New Insights on the Drell-Yan Angular Distributions

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Workshop on “Mapping Parton Distribution
Amplitudes and Functions”
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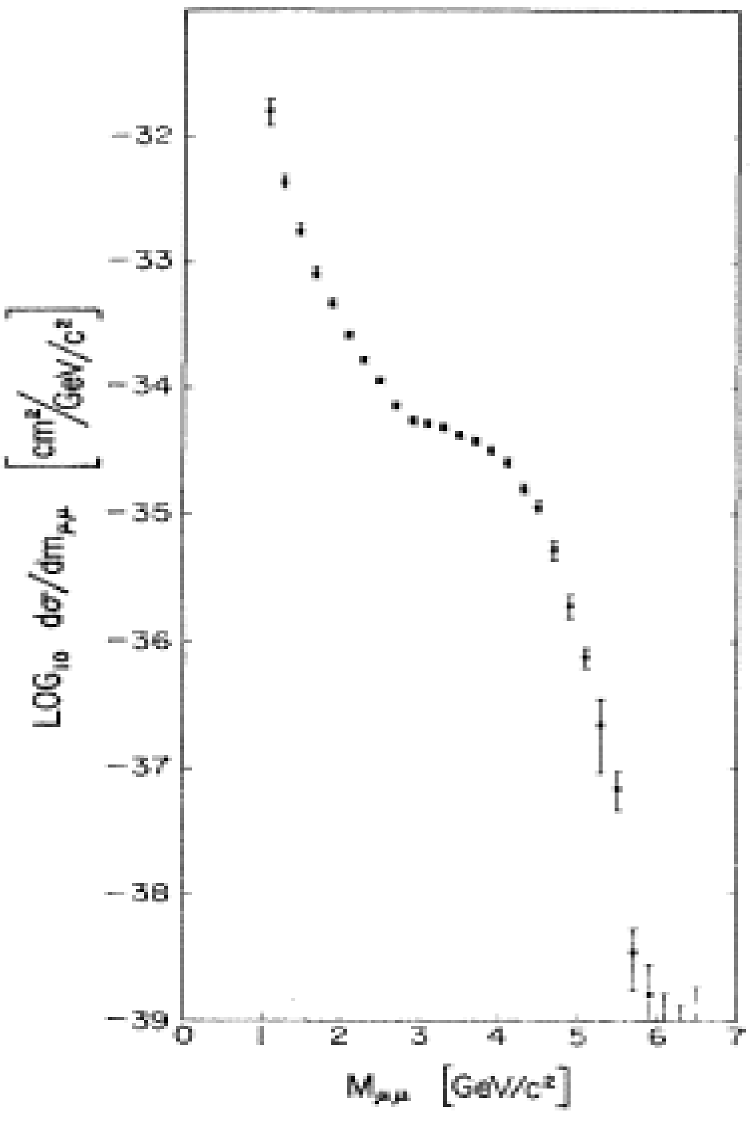
Based on the papers of JCP, Wen-Chen Chang, Evan McClellan,
Oleg Teryaev, Phys. Lett. B758 (2016) 384, PRD 96 (2017) 054020,
and arXiv:1808.04398

First Dimuon Experiment



$$p + U \rightarrow \mu^{+} + \mu^{-} + X$$ 29 GeV proton

Lederman et al. PRL 25 (1970) 1523

Experiment originally designed to search for neutral weak boson (Z^0)

Missed the J/Ψ signal !

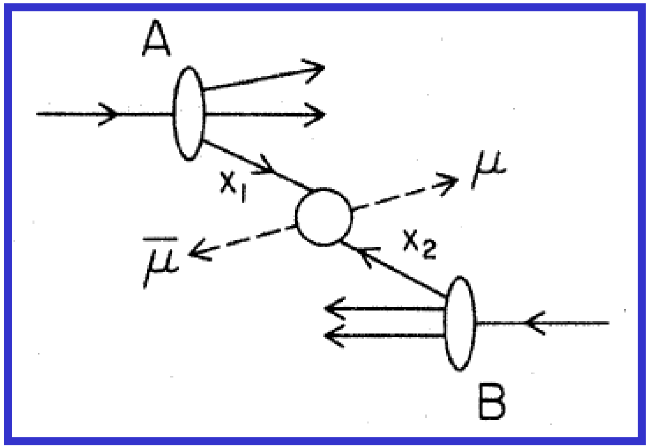
“Discovered” the Drell-Yan process

The Drell-Yan Process

MASSIVE LEPTON-PAIR PRODUCTION IN HADRON-HADRON COLLISIONS AT HIGH ENERGIES*

Sidney D. Drell and Tung-Mow Yan
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305
(Received 25 May 1970)

On the basis of a parton model studied earlier we consider the production process of large-mass lepton pairs from hadron-hadron inelastic collisions in the limiting region, $s \to \infty$, Q^2/s finite, Q^2 and s being the squared invariant masses of the lepton pair and the two initial hadrons, respectively. General scaling properties and connections with deep inelastic electron scattering are discussed. In particular, a rapidly decreasing cross section as $Q^2/s \to 1$ is predicted as a consequence of the observed rapid falloff of the inelastic scattering structure function νW_2 near threshold.



$$\left(\frac{d^2\sigma}{dx_1 dx_2}\right)_{D.Y.} = \frac{4\pi\alpha^2}{9 s x_1 x_2} \sum_a e_a^2 \left[q_a(x_1)\bar{q}_a(x_2) + \bar{q}_a(x_1) q_a(x_2) \right]$$

Naive Drell-Yan and Its Successor*

T-M. Yan
Floyd R. Newman Laboratory of Nuclear Studies
Cornell University
Ithaca, NY 14853

February 1, 2008

Abstract

We review the development in the field of lepton pair production since proposing parton-antiparton annihilation as the mechanism of massive lepton pair production. The basic physical picture of the Drell-Yan model has survived the test of QCD, and the predictions from the QCD improved version have been confirmed by the numerous experiments performed in the last three decades. The model has provided an active theoretical arena for studying infrared and collinear divergences in QCD. It is now so well understood theoretically that it has become a powerful tool for new phyiscs information such as precision measurements of the W mass and lepton and quark sizes.

*Talk given at the Drell Fest, July 31, 1998, SLAC on the occasion of Prof. Sid Drell's retirement.

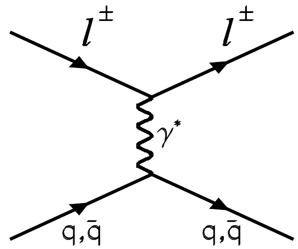
"… our original crude fit did not even remotely resemble the data. Sid and I went ahead to publish our paper because of the model's simplicity…"

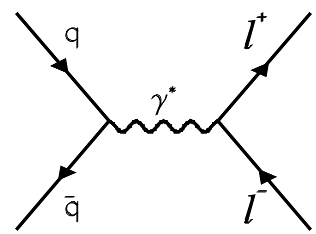
"… the successor of the naïve model, the QCD improved version, has been confirmed by the experiments…"

"The process has been so well understood theoretically that it has become a powerful tool for precision measurements and new physics."

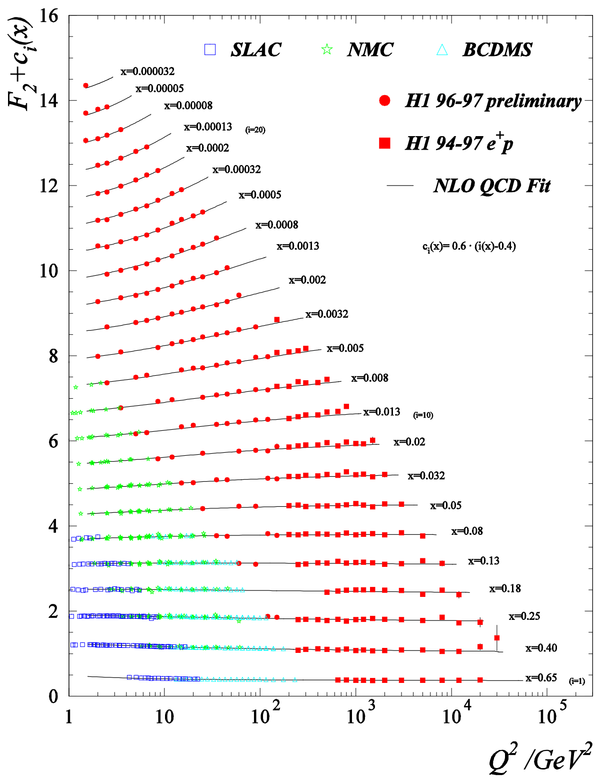
Complementarity between DIS and Drell-Yan

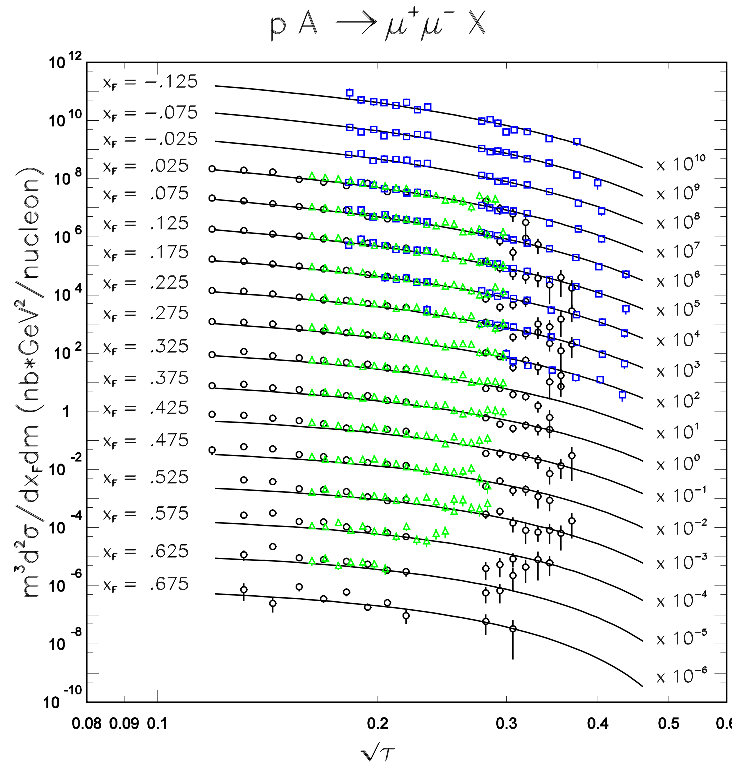
DIS





Drell-Yan





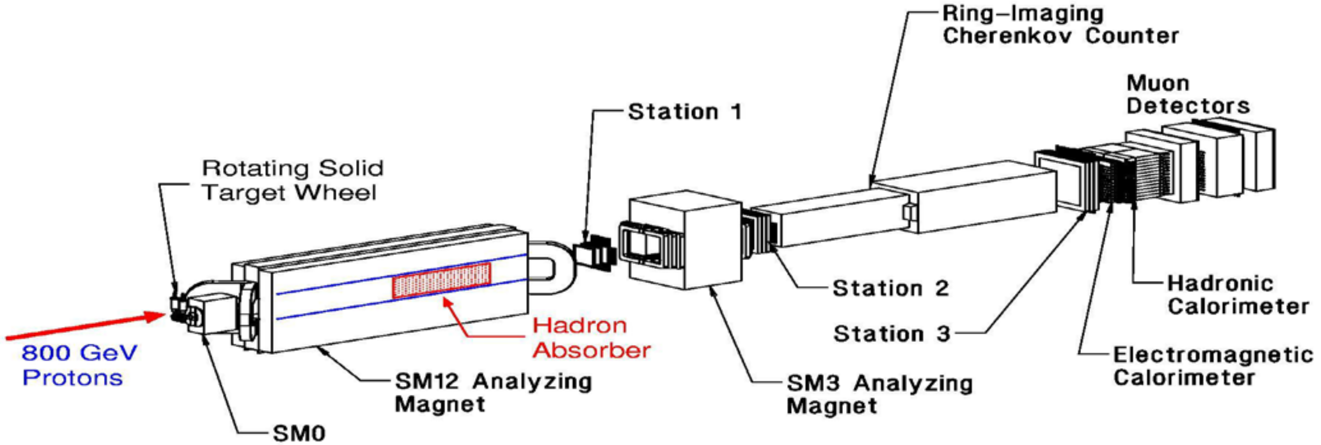
Ann.Rev.Nucl. Part. Sci. 49 (1999) 217;

Peng and Qiu, Prog. Part. Nucl. Phys. 76 (2014)43

Both DIS and Drell-Yan process are tools to probe the quark and antiquark structure in hadrons (factorization, universality)

Fermilab Dimuon Spectrometer

(E605 / 772 / 789 / 866 / 906 /1039)



1) Fermilab E772 (proposed in 1986 and completed in 1988)
   "Nuclear Dependence of Drell-Yan and Quarkonium Production"
2) Fermilab E789 (proposed in 1989 and completed in 1991)
   "Search for Two-Body Decays of Heavy Quark Mesons"
3) Fermilab E866 (proposed in 1993 and completed in 1996)
   "Determination of $\bar{d} / \bar{u}$ Ratio of the Proton via Drell-Yan"
4) Fermilab E906 (proposed in 1999, completed in 7/2017)
   "Drell-Yan with the FNAL Main Injector"
5) Fermilab E1039 (proposed in 2015, expected to start in 2019)
   "Drell-Yan with transversely polarized target"

EXPERIMENT E789- Moving Cable at Meson.
"The Snake".

