Plotting pion's light front wave function

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Mapping Parton Distribution Amplitude and Function,
2018/09/10, ECT*, Trento, Italy

FACT-I

Particle Data Group

Citation: C. Patrignani *et al.* (Particle Data Group), Chin. Phys. C, **40**, 100001 (2016) and 2017 update

g or gluon

$I(J^P) = 0(1^-)$

SU(3) color octet

Mass $m = 0$. Theoretical value. A mass as large as a few MeV may not be precluded, see YNDURAIN 95.

VALUE	*DOCUMENT ID*		*TECN*	*COMMENT*
• • • We do not use the following data for averages, fits, limits, etc. • • •				
	ABREU	92E	DLPH	Spin 1, not 0
	ALEXANDER	91H	OPAL	Spin 1, not 0
	BEHREND	82D	CELL	Spin 1, not 0
	BERGER	80D	PLUT	Spin 1, not 0
	BRANDELIK	80C	TASS	Spin 1, not 0

gluon REFERENCES

YNDURAIN	95	PL B345 524	F.J. Yndurain	(MADU)
ABREU	92E	PL B274 498	P. Abreu *et al.*	(DELPHI Collab.)
ALEXANDER	91H	ZPHY C52 543	G. Alexander *et al.*	(OPAL Collab.)
BEHREND	82D	PL B110 329	H.J. Behrend *et al.*	(CELLO Collab.)
BERGER	80D	PL B97 459	C. Berger *et al.*	(PLUTO Collab.)
BRANDELIK	80C	PL B97 453	R. Brandelik *et al.*	(TASSO Collab.)

In QCD: Gluons become massive!

Nankai University

PHYSICAL REVIEW VOLUME 125, NUMBER 1 JANUARY 1, 1962

Gauge Invariance and Mass

JULIAN SCHWINGER
Harvard University, Cambridge, Massachusetts, and University of California, Los Angeles, California
(Received July 20, 1961)

It is argued that the gauge invariance of a vector field does not necessarily imply zero mass for an associated particle if the current vector coupling is sufficiently strong. This situation may permit a deeper understanding of nucleonic charge conservation as a manifestation of a gauge invariance, without the obvious conflict with experience that a massless particle entails.

- Schwinger 1962

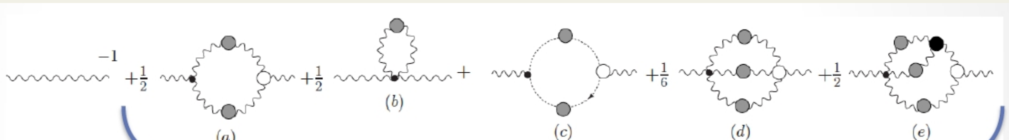
PHYSICAL REVIEW D VOLUME 26, NUMBER 6 15 SEPTEMBER 1982

Dynamical mass generation in continuum quantum chromodynamics

John M. Cornwall
Department of Physics, University of California, Los Angeles, California 90024
(Received 30 April 1982)

- Cornwall 1982

$\Delta^{-1}_{\mu\nu}(q) =$



$\Pi_{\mu\nu}(q)$

$\Pi_{\mu\nu}(q) = P_{\mu\nu}(q)\Pi(q)$

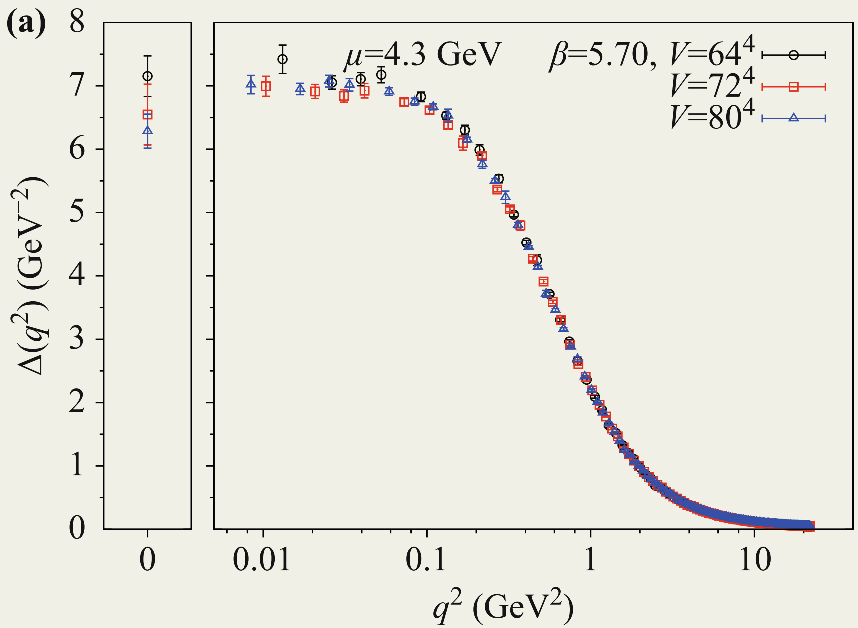
$P_{\mu\nu}(q) = g_{\mu\nu} - q_\mu q_\nu / q^2$

- Binosi & Papavassiliou

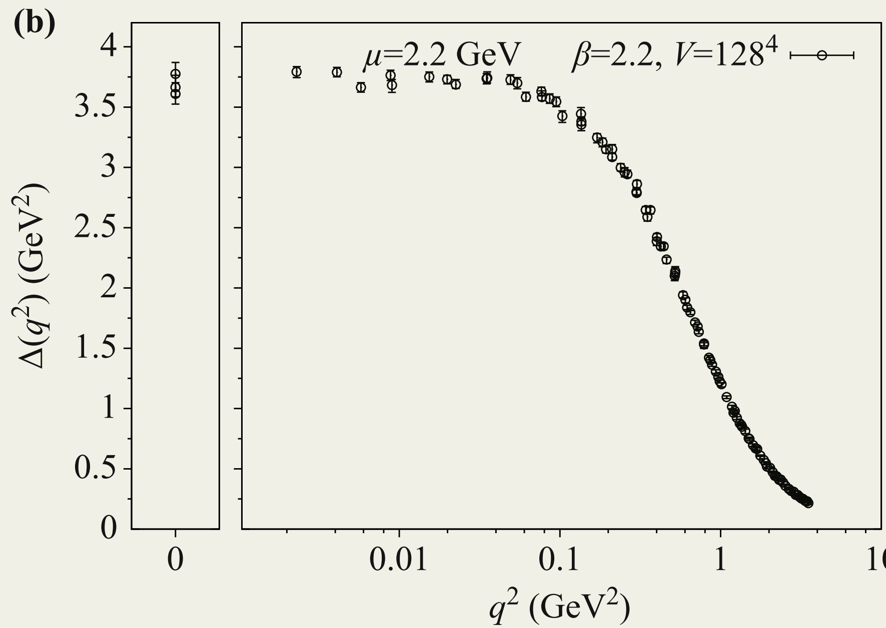
Phys. Rept. 479 (2009)1-152
Pinch Technique: Theory and Applications

On the Lattice

a) SU(3)



b)SU(2)



I. L. Bogolubsky, E. M. Ilgenfritz, M. Muller-Preussker, and A. Sternbeck, Lattice gluodynamics computation of Landau gauge Green's functions in the deep infrared, *Phys. Lett. B* 676, 69 (2009), arXiv: 0901.0736 [hep-lat]

I. L. Bogolubsky, E. M. Ilgenfritz, M. Muller-Preussker, and A. Sternbeck, The Landau gauge gluon and ghost propagators in 4D SU(3) gluodynamics in large lattice volumes, arXiv: 0710.1968 [hep-lat]

A. Cucchieri and T. Mendes, Numerical test of the Gribov–Zwanziger scenario in Landau gauge, *PoS QCD-TNT* 09, 026 (2009), arXiv: 1001.2584 [hep-lat]

