Holographic Distribution Amplitudes & Functions

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Mapping Parton Distribution Amplitudes and Functions
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Overview

The holographic light-front Schrödinger Equation

Brodsky & de Téramond (PRL, 2009) | Brodsky, de Téramond, Dosch & Erlich (Phys. Rep. 2015)

$$\left(-\frac{\mathrm{d}^2}{\mathrm{d}\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U_{\mathrm{eff}}(\zeta)\right)\phi(\zeta) = M^2\phi(\zeta)$$

- $\zeta^2 = x(1-x)b_\perp^2$: key light-front variable for bound states.
- $x = k^+/P^+$: light-front momentum fraction of quark.
- $b_\perp$: transverse distance between quark and antiquark.
- U_{eff} : effective (includes coupling to higher Fock sectors) $q\bar{q}$ potential.

Meson light-front wavefunction :

$$\Psi(x,\zeta,\varphi) = \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}} f(x) e^{iL\varphi}$$

Assumptions behind the holographic LFSE

The holographic light-front SE is derived in light-front QCD with

- No quantum loops (no Λ_{QCD})
- Massless quarks ($m_f \to 0$)

} semiclassical approximation

- No mass scale → Conformal invariance → Gravity dual in higher dimensional anti-de Sitter space

Brodsky, de Téramond, Dosch & Erlich (Phys. Rep. 2015)

Maldacena: AdS=CFT

Light-front holography dictionary

Physical spacetime	Anti de Sitter spacetime
LF transverse distance	Fifth dimension in AdS
ζ	z_5
Angular momentum	Spin + (AdS mass×radius)
L^2	$(\mu R)^2 + (2 - J)^2$

LFSE maps onto wave equation for weakly coupled spin-J string modes in warped AdS with

$$U_{\rm eff}(\zeta) = \frac{1}{2}\varphi''(z) + \frac{1}{4}\varphi'(z)^2 + \frac{2J-3}{2z}\varphi'(z)$$

$\varphi(z)$: dilaton field that breaks conformal invariance in AdS

A unique confinement potential

- The dilaton field that distorts pure AdS geometry drives the confinement dynamics in physical spacetime
- A quadratic dilaton $\varphi = \kappa^2 z^2$ gives

$$U_{\rm eff}(\zeta) = \overbrace{\kappa^4 \zeta^2}^{\text{Conformal SB}} + \overbrace{2\kappa^2 (J-1)}^{\text{AdS/QCD}}$$

- Only harmonic potential preserves the conformal invariance of the underlying QCD action while introducing a mass scale in LF Hamiltonian: dAFF mechanism for breaking conformal symmetry in QM
- AdS/QCD mapping of EM form factors: $f(x) = \sqrt{x(1-x)}$

dAFF: de Alfaro, Furbini & Furlan (1976)

dAFF in LFQCD: Brodsky, de Téramond & Dosch (Phys. Lett. 2013)

Predicting a massless pion

Solving the holographic LF Schrödinger Equation

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U_{\mathrm{eff}}(\zeta)\right)\phi(\zeta) = M^2\phi(\zeta)$$

with

$$U_{\mathrm{eff}}(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(J-1)$$

gives

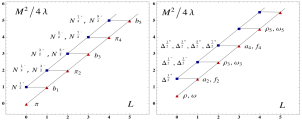
$$M^2 = (4n+2L+2)\kappa^2 + 2\kappa^2(J-1)$$

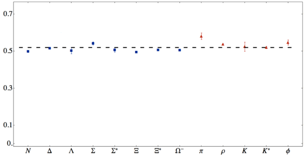
- Lightest bound state ($n = L = J = 0$) is massless ($M = 0$)
- Identify with pion (chiral symmetry breaking)
- If $U_{\mathrm{eff}} = \kappa^4\zeta^p + 2\kappa^2(J-1)$, $M_\pi = 0$ only for $p = 2$

Fixing the fundamental AdS/QCD scale

Regge slopes global fits

Reference	Fit	κ[MeV]
Brodsky et al. [Phys. Rep. 2015]	Mesons	540–590
Brodsky et al. [IJMD, 2016]	Mesons & Baryons	523