Angular Distribution in the "Naïve" Drell-Yan

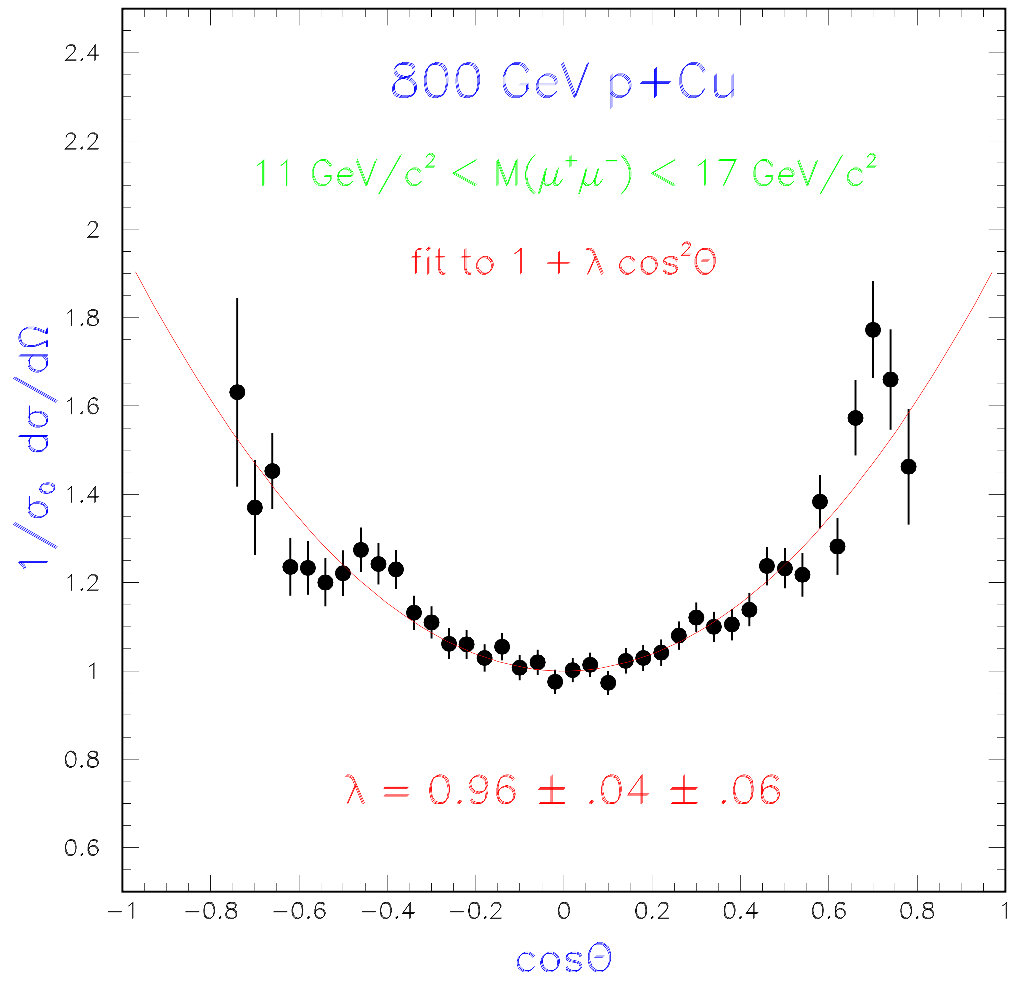
VOLUME 25, NUMBER 5 PHYSICAL REVIEW LETTERS 3 AUGUST 1970

(3) The virtual photon will be predominantly transversely polarized if it is formed by annihilation of spin-$\frac{1}{2}$ parton-antiparton pairs. This means a distribution in the di-muon rest system varying as $(1+\cos^2\theta)$ rather than $\sin^2\theta$ as found in Sakurai's[10] vector-dominance model, where θ is the angle of the muon with respect to the timelike photon momentum. The model used in Fig.

Drell-Yan angular distribution

Lepton Angular Distribution of "naïve" Drell-Yan:

$$\frac{d\sigma}{d\Omega} = \sigma_0 (1 + \lambda \cos^2 \theta); \quad \lambda = 1$$

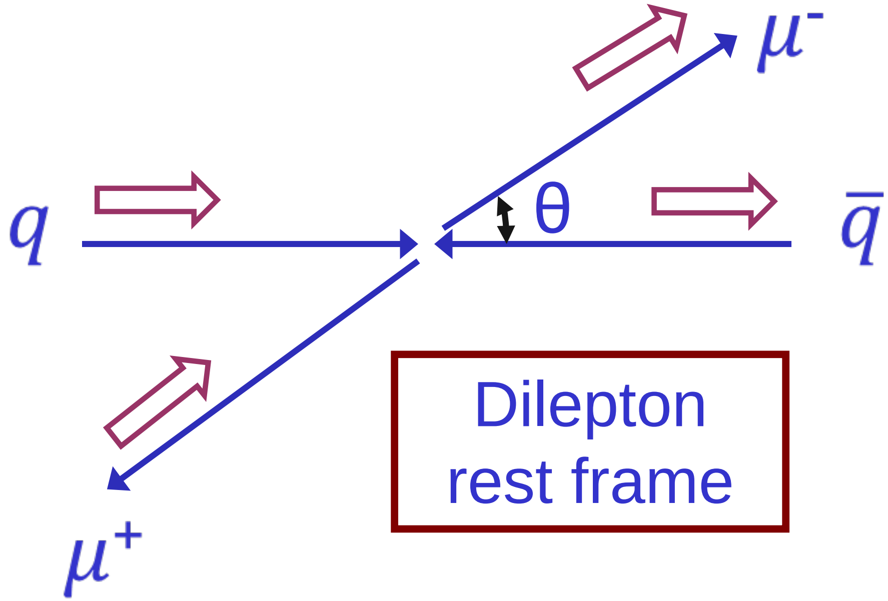


Data from Fermilab E772

(Ann. Rev. Nucl. Part. Sci. 49 (1999) 217-253)

Why is the lepton angular distribution $1+\cos^2\theta$?

Helicity conservation and parity



Dilepton rest frame

$RL \to RL$

$$d\sigma \sim (1+\cos\theta)^2$$

$RL \to LR$

$$d\sigma \sim (1-\cos\theta)^2$$

$LR \to LR$

$$d\sigma \sim (1+\cos\theta)^2$$

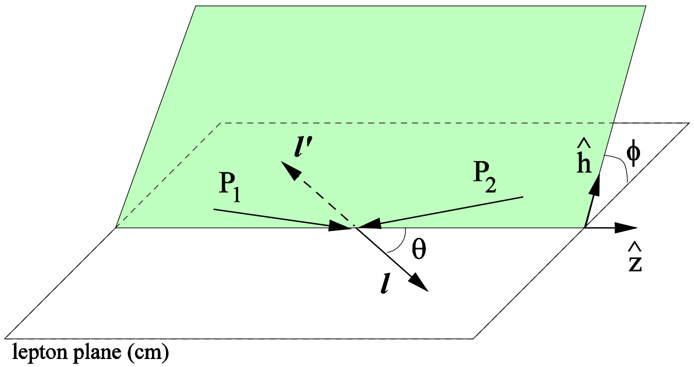
$LR \to RL$

$$d\sigma \sim (1-\cos\theta)^2$$

Adding all four helicity configurations:

$$d\sigma \sim 1+\cos^2\theta$$

Drell-Yan lepton angular distributions



Θ and Φ are the decay polar and azimuthal angles of the μ^- in the dilepton rest-frame

Collins-Soper frame

A general expression for Drell-Yan decay angular distributions:

$$\left(\frac{1}{\sigma}\right)\left(\frac{d\sigma}{d\Omega}\right)=\left[\frac{3}{4\pi}\right]\left[1+\lambda\cos^2\theta+\mu\sin 2\theta\cos\phi+\frac{\nu}{2}\sin^2\theta\cos 2\phi\right]$$

Lam-Tung relation: $1-\lambda=2\nu$

– Reflect the spin-1/2 nature of quarks

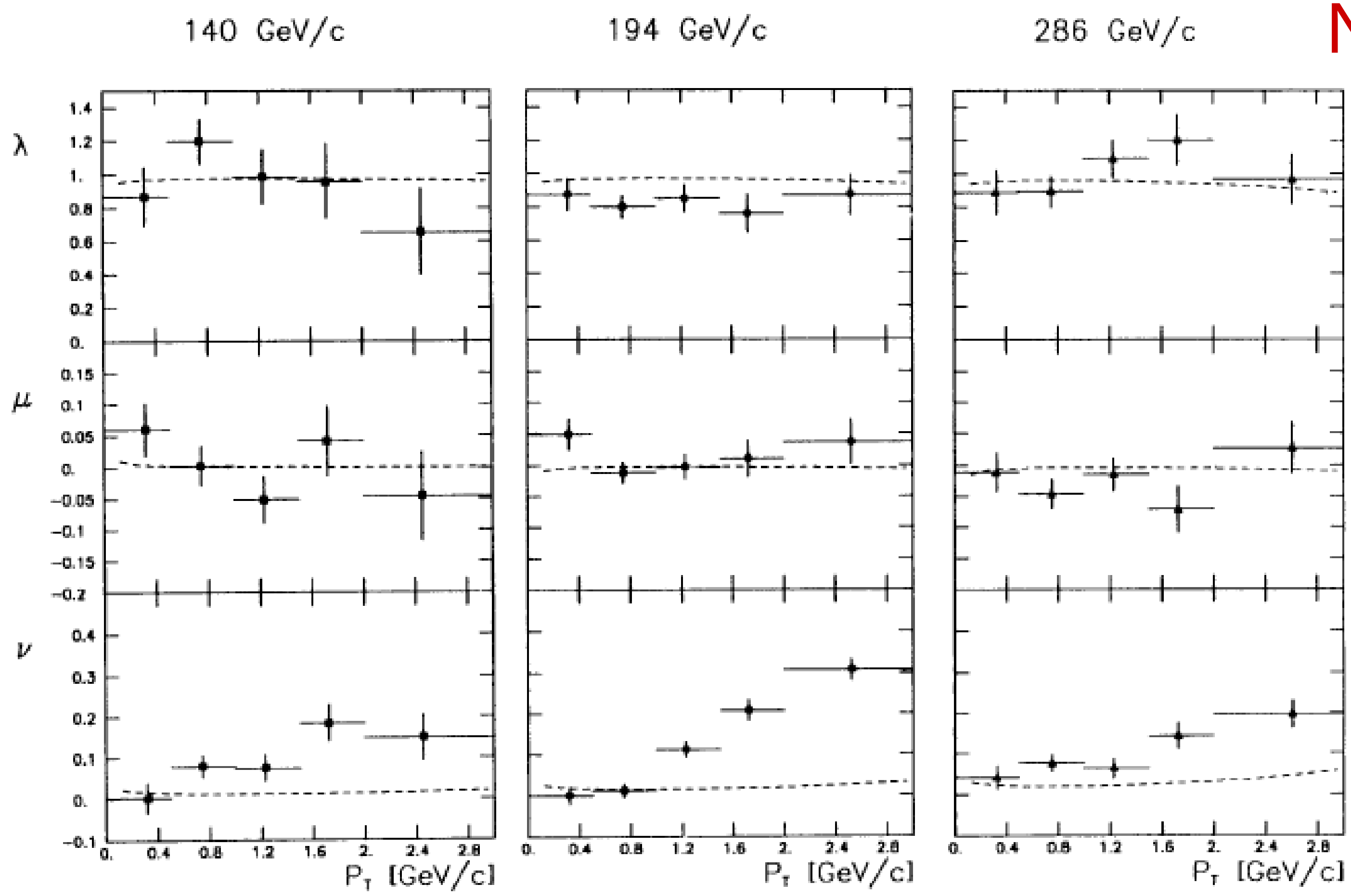
(analog of the Callan-Gross relation in DIS)