Gluon Propagator

Spectral representation

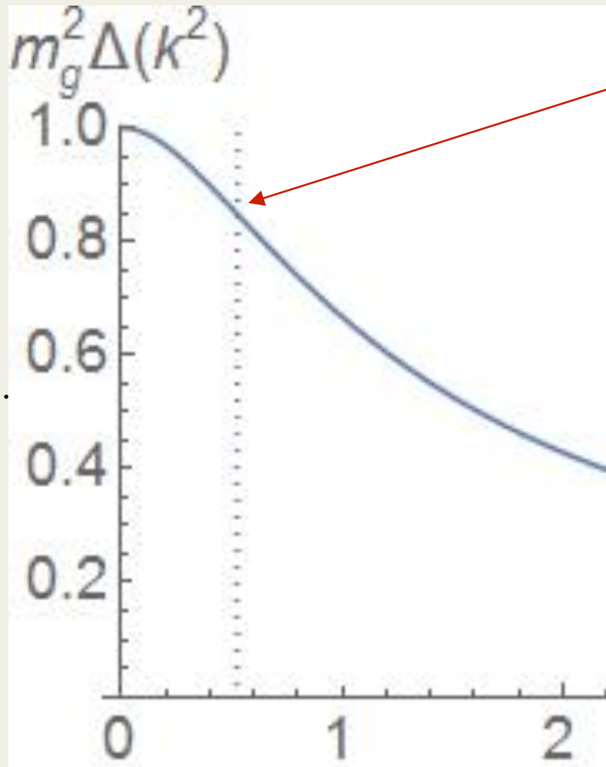
$$\Delta(q^2) = \int_0^\infty \mathrm{d}\sigma \frac{\rho(\sigma)}{q^2+\sigma},$$

if

$$\Delta''(q_\star^2) = 2\int_0^\infty \mathrm{d}\sigma \frac{\rho(\sigma)}{(q_\star^2+\sigma)^3} = 0.$$

Non positive definite

$m_g \approx 0.5$ GeV

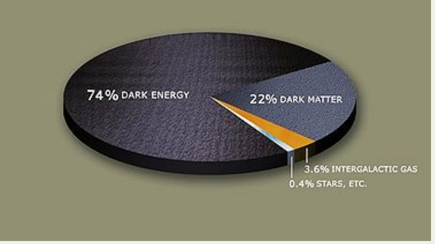


$r_C \approx 0.5$ fm existing an inflection point:

$r<r_c$ Parton structure;

$r>r_c$ Propagating mode change, massive

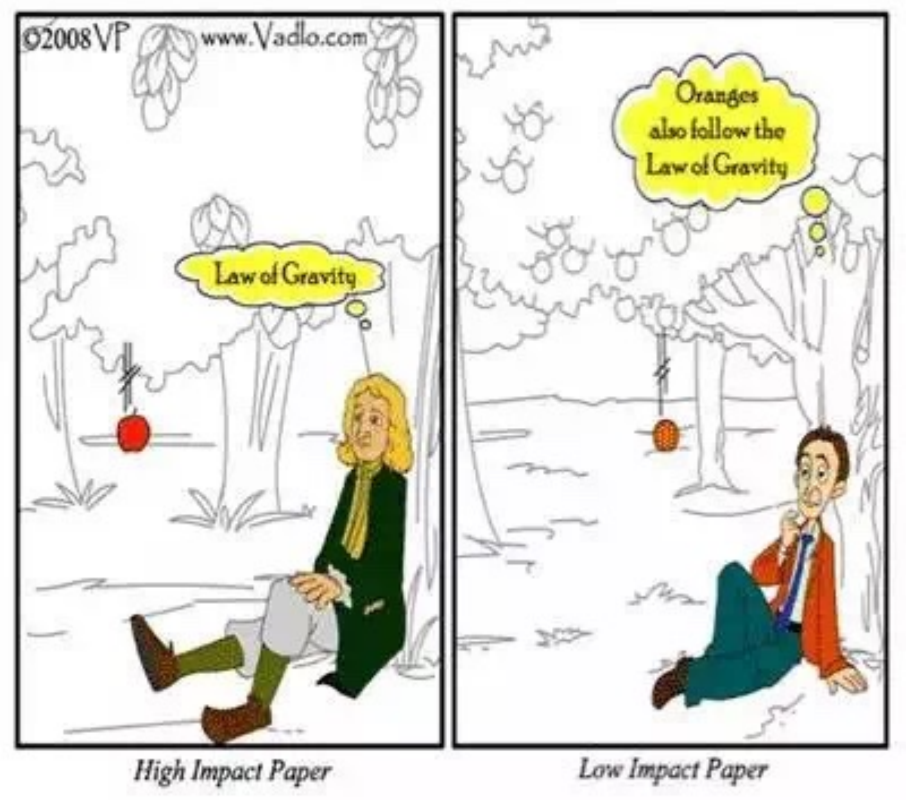
胶子=汉尼拔



南開大學 Nankai University

- ……
- Can we quantitatively understand quark and gluon confinement in quantum chromodynamics and the existence of a mass gap?

Quantum chromodynamics, or QCD, is the theory describing the strong nuclear force. Carried by gluons, it binds quarks into particles like protons and neutrons. Apparently, the tiny subparticles are permanently confined: one can't pull a quark or a gluon from a proton because the strong force gets stronger with distance and snaps them right back inside.



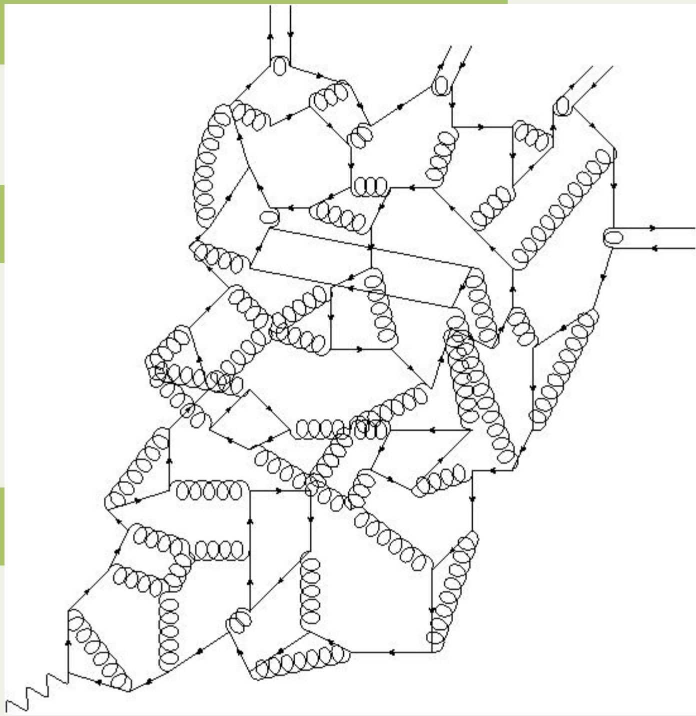
Gluon mass scale

$m_g \approx 0.5$ GeV : A dynamical mass scale generation

$r_C \approx 0.5$: maxium wavelength

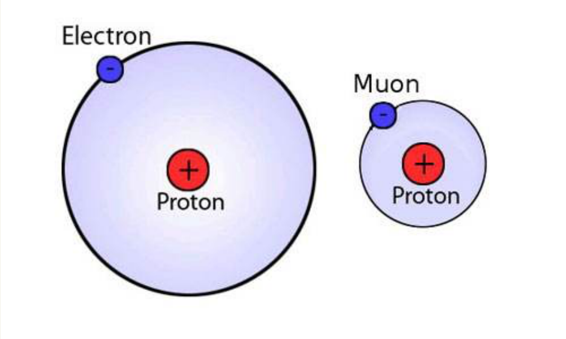
Quark and Hadron

Observables???



FACT-II

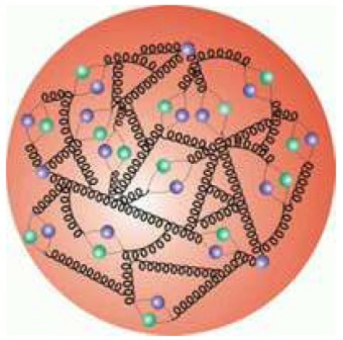
Bound state and quantum field theory



QED

Trace anomaly

- All renormalisable four-dimensional theories possess a trace anomaly;
- The size of the trace anomaly in QED must be great deal smaller than that in QCD.



QCD

Field theory Successful:

- Nonrelativistic quantum mechanics to handle bound state;
- Perturbation theory to handle relativistic effects

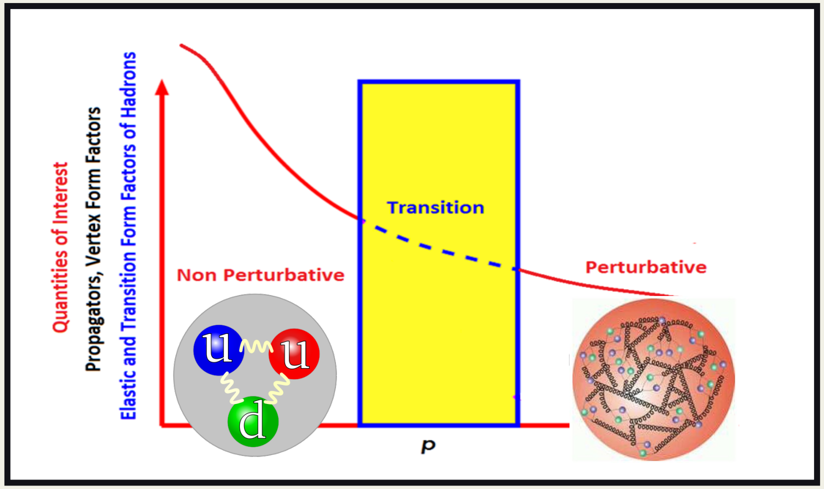
Field theory not Successful yet:

- Growth of the running coupling constant in the infrared region;
- Confinement;
- Dynamical Chiral Symmetry Breaking;
- Possible nontrivial vacuum structure in hadron

Hadron bound state problem

Constituent quark model-> intuitive understanding of many low energy observables.

