Towards lattice-assisted hadron physics calculations based on gauge-fixed n-point functions

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in collaboration with

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"Mapping Parton Distribution Amplitudes and Functions"

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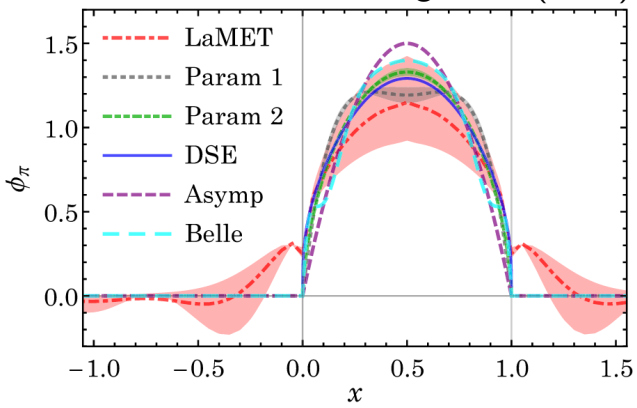
Motivation

Research in hadron physics / QCD thermodynamics

- lattice QCD currently preferred tool to provide theoretical estimates
- full control over systematic error, hard/expensive in practice
- New: PDFs and PDAs available via quasi-function/amplitudes
- Many new studies recently, requires much effort
  (see, e.g., LaMET, ETMC or RQCD approach)

Example:

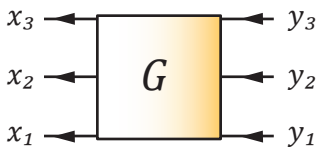
LaMET: Zhang et al. (2017)



Lattice is not the only nonperturbative framework

- Bound-state / Dyson-Schwinger equations
- Functional Renormalization group
- Pros/Cons different to lattice
- Input: nonperturbative n-point functions (fixed gauge)
- **Problem**: truncation of infinite system of equations / of effective action
  (control of systematic error difficult without external input)

Hadron physics calculations



Hadron properties encoded in QCD's n-point functions

[follow review Eichmann et al., Prog.Part.Nucl.Phys 91 (2016) 1]

- information contained in many n-point functions, varying effort to get them
- bound states / resonances = color singlets, poles in n-point functions
- **Example:** quark-antiquark 6-point function

$$G^{\alpha\beta\gamma}_{\delta\eta\rho}(x_1,x_2,x_3|y_1,y_2,y_3) := \langle 0|\, T\psi_\alpha(x_1)\psi_\beta(x_2)\psi_\gamma(x_3)\ \bar\psi_\delta(y_1)\bar\psi_\eta(y_2)\bar\psi_\rho(y_3)|0\rangle$$

spectral decomposition in momentum space

$$G^{\alpha\beta\gamma}_{\delta\eta\rho}(p_f,q_f,P|p_i,q_i,P) \simeq \sum_n \frac{\Psi^{(n)}_{\alpha\beta\gamma}(p_f,q_f,P)\,\bar\Psi^{(n)}_{\delta\eta\rho}(p_i,q_i,P)}{P^2+m_n^2} + \dots$$

- p, q... relative momenta, P ... total momentum
- G and $\Psi^{(n)}$ may be gauge-dependent, but **poles** $P^2 = -m_n^2$ gauge-**independent**
- Pole residue = Bethe-Salpeter wave function $\Psi^{(n)}$
  (coordinate space)

$$\Psi^{(n)}_{\alpha\beta\gamma}(x_1,x_2,x_3,P) = \langle 0|\, T\psi_\alpha(x_1)\psi_\beta(x_2)\psi_\gamma(x_3)|n\rangle$$

Hadron physics calculations

Spectroscopy with lattice QCD

- define gauge-invariant interpolating fields $h(x)$ and $\bar{h}(y)$ at

$$x_1 = x_2 = x_3 = x \quad \text{and} \quad y_1 = y_2 = y_3 = y$$

- extract poles from <u>2-point</u> correlator

$$C(x-y) = \langle 0 | T \underbrace{[\Gamma^{\alpha\beta\gamma}\psi_\alpha\psi_\beta\psi_\gamma](x)}_{h(x)} \underbrace{[\bar{\Gamma}^{\delta\eta\rho}\bar{\psi}_\delta\bar{\psi}_\eta\bar{\psi}_\rho](y)}_{\bar{h}(y)} | 0 \rangle$$

- Spectral decomposition

$$C(\vec{P},t) = \int \frac{d^3\vec{x}}{(2\pi)^4}\, e^{ixP}\, C(\vec{x},t) \quad \xrightarrow{t \gg 0} \quad \frac{e^{-E_0|t|}}{2E_0} |r_0|^2 u_0(\vec{P})\, \bar{u}_0(\vec{P}) \quad + \quad \cdots$$

- time-like pole in momentum space = exponential Euclidean time decay
- baryon mass from exponential decay of $C(\vec{P},t)$
- Pole residues are simple:

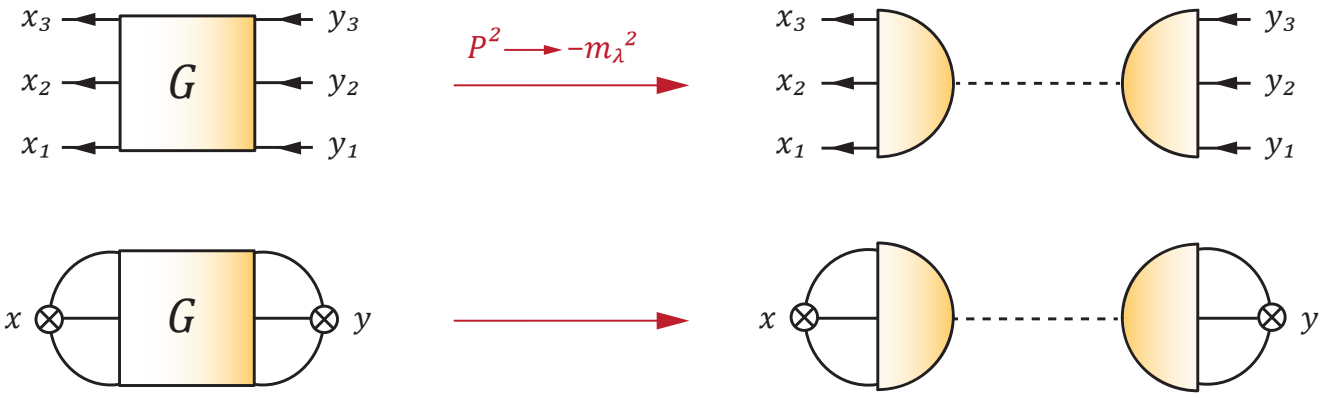
$$\Gamma^{\alpha\beta\gamma}\Psi^{(n)}_{\alpha\beta\gamma}(x,x,x,P) = \langle 0|h(x)|n\rangle = \langle 0|h(0)|n\rangle\, e^{-ixP} = r_n\, u_n(\vec{P})\, e^{-ixP}$$

Hadron physics calculations

Spectroscopy with lattice QCD (put into graphs)

quark-antiquark 6-point function

residue at pole



2-point correlator (lattice)

pole of C(x,y) = decay in Euclidean t

Figure from [Eichmann et al. Prog.Part.Nucl.Phys. 91 (2016) 1, Fig.3.5]

Graphs

- Square = 6-point quark-antiquark function
- half circle = BS wave function (residue)

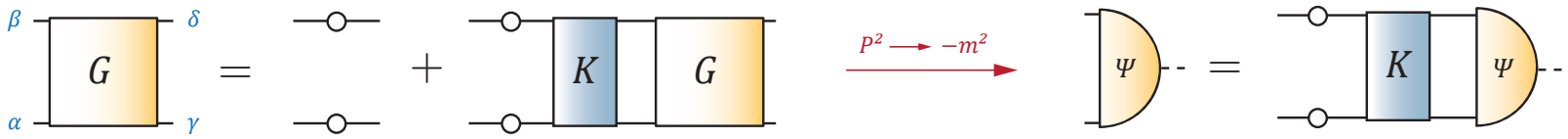
Hadron physics calculations

Functional approach (solving a bound-state equation)