– Insensitive to QCD - corrections

Decay angular distributions in pion-induced Drell-Yan

$$\left(\frac{1}{\sigma}\right)\left(\frac{d\sigma}{d\Omega}\right)=\left[\frac{3}{4\pi}\right]\left[1+\lambda\cos^2\theta+\mu\sin 2\theta\cos\phi+\frac{\nu}{2}\sin^2\theta\cos 2\phi\right]$$

NA10 $\pi^- + \mathbf{W}$



Z. Phys.

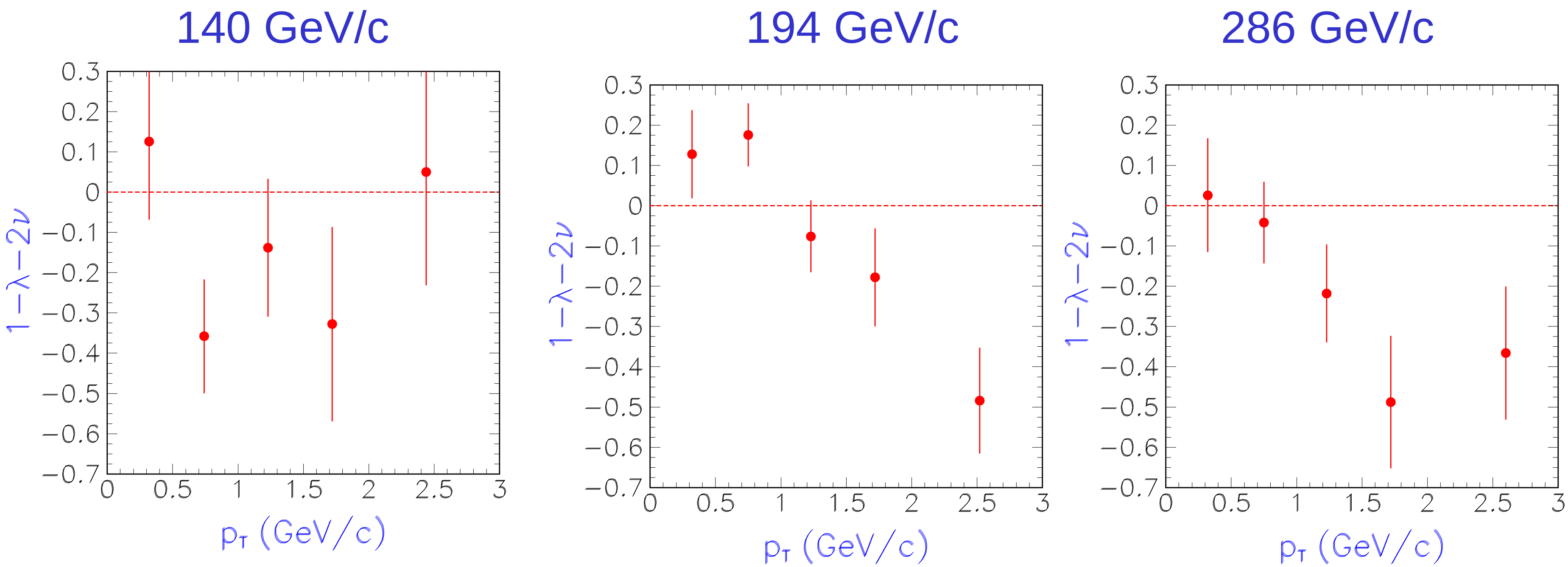
37 (1988) 545

Dashed curves are from pQCD calculations

$\nu \neq 0$ and ν increases with p_T

Decay angular distributions in pion-induced Drell-Yan

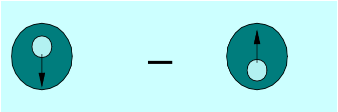
Is the Lam-Tung relation (1-λ-2ν=0) violated?



Data from NA10 (Z. Phys. 37 (1988) 545)

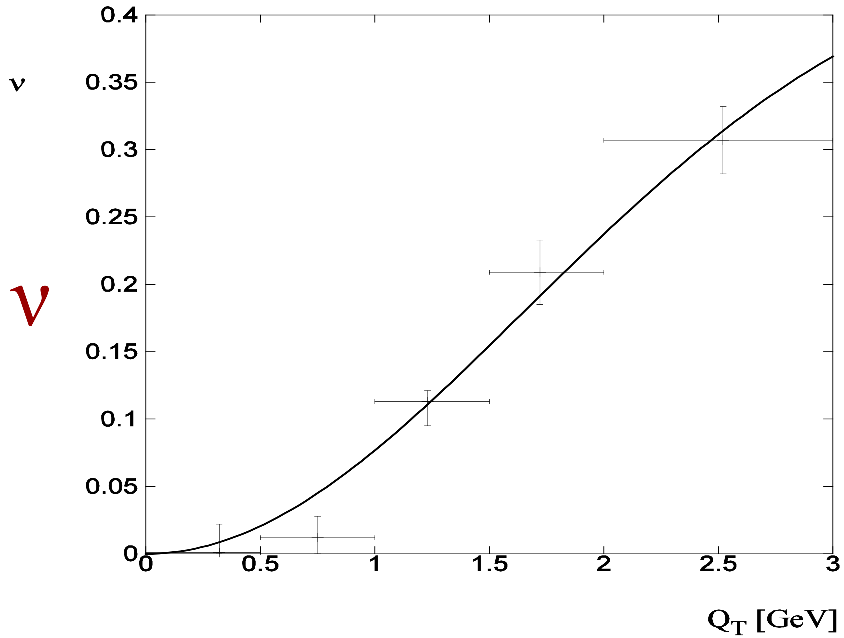
Violation of the Lam-Tung relation suggests interesting new origins (Brandenburg, Nachtmann, Mirkes, Brodsky, Khoze, Müller, Eskolar, Hoyer,Väntinnen, Vogt, etc.)

Boer-Mulders function $h_1^\perp$



- Boer pointed out that the cos2ϕ dependence can be caused by the presence of the Boer-Mulders function.

- $h_1^\perp$ can lead to an azimuthal dependence with $\nu \propto \left(\frac{h_1^\perp}{f_1}\right)\left(\frac{\overline{h}_1^\perp}{\overline{f}_1}\right)$



$$h_1^\perp(x, k_T^2) = \frac{\alpha_T}{\pi} c_H \frac{M_C M_H}{k_T^2 + M_C^2} e^{-\alpha_T k_T^2} f_1(x)$$

$$\nu = 16\kappa_1 \frac{Q_T^2 M_C^2}{(Q_T^2 + 4M_C^2)^2}$$

Boer, PRD 60 (1999) 014012

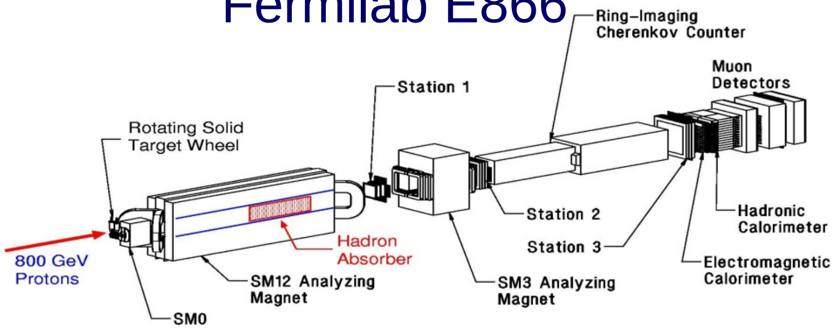
κ_1=0.47, M_C=2.3 GeV

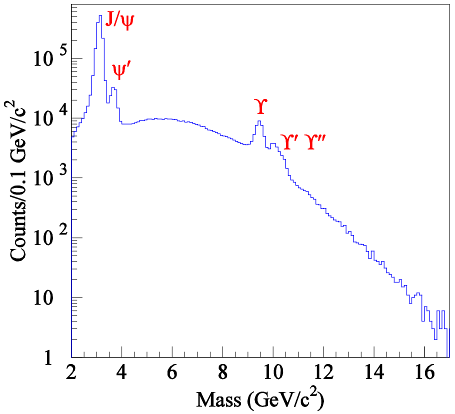
ν>0 implies valence BM functions for pion and nucleon have same signs

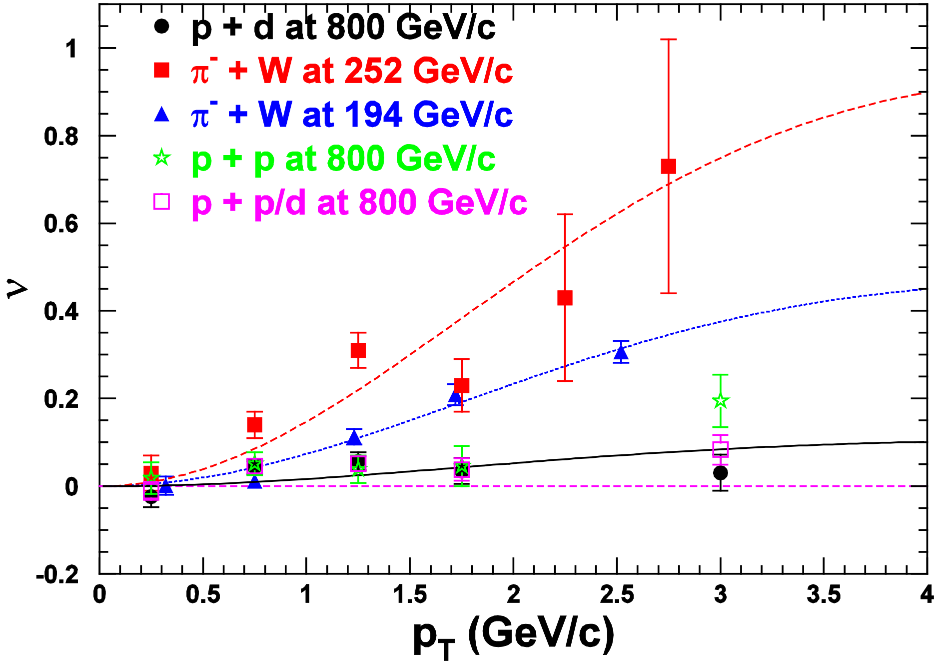
Azimuthal cos2Φ Distribution in p+d Drell-Yan

Lingyan Zhu et al., PRL 99 (2007) 082301;
PRL 102 (2009) 182001

Fermilab E866







With Boer-Mulders function $h_1^{\perp}$:

$\nu(\pi^- W \rightarrow \mu^+\mu^- X) \sim$ [valence $h_1^{\perp}(\pi)$] * [valence $h_1^{\perp}(p)$]