Universal $\kappa = \lambda^2 \sim 0.5$ GeV

Restoring dependence on quark masses

- In momentum space, 'complete' invariant mass of $q\bar{q}$ pair

$$M_{q\bar{q}}^2 = \frac{\mathbf{k}_\perp^2}{x(1-x)} \to \frac{\mathbf{k}_\perp^2 + m_f^2}{x(1-x)}$$

Brodsky, de Téramond, Subnuclear Series Proc. (2009)

- Meson wavefunction becomes

$$\Psi(x,\zeta^2) = \mathcal{N}\sqrt{x(1-x)}\exp\left[-\frac{\kappa^2\zeta^2}{2}\right]\exp\left[-\frac{m_f^2}{2\kappa^2 x(1-x)}\right]$$

- m_f are effective quark masses (because of coupling of valence sector to higher Fock sectors).

Restoring dependence on quark spins

- In semiclassical approximation :

$$\Psi(x,\mathbf{k}_\perp^2,h,\bar{h}) = \Psi(x,\mathbf{k}_\perp^2) \times S_{h,\bar{h}}$$

where

$$S^{\pi}_{h,\bar{h}} = \frac{1}{\sqrt{2}} h\delta_{h,-\bar{h}}$$

and

$$S^{\rho}_{h,\bar{h}} = \frac{1}{\sqrt{2}} \delta_{h,-\bar{h}}$$

This leads to degenerate decay constants (and DAs) for the π and ρ mesons, in contradiction with experiment.

Dynamical spin structure

More realistically

$$\Psi(x, \mathbf{k}_\perp) \to \Psi(x, \mathbf{k}_\perp, h, \bar{h}) = \Psi(x, \mathbf{k}_\perp) \times S_{h\bar{h}}(x, \mathbf{k}_\perp)$$

Vector meson:

$$S^{\lambda}_{h\bar{h}}(x, \mathbf{k}_\perp) = \frac{\bar{v}_{\bar{h}}(x, \mathbf{k}_\perp)}{\sqrt{1-x}} [\epsilon^{\lambda}_{V} \cdot \gamma] \frac{u_h(x, \mathbf{k}_\perp)}{\sqrt{x}}$$

Pseudoscalar meson:

$$S_{h\bar{h}}(x, \mathbf{k}_\perp) = \frac{\bar{v}_{\bar{h}}(x, \mathbf{k}_\perp)}{\sqrt{1-x}} \left[\frac{M_P}{2P^+} \gamma^+ \gamma^5 + B \gamma^5 \right] \frac{u_h(x, \mathbf{k}_\perp)}{\sqrt{x}}$$

$B \to 0$: no dynamical spin effects

Diffractive ρ electroproduction in the dipole model

PRL **109,** 081601 (2012) PHYSICAL REVIEW LETTERS

AdS/QCD Holographic Wave Function for the ρ Meson and Diffractive ρ Meson Electroproduction

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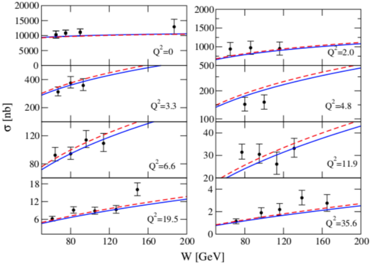
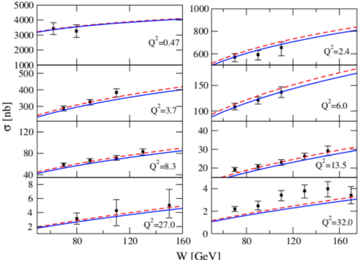
(Received 5 April 2012; published 20 August 2012)

We show that anti–de Sitter/quantum chromodynamics generates predictions for the rate of diff ρ-meson electroproduction that are in agreement with data collected at the Hadron Electro Accelerator electron–proton collider.

