Nucleon matrix elements from lattice QCD

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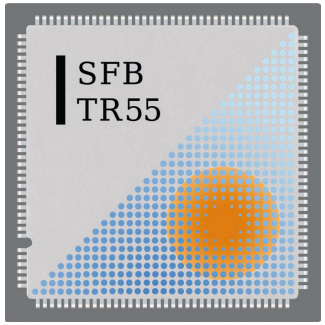
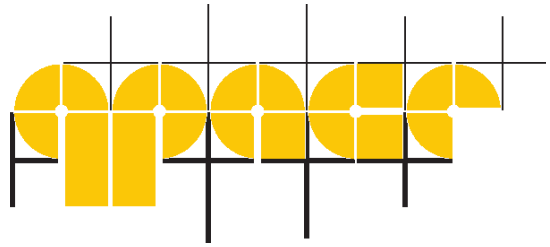
RQCD: G S Bali, S Bürger, M Göckeler, S Piemonte, R Rödl, S Schäfer, J Simeth, W Söldner, A Sternbeck, S Weishäupl, T Wurm, ...





"Mapping Parton Distribution Amplitudes and Functions", ECT*, Trento, 10.09.2018

Outline

Orientation, general considerations

Helicity

- Isovector: $g_A = \Delta u + \Delta\overline{u} - \Delta d - \Delta\overline{d} = \langle 1 \rangle_{\Delta u^+ - \Delta d^+}$
- Flavour separated: $\langle 1 \rangle_{\Delta q^+}$
- Isovector: $\langle x \rangle_{\Delta u^- - \Delta d^-}$

Transversity

- Isovector: $g_T = \delta u - \delta\overline{u} - (\delta d - \delta\overline{d}) = \langle 1 \rangle_{\delta u^- - \delta d^-}$
- Flavour separated: $\langle 1 \rangle_{\delta q^-}$
- Isovector: $\langle x \rangle_{\delta u^+ - \delta d^+}$

Unpolarised

- Isovector: $\langle x \rangle_{u^+ - d^+}$

Discussion of systematics for the example of g_A.

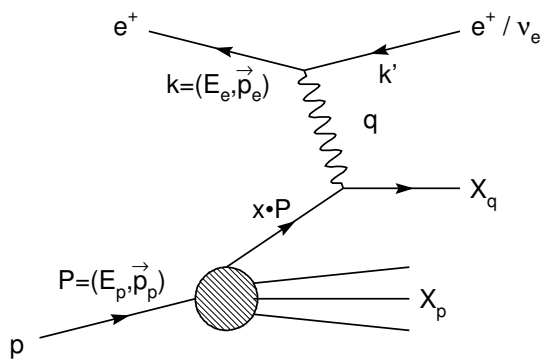
Constraining PDF fits with lattice moments

SU(3) flavour symmetry

Summary/Outlook.

Orientation

Compute lattice matrix elements of local operators $\langle N|\mathcal{O}|N\rangle$ in the isospin limit.



DIS cross-section

$$\sigma \sim L^{\mu\nu}(k,q)W_{\mu\nu}(p,q)$$

Hadronic tensor

$$W^{\gamma}_{\mu\nu}(p,q) = \frac{1}{4\pi}\int \mathrm{d}^4 z\, e^{-iqz}\langle N(p)|j_\mu(z)j^\dagger_\nu(0)|N(p)\rangle$$

Unpolarised ($x = Q^2/(2pq)$, $Q^2 = -q^2$)

$$W_{\gamma,\mu\nu}(p,q) = \left(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu}\right)F_1(x,Q) + \left(p^\mu - \frac{pq}{q^2}q^\mu\right)\left(p^\nu - \frac{pq}{q^2}q^\nu\right)\frac{F_2(x,Q)}{pq}$$

Polarised

$$W_{\gamma,\mu\nu}(p,q) = i\epsilon^{\mu\nu\rho\sigma}q_\rho\left(\frac{s_\sigma}{pq}\left(g_1(x,Q)+g_2(x,Q)\right) - \frac{q\cdot p_\sigma}{(pq)^2}g_2(x,Q)\right)$$

OPE: Wilson coefficients $c_{1,n}$, forward matrix elements $\langle x^n\rangle_q$

$$2\int dx\, x^{n-1}F_1(x,Q) = \sum_{q=u,d,s} c^{(q)}_{1,n}(\mu^2/Q^2,\alpha_S(\mu))\langle x^n\rangle_q$$

Similarly, moments of $F_2 \to \langle x^n\rangle_q$, $g_{1,2} \to \langle x^n\rangle_{\Delta q}, d_n$

Unpolarised:

$$\langle x\rangle_{q^+}(\mu) = \sum_{q=u,d,s,g} Z_q(\mu)\langle x\rangle^{latt}_{q^+}, \qquad \langle x\rangle^{latt}_{q^+} \propto \langle N|\bar{q}[\gamma_4 \overleftrightarrow{D}_4 - \frac{1}{3}\boldsymbol{\gamma}\cdot\overleftrightarrow{\mathbf{D}}]q|N\rangle^{latt}$$

$$\langle x\rangle^{latt}_{g} \propto \langle N|\mathrm{Tr}(F_{4\sigma}F_{4\sigma}) - \frac{1}{3}\mathrm{Tr}(F_{j\sigma}F_{j\sigma})|N\rangle^{latt}$$

Polarised:

$$\langle \mathbb{1}\rangle_{\Delta q^+}(\mu) = \sum_{q=u,d,s} Z'_q(\mu)\langle \mathbb{1}\rangle^{latt}_{\Delta q^+}, \qquad \langle \mathbb{1}\rangle^{latt}_{\Delta q^+} \propto \langle N,s|\bar{q}\gamma_i\gamma_5 q|N,s\rangle^{latt}$$

$$\langle x\rangle_{\Delta q^-}(\mu) = \sum_{q=u,d,s} Z''_q(\mu)\langle x\rangle^{latt}_{\Delta q^-}, \qquad \langle x\rangle^{latt}_{\Delta q^-} \propto \langle N,s|\bar{q}\gamma_5\gamma_{\{i}\overleftrightarrow{D}_{j\}}\, q|N,s\rangle^{latt}$$

Structure functions related to PDFs q_i: $F(x,Q) = \sum_i C_i\left[x,\alpha_s(\mu_F)\right] \otimes q_i\left(x,\mu_F\right) + \cdots$

Access Mellin moments:

$$\langle x^{n-1}(\mu)\rangle_q = \int_0^1 \mathrm{d}x\, x^{n-1}\left[q(x,\mu) + (-)^n\bar{q}(u,\mu)\right]$$

$$\langle x^{n-1}(\mu)\rangle_{\Delta q} = \int_0^1 \mathrm{d}x\, x^{n-1}\left[\Delta q(x,\mu) - (-)^n\Delta\bar{q}(x,\mu)\right]$$

Also transversity: $\langle \mathbb{1}\rangle_{\delta q^-}(\mu), \langle x\rangle_{\delta q^+}(\mu) \to$ moments of $\delta q \pm \delta\bar{q}$.

At present $n = 1, 2$ calculated due to problems with:

Statistics:

Higher n require matrix elements of operators with more derivatives.

Signal/noise deteriorates.

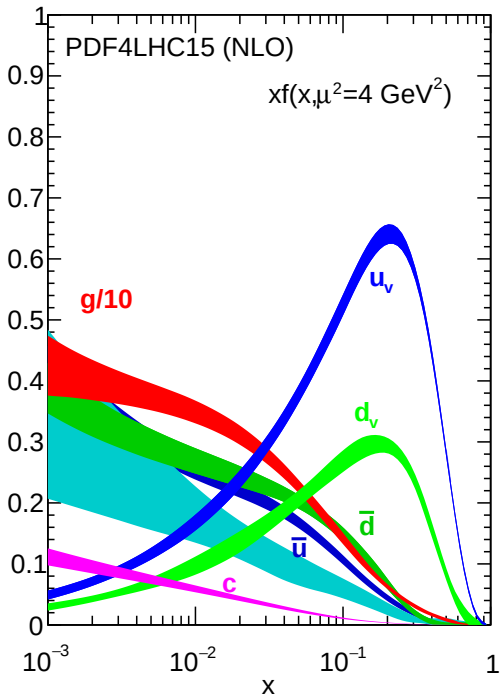
Renormalisation:

Mixing of operators with lower dimension ($n' = n - \ell$) under renormalisation is possible due to reduced lattice symmetry.

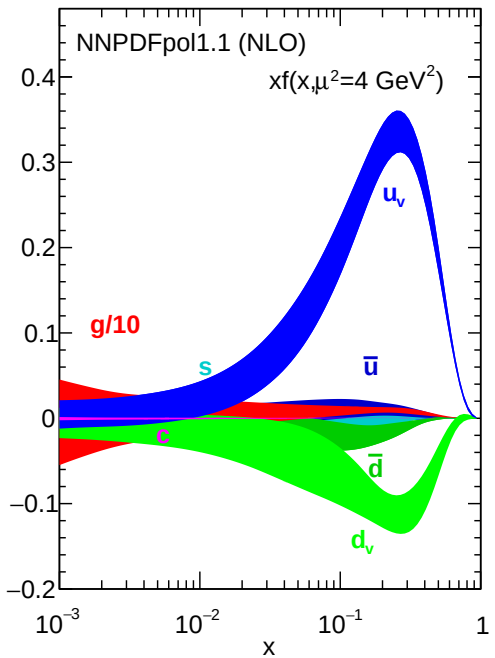
This leads to power divergences $a^{-\ell}$. Cannot be avoided for $n > 3$.

Global PDF fits

Unpolarised

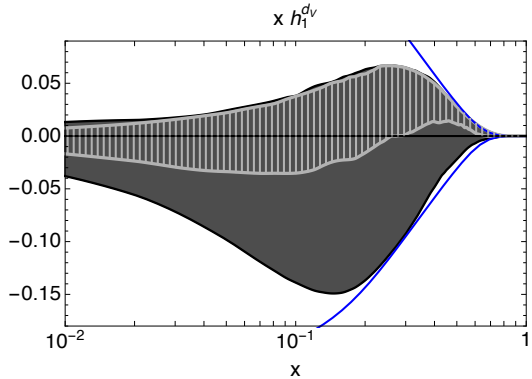


Polarised



Transversity

[Radici,1802.05212]



PDFs, q_i, extracted: data + functional form + DGLAP.

Choose: data, functional form, range for fit, additional experimental input (e.g. g_A), assumptions (SU(3) symmetry), ...

General considerations: $\langle N|\bar{q}\Gamma q|N\rangle$

(Isospin symmetric limit) Isovector combinations only connected. Isoscalar also disconnected.

Systematics:

- Excited state pollution.
- Renormalisation+ improvement: for $\vec{p} = \vec{p}' = 0$/some operators/actions $c_{\mathcal{O}} = 0$ or $b_{\mathcal{O}} = 0$
  $\mathcal{O}^{\overline{\text{MS}}}(\mu) = Z^{\overline{\text{MS}},latt}(a\mu)\left[(1 + b_{\mathcal{O}} am_q)\mathcal{O}^{latt} + ac_{\mathcal{O}}\mathcal{O}_1^{latt}\right]$
- Volume: exponentially suppressed $\sim m_\pi^2 e^{-Lm_\pi}/(m_\pi L)^{3/2}$, $Lm_\pi > 4$.
- Discretisation effects: $\mathcal{O}(a)$ or $\mathcal{O}(a^2)$.
- Physical point extrapolation: chiral pert. (inspired) $m_\pi \to m_\pi^{phys}$.

Status from the meson sector: FLAG 2016 ($N_f = 2 + 1$)

- $K \to \pi \ell \nu$, form factor at $q^2 = 0$, $f_+(0) = 0.9677(27)$
- $f_K / f_\pi = 1.192(5)$
- Bag parameter $B_K^{\overline{\mathrm{MS}}}(2\ GeV) = 0.5570(71)$
- ...

Difficulties in the baryon sector:

- Signal vs noise decays with $e^{-(E - 3m_\pi/2)(t_f - t_i)}$.
- Excited state pollution, $E_n = (N)^+, (N\pi)^-, (N\pi\pi)^+, \ldots$

$$\langle \mathcal{N}(t_f) J(t) \overline{\mathcal{N}}(t_i) \rangle = \sum_{n,m} \langle 0|\mathcal{N}|N_n\rangle \langle N_n|J|N_m\rangle \langle N_m|\overline{\mathcal{N}}|0\rangle e^{-E_m(t - t_i)} e^{-E_n(t_f - t)}$$

  Ground state dominance in the limit of large $t_f - t_i$ and $t_f - t$.
- Volume, exponentially suppressed $\sim m_\pi^2 e^{-L m_\pi} / (m_\pi L)^{3/2}$.
- Physical point extrapolation: range of convergence of baryon chiral PT not known.