Minimum number of constituents required



Feynman's parton model-> intuitive understanding of high-energy phenomena.

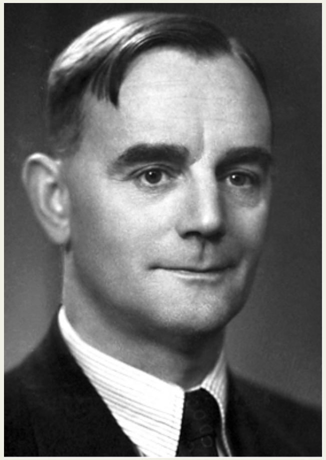
Constituent picture; Probabilistic interpretation of distribution functions

QCD vacuum in the hadron is very complicated medium
Individual quarks and gluons are lost in the sea

Both the constituent quark model and the parton model are put in peril by QCD with a possible complicated vacuum structure.

Why Pion-----Messager of QCD

- Pion is Massless...

• In October 1934**, Hideki Yukawa** predicated the existence of a “heavy quantum” meson, exchanging nuclear force between neutrons and protons.
• It was discovered by **Cecil Powel** in 1949 in cosmic ray tracks in a photographic emulsion.
• Pion was nicely accommodated in the Eight Fold way of **Murray Gell-Mann** in 1961.
• **Yoichiro Nambu** associated it with CSB in 1960.

Pion's dichotomy
Goldstone boson and Bound State

Maris, Roberts and Tandy, Phys. Lett. **B420**(1998) 267-273

- Pion's Bethe-Salpeter amplitude

  Solution of the Bethe-Salpeter equation

$$\Gamma_{\pi^j}(k;P) = \tau^{\pi^j}\gamma_5\left[iE_\pi(k;P) + \gamma\cdot P F_\pi(k;P) + \gamma\cdot k\, k\cdot P\, G_\pi(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, H_\pi(k;P)\right]$$

- Dressed-quark propagator $S(p) = \frac{1}{i\gamma\cdot p\, A(p^2) + B(p^2)}$

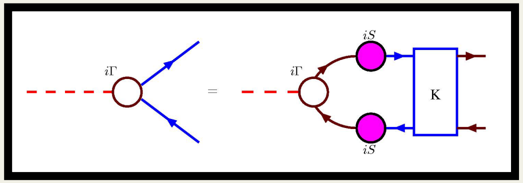
- Axial-vector Ward-Takahashi identity entails(chiral limit)

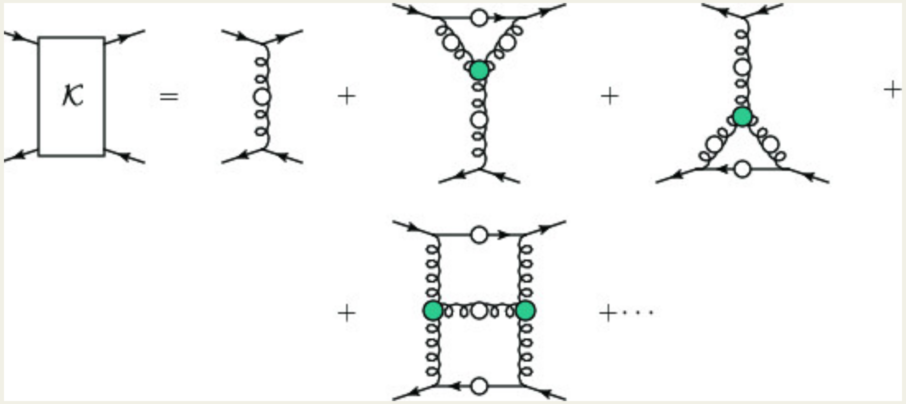
$$f_\pi E(k;P|P^2=0) = B(k^2) + (k\cdot P)^2 \frac{d^2B(k^2)}{d^2k^2} + \ldots$$

- Given the dichotomy of pion the fine-tuning should not play any role in an explanation of pion properties;
- Descriptions of pion within frameworks that cannot faithfully express symmetries and their breaking patterns(such as constituent-quark models) are unreliable;
- Hence, pion properties are an almost direct measure of the dressed-quark mass function.

Dyson-Schwinger Equation scope

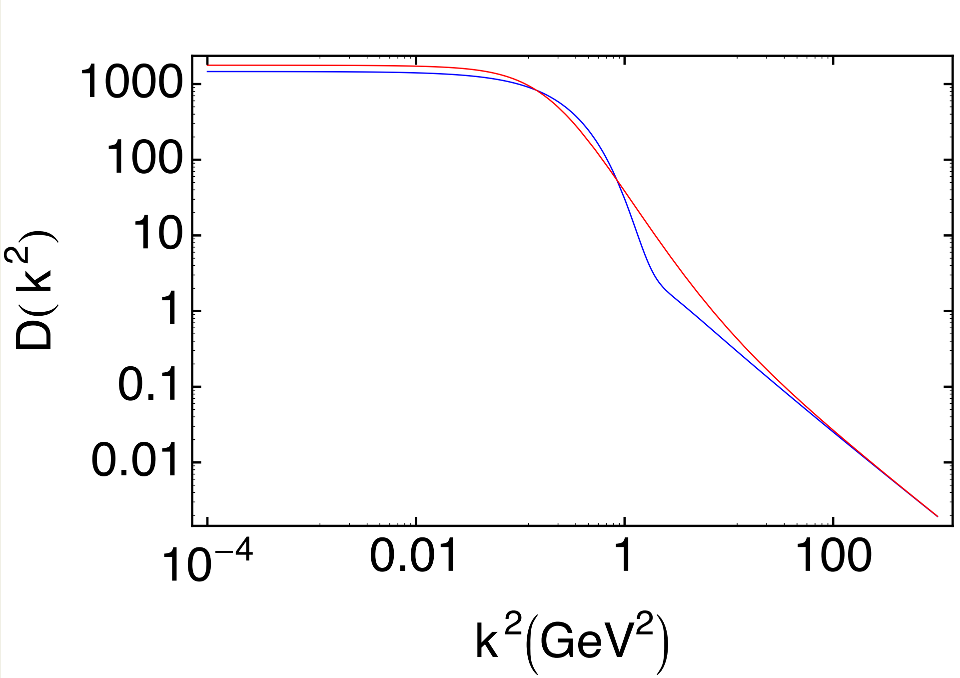
Bethe-Salpeter Equations





Modeling Interaction

RGI interaction



- Landau Gauge
- Blue line: QC
- Red line: New
- m_g=0.5GeV

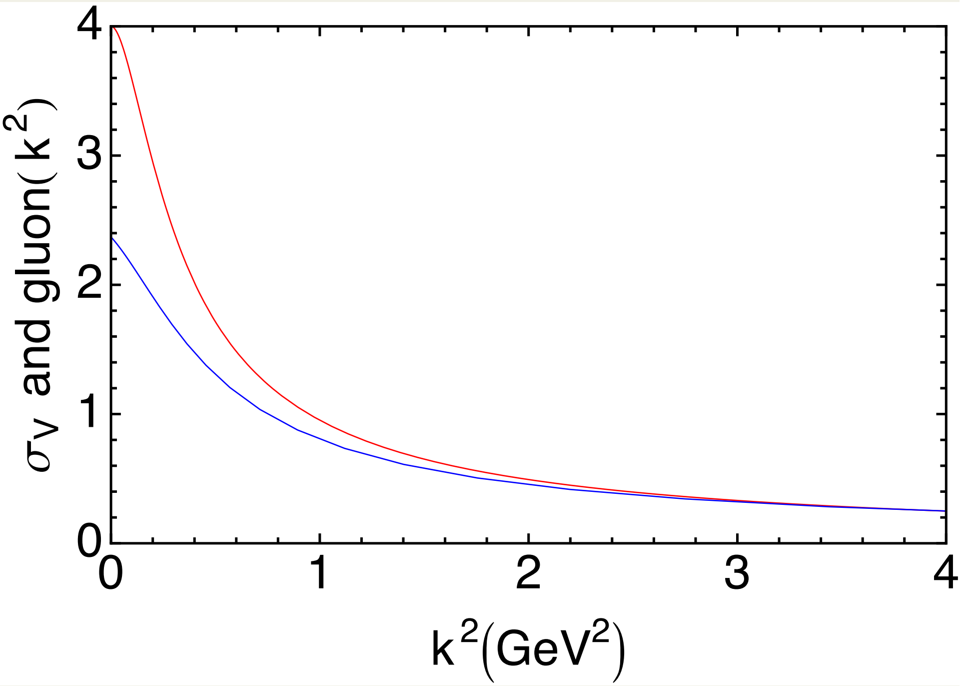
$$D_{\mu\nu}(k) = \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) D(k^2)$$