- could apply same approach as lattice QCD
- much simpler: solve self-consistent relation for hadron wave functions Ψ
  (Ψ = residue of pole of n-point function, full information about hadron on its pole)
- resulting equations known as hadron bound-state equations
  Bethe-Salpether / Faddeev equations (for mesons / baryons)

Meson (4-point function = two-particle bound state)

- Dyson equation: $G = G_0 + KG$ $\xrightarrow{P^2 \to -m^2}$ Ψ satisfies BS equation
  (compact notation, Eichmann et al. Prog.Part.Nucl.Phys. 91 (2016) 1, Fig.3.7])



- G_0 ... nonperturbative quark and antiquark propagators (no interaction)
- K ... 4-quark scattering kernel (interaction)

Bethe-Salpether equation for meson amplitude

Meson-BS amplitude

- amputated wave function fulfills $\Gamma = KG_0\Gamma$, i.e.,

$$\Gamma_{\alpha\beta}(p,P) = \int \frac{d^4q}{(2\pi)^4} \, K_{\alpha\gamma,\delta\beta}(p,P;q) \, \{S(q_+) \, \Gamma(q,P) \, S(-q_-)\}_{\gamma\delta}$$

- $\Gamma = 4 \times 4$ Dirac matric, for mesons (J^P) with $J > 0$: $\Gamma \to \Gamma^{\mu_1 \cdots \mu_n}$
- $S =$ nonperturbative quark propagators

Can solve it at least in some truncation (e.g., rainbow-ladder)

- Eigenvalue problem: $\Gamma = \lambda(P^2) \, KG_0\Gamma$
- For all P_n^2 with $\lambda(P_n^2) = 1$ read off mass: $m_n^2 = -P_n^2$ ($m_1 \ldots$ ground state)
- Properties of hadron $(P^2 = -m^2)$ from eigenvector Γ with suitable base $\tau^{(i)}$

$$\Gamma_{\alpha\beta}(p,P) = \sum_i f_i(p^2, p\cdot P; -m^2) \, \tau^{(i)}_{\alpha\beta}(p,P)$$

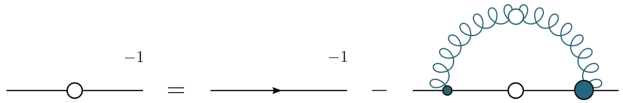
- BSE = infinite system of coupled integral equations for FF $f_i(p^2, p\cdot P; -m^2)$

Bethe-Salpether equation for meson amplitude

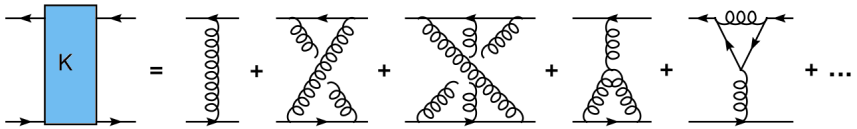
Meson-BS amplitude

$$\Gamma_\pi(p,P) = \int_q^\Lambda \mathcal{K}_{\alpha\gamma,\delta\beta}(p,q,P)\left\{S(q_+)\,\Gamma_\pi(q,P)\,S(-q_-)\right\}_{\gamma\delta}$$

Quark Propagator $\mathcal{S}$ (DSE)

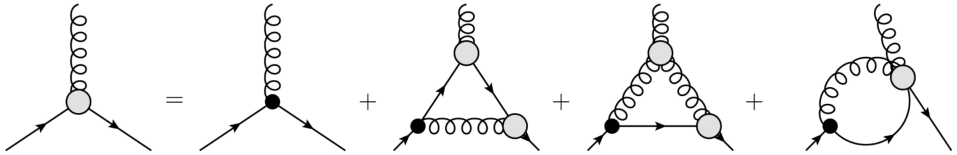


Scattering Kernel $\mathcal{K}$



Quark-Gluon-Vertex (DSE)

(infinite tower of equations)



Gluon propagator (DSE)

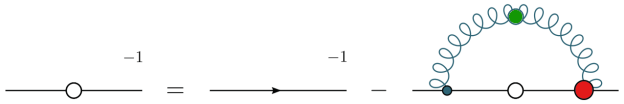
Bethe-Salpether equation for meson amplitude

in Rainbow-Ladder truncation

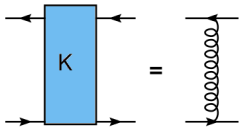
Meson-BS amplitude

$$\Gamma_\pi(p,P) = \int_q^\Lambda \mathcal{K}_{\alpha\gamma,\delta\beta}(p,q,P)\left\{S(q_+)\,\Gamma_\pi(q,P)\,S(-q_-)\right\}_{\gamma\delta}$$

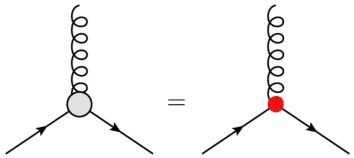
Quark Propagator $\mathcal{S}$ (truncated DSE)



Scattering Kernel $\mathcal{K}$



Quark-Gluon-Vertex (prop. to tree-level)



$$\simeq \gamma_\mu T^a \cdot \Gamma(k^2)$$

Gluon propagator (effective propagator)

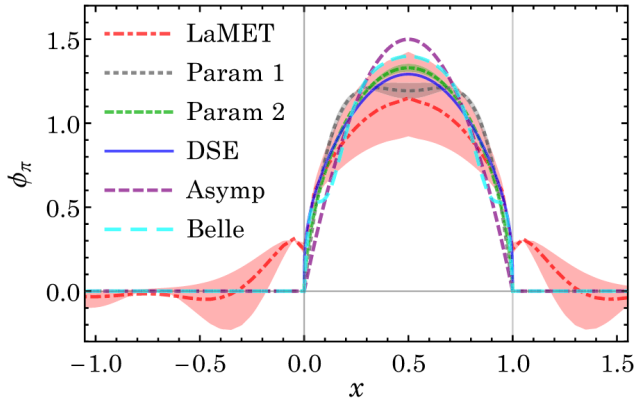
$$D_{\mu\nu}(k) = \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\frac{Z(k^2)}{k^2}$$

Hadron properties from bound-state amplitude

Bound-state amplitude gives access to . . .

- form factors: electromagnetic, transition
- PDAs, PDFs, GPDs, . . .

LaMET: Zhang et al. (2017)



Ex 1: pion DA [e.g. Chang et al., PRL110(2013)092001]
(projection onto light front)

$$\phi_\pi(x) = \frac{1}{F_\pi} \, \mathrm{Tr} \, Z_2 \int_q \delta^x_\zeta(q_+) \, \gamma\cdot\zeta \, \gamma_5 \, \Gamma_\pi(k,P)$$

- DSE/BSE calculation via ~ 50 Mellin moments
- Lattice calculation, either via moments (~ 2) or directly (e.g., LaMET, RQCD)

Ex 2: pion form factor $F_\pi(Q^2)$
(impulse approximation)

[Maris/Tandy (2000)]
$(q_\pm = q \pm Q/2, k_\pm = q \pm Q/4)$

$$P_\mu F_\pi(Q^2) = -\int_q \mathrm{Tr}\left[\Gamma_\pi(k_+,-P_+) \, S(q) \, i \, \underbrace{\Gamma_\mu(q_+;Q)}_{\text{quark-photon vertex}} \, S(q+Q) \, \Gamma_\pi(k_-,-P_-) \right]$$

