



## Exclusive quarkonium-production studies for the EIC and LHC

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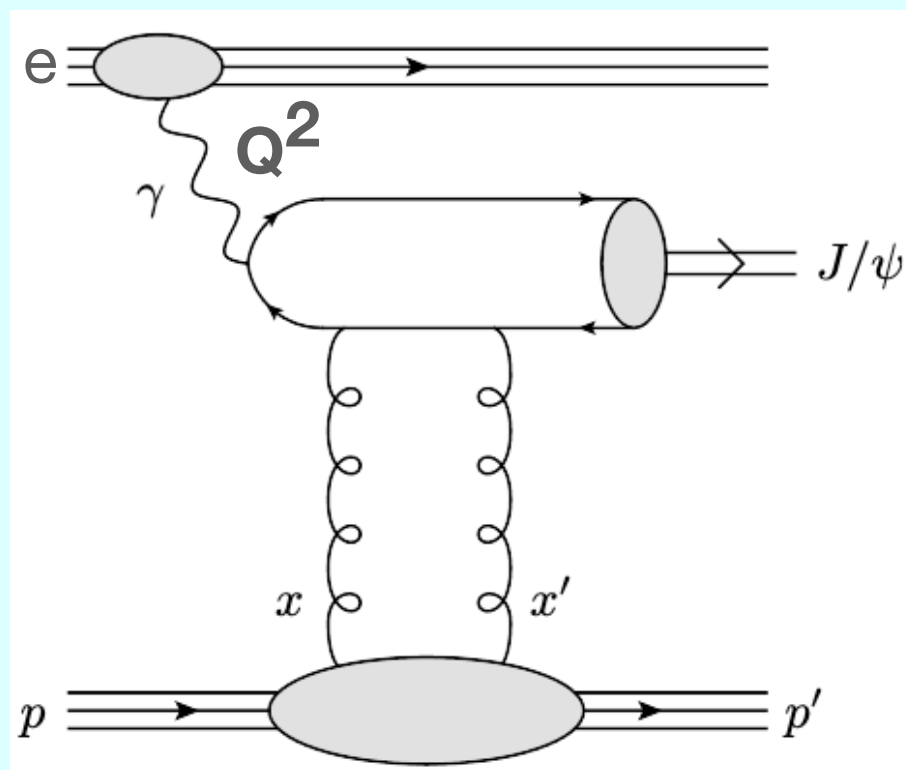
# Synergies

In the spirit of the workshop, I will discuss the synergies of

two aspects of **exclusive quarkonium production** related to EIC and LHC physics:



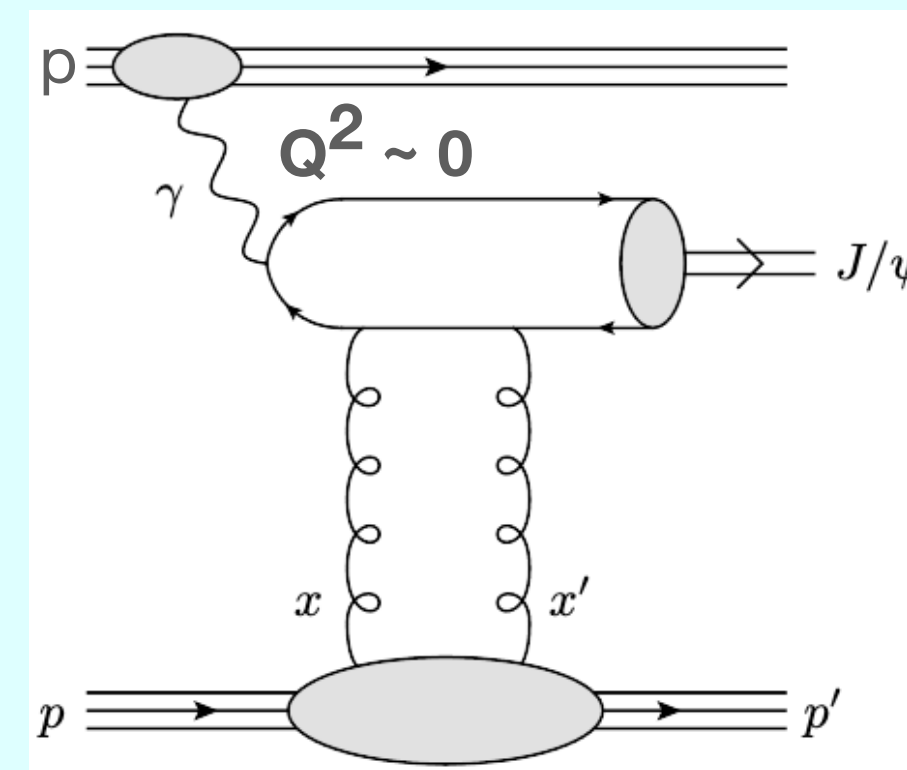
EIC (ep/eA) & HERA



Tagged e -> Lever arm in the virtuality  $Q^2$  at **moderate-to-low(ish) x** reach on target

photo- ( $Q^2 < 1 \text{ GeV}^2$ ) and electroproduction ( $Q^2 > 1 \text{ GeV}^2$ )

LHC (pp, pA, AA)



UPCs at the LHC -> flux of on-shell  $Q^2 \sim 0$  photons and **low-x** reach on target

photoproduction

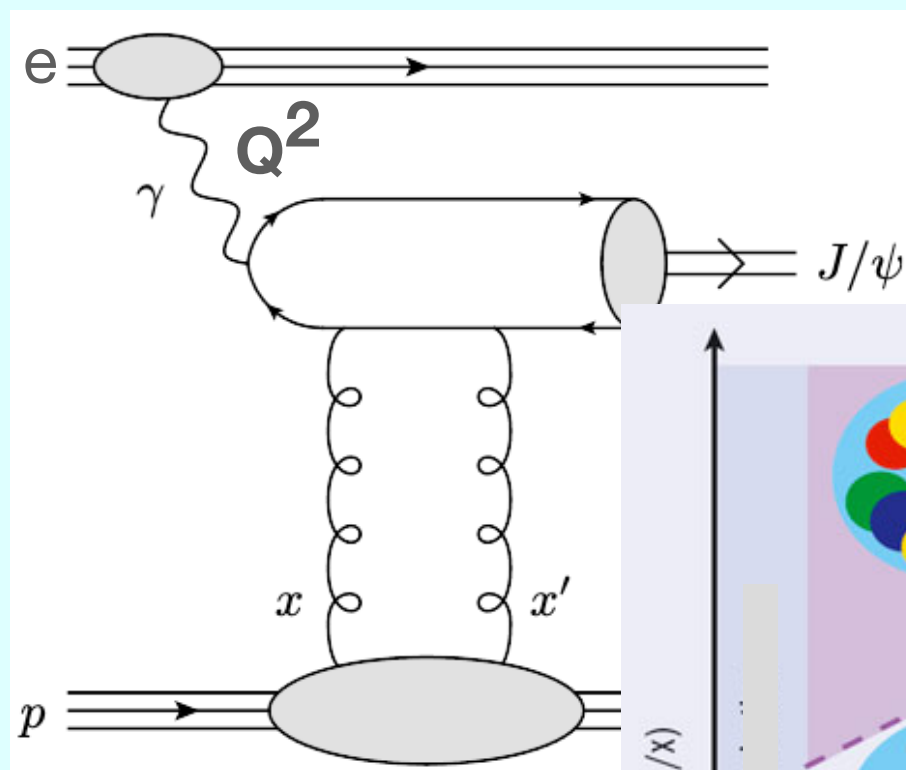
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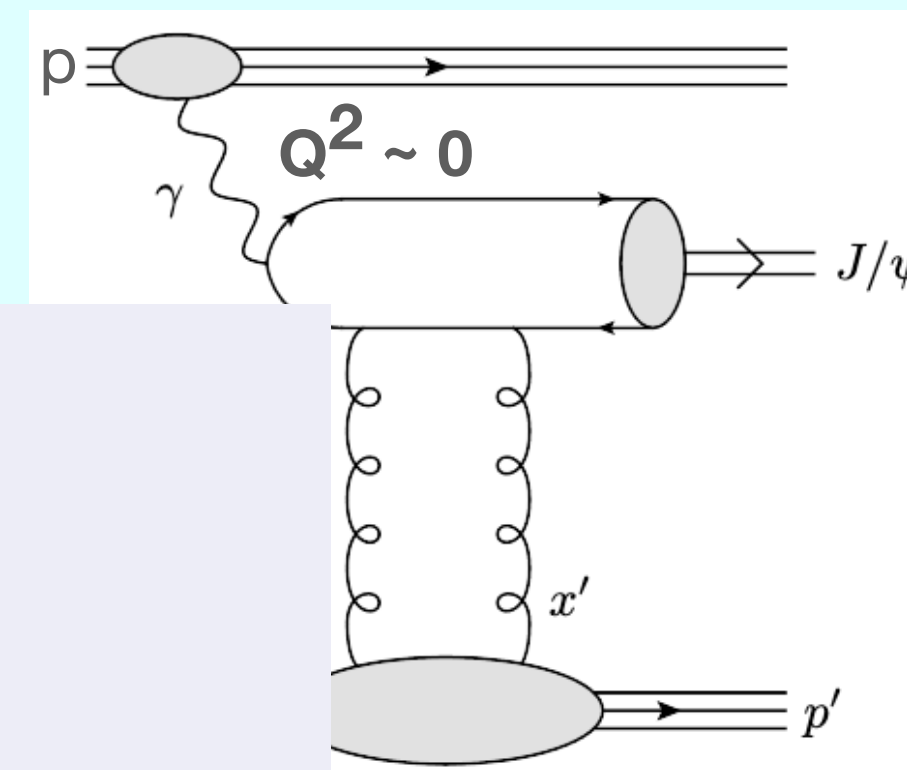
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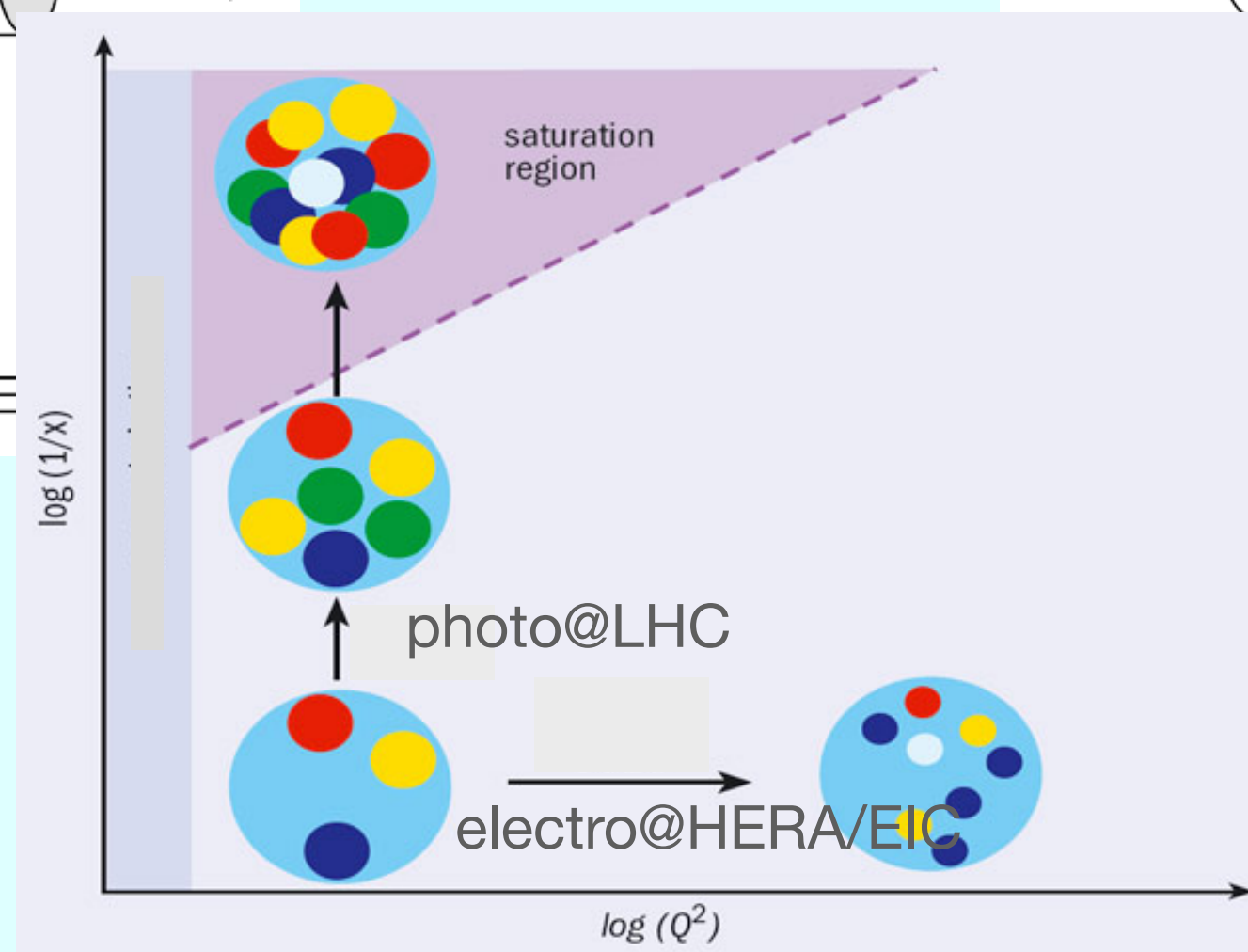
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Complementary regions in x,Q plane

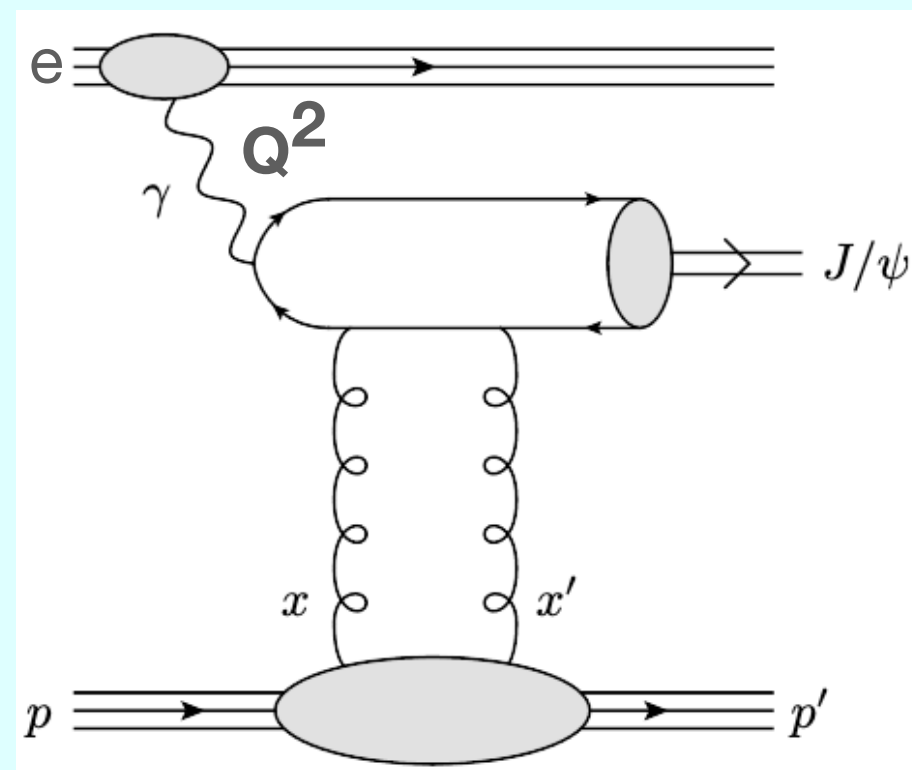
# Outline

In the spirit of the workshop, I will discuss the synergies of

two aspects of **exclusive quarkonium production** related to EIC and LHC physics:



## 1. EIC (ep/eA) & HERA



- **Theory** and **phenomenology** of exclusive quarkonium electroproduction at the EIC

- Kinematics and set-up
- Results
- Phenomenology

- Based on results in [CAF et al. JHEP 08 \(2021\) 150](#) and those to appear elsewhere

# Outline

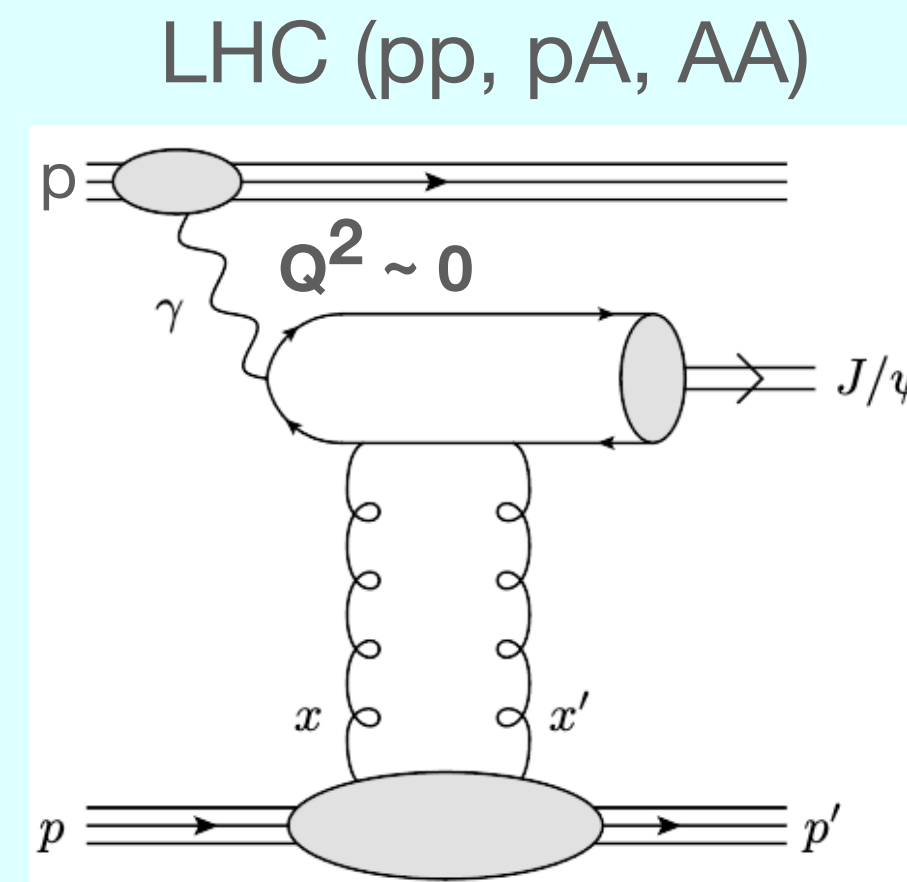
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2.