(Discretisation effects)

Helicity

Isovector: $g_A = \Delta u + \Delta \overline{u} - \Delta d - \Delta \overline{d} = \langle 1 \rangle_{\Delta u^+ - \Delta d^+}$
Flavour separated: $\langle 1 \rangle_{\Delta q^+}$
Isovector: $\langle x \rangle_{\Delta u^- - \Delta d^-}$

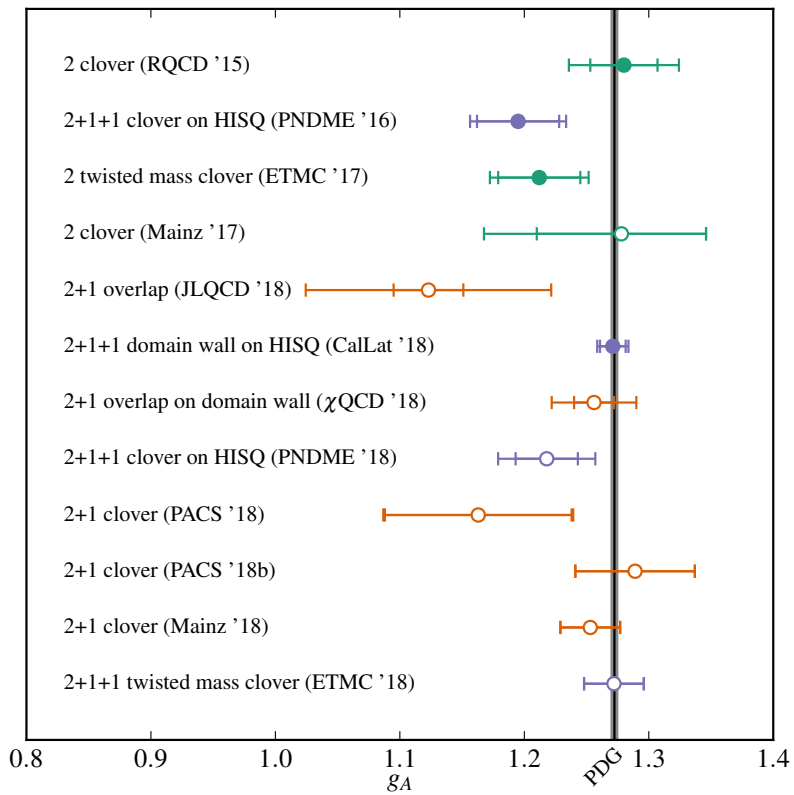
Isovector (axial charge): $g_A = \langle 1 \rangle_{\Delta u^+ - \Delta d^+}$

PDG 2018: $\mathbf{g_A}/\mathbf{g_V} = 1.2724(23)$ (assuming SM) from ultra-cold neutron β decay experiments. $g_V = 1$ in the isospin limit.

Benchmark quantity: demonstration of lattice techniques, known to be sensitive to:

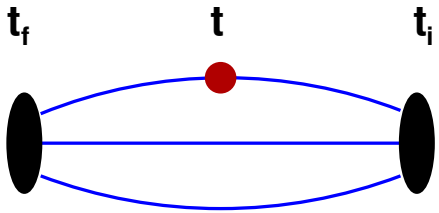
- excited state contamination,
- spatial volume,
- quark mass, ...

Left: J. Green plenary talk at Lattice 2018

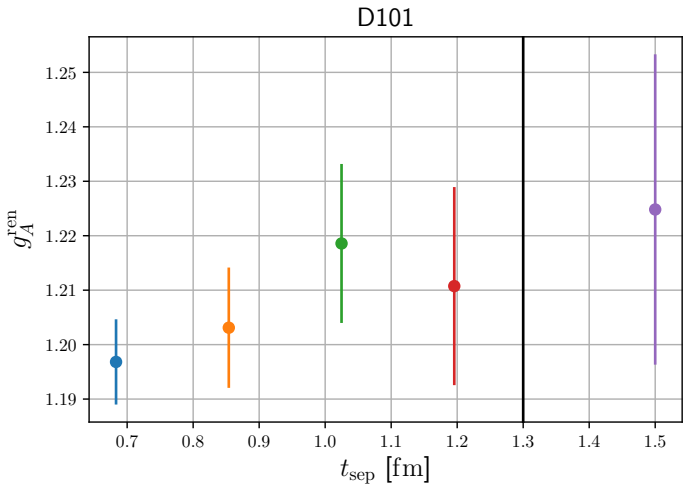


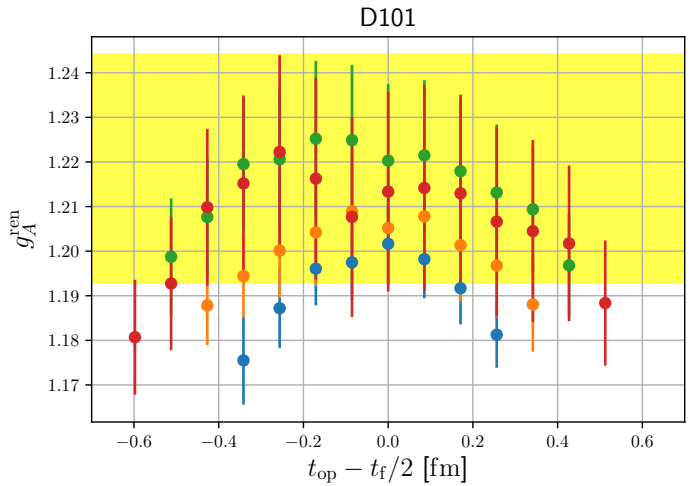
Excited states

Ground state dominance in the limit of large $t_f - t_i$ and $t_f - t$.



Generate 3pt functions at several $t_{sep} = t_f - t_i$, typically, 3 to 5 t_{seps}. More measurements per configuration for larger $t_f - t_i$ (signal to noise).

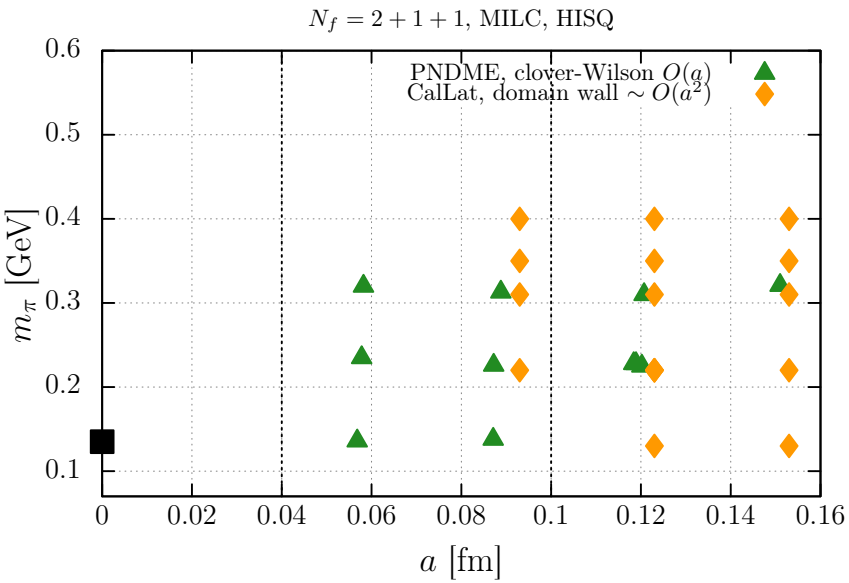


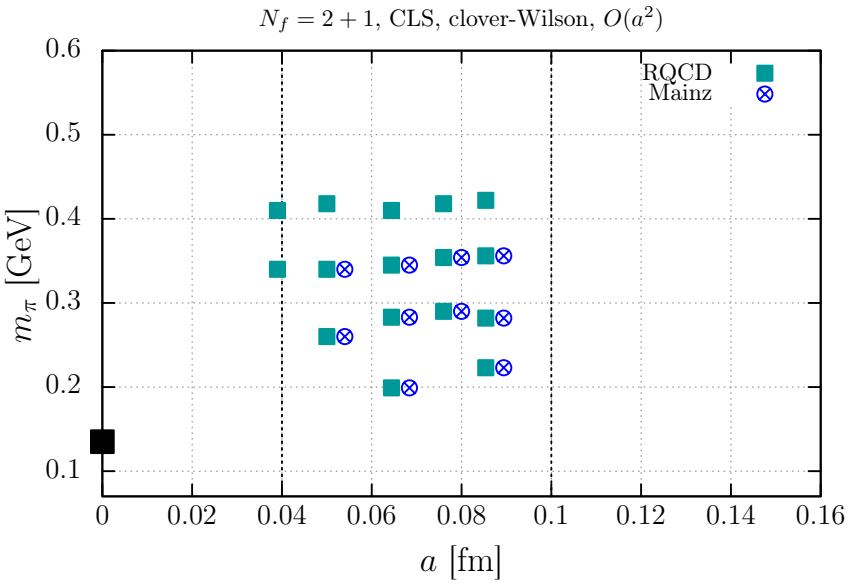


RQCD: T. Wurm, $m_\pi = 220$ MeV, $a = 0.086$ fm.

Renormalise lattice matrix elements to $\overline{\rm MS}$ using Rome-Southampton non-perturbative technique [Martinelli,hep-lat/9411010].

Landscape of ensembles

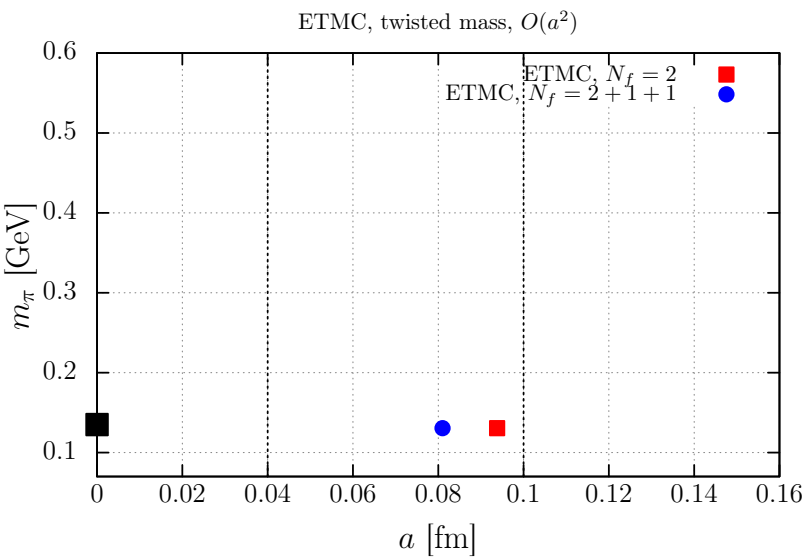




Leading discretisation effects depend on the quantity.

Multiple volumes at fixed β and m_π are also generated.

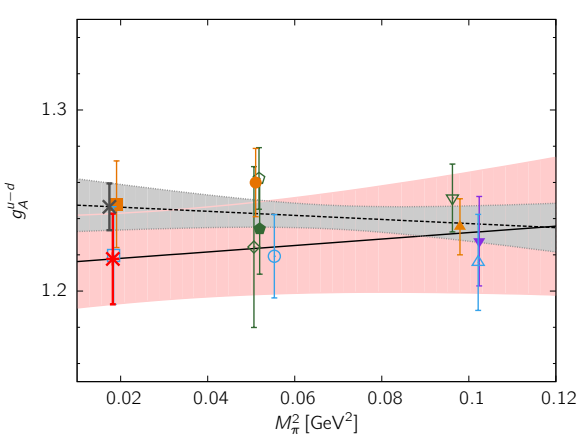
Also,
$N_f = 2+1$, JLQCD, LHPC, PACS, QCDSF, RBC/UKQCD.
$N_f = 2$, Mainz, RQCD, QCDSF, ...

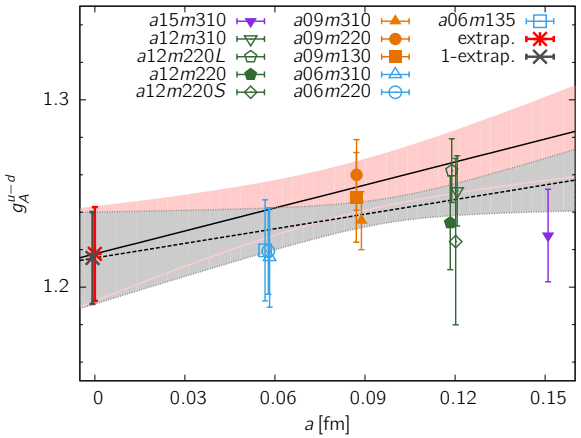


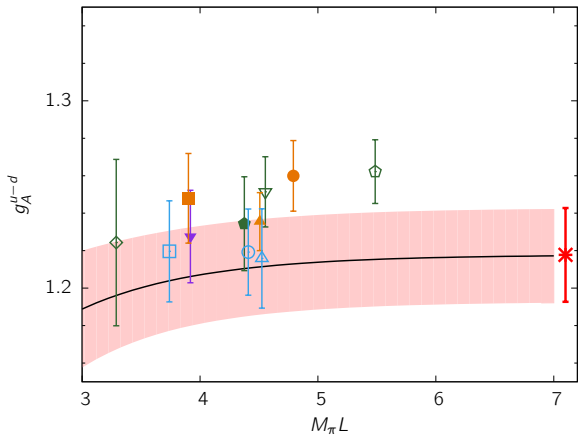
Chiral, continuum and volume extrapolation

[PNDME,1806.09006] $N_f = 2+1+1$, clover on HISQ, $O(a)$. $g_A = 1.218(25)(30)$

$$g_A = c_1 + c_2 a + c_3 M_\pi^2 + c_4 M_\pi^2 e^{-M_\pi L} \qquad \text{Also considered:} \qquad c_3^{\log} M_\pi^2 \ln(M_\pi/M_\rho)^2$$

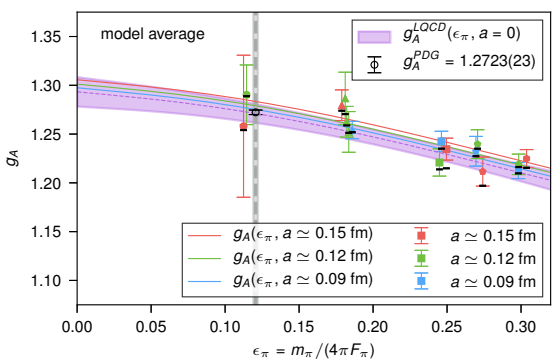


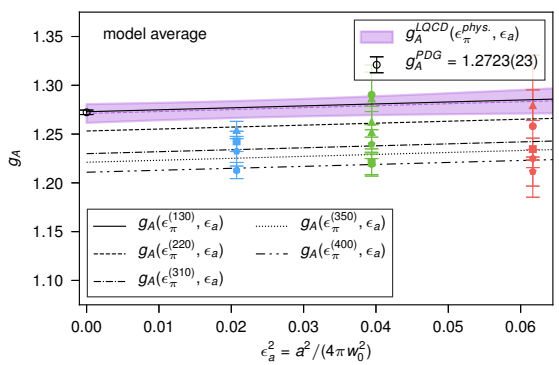


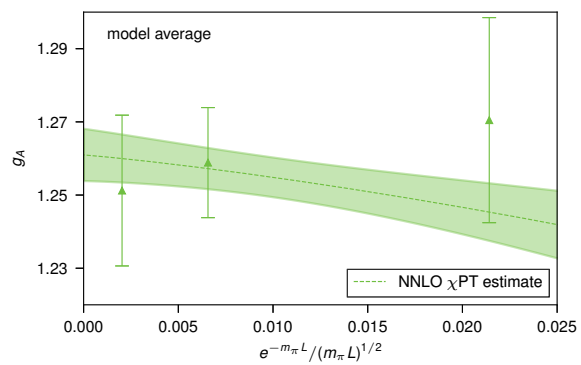


[CalLat,1805.12130] $N_f = 2+1+1$, domain wall on HISQ $\sim O(a^2)$. $g_A = 1.271(13)$

Considered: NNLO ChPT, Taylor expansion up to M_π^4, disc. $O(a^2)$, residual $O(a)$, finite volume.