Lattice-assisted bound-state equations

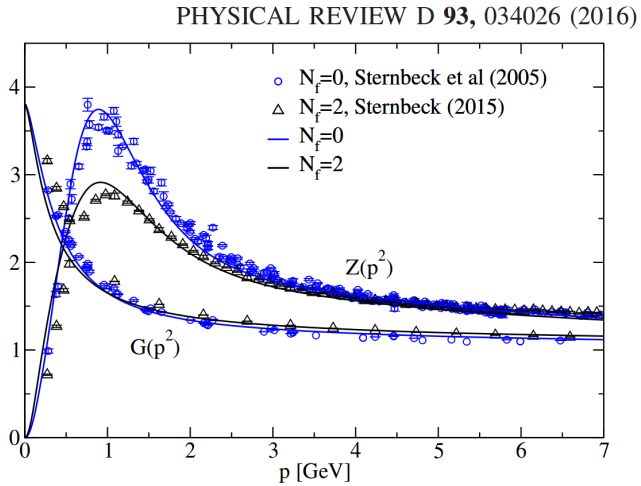
Lattice can help to control systematic error of truncation

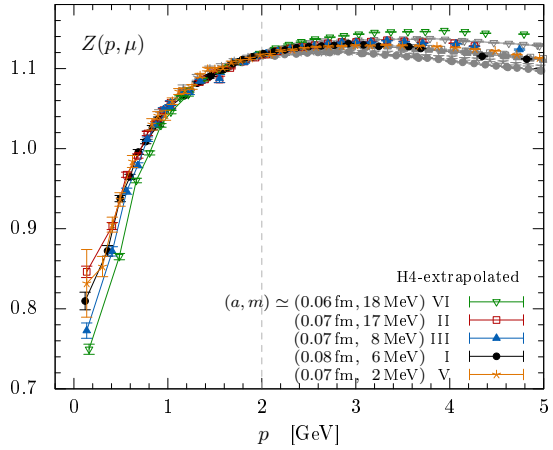
- Lattice QCD + gauge-fixing: access to n-point functions
- Helped to settle momentum dependence of 2-point functions Landau gauge

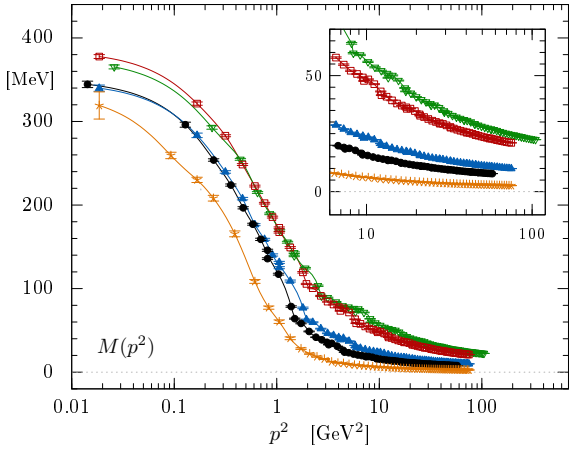
$$D_{\mu\nu}(p) = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\frac{Z(p^2)}{p^2}, \qquad G(p) = \frac{J(p^2)}{p^2}, \qquad S(p) = \frac{Z(p^2)}{i\not{p} + M(p^2)}$$

Lattice results are untruncated

new arXiv:1809.02541







gluon, ghost, quark wave and quark mass dressing

Lattice-assisted bound-state equations

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Lattice results are untruncated

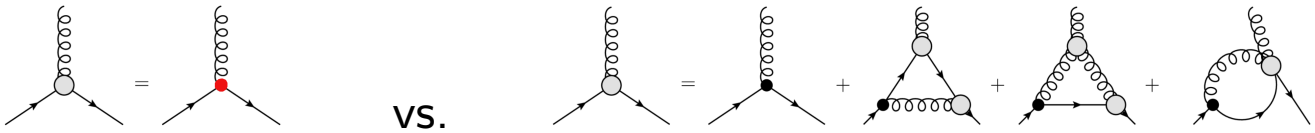
- but discretization and volumes effects should not be ignored
- $H(4)$ symmetry causes deviations at large p^2: $F(p^2) \to F(p)$
- Wilson term changes momentum behavior $\propto O(a^2p^2)$
- Challenge: continuum- and infinite-volume extrapolated results
- very large & fine lattices are required

Good news: Lattice methods currently keep up with required precision

Projects on 3-point functions in Landau gauge

Most important: Lattice-QCD input for 3-point functions:

- **Quark-gluon / Triple-gluon vertex** *AS* et al.(2016) arXiv:1702.00612
  (a) in collaboration with Kızılersü, Oliveira, Silva, Skullerud, Williams (work in progress)
  (b) BSc. project with P. Balduf (FSU Jena) (to appear)



- **BS wave functions** (work in progress)

$$\Gamma_\pi(p,P) = \int_q^\Lambda \mathcal{K}_{\alpha\gamma,\delta\beta}(p,q,P)\{S(q_+)\,\Gamma_\pi(q,P)\,S(-q_-)\}_{\gamma\delta}$$

- **Tensor structure of quark bilinears**

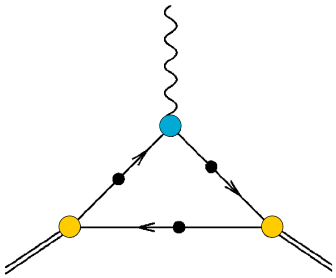
$$G(x,y) = \langle \psi(x)\bar{\psi}(z)\,\Lambda\,\psi(z)\bar{\psi}(y)\rangle \qquad \text{where} \quad \Lambda = \gamma_\mu,\ \gamma_5\gamma_\mu,\ \sigma_{\mu\nu},\ldots$$

  - Full tensor structure of underlying vertex, Example: **Quark-photon vertex**
  - Reuse/new data from/for RI'(S)MOM renormalization program

Hadron properties from bound-state amplitude

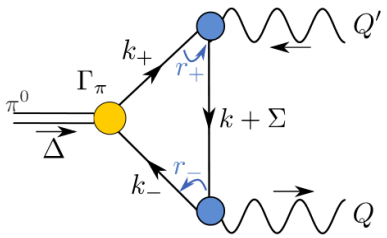
Ex 2: pion form factor $F_\pi(Q^2)$

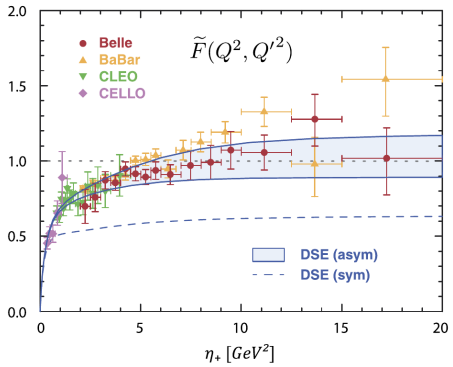
(impulse approximation)



Ex 3: Meson transition form factor $\pi^0 \to \gamma^*\gamma^*$

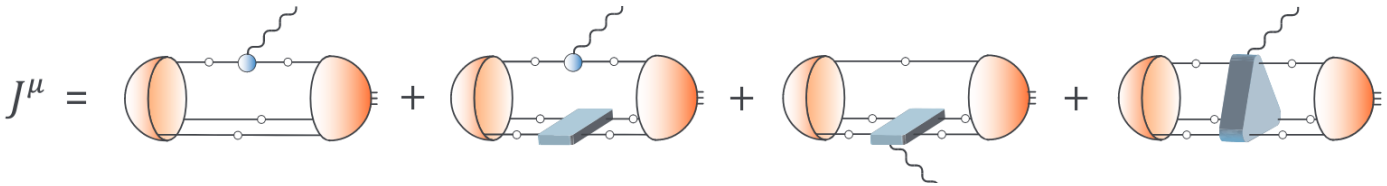
(Weil et al. (2017), impulse approximation)





Ex 4: Nucleon electromagnetic current

(Eichmann, PRD84 (2011) 014014)



Require

- nonperturbative quark propagators (quarks DSE | lattice data → this talk)
- BS amplitudes Γ (truncated BSE | lattice: work in progress)
- Quark-photon vertex (quark-photon BSE | first lattice data → this talk)

