Parton distribution amplitudes of neutral pseudoscalar mesons

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Mapping Parton Distribution Amplitudes and Functions.
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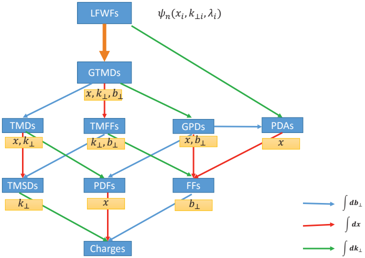
The quark structure of hadrons

- Hadron structure functions:
  LFWFs, GPDs, TMDs, PDFs, PDAs,
  FFs etc..[1]

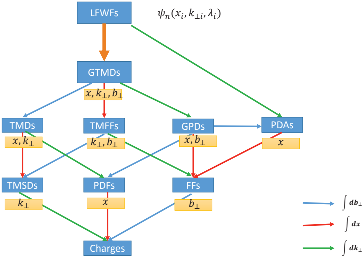


- Form factors: the closest thing we have to a snapshot.
  - e.g. $F(Q^2)$:
    momentum transfer Q.
- The 1D picture of how quarks move within a hadron:
  - PDFs and PDAs.
  - e.g. $q(x)$:
    longitudinal momentum fraction x.
- A multidimensional view of hadron structure:
  - LFWFs, GPDs, TMDs etc..
  - e.g. $\psi(x, k_\perp)$:
    longitudinal x and transverse momentum fraction $k_\perp$.

[1] C. Lorce and B. Pasquini. Wigner distributions and quark orbital angular momentum. Int. J. Mod. Phys. Conf. Ser. 20 (2012) 84-91.

The quark structure of hadrons

- Hadron structure functions: LFWFs, GPDs, TMDs, PDFs, PDAs, FFs etc..[1]



- LFWFs$\Longrightarrow$Leading-twist PDAs

$$\phi(x) = \frac{1}{16\pi^3} \int dk_\perp^2 \, \psi^{\uparrow\downarrow}(x, k_\perp^2) \,. \tag{1}$$

- LFWFs$\Longrightarrow$Unpolarised TMDs

$$f_1(x, k_\perp^2) = \frac{1}{16\pi^3} \sum_{\lambda_q, \lambda_{\bar{q}}} |\psi^{\lambda_q, \lambda_{\bar{q}}}(x, k_\perp^2)|^2$$
$$= \frac{1}{16\pi^3} (|\psi^{\uparrow\downarrow}|^2 + |\psi^{\downarrow\uparrow}|^2 + |\psi^{\uparrow\uparrow}|^2 + |\psi^{\downarrow\downarrow}|^2) \tag{2}$$

- Unpolarised TMDs$\Longrightarrow$PDFs

$$q(x) = \int dk_\perp^2 \, f_1(x, k_\perp^2) \tag{3}$$

[1] C. Lorce and B. Pasquini. Wigner distributions and quark orbital angular momentum. Int. J. Mod. Phys. Conf. Ser. 20 (2012) 84-91.

PDAs in theory

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- Leading-twist PDAs

$$\phi(x) = \frac{1}{16\pi^3} \int dk_\perp^2 \psi^{\uparrow\downarrow}(x, k_\perp^2)\,. \quad (4)$$

- PDAs in theory
  - QCD sum rules:
    - ★ V.L. Chernyak and A.R. Zhitnitsky, Nucl. Phys. B 201, 492 (1982).
    - ★ P.Ball, and V.M. Braun, Phys.Rev. D54 (1996) 2182-2193, Nucl.Phys. B543 (1999) 201-238 .
    - ★ V.M. Braun, S.E. Derkachov, G.P. Korchemsky, and A.N. Manashov, Nucl.Phys. B553 (1999) 355-426.
    - ★ A.P. Bakulev, S.V. Mikhailov, and N.G. Stefanis, Phys.Lett. B508 (2001) 279-289, Phys.Lett. B590 (2004) 309-310.
  - Light-front QCD:
    - ★ G.P. Lepage and S.J. Brodsky, Phys. Lett. B 87, 359 (1979).
    - ★ S.J. Brodsky, and G.F. de Teramond, Phys.Rev.Lett. 96 (2006) 201601.
  - NJL model:
    - ★ E.R. Arriola, and W. Broniowski, Phys.Rev. D66 (2002) 094016.
  - Instanton model:
    - ★ A.E. Dorokhov, JETP Lett. 77 (2003) 63-67, Pisma Zh.Eksp.Teor.Fiz. 77 (2003) 68-72.
  - Lattice QCD:
    - ★ G. Martinelli, and C.T. Sachrajda, Phys.Lett. B190 (1987) 151-156, Phys.Lett. B217 (1989) 319-324.
    - ★ D. Daniel, R. Gupta, and D.G. Richards, Phys.Rev. D43 (1991) 3715-3724.
    - ★ V.M. Braun, M. Gockeler, R. Horsley, H. Perlt, D. Pleiter, P.E.L. Rakow, G. Schierholz, A. Schiller, W. Schroers, H. Stuben, and J.M. Zanotti, Phys.Rev. D74 (2006) 074501.
    - ★ UKQCD Collaboration, Phys.Lett. B641 (2006) 67-74.
  - Dyson-Schwinger Equations:
  - etc.