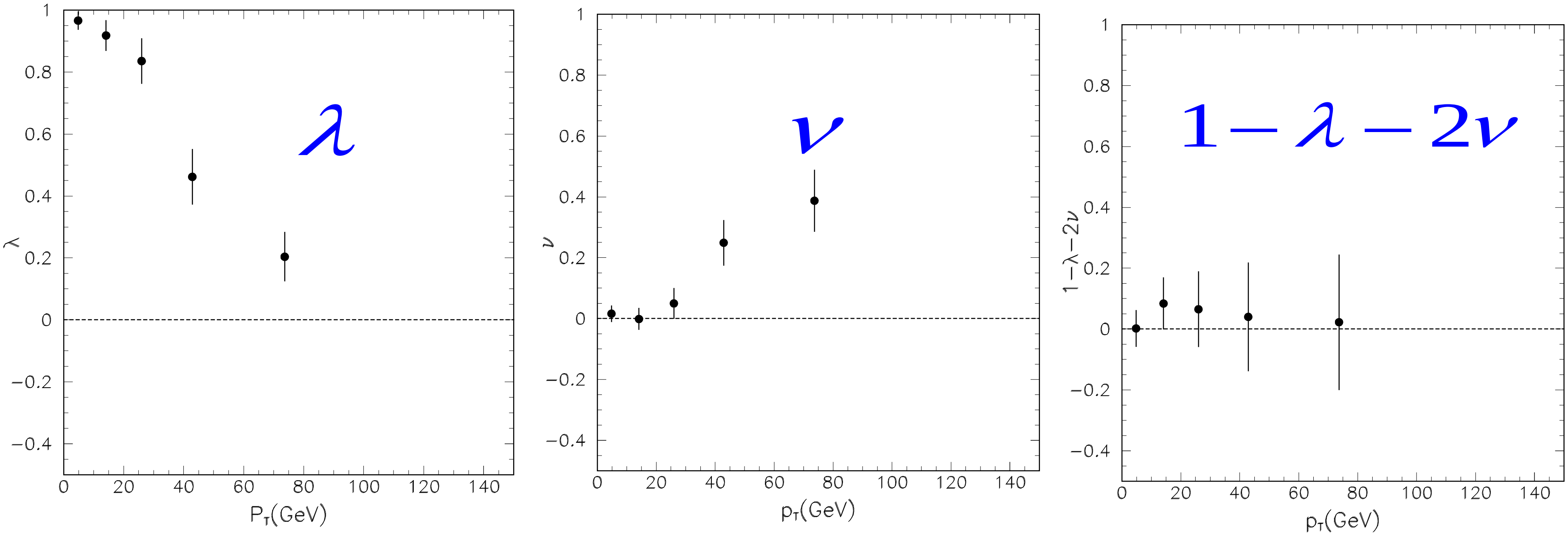
$\nu(pd \rightarrow \mu+\mu- X) \sim$ [valence $h_1^{\perp}(p)$] * [sea $h_1^{\perp}(p)$]

Sea-quark BM function is much smaller than valence BM function

Lam-Tung relation from CDF Z-production

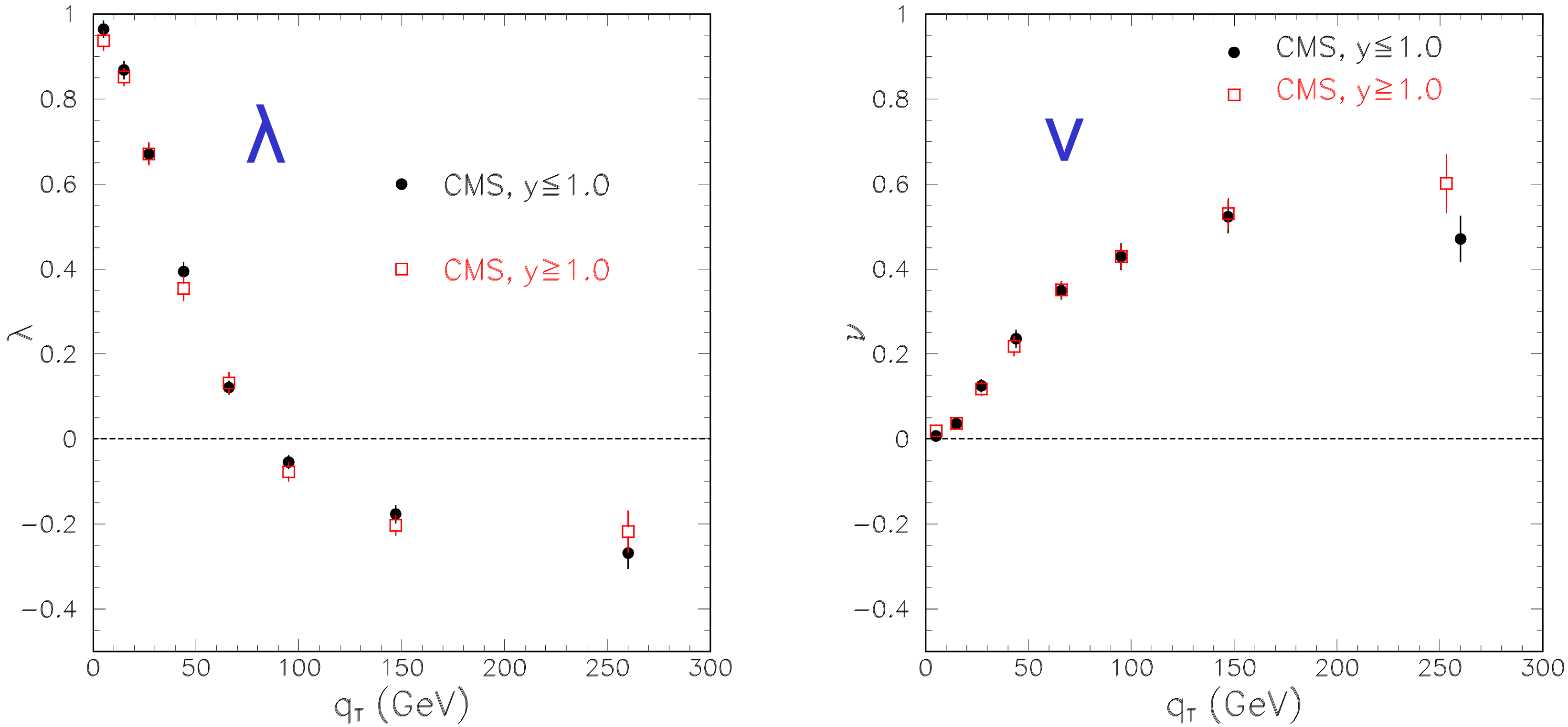
$$p+\bar{p}\rightarrow e^{+}+e^{-}+X \text{ at } \sqrt{s}=1.96\,\text{TeV}$$

arXiv:1103.5699 (PRL 106 (2011) 241801)



- Strong p_T (q_T) dependence of λ and ν
- Lam-Tung relation (1-λ = 2ν) is satisfied within experimental uncertainties (TMD is not expected to be important at large p_T)

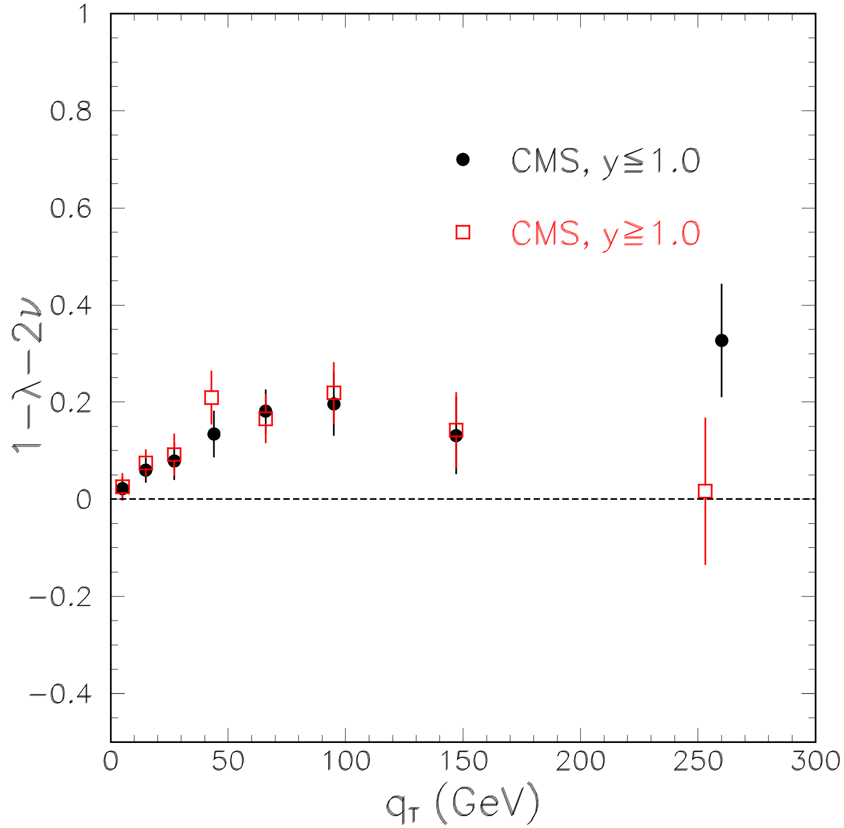
Recent CMS (ATLAS) data for *Z*-boson production in *p*+*p* collision at 8 TeV



(arXiv:1504.03512, PL B 750 (2015) 154)

- Striking q_T dependencies for λ and ν were observed at two rapidity regions
- Is Lam-Tung relation violated?

Recent data from CMS for Z-boson production in $p+p$ collision at 8 TeV



- Yes, the Lam-Tung relation is violated ($1-\lambda > 2\nu$)!
- Can one understand the origin of the violation of the Lam-Tung relation?

Interpretation of the CMS Z-production results

$$\frac{d\sigma}{d\Omega} \propto (1+\cos^2\theta) + \frac{A_0}{2}(1-3\cos^2\theta) + A_1 \sin 2\theta \cos\phi$$

$$+ \frac{A_2}{2}\sin^2\theta \cos 2\phi + A_3 \sin\theta \cos\phi + A_4 \cos\theta$$

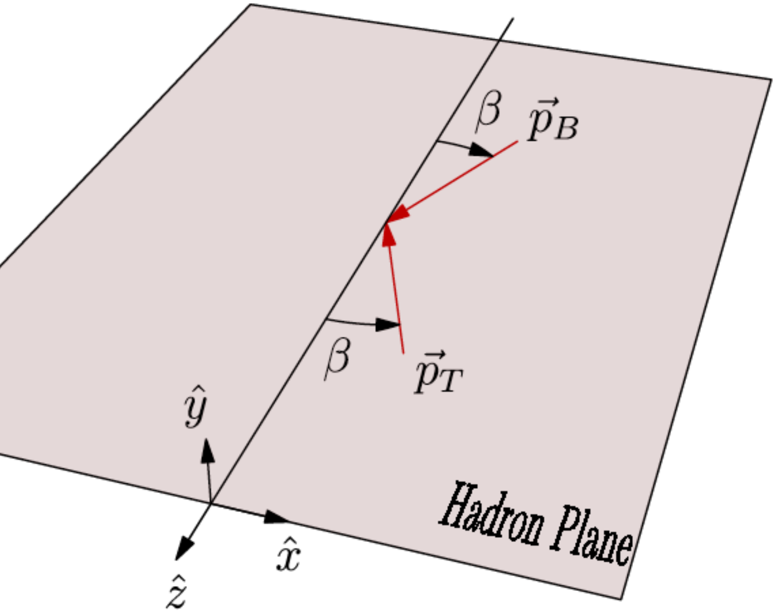
$$+ A_5 \sin^2\theta \sin 2\phi + A_6 \sin 2\theta \sin\phi + A_7 \sin\theta \sin\phi$$

Questions:

- How is the above expression derived?
- Can one express $A_0 - A_7$ in terms of some quantities?
- Can one understand the q_T dependence of A_0, A_1, A_2, etc?
- Can one understand the origin of the violation of Lam-Tung relation?

How is the angular distribution expression derived?

Define three planes in the Collins-Soper frame



1) Hadron Plane

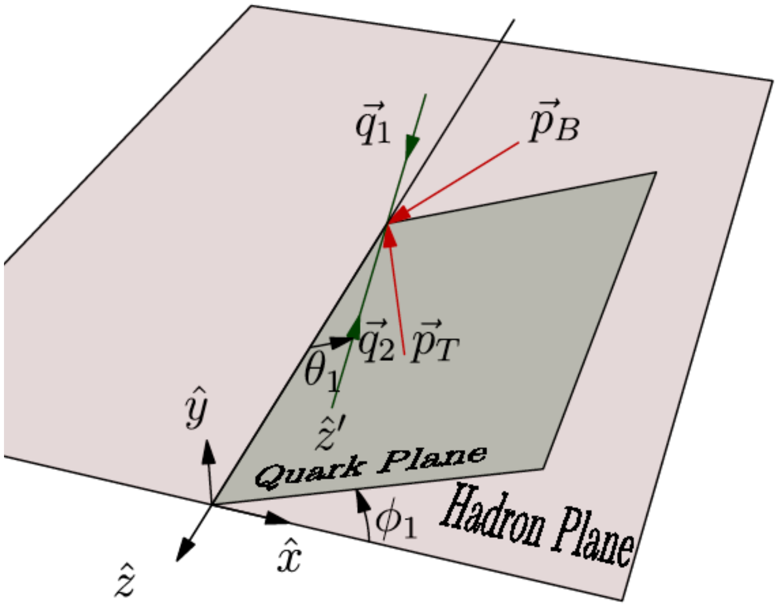
- Contains the beam $\vec{P}_B$ and target $\vec{P}_T$ momenta
- Angle β satisfies the relation $\tan\beta = q_T / Q$

- Q is the mass of the dilepton (Z)
- when $q_T \to 0,\ \beta \to 0^\circ$;

  when $q_T \to \infty,\ \beta \to 90^\circ$

How is the angular distribution expression derived?