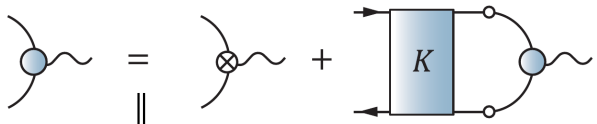
Quark-Photon (γ_μ) vertex as solution of BS equation

QCD dressing of tensor structure

- key to any BS/DSE calculation of electromagnetic properties of hadrons
  (elastic and transition form factors, HVP, ...)

$$\Gamma_\mu(k,Q) = \underbrace{i\gamma_\mu\lambda_1 + 2k^\mu[i\not{k}\,\lambda_2 + \lambda_3]}_{\Gamma_\mu^{\mathrm{BC}}} + \sum_{j=1}^{8} i f_j\, T_\mu^{(j)}(k,Q)$$

- Γ_μ satisfies <u>inhomogeneous BS equation</u> and Γ_μ^{BC} vector WT identity



$$Q_\mu\Gamma_\mu = S^{-1}(k_-) - S^{-1}(k_+)$$

$$\Gamma_\mu(k,Q) = Z_2\gamma_\mu - Z_2^2\frac{4}{3}\int_q [S(q+Q/2)\,\Gamma_\mu(q,Q)\,S(q-Q/2)]\,K(k-q)$$

- In practice: "rainbow-ladder" truncated quark DSE and kernel ...systematic error?

$$K_{\rho\sigma,\alpha\beta}(k) = G(k^2)\,T^k_{\mu\nu}\gamma_\mu^{\alpha\rho}\gamma_\nu^{\sigma\beta} \qquad T^k_{\mu\nu}G(k^2)\ldots\text{eff. gluon propagator}$$

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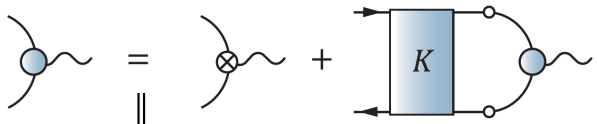
- Γ_μ satisfies <u>inhomogeneous BS equation</u> and Γ_μ^{BC} vector WT identity



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- In practice: "rainbow-ladder" truncated quark DSE and kernel ...systematic error?

$$K_{\rho\sigma,\alpha\beta}(k) = G(k^2)\,T^k_{\mu\nu}\gamma_\mu^{\alpha\rho}\gamma_\nu^{\sigma\beta} \qquad T^k_{\mu\nu}\,G(k^2)\ldots\text{eff. gluon propagator}$$

Quark-Photon (γ_μ) vertex from lattice QCD

Monte Carlo averages: quark-γ_μ bilinear and propagator in Landau gauge
(Inverse of D_U via momentum sources, numerically demanding)

$$(1) \qquad G_\mu(k,Q) = \sum_{x,y,z} e^{ik_+(x-z)} e^{ik_-(z-y)} \left\langle [D_U^{-1}]_{xz} \gamma_\mu [D_U^{-1}]_{zx} \right\rangle_U$$

$$(2) \qquad S(k_\pm) = \sum_{x,y} e^{ik_\pm(x-y)} \left\langle [D_U^{-1}]_{xy} \right\rangle_U \qquad \text{with} \quad k_\pm = k \pm \frac{Q}{2}$$

Vertex from amputated 3-point function

$$\Gamma_\mu(k,Q) = S^{-1}\left(k - \frac{Q}{2}\right) G_\mu(k,Q)\, S^{-1}\left(k + \frac{Q}{2}\right)$$

Form factors from solving

$$\Gamma_\mu(k,Q) = i\gamma_\mu \lambda_1 + 2k^\mu [i\not{k}\, \lambda_2 + \lambda_3] \; + \; \sum_{j=1}^{8} i f_j \, T_\mu^{(j)}(k,Q)$$

Parameters of our gauge field ensembles ($N_f = 2$)

Lattice action

- Wilson gauge action
- Wilson clover fermions
- Landau gauge
  (after thermalization)

Can study:

- quark mass dependence
- discret./ volume effects

β	κ	$L_s^3 \times L_t$	a [fm]	m_b [MeV]	m_π [MeV]
5.20	0.13584	$32^3 \times 64$	0.08	14	411
5.20	0.13596	$32^3 \times 64$	0.08	6	280
5.29	0.13620	$32^3 \times 64$	0.07	17	422
5.29	0.13632	$32^3 \times 64$	0.07	8	295
5.29	0.13632	$64^3 \times 64$	0.07	8	290
5.29	0.13640	$64^3 \times 64$	0.07	2	150
5.40	0.13647	$32^3 \times 64$	0.06	18	426
5.40	0.13660	$48^3 \times 64$	0.06	7	260

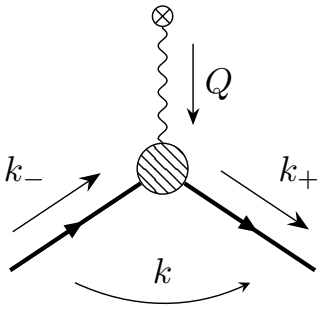
Consider:

- local vector current: $\bar{\psi}_x \gamma_\mu \psi_x$
- twisted boundary condition for quarks

Acknowledgements

- $N_f = 2$ configurations provided by RQCD collaboration (Regensburg)
- Gauge-fixing and calculation of propagators at the HLRN, LRZ and FSU Jena

Quark-Photon vertex: discrete lattice momenta



Momenta quarks: $k_\pm = k \pm Q/2$

- twisted b.c. for quarks: $\psi(x + \hat{\mu} L_\mu) = e^{i\theta_\mu}\psi(x)$

$$ak_\mu = \frac{2\pi n_\mu}{L_\mu} + \frac{\theta_\mu}{L_\mu} \equiv \frac{2\pi}{L_\mu}\left(n_\mu + \frac{\tau_\mu}{2}\right) \qquad \text{twist angle:} \quad \theta_\mu = \pi\tau_\mu/L_\mu$$

- **symmetric** setup (preferred but no access to $\lambda_2, \lambda_3, f_5, f_7$)

$$Q^2 = k_-^2 = k_+^2, \qquad n^- = n\,(1,1,0,0) + (\tau,\tau,0,0)\,, \qquad z \equiv \frac{k\cdot Q}{|k||Q|}$$