Leading twist

- Twist: $t = l - s$, l: the scaling dimension, s: spin projection.[1]
- ψ: l=3/2, ψ_+: s=1/2, ψ_-: s=-1/2.

$$\psi_+ = \frac{1}{2}\gamma_-\gamma_+\psi\,, \quad \psi_- = \frac{1}{2}\gamma_+\gamma_-\psi \tag{5}$$

- Operator $\overline{\psi}\gamma_\mu\psi$:

$$\begin{aligned} twist-2: &\quad \overline{\psi}_+\gamma_+\psi_+ \\ twist-3: &\quad \overline{\psi}_+\gamma_\perp\psi_- + \overline{\psi}_-\gamma_\perp\psi_+ \\ twist-4: &\quad \overline{\psi}_-\gamma_-\psi_- \end{aligned} \tag{6}$$

- Matrix elements:

$$\langle 0|\psi(-z)\hat{O}\psi(z)|\pi(P)\rangle \tag{7}$$

  - $\hat{O}$: operator of twist $t = 2, 3, 4$.
- Twist-2 operator: $\overline{\psi}_+\hat{O}\psi_+$, and $\hat{O} \in \{\gamma_+, \gamma_+\gamma_5, \sigma_{+\perp}, \sigma_{+\perp}\gamma_5\}$.
- Twist-2 PDA:

$$\begin{aligned} &\langle 0|\psi(-z)\gamma_5\gamma_\mu\psi(z)|\pi(P)\rangle \\ =&f_\pi P_\mu \int_0^1 dx e^{-i(2x-1)z\cdot P}\phi(x) \end{aligned} \tag{8}$$

[1] V. Braun, G. Korchemsky and D. Mueller. The uses of conformal symmetry in QCD. Prog. Part. Nucl. Phys. 2003.

Leading twist PDAs

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- Matrix elements:

$$\langle 0|\psi(-z)\gamma_5\gamma.n\psi(z)|\pi(P)\rangle = f_\pi n\cdot P\int_0^1 dxe^{-i(2x-1)z\cdot P}\phi(x)\,,$$
$$= tr_{CD}Z_2\int_{dq}^{\Lambda} e^{-iz\cdot q-iz\cdot(q-P)}\gamma_5\gamma.n\chi(q;P)\,. \quad (9)$$

- Projecting Bethe-Salpeter wave function onto the light front:

$$f_\pi\phi(x) = tr_{CD}Z_2\int_{dq}^{\Lambda}\delta(n\cdot q_+ - xn\cdot P)\gamma_5\gamma\cdot n\chi(q;P)\,. \quad (10)$$

  - n, light-like four-vector, $n^2 = 0$.
  - f_π, decay constant.
  - $\chi(q;P)$, Bethe-Salpeter wave function, the solution of Bethe-Salpeter equation.

Partonic Structure of Neutral Pseudoscalars

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- The flavor-chiral symmetries:

$$SU(3)_R \otimes SU(3)_L \otimes U(1)_V \otimes U(1)_A\,,$$
$$SU(3)_R \otimes SU(3)_L \to SU(3)_V\,.$$

- π^0.
  - Nambu-Goldstone boson and bound-state.
  - Dynamical chiral symmetry breaking(DCSB).
- η, η'.
  - $U(1)_A$: non-Abelian axial anomaly.
  - Flavor mixing with strange quark.
- η_c, η_b.
  - Heavy quarkonium.