- Universal $\kappa = 0.54$ GeV
- $m_f = 0.14$ GeV
- HERA data

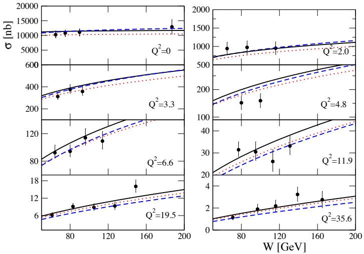


(a) H1

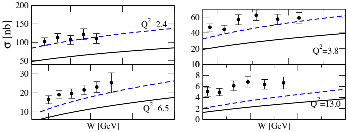


Diffractive ϕ electroproduction in the dipole model

M. Ahmady, N. Sharma, R.S, Phys. Rev. D94 (2016) 074018



- Universal $\kappa = 0.54$ GeV
- Best fit with $m_f = 0.047$ GeV and $m_s = 0.14$ GeV
- HERA data



Holographic vector meson DAs in B physics

Work done with M. Ahmady et al.

- Isospin asymmetry in $B \to K^* \gamma$ PRD 88 (2013) 014042
- Power-suppressed contributions in $\bar{B}^0 \to \rho^0 \gamma$ PRD 87 (2013) 054013
- Branching ratio of $B \to \rho l \nu_l$ PRD 88 (2013) 074031
- Isospin asymmetry in $B \to K^* \mu^+ \mu^-$ PRD 90 (2014) 074010
- FB asymmetry in $B \to K^* \mu^+ \mu^-$ PRD 92 (2015) 114028
- Branching ratio of $B \to K^* \nu \bar{\nu}$ PRD 98 (2018) 053002

A notable advantage

Holographic DAs avoid the problem of end-point divergences when computing power-suppressed corrections.

Holographic pion wavefunction

M. Ahmady, F. Chishtie, RS, PRD 95 (2017) 074008

Momentum space

$$\Psi^{\pi}_{h,\bar{h}}(x,\mathbf{k}_\perp) = \left[(M_\pi x\bar{x} + Bm_f)h\delta_{h,-\bar{h}} - Bke^{-ih\theta_k}\delta_{h,\bar{h}}\right]\frac{\Psi(x,\mathbf{k}_\perp^2)}{x\bar{x}}$$

Impact space

$$\Psi^{\pi}_{h,\bar{h}}(x,\mathbf{b}_\perp) = \left[(M_\pi x\bar{x} + Bm_f)h\delta_{h,-\bar{h}} + \overbrace{iBh\partial_b\delta_{h,\bar{h}}}^{\text{same helicities}}\right]\frac{\Psi(x,\zeta^2)}{x\bar{x}}$$

Pion observables

Data on pion decay constant, radius, EM and transition form factors all prefer $B \geq 1$ with $\kappa = 0.523$ GeV and $m_f = 0.330$ GeV

Dynamical spin effects in pion

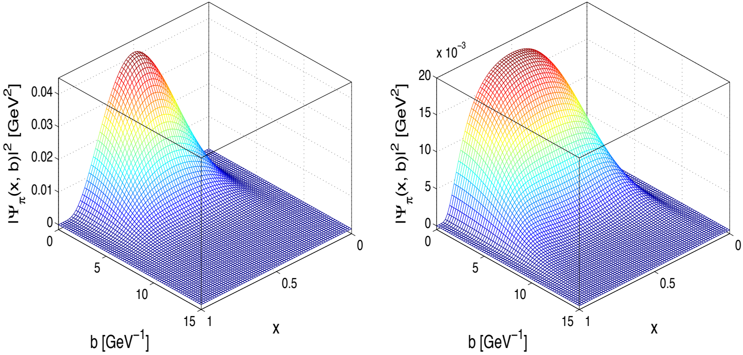


Figure: The holographic pion light-front wavefunction squared with $B = 0$ (left) and with $B \gg 1$ (right). $\kappa = 0.523$ GeV. $m_f = 0.330$ GeV.

Charge radii of pion and kaon

Extension to pseudoscalar octet + evolution effects:

M. Ahmady, C. Mondal, R.S, PRD (2018) 034010

$$\sqrt{\langle r_P^2 \rangle} = \left[\frac{3}{2} \int \mathrm{d}x \mathrm{d}^2\mathbf{b}_\perp [b_\perp(1-x)]^2 |\Psi_P(x, \mathbf{b}_\perp)|^2 \right]^{1/2}$$