$$z = 0 \qquad n^+ = n\,(0,1,1,0) + (0,\tau,\tau,0)$$

- **asymmetric** setup

$$Q^2 = k_-^2 > k_+^2, \qquad n^- = n\,(2,1,0,0) + (2\tau,\tau,0,0)$$

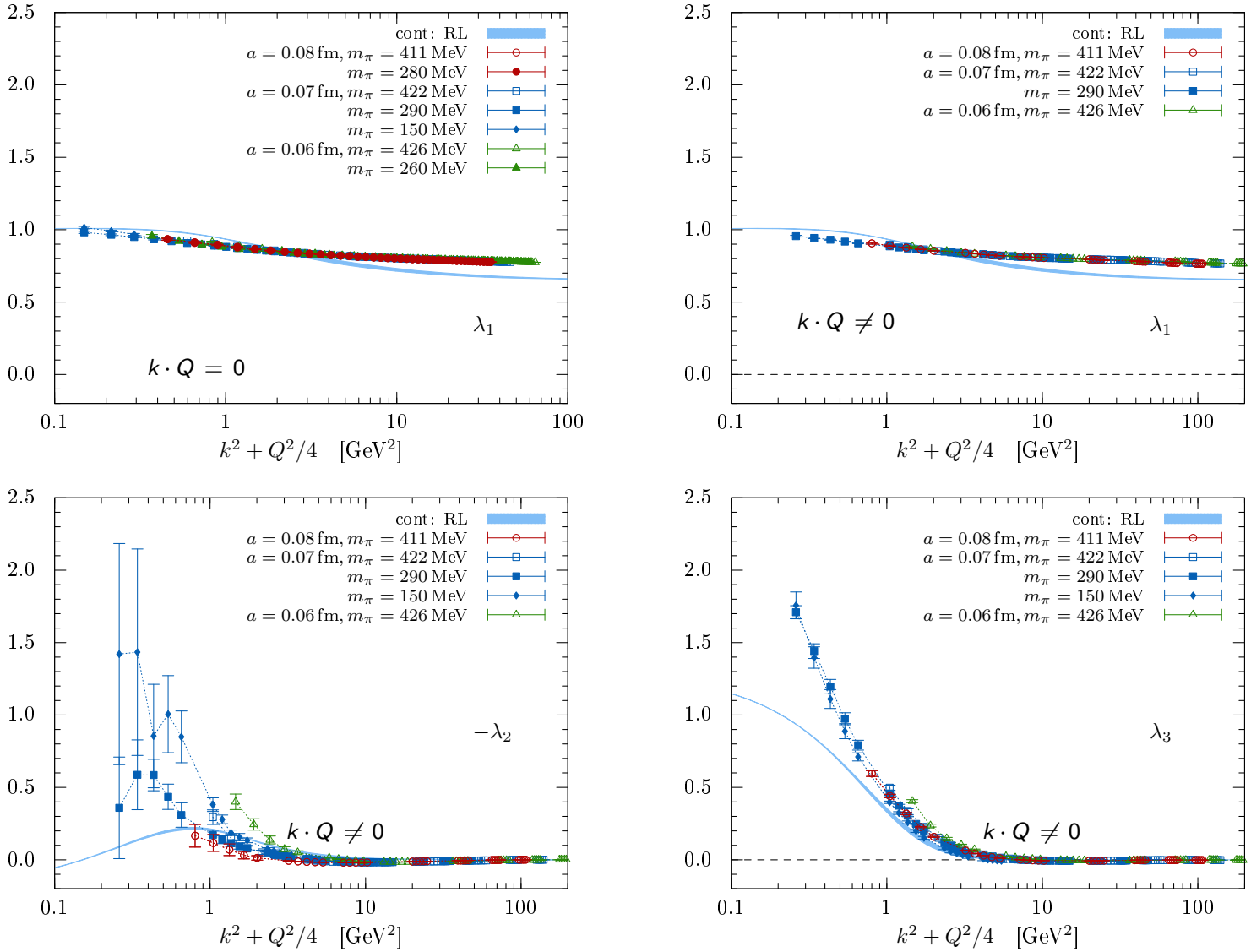
$$z = \frac{1}{\sqrt{5}} \qquad n^+ = n\,(0,1,1,0) + (0,\tau,\tau,0)$$

- discrete values: $n = 1, 2, \ldots, L_s/4$ and twists $\tau = 0, 0.4, 0.8, 1.2$ and 1.6
(gives smooth interpolation)

Lattice results

$$\Gamma_\mu = i\gamma_\mu\lambda_1 + 2k^\mu[i\not{k}\,\lambda_2 + \lambda_3] \;+\; \sum_{j=1}^{8} i f_j T_\mu^{(j)}(k,Q)$$

(preliminary) vs. continuum (rainbow-ladder)



(Rainbow-ladder results is from G. Eichmann)

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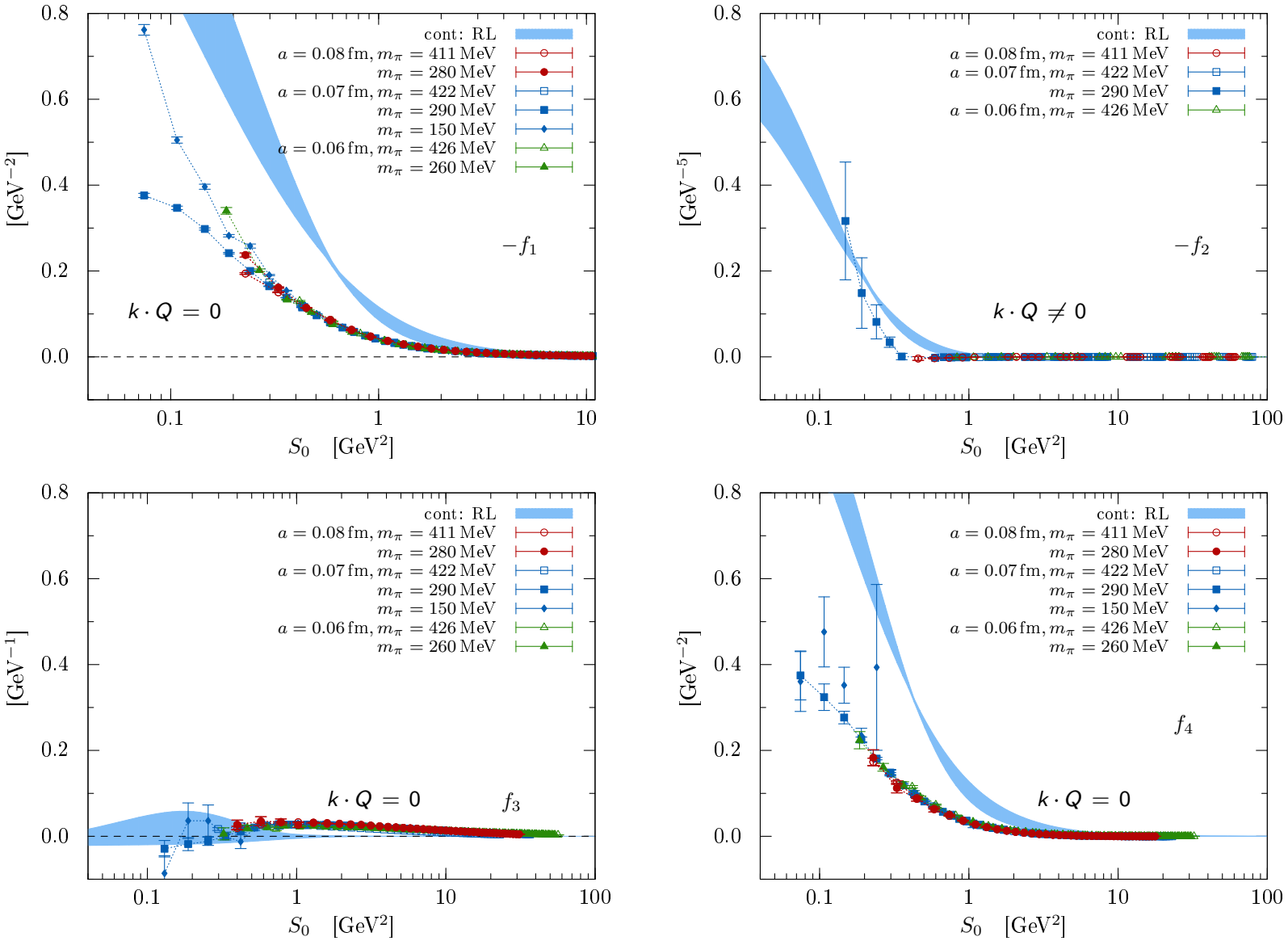
$S_0 = k^2/3 + Q^2/4$