- Particles: π^0, η, η', η_c, η_b.
- Physical quantities:
  - Leading-twist Parton distribution amplitudes (PDAs).
- The science questions:
  - $u\bar{u} \to b\bar{b}$: a picture connects Goldstone mode with heavy-heavy systems.
  - η, η': flavor mixing states.

$$f_\pi \phi(x) = tr_{CD} Z_2 \int_{dq}^{\Lambda} \delta(n\cdot q_+ - x n\cdot P)\gamma_5 \gamma\cdot n \chi(q;P)\,. \quad (11)$$

Methods computing PDAs with DSEs

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- Moments: $\langle x^m \rangle = \int_0^1 dx x^m \phi(x)$
  - Perturbation theory integral representations (PTIRs)[1]:
    - ★ Infinite number of Mellin moments.
    - ★ Combine denominators $\Rightarrow$ the integral over feynman parameters.
    - ★ Represent the Bethe-Sapeter wave function with parameters.
  - "Brute-force" approach[2]:
    - ★ Limited number of Mellin moments.

- Spectral function: $\chi(q, P) = \int_{-1}^{1} dz \int_0^\infty d\gamma \frac{g(z,\gamma)}{(q^2 + zq\cdot P + \frac{1}{4}P^2 + M^2 + \gamma)^3}$
  - Maximum entropy method (MEM)[3]:
    - ★ Well-known method to solve the ill-posed inversion problem.
    - ★ Extract the weight function of Bethe-Salpeter wave function.

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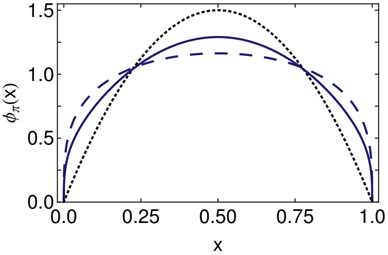
[1] L. Chang, I.C. Cloet, J.J. Cobos-Martinez, C.D. Roberts, S.M. Schmidt, and P.C. Tandy, Phys. Rev. Lett. 110, 132001 (2013),1301.0324.

[2] M. Ding, F. Gao, L. Chang, Y.X. Liu, and C.D. Roberts, Leading-twist parton distribution amplitudes of S-wave heavy-quarkonia, Phys.Lett. B753 (2016) 330-335.

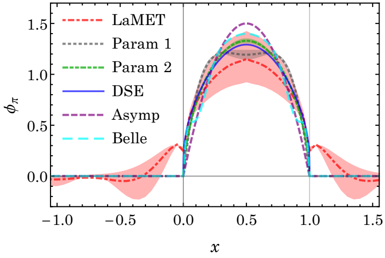
[3] F. Gao, L. Chang, and Y.X. Liu, A novel algorithm for extracting the parton distribution amplitude from the Euclidean Bethe-Salpeter wave function. arXiv:1611.03560.

Pion PDA

- Pion PDA from DSE[1,2]:



- Pion PDA from Lattice[2]:

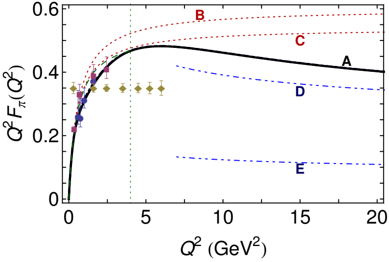


[1] L. Chang, I.C. Cloet, J.J. Cobos-Martinez, C.D. Roberts, S.M. Schmidt, and P.C. Tandy, Phys. Rev. Lett. 110, 132001 (2013).

[2] Jian-Hui Zhang, Jiunn-Wei Chen, Xiangdong Ji, Luchang Jin, and Huey-Wen Lin, arXiv:1702.00008v2.

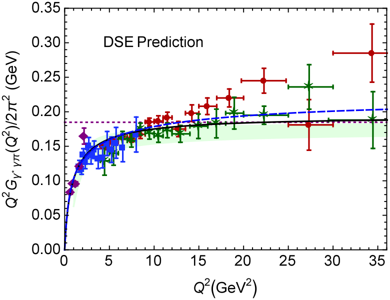
Pion EFF and TFF

- Pion elastic form factors[1]:



$$Q^2 F_\pi(Q^2) \overset{Q^2 > Q_0^2}{\approx} 16\pi f_\pi^2 \alpha_s(Q^2) \left(\frac{1}{3} \int_0^1 dx \frac{\phi_\pi(x; Q^2)}{x} \right)^2 \quad (12)$$

- Pion transition form factor[2]:



$$Q^2 G_\pi(Q^2) \overset{Q^2 > Q_0^2}{\approx} 4\pi^2 f_\pi \frac{1}{3} \int_0^1 dx \frac{\phi_\pi(x; Q^2)}{x} \quad (13)$$

[1] L. Chang, I.C. Cloët, C.D. Roberts, S.M. Schmidt, and P.C. Tandy, Phys. Rev. Lett. 111 (2013) no.14, 141802.

[2] K. Raya, L. Chang, A. Bashir, J.J. Cobos-Martinez, L.X. Gutiérrez-Guerrero, C.D. Roberts, and P.C. Tandy. Phys.Rev. D93 (2016) no.7, 074017.