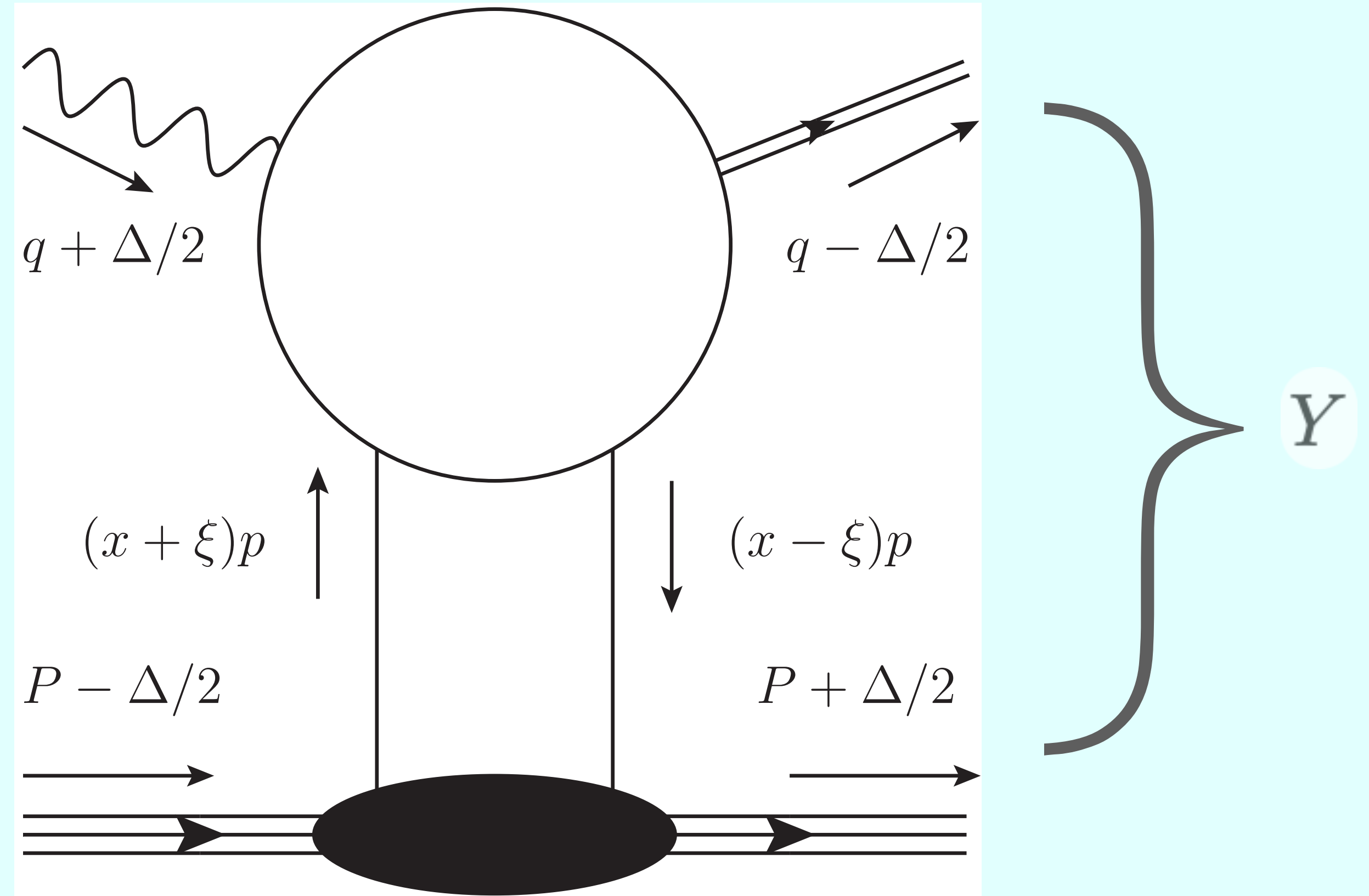
- Exclusive quarkonium photoproduction at the LHC as **constraints** on the low- $x$  and low-scale gluon PDF
  - Inclusion of framework into xFitter
  - Two ongoing studies



# 1. Exclusive quarkonium electroproduction at the EIC

# Notation and collinear factorisation

- Fluctuation of *space-like* incoming photon into pair of heavy quarks
- Pair interacts with proton (or nucleus) via two-parton colour-singlet exchange
- Modelling of heavy quark pair recombination into *time-like* exclusive vector meson made within NRQCD
- Decompose the three linearly independent momenta in a high-energy Sudakov basis spanned by  $\{p, n, \Delta_\perp\}$
- Define analogy of Bjorken variable
- Proton (or nucleus) recoils *slightly* with net momentum exchange  $2\xi$  along light-cone direction in Bjorken limit



$$\begin{aligned}
 P^\mu &= p^\mu + \frac{M_N^2 - t/4}{2} n^\mu, \\
 q^\mu &= -\zeta p^\mu + \frac{\tilde{Q}^2}{2\zeta} n^\mu, \\
 \Delta^\mu &= -2\xi p^\mu + \xi(M_N^2 - t/4)n^\mu + \Delta_\perp^\mu
 \end{aligned}$$

Bjorken limit and leading twist exp.

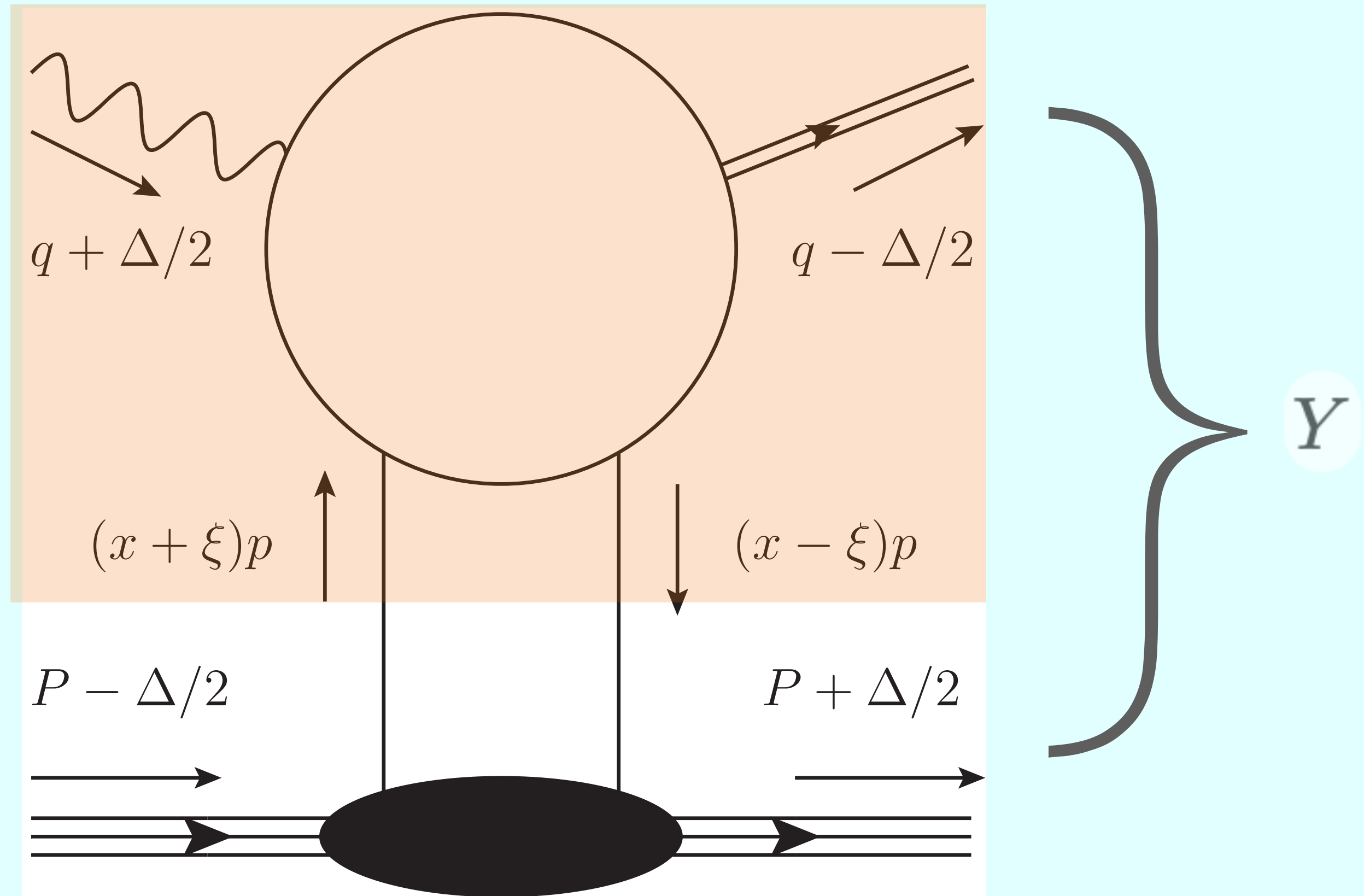
$$x_B = \frac{\tilde{Q}^2}{2P \cdot q}$$

$$\begin{aligned}
 \lim_{\tilde{Q}^2 \rightarrow \infty, x_B \text{ fixed}} P^\mu &= p^\mu, \\
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Hard-scattering:



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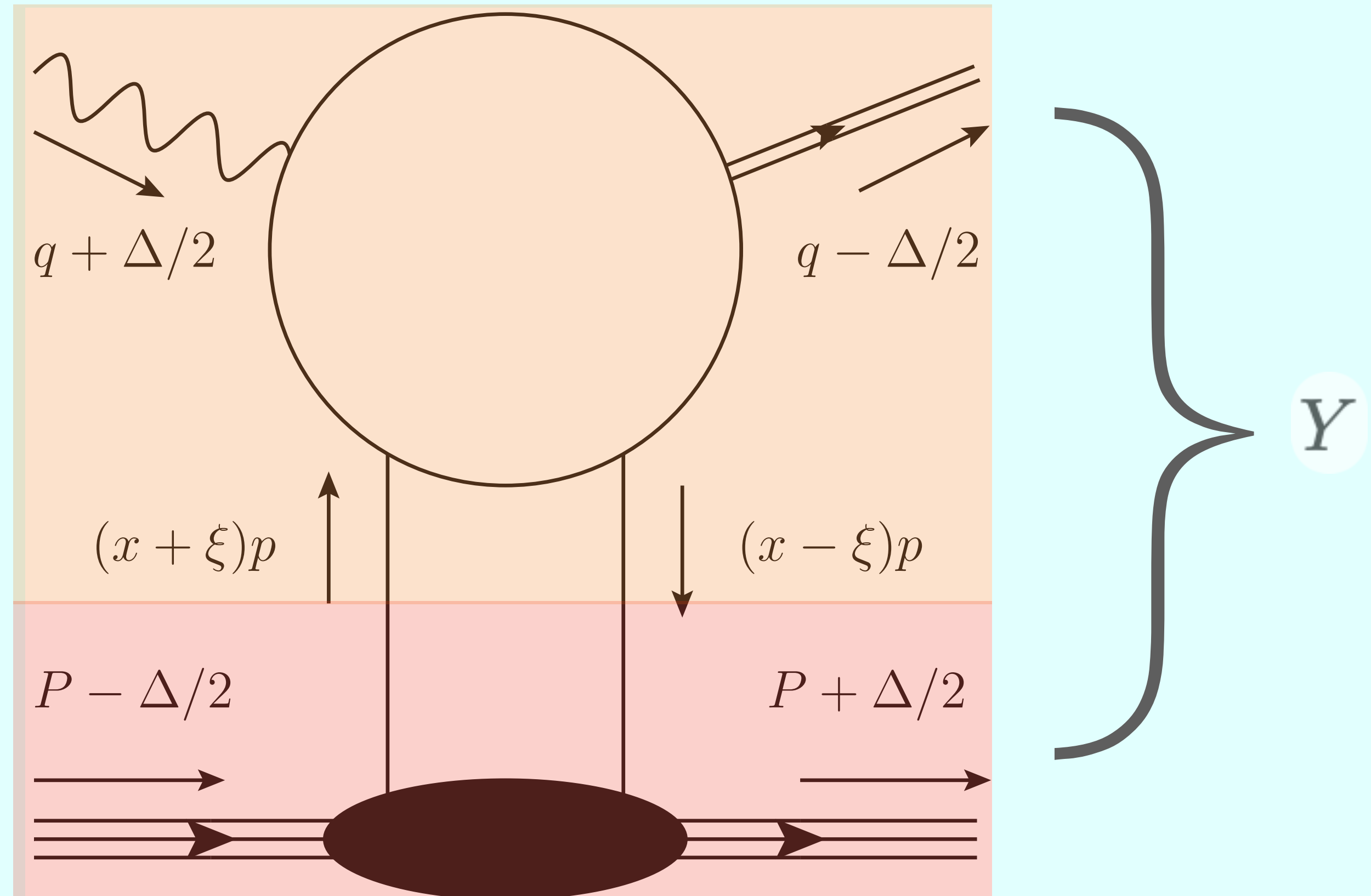


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Hard-scattering:

GPDs:



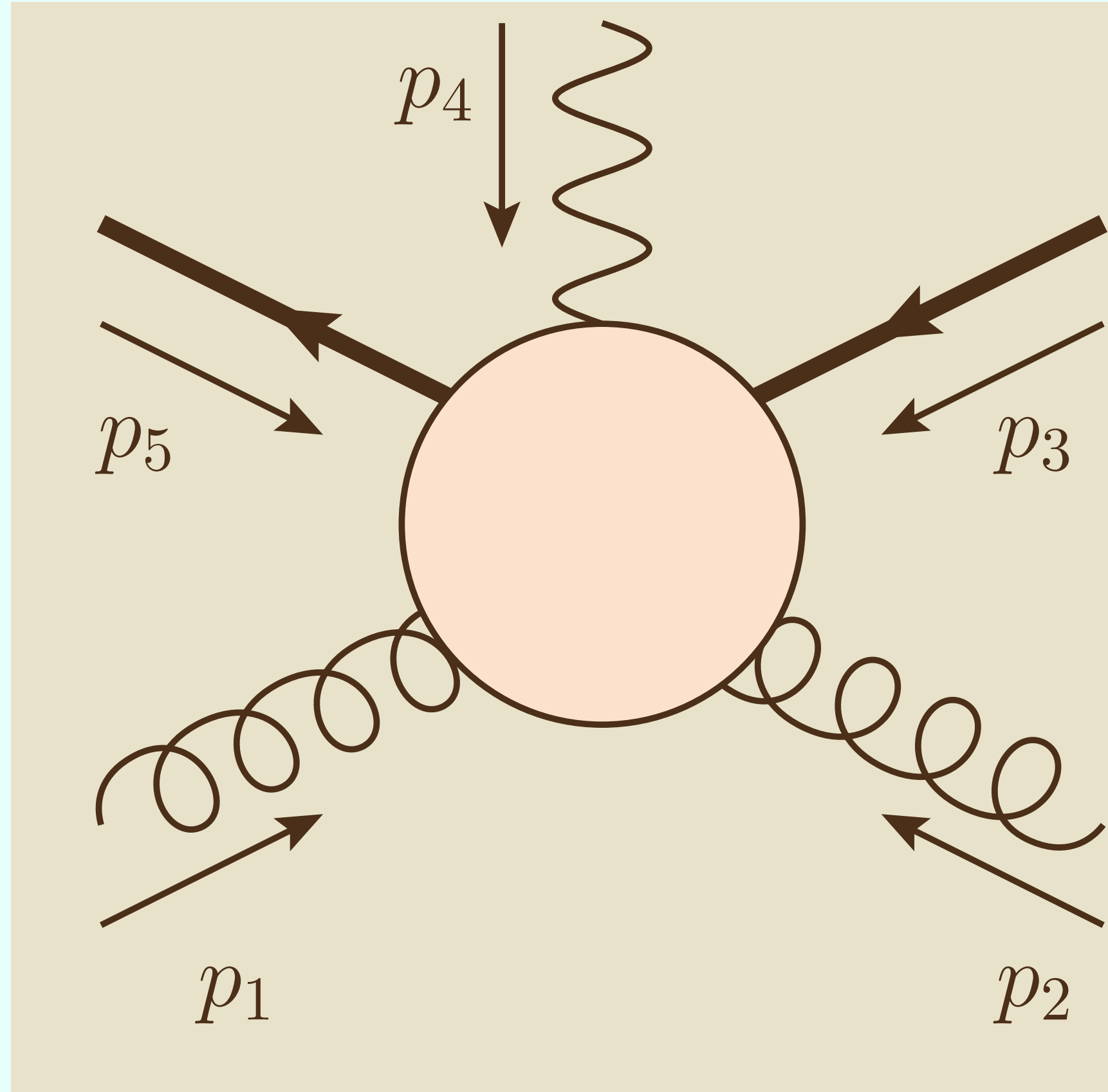
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# Kinematics