Define three planes in the Collins-Soper frame



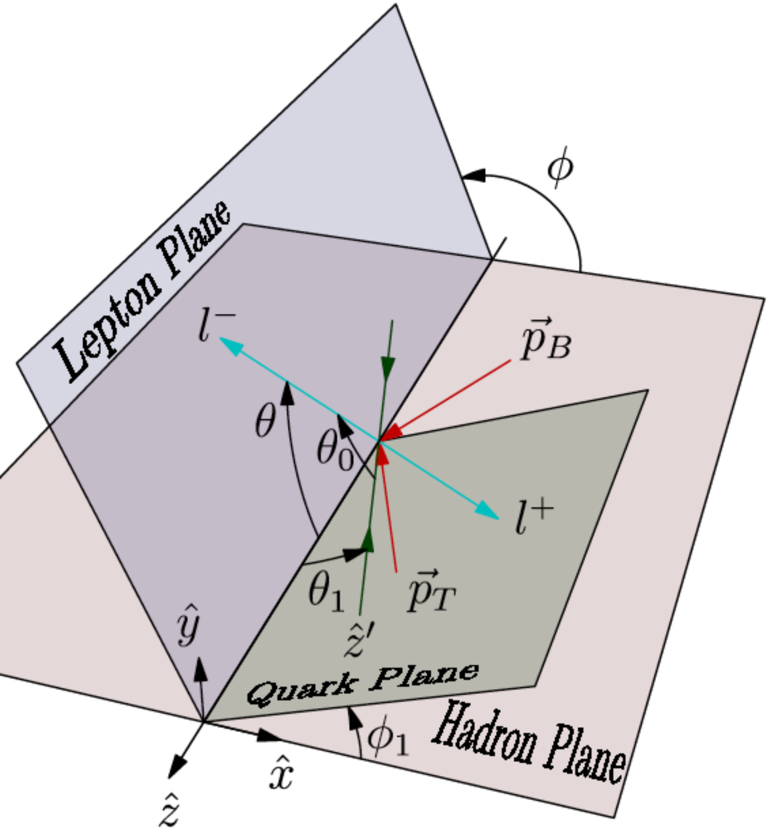
1) Hadron Plane

- Contains the beam $\vec{P}_B$ and target $\vec{P}_T$ momenta
- Angle β satisfies the relation $\tan\beta = q_T / Q$

2) Quark Plane

- q and $\overline{q}$ have head-on collision along the $\hat{z}'$ axis
- $\hat{z}'$ and $\hat{z}$ axes form the quark plane
- $\hat{z}'$ axis has angles θ_1 and ϕ_1 in the C-S frame

How is the angular distribution expression derived?



Define three planes in the Collins-Soper frame

1) Hadron Plane

- Contains the beam $\vec{P}_B$ and target $\vec{P}_T$ momenta
- Angle β satisfies the relation $\tan\beta = q_T / Q$

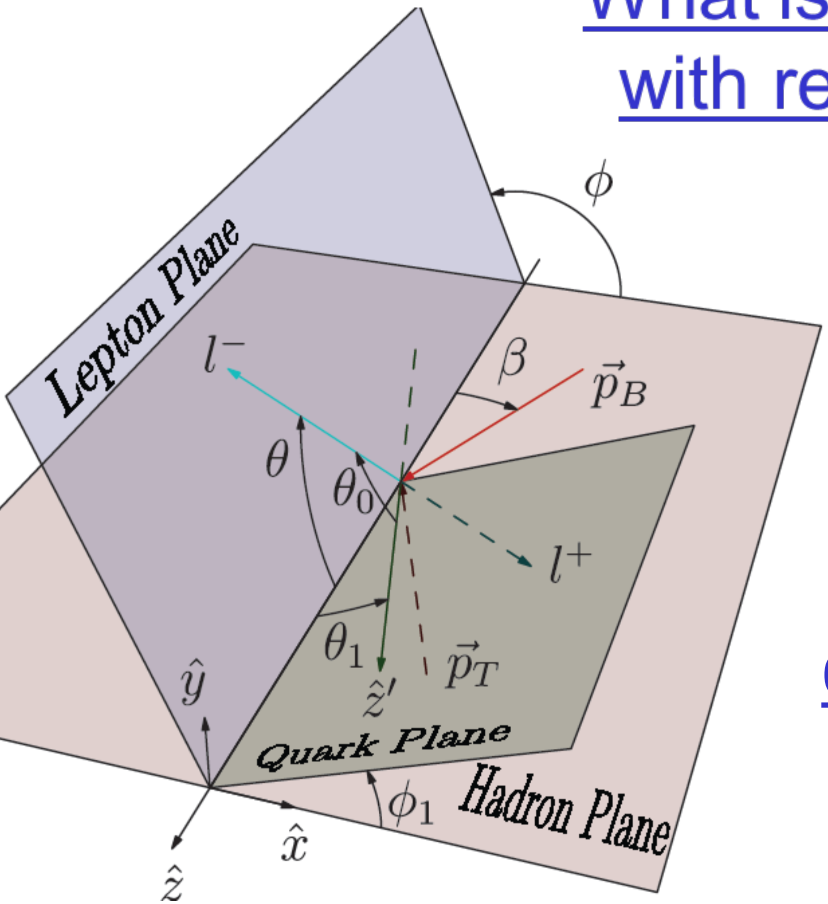
2) Quark Plane

- q and $\overline{q}$ have head-on collision along the $\hat{z}'$ axis
- $\hat{z}'$ axis has angles θ_1 and ϕ_1 in the C-S frame

3) Lepton Plane

- l^- and l^+ are emitted back-to-back with equal $|\vec{P}|$
- l^- and $\hat{z}$ form the lepton plane
- l^- is emitted at angle θ and ϕ in the C-S frame

How is the angular distribution expression derived?



What is the lepton angular distribution with respect to the $\hat{z}'$ (natural) axis?

$$\frac{d\sigma}{d\Omega} \propto 1 + a\cos\theta_0 + \cos^2\theta_0$$

Azimuthally symmetric !

How to express the angular distribution in terms of θ and φ?

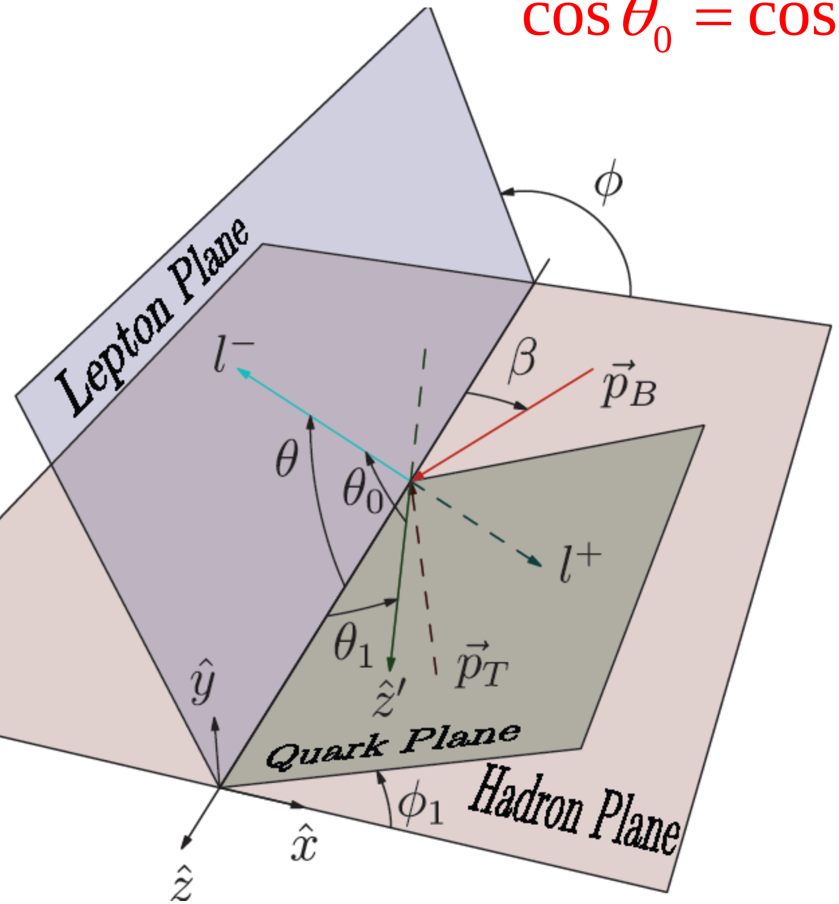
Use the following relation:

$$\cos\theta_0 = \cos\theta\cos\theta_1 + \sin\theta\sin\theta_1\cos(\phi - \phi_1)$$

How is the angular distribution expression derived?

$$\frac{d\sigma}{d\Omega} \propto 1 + a\cos\theta_0 + \cos^2\theta_0$$

$$\cos\theta_0 = \cos\theta\cos\theta_1 + \sin\theta\sin\theta_1\cos(\phi - \phi_1)$$



$$\begin{aligned}\frac{d\sigma}{d\Omega} \propto\ & (1+\cos^2\theta) + \frac{\sin^2\theta_1}{2}(1 - 3\cos^2\theta) \\ & + (\frac{1}{2}\sin 2\theta_1 \cos\phi_1)\sin 2\theta\cos\phi \\ & + (\frac{1}{2}\sin^2\theta_1\cos 2\phi_1)\sin^2\theta\cos 2\phi \\ & + (a\sin\theta_1\cos\phi_1)\sin\theta\cos\phi + (a\cos\theta_1)\cos\theta \\ & + (\frac{1}{2}\sin^2\theta_1\sin 2\phi_1)\sin^2\theta\sin 2\phi \\ & + (\frac{1}{2}\sin 2\theta_1\sin\phi_1)\sin 2\theta\sin\phi \\ & + (a\sin\theta_1\sin\phi_1)\sin\theta\sin\phi .\end{aligned}$$

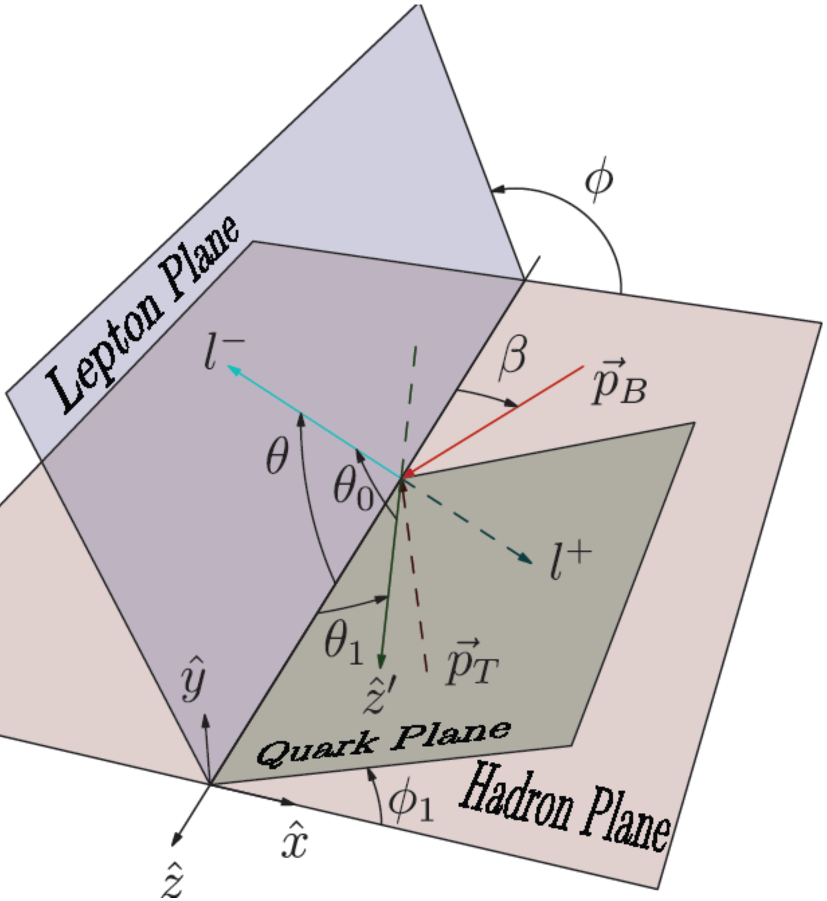
All eight angular distribution terms are obtained!