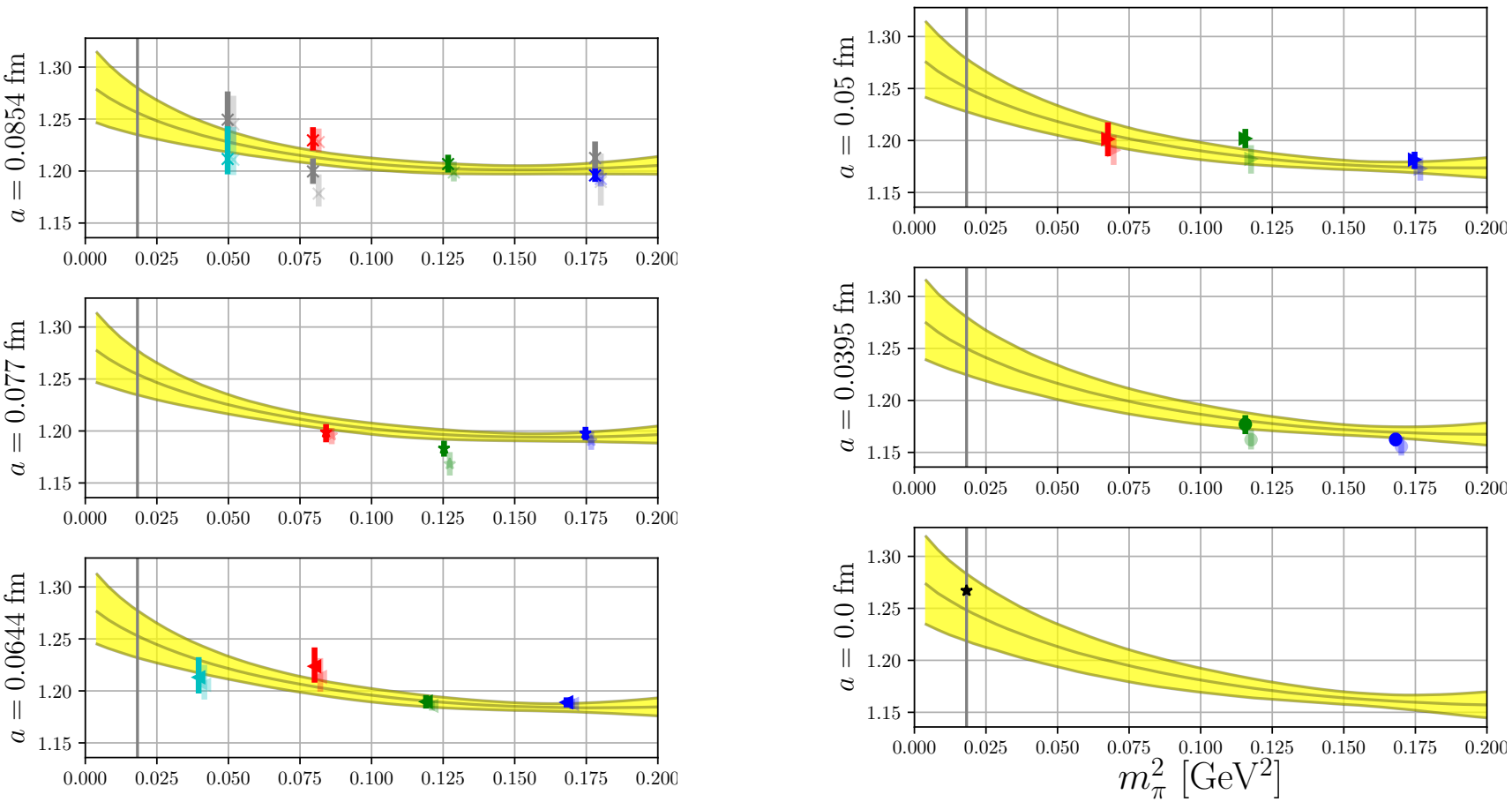




Chiral, continuum and volume extrapolation

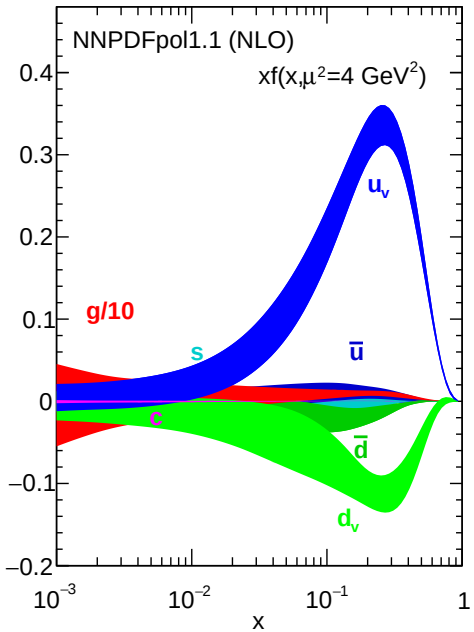
Mainz preliminary Lattice 2018 talk by K. Ottnad $(g_A)^{ABDE} = 1.253(24)$ (single fit with $M_\pi = 200 - 350$ MeV). $g_A = A + BM_\pi^2 + CM_\pi^2 \ln(M_\pi) + Da^2 + EM_\pi^2 e^{-M_\pi L}$

RQCD very preliminary (single fit with $M_\pi = 200 - 410$ MeV)



Flavour separated $\langle 1 \rangle_{\Delta q^+}$

[PDFLattice,1711.07916]



PDF fits to polarised DIS and SIDIS data.

$$\langle 1 \rangle_{\Delta q} = \int_0^1 dx \, \left(\Delta q(x, \mu^2) - \Delta \overline{q}(x, \mu^2) \right)$$

$$\langle x \rangle_{\Delta q} = \int_0^1 dx \, x \left(\Delta q(x, \mu^2) - \Delta \overline{q}(x, \mu^2) \right)$$

$\mu^2 = 4$ GeV^2	NNPDFpol1.1	DSSV08	JAM15	JAM17
$\langle 1 \rangle_{\Delta u^+ - \Delta d^+}$	1.250(16)	1.260(18)	1.314(6)	1.240(41)
$\langle 1 \rangle_{\Delta u^+}$	0.794(46)	0.814(12)	0.831(21)	0.812(22)
$\langle 1 \rangle_{\Delta d^+}$	−0.453(52)	−0.456(11)	−0.476(22)	−0.428(31)
$\langle 1 \rangle_{\Delta s^+}$	−0.120(81)	−0.112(23)	−0.109(20)	−0.038(96)
$\langle x \rangle_{\Delta u^- - \Delta d^-}$	0.195(14)	0.203(9)	—	0.241(26)

(Ji) spin sum rule: $\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_\psi + J_g$, where $\Delta\Sigma = \Delta u + \Delta d + \Delta s$

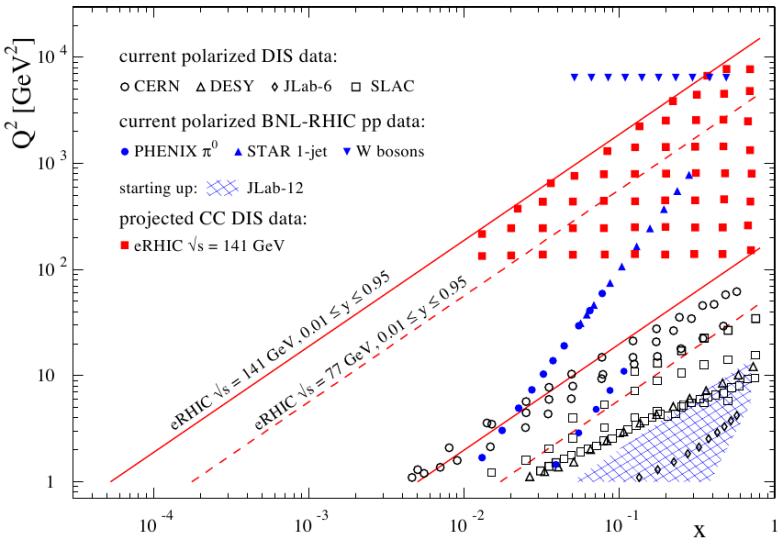
Flavour separated $\langle 1 \rangle_{\Delta q^+}$

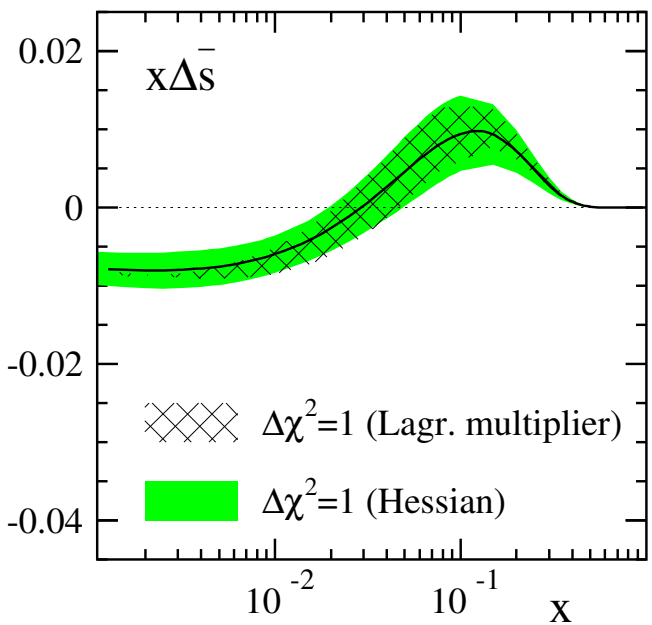
Minimal x in expt. around 10^{-2}.

c.f. unpolarised case:

$x_{min} < 10^{-4}$.

Right: [Aschenauer,1409.1633]





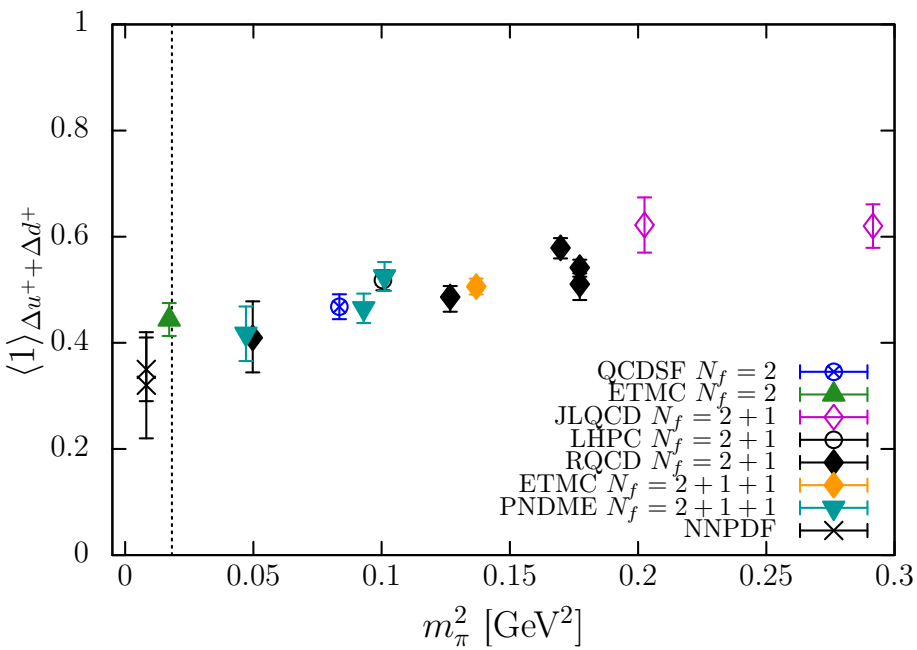
[DSSV,0904.3821] $Q^2 = 10$ GeV^2

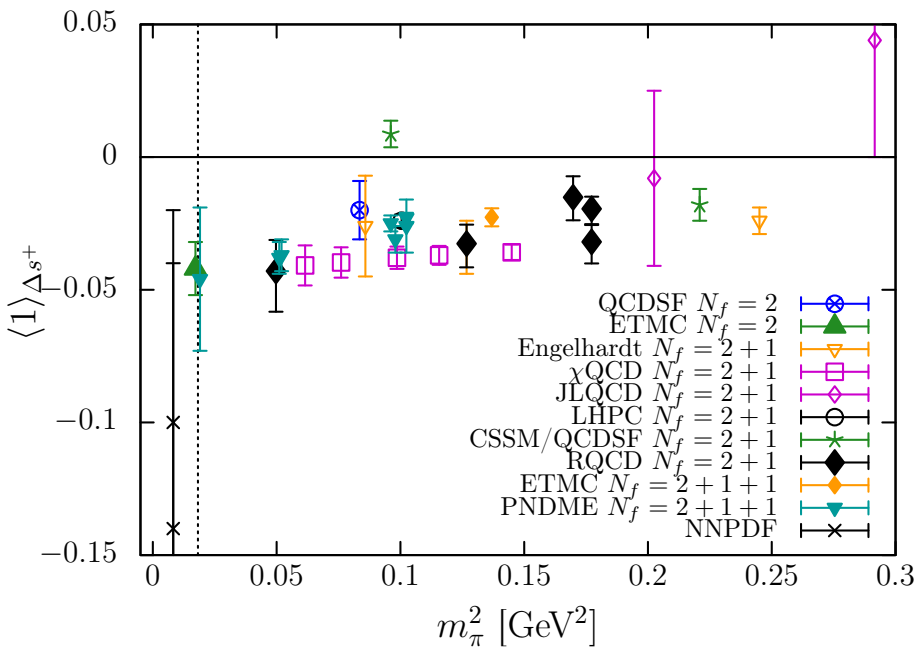
Larger negative values of
$\Delta s + \Delta\bar{s} = 2\Delta\bar{s}$
are due to regions outside of the experimental range.

