent" of external states*
- Admits simple computational methodology
  - *Coordinate-space currents computationally demanding in lattice QCD*

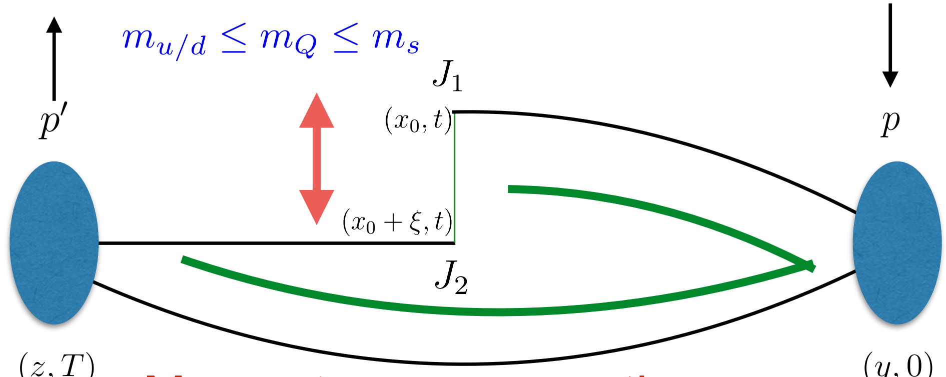
# Quasi-Distribution of Pion

$m_\pi \simeq 300\,\mathrm{MeV}$

LP3, arXiv:1804.01483



# Computational Setup




c.f. Bali et al, arXiv:1807.07073

$$\langle \Pi(-p') | \mathcal{O}_{J_1}(x_0) \mathcal{O}_{J_2}(\xi) | \Pi(-p') \rangle =$$
$$= \sum_{y,z} e^{i(p'.z - p.y)} \langle \bar{q}\, \Gamma_\Pi\, q(z,T)\, \bar{Q}\, J_2\, Q(x_0+\xi, t)\, \bar{q}\, J_1\, q(x_0, t)\, \bar{q}\, \Gamma_\Pi\, q(y, 0) \rangle$$
$$= \sum_{y,z} e^{i(p'.z - p.y)} \mathrm{Tr}[J_2\, G_Q(x_0+\xi, t; x_0, t)\, J_1\, G(x_0, t; y, 0)\, \Gamma_\Pi G(y, 0; z, T) \Gamma_\Pi\, G(z, T; x_0+\xi, t)]$$

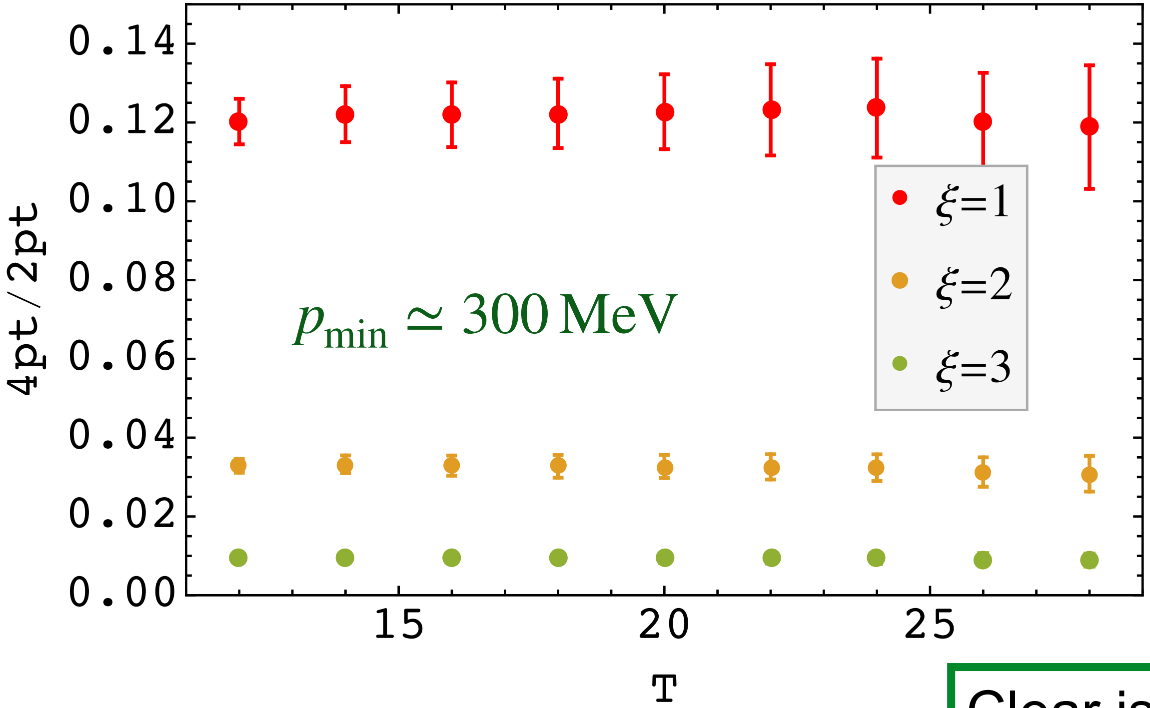
Straightforward computational setup using sequential-source method:

$$D(Z, T; w) H(w; x_0, t) = \sum_y e^{-ip\cdot y} \Gamma_\Pi G(y, 0; x_0, t)$$
$$D(s; w) \tilde{H}(w; x_0, t) = \sum_z e^{ip\cdot z} \Gamma_\Pi H(z, T; x_0, t)$$

# Demonstration

- 2+1 Flavor clover-fermion action
- 10 different current-current combinations

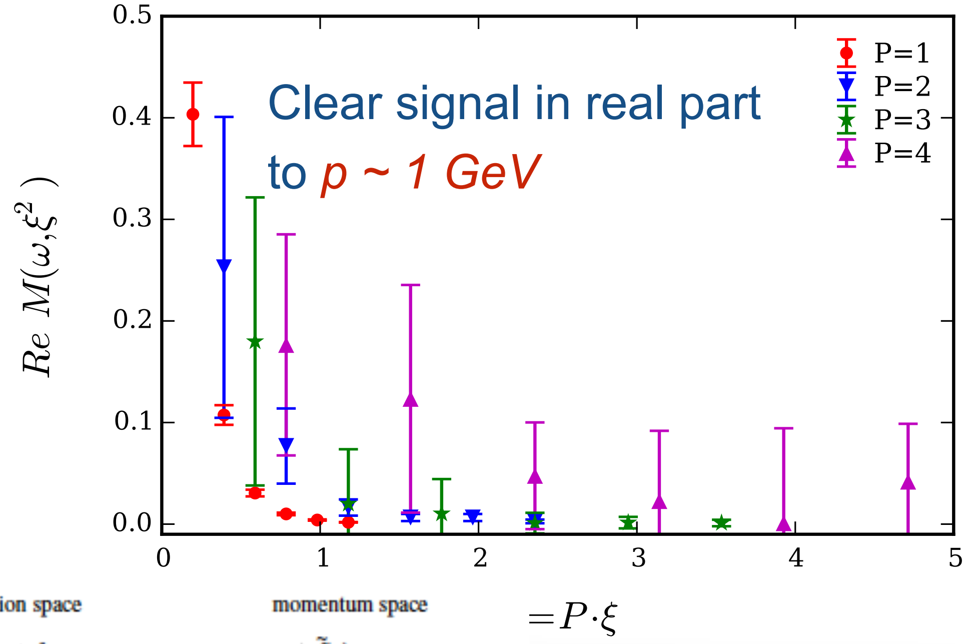
Lattice details: $32^3 \times 96, a \simeq 0.127\,\text{fm}, m_\pi \simeq 430\,\text{MeV}, 450\,\text{Configs}$