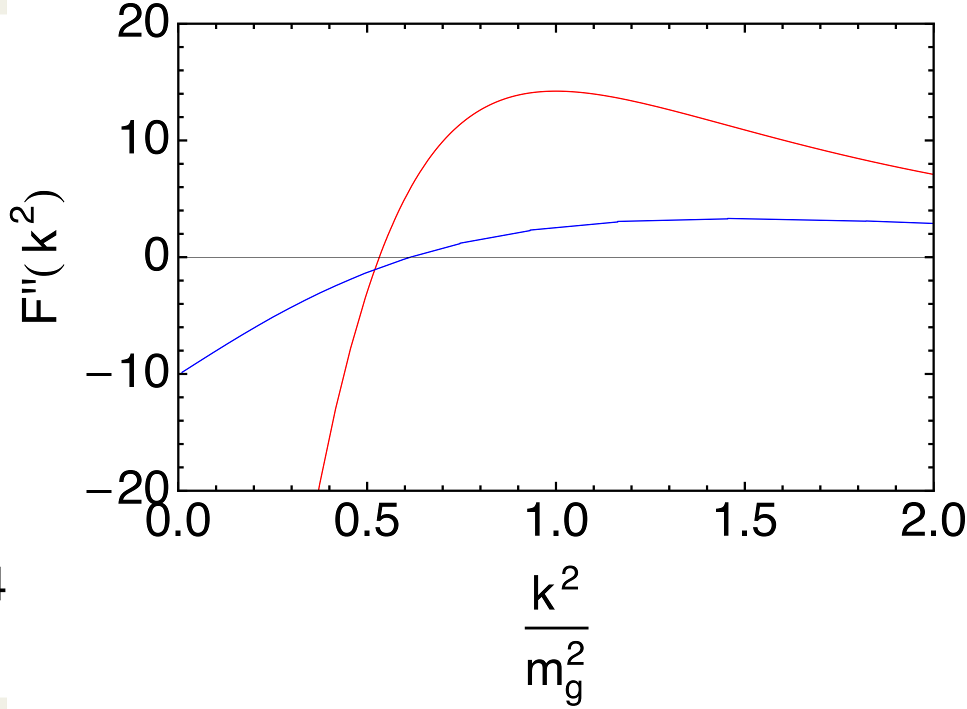
$$D_{QC}(s) = 8\pi^2 \frac{d^3}{\omega^5} \mathrm{e}^{-\frac{s}{\omega^2}} + \frac{8\pi^2\gamma_m}{\ln\left(\tau + \left(1 + \frac{s}{\Lambda_{QCD}^2}\right)^2\right)} \frac{1 - \mathrm{e}^{-s}}{s}$$

vs

$$D_C(s) = \frac{8\pi^2\gamma_m}{\ln\left(\tau + \left(1 + \frac{s}{\Lambda_{QCD}^2}\right)^2\right)} \frac{1}{s + \frac{m_g^4}{s+m_g^2}} \left(1 + \frac{d}{s + \frac{m_g^4}{s+m_g^2}}\right)$$

Quark Propagator $S(k) = -i\gamma \cdot k\sigma_V(k^2) + \sigma_S(k^2)$



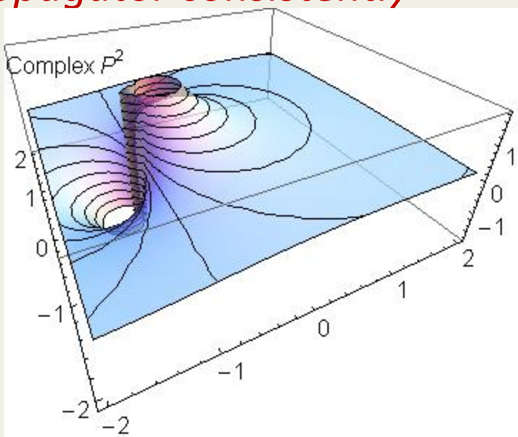


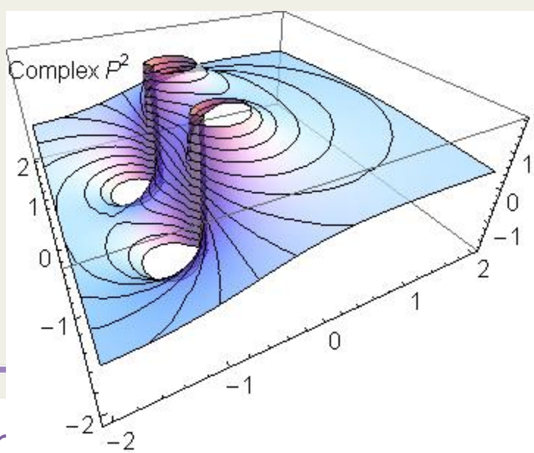
- k^2 dependence of quark propagator and gluon mass
- Red line: running gluon propagagor
- Blue line: vector part of propagator

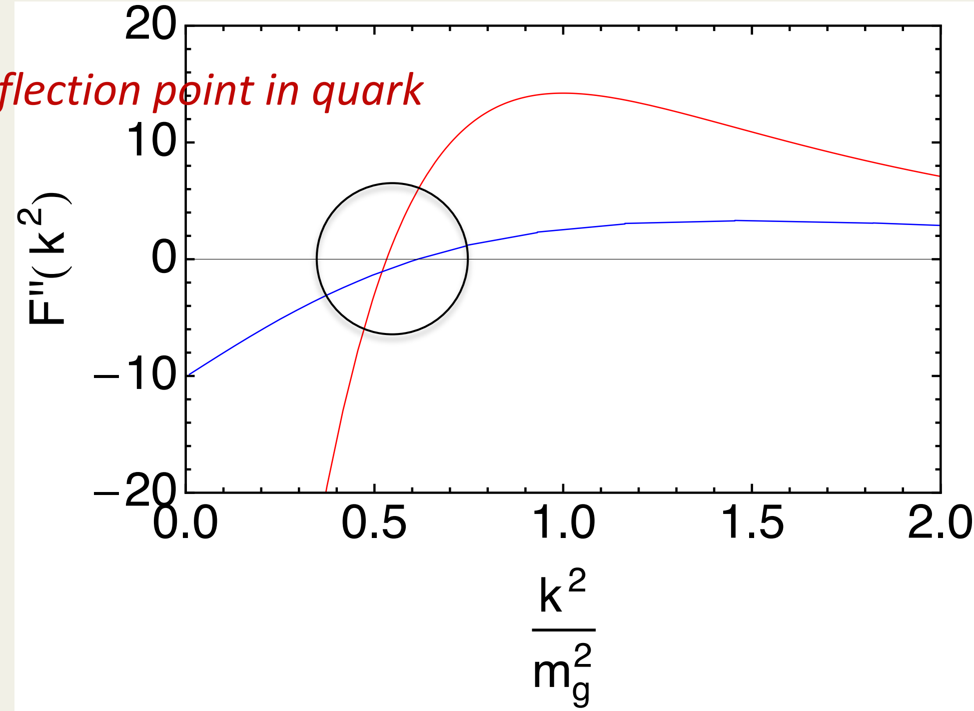
- Inflection points
- Red line: running gluon propagagor
- Blue line: vector part of propagator

Quark Propagator $S(k) = -i\gamma \cdot k\sigma_V(k^2) + \sigma_S(k^2)$

- *at around $k^2 \approx 0.5\ mg^2$ there exists an inflection point in quark propagator consistently*



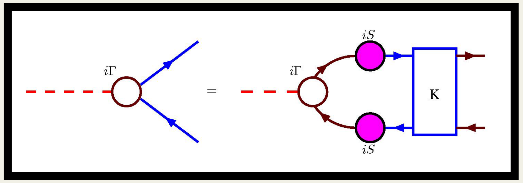




- Inflection points
- Red line: running gluon propagator
- Blue line: vector part of propagator

Pion Bethe-Salpeter Wave Function

- Solving BSEs



- Eigen equation

- General forms of wave function

$$\chi_\pi(k;P) = \gamma_5\left(iE(k;P) + \gamma\cdot PF(k;P) + \gamma\cdot kG(k;P) + \sigma_{\mu\nu}k_\mu P_\nu H(k;P)\right)$$