Flavour separated $\langle 1 \rangle_{\Delta q^+}$ at $\mu = 2\,\mathrm{GeV}$, $\overline{\mathrm{MS}}$

C.f. isovector case:

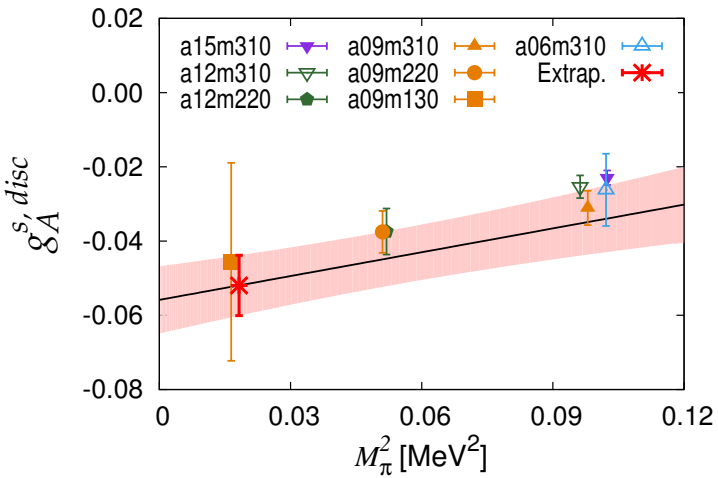
- Additional disconnected diagram.
- Mixing under renormalisation $\langle \mathbb{1} \rangle_{\Delta q}(\mu) = \sum_{q=u,d,s} Z'_q(\mu) \langle \mathbb{1} \rangle^{latt}_{\Delta q}$.
  Singlet combination acquires an anomalous dimension due to the $\mathrm{U}_A(1)$ anomaly, $Z^{n.s.}/Z^s = 1 + O(\alpha_S^2)$.
  $a_3 = g_A = \Delta u - \Delta d$, $a_8 = \Delta u + \Delta d - 2\Delta s$, $a_0(Q^2) = \Delta u + \Delta d + \Delta s$



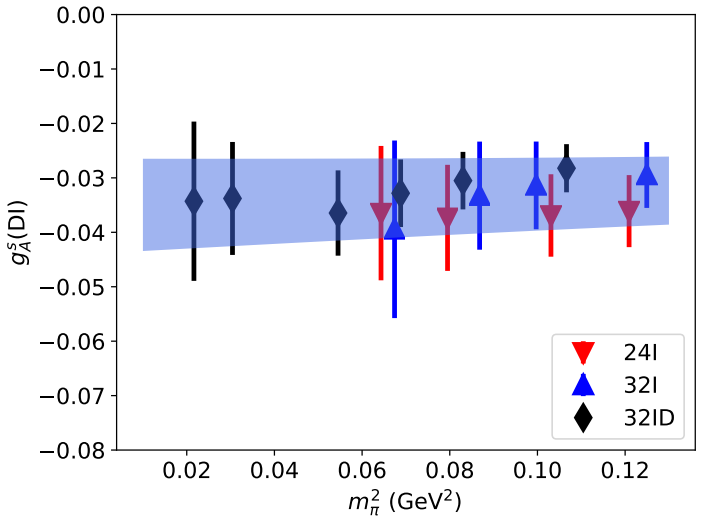


Flavour separated $\langle 1 \rangle_{\Delta q^+}$ at $\mu = 2\,\text{GeV}$, $\overline{\text{MS}}$

Continuum and chiral extrapolation performed in some studies.



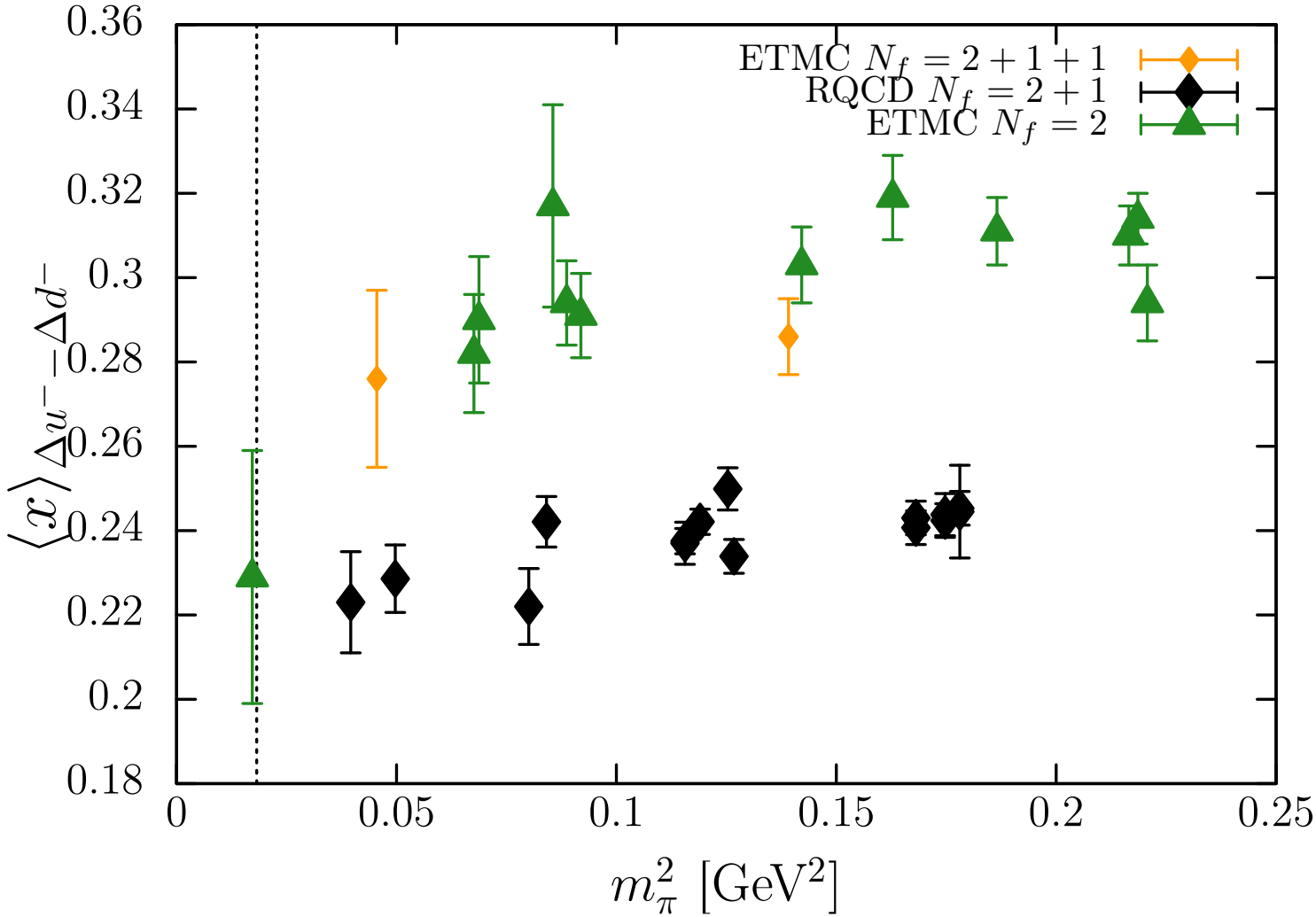
[PNDME,1806.10604]:
$\langle 1 \rangle_{\Delta u^+} = 0.777(25)$,
$\langle 1 \rangle_{\Delta d^+} = -0.438(18)$
$\langle 1 \rangle_{\Delta s^+} = -0.053(8)$



[χQCD,1806.08366]:
$\langle 1 \rangle_{\Delta u^+} = 0.846(18)(32)$,
$\langle 1 \rangle_{\Delta d^+} = -0.410(16)(18)$
$\langle 1 \rangle_{\Delta s^+} = -0.035(8)(7)$

Also [ETMC,1706.02973] M_π^{phys}, single $a = 0.09$ fm: $\langle 1 \rangle_{\Delta u^+} = 0.820(26)(4)$, $\langle 1 \rangle_{\Delta d^+} = -0.386(16)(6)$ $\langle 1 \rangle_{\Delta s^+} = -0.042(10)(2)$

Isovector: $\langle x \rangle_{\Delta u^- - \Delta d^-}$ at $\mu = 2\,\mathrm{GeV}$, $\overline{\mathrm{MS}}$



C.f. NNPDFpol1.1 0.195(14), DSSV08 0.203(9), JAM17 0.241(26).

Earlier studies by [LHPC,1001.3620], [QCDSF,1101.2326], [RBC,1003.3387].

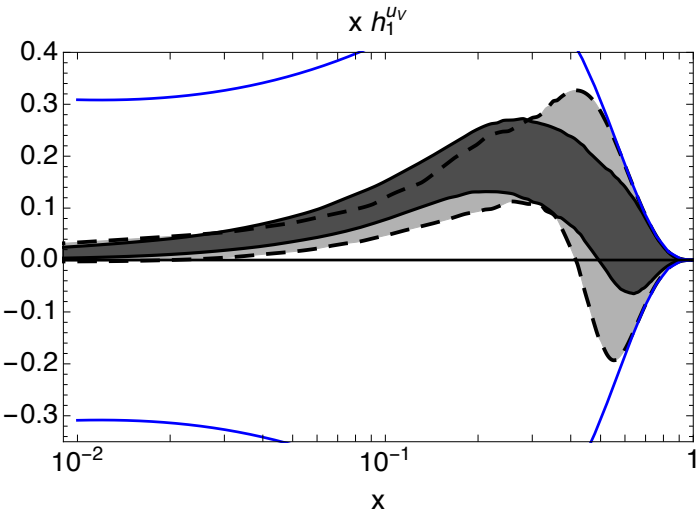
Transversity

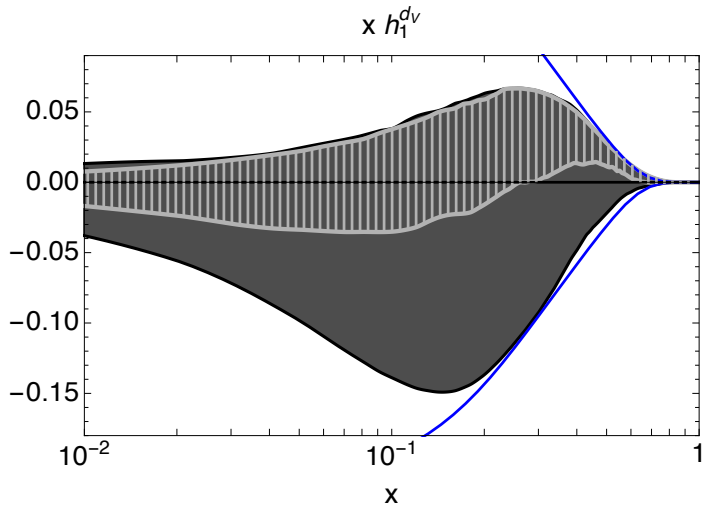
Isovector: $g_T = \delta u - \delta \overline{u} - (\delta d - \delta \overline{d}) = \langle 1 \rangle_{\delta u^- - \delta d^-}$
Flavour separated: $\langle 1 \rangle_{\delta q^-}$
Isovector: $\langle x \rangle_{\delta u^+ - \delta d^+}$

Transversity

Difficult to extract experimentally.

[Radici,1802.05212]: global analysis of $\pi\pi$ production in DIS and pp.