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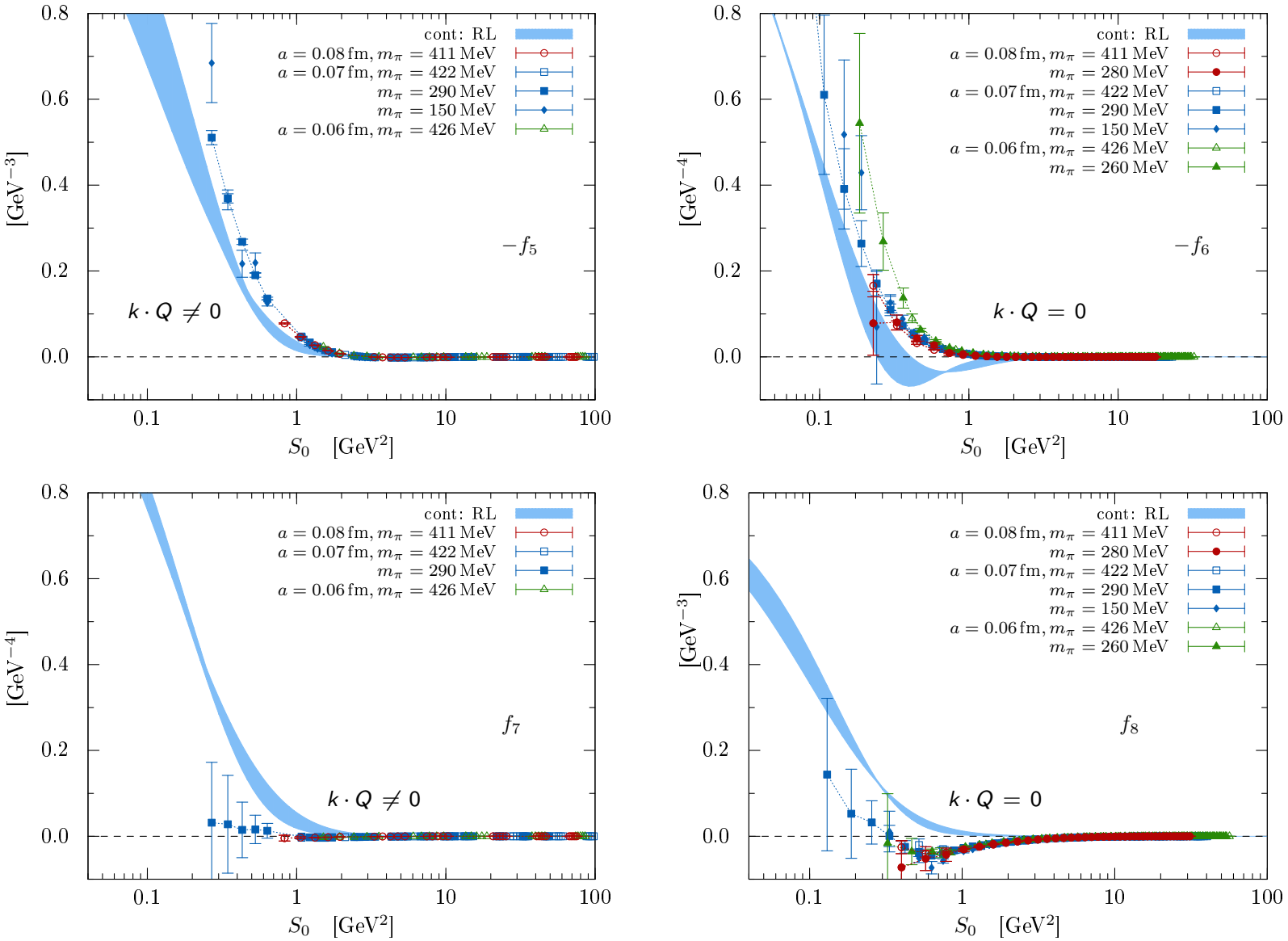
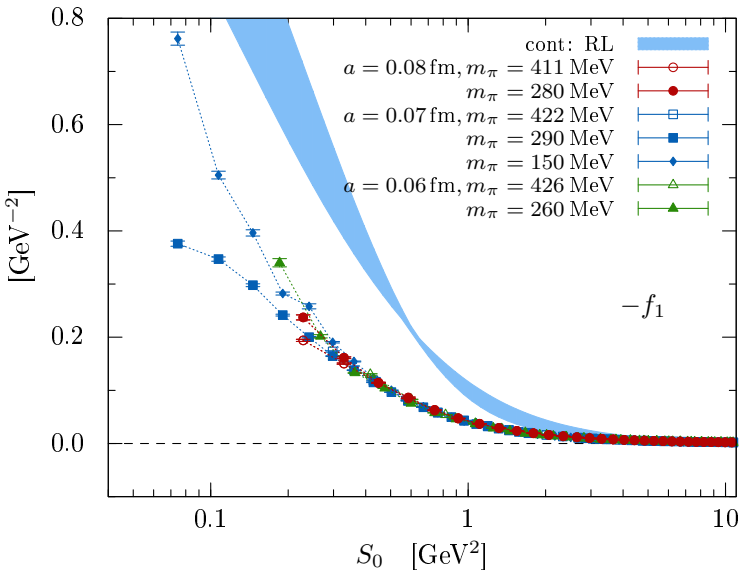


(Rainbow-ladder results is from G. Eichmann)

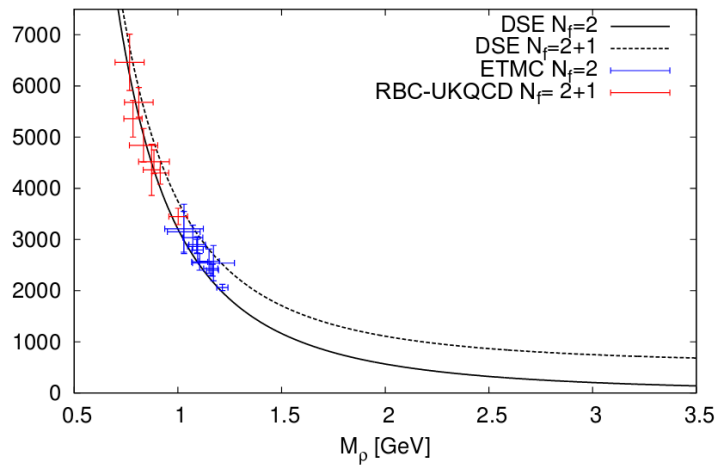
Improvement of lattice discretization

Lattice results for form factors

- volume, discretization effects smaller than deviations to RL-results
- $k \cdot Q$ dependence small
- lattice results sufficient for currently required precision
- good hint for improved truncations



Hadron vacuum polarization



Quark-Photon-Vertex

- contains information about QCD contributions to $a_\mu = \frac{1}{2}(g_\mu - 2)$ (LbL and HVP)

$$a_\mu^{HVP} = \frac{\alpha}{\pi} \int_0^1 dx(1-x) \left[-e^2 \Pi_R \left(\frac{x^2 m_\mu^2}{1-x} \right) \right]$$

HVP: Fourier transform of hadronic part of vector-vector current correlator

$$\Pi^{\mu\nu}(Q) = \int d^4x \, e^{iQ\cdot x} \langle 0 | \, \mathrm{T} j^\mu(x) j^\nu(0) \, | 0 \rangle = \left(Q^2 \delta_{\mu\nu} + Q_\mu Q_\nu \right) \Pi(Q^2)$$

$$= \mathrm{Tr} \int \frac{d^4k}{(2\pi)^4} \, Z_2 \, i\gamma^\mu \, S(k_+) \, \Gamma^\nu(k, Q) \, S(k_-)$$

Requires nonperturbative

- quark propagator (quarks DSE | lattice data)
- Quark-photon vertex (quark-photon BSE | first lattice data)

Improvement of lattice discretization

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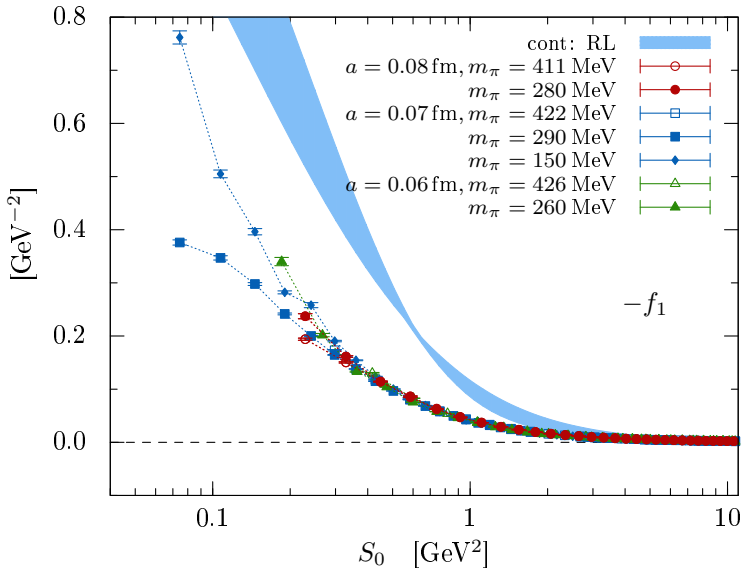