- Rest frame

$$P_\mu = \{0,0,0,im_\pi\}$$

$$k_\mu = \sqrt{k^2}\left\{0,0,\sqrt{1-z^2},z\right\}$$

E,F,G,H are two dimensional functions with respect to k^2 and z

- Numerical tricks:

$$D\left((p-q)^2\right) = D(p^2+q^2-2\sqrt{p^2q^2}\cos(\theta)) = \sum_{n=0}^{N_c} D_n(p^2,q^2)U_n\left(\cos(\theta)\right)$$

$$\int d^4l\delta^4(l)\frac{k\cdot lp\cdot l}{l^2}f(k,p,l) = \frac{1}{4}\int d^4l\delta^4(l)k\cdot pf(k,p,l)$$

$$\mathcal{F}(k;P) = \sum_{n=0}^{M}\mathcal{F}_n(k^2)U_n(z)$$

Properties of Pion BS Wave Function

$$\chi_\pi(k;P) = \gamma_5 \left(iE(k;P) + \gamma\cdot PF(k;P) + \gamma\cdot kG(k;P) + \sigma_{\mu\nu}k_\mu P_\nu H(k;P)\right)$$

- Ultraviolet Behaviors(up to logarithm)

$$\mathcal{F}_n(k^2) \propto \frac{1}{k^{4+\frac{n}{2}}}$$

Note-I: $$f_\pi E(k;P|P^2=0) = B(k^2) + (k\cdot P)^2 \frac{d^2B(k^2)}{d^2k^2} + \ldots$$

Note-II: Nakanishi reprensation

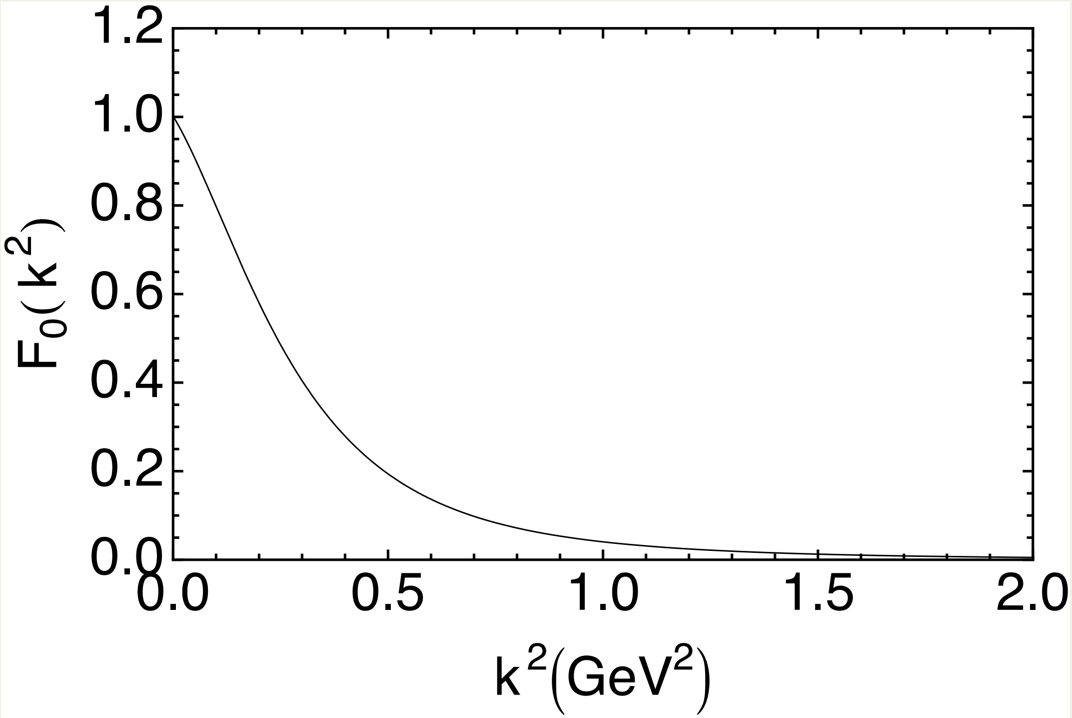
$$F(k;P) = \int_{-1}^{1} dz \int_0^\infty d\gamma \frac{\rho(\gamma,z)}{(k^2 + zk\cdot P + M^2 + \gamma)^3}$$

- Infrared Behaviors

Properties of Pion BS Wave Function

$$\chi_\pi(k;P) = \gamma_5\left(iE(k;P) + \gamma\cdot PF(k;P) + \gamma\cdot kG(k;P) + \sigma_{\mu\nu}k_\mu P_\nu H(k;P)\right)$$

- Infrared Behaviors

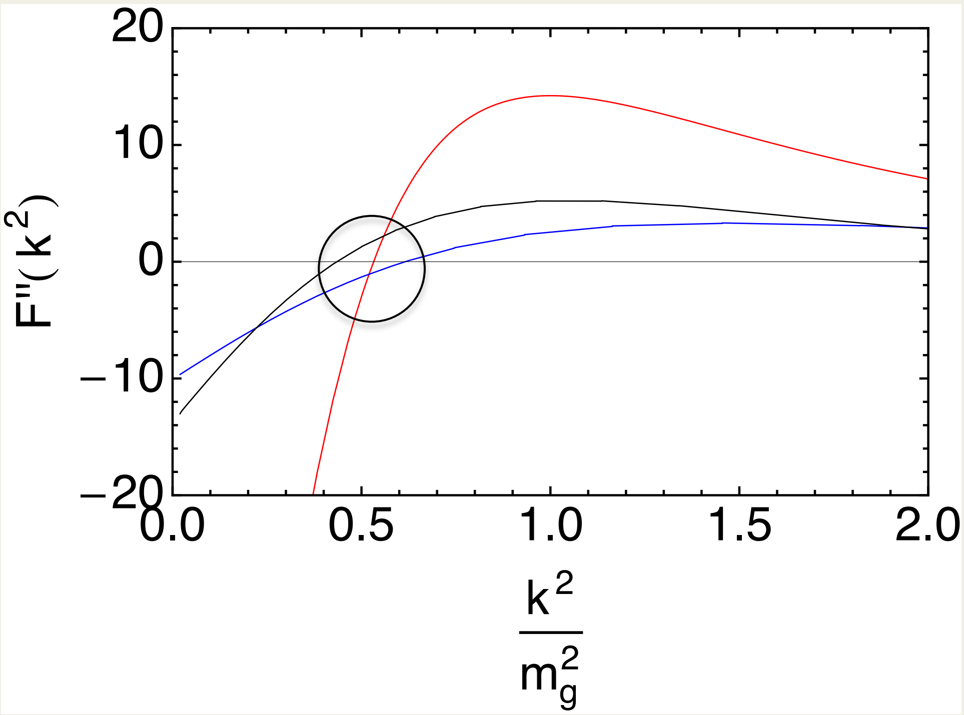


k^2 dependence of first Chebyshev moments of F(k;P)

Properties of Pion BS Wave Function

$$\chi_\pi(k;P) = \gamma_5\left(iE(k;P) + \gamma\cdot PF(k;P) + \gamma\cdot kG(k;P) + \sigma_{\mu\nu}k_\mu P_\nu H(k;P)\right)$$

- Infrared Behaviors



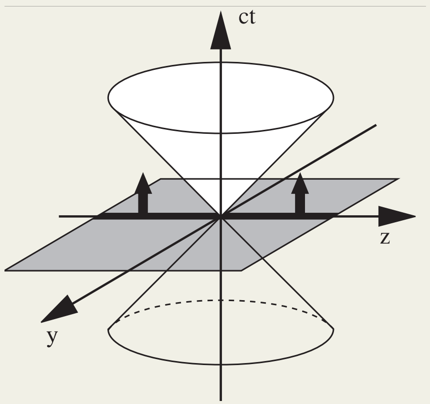
- Inflection points
- Black line: F function
- Red line: running gluon propagagor
- Blue line: vector part of propagator

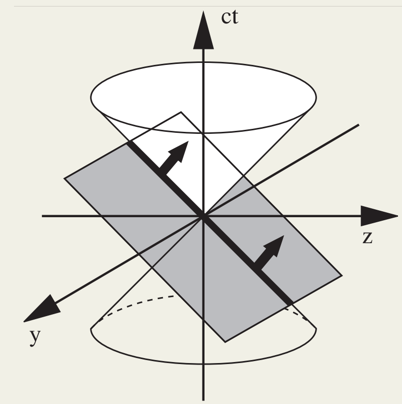
Projecting BSWs to Light Front

equal-time dynamics vs light-front dynamics

Dirac 1949

$$t \equiv x^0 \qquad\qquad t \equiv x^+ = x^0 + x^3$$





Valence quark picture

Definitive of a hadron – it’s how we tell a proton from a neutron

Expresses charge; flavour; baryon number; and other Poincaré-invariant macroscopic quantum numbers