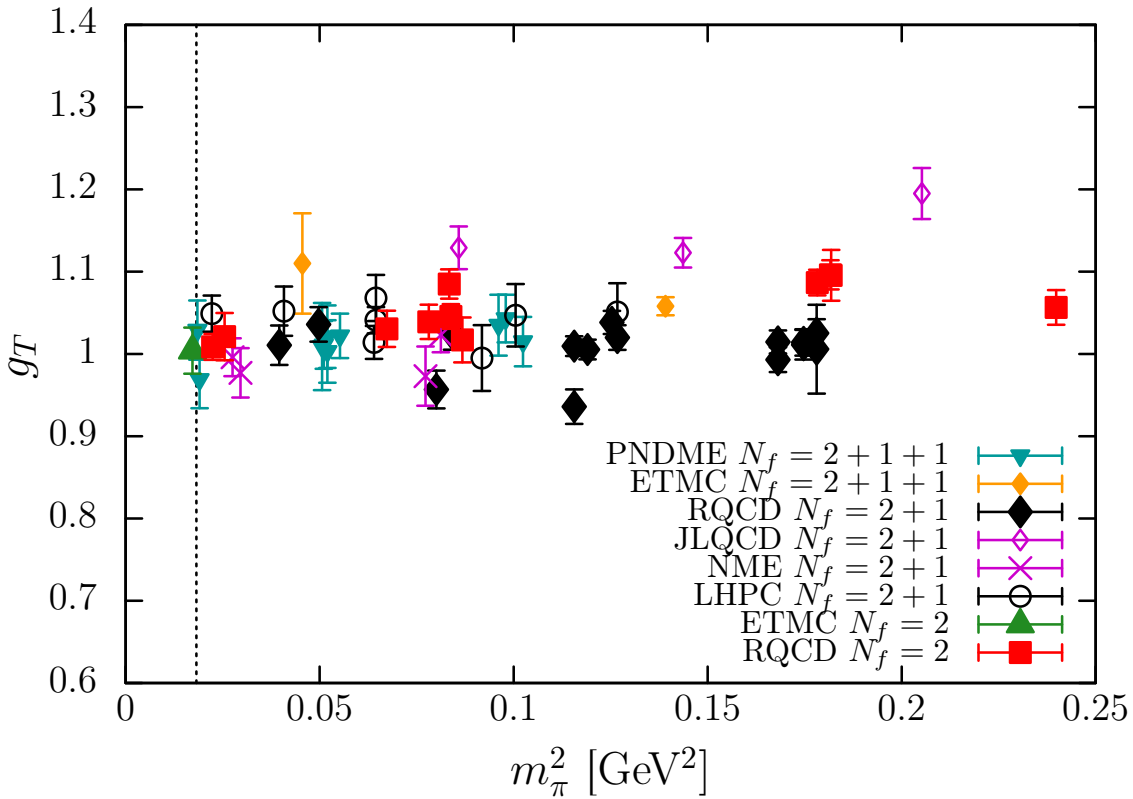


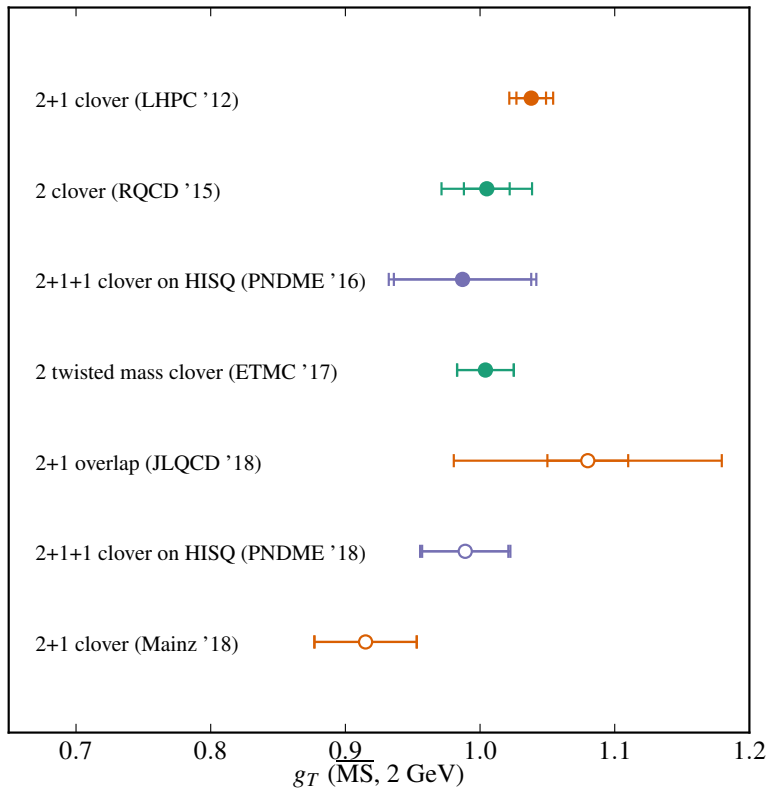
Find for $Q^2 = 4$ GeV^2:

$\langle 1 \rangle_{\delta u^-} = 0.39(10)$ and $\langle 1 \rangle_{\delta d^-} = -0.11(26)$ with $g_T = \langle 1 \rangle_{\delta u^- - \delta d^-} = 0.53(25)$.

Isovector (tensor charge): $g_T = \langle 1 \rangle_{\delta u^- - \delta d^-}$ at $\mu = 2$ GeV in $\overline{\text{MS}}$

Mild dependence on a, M_π, V. Overall uncertainty $< 10\%$.





Plenary talk by J. Green at Lattice 2018

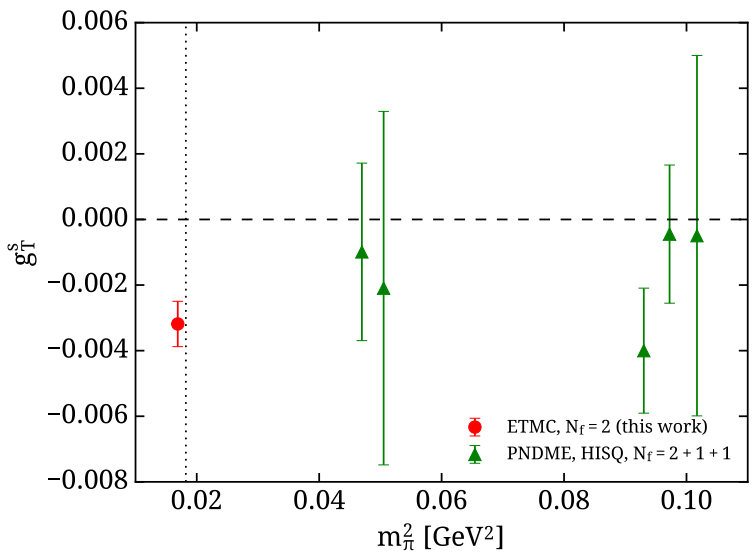
Extrapolation in a, M_π, V: [PNDME,1806.09006], Mainz Lattice 2018

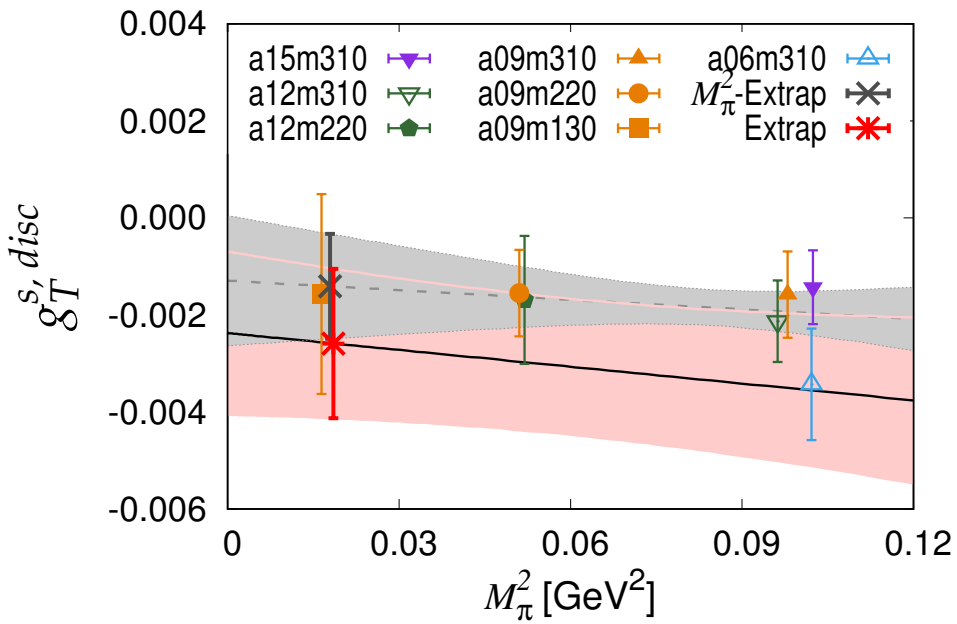
M_π^{phys}, single $a = 0.09$ fm: [ETMC,1703.08788]

Flavour separated $\langle 1 \rangle_{\delta q^-}$ at $\mu = 2$ GeV in $\overline{\text{MS}}$

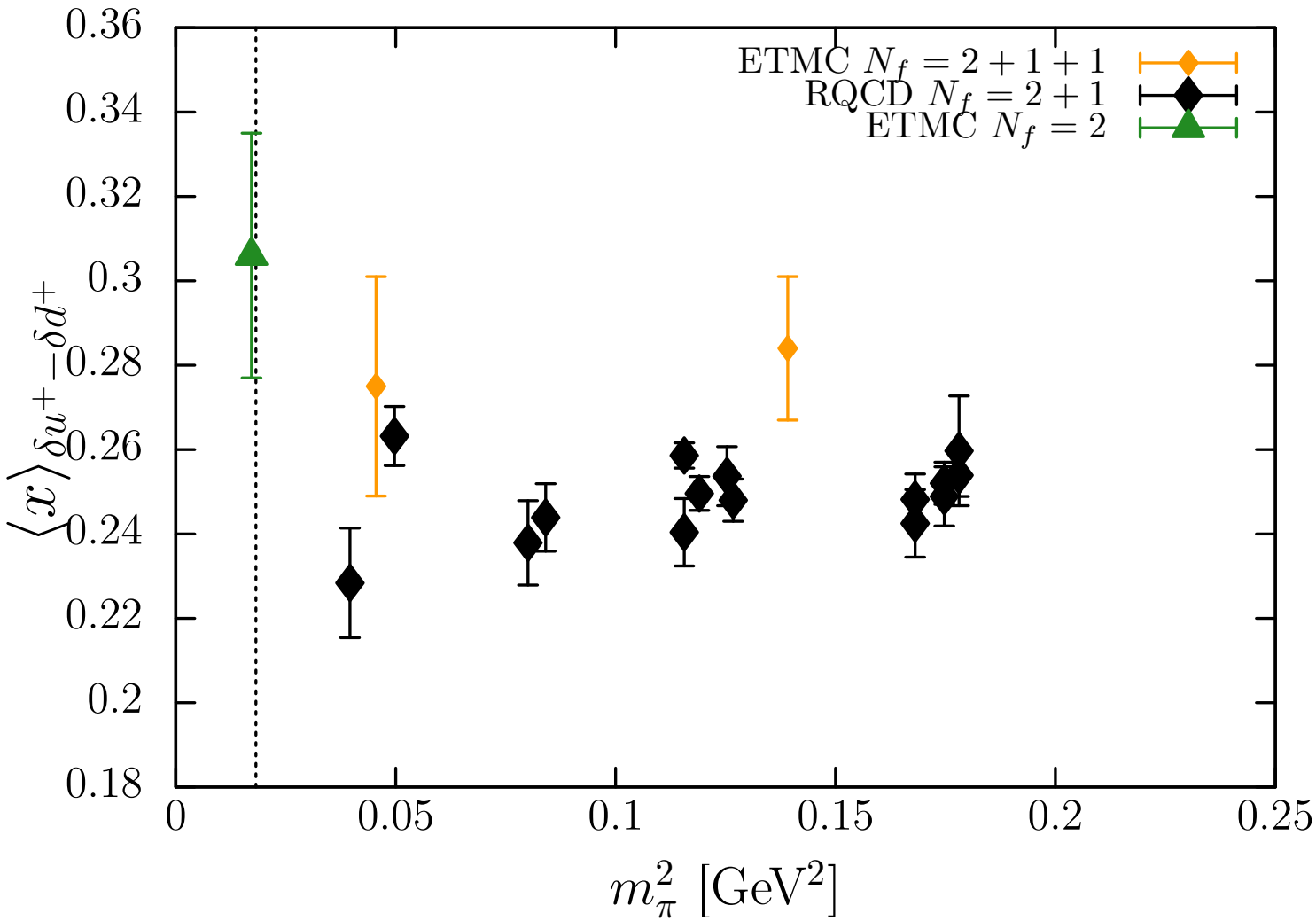
Disconnected diagram and mixing under renormalisation ($Z^{n.s.}/Z^s = 1 + O(\alpha^3)$).

	$\langle 1 \rangle_{\delta u^-}$	$\langle 1 \rangle_{\delta d^-}$	$\langle 1 \rangle_{\delta s^-}$
[PNDME,1808.07597]	0.78(3)	−0.20(1)	−0.0027(16)
[ETMC,1703.08788]	0.78(2)	−0.22(2)	−0.0032(07)





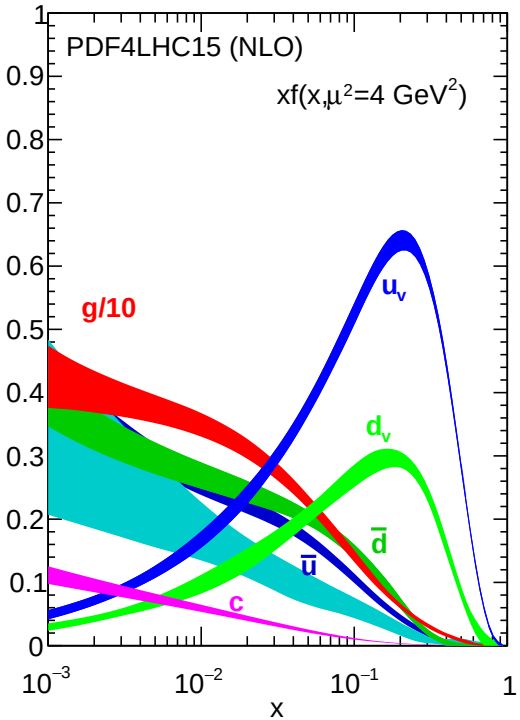
Isovector: $\langle x \rangle_{\delta u^+ - \delta d^+}$ at $\mu = 2\,\mathrm{GeV}$, $\overline{\mathrm{MS}}$



Unpolarised

Isovector: $\langle x \rangle_{u^+ - d^+}$

Unpolarised PDFs



$$\langle x \rangle_{q^+} = \int_0^1 dx\, x \left(q(x,\mu^2) + \overline{q}(x,\mu^2) \right))$$