**Bjorken limit simplification:** *three* independent variables, basis of *two* light-like vectors and  $\mathbf{P}$  defines the collinear direction

Let us now impose that the incoming photon (outgoing HVM) is off-shell (on-shell). This means

$$(q^\mu + \Delta^\mu/2)^2 = -\tilde{Q}^2 \left(1 + \frac{\xi}{x_B}\right) = Q^2, \quad (5.10)$$

$$(q^\mu - \Delta^\mu/2)^2 = -\tilde{Q}^2 \left(1 - \frac{\xi}{x_B}\right) = M^2, \quad (5.11)$$

and therefore,

$$\tilde{Q}^2 = -\frac{Q^2 + M^2}{2} \quad \text{and} \quad \frac{\xi}{x_B} = \frac{Q^2 - M^2}{Q^2 + M^2}, \quad (5.12)$$

$$\begin{aligned} p_1^\mu &= (1 + r_1/r_3)p^\mu \\ p_2^\mu &= -(1 - r_1/r_3)p^\mu \\ p_3^\mu = p_5^\mu &= \frac{(r_2 - r_1)}{2r_3}p^\mu + \frac{r_3}{2}n^\mu \\ p_4^\mu &= -\frac{(r_1 + r_2)}{r_3}p^\mu - r_3n^\mu \end{aligned}$$

$$r_{1,2} = \frac{Q^2 \mp 4m^2}{4} \quad \text{and} \quad r_3 = \frac{Q^2 - 4m^2}{4\hat{\zeta}}$$

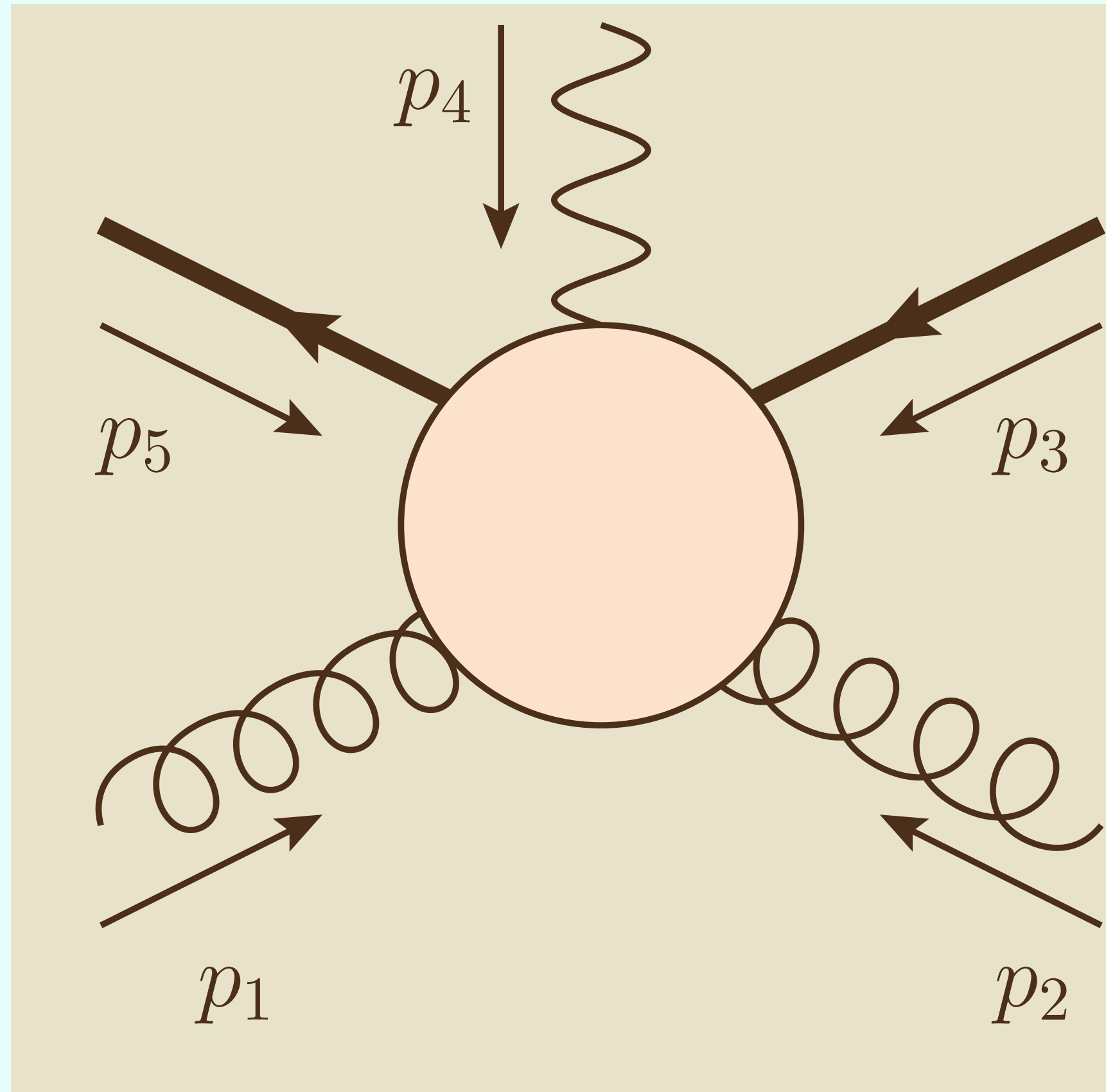
Compute to NLO in pQCD:

$$\gamma^*(p_4) + g(p_1) \rightarrow Q(-p_5) + \bar{Q}(-p_3) + g(-p_2)$$

Loop-induced Quark Process:

$$\gamma^*(p_4) + q(p_1) \rightarrow Q(-p_5) + \bar{Q}(-p_3) + q(-p_2)$$

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## Heavy Vector-Meson spin projection

- S-wave, spin-triplet LO NRQCD projection:

$i, j$ : colour indices,  
 $\alpha, \beta$ : spinor indices

$$v_\alpha^i(-p_3) \bar{u}_\beta^j(-p_5) \rightarrow \delta^{ij} \langle O_1 \rangle_V^{\frac{1}{2}} [e_S^* \dots]_{\alpha\beta}$$

$$p_3 = p_5$$

Colour correlated heavy quark-antiquark pair.

$\langle O_1 \rangle_V$  - NRQCD matrix element  $\propto \Gamma_{ee}$

# Generalised Parton Distribution (GPD) spin projections

Non-perturbative contributions of the quark and gluon amplitudes can be expressed in terms of GPDs contained in so-called Parton density matrices

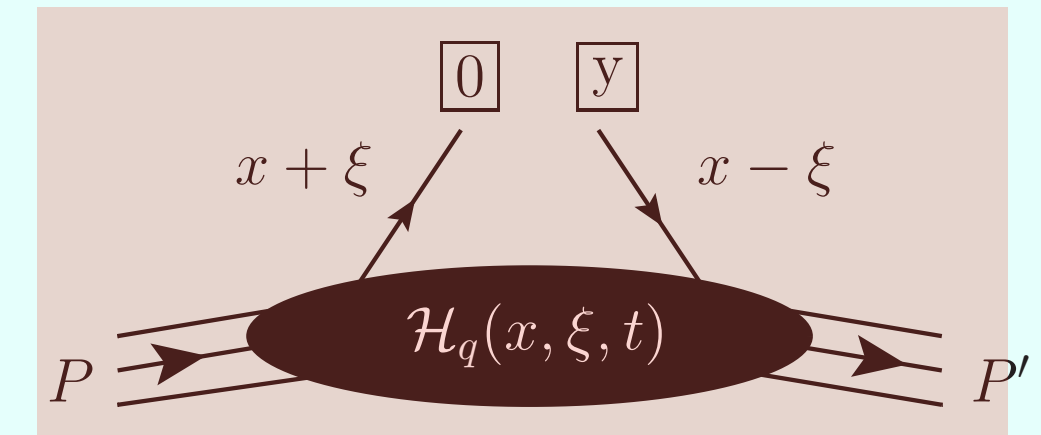
Ji [hep-ph/9801260](#)  
[hep-ph/9603249](#)

Radyushkin [hep-ph/9604317](#)  
[hep-ph/9605431](#)

**Quark:** Quark GPD contraction implemented as a spin projection of the *on-shell* quark scattering matrix

$$M_{\alpha\beta}^q(x, \xi) = 2 \int \frac{d\lambda}{2\pi} e^{-i\lambda x} \langle P + \frac{\Delta}{2} | \bar{\psi}_\beta^q \left( \frac{\lambda}{2} \right) W \left[ -\frac{\lambda}{2}, \frac{\lambda}{2} \right] \psi_\alpha^q \left( -\frac{\lambda}{2} \right) | P - \frac{\Delta}{2} \rangle$$

$$= \mathbf{F}^q(x, \xi) \not{x} + \tilde{F}^q(x, \xi) (\gamma_5 \not{x}) + \dots$$



$$\langle P' | \bar{\psi}_q(y) \mathcal{P} \{ \} \psi_q(0) | P \rangle$$



$$u_\alpha^i(p_1) \bar{u}_\beta^j(-p_2) \rightarrow \frac{\delta^{ij}}{N_c} F^q(x, \xi) \not{x}_{\alpha\beta}$$



No net colour flow  
Avg. over no. of colours  
Results in trace at **amplitude** level

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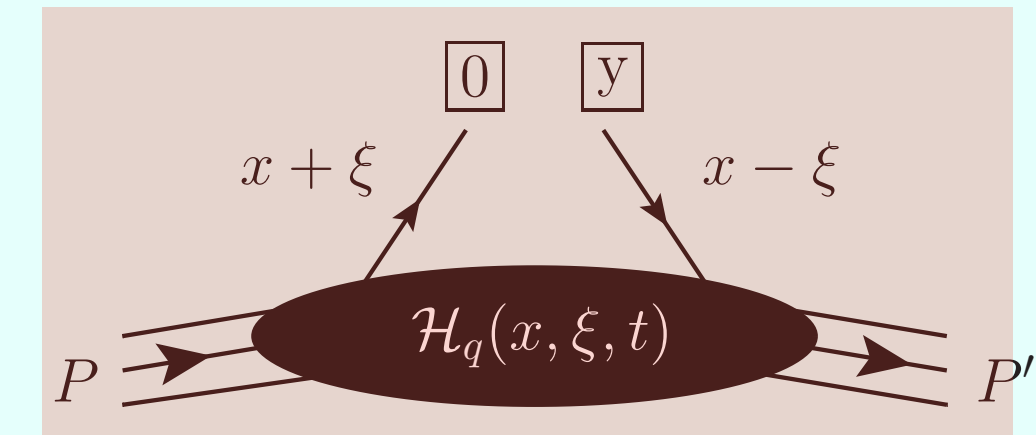
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$$\langle P' | \bar{\psi}_q(y) \mathcal{P} \{ \psi_q(0) | P \rangle$$

$$\longrightarrow u_\alpha^i(p_1) \bar{u}_\beta^j(-p_2) \longrightarrow \frac{\delta^{ij}}{N_c} F^q(x, \xi) \not{x} \longleftarrow$$

No net colour flow  
Avg. over no. of colours  
Results in trace at **amplitude** level

**Gluon:** Gluon GPD contraction implemented as a spin projection of the *on-shell* gluon scattering matrix

$$\epsilon_1^\mu \epsilon_2^{*\nu} \longrightarrow \frac{\delta_{ab}}{(N_c^2 - 1)} \frac{1}{d - 2} \frac{F^g(x, \xi)}{(x - \xi + i\varepsilon)(x + \xi - i\varepsilon)} g_\perp^{\mu\nu}$$

See [hep-ph/0204191](https://arxiv.org/abs/hep-ph/0204191)  
for more discussion  
about +ie  
prescription

# Tensor Decomposition

- Strip off polarisation vectors and work with amputated amplitude

$$T^{(\mu\nu)} = Ag^{\mu\nu} + Bp^\mu n^\nu + Cn^\mu p^\nu + Dp^\mu p^\nu + En^\mu n^\nu$$

- Impose local current conservation at the photon vertex (Ward identity)  $p_{4,\mu} T^{(\mu\nu)} = 0$  and  $K_\nu T^{(\mu\nu)} = 0$

$$T^{(\mu\nu)} = -g_\perp^{\mu\nu} T_\perp + \left( \frac{p_4 \cdot p}{p_4 \cdot n} n^\mu - p^\mu \right) \left( \frac{K \cdot p}{K \cdot n} n^\nu - p^\nu \right) \frac{\tilde{T}_L}{4} = -g_\perp^{\mu\nu} T_\perp + \ell^{\mu\nu} T_L$$

- Same non-axial terms one derives in **Generalised Deeply Virtual Compton Scattering** (GDVCS), e.g. see [1212.6674](#)

$$\varepsilon_{\pm,\mu}^\gamma \varepsilon_{\pm,\nu}^{V*} T^{(\mu\nu)} = -\varepsilon_{\pm,\mu}^\gamma \varepsilon_{\pm,\nu}^{V*} g_\perp^{\mu\nu} T_\perp = -\varepsilon_\pm^\gamma \cdot \varepsilon_\pm^{V*} T_\perp = T_\perp \quad \& \quad \varepsilon_{L,\mu}^\gamma \varepsilon_{L,\nu}^{V*} T^{(\mu\nu)} = \varepsilon_{L,\mu}^\gamma \varepsilon_{L,\nu}^{V*} \ell^{\mu\nu} T_L = T_L$$

- With this choice of normalisation of  $T^{\{\mu\nu\}}$  the process transverse and longitudinal form factors coincide with the **helicity amplitudes**

$$A^{\pm\pm} = \varepsilon_{\pm,\mu}^\gamma \varepsilon_{\pm,\nu}^{V*} T^{(\mu\nu)} = T_\perp$$

$$A^{00} = \varepsilon_{L,\mu}^\gamma \varepsilon_{L,\nu}^{V*} T^{(\mu\nu)} = T_L$$

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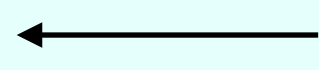
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- Same non-axial terms one derives in **Generalised Deeply Virtual Compton Scattering** (GDVCS), e.g. see [1212.6674](#)
- In collinear factorisation

$$T^{(\mu\nu)} = -g_\perp^{\mu\nu} \int_{-1}^1 \frac{dx}{x} \left[ \sum_q F^q(x, \tilde{\xi}) C_{\perp,q} \left( \frac{\tilde{\xi}}{x}, Q^2 \right) + C_{\perp,g} \left( \frac{\tilde{\xi}}{x}, Q^2 \right) \frac{F^g(x, \tilde{\xi})}{x} \right] \\ + \ell^{\mu\nu} \int_{-1}^1 \frac{dx}{x} \left[ \sum_q F^q(x, \tilde{\xi}) C_{L,q} \left( \frac{\tilde{\xi}}{x}, Q^2 \right) + C_{L,g} \left( \frac{\tilde{\xi}}{x}, Q^2 \right) \frac{F^g(x, \tilde{\xi})}{x} \right]$$

+ ...

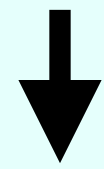


Beyond leading twist, outwith t=0 chiral-even theory, appearing in polarised scattering or incorporation of nucleon-mass effects...