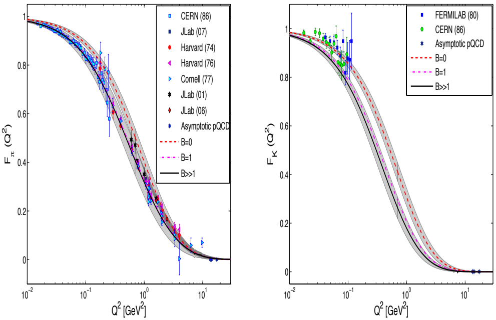
- $\pi^\pm$: $B \geq 1$ preferred
- $K^\pm$: $B = 0$ preferred ($m_s = 0.500$ GeV)

P	B	$\sqrt{\langle r_P^2 \rangle}_{\text{Th.}}$ [fm]	$\sqrt{\langle r_P^2 \rangle}_{\text{Exp.}}$ [fm]
$\pi^\pm$	0	0.543 ± 0.022	0.672 ± 0.008
	1	0.673 ± 0.034	
	$\gg 1$	0.684 ± 0.035	
$K^\pm$	0	0.615 ± 0.038	0.560 ± 0.031
	1	0.778 ± 0.065	
	$\gg 1$	0.815 ± 0.070	

Spacelike EM form factors of pion and kaon

Drell & Yan (PRL, 70); West (PRL, 70)

$$F_{\rm EM}(Q^2) = 2\pi \int {\rm d}x{\rm d}b_\perp \; b_\perp \; J_0[(1-x)b_\perp Q] \; |\Psi^P(x,\mathbf{b}_\perp)|^2$$



Left: $\pi^\pm$, $B \geq 1$ preferred. Right: $K^\pm$, $B = 0$ preferred.

Decay constants of pion and kaon

$$\langle 0|\bar{\Psi}\gamma^{\mu}\gamma_5\Psi|P\rangle = f_P p^{\mu}$$

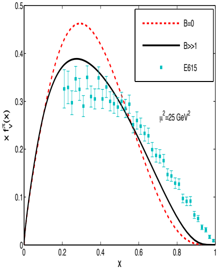
$$f_P = 2\sqrt{\frac{N_c}{\pi}}\int \mathrm{d}x\{((x(1-x)M_\pi^2) + B(xm_q + \bar{x}m_{\bar{q}})M_P\}\frac{\Psi^P(x,\zeta)}{x(1-x)}\bigg|_{\zeta=0}$$

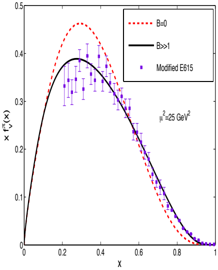
- $\pi^{\pm}$: $B \geq 1$ preferred
- $K^{\pm}$: $B = 0$ preferred.

P	B	$f_P^{\mathrm{Th.}}$ [MeV]	$f_P^{\mathrm{Exp.}}$ [MeV]
$\pi^{\pm}$	0	162 ± 8	$130 \pm 0.04 \pm 0.2$
	1	138 ± 5	
	$\gg 1$	135 ± 6	
$K^{\pm}$	0	156 ± 8	156 ± 0.5
	1	142 ± 7	
	$\gg 1$	135 ± 6	

Holographic pion PDF versus E615 Drell-Yan data

$$f_v^\pi(x) = \int \mathrm{d}^2\mathbf{b}_\perp \sum_{h,\bar{h}} |\Psi_{h\bar{h}}(x, \mathbf{b}_\perp)|^2$$





Original E615 DY data — Re-analyzed E615 DY data

- DGLAP evolution from $\mu_0 = 0.316$ GeV to $\mu = 5$ GeV
- Reanalysis of E615 data: Aicher, Schafer, and Vogelsang, PRL 105, 252003 (2010)

Pion Distribution Amplitude

$$\langle 0|\bar{\Psi}_d(z)\gamma^+(1-\gamma_5)\Psi_u(0)|\pi^+\rangle = f_\pi P^+ \int dx e^{ix(P\cdot z)}\varphi_\pi(x,\mu)$$

- Lattice QCD:

  R. Arthur et al. PRD 83 (2011) 074505 (2011); V. M. Braun et al. PRD92 (2015) 014504

- Dyson-Schwinger Equations:

  Chang et al. PRL 110 (2013) 132001

- QCD Sum Rules:

  P. Ball and R. Zwicky, PRD 71 (2005) 014015; A. P. Bakulev et al. Phys. Lett. B590 (2004) 309

- Platykurtic:

  N. G. Stefanis Phys. Lett. B738 (2014) 483

- Light-front Quark Model:

  H. M. Choi and C. R. Ji, PRD 75 (2007) 034019

- Light-front holography:

  S. J. Brodsky, F. G. Cao and G. F de Téramond, PRD 84 (2011) 075012

Perturbative evolution of DAs

Efremov-Radyushkin-Lepage-Brodsky (ERLB) equations

$$\varphi(x,\mu) = 6x\bar{x}\sum_{n=0}^{\infty} C_n^{3/2}(2x-1)a_n(\mu)$$

where

$$a_n(\mu) = \frac{2}{3}\frac{2n+3}{(n+1)(n+2)}\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\gamma_n^0/2\beta_0}\int_0^1 \mathrm{d}x C_n^{3/2}(2x-1)\varphi(x,\mu_0)\ .$$

with

$$\gamma_n^{(0)} = -2C_F\left[3+\frac{2}{(n+1)(n+2)} - 4\sum_k^{n+1}\frac{1}{k}\right]$$

and

$$\beta_0 = \frac{11}{3}C_A - \frac{2}{3}n_f\ .$$

Efremov and Radyushkin, Phys. Lett. B94 (1980) 245; Lepage and Brodsky, Phys. Rev. D22 (1980) 2157

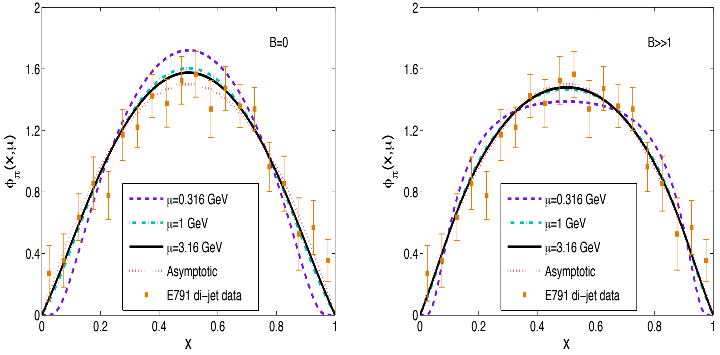
Moments of holographic pion DA

$$f_\pi \varphi_\pi(x,\mu) = 2\sqrt{\frac{N_c}{\pi}} \int \mathrm{d}b_\perp \, b_\perp J_1(\mu b_\perp)(x\bar{x}M_\pi + Bm_f)\frac{\Psi(x,\zeta)}{x\bar{x}}$$

Pion DA	μ [GeV]	$\langle \xi_2 \rangle$	$\langle \xi_4 \rangle$
Asymptotic	∞	0.200	0.085
LFH ($B=0$)	$1,2$	$0.180, 0.185$	$0.067, 0.071$
LFH ($B \geq 1$)	$1,2$	$0.200, 0.200$	$0.085, 0.085$
Platykurtic	2	$0.220^{+0.009}_{-0.006}$	$0.098^{+0.008}_{-0.005}$
Dyson-Schwinger[RL,DB]	2	$0.280, 0.251$	$0.151, 0.128$
Lattice (2010)	2	0.28(1)(2)	
Lattice (2015)	2	0.2361(41)(39)	

- ERLB evolution from initial scale $\mu_0 = 0.316$ GeV
- Dynamical spin enhances evolution to asymptotic DA