Lattice artefacts

- lattice QCD can provide untruncated result as input
- but discretization and volumes effects cannot be ignored
- $H(4)$ symmetry causes deviations at large k^2, Q^2: $f(k^2, Q^2) \to f(k, Q)$
- Wilson term changes momentum behavior $\propto O(a^2 p^2)$
- lattice offshell-improvement to better reach continuum-extrapolated results

Vector WT identity and the lattice

Continuum: vector Ward-Takahashi identity (vWTI)

$$Q_\mu G_\mu(k,Q) = S(k_-) - S(k_+) \qquad \text{with} \quad G_\mu = S(k_-)\,\Gamma_\mu(k,Q)\,S(k_+)$$

Lattice: vWTI for Wilson fermions and naive current $(k_\pm = k \pm Q/2)$

- 3-point fcts.:

$$G_\mu(k,Q) = \sum_{x,y,z} e^{ik_-(x-z)} e^{-ik_+(y-z)} \left\langle \psi_x V_\mu(z)\,\bar\psi_y \right\rangle$$

$$= Z_V(g^2) \overbrace{\sum_{x,y,z} e^{ik_-(x-z)} e^{-ik_+(y-z)} \left\langle \psi_x \tilde V_\mu(z)\,\bar\psi_y \right\rangle}^{\tilde G_\mu(k,Q)} + O(a)$$

- point-split vector current satisfies WTI [Karsten/Smit (1981)]

$$V^f_\mu(z) = \frac{1}{2}\left(\bar\psi_z[\gamma_\mu - 1]U_{x\mu}\frac{\lambda^f}{2}\psi_{z+a\hat\mu} + \bar\psi_{z+a\hat\mu}[\gamma_\mu + 1]U^\dagger_{x\mu}\frac{\lambda^f}{2}\psi_z \right)$$

$$= \bar\psi_z\gamma_\mu\frac{\lambda^f}{2}\psi_z + aK_\mu(z) \equiv Z_V(g_0^2)\,\tilde V^f_\mu(z) + O(a)$$

Vector WT identity and the lattice

Continuum: vector Ward-Takahashi identity (vWTI)

$$Q_\mu G_\mu(k,Q) = S(k_-) - S(k_+) \qquad \text{with} \quad G_\mu = S(k_-)\,\Gamma_\mu(k,Q)\,S(k_+)$$

Lattice: vWTI for Wilson fermions and naive current

$$Z_V(g_0^2)\cdot Q_\mu \tilde{G}_\mu(k,Q) = S(k_-) - S(k_+) + O(a;k,Q)$$

Test: calculate $\lambda_1(k,Q)$ twice

1. from vertex data

$$\Gamma_\mu = S^{-1}(k_-)\,\tilde{G}_\mu(k,Q)\,S^{-1}(k_+) \quad = \quad i\gamma_\mu\lambda_1 + 2k^\mu[i\not{k}\,\lambda_2 + \lambda_3] \;+\; \sum_{j=1}^{8} i f_j\, T_\mu^{(j)}(k,Q)$$

2. from data for quark propagator S

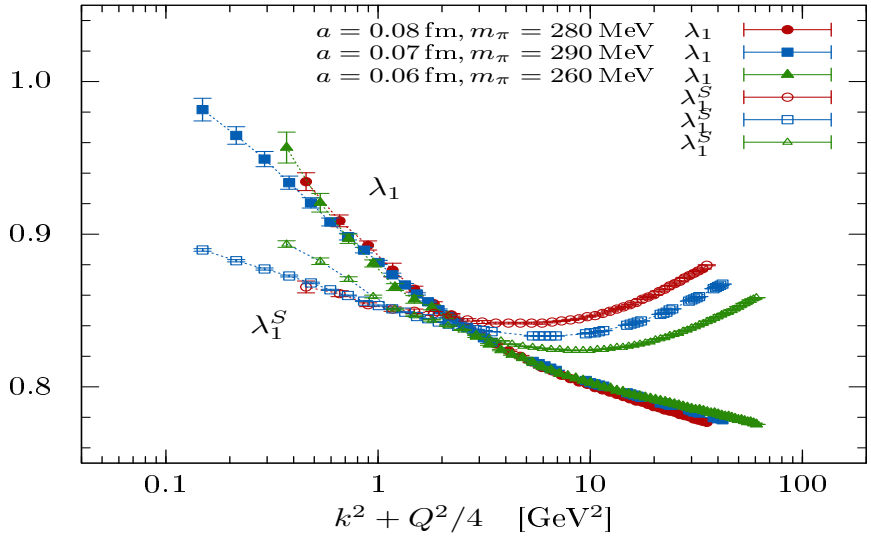
$$\lambda_1^S = \frac{A(k_+^2) + A(k_-^2)}{2} \qquad \text{where} \quad S^{-1}(p) = i\gamma_\mu \sin ap_\mu\, A(p) + B(p)$$

Vector WT identity and the lattice: λ_1 vs. λ_1^S

Find

- deviations grow with $k^2 + Q^2/4$
- discret. effects larger for λ_1^S
- surprised about deviations at small $k_\pm^2$
- Which is correct λ_1 or λ_1^S?

($\approx$ same m_b, $a = 0.08 \ldots 0.06$ fm)



Test: calculate $\lambda_1(k, Q)$ twice

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2. from data for quark propagator S

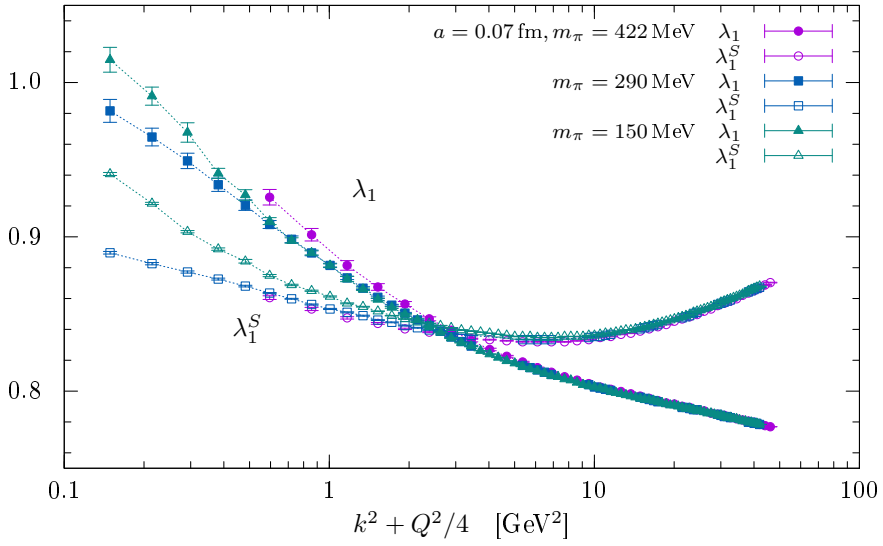
$$\lambda_1^S = \frac{A(k_+^2) + A(k_-^2)}{2} \qquad \text{where} \quad S^{-1}(p) = i\gamma_\mu \sin a p_\mu\, A(p) + B(p)$$

Vector WT identity and the lattice: λ_1 vs. λ_1^S

Find

- deviations grow with $k^2 + Q^2/4$ and m
- discret. effects larger for λ_1^S
- surprised about deviations at small $k_\pm^2$
- Which is correct λ_1 or λ_1^S?