# Tool-chain

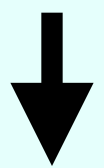
**QGRAF** [J. Comput. Phys. 105 \(1993\) 279–289](#)

Generate diagrams



**FORM 4.2** [1707.06453](#)

Main workflow,  $D$ -dim traces, colour algebra & assignment of diagrams to an ‘Auxiliary topology’



**FORM 4.2 / MATHEMATICA**



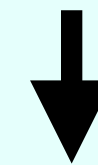
**REDUZE2** [1201.4330](#)

Integral family, propagators related to each other via shift of loop momentum

Passarino-Veltman (PV) & Linear reduction (LR)

$p_1 \propto p_2$  => linear dependence to cure  
 $p_3 = p_5$  => generalised partial-fractioning

Reduze database for each topology  
 Scalar integrals -> Master integrals



**FORM 4.2**

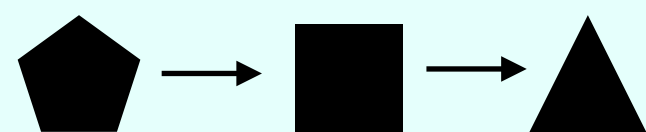
Insert Master integrals

$$I^{\mu\nu}(v_1, \dots, v_N) = \int [dl] \frac{l^\mu l^\nu}{D_1^{v_1} \dots D_N^{v_N}} \rightarrow I(\tilde{v}_1, \dots, \tilde{v}_N) = \int [dl] \frac{\mathcal{N}(l \cdot l, p_i \cdot l)}{D_1^{\tilde{v}_1} \dots D_N^{\tilde{v}_N}}$$

(PV)

$$I(\hat{v}_1, \dots, \hat{v}_N) = \int [dl] \frac{1}{D_1^{\hat{v}_1} \dots D_N^{\hat{v}_N}}$$

(LR)





**Here:** CAF et al. JHEP 08 (2021) 150

$$A_{\text{HE}}^{\pm\pm\pm} = -\frac{2m}{\sqrt{-Q^2}} A_{\text{HE}}^{00} \sim -\frac{4i\pi^2 g_e e_q 4m^2}{N_c \xi (4m^2 - Q^2)} \left( \frac{\langle O_1 \rangle_V}{m^3} \right)^{1/2} \\ \times \left[ \alpha_s F^g(\xi, \xi) + \frac{\alpha_s^2 N_c}{\pi} \ln \left( \frac{4m^2 - Q^2}{4\mu_F^2} \right) \int_{\xi}^1 \frac{dX}{X} F^g(X, \xi) \right. \\ \left. + \frac{\alpha_s^2 C_F}{\pi} \ln \left( \frac{4m^2 - Q^2}{4\mu_F^2} \right) \int_{\xi}^1 dX (F^q(X, \xi) - F^q(-X, \xi)) \right].$$

- High-energy limit: where the photon-parton system centre-of-mass energy is so much greater than the mass of heavy vector meson  $W^2 \gg M^2$
- Apt choice of factorisation scale allows for resummation of large double logarithms at small x:

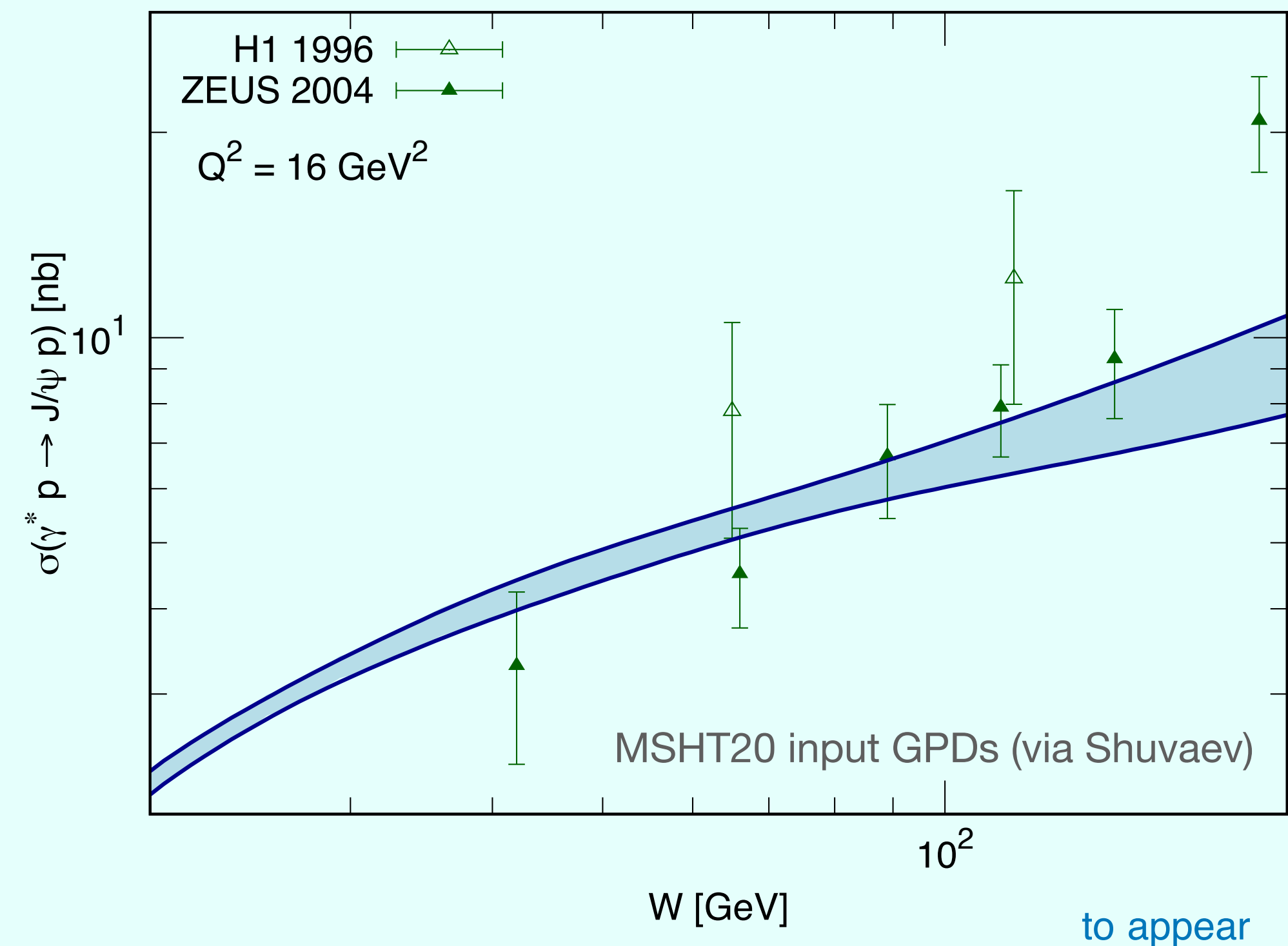
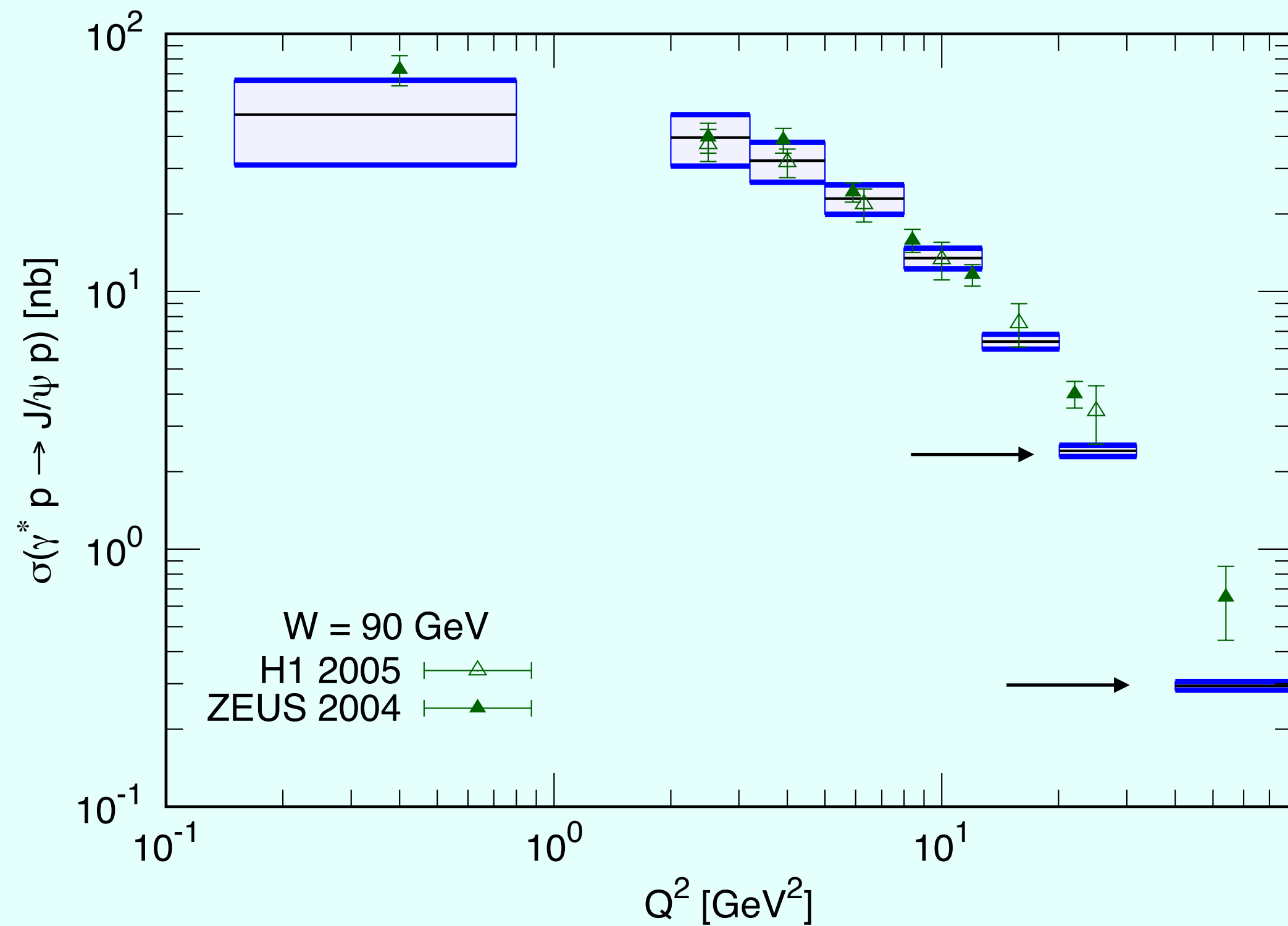
$$\sim \alpha_s^2 \ln(1/\xi) \ln(\bar{Q}^2 / \mu_F^2)$$

Ivanov et al (hep-ph/0401131):

$$A_{\text{HE}}^{\pm\pm\pm} \sim -\frac{4i\pi^2 g_e e_q}{N_c \xi} \left( \frac{\langle O_1 \rangle_V}{m^3} \right)^{1/2} \\ \times \left[ \alpha_s F^g(\xi, \xi) + \frac{\alpha_s^2 N_c}{\pi} \ln \left( \frac{m^2}{\mu_F^2} \right) \int_{\xi}^1 \frac{dX}{X} F^g(X, \xi) \right. \\ \left. + \frac{\alpha_s^2 C_F}{\pi} \ln \left( \frac{m^2}{\mu_F^2} \right) \int_{\xi}^1 dX (F^q(X, \xi) - F^q(-X, \xi)) \right].$$

- $Q^2 \rightarrow 0$  limit is smooth and maps onto Photoproduction calculation by Ivanov et al.
- $m^2 \rightarrow 0$  limit cannot be taken because would introduce a collinear divergence in the photon quark-antiquark vertex

# Phenomenology

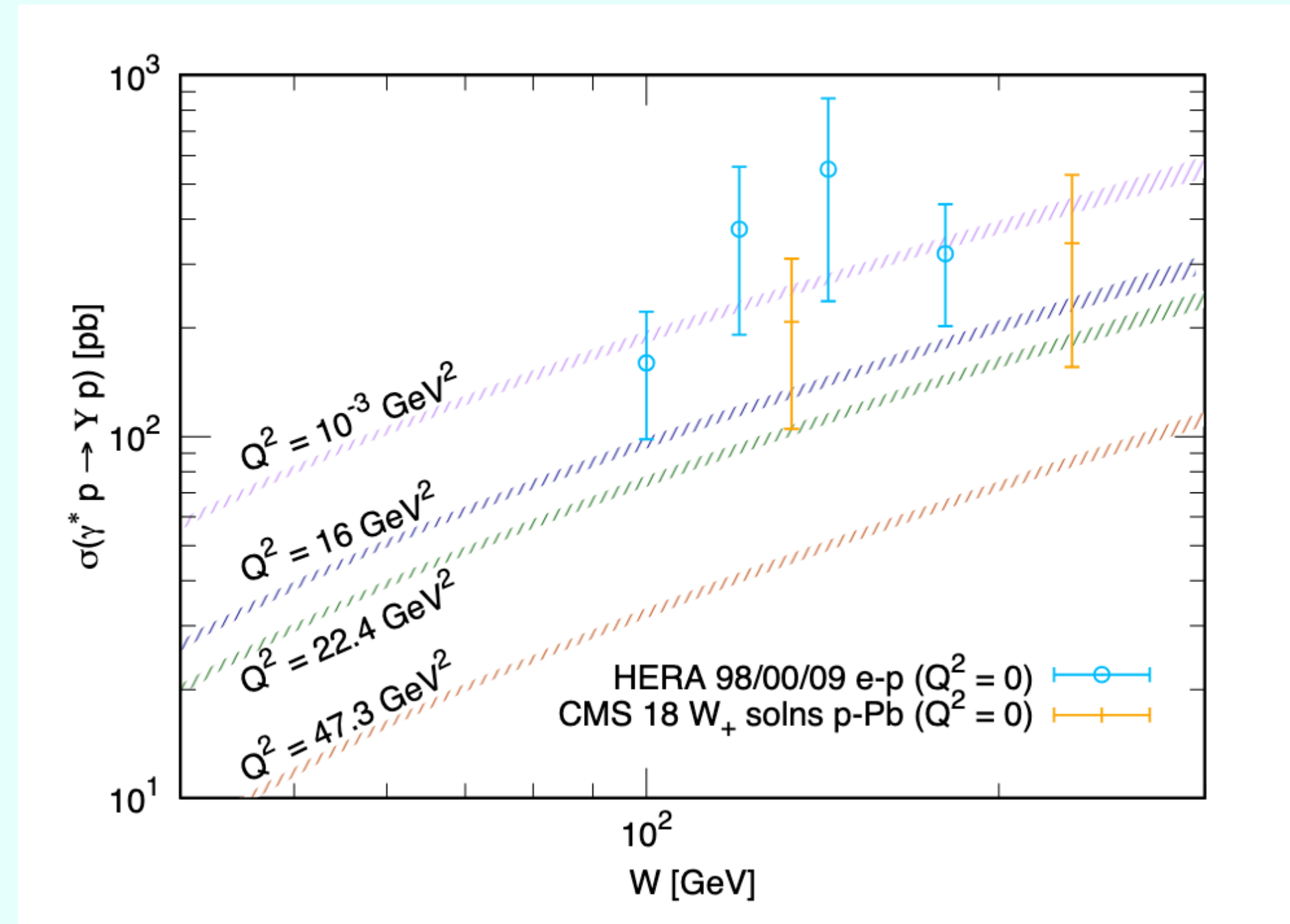
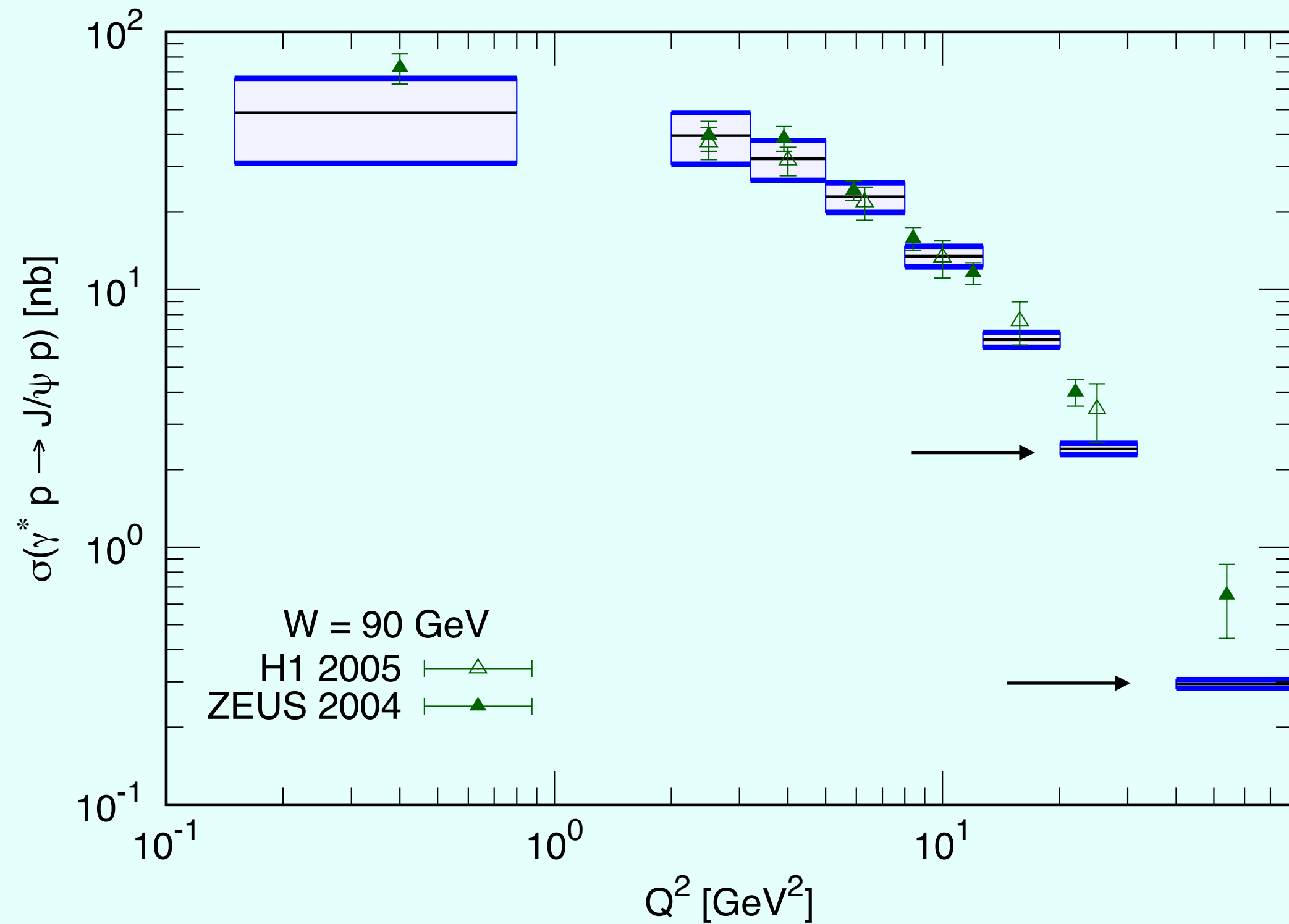


$$C_q^{(1)}|_{Q^2 \rightarrow \infty} \sim \ln(Q^2/m^2)$$

$$C_g^{(1)}|_{Q^2 \rightarrow \infty} \sim \ln(Q^2/m^2)^2$$

- Need for resummation evident in the data already?
- Errors shown are reflective of the PDF error only, factorisation scale dependency large at low  $Q^2$  in conventional approach, alleviated through  $Q_0$  subtraction or NLO CF + DLA HEF (see later, not shown here). At large scales, this dependency small.
- EIC will provide increased data coverage, complementing HERA and help resolve some discrepancies within current statistic-limited HERA data