Holographic pion DA versus E971 di-jet data



- Holographic DAs almost asymptotic at 2 GeV
- Consistent with E971 di-jet data

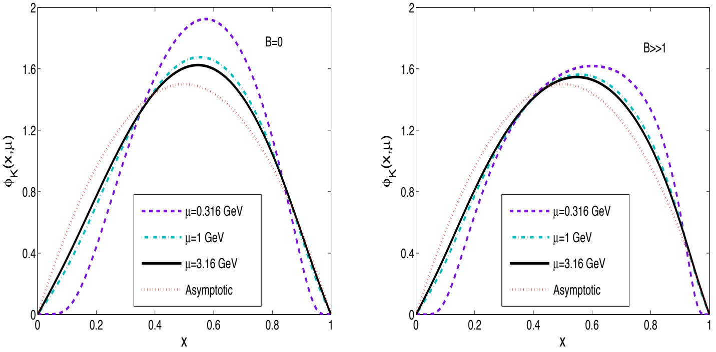
Moments of holographic kaon DA

$$f_K \varphi_K(x,\mu) = 2\sqrt{\frac{N_c}{\pi}} \int \mathrm{d}b_\perp b_\perp J_1(\mu b_\perp)(x\bar{x}M_\pi + B(xm_f + \bar{x}m_s))\frac{\Psi(x,\zeta)}{x\bar{x}}$$

Kaon DA	μ [GeV]	$\langle\xi_1\rangle$	$\langle\xi_2\rangle$	$\langle\xi_3\rangle$	$\langle\xi_4\rangle$
Asymptotic	∞	0	0.200	0	0.085
Holographic ($B=0$)	2	0.047	0.180	0.021	0.067
Holographic ($B \gg 1$)	2	0.081	0.195	0.034	0.081
Lattice	2	0.036(2)	0.26(2)		
Sum Rules	1	0.036	0.286	0.015	0.143
Dyson-Schwinger[RL]	2	0.11	0.24	0.064	0.12
Dyson-Schwinger[DB]	2	0.040	0.23	0.021	0.11

- ERLB evolution from initial scale $\mu_0 = 0.316$ GeV
- Dynamical spin has opposite effects on even and odd moments

Holographic kaon DA



- Left: $B = 0$. Right: $B \gg 1$.
- ERLB evolution from initial scale $\mu_0 = 0.316$ GeV. Kaon DA (unlike pion DA) still distinct from asymptotic DA at $\mu = 2$ GeV.

Meson-to-photon transition form factors

Related to the inverse moment of the DA: Lepage & Brodsky (80)

$$F_{\gamma\pi}(Q^2) = \left(\frac{\hat{e}_u^2 - \hat{e}_d^2}{\sqrt{2}}\right) I(Q^2; m_f, M_\pi)$$

$$F_{\eta\gamma} = \cos\theta F_{\eta_8\gamma} - \sin\theta F_{\eta_1\gamma}$$
$$F_{\eta'\gamma} = \sin\theta F_{\eta_8\gamma} + \cos\theta F_{\eta_1\gamma}$$

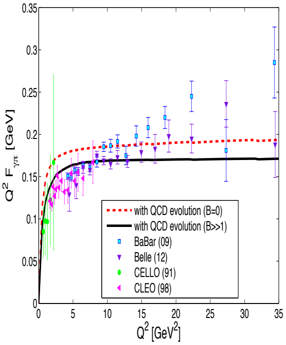
$$F_{\eta_1\gamma}(Q^2) = \left(\frac{\hat{e}_u^2 + \hat{e}_d^2}{\sqrt{3}}\right) I(Q^2; m_q, M_{\eta_1}) + \frac{\hat{e}_s^2}{\sqrt{3}} I(Q^2; m_s, M_{\eta_1})$$

$$F_{\eta_8\gamma}(Q^2) = \left(\frac{\hat{e}_u^2 + \hat{e}_d^2}{\sqrt{6}}\right) I(Q^2; m_q, M_{\eta_8}) - 2\frac{\hat{e}_s^2}{\sqrt{6}} I(Q^2; m_s, M_{\eta_8})$$

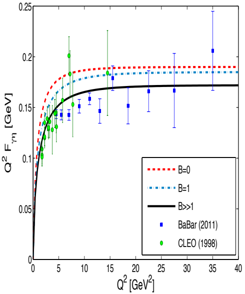
$$I(Q^2; m_f, M_P) = 2\int_0^1 dx \frac{\varphi_P(x, xQ; m_f, M_P)}{xQ^2}$$

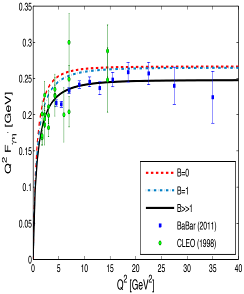
$\pi \to \gamma$ transition form factor

- $B \geq 1$ clearly preferred
- ERBL evolution
- Cannot accommodate BaBar (2009) anomaly



$\eta^{(\prime)} \to \gamma$ transition form factor

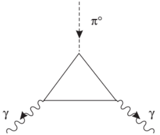




$B \gg 1$ favoured for both η and η'. Mixing angle: $\theta = 14.1^\circ$.