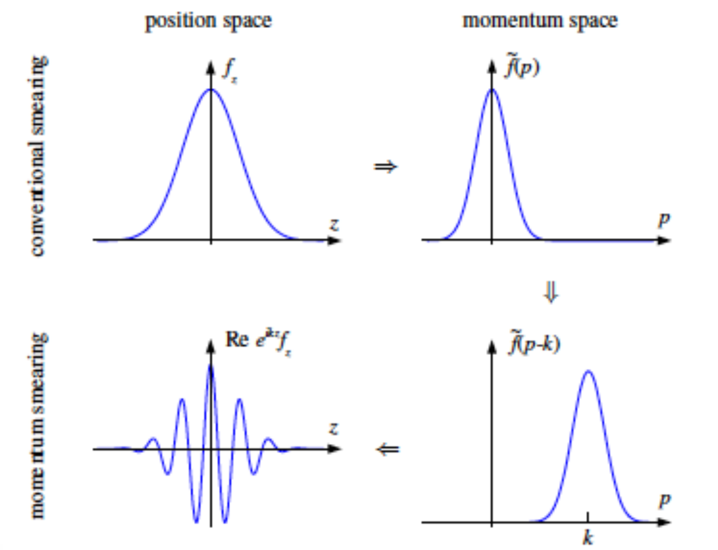



$J_1 = V, J_2 = A$

Clear isolation of pion matrix element

# Challenges of Higher Momenta



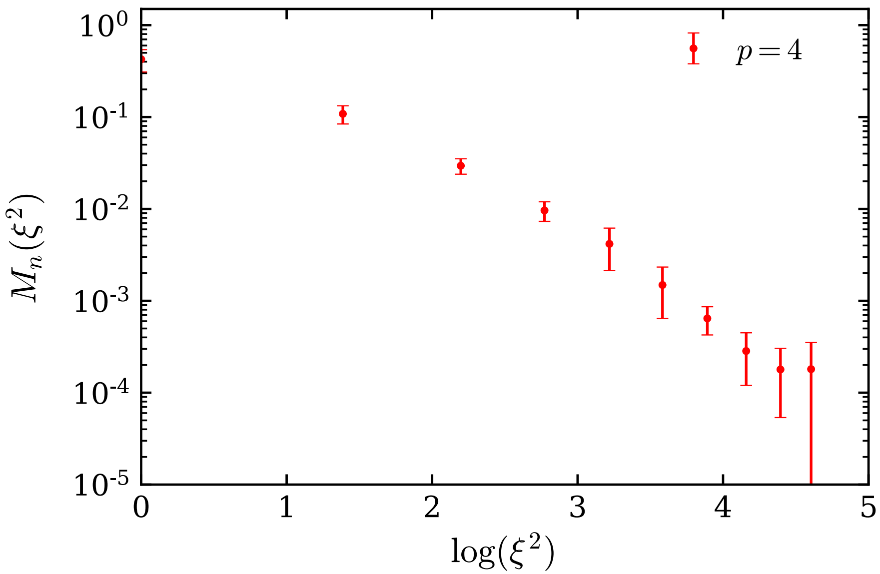



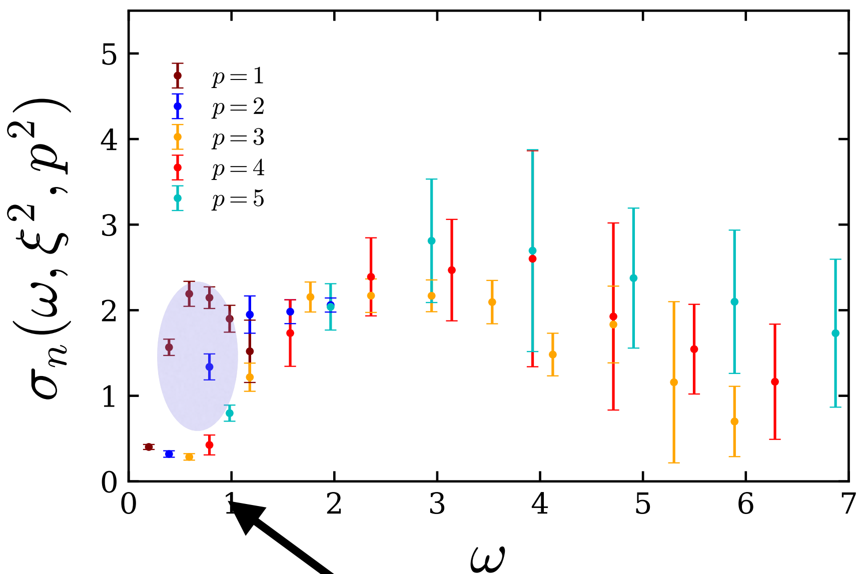

**Boosted interpolating operators**

Bali *et al.*, Phys. Rev. D 93, 094515 (2016)

# Preliminary Results

V-V current correlation




Only about 1/3 statistics of p=3,4,5 data analyzed