# Phenomenology



to appear

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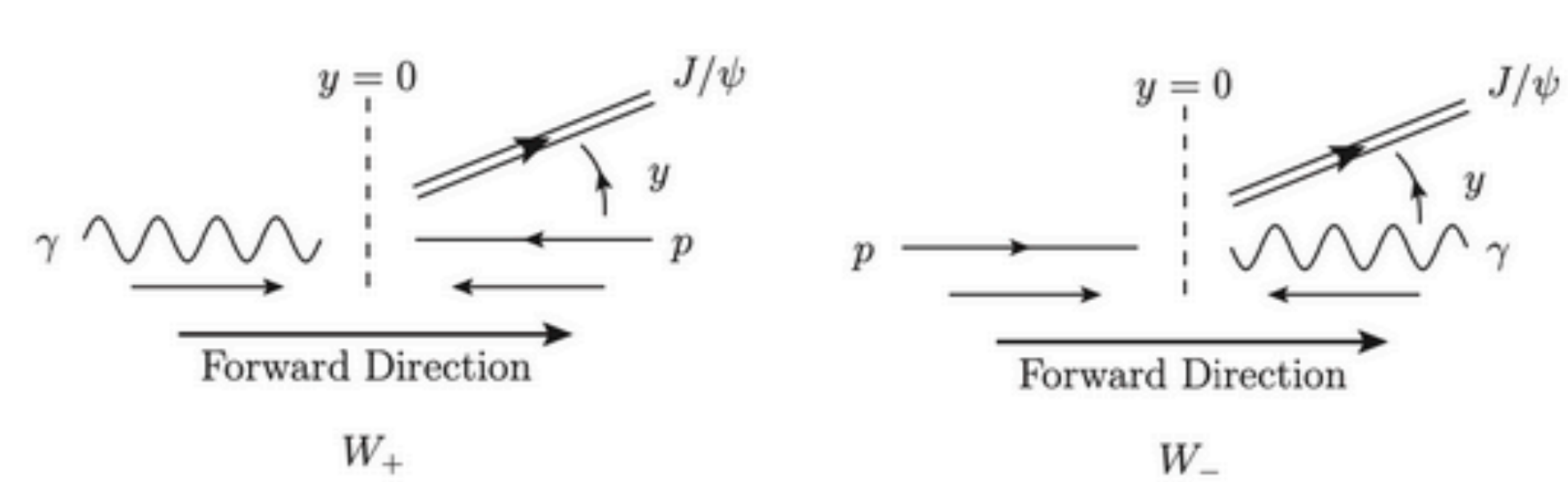
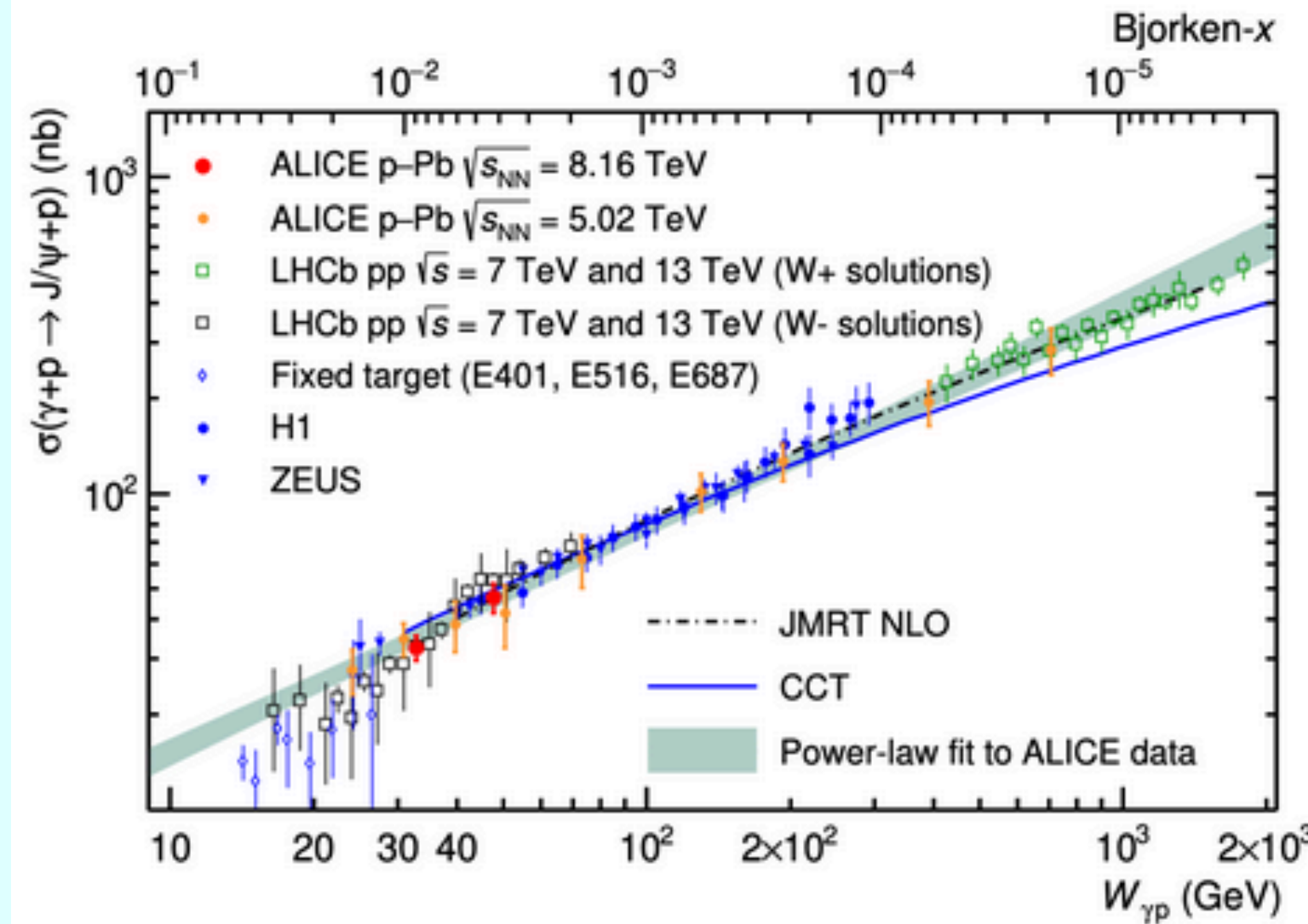
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## 2. Exclusive quarkonium photoproduction at the LHC

# eA vs. hadron-hadron

## Exclusive J/psi photoproduction to date (fixed target+ ep, pp, pPb)



### Unfolding at LHCb:

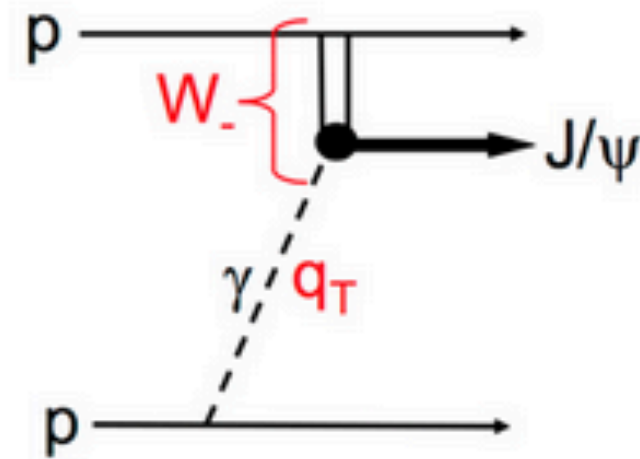
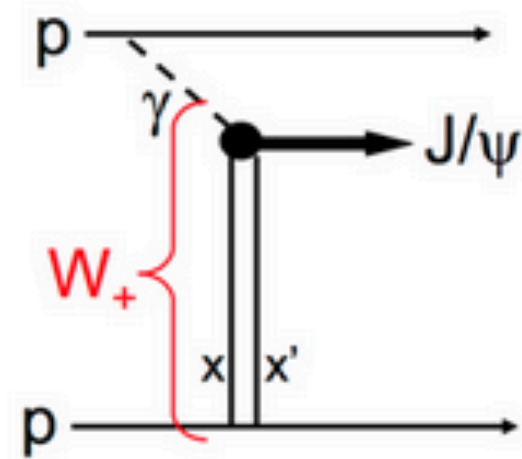
LHCb data

$$\frac{d\sigma(pp)}{dy} = S^2(W_+) \left( k_+ \frac{dn}{dk_+} \right) \sigma_+(\gamma p) + S^2(W_-) \left( k_- \frac{dn}{dk_-} \right) \sigma_-(\gamma p)$$

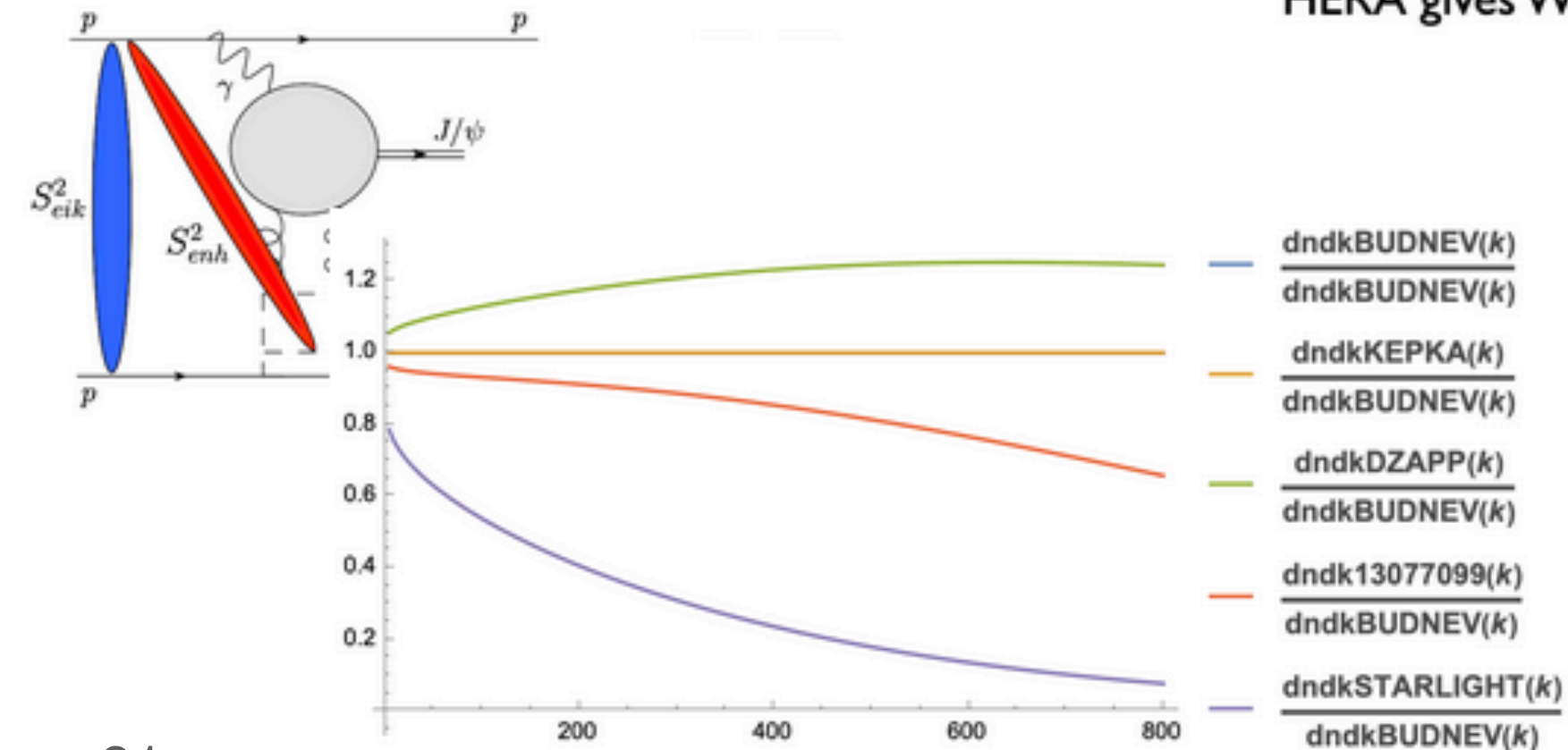
survival probability factors      LHCb 'data'      photon flux

HERA gives W-

pp@LHCb: [1401.3288](#), [1806.04079](#)



W+ /W- ambiguity:  $W_{\pm}^2 = M_{J/\psi} \sqrt{s} e^{\pm|y|}$



# Probe of nucleon gluon PDF

## Probe of nucleon gluon PDF

$$\left. \frac{d\sigma}{dt}(\gamma^* p \rightarrow J/\psi p) \right|_{t=0} = \frac{\Gamma_{ee}^{J/\psi} M_{J/\psi}^3 \pi^3}{48\alpha_{em}} \left[ \frac{\alpha_s(\bar{Q}^2)}{\bar{Q}^4} R_g x g(x, \bar{Q}^2) \right]^2 \left( 1 + \frac{Q^2}{M_{J/\psi}^2} \right)$$

Sensitive to GPD not PDF! but can relate PDF & GPD at low x reliably via the so-called Shuvaev transform [CAF, Jones, Martin, Ryskin, Teubner 1908.08398 & 2006.13857](#)

(1) UPC  $\rightarrow$  large W photoproduction (2)  $\rightarrow$  constraints on gluon PDF

In pPb, survival factor close to unity and there is less modelling dependence in (1) as compared to pp

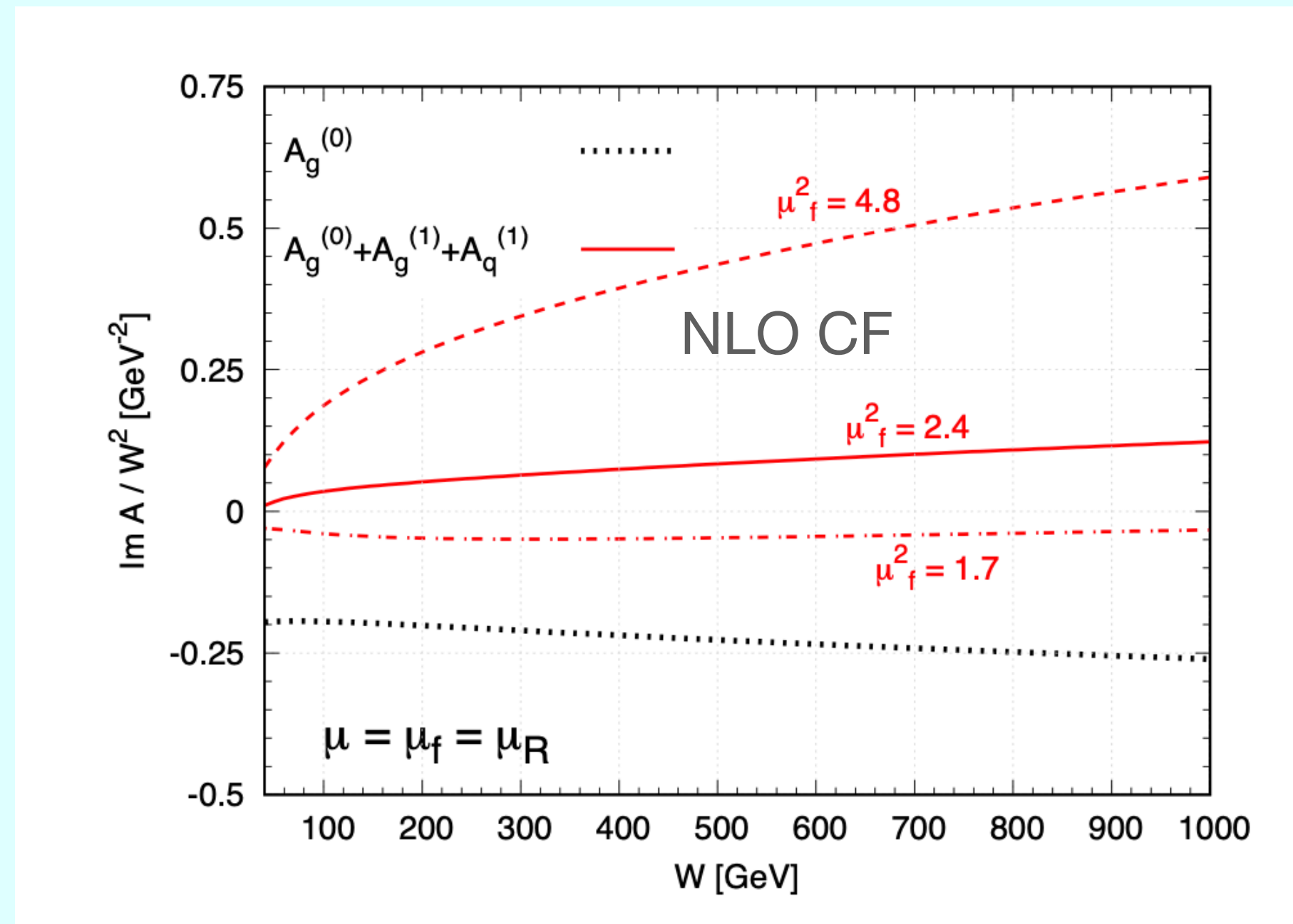
There will be data from EIC in eA where such modelling does not play a role but then the energy range is limited. In pp/pPb at the LHC we can access larger W, in pPb we can push the **precision** of (2), i.e. that of low x and low scale exclusive quarkonium data as constraints on the gluon PDF