PDAs of η_c and η_b

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- Moments of PDAs:

$$\langle x^m \rangle = \int_0^1 dx x^m \phi_{\eta_c/\eta_b}(x) = \frac{1}{f_{\eta_c/\eta_b}} tr_{CD} Z_2 \int_{dq}^{\Lambda} \frac{(n\cdot q_+)^m}{(n\cdot P)^{m+1}} \gamma_5 \gamma\cdot n \chi_{\eta_c/\eta_b}(q;P)\,. \quad (14)$$

  - "Brute-force" approach.
  - Calculate directly, limited number of moments.
  - A factor $1/(1+q^2r^2)^{\frac{m}{2}}$ is introduced for $\langle x^m \rangle$, and each moment is a function of r, with reliable results extrapolated to $r^2 = 0$.
  - Reconstruct the PDAs from their moments.

- Quarkonia properties
  - Current-quark masses were chosen in order to fit $m_{\eta_c} = 2.98$GeV, $m_{\eta_b} = 9.39$GeV.
  - Decay constants

	DSEs	*expt.*[1]	*lQCD*[2]	*CQM*[3]
f_{η_c}	0.262	0.238	0.279	0.841
f_{η_b}	0.543		0.472	0.728

[1] K. Olive et al. Particle data group collaboration. Chin. Phys. C, 2014.

[2] C.McNeile et al. Heavy meson masses and decay constants from relativistic heavy quarks in full lattice QCD. Physical Review D, 2012.

[3] J.Segovia et al.$J^{PC} = 1^{--}$ hidden charm resonances. Physical Review D, 2008.

PDAs of η_c and η_b

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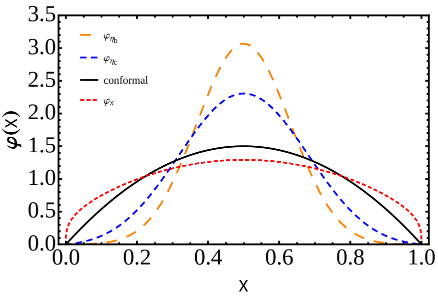
- $\phi_{\eta_c}(x), \phi_{\eta_b}(x)$[4]
  - Piecewise convex-concave-convex function.
  - Deviate noticeably from $\phi_{NRQCD}(x) = \delta(x - 1/2)$.
  - Ordering of PDAs peak heights and widths: ($<_N$ means narrower than) $\phi^{asy}(x) = 6x(1 - x)$.

$$\phi_{\eta_b} <_N \phi_{\eta_c} <_N \phi^{asy} <_N \phi_\pi$$

  - $\Lambda_{QCD}/m_q(\zeta) \to 0$, $\phi(x) \to \delta(x - 1/2)$.
  - Critical current quark mass $m_q^c(\zeta = 2GeV) = 0.15\text{GeV}$, $\phi(x) = \phi^{asy}(x)$.



4 M. Ding, F. Gao, L. Chang, Y.-X. Liu, and C. D. Roberts, Phys. Lett. B 753, 330 (2016).

PDAs evolution

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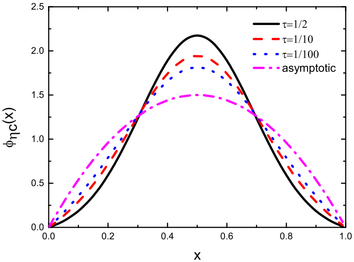
- PDA of η_c evolution with $\tau = 1/\zeta$



- PDA evolution with renormalisation scale: ERBL evolution[1,2]

$$\phi(x;\tau) = \phi^{asy}(x)\left[1 + \sum_{j=2,4,\ldots}^{\infty} a_j^{3/2}(\tau) C_j^{(3/2)}(2x-1)\right],$$

$$a_j^{3/2}(\tau) = a_j^{3/2}(\tau_2)\left[\frac{\alpha_s(\tau_2)}{\alpha_s(\tau)}\right]^{\gamma_j^{(0)}/\beta_0}. \quad (15)$$

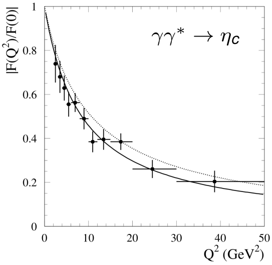
- $\Lambda_{QCD}/\zeta \to 0$, $\phi(x) \to \phi^{asy}(x)$.