Radiative decay widths

- Using Adler-Bell-Jackiw anomaly relations



- π^0 : $B \geq 1$ preferred
- η : $B = 0$ preferred
- η' : $B \gg 1$ preferred

P	B_l	B_s	$\Gamma^{\text{Th.}}_{P\to 2\gamma}$[KeV]	$\Gamma^{\text{Exp.}}_{P\to 2\gamma}$[KeV]
π^0	0	-	$(5.62 \pm 0.57) \times 10^{-3}$	$(7.82 \pm 0.22) \times 10^{-3}$
	1	-	$(7.72 \pm 0.62) \times 10^{-3}$	
	$\gg 1$	-	$(8.13 \pm 0.68) \times 10^{-3}$	
η	0	0	0.542 ± 0.082	0.516 ± 0.018
	1	0	0.600 ± 0.056	
	1	1	0.622 ± 0.055	
	$\gg 1$	0	0.663 ± 0.061	
	$\gg 1$	$\gg 1$	0.710 ± 0.059	
η'	0	0	3.51 ± 0.48	4.28 ± 0.19
	1	0	3.88 ± 0.49	
	1	1	3.94 ± 0.49	
	$\gg 1$	0	4.51 ± 0.56	
	$\gg 1$	$\gg 1$	4.73 ± 0.57	

Importance of dynamical spin effects less clear for η-η'

Pion TMDs

$$f_{1,\pi}(x, k_\perp^2) = \frac{1}{2}\int \mathrm{d}z^- \mathrm{d}^2 z_\perp e^{iz\cdot k}\langle \pi|\bar{\Psi}(0)[0,z]\gamma^+\Psi(z)|\pi\rangle_{z^+=0}$$

$$h_{1,\pi}^{\perp}(x, k_\perp^2) = \frac{\epsilon^{ij}k_\perp^j}{k_\perp^2}\frac{M_\pi}{2}\int \mathrm{d}z^- \mathrm{d}^2 z_\perp e^{iz\cdot k}\langle \pi|\bar{\Psi}(0)[0,z]i\sigma^{i+}\gamma^5\Psi(z)|\pi\rangle_{z^+=0}$$

To first order in the strong coupling of gauge link

$$f_{1,\pi}(x, \mathbf{k}_\perp^2) = \frac{1}{16\pi^3}\sum_{h,\bar{h}}|\Psi_{h,\bar{h}}(x, \mathbf{k}_\perp^2)|^2$$

$$\frac{k_\perp^2}{M_\pi}h_{1,\pi}^{\perp}(x, \mathbf{k}_\perp^2) = \int \frac{\mathrm{d}^2\mathbf{k}'}{16\pi^3}G(\mathbf{k}_\perp, \mathbf{k}'_\perp)\sum_{h,\bar{h}}\Psi^*_{-h,\bar{h}}(x, \mathbf{k}'_\perp)he^{ih\theta_{k_\perp}}k_\perp\Psi_{h,\bar{h}}(x, \mathbf{k}_\perp$$

where for rescattering via a single-gluon exchange

$$G(\mathbf{k}_\perp, \mathbf{k}'_\perp) = \frac{\alpha_s C_F}{2\pi}\frac{1}{(\mathbf{k}_\perp - \mathbf{k}'_\perp)^2}$$

Holographic TMD PDF

Unpolarized TMD:

$$f_{1,\pi}(x,\mathbf{k}_\perp^2) = \frac{\mathcal{N}^2}{16\pi^3(x\bar{x})^3}[(M_\pi x\bar{x}+Bm_f)^2+B^2\mathbf{k}_\perp^2]\exp\left(-\frac{\mathbf{k}_\perp^2+m_f^2}{\kappa^2 x\bar{x}}\right)$$