# Probe of nucleon gluon PDF

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$$\left. \frac{d\sigma}{dt}(\gamma^* p \rightarrow J/\psi p) \right|_{t=0} = \frac{\Gamma_{ee}^{J/\psi} M_{J/\psi}^3 \pi^3}{48\alpha_{\text{em}}} \left[ \frac{\alpha_s(\bar{Q}^2)}{\bar{Q}^4} R_g x g(x, \bar{Q}^2) \right]^2 \left( 1 + \frac{Q^2}{M_{J/\psi}^2} \right)$$

Framework: NLO collinear factorisation (CF) with Shuvaev

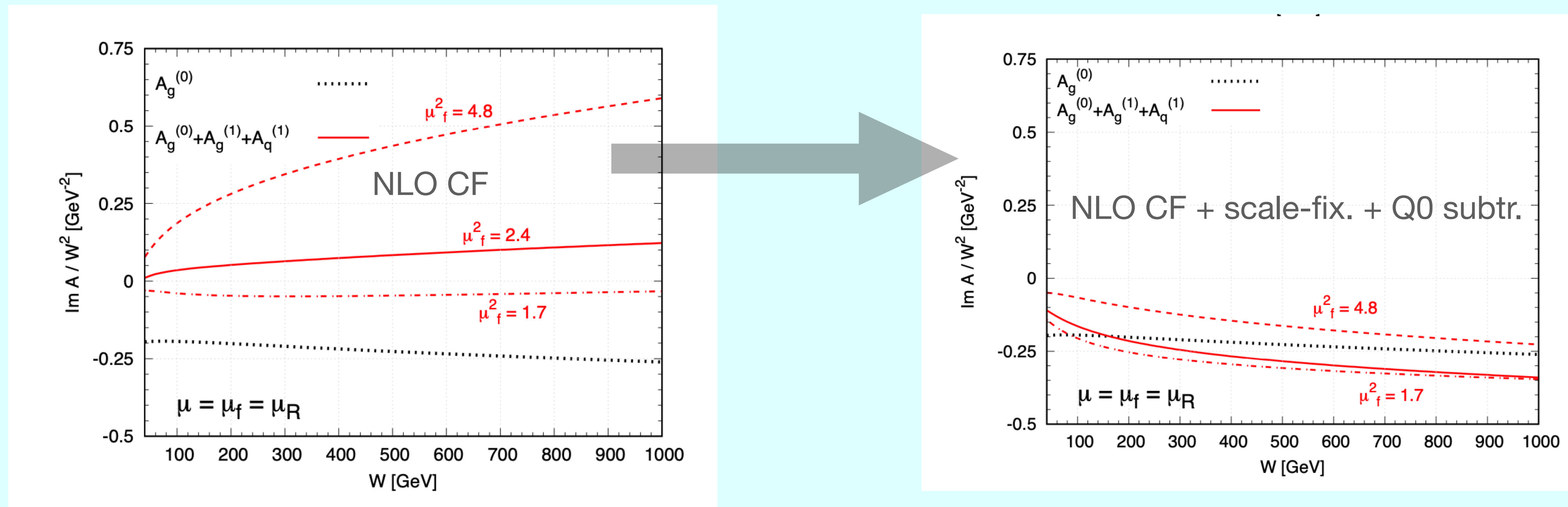


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Framework: NLO collinear factorisation (CF) with Shuvaev + scale-fixing + Q0 subtr. [1908.08398](#) & [2006.13857](#)



See also CAF, Lansberg, Nabeebaccus, Nefedov, Sznajder, Wagner for complementary approach (c.f. Saad talk), to appear

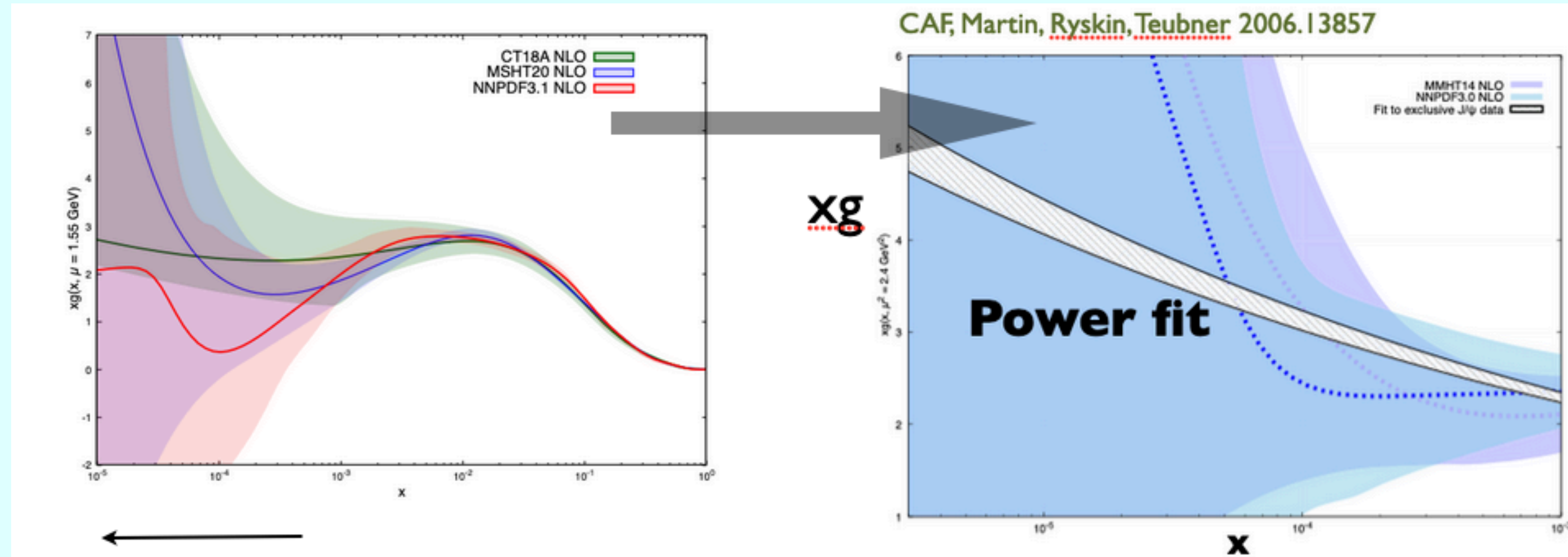


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Framework: NLO collinear factorisation (CF) with Shuvaev. + scale-fixing + Q0 subtr. [1908.08398](#) & [2006.13857](#)



pp@LHCb excl. J/psi UPCs currently  
 probes down to  $x \sim 3 \times 10^{-6}$   
 ...unconstrained domain in PDF fits!

Standalone fits + reweighting studies using pp  
 data, but not yet considered in a larger fitting  
 framework.....

# xFitter implementation



Public PDF fitting tool to perform a variety of tasks:  
 **$\alpha_s$  extraction, PDF reweightings, fits...**

- QCDNUM/APFEL for QCD evolution
  - MINUIT/CERES for minimisation in various mutually consistent approaches
  - interfaces to independent codes
  - various output formats (e.g. LHAPDF6,..)
  - Default config. based on set-up of HERAPDF2.0
- 
- Incorporate new '**JPSI**' reaction via xFitter's ReactionTheory class
    - i) PDF profiling -> exclusive J/psi datasets + theory
    - ii) PDF fitting -> gluon PDF pseudodata + HERA DIS RunI+II datasets
    - ii) PDF fitting -> exclusive J/psi+HERA DIS RunI+II datasets + theory

# (i) PDF profiling:

NNPDF30\_nlo\_as\_0118

$N_{\text{rep}} = 1000$

profiled with LHCb 13  
TeV excl. J/psi data

1806.04079

$$N_{\text{eff}} = \exp \left( \frac{1}{N_{\text{rep}}} \sum_{k=1}^{N_{\text{rep}}} w_k \ln(N_{\text{rep}}/w_k) \right)$$

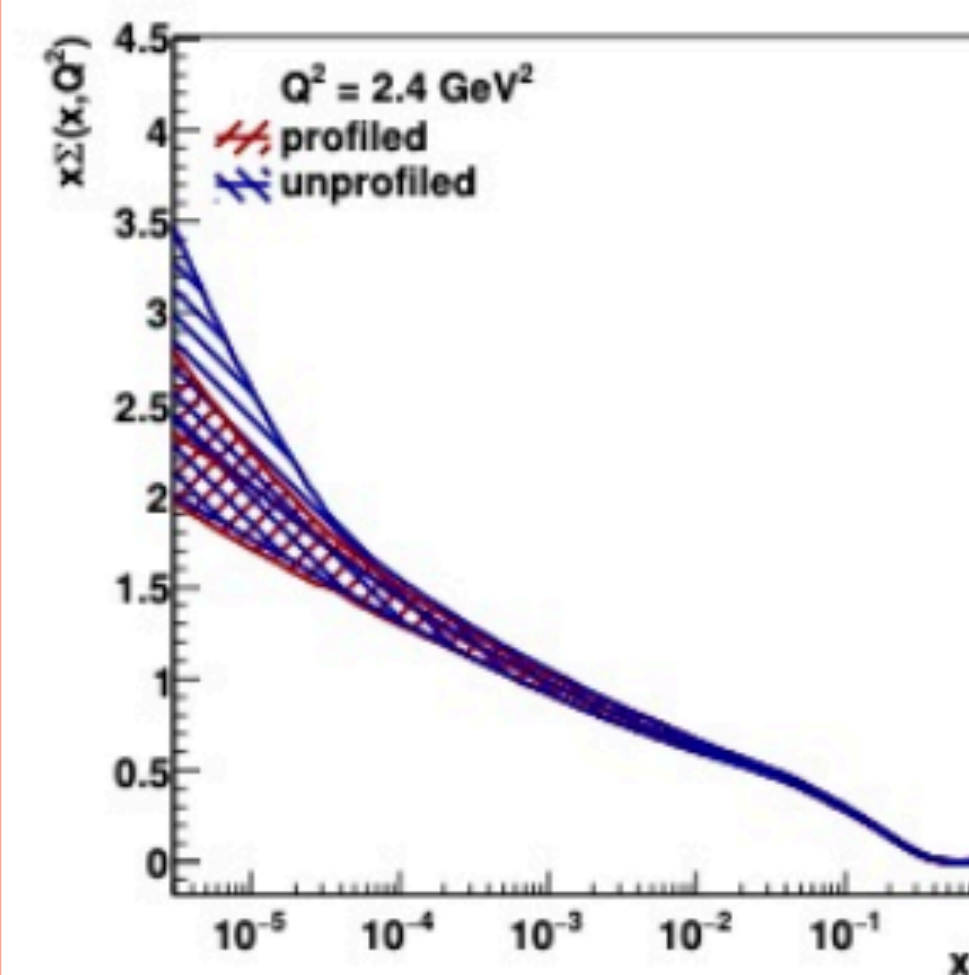
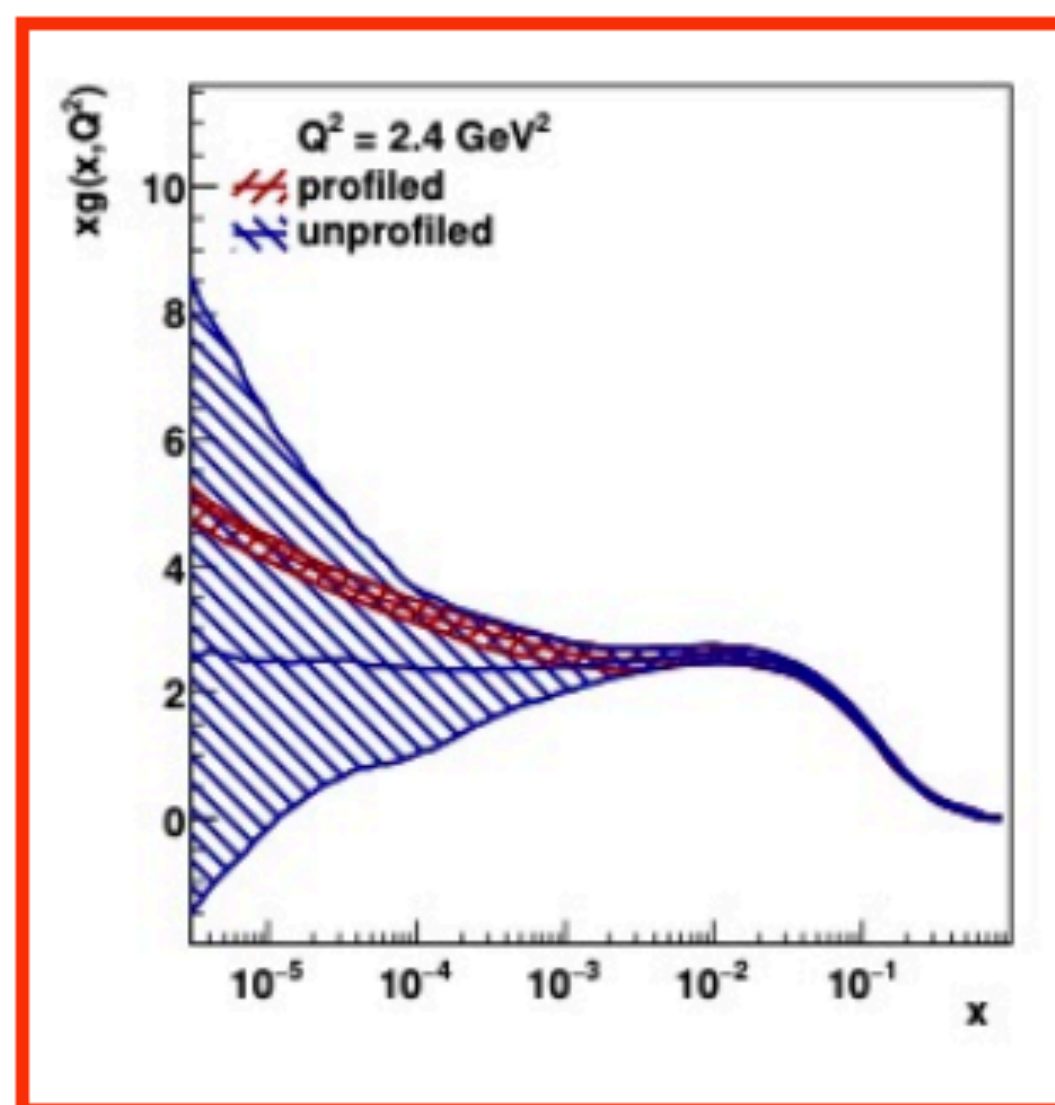
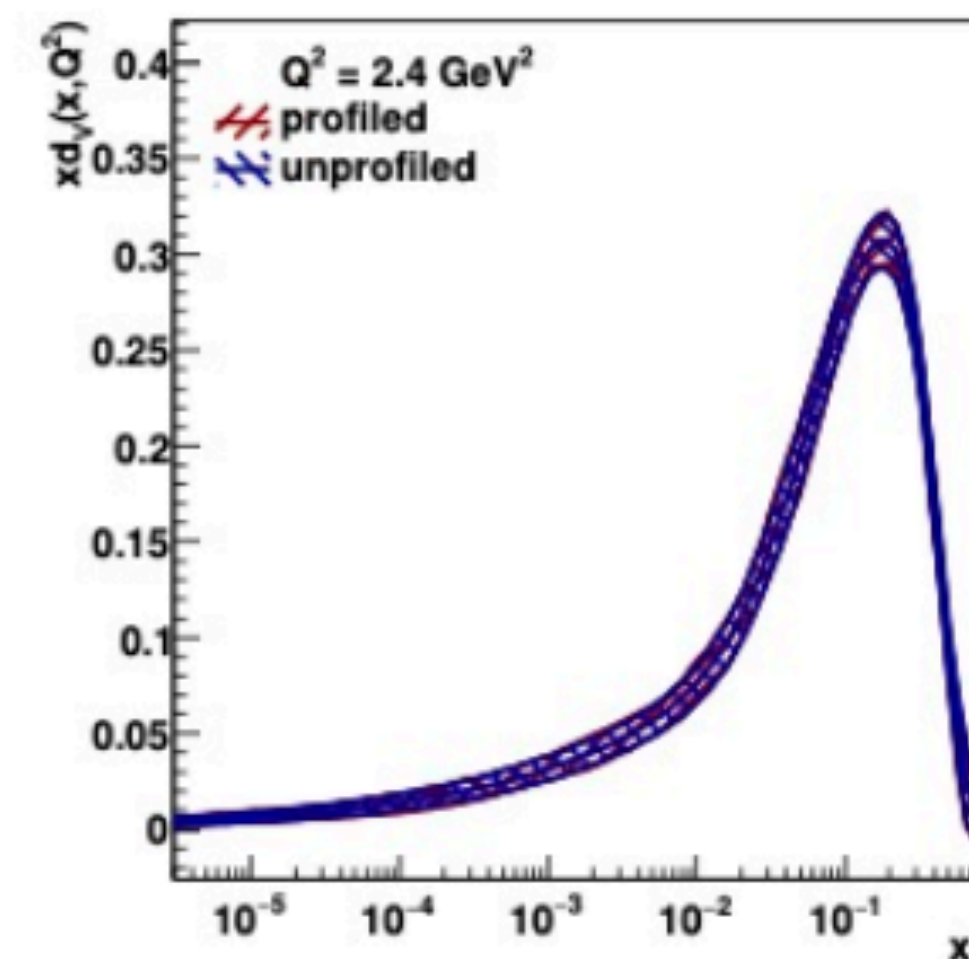
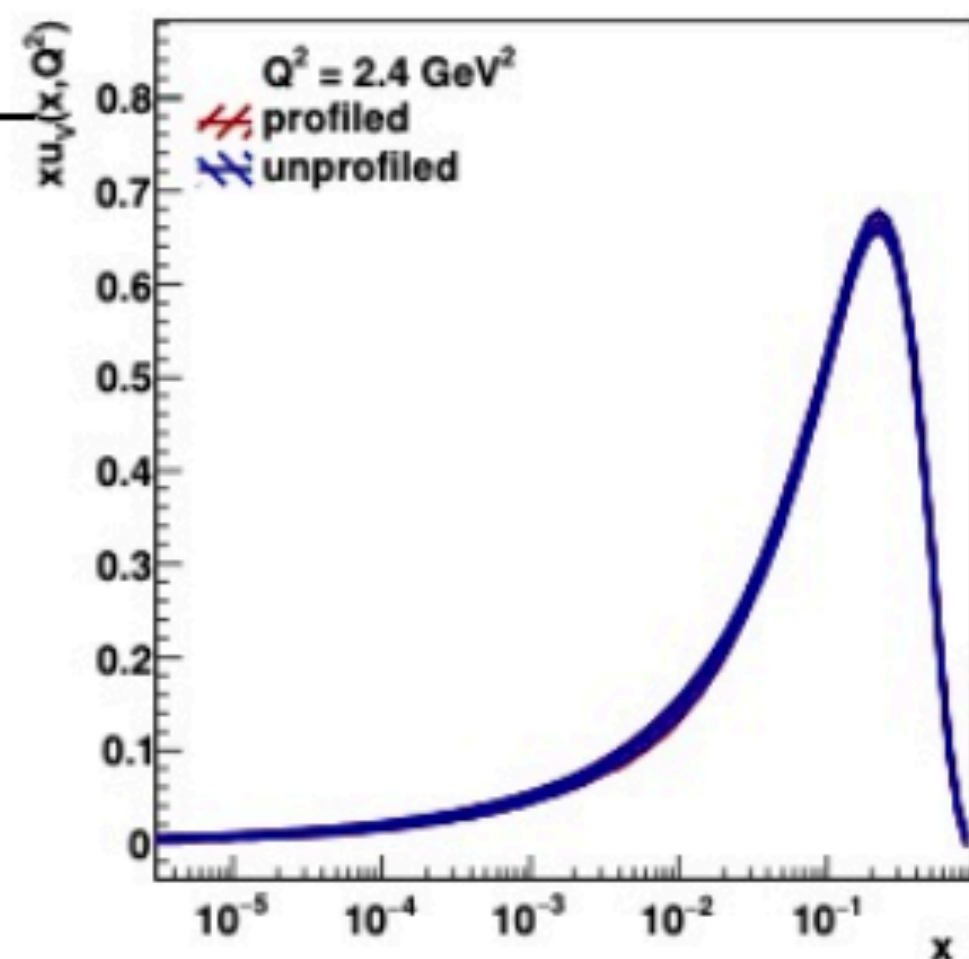
= 63  $\ll$   $N_{\text{rep}}$