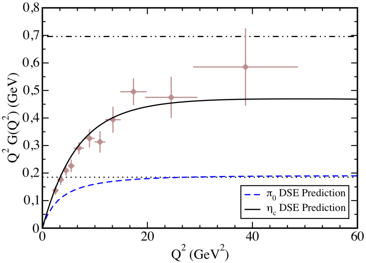
[1] G.P.Lepage and S.J.Brodsky. Exclusive processes in perturbative quantum chromodynamics. Physical Review D, 1980.

[2] A.Efremov and A.Radyushkin. Factorization and asymptotic behaviour of pion form factor in QCD. Physics Letters B, 1980.

TFF of η_c

- Transition form factor $\gamma\gamma^* \to \eta_c$ in BaBar[1].
- TFF from DSEs[2].





- Asymptotic behaviour can be analyzed by PDAs:

$$\lim_{Q^2 \to \infty} Q^2 G_{\eta_c}(Q^2) = 4\pi^2 \int_0^1 dx \, \frac{\frac{4}{9} f_{\eta_c} \phi_{\eta_c}(x)}{1 - x} \tag{16}$$

[1] BaBar Collaboration. Measurement of the $\gamma\gamma^* \to \eta_c$ transition form factor. Phys.Rev. D81 (2010) 052010.

[2] K. Raya, M. Ding, A. Bashir, L. Chang and C.D. Roberts. Partonic structure of neutral pseudoscalars via two photon transition form factors, Phys.Rev. D95 (2017) 074014.

Flavor mixing in η and η'

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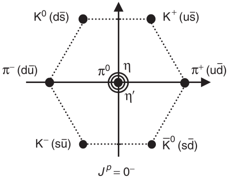
- Quark model[1]:
  - One of the octet: $|\eta\rangle = \frac{1}{\sqrt{6}}(|u\bar{u}\rangle + |d\bar{d}\rangle - 2|s\bar{s}\rangle)$
  - Singlet: $|\eta'\rangle = \frac{1}{\sqrt{3}}(|u\bar{u}\rangle + |d\bar{d}\rangle + |s\bar{s}\rangle)$



- SU(3) flavour symmetry is broken: $m_s > m_{u/d}$.
- Physical $\eta - \eta'$ states are mixtures of the octet and singlet states.
- $\eta - \eta'$ mixing scheme with quark flavor basis[2].

$$\begin{pmatrix} |\eta\rangle \\ |\eta'\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}}(|u\bar{u}\rangle + |d\bar{d}\rangle) \\ |s\bar{s}\rangle \end{pmatrix}, \tag{17}$$

- Ideal mixing: $\theta = 0$, $|\eta\rangle = \frac{1}{\sqrt{2}}(|u\bar{u}\rangle + |d\bar{d}\rangle)$, $|\eta'\rangle = |s\bar{s}\rangle$.

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[1] M. Thomson. Modern particle physics. Cambridge University Press, 2013.

[2] T. Feldmann, P. Kroll, and B. Stech, Phys. Rev. D58,114006 (1998), Phys. Lett. B449, 339 (1999).

PDAs of η and η'

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- Projecting Bethe-Salpeter wave function onto to the light front: ϕ_η^q, ϕ_η^s, $\phi_{\eta'}^q$, $\phi_{\eta'}^s$.

$$f^{q,s}_{\eta,\eta'}\phi^{q,s}_{\eta,\eta'}(x) = Z_2 tr\int_q^\Lambda \delta(n\cdot q_+ - xn\cdot P)\gamma_5\gamma\cdot n\chi^{q,s}_{\eta,\eta'}(q,P) \quad (18)$$

- Bethe-Salpeter wave function in the quark flavor basis

$$\chi_{\eta,\eta'}(k,P) = \mathbb{F}^q\chi^q_{\eta,\eta'}(k,P) + \mathbb{F}^s\chi^s_{\eta,\eta'}(k,P)\,, \quad (19)$$

- q: $u-$ $d-$ quark, and s the strange quark.

$$\mathbb{F}^q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}\,, \quad \mathbb{F}^s = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sqrt{2} \end{pmatrix}\,. \quad (20)$$

- S: dressed-quark propagator; Γ: Bethe-Salpter amplitude.