$$
\begin{aligned}
\frac{d\sigma}{d\Omega} \propto\ & (1+\cos^2\theta) + \frac{\sin^2\theta_1}{2}(1-3\cos^2\theta) \\
& + (\frac{1}{2}\sin 2\theta_1 \cos\phi_1)\sin 2\theta\cos\phi \\
& + (\frac{1}{2}\sin^2\theta_1 \cos 2\phi_1)\sin^2\theta\cos 2\phi \\
& + (a\sin\theta_1\cos\phi_1)\sin\theta\cos\phi + (a\cos\theta_1)\cos\theta \\
& + (\frac{1}{2}\sin^2\theta_1\sin 2\phi_1)\sin^2\theta\sin 2\phi \\
& + (\frac{1}{2}\sin 2\theta_1\sin\phi_1)\sin 2\theta\sin\phi \\
& + (a\sin\theta_1\sin\phi_1)\sin\theta\sin\phi .
\end{aligned}
$$

$$
\begin{aligned}
\frac{d\sigma}{d\Omega} \propto\ & (1+\cos^2\theta) + \frac{A_0}{2}(1-3\cos^2\theta) \\
& + A_1\sin 2\theta\cos\phi \\
& + \frac{A_2}{2}\sin^2\theta\cos 2\phi \\
& + A_3\sin\theta\cos\phi + A_4\cos\theta \\
& + A_5\sin^2\theta\sin 2\phi \\
& + A_6\sin 2\theta\sin\phi \\
& + A_7\sin\theta\sin\phi
\end{aligned}
$$

$A_0 - A_7$ are entirely described by θ_1, ϕ_1 and a

Angular distribution coefficients $A_0 - A_7$



$$A_0 = \langle \sin^2\theta_1 \rangle$$

$$A_1 = \frac{1}{2}\langle \sin 2\theta_1 \cos\phi_1 \rangle$$

$$A_2 = \langle \sin^2\theta_1 \cos 2\phi_1 \rangle$$

$$A_3 = a\langle \sin\theta_1 \cos\phi_1 \rangle$$

$$A_4 = a\langle \cos\theta_1 \rangle$$

$$A_5 = \frac{1}{2}\langle \sin^2\theta_1 \sin 2\phi_1 \rangle$$

$$A_6 = \frac{1}{2}\langle \sin 2\theta_1 \sin\phi_1 \rangle$$

$$A_7 = a\langle \sin\theta_1 \sin\phi_1 \rangle$$

Some implications of the angular distribution coefficients $A_0 - A_7$

$$A_0 = \left\langle \sin^2\theta_1 \right\rangle$$

$$A_1 = \frac{1}{2}\left\langle \sin 2\theta_1 \cos\phi_1 \right\rangle$$

$$A_2 = \left\langle \sin^2\theta_1 \cos 2\phi_1 \right\rangle$$

$$A_3 = a\left\langle \sin\theta_1 \cos\phi_1 \right\rangle$$

$$A_4 = a\left\langle \cos\theta_1 \right\rangle$$

$$A_5 = \frac{1}{2}\left\langle \sin^2\theta_1 \sin 2\phi_1 \right\rangle$$

$$A_6 = \frac{1}{2}\left\langle \sin 2\theta_1 \sin\phi_1 \right\rangle$$

$$A_7 = a\left\langle \sin\theta_1 \sin\phi_1 \right\rangle$$

- $A_0 \geq A_2$ (or $1 - \lambda - 2\nu \geq 0$)
- Lam-Tung relation ($A_0 = A_2$) is satisfied when $\phi_1 = 0$
- Forward-backward asymmetry, a, is reduced by a factor of $\left\langle \cos\theta_1 \right\rangle$ for A_4
- A_5, A_6, A_7 are odd function of ϕ_1 and must vanish from symmetry consideration
- Some equality and inequality relations among $A_0 - A_7$ can be obatined

Some implications of the angular distribution coefficients $A_0 - A_7$

$$A_0 = \langle \sin^2\theta_1 \rangle$$

$$A_1 = \frac{1}{2}\langle \sin 2\theta_1 \cos\phi_1 \rangle$$

$$A_2 = \langle \sin^2\theta_1 \cos 2\phi_1 \rangle$$

$$A_3 = a\langle \sin\theta_1 \cos\phi_1 \rangle$$

$$A_4 = a\langle \cos\theta_1 \rangle$$

$$A_5 = \frac{1}{2}\langle \sin^2\theta_1 \sin 2\phi_1 \rangle$$

$$A_6 = \frac{1}{2}\langle \sin 2\theta_1 \sin\phi_1 \rangle$$

$$A_7 = a\langle \sin\theta_1 \sin\phi_1 \rangle$$

Some bounds on the coefficients can be obtained

$$0 < A_0 < 1$$

$$-1/2 < A_1 < 1/2$$

$$-1 < A_2 < 1$$

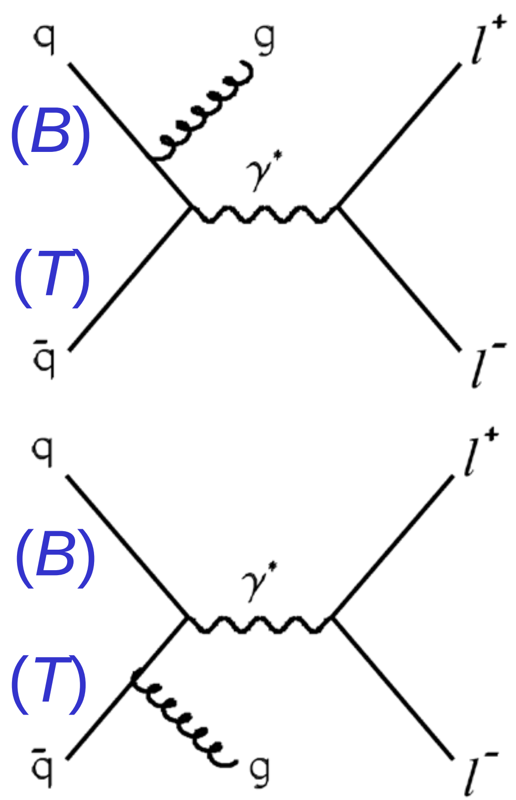
$$-a < A_3 < a$$

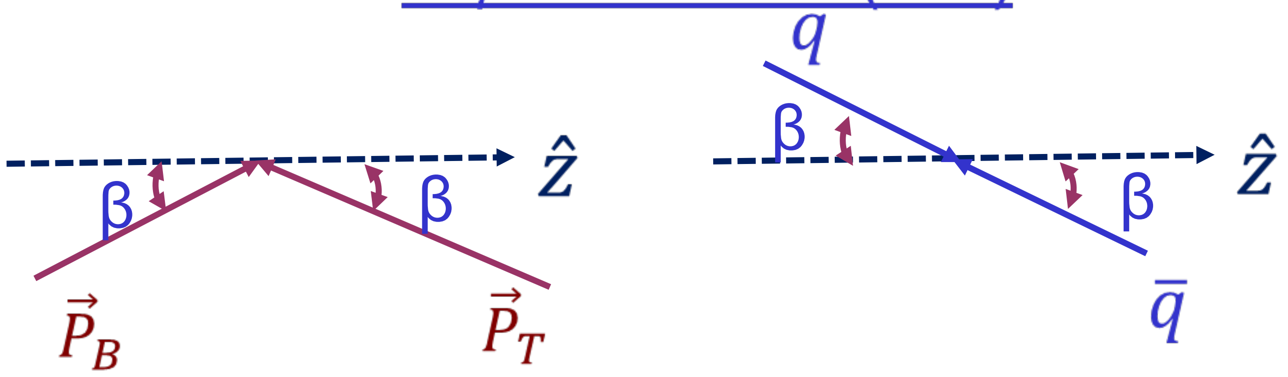
$$-a < A_4 < a$$

What are the values of θ_1 and ϕ_1 at order α_s?

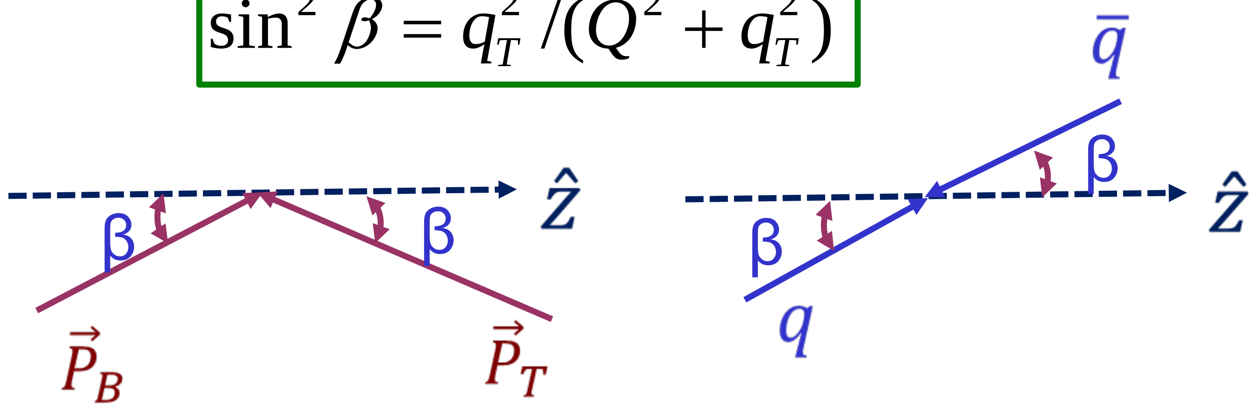
1) $q\bar{q} \rightarrow \gamma^*(Z^0)g$

In γ^* rest frame (C-S)





$$\sin^2\beta = q_T^2/(Q^2 + q_T^2)$$



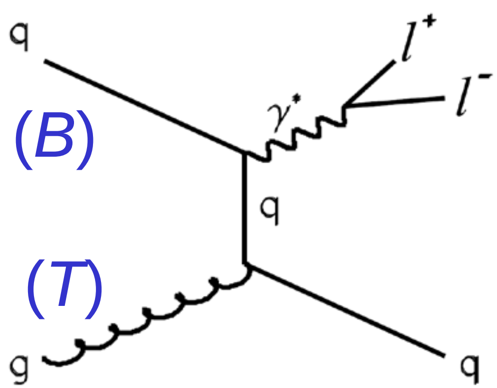
$\theta_1 = \beta$ and $\phi_1 = 0$; $A_0 = A_2 = \sin^2\beta$

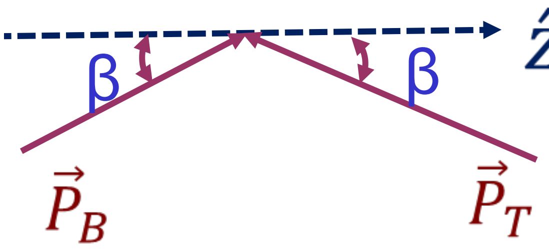
$$\lambda = \frac{2-3A_0}{2+A_0} = \frac{2Q^2 - q_T^2}{2Q^2 + 3q_T^2}; \qquad \nu = \frac{2A_2}{2+A_0} = \frac{2q_T^2}{2Q^2 + 3q_T^2}$$

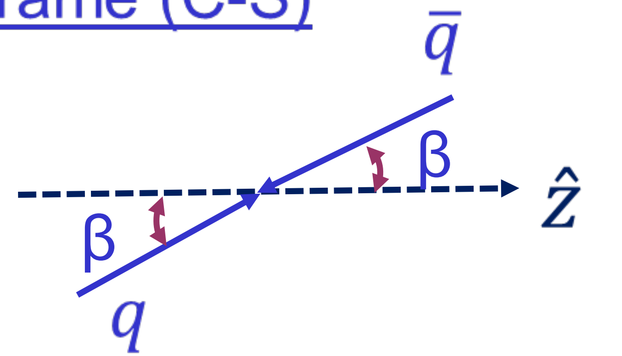
What are the values of θ_1 and ϕ_1 at order α_s?