PDF4LHC [Butterworth,1510.03865]: NNPDF3.0, CT14, MMHT2014 data sets

[PDFLattice,1711.07916] $\mu^2 = 4$ GeV^2

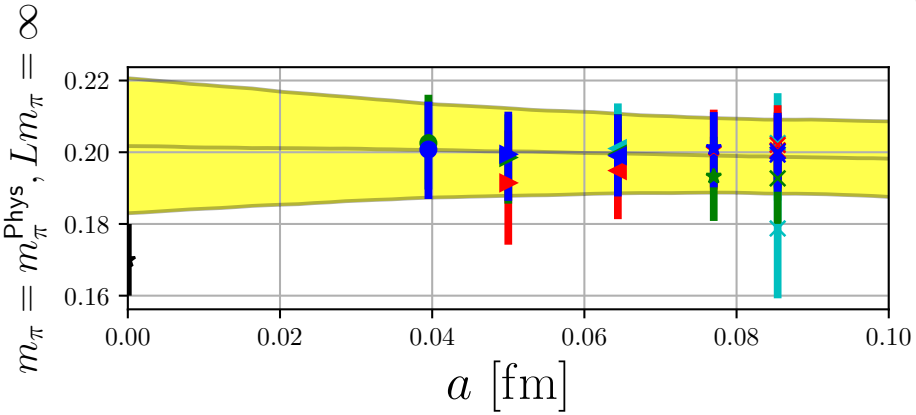
	NNPDF3.1	CT14	MMHT2014	ABMP2016	CJ15	HERAPDF2.0
$\langle x \rangle_{u^+-d^+}$	0.152(3)	0.158(4)	0.151(4)	0.167(4)	0.152(2)	0.188(3)
$\langle x \rangle_{u^+}$	0.348(4)	0.348(3)	0.348(5)	0.353(3)	0.348(1)	0.372(4)
$\langle x \rangle_{d^+}$	0.196(3)	0.190(3)	0.197(5)	0.186(3)	0.196(1)	0.185(7)
$\langle x \rangle_{s^+}$	0.039(3)	0.035(5)	0.035(9)	0.041(2)	—	0.035(11)
$\langle x \rangle_{g}$	0.410(4)	0.416(5)	0.411(9)	0.412(4)	0.416(1)	0.401(10)

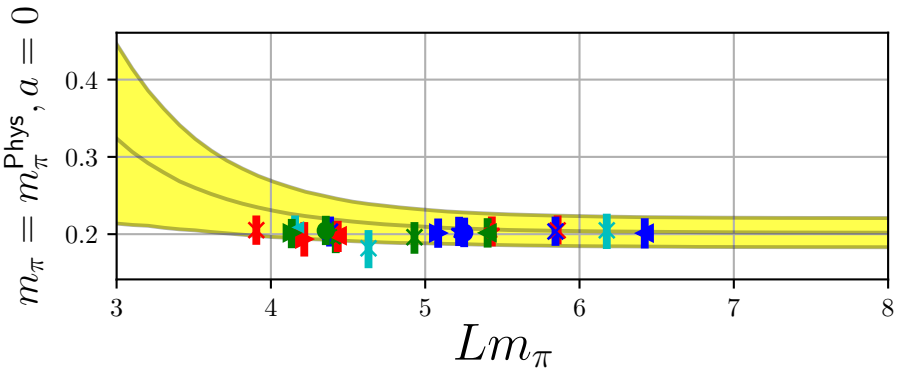
PDF4LHC:

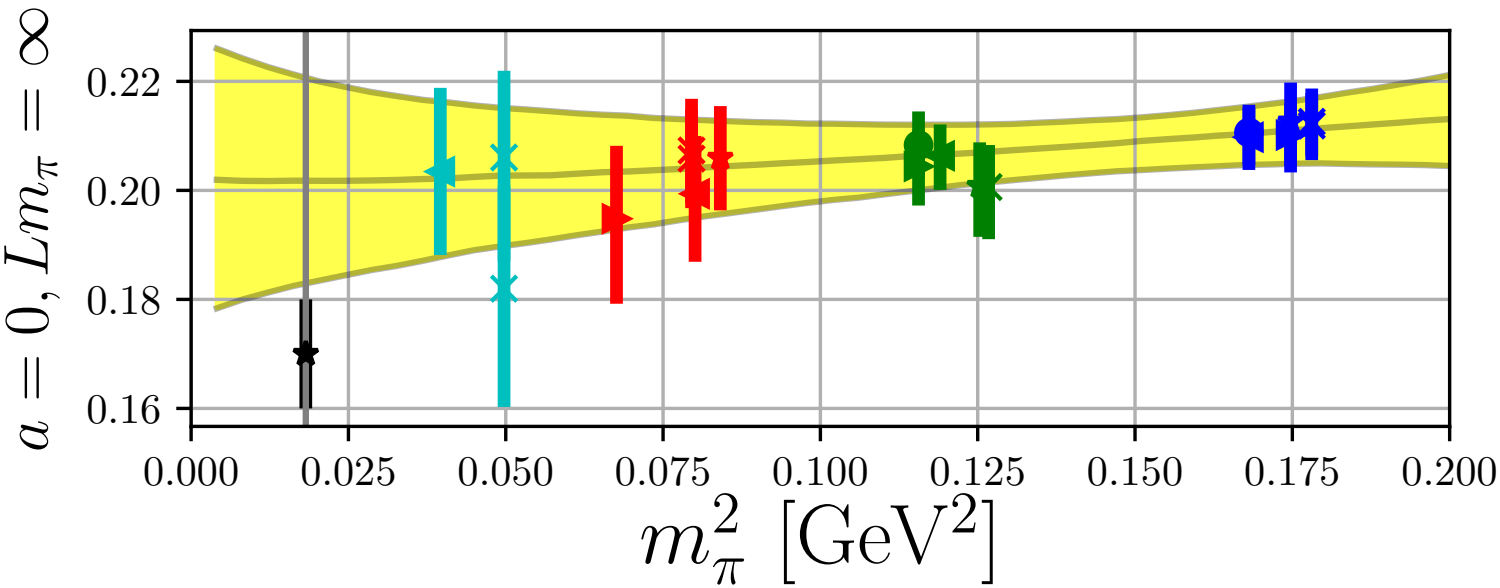
$\langle x \rangle_{u^+-d^+} = 0.155(5)$, $\langle x \rangle_{u^+} = 0.347(5)$ $\langle x \rangle_{d^+} = 0.193(6)$ $\langle x \rangle_{s^+} = 0.036(6)$ $\langle x \rangle_{g^+} = 0.414(9)$

Isovector $\langle x \rangle_{u^+-d^+}$ ($\overline{\text{MS}}$ at $\mu = 2\,\text{GeV}$)

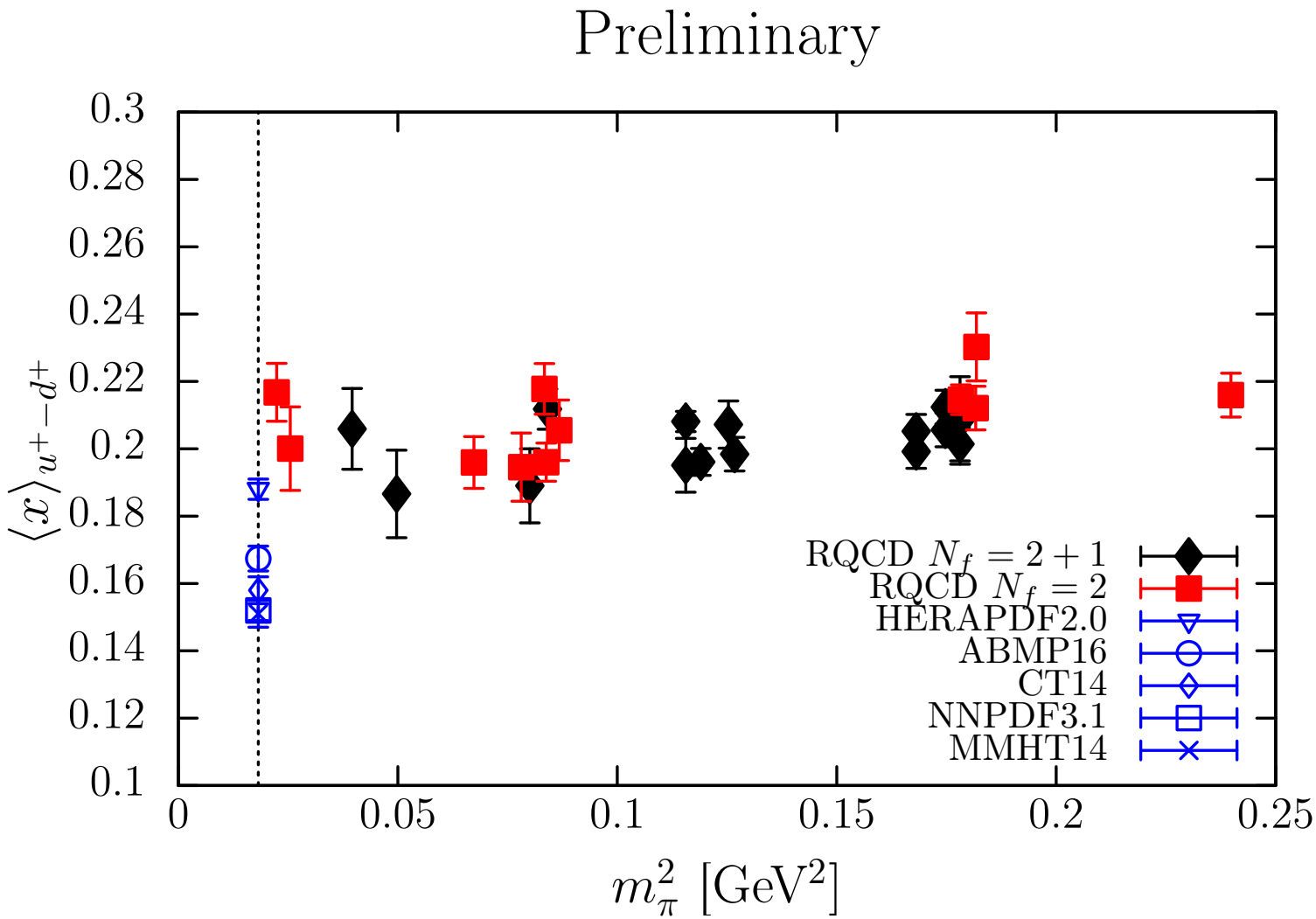
RQCD: T Wurm et al







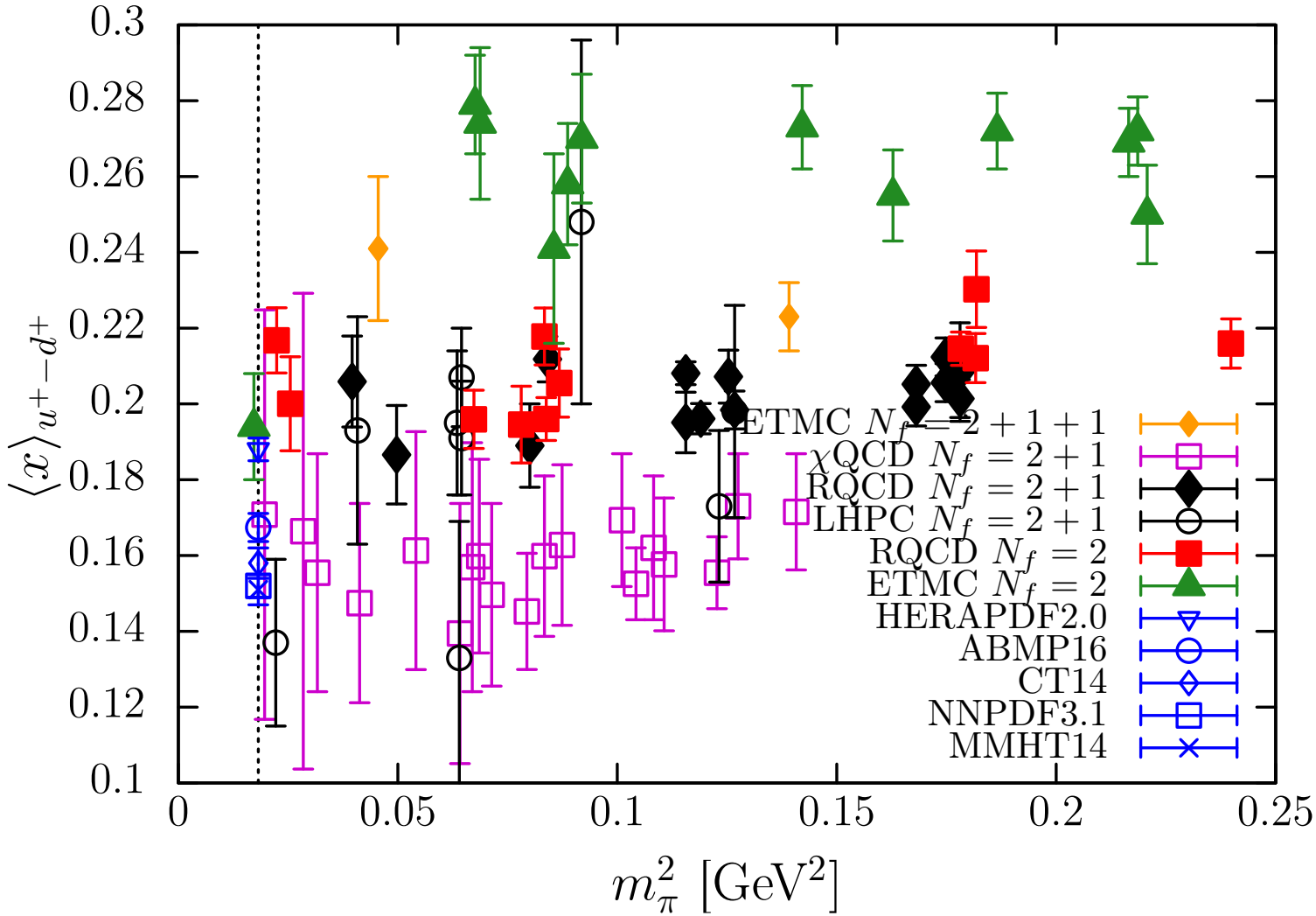
Isovector $\langle x \rangle_{u^+-d^+}$ from RQCD



$N_f = 2$: [RQCD,1408.6850]]

Global fits range from 0.151(4) (MMHT14) to 0.188(3) (HERAPDF2.0).