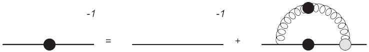
$$\chi^{q,s}_{\eta,\eta'}(k,P) = S^{q,s}(k_+)\Gamma^{q,s}_{\eta,\eta'}(k,P)S^{q,s}(k_-)\,, \quad (21)$$

Bethe-Salpeter wavefucntion

- *S*: dressed-quark propagator.

$$S^{q,s}(k) = Z^{q,s}(k^2,\zeta)/[i\gamma\cdot k + M^{q,s}(k^2,\zeta)]\,, \tag{22}$$

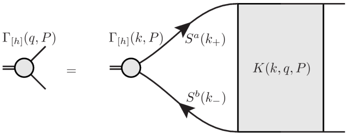
$$S^{-1}(k) = Z_2(i\gamma\cdot k + m^{bm}) + Z_2^2\int_q^\Lambda g^2 D_{\mu\nu}(k-q)\frac{\lambda^a}{2}S(q)\Gamma^a_\nu(q,k)\,. \tag{23}$$



- Γ: Bethe-Salpeter amplitude.

$$\Gamma_{\eta,\eta'}(k,P) = \mathbb{F}^q\Gamma^q_{\eta,\eta'}(k,P) + \mathbb{F}^s\Gamma^s_{\eta,\eta'}(k,P)\,, \tag{24}$$

$$[\Gamma_{\eta,\eta'}(k,P)]_{tu} = Z_2^2\int_q^\Lambda [S(q_+)\Gamma_{\eta,\eta'}(q,P)S(q_-)]_{sr}K^{rs}_{tu}(q,k,P)\,. \tag{25}$$



Anomaly Bethe-Salpeter kernel

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- Axial-vector Ward-Takahashi Identity:

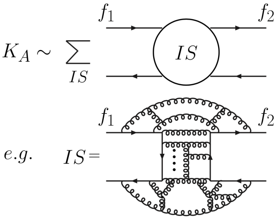
$$P_\mu \Gamma^a_{5\mu}(k;P) = S^{-1}(k_+) i\gamma_5 \mathbb{F}^a + i\gamma_5 \mathbb{F}^a S^{-1}(k_-) - 2iM^{ab}\Gamma^b_5(k;P) - \mathbb{A}^a(k;P)\,, \qquad (26)$$

- Introducing non-Abelian anomaly in BS kernel[1]

$$K^{rs}_{tu}(q,k,P) = (K_L)^{rs}_{tu}(q,k,P) + (K_A)^{rs}_{tu}(q,k,P)\,,$$

$$(K_L)^{rs}_{tu}(q,k,P) = -D_{\mu\nu}(k-q)[\gamma_\mu \frac{\lambda^a}{2}]_{ts}[\gamma_\nu \frac{\lambda^a}{2}]_{ru}\,,$$

$$(K_A)^{rs}_{tu}(q,k,P) = -\xi(k-q)\{\cos\theta^2_\xi[\zeta\gamma_5]_{rs}[\zeta\gamma_5]_{tu} - \sin\theta^2_\xi \frac{1}{M_u^2}[\zeta\gamma\cdot P\gamma_5]_{rs}[\zeta\gamma\cdot P\gamma_5]_{tu}\} \qquad (27)$$

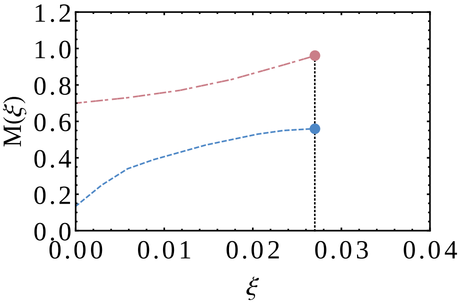


[1] M. S. Bhagwat, L. Chang, Y.-X. Liu, C. D. Roberts, and P. C. Tandy, Flavour symmetry breaking and meson masses, Phys. Rev. C76, 045203 (2007), 0708.1118.

Anomaly strength and η-η' masses

- The anomaly strength $\xi = 0$,
  $m_\eta(\xi = 0) = m_\pi = 0.134\text{GeV}$,
  $m_{\eta'}(\xi = 0) = m_{s\bar{s}} = 0.70\text{GeV}$.
- The anomaly strength ξ increase,
  $m_\eta(\xi)$ and $m_{\eta'}(\xi)$ increase.
- The anomaly strength $\xi = \xi_c$,
  $m_\eta(\xi = \xi_c) = m_\eta$,
  $m_{\eta'}(\xi = \xi_c) = m_{\eta'}$.

- $m_\eta(\xi)$ and $m_{\eta'}(\xi)$ with the dependence of the non-Abelian anomaly strength ξ.
  Curves: *dashed*, $m_\eta(\xi)$; *dash-dotted*, $m_{\eta'}(\xi)$.