2) $qg \to \gamma^*(Z^0)q$

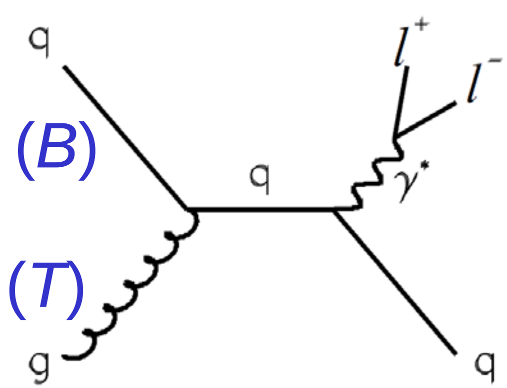
In γ^* rest frame (C-S)

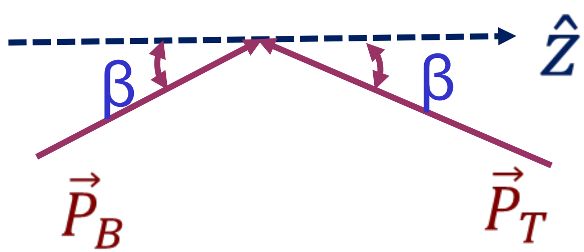


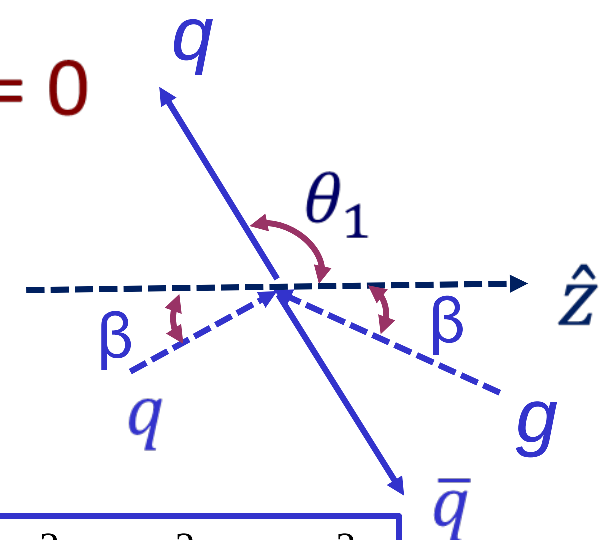




$\theta_1 = \beta$ and $\phi_1 = 0$





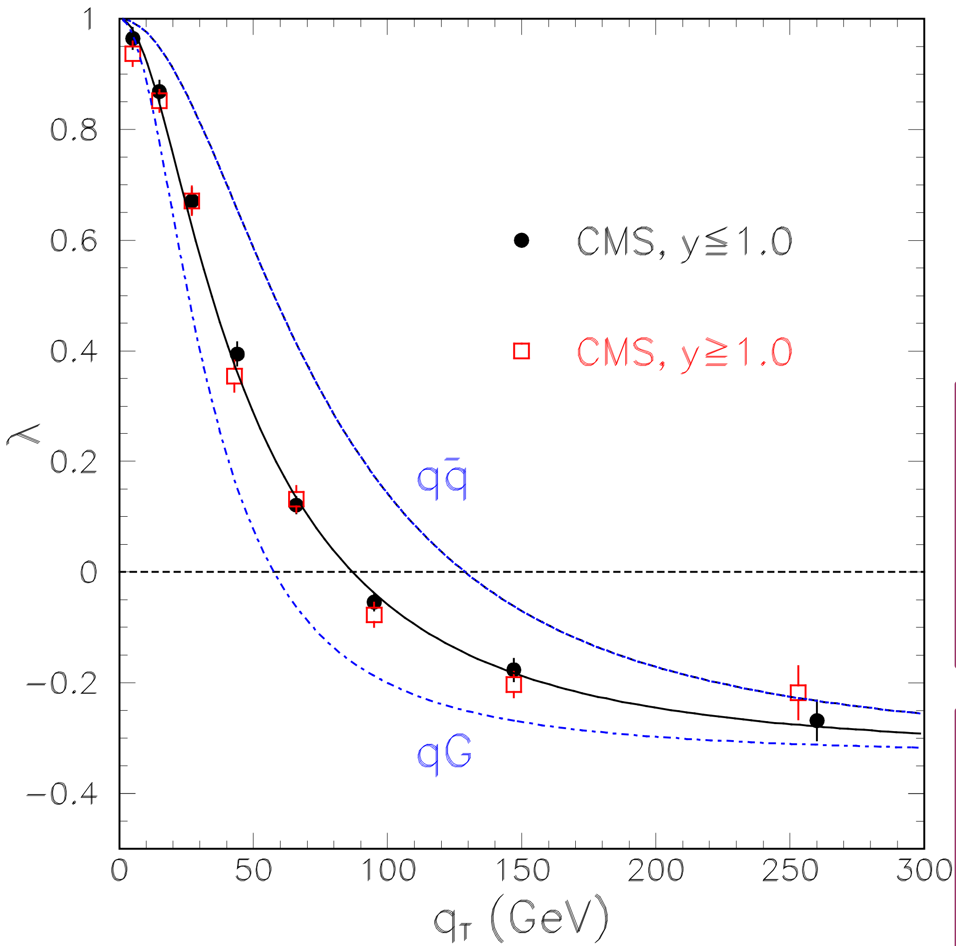


$\theta_1 > \beta$ and $\phi_1 = 0$; $A_0 = A_2 \approx 5q_T^2/(Q^2 + 5q_T^2)$

$$\lambda = \frac{2 - 3A_0}{2 + A_0} = \frac{2Q^2 - 5q_T^2}{2Q^2 + 15q_T^2}; \quad \nu = \frac{2A_2}{2 + A_0} = \frac{10q_T^2}{2Q^2 + 15q_T^2}$$

Compare with CMS data on λ

(*Z* production in *p*+*p* collision at 8 TeV)



$$\lambda = \frac{2Q^2 - q_T^2}{2Q^2 + 3q_T^2} \quad \text{for} \quad q\bar{q} \to Zg$$

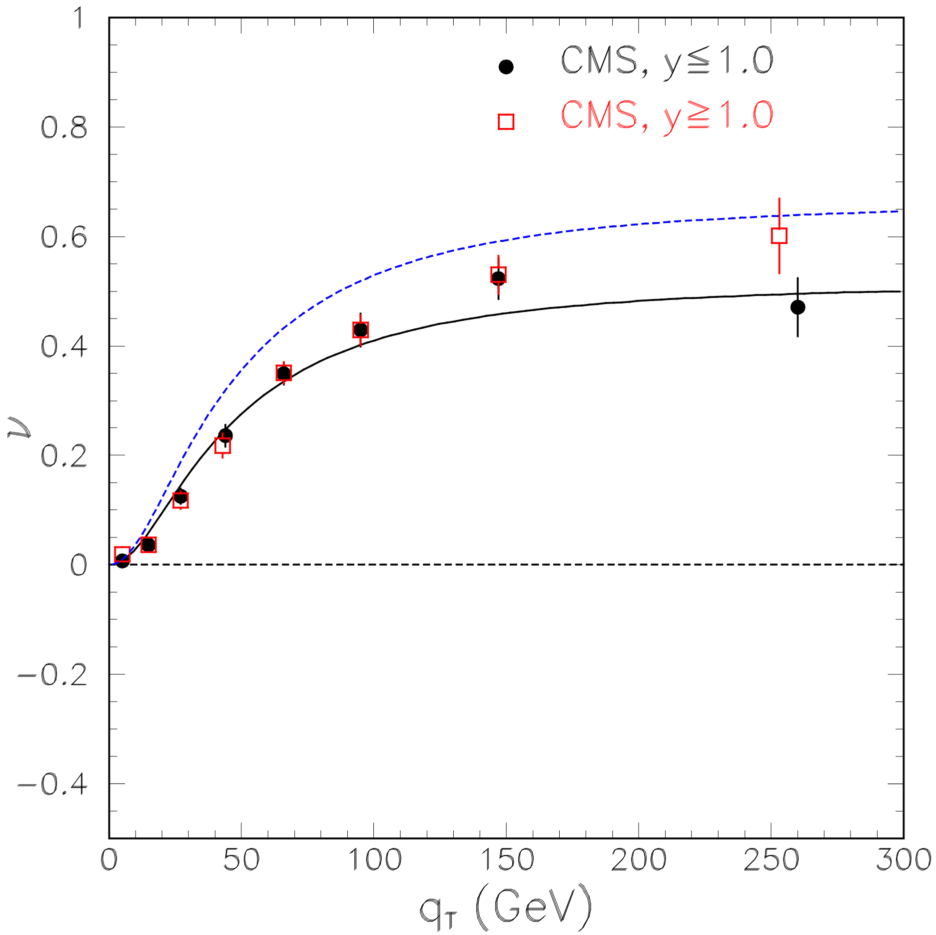
$$\lambda = \frac{2Q^2 - 5q_T^2}{2Q^2 + 15q_T^2} \quad \text{for} \quad qG \to Zq$$

For both processes
$\lambda = 1$ at $q_T = 0$ ($\theta_1 = 0^o$)
$\lambda = -1/3$ at $q_T = \infty$ ($\theta_1 = 90^o$)

Data can be well described with a mixture of 58.5% qG and 41.5% $q\bar{q}$ processes

Compare with CMS data on ν

(Z production in p+p collision at 8 TeV)



$$\nu = \frac{2q_T^2}{2Q^2 + 3q_T^2} \quad \text{for} \quad q\bar{q} \to Zg$$

$$\nu = \frac{10q_T^2}{2Q^2 + 15q_T^2} \quad \text{for} \quad qG \to Zq$$

Dashed curve corresponds to a mixture of 58.5% qG and 41.5% $q\bar{q}$ processes

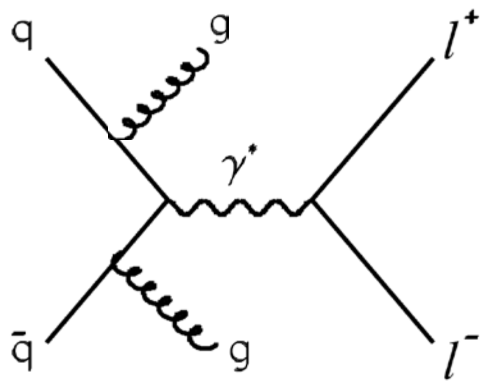
Solid curve corresponds to

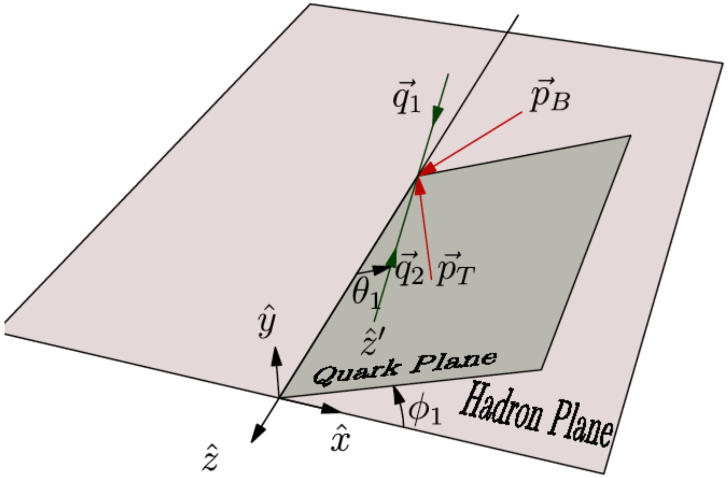
$$\left\langle \sin^2\theta_1 \cos 2\phi_1 \right\rangle / \left\langle \sin^2\theta_1 \right\rangle = 0.77$$