Parton physics involves time-dependent dynamics

Projecting BSWs on Light Front

DSE group 2013-2018

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3, Distribution amplitudes of light-quark mesons from lattice QCD
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14, Parton distribution amplitudes: Revealing correlations within the proton and Roper
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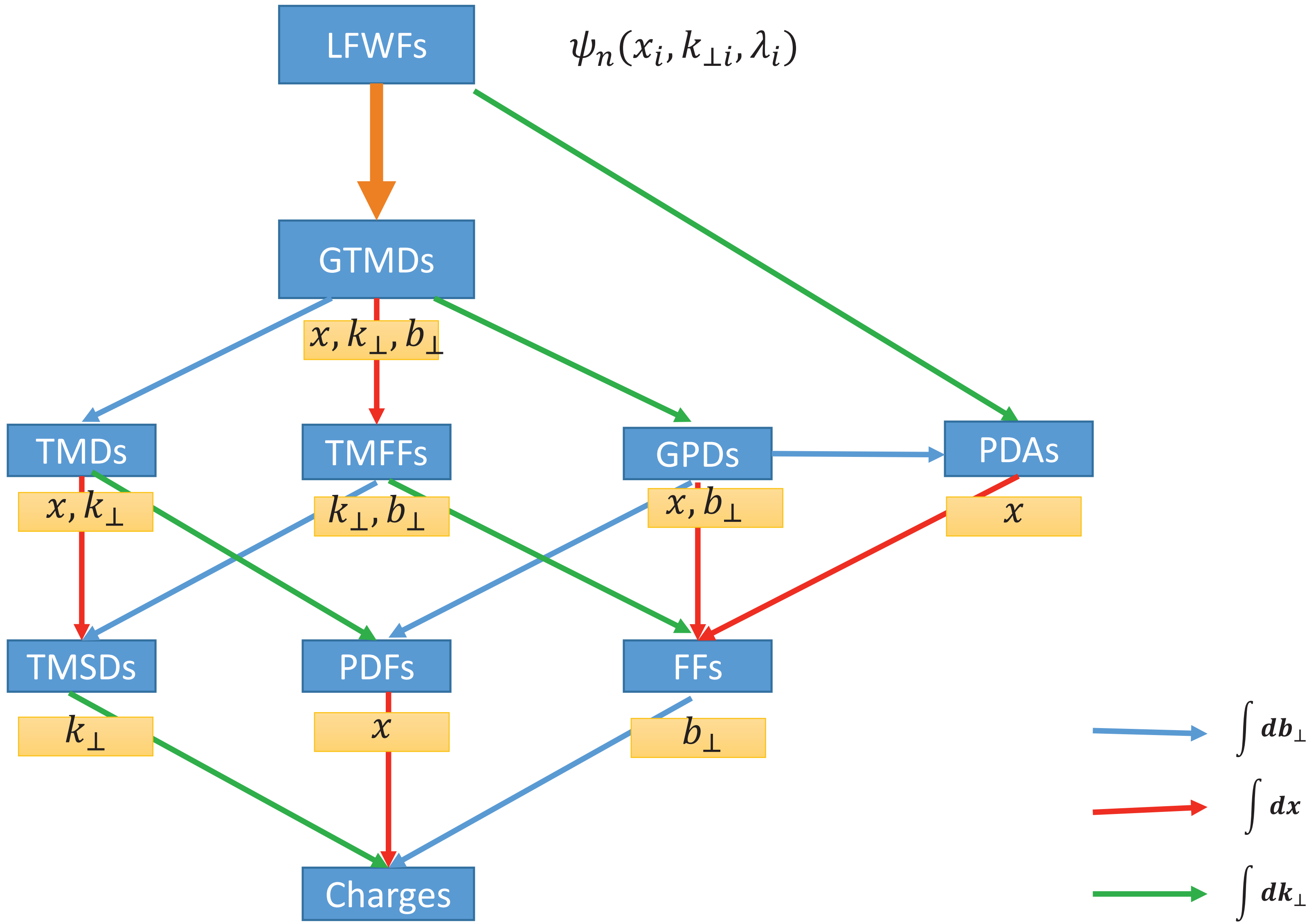
Phys. Rev. D 97 (2018) 094014

Keywords: Amplitudes
Nakanishi representation
Moments


LFWFs
$\psi_n(x_i, k_{\perp i}, \lambda_i)$
GTMDs
$x, k_\perp, b_\perp$
TMDs
TMFFs
GPDs
PDAs
$x, k_\perp$
$k_\perp, b_\perp$
$x, b_\perp$
x
TMSDs
PDFs
FFs
$k_\perp$
x
$b_\perp$
Charges
$\int db_\perp$
$\int dx$
$\int dk_\perp$

Pion's LFWF definition

$$\psi_{\uparrow\downarrow}(x,k_\perp^2)=\frac{1}{f_\pi}\int\frac{d^2k_\parallel}{\pi}\delta(n\cdot k_+-xn\cdot P)\mathrm{tr}_{\mathrm{CD}}[\gamma_5\gamma\cdot n\chi(k_+,k_-)] \tag{1}$$

and

$$k_\perp^i\psi_{\uparrow\uparrow}(x,k_\perp^2)=\frac{1}{f_\pi}\int\frac{d^2k_\parallel}{\pi}\delta(n\cdot k_+-xn\cdot P)\mathrm{tr}_{\mathrm{CD}}[\gamma_5\sigma_{ni}\chi(k_+,k_-)] \tag{2}$$

with $k_\pm=k\pm\frac{P}{2}$, $\sigma_{ni}=\frac{I}{2}(\gamma\cdot n\gamma_i-\gamma_i\gamma\cdot n)$ and χ is the pion BS wave function. Where $\psi_{\uparrow\downarrow}(x,k_\perp^2)$ denotes the pion light front wave function with anti parallel quark helicity and $\psi_{\uparrow\uparrow}(x,k_\perp^2)$ the parallel quark helicity. For finite x and $k_\perp$ the above integration is convergent

LFWF's moments

$$\int_0^1 dx(2x-1)^m\psi(x,k_\perp)=\frac{1}{f_\pi}\int\frac{d^2k_\parallel}{\pi}\frac{1}{n\cdot P}(\frac{2n\cdot k}{n\cdot P})^m\mathrm{tr}_{\mathrm{CD}}[\gamma_5\gamma\cdot n\chi(k_+,k_-)]$$

with $k_\mu=\{0,\sqrt{k_\perp^2},\sqrt{k_\parallel^2}\sqrt{1-z_1^2},\sqrt{k_\parallel^2}z_1\}$

We have

$$\mathcal{F}(k;P)=\sum_{n=0}^{M}\mathcal{F}_n(k^2)U_n(z)=\sum_{n=0}^{M}\mathcal{F}_n(k_\perp^2+k_\parallel^2)U_n(\sqrt{\frac{k_\parallel^2}{k_\perp^2+k_\parallel^2}}z_1)$$

$$x^\alpha(1-x)^\alpha$$

$$\langle(2x-1)^2\rangle$$

- Hadronic scale;
- Small pion mass;

Does Matter!

Distribution amplitudes of light-quark mesons from lattice QCD

Jorge Segovia (Argonne, PHY) , Lei Chang (Adelaide U.) , Ian C. Cloët, Craig D. Roberts (Argonne, PHY) , Sebastian M. Schmidt (IAS, Julich) , Hong-shi Zong (Nanjing U. & Beijing, Inst. Theor. Phys. & Purple Mountain Observ.)