η and η' masses and decay constants

- Meson mass:
  - $m_\eta = 0.56\text{GeV}$ and $m_{\eta'} = 0.96\text{GeV}$, in experiment 0.55GeV, 0.96GeV, respectively.
- Decay constants:

	f_η^q	f_η^s	$f_{\eta'}^q$	$f_{\eta'}^s$
herein-direct	0.072	-0.092	0.070	0.104
herein-fit	0.074	-0.094	0.068	0.101
phen.[1 2 3]	0.090(13)	-0.093(28)	0.073(14)	0.094(8)

$$\begin{pmatrix} f_\eta^q & f_\eta^s \\ f_{\eta'}^q & f_{\eta'}^s \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} f^q \\ f^s \end{pmatrix} , \qquad (28)$$

$$\begin{aligned} \theta &= 42.8^\circ . \\ f^q &= 0.101\,GeV = 1.08 f_\pi \\ f^s &= 0.138\,GeV = 1.49 f_\pi . \end{aligned} \qquad (29)$$

- octet and singlet basis herein:
  $\theta_8 = -20^\circ,\ \theta_0 = -2.9^\circ,\ f^8 = 1.38 f_\pi,\ f^0 = 1.25 f_\pi$.
  Compared with phenomenological analyses[1 2 3]:
  $\theta_8 = -18(6)^\circ,\ \theta_0 = -6(6)^\circ,\ f^8 = 1.34(8) f_\pi,\ f^0 = 1.25(10) f_\pi$.

[1] T. Feldmann, P. Kroll, and B. Stech, Phys. Lett. B 449, 339 (1999).

[2] M. Benayoun, L. DelBuono and H. B. O'Connell, Eur. Phys. J. C 17, 593 (2000).

[3] F. De Fazio and M. R. Pennington, JHEP 07, 051 (2000).

PTIRs

- Moments of PDAs:

$$\langle x^m \rangle = \int_0^1 dx x^m \phi_{\eta,\eta'}^{q,s}(x) = \frac{1}{f_{\eta,\eta'}^{q,s}} tr_{CD} Z_2 \int_{dq}^{\Lambda} \frac{(n\cdot q_+)^m}{(n\cdot P)^{m+1}} \gamma_5 \gamma\cdot n \chi_{\eta,\eta'}^{q,s}(q;P)\,. \tag{30}$$

  - Perturbation theory integral representations (PTIRs)[1].
  - Represent the Bethe-Sapeter wave function with parameters.
  - Infinite number of Mellin moments.
  - Reconstruct the PDAs from their moments.

$$S(k) = \Sigma_{j=1}^{jm}\left[\frac{z_j}{i\gamma\cdot k + m_j} + \frac{z_j^*}{i\gamma\cdot k + m_j^*}\right] \tag{31}$$

$$\mathcal{F}(k;P) = \mathcal{F}^i(k;P) + \mathcal{F}^u(k;P)\,,$$

$$\mathcal{F}^i(k,P) = c_{\mathcal{F}}^i \int_{-1}^1 dz \rho_{\nu_{\mathcal{F}}^i}(z)[a_{\mathcal{F}} \hat{\Delta}_{\Lambda_{\mathcal{F}}^i}^4(k_z^2)\,,$$

$$+ a_{\mathcal{F}}^- \hat{\Delta}_{\Lambda_{\mathcal{F}}^i}^5(k_z^2)]\,,$$

$$\mathcal{F}^u(k;P) = c_{\mathcal{F}}^u \int_{-1}^1 dz \rho_{\nu_{\mathcal{F}}^u}(z) \hat{\Delta}_{\Lambda_{\mathcal{F}}^u}^{l_{\mathcal{F}}^u}(k_z^2)\,. \tag{32}$$

	$z1$	$m1$	$z2$	$m2$
q	(0.37,0.32)	(0.52,0.29)	(0.12,0.11)	(-1.31,-0.90)
s	(0.41,0.32)	(0.74,0.39)	(0.12,0.10)	(-1.57,-0.95)

	c^i	c^u	ν^i	ν^u	a	Λ^i	Λ^u	l^u
E_η^q	0.94	0.06	-0.60	1.0	2.75	1.35	1.0	1.1
F_η^q	0.61	0.006	3.6	1.0	2.75	1.07	1.0	1.1
G_η^q	0.36	0.03	0.10	1.0	2.75	1.10	1.0	2.2
E_η^s	-2.12	-0.12	-0.40	1.0	2.75	1.35	1.0	1.1
F_η^s	-0.80	-0.01	1.20	1.0	2.75	1.18	1.0	1.1
G_η^s	-0.22	-0.04	0.10	1.0	2.75	1.30	1.0	2.2
$E_{\eta'}^q$	0.93	0.07	-0.40	1.0	2.75	1.30	1.0	1.1
$F_{\eta'}^q$	0.64	0.007	0.40	1.0	2.75	1.12	1.0	1.1
$E_{\eta'}^s$	1.94	0.19	-0.22	1.0	2.75	1.53	1.0	1.1
$F_{\eta'}^s$	0.86	0.02	1.60	1.0	2.75	1.30	1.0	1.1

[1] N. Nakanishi. Perturbation-Theoretical Integral Representation and the High-Energy Behavior of the Scattering Amplitude II*. Physical Review, 133, 5B, 1964.