p=1 (0.3 GeV) data deviates

# Comparison with Phenomenology

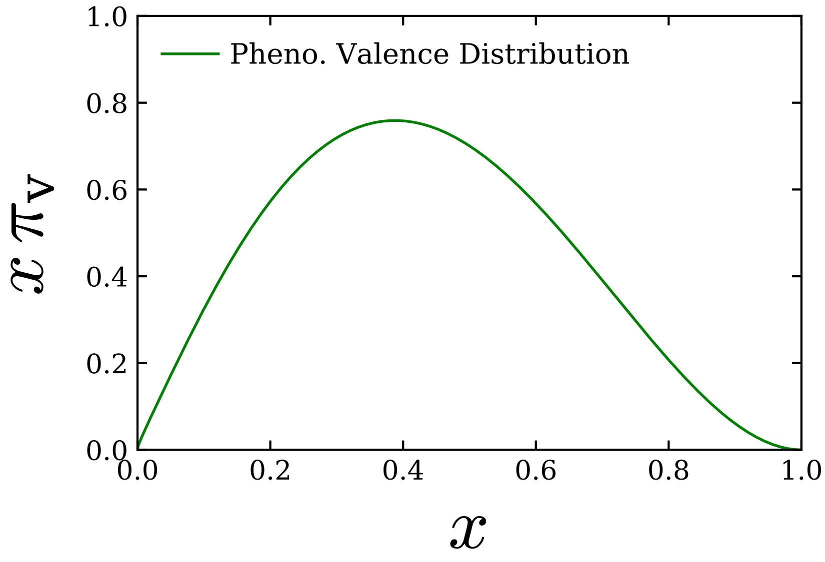
Does the calculated matrix lead to consistent description of pion PDF ?

$$\sigma_n(\omega,\xi^2,P^2) = \sum_a \int_{-1}^{1} \frac{dx}{x} f_a(x,\mu^2) K_n^a(x\omega,\xi^2,P^2,\mu^2) + \mathcal{O}(\xi^2\Lambda^2)$$

*Non flavor-changing*

$$\sigma_{S,V,\tilde{V}} = -\frac{2}{\pi^2}\sum_a \int_{-1}^{1} dx\, f_a(x)\omega \sin(x\omega)$$