Nov 6, 2013 - 6 pages

Phys.Lett. B731 (2014) 13-18
(2014-04-04)
DOI: 10.1016/j.physletb.2014.02.006
e-Print: arXiv:1311.1390 [nucl-th] | PDF

Second moment

$$\langle (2x-1)^2 \rangle$$

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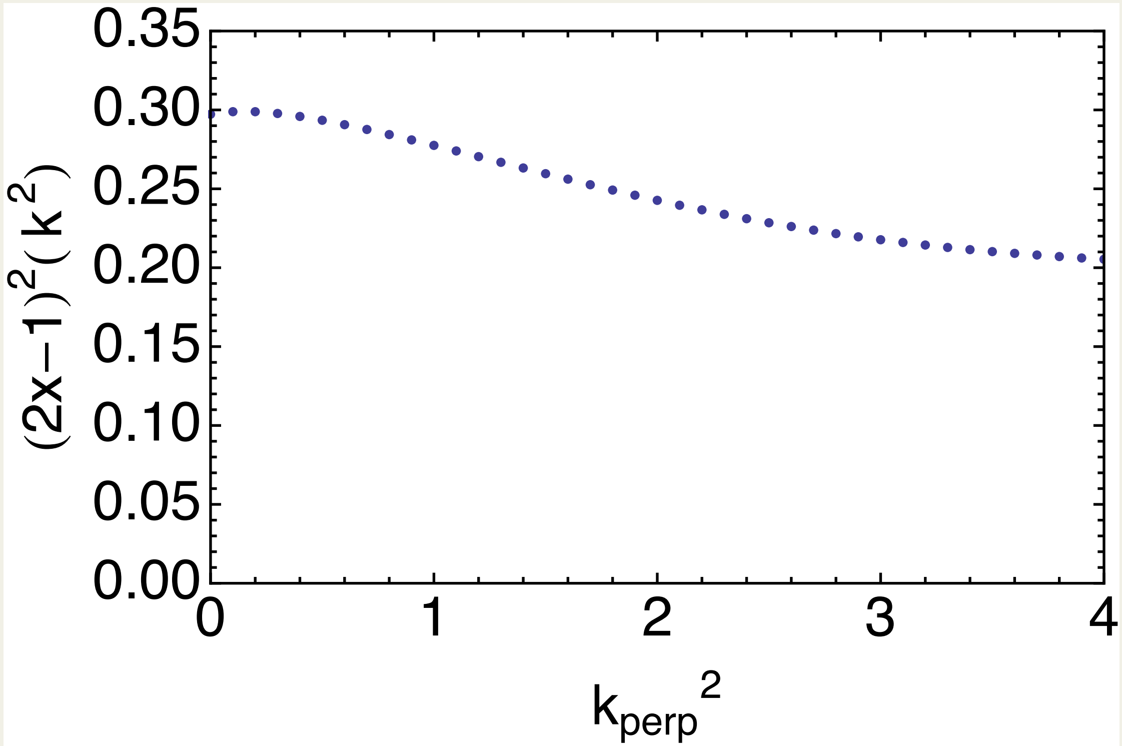
$$x^{0.182}(1-x)^{0.182}$$



$x(1-x)$

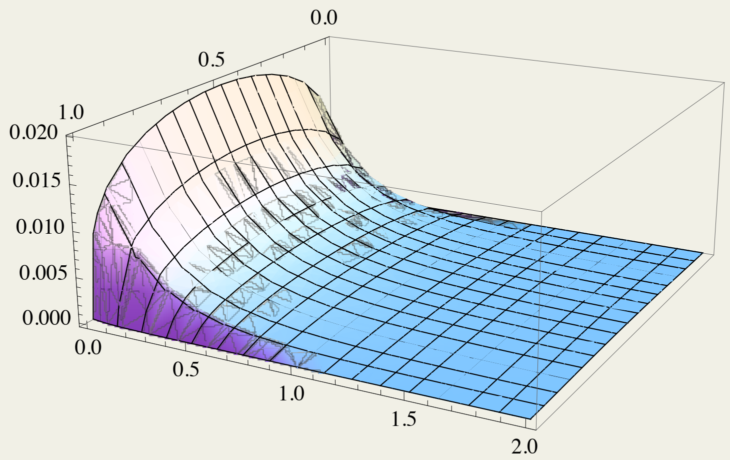
Infrared soft

Ultraviolet hard

BS wave function tells more good story about the light front wave function

Plotting Pion's LFWF

Suppose
$$\psi_{\uparrow\downarrow}(x,k_\perp^2)=\mathcal{N}\mathcal{C}(k_\perp^2)\frac{\Gamma[2+2\alpha(k_\perp^2)]}{\Gamma[1+\alpha(k_\perp^2)]^2}(x(1-x))^{\alpha(k_\perp^2)}$$



Overlap representation

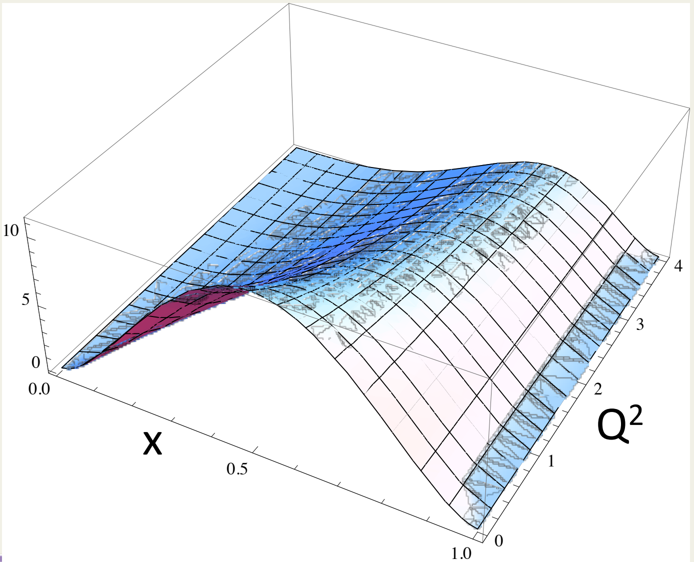
The overlap represention for the GPD can be written as

$$\mathcal{H}(x,Q^2)=\int d^2k_\perp \frac{1}{16\pi^3}\psi(x,k_\perp+(1-x)q_\perp)\psi(x,k_\perp)$$

$$\langle\Gamma_i|\chi_j\rangle \propto \langle (S^{-1}\chi S^{-1})_i|\chi_j\rangle \propto \delta_{ij}$$

$$\mathcal{H}(x,Q^2)=\int d^2k_\perp \frac{1}{16\pi^3}\boxed{x(1-x)}\psi(x,k_\perp+(1-x)q_\perp)\psi(x,k_{k_\perp})$$

Consistent to triangle diagram calculation for the electromagnetic form factor



Consistent to C. Mezrag etal, arXiv:1411.6634

Electromagnetic Form Fator

$$F(x) = \int dx \mathcal{H}(x, Q^2 = 0) = \int dx \int d^2 k_\perp \frac{1}{16\pi^3} x(1-x)\psi(x, k_\perp + (1-x)q_\perp)\psi(x, k_{k_\perp})$$

- Leading-twist valence-parton light-front wave function
- Direct calculation of $F_\pi(Q^2)$ via overlap representation of GPD
- No assumption of validity of collinear factorisation
- Computational verification … good approximation on $Q^2 > 10$ GeV^2

$$\exists Q_0 > \Lambda_{\rm QCD} \mid Q^2 F_\pi(Q^2) \overset{Q^2 > Q_0^2}{\approx} 16\pi\alpha_s(Q^2) f_\pi^2 w_\varphi^2, \quad (1)$$

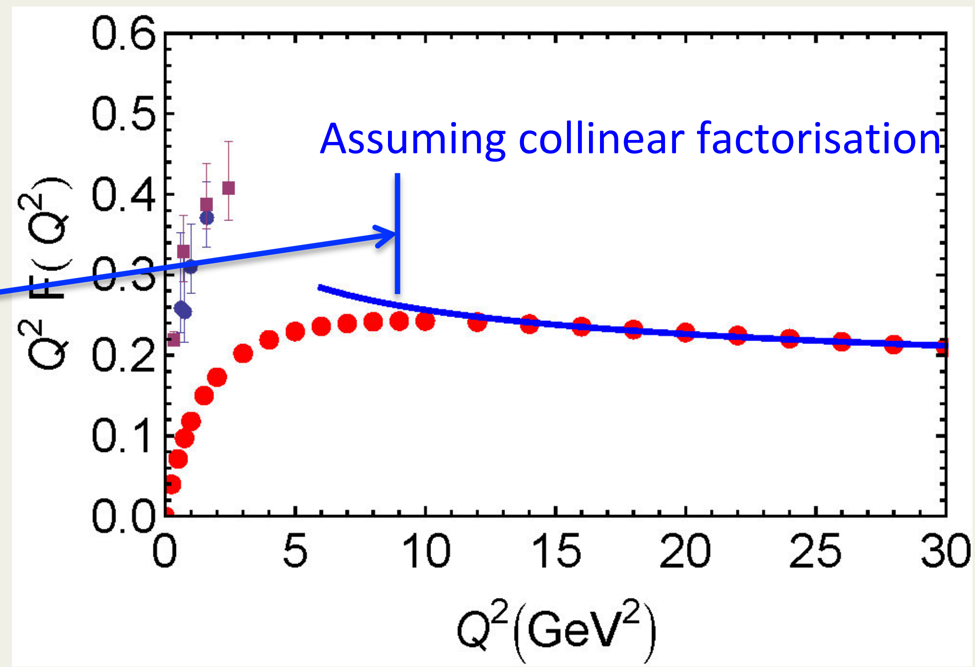
where $f_\pi = 92.2\,\text{MeV}$ is the pion decay constant [30],

$$\alpha_s(Q^2) = 4\pi/[\beta_0 \ln(Q^2/\Lambda^2_{\rm QCD})], \quad (2)$$

$\beta_0 = 11 - (2/3)n_f$ (n_f is the number of active quark flavours), is the leading-order expression for the strong running coupling, and