Condition  $N_{\text{eff}} \ll N_{\text{rep}}$   
expected here

Precursor to full fit

$$\langle O \rangle = \frac{1}{N_{\text{rep}}} \sum_{k=1}^{N_{\text{rep}}} O(f_k),$$

$$\langle O^{\text{new}} \rangle = \frac{1}{N_{\text{rep}}} \sum_{k=1}^{N_{\text{rep}}} w_k O(f_k)$$



## (ii) Fitting: approach 1: gluon pseudodata

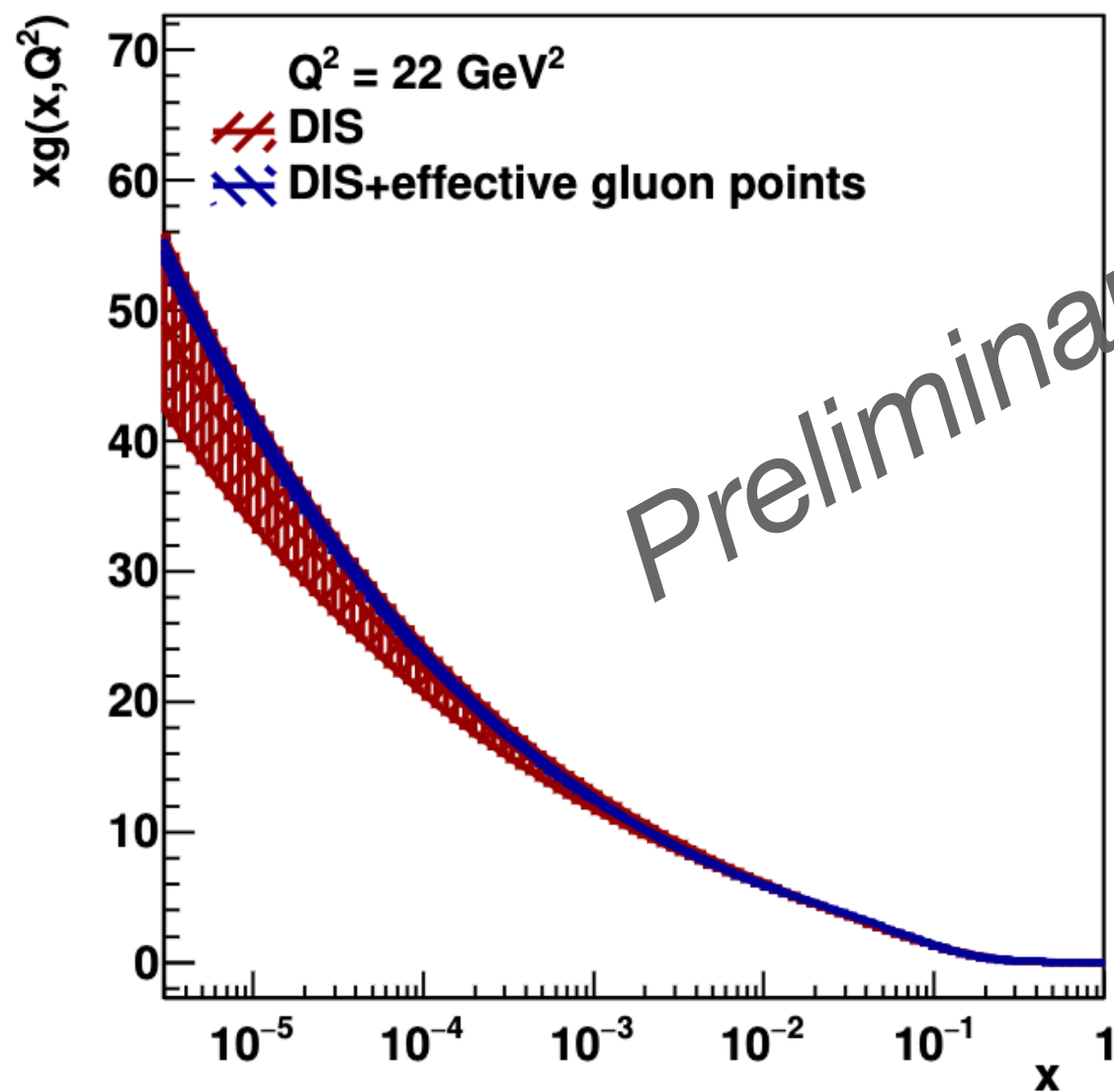
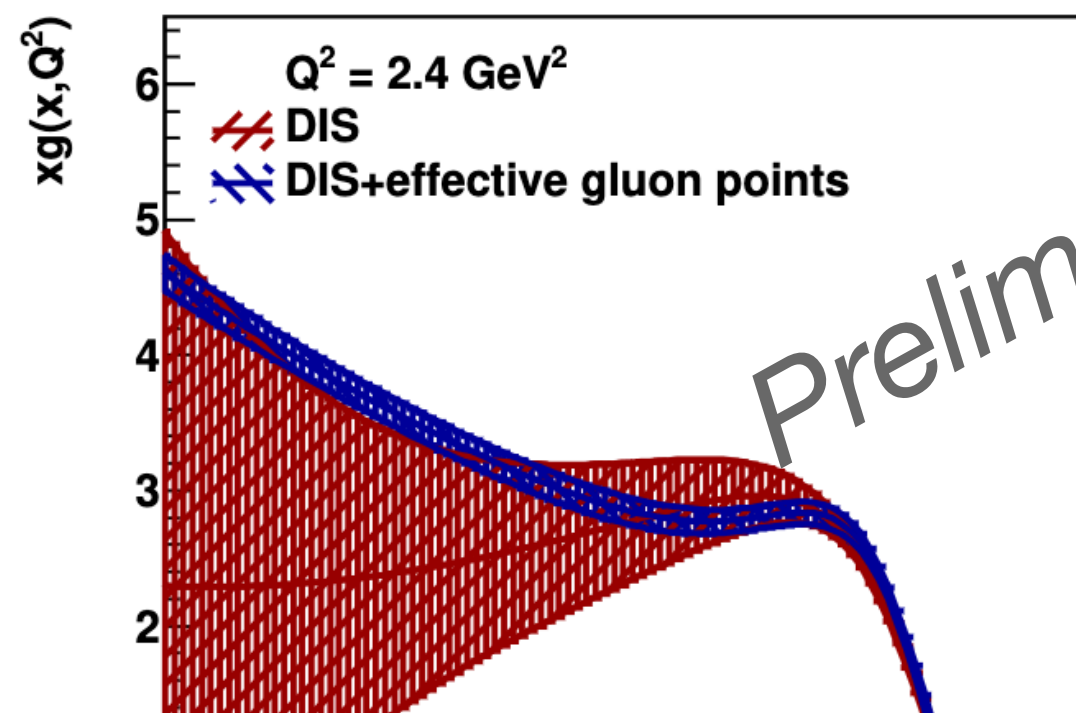
- Generate effective gluon PDF pseudodata from experimental data:

taken from 2006.13857

$$g_{\text{eff}}(x_i, \mu_{\text{opt}}) = g_{\text{fit}}(x_i, \mu_{\text{opt}}) \sqrt{\frac{\sigma_+(\text{data})_i}{\sigma_+(\text{fit})_i}}$$
$$\delta g_{\text{eff}}(x_i) = \frac{1}{2} g_{\text{eff}}(x_i) \frac{\delta \sigma_+(\text{data})_i}{\sigma_+(\text{data})_i}$$

## (ii) Fitting: approach 1: gluon pseudodata: Results

- Generate effective gluon PDF pseudodata from experimental data:



taken from 2006.13857

$$g_{\text{eff}}(x_i, \mu_{\text{opt}}) = g_{\text{fit}}(x_i, \mu_{\text{opt}}) \sqrt{\frac{\sigma_+(\text{data})_i}{\sigma_+(\text{fit})_i}}$$

$$\delta g_{\text{eff}}(x_i) = \frac{1}{2} g_{\text{eff}}(x_i) \frac{\delta \sigma_+(\text{data})_i}{\sigma_+(\text{data})_i}$$

Dataset	$\chi^2_{\text{min}}/\text{d.o.f}$ (DIS)	$\chi^2_{\text{min}}/\text{d.o.f}$ (DIS+eff. gluon pts.)
HERA1+2 NCep 820	80/73	79/73
HERA1+2 NCep 460	220/207	220/207
HERA1+2 CCep	43/39	44/39
HERA1+2 NCem	221/159	220/159
HERA1+2 CCem	54/42	56/42
HERA1+2 NCep 575	223/257	227/257
HERA1+2 NCep 920	465/391	470/391
LHC excl. $J/\psi$ $pp$ 7 TeV	N/A	8.95/10
LHC excl. $J/\psi$ $pp$ 13 TeV	N/A	3.51/10
LHC excl. $\Upsilon$ $pp$ 7,8 TeV	N/A	3.23/3
Total $\chi^2_{\text{min}}/\text{d.o.f}$	1412/1154 $\sim$ 1.22	1444/1177 $\sim$ 1.23

## (ii) Fitting: approach 2: cross section

Profiling: **precomputed** GPD grids for each LHAPDF member set

Fitting: Generate JPSI theory prediction using an input GPD grid  $\mathbf{G}^{(0)}$  constructed from a given LHAPDF member set  $\mathbf{S}^{(0)}$

After each fit iteration  $i$ ,

1. xFitter outputs an updated member set  $\mathbf{S}^{(i)}$  in LHAPDF format
2. which is interfaced to an independent GPD routine to produce a corresponding updated GPD grid  $\mathbf{G}^{(i)}$
3. which can be used for the JPSI theory prediction in iteration  $i+1$
4. perform iteration  $i+1$
5. repeat steps 1)–4) until convergence of fit






→ **feasible?**

## (ii) Fitting: approach 2: cross section

Profiling: **precomputed** GPD grids for each LHAPDF member set

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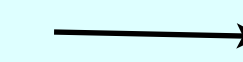
1. xFitter outputs an updated member set  $\mathbf{S}^{(i)}$  in LHAPDF format  
↓  adaptation of xFitter code needed...doable
  2. which is interfaced to an independent GPD routine to produce a corresponding updated GPD grid  $\mathbf{G}^{(i)}$   
↓  time costly, could optimise required GPD grid output, granularity of LHAPDF x-grid...
  3. which can be used for the JPSI theory prediction in iteration  $i+1$   
↓  adaptation of xFitter code needed...doable
  4. perform iteration  $i+1$   
↓ 
  5. repeat steps 1)–4) until convergence of fit
-  **Observation: amplitude highly peaked at  $x \sim x_i \Rightarrow 2D \rightarrow 1D$  numerical integration**

## (ii) Fitting: approach 2: cross section: 2D->1D simplification

Can reduce computationally intensive Shuvaev transform routine to **simpler** 1D numerical integration at the the point  $\mathbf{x}=\mathbf{x}_i$ :

$$H_g(x/2, x/2) = \frac{4x}{\pi} \int_{x/4}^1 dy y^{1/2} (1-y)^{1/2} g\left(\frac{x}{4y}\right)$$

For given input gluon distribution, this gives the result of the full Shuvaev transform at the point  $\mathbf{x}=\mathbf{x}_i$



For  $(\mathbf{x}-\mathbf{x}_i) > 0.1(\mathbf{x}+\mathbf{x}_i)$ , this is already a 10% deviation from the full result

Example: further assuming **pure-power behaviour** of gluon PDF gives famous **Rg** formula commonly used

Computing this on the fly a more tractable exercise than

In progress...