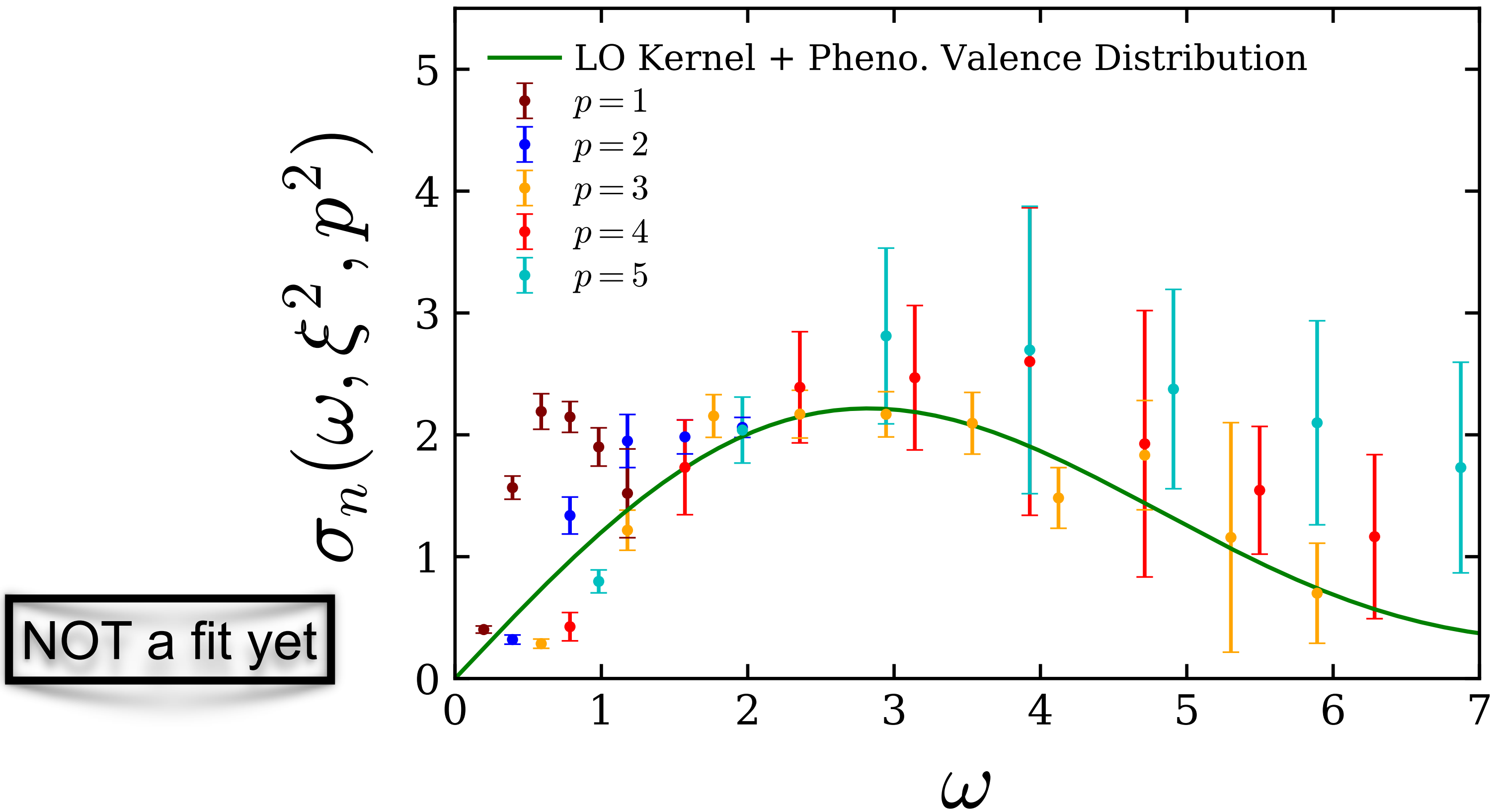
*Tree-level*




$$K_S^q(x\omega,\xi^2,0,0) = \frac{i}{\pi^2} x\omega \left(e^{ix\omega} - e^{-ix\omega}\right)$$