(same a, $m_b = 2 \ldots 17$ MeV)



Test: calculate $\lambda_1(k, Q)$ twice

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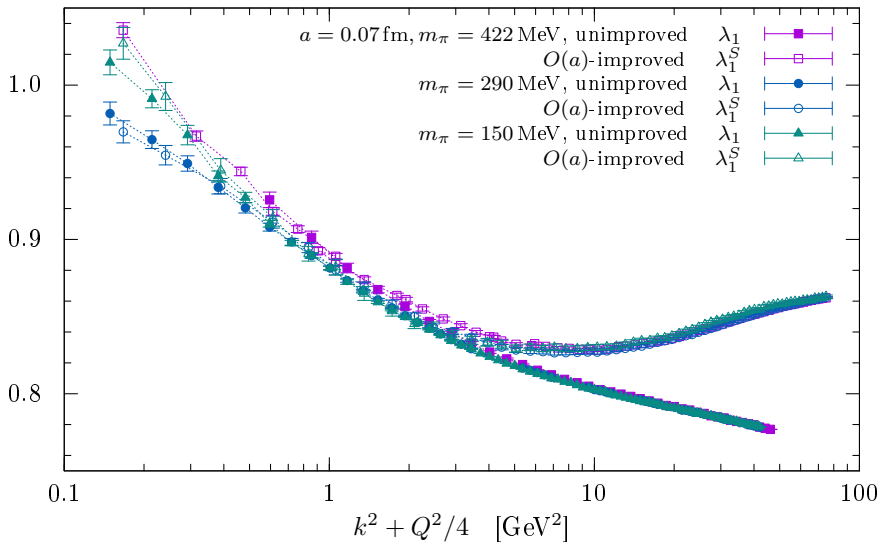
Vector WT identity and the lattice: λ_1 vs. λ_1^S

(same a, $m_b = 2 \ldots 17$ MeV)



Find

- deviations grow with $k^2 + Q^2/4$ and m
- discret. effects larger for λ_1^S
- surprised about deviations at small $k_\perp^2$
- Which is correct λ_1 or λ_1^S?
  cancelation effects improve λ_1

First data for O(a)-improved propagator

[Oliveira, Silva, Skullerud, *A.S.*, arXiv:1809.02541]

$$S_{\mathrm{rot}}(k) = (1 + 2b_q am) \cdot \sum_{x,y} e^{-ik(x-y)} \left\langle L_x(U)\, M_{xy}^{-1}(U)\, R_y(U) \right\rangle_U$$

Improves WTI!

- M is Wilson clover-fermion matrix
- left/right rotation: $L_x(U) \equiv \left[1 - c_q a \overrightarrow{\not{D}}_x(U)\right], \quad R_y(U) \equiv \left[1 + c_q a \overleftarrow{\not{D}}_y(U)\right]$
- has smaller $O(a^2)$ effects than other definitions, see e.g. QCDSF (2001)

Summary and Outlook

First lattice data for Quark-Photon vertex (ever)

- Volume, discretization effects smaller than deviations to continuum
- give good hints for improving currently used truncations

Future: high precision

- lattice implementation should be O(a)-improved (off-shell)
- work out lattice corrections (nothing available), improve statistics at low momentum

Other vertices: full tensor structure also needed

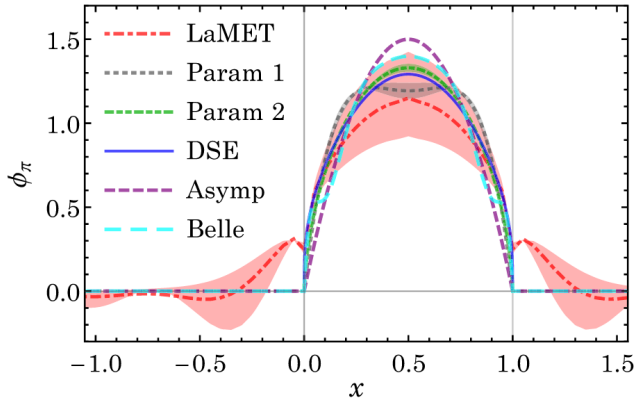
- $G(x,y) = \langle \psi(x)\bar{\psi}(z)\, \Lambda\, \psi(z)\bar{\psi}(y) \rangle$ where $\Lambda = \gamma_\mu,\ \gamma_5\gamma_\mu,\ \sigma_{\mu\nu}, \ldots$
  - $\boldsymbol{\sigma_{\mu\nu}}$ **vertex** for proton tensor charges
    see, e.g., [recent paper Wang et al. (2018)]
  - **Axialvector vertex** ($\boldsymbol{\gamma_5\gamma_\mu}$) required for axial form factors
    see, e.g., [Eichmann & Fischer, EPJA48 (2012) 9]
  - **Pseudoscalar vertex** ... all the usual lattice hadron physics quantities.
- Data taken, work in progress

Hadron properties from bound-state amplitude

Bound-state amplitude gives access to . . .

- form factors: electromagnetic, transition
- PDAs, PDFs, GPDs, . . .

LaMET: Zhang et al. (2017)



Ex 1: pion DA [e.g. Chang et al., PRL110(2013)092001]
(projection onto light front)

$$\phi_\pi(x) = \frac{1}{F_\pi} \operatorname{Tr} Z_2 \int_q \delta^x_\zeta(q_+)\, \gamma\cdot\zeta\, \gamma_5\, \Gamma_\pi(k,P)$$

- DSE/BSE calculation via ~ 50 Mellin moments
- Lattice calculation, either via moments (~ 2) or directly (e.g., LaMET, RQCD)

Ex 2: pion form factor $F_\pi(Q^2)$
(impulse approximation)

[Maris/Tandy (2000)]
$(q_\pm = q \pm Q/2,\, k_\pm = q \pm Q/4)$

$$P_\mu F_\pi(Q^2) = -\int_q \operatorname{Tr}\Big[\Gamma_\pi(k_+,-P_+)\; S(q)\; i\, \underbrace{\Gamma_\mu(q_+;Q)}_{\text{quark-photon vertex}}\; S(q+Q)\; \Gamma_\pi(k_-,-P_-)\Big]$$

Summary and Outlook: Hadronic wave functions on the lattice

Meson-BS amplitude

$$\Gamma_\pi(p,P) = \int_q^\Lambda \mathcal{K}_{\alpha\gamma,\delta\beta}(p,q,P)\left\{S(q_+)\,\Gamma_\pi(q,P)\,S(-q_-)\right\}_{\gamma\delta}$$

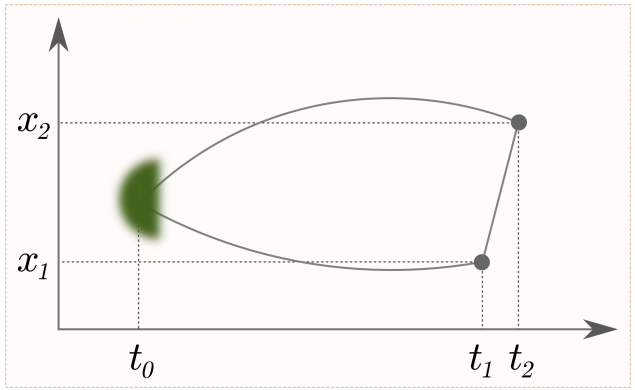
Meson-BS wave function

- momentum space

$$\phi_\pi(p,P) = S^{-1}(q_+)\,\Gamma_\pi(p,P)\,S^{-1}(q_-)$$

- coordinate space

$$\phi_\pi(x_1-x_2,P) = \langle 0|\,T[\psi_1(x_1)\,\bar\psi_2(x_2)]\,|\pi(P)\rangle$$



- caveat: restriction to (almost) equal time
- similar as RQCD's quantity for calculation of PDAs

Aim: map out (part of) tensor structure e.g., for pion of isospin k

$$\phi_{\pi_k}(r,P) = \frac{\sigma_k}{2}\gamma_5\Big[-if_1 + i\gamma_\mu P_\mu f_2 + \gamma_\mu x_\mu f_3 + \sigma^{\mu\nu}P_\mu x_\nu f_4\Big],$$

- FF f_j = functions of Lorentz-invariants: $x\cdot P = \vec{r}\cdot\vec{P},\ x^2 = r^2,\ P^2 = m_\pi^2$

Thank you for your attention!