**Full Transform:**

$$\mathcal{H}_q(x, \xi) = \int_{-1}^1 dx' \left[ \frac{2}{\pi} \text{Im} \int_0^1 \frac{ds}{y(s)\sqrt{1-y(s)x'}} \right] \frac{d}{dx'} \left( \frac{q(x')}{|x'|} \right),$$

$$\mathcal{H}_g(x, \xi) = \int_{-1}^1 dx' \left[ \frac{2}{\pi} \text{Im} \int_0^1 \frac{ds(x + \xi(1-2s))}{y(s)\sqrt{1-y(s)x'}} \right] \frac{d}{dx'} \left( \frac{g(x')}{|x'|} \right),$$

$$y(s) = \frac{4s(1-s)}{x + \xi(1-2s)}.$$

[ Shuvaev et. al 1999 ]

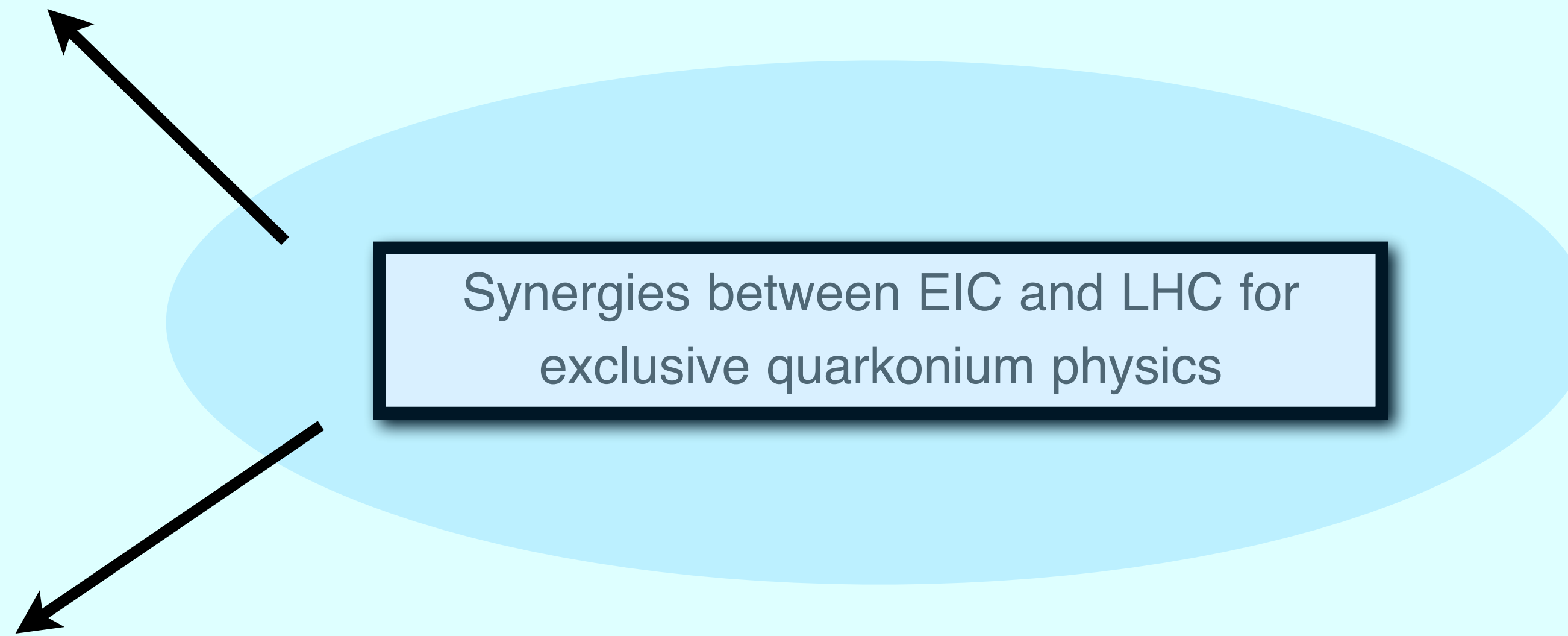


# Summary

**EIC:** Exclusive quarkonium **electro**production from HERA and EIC

Confront with theoretical predictions at NLO

Need for resummation at large photon virtualities or lack of data statistics -> more resolving power from EIC



**LHC:** Exclusive quarkonium **photo**production from LHC

Integration of NLO framework into xFitter to analyse exclusive data constraints within a larger fitting machinery.

Proof of concept with first numerical insights

# Heavy Vector-Meson spin projection

- Amplitude for production of two *on-shell heavy* quarks is computed and projected onto outgoing heavy quarkonium state
- To leading order in the NRQCD expansion, the momenta of the quark and antiquarks are equal
- The S-wave, spin-triplet projection can be written to leading order in the heavy quark-antiquark relative velocity as

**i,j:** colour indices,  
**alpha, beta:** spinor indices

$$v_{\alpha}^i(-p_3) \bar{u}_{\beta}^j(-p_5) \rightarrow \delta^{ij} \langle O_1 \rangle_V^{\frac{1}{2}} [\not{\epsilon}_S \dots]_{\alpha\beta}$$

Colour correlated heavy quark-antiquark pair.

$\langle O_1 \rangle_V$  – NRQCD matrix element  $\propto \Gamma_{ee}$

# Tensor Decomposition

- Strip off polarisation vectors and work with amputated amplitude

$$T^{(\mu\nu)} = Ag^{\mu\nu} + Bp^\mu n^\nu + Cn^\mu p^\nu + Dp^\mu p^\nu + En^\mu n^\nu$$

- Impose local current conservation at the photon vertex (Ward identity)  $p_{4,\mu} T^{(\mu\nu)} = 0$

- As well as the relation  $K_\nu T^{(\mu\nu)} = 0$

$$v_\alpha^i(-p_3) \bar{u}_\beta^j(-p_5) \rightarrow \delta^{ij} \langle O_1 \rangle_V^{\frac{1}{2}} [\cancel{e}_S^* \dots]_{\alpha\beta}$$

*Ivanov et al (hep-ph/0401131):*

Modification to the HVM projector allows for gauge dependent terms to cancel at the diagram level

$$T^{(\mu\nu)} = -g_\perp^{\mu\nu} T_\perp + \left( \frac{p_4 \cdot p}{p_4 \cdot n} n^\mu - p^\mu \right) \left( \frac{K \cdot p}{K \cdot n} n^\nu - p^\nu \right) \frac{\tilde{T}_L}{4} = -g_\perp^{\mu\nu} T_\perp + \ell^{\mu\nu} T_L$$

$$g_\perp^{\mu\nu} = g^{\mu\nu} - p^\mu n^\nu - p^\nu n^\mu \quad \ell^{\mu\nu} = \frac{N}{4} \left( \frac{p_4 \cdot p}{p_4 \cdot n} n^\mu - p^\mu \right) \left( \frac{K \cdot p}{K \cdot n} n^\nu - p^\nu \right) \quad \text{and} \quad T_L = \tilde{T}_L / N$$

- Same decomposition one derives in **Generalised Deeply Virtual Compton Scattering** (GDVCS), e.g. see 1212.6674

# Linear reduction

- Problem of linear dependencies in diagram propagators due to external constraints

$$p_1 \propto p_2$$

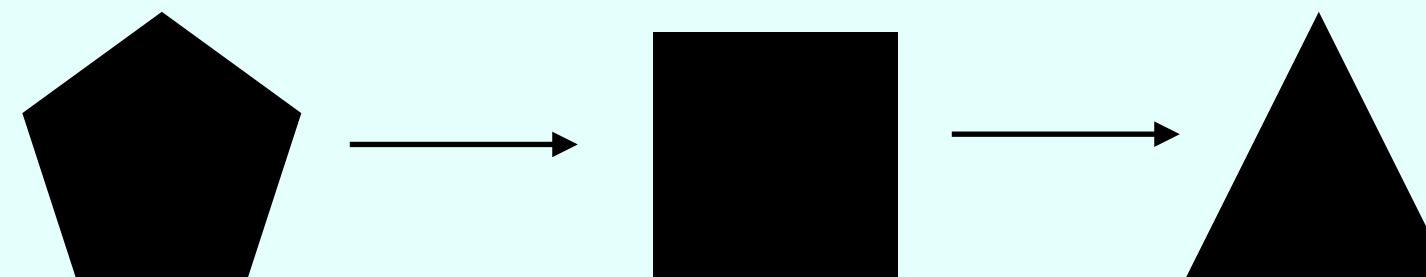
$$p_3 = p_5$$

- Need to combat this linear dependence before using REDUZE2 otherwise leads to incomplete reduction

- Implement generalised partial-fractioning routine in line with the **Leonartas Algorithm** (dates back to the 70s)

Soviet Math. (Iz. VUZ) 22 (1978) 35–38

- Linear relations amongst propagators used to iteratively remove this linear dependence



$$\# \text{ props of triangle} = L(L + 1)/2 + LE$$

L: # of loop momenta

E: # of linearly independent external vectors

$$I^{\mu\nu}(v_1, \dots, v_N) = \int [dl] \frac{l^\mu l^\nu}{D_1^{v_1} \dots D_N^{v_N}}$$

Starting tensor integrals, need at most a basis decomposition for rank-two tensor

Rank 1:  $l^\mu = (n \cdot l)p^\mu + (p \cdot l)n^\mu$

Rank 2:  $l^\mu l^\nu = g^{\mu\nu}T_{00} + p^\mu p^\nu T_{11} + p^\mu n^\nu T_{12} + n^\mu p^\nu T_{21} + n^\mu n^\nu T_{22}$

$$T_{ij} \equiv T_{ij}(l \cdot l, p \cdot l, n \cdot l)$$

# Linear reduction cont.

$$I^{\mu\nu}(v_1, \dots, v_N) = \int [dl] \frac{l^\mu l^\nu}{D_1^{v_1} \dots D_N^{v_N}}$$

↓ (a)

$$I(\tilde{v}_1, \dots, \tilde{v}_N) = \int [dl] \frac{\mathcal{N}(l \cdot l, p_i \cdot l)}{D_1^{v_1} \dots D_N^{v_N}}$$

↓ (b) re-write scalar products

$$I(\hat{v}_1, \dots, \hat{v}_N) = \int [dl] \frac{1}{D_1^{\hat{v}_1} \dots D_N^{\hat{v}_N}}$$

↓

Pass to linear reduction stage

## Linear reduction algorithm:

For each integral structure of the form  $C \cdot D_1^{-\hat{v}_1} \dots D_N^{-\hat{v}_N}$  check for a linear dependency amongst the propagators

IF not, pass structure to REDUZE2, ready for integral reduction

IF so, need to linearly decompose

Follow:

Linear dependence => there exists a relation of the form

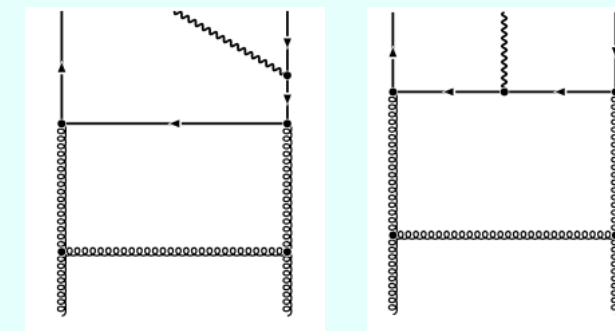
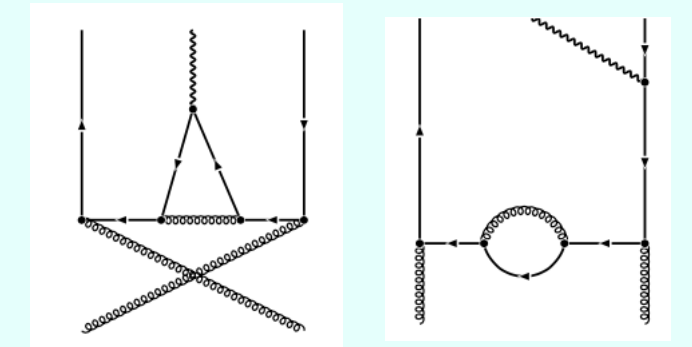
$$\sum_i A_i D_i + B = 0$$

If B is not zero, then find resolution of unity as

$$1 = -\frac{1}{B} \sum_i A_i D_i$$

Otherwise, if B = 0, find

$$1 = -\frac{1}{A_k D_k} \sum_{i \neq k} A_i D_i$$



**Upshot:** structure with  $N$  propagators now expressed as a linear combination of structures containing  $N-1$  propagators. Repeat until linear dependence removed.

**Full Transform:**

$$\mathcal{H}_q(x, \xi) = \int_{-1}^1 dx' \left[ \frac{2}{\pi} \operatorname{Im} \int_0^1 \frac{ds}{y(s) \sqrt{1 - y(s)x'}} \right] \frac{d}{dx'} \left( \frac{q(x')}{|x'|} \right),$$

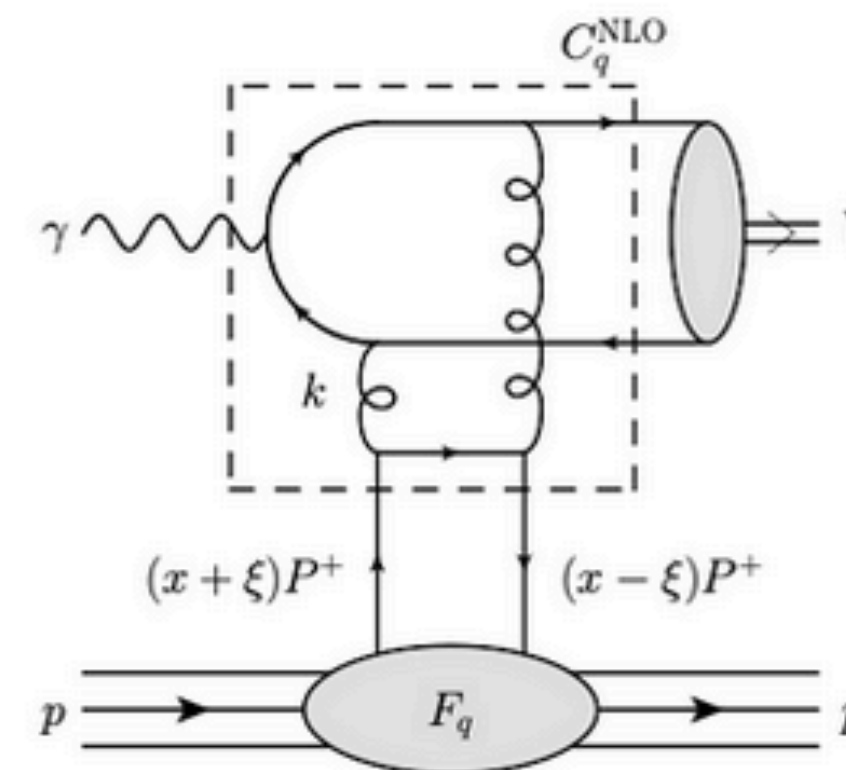
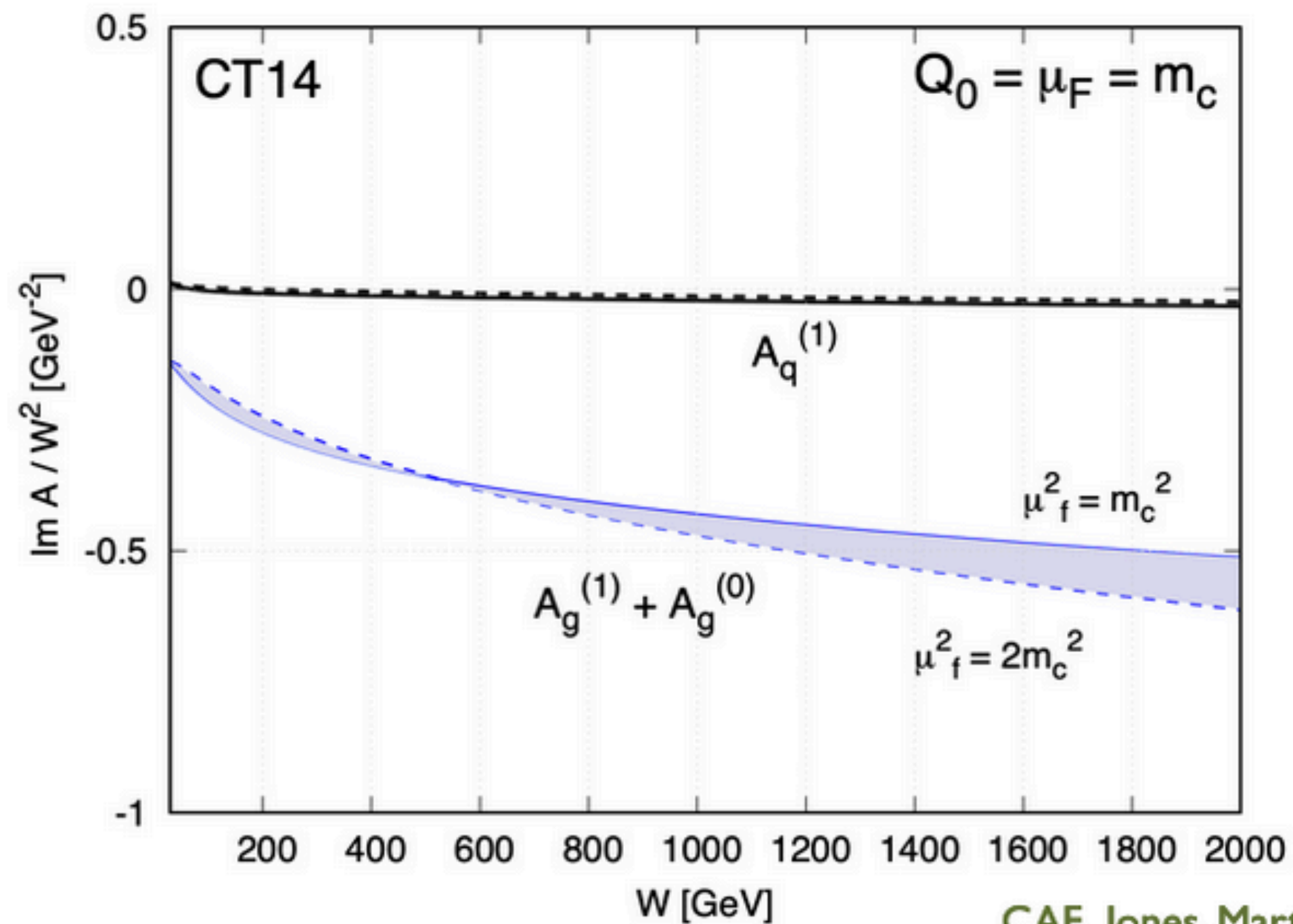
$$\mathcal{H}_g(x, \xi) = \int_{-1}^1 dx' \left[ \frac{2}{\pi} \operatorname{Im} \int_0^1 \frac{ds(x + \xi(1 - 2s))}{y(s) \sqrt{1 - y(s)x'}} \right] \frac{d}{dx'} \left( \frac{g(x')}{|x'|} \right),$$

$$y(s) = \frac{4s(1 - s)}{x + \xi(1 - 2s)}.$$

[ Shuvaev et. al 1999 ]

# Interplay of quark and gluons at NLO

After  $Q_0$  subtraction:



CAF, Jones, Martin, Ryskin, Teubner, 1908.08398

Quark contribution separated from hard scattering by at least *one* step of DGLAP evolution and is therefore removed after imposition of  $Q_0$  subtraction (as reflected in the numerics)

—————> **Gluon driven like at LO**

# Constraints from inclusive D meson production data