$$f(x) \simeq A x^\alpha (1-x)^\beta (1 + \gamma\sqrt{x} + \delta x)$$

# Preliminary Results - II

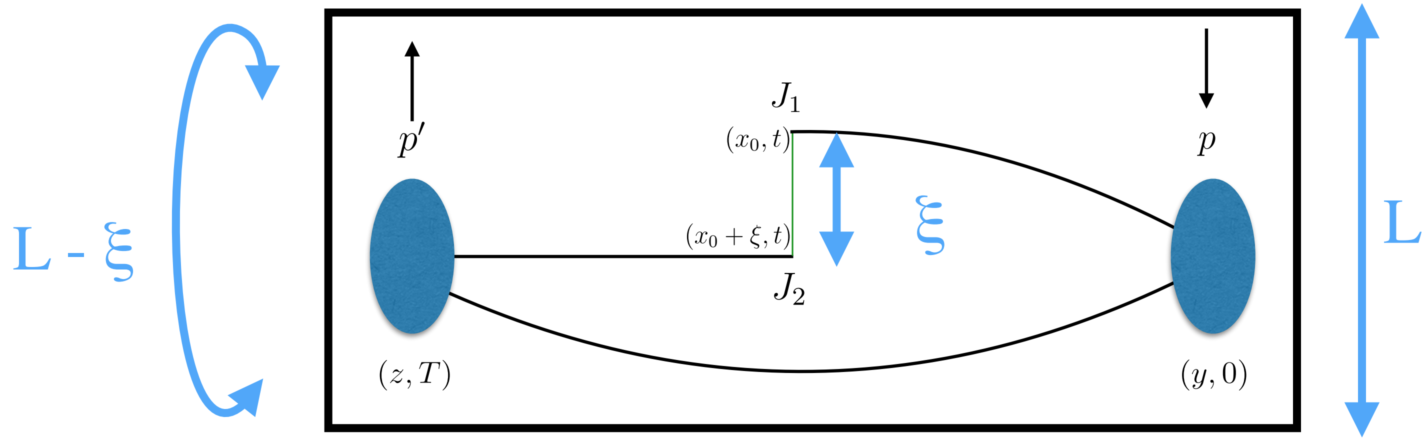


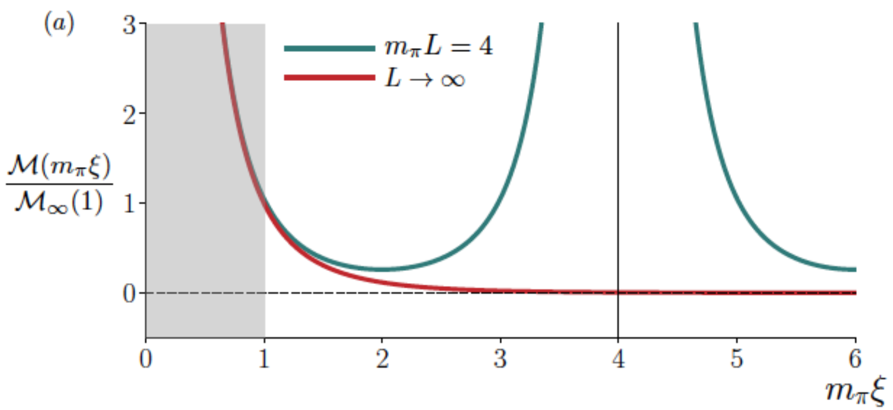

*K's being calculated at LO and NLO for different currents*

A combined fit to many LCSs on an ensemble will lead to precise determination of PDFs