PDAs of η and η'

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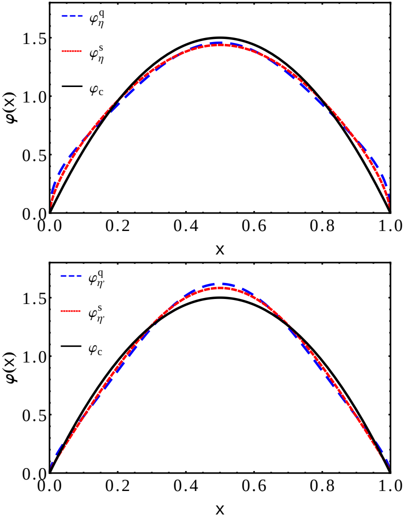
- ϕ_η^q, ϕ_η^s, and $\phi_{\eta'}^q$, $\phi_{\eta'}^s$
- Ordering of PDAs peak heights and widths: ($<_N$ means narrower than) $\phi_c(x) = 6x(1-x)$.

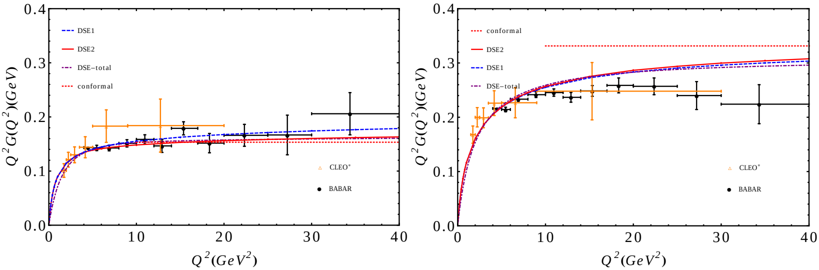
$$\phi_{\eta'}^q, \phi_{\eta'}^s <_N \phi_c <_N \phi_\eta^q, \phi_\eta^s \qquad (33)$$

- Mass effect dominate.
- $\phi_{\eta,\eta'}^q$ behaves almost equivalently with $\phi_{\eta,\eta'}^s$.



TFFs of η and η'

- Transition form factor $\gamma\gamma^* \to \eta, \eta'$[1,2,3].



- Asymptotic behaviour can be analyzed by PDAs:

$$\lim_{Q^2\to\infty} Q^2 G_{\eta,\eta'}(Q^2) = 4\pi^2 \int_0^1 dx \frac{\frac{5}{9} f^q_{\eta,\eta'} \phi^q_{\eta,\eta'}(x) + \frac{\sqrt{2}}{9} f^s_{\eta,\eta'} \phi^s_{\eta,\eta'}(x)}{1-x} \tag{34}$$

[1] CELLO Collaboration, A Measurement of the π^0, η and η' electromagnetic form factors, Z.Phys. C49 (1991) 401-410.

[2] CLEO Collaboration, Measurements of the meson photon transition form factors of light pseudoscalar mesons at large momentum transfer, Phys.Rev. D57 (1998) 33-54.

[3] BaBar Collaboration. Measurement of the $\gamma\gamma^* \to \eta$ and $\gamma\gamma^* \to \eta'$ transition form factors. Phys.Rev.D84 (2011) 052001.

Summary and outlook

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- Summary:
  - Leading twist PDA of π.
  - Leading twist PDAs of η_c and η_b:
    - ★ Piecewise convex-concave-convex function.
    - ★ Deviate noticeably from $\delta(x - 1/2)$.
    - ★ Narrower than conformal distribution $6x(1 - x)$.
    - ★ Predict $\gamma\gamma^* \to \eta_c$ transition form factor G_{η_c} with large Q^2.
  - Leading twist PDAs of η and η':
    - ★ PDAs of η are broader than conformal distribution $6x(1 - x)$.
    - ★ PDAs of η' are narrower than conformal distribution $6x(1 - x)$.
    - ★ Predict $\gamma\gamma^* \to \eta, \eta'$ transition form factor $G_{\eta,\eta'}$ with large Q^2.
- Outlook:
  - PDFs, GPDs, TMDs, etc.
- Thanks!