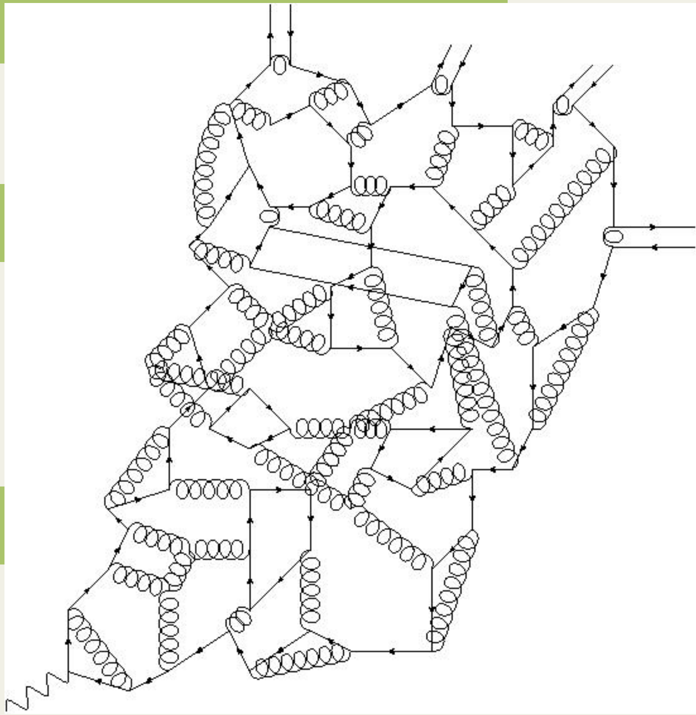
$$w_\varphi = \frac{1}{3}\int_0^1 dx\, \frac{1}{x}\varphi_\pi(x)\,, \quad (3)$$

where $\varphi_\pi(x)$ is the pion's valence-quark parton distribution amplitude (PDA). The value of Q_0 is not pre-

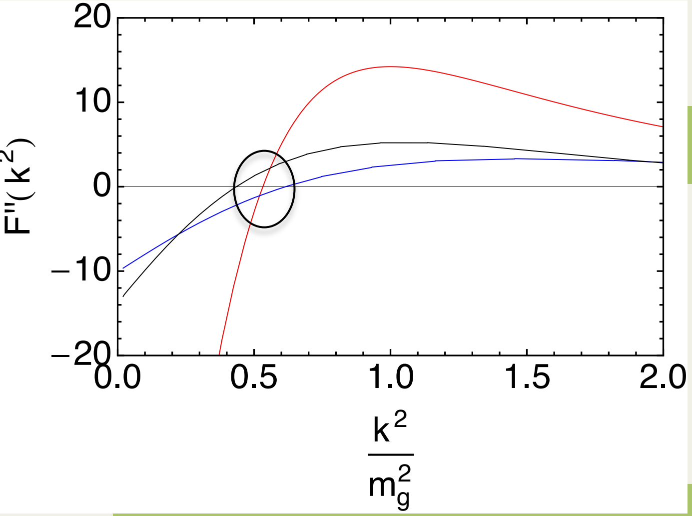


Gluon mass scale----LFWFs

$m_g \approx 0.5\ GeV$: A dynamical mass scale generation

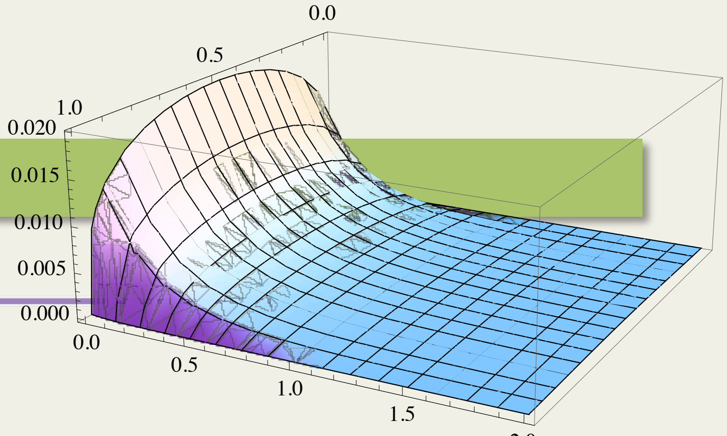


$r_C \approx 0.5$: *maxium wavelength*



Quark and Hadron

LFWFs



Gluon mass scale----LFWFs

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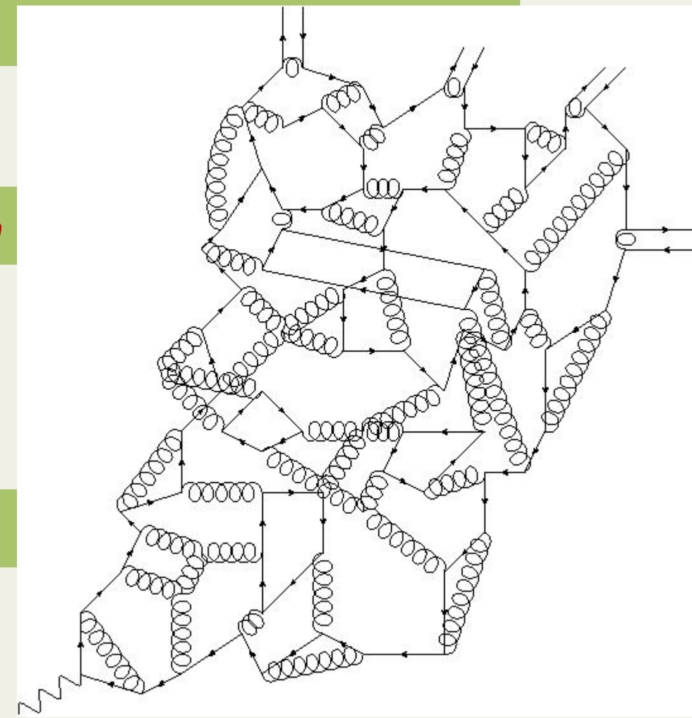

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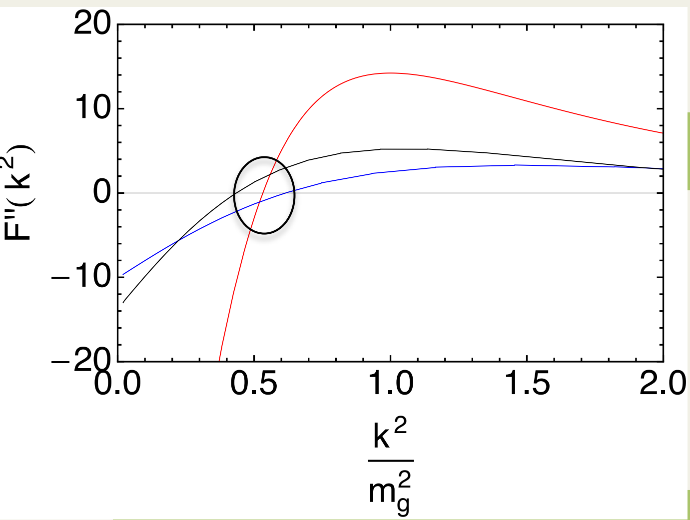
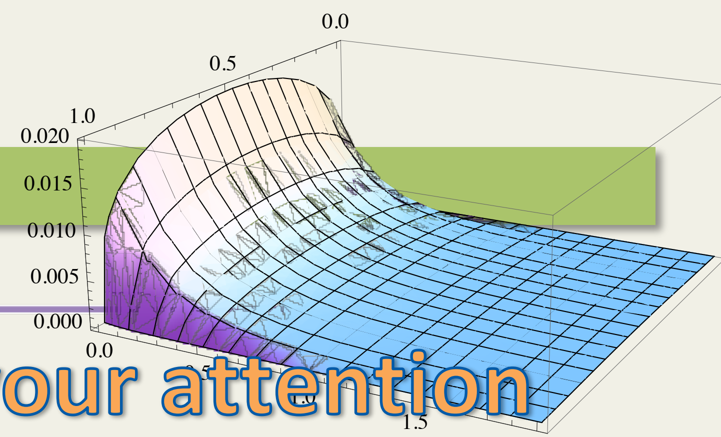


Quark and Hadron

LFWFs



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Thanks for your attention