# Finite Volume Effects

Briceno, Guerrero, Hansen and Monahan, arXiv:1805.01304



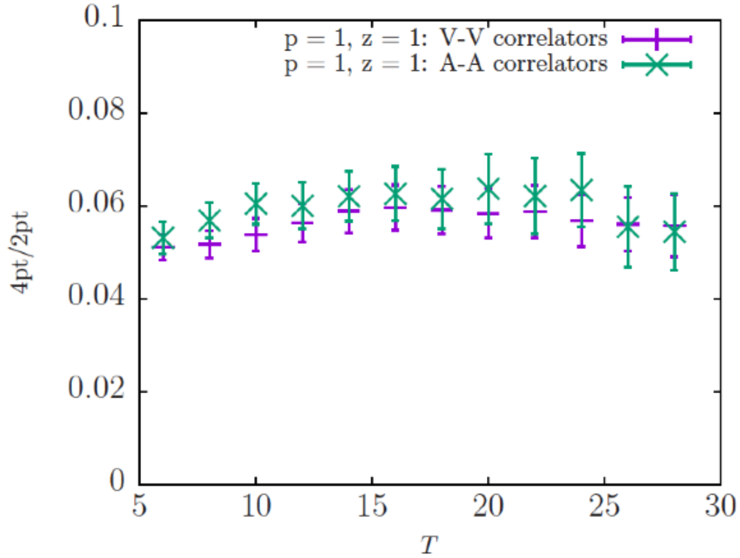




Typically $m_\pi L \simeq 4$

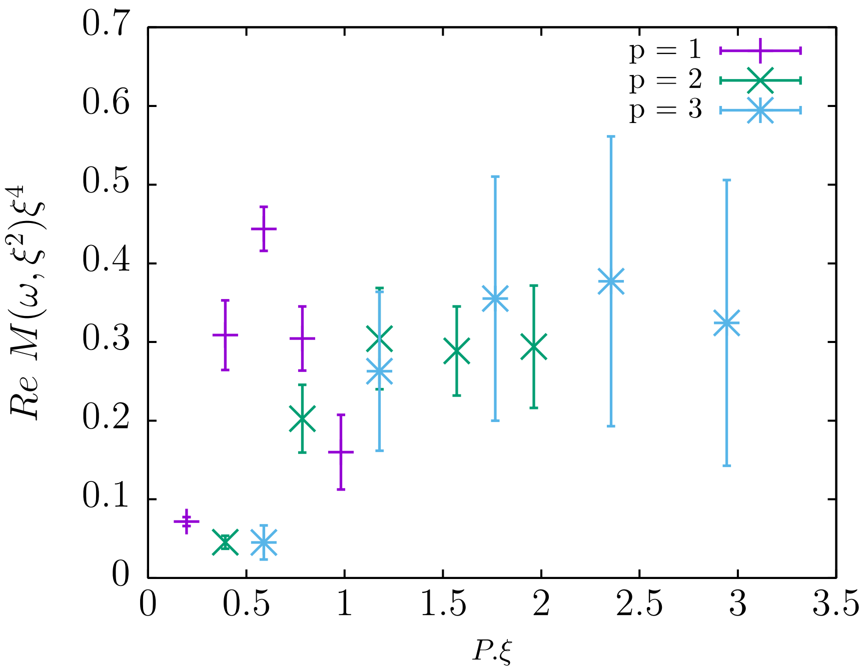
Future? $\left\{ \begin{array}{l} \xi \text{ shortdistance} \\ m_\pi \to m_\pi^{\text{phys}} \end{array} \right.$

# Kaon PDF…

- Calculations less advanced with more limited statistics




V-V and A-A comparable



# Summary

- Calculation of current-current correlators for pion *and kaon* in progress for variety of local operators + “pseudo-PDFs”. Global fit to lattice “cross sections”!
- Renormalization straightforward.
- Important to understand finite-volume effects -

$$24^3 \times 96, a \simeq 0.127\,\mathrm{fm}, m_\pi \simeq 430\,\mathrm{MeV}$$

Projected calculations with

$$m_\pi \approx 380 \text{ MeV}, \;\; a \approx 0.09 \text{ fm } (32^3 \times 64$$
$$m_\pi \approx 170 \text{ MeV } (64^3 \times 128)$$

- Application to nucleon
  - No straight-forward application of “sequential-source” method
  - Alternative approaches in progress