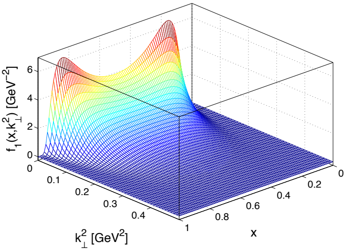
Boer-Mulders TMD:

$$\begin{aligned} h_{1,\pi}^{\perp}(x,\mathbf{k}_\perp^2) &= \frac{B\mathcal{N}^2 M_\pi C_F\alpha_s}{8\pi^4}\frac{(M_\pi x\bar{x}+Bm_f)}{(x\bar{x})^2} \\ &\times \left(\frac{\kappa^2}{\mathbf{k}_\perp^2}\right)\exp\left(-\frac{\mathbf{k}_\perp^2+2m_f^2}{2\kappa^2 x\bar{x}}\right)\left[1-\exp\left(-\frac{\mathbf{k}_\perp^2}{2\kappa^2 x\bar{x}}\right)\right] \end{aligned}$$

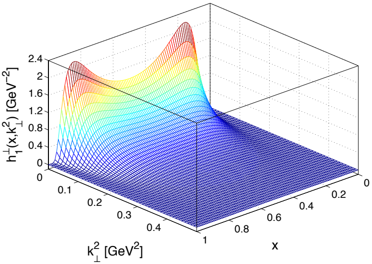
If $B=0$, then $h_{1,\pi}^{\perp}(x,\mathbf{k}_\perp^2)=0$

Dynamical spin effects → Boer-Mulders effects

Holographic pion TMDs



Unpolarized TMD

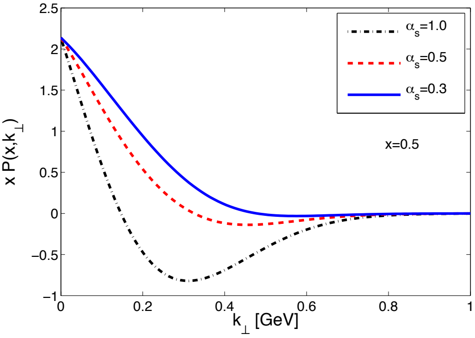


Boer-Mulders TMD

Positivity constraint

Positivity bound on TMDs

$$P(x, \mathbf{k}_\perp^2) \equiv f_{1,\pi}(x, \mathbf{k}_\perp^2) - \frac{k_\perp}{M_\pi} |h_{1,\pi}^\perp(x, \mathbf{k}_\perp^2)| \geq 0$$



hTMDs versus lattice data

A lattice calculation by Engelhardt et al. (PRD, 2016) predicts

$$\langle k_\perp \rangle (b_\perp^2) = M_\pi \frac{\tilde{h}_{1,\pi}^{\perp[1](1)}(b_\perp^2)}{\tilde{f}_{1,\pi}^{[1](0)}(b_\perp^2)}$$

$$\tilde{f}^{[m](n)}(b_\perp^2) = \frac{2\pi n!}{M_\pi^{2n}} \int \mathrm{d}x \, x^{m-1} \int \mathrm{d}k_\perp k_\perp \left(\frac{k_\perp}{b_\perp} \right)^n J_n(b_\perp k_\perp) f(x, k_\perp^2)$$

at $M_\pi = 518$ MeV.

$b_\perp$ [fm]	Lattice 1 [GeV]	Lattice 2 [GeV]	LFHQCD [GeV]
0.27	0.138(28)	0.133(19)	0.0870
0.34	0.128(29)	0.121(16)	0.0867
0.36	0.145(25)	0.148(15)	0.0886

Holographic Boer-Mulders TMD

Preliminary results on holographic pion TMDs:

- With $\alpha_s \leq 0.3$, positivity bound is satisfied and LO truncation justified but lattice data underestimated.
- Perturbative single-gluon exchange is just a first approximation.
- Need to consider non-perturbative, eikonal gluons: Gamberg & Schlegel: Phys. Lett. B685 (2010) 95.

Summary & Outlook

- Universal holographic light-front wavefunction for all mesons
- Spin structure distinguishes between pseudoscalar and vector mesons
- Dynamical spin effects vary in pseudoscalars and are required by the pion data
- Spin-improved holographic pion PDF fits (re-analyzed) E615 Drell-Yan data
- Work in progress for holographic pion TMDs (and GPDs)

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