$q - \bar{q}$ axis is non-coplanar relative to the hadron plane

Origins of the non-coplanarity

1) Processes at order α_s^2 or higher

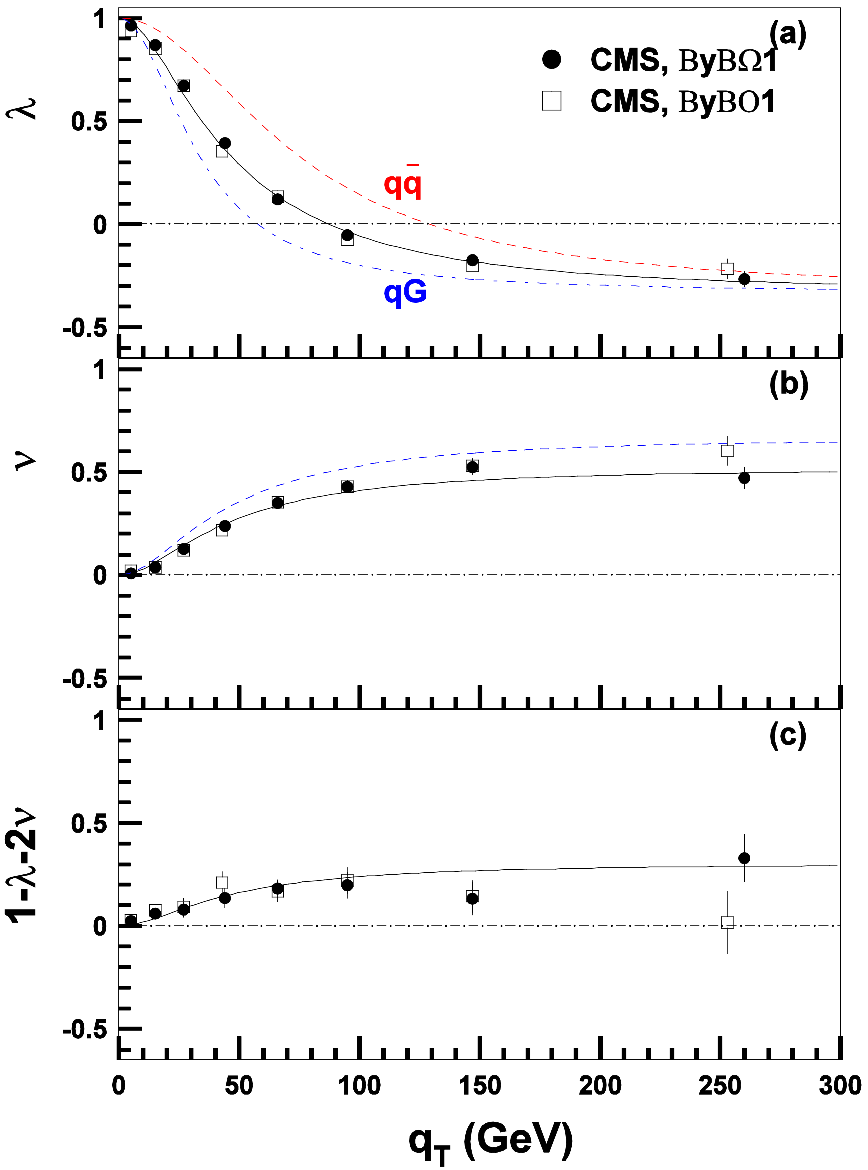




2) Intrinsic k_T from interacting partons

(Boer-Mulders functions in the beam and target hadrons)

Compare with CMS data on Lam-Tung relation



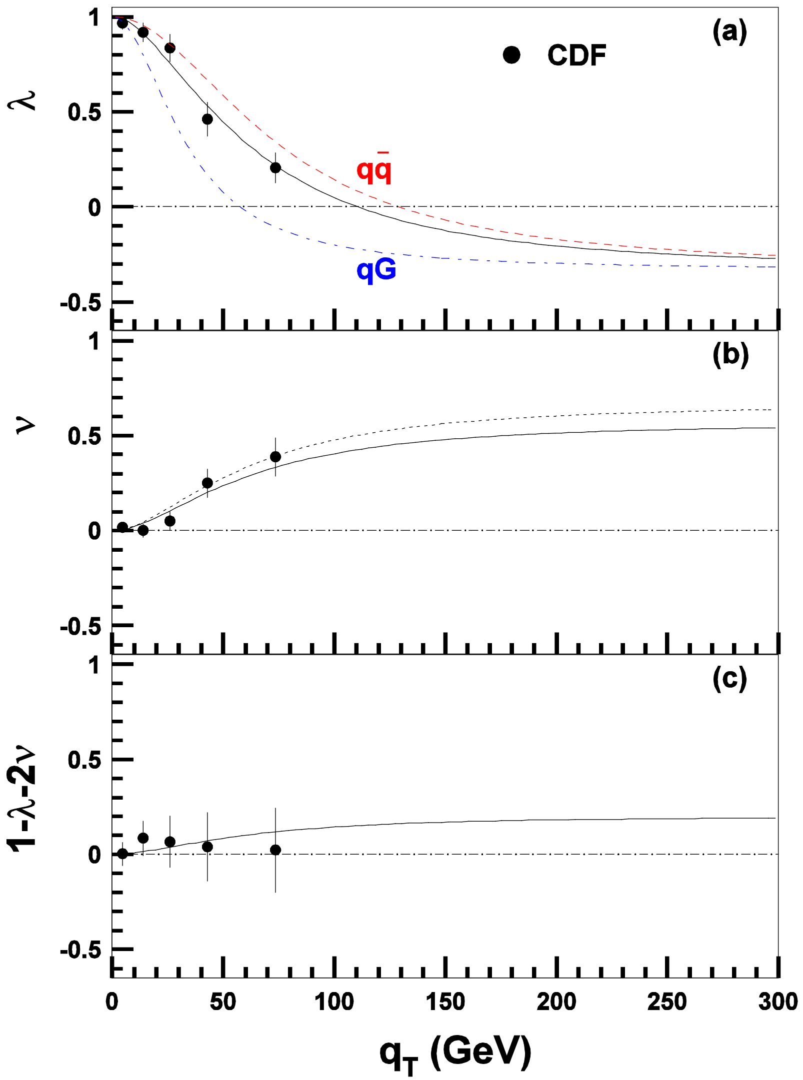
Solid curves correspond to a mixture of 58.5% qG and 41.5% $q\bar{q}$ processes, and

$$\langle \sin^2\theta_1 \cos 2\phi_1 \rangle / \langle \sin^2\theta_1 \rangle = 0.77$$

Violation of Lam-Tung relation is well described

Compare with CDF data

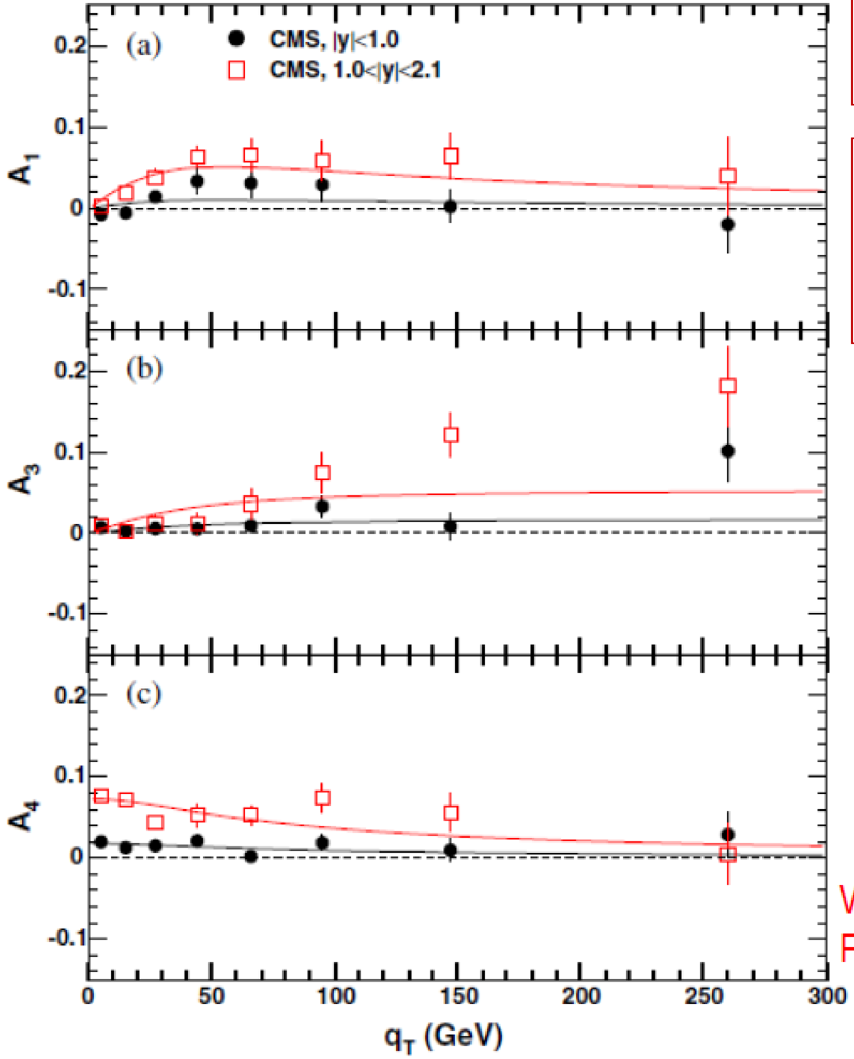
(Z production in $p + \bar{p}$ collision at 1.96 TeV)



Solid curves correspond to a mixture of 27.5% qG and 72.5% $q\bar{q}$ processes, and $\langle \sin^2\theta_1 \cos 2\phi_1 \rangle / \langle \sin^2\theta_1 \rangle = 0.85$

Violation of Lam-Tung relation is not ruled out

Compare with CMS data on A_1, A_3 and A_4



$$A_1 = r_1 \left[f \frac{q_T Q}{Q^2 + q_T^2} + (1 - f) \frac{\sqrt{5} q_T Q}{Q^2 + 5 q_T^2} \right]$$

$$A_3 = r_3 \left[f \frac{q_T}{\sqrt{Q^2 + q_T^2}} + (1 - f) \frac{\sqrt{5} q_T}{\sqrt{Q^2 + 5 q_T^2}} \right]$$

$$A_4 = r_4 \left[f \frac{Q}{\sqrt{Q^2 + q_T^2}} + (1 - f) \frac{Q}{\sqrt{Q^2 + 5 q_T^2}} \right]$$

Rapidity of A_1, A_3 and A_4 are well described

W.C. Chang, R.E. McClellan, J.C. Peng, O. Teryaev
Phys. Rev. D 96, 054020 (2017)

Future prospects

- Extend this study to W-boson production
  - Preliminary results show that the data can be well described
- Extend this study to fixed-target Drell-Yan data
  - Extraction of Boer-Mulders functions must take into account the QCD effects
- Extend this study to dihadron production in $e^- e^+$ collision (inverse of the Drell-Yan)
  - Analogous angular distribution coefficients and analogous Lam-Tung relation

Future prospects

- Extend this study to semi-inclusive DIS at high p_T (involving two hadrons and two leptons)
  - Relevant for EIC measurements
- Rotational invariance, equality, and inequality relations formed by various angular distribution coefficients
  - See preprint with D. Boer, arXiv: 1808.04398
- Comparison with pQCD calculations
  - Preprint under preparation
  - Lambertson and Vogelsang, PRD 93 (2016) 114013

Summary

- The lepton angular distribution coefficients A_0-A_7 are described in terms of the polar and azimuthal angles of the $q-\bar{q}$ axis.
- The striking q_T dependence of A_0 (or equivalently, λ) can be well described by the mis-alignment of the $q-\bar{q}$ axis and the Collins-Soper *z*-axis.
- Violation of the Lam-Tung relation ($A_0 \neq A_2$) is described by the non-coplanarity of the $q-\bar{q}$ axis and the hadron plane. This can come from order α_s^2 or higher processes or from intrinsic k_T.
- This study can be extended to fixed-target Drell-Yan data.