Isovector $\langle x \rangle_{u^+-d^+}$: current status



Present errors after extrapolations $\sim 10\% \Rightarrow$ still some work to do.

χQCD results should be increased by a factor ~ 1.25.

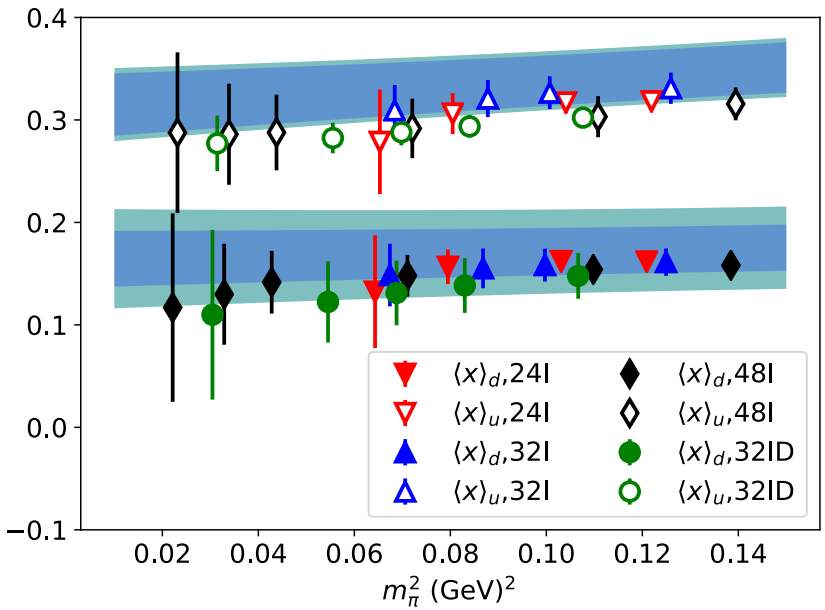
Flavour separated $\langle x \rangle_q$

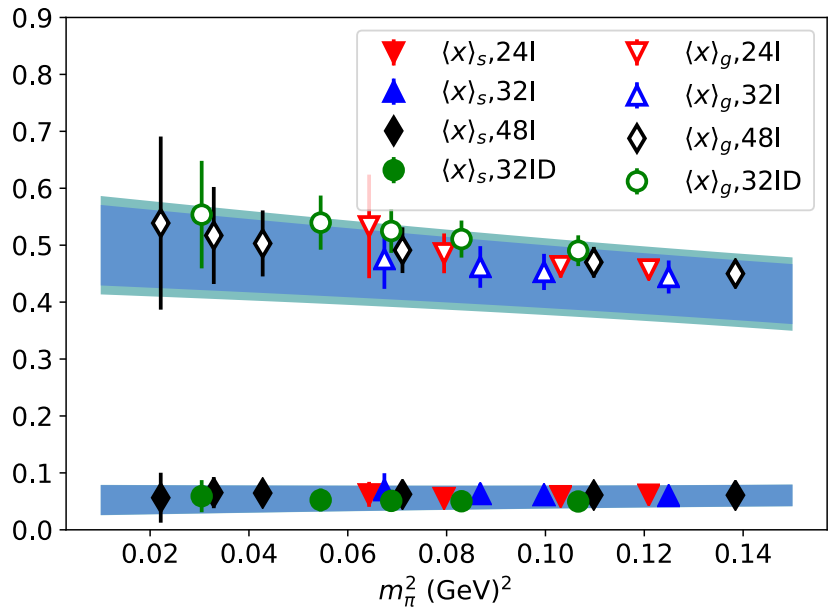
[ETMC,1706.02973] $N_f = 2$, M_π^{phys}, single $a = 0.09$ fm.

$$\langle x \rangle_u = 0.45(7) \qquad \langle x \rangle_d = 0.26(7) \qquad \langle x \rangle_s = 0.09(4) \qquad \langle x \rangle_g = 0.27(3)$$

Momentum sum rule: $\sum_q \langle x \rangle_{q^+} + \langle x \rangle_g = 1.07(16)$

[χQCD,1808.08677] $N_f = 2 + 1$, $a = 0.14, 0.11, 0.08$ fm, $M_\pi = 337, 300, 170$ MeV.





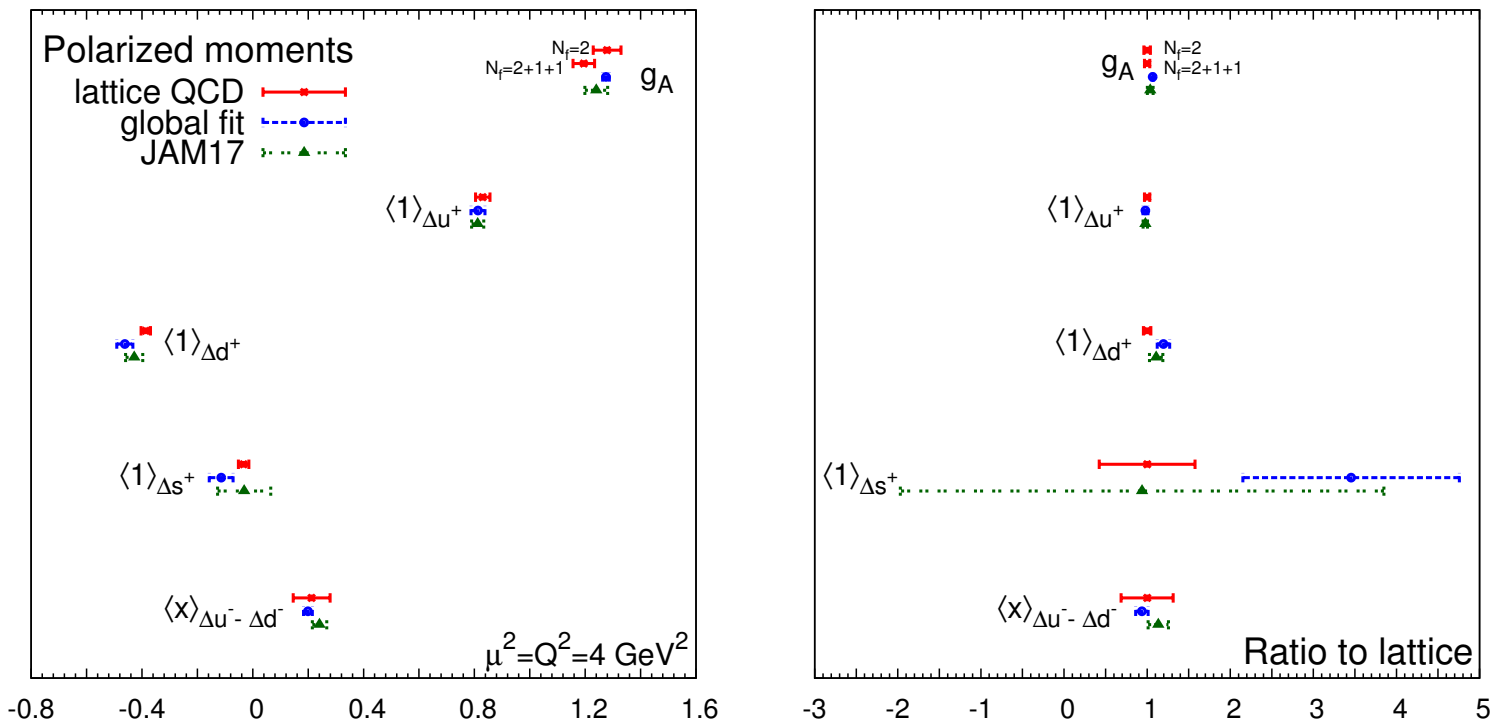
Momentum sum rule enforced for every (a, m_π). Normalisation factor $Z = (\sum_{u,d,s,g} \langle x \rangle)^{-1} \sim 0.8$. Previously, [$\chi$QCD,1805.00531] $N_f = 2 + 1$, $m_\pi = 140$ MeV, $a = 0.11$ fm, $\langle x \rangle_g = 0.42(4)(11)$.

Lattice constraints on PDF fits

PDFLattice [Lin,1711.07916]:

- Assess systematics of different lattice determinations of moments of polarised and unpolarised PDFs. Provide lattice averages.
- Compare lattice and PDF results for the moments.

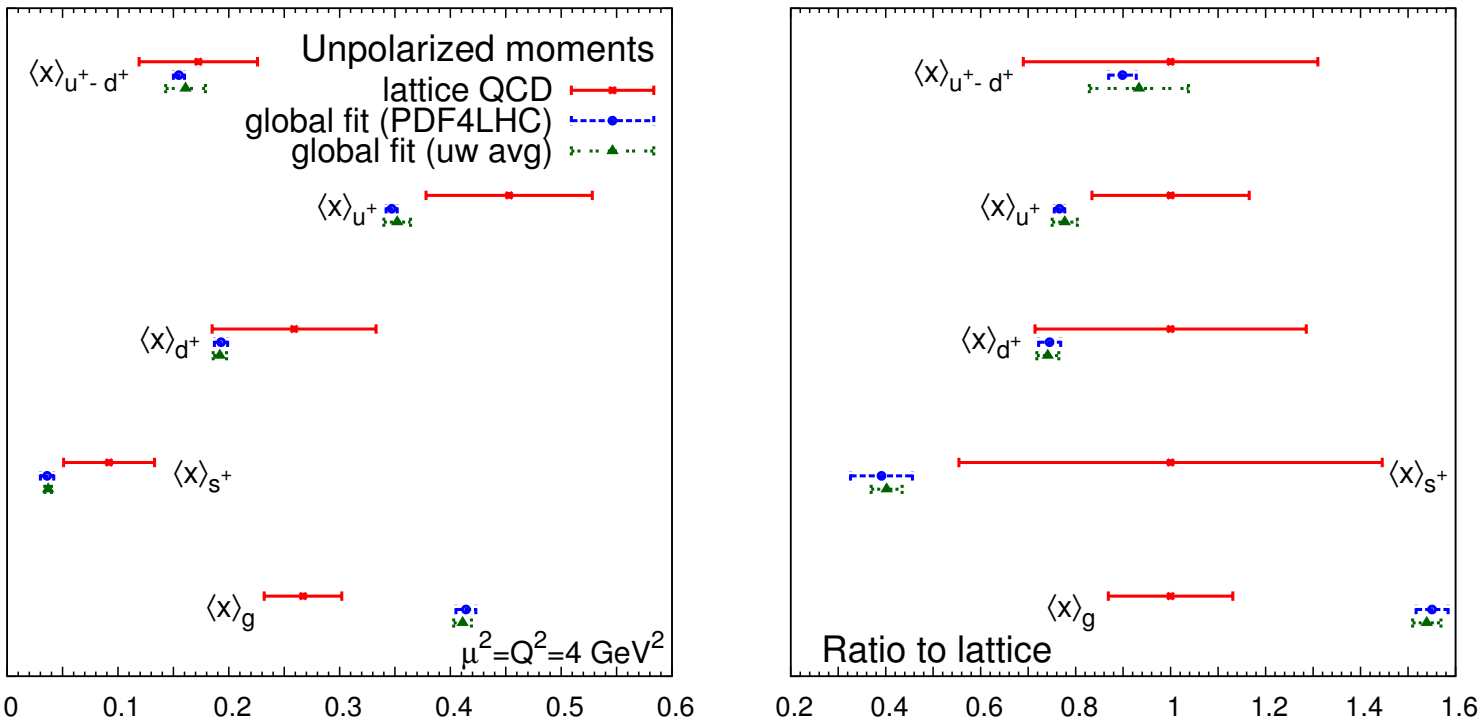
Comparison with moments from unweighted average of NNPDFpol1.1, DSSV08 and JAM15 PDF sets (global fit) and JAM17.



$\langle 1 \rangle_{\Delta u^+, \Delta d^+}$, single calculation. $\langle 1 \rangle_{\Delta s^+}$, $\langle x \rangle_{\Delta u^+ - \Delta d^+}$ a range.

Lattice constraints on PDF fits

Comparison with moments from PDF4LHC and unweighted average of moments of NNPDF3.1, CT14, MMHT2014, ABMP2016, CJ15 and HERAPDF2.0 (uw avg).



$\langle x \rangle_{u^+,d^+,s^+,g^+}$, single calculation. $\langle x \rangle_{u^+-d^+}$ a range.

Also considered constraining power of lattice results on PDFs.

FLAG nucleon matrix elements

Nucleon matrix element working group:

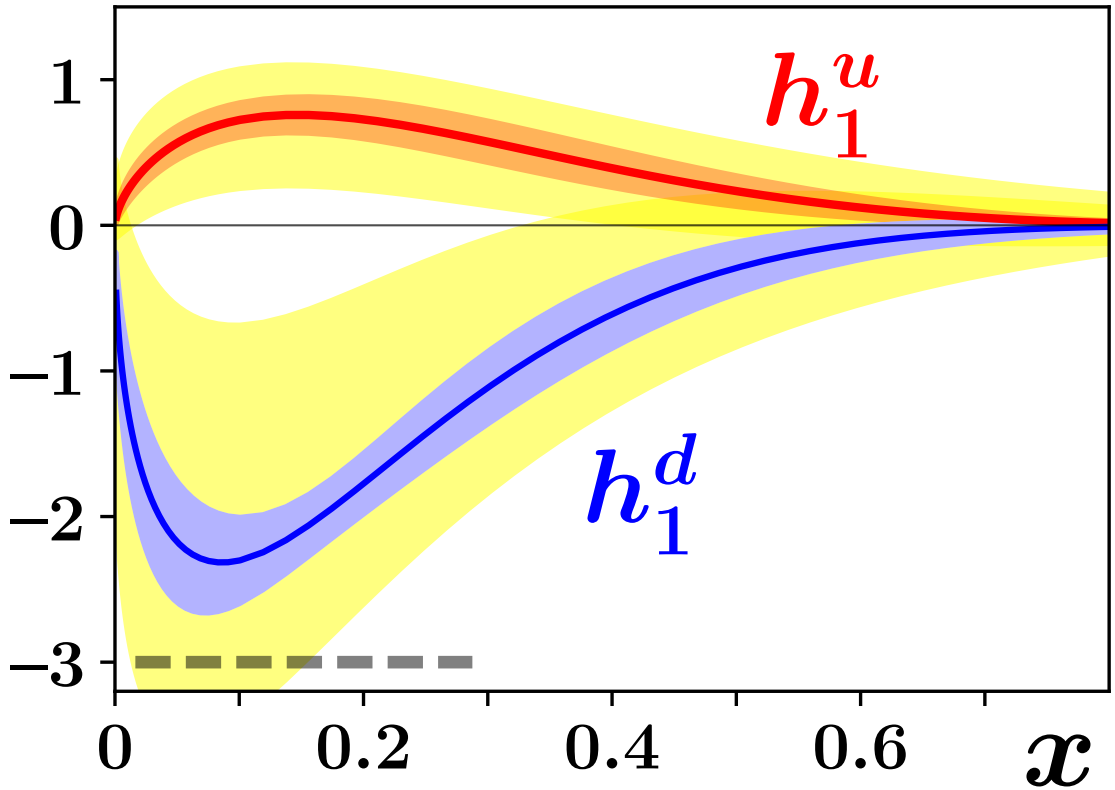
S. C (RQCD), Amy Nicholson (CalLat), Rajan Gupta (PNDME), Hartmut Wittig (Mainz).

Averaging:

- Isovector charges: $g_A = \langle 1 \rangle_{\Delta u^+ - \Delta d^+}$, g_S, $g_T = \langle 1 \rangle_{\delta u^- - \delta d^-}$
- Flavour singlets: $\langle 1 \rangle_{\Delta u^+, \Delta d^+, \Delta s^+}$, $\langle 1 \rangle_{\delta u^-, \delta d^-, \delta s^-}$
- $\sigma_{\pi N}$, σ_s

Global analysis of transversity with lattice constraints

Transversity PDFs $h_1^u(x) = \delta u(x)$, $h_1^d(x) = \delta d(x)$, assuming $\delta\bar{u} = \delta\bar{d} = \delta\bar{s} = \delta s$



From JAM: [H-W Lin,1710.09858]]:

Fit SIDIS Collins asymmetry data ($A_{UT}^{\sin(\phi_h+\phi_s)}$) alone (yellow),
and with $\delta u - \delta\bar{u} - \delta d + \delta\bar{d}$ constrained to $g_T = 1.01(6)$ (red+blue)

(average of [LHPC,1206.4527]], [PNDME,1606.07049]], [RQCD,1412.7336]])

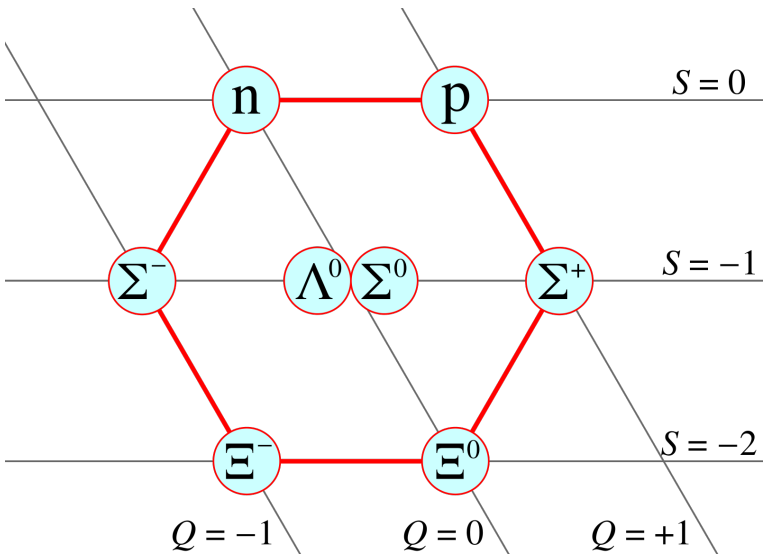
SU(3) flavour symmetry breaking in the baryon octet

So far only considered matrix elements between nucleon states $\langle N|\mathcal{O}|N\rangle$.

Global fits for helicity PDFs fix $a_3 = g_A = \Delta u - \Delta d$ and $a_8 = \Delta u + \Delta d - 2\Delta s$ from experiment.

Assume SU(3) flavour symmetry to extract a_8 from hyperon weak decays.

Symmetry relates $N \to N$, $\Sigma \to \Sigma$, $\Xi \to \Xi$, $\Xi \to \Sigma$, $\Xi \to \Lambda$, $\Lambda \to N$ ....



Isospin $I = 1, \frac{1}{2}$ and hypercharge Y.

$$g_A^B = \langle B|(\overline{u}\gamma_0\gamma_5 u - \overline{d}\gamma_0\gamma_5 d)|B\rangle$$
$$= 2I_3(F + YD)$$

$I = 0$

$$g_A^\Lambda = \langle \Lambda|(\overline{u}\gamma_0\gamma_5 d)|\Sigma\rangle$$
$$= 2D$$

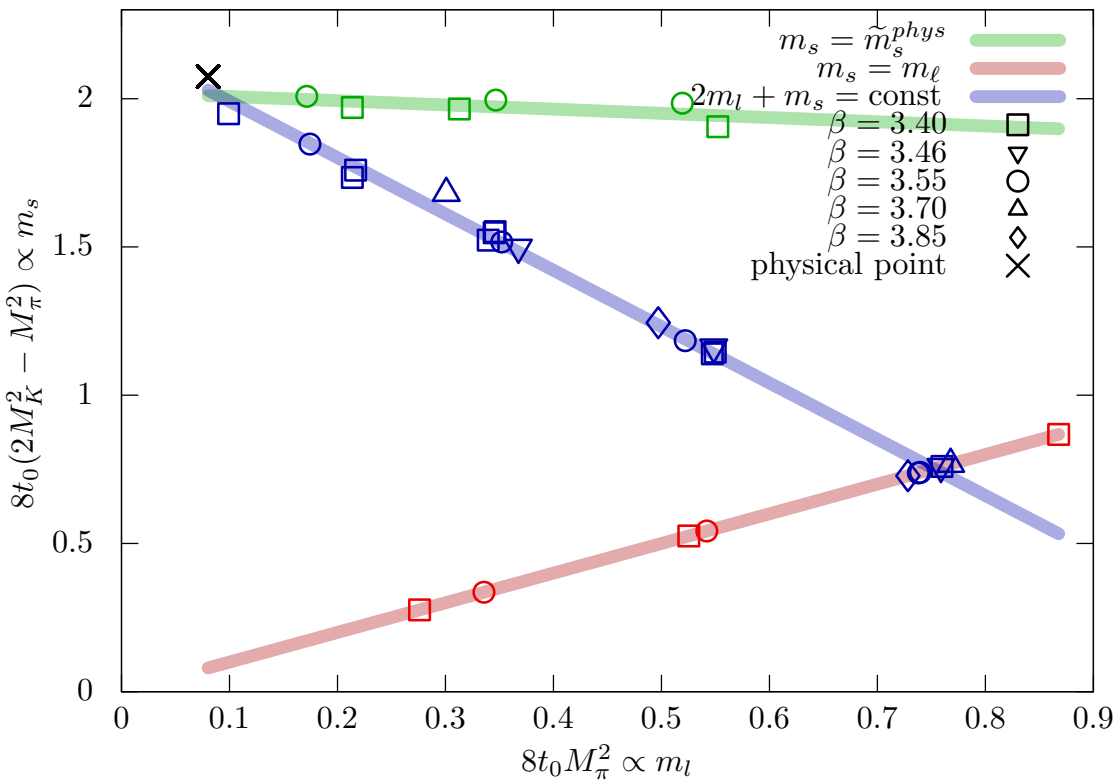
$g_A^p = a_3 = F + D$, $g_A^{\Sigma^+} = 2F$, $g_A^{\Xi^0} = F - D$, want $a_8 = 3F - D$

SU(3) flavour symmetry breaking in the baryon octet

Full study involving $\overline{d}\Gamma u$, $\overline{d}\Gamma s$, $\overline{u}\Gamma u - \overline{d}\Gamma d$, $\overline{u}\Gamma u + \overline{d}\Gamma d - 2\overline{s}\Gamma s$ for N, Σ, Λ and Ξ, considered by [QCDSF,1212.2564].

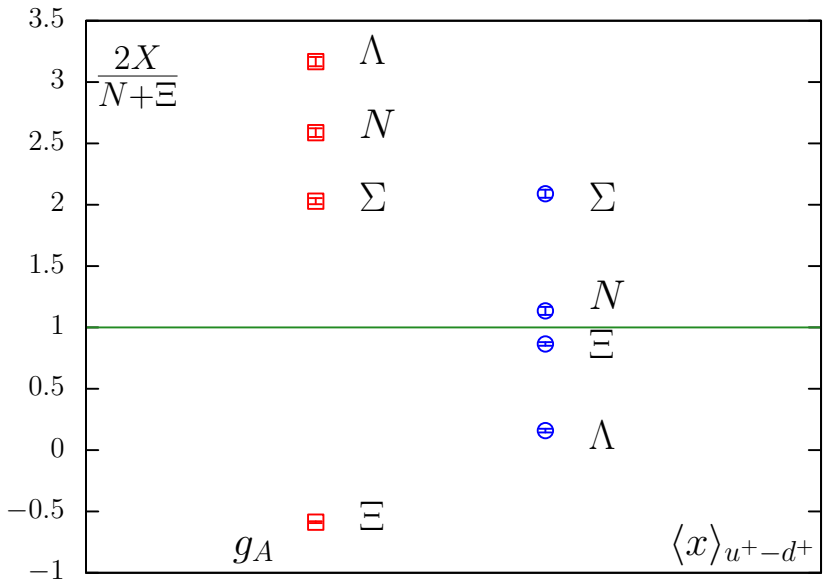
[QCDSF,1102.5300] pioneering approach: simulations with $m_u + m_d + m_s$ held constant.

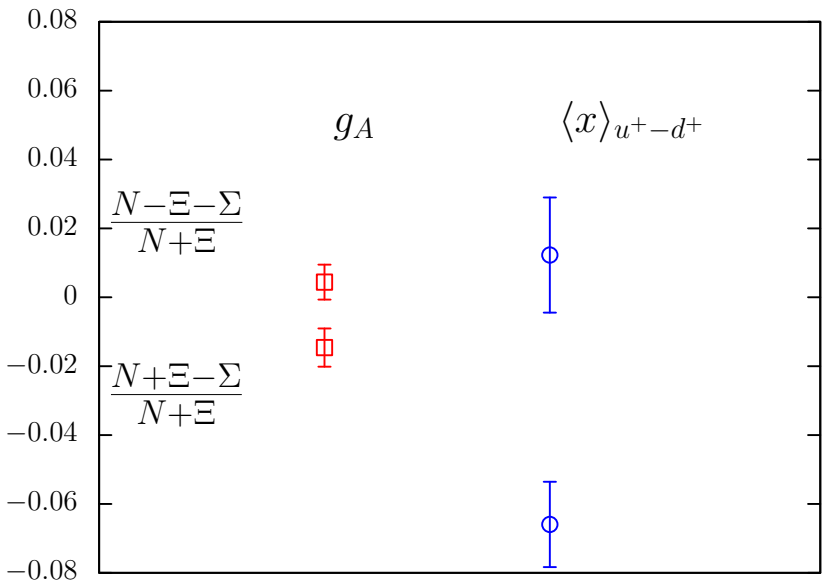
Also implemented by the Coordinated Lattice Simulations (CLS) effort.



SU(3) flavour symmetry breaking in the baryon octet

Work in progress: $m_\pi = 356$ MeV, $m_K = 442$ MeV, $a = 0.086$ fm





SU(3) limit: matrix elements follow $N = F + D$, $\Sigma = 2F$, $\Xi = F - D$, $\Lambda = 2D$.
Away from this limit: corrections start at $O(m_l)$, $m_l = m_{u/d} - m_s$.

Earlier SU(3) breaking studies: [QCDSF,hep-lat/0208017], [Lin,0712.1214], [Sasaki,0811.1406], [Erkol,0911.2447], [QCDSF,hep-lat/0208017], [Sasaki,PoS2016].

Summary and Outlook

- Significant progress towards precision determinations of nucleon matrix elements: a, m_π and V extrapolations. Simulations at physical m_π.
- Systematics are being explored $\rightarrow g_A = \langle 1 \rangle_{\Delta u^+ - \Delta d^+}$, $g_T = \langle 1 \rangle_{\delta u^+ - \delta d^+}$ and individual quark flavours $\langle 1 \rangle_{\Delta u^+, \Delta d^+, \Delta s^+}$, $\langle 1 \rangle_{\delta u^+, \delta d^+, \delta s^+}$.
- Lowest moments for helicity and transversity could be used to constrain PDF fits.
- First attempts to assess lattice systematics and provide lattice averages. $\rightarrow$ FLAG Nucleon Matrix Elements working group.
- Lattice constraints have been used in a global analysis of transversity.
- Higher moments: matrix elements involving derivatives require more work, however, $\langle x^2 \rangle_{u^- - d^-}$, $\langle x^2 \rangle_{\Delta q^+}$, $\langle x^2 \rangle_{\delta q^-}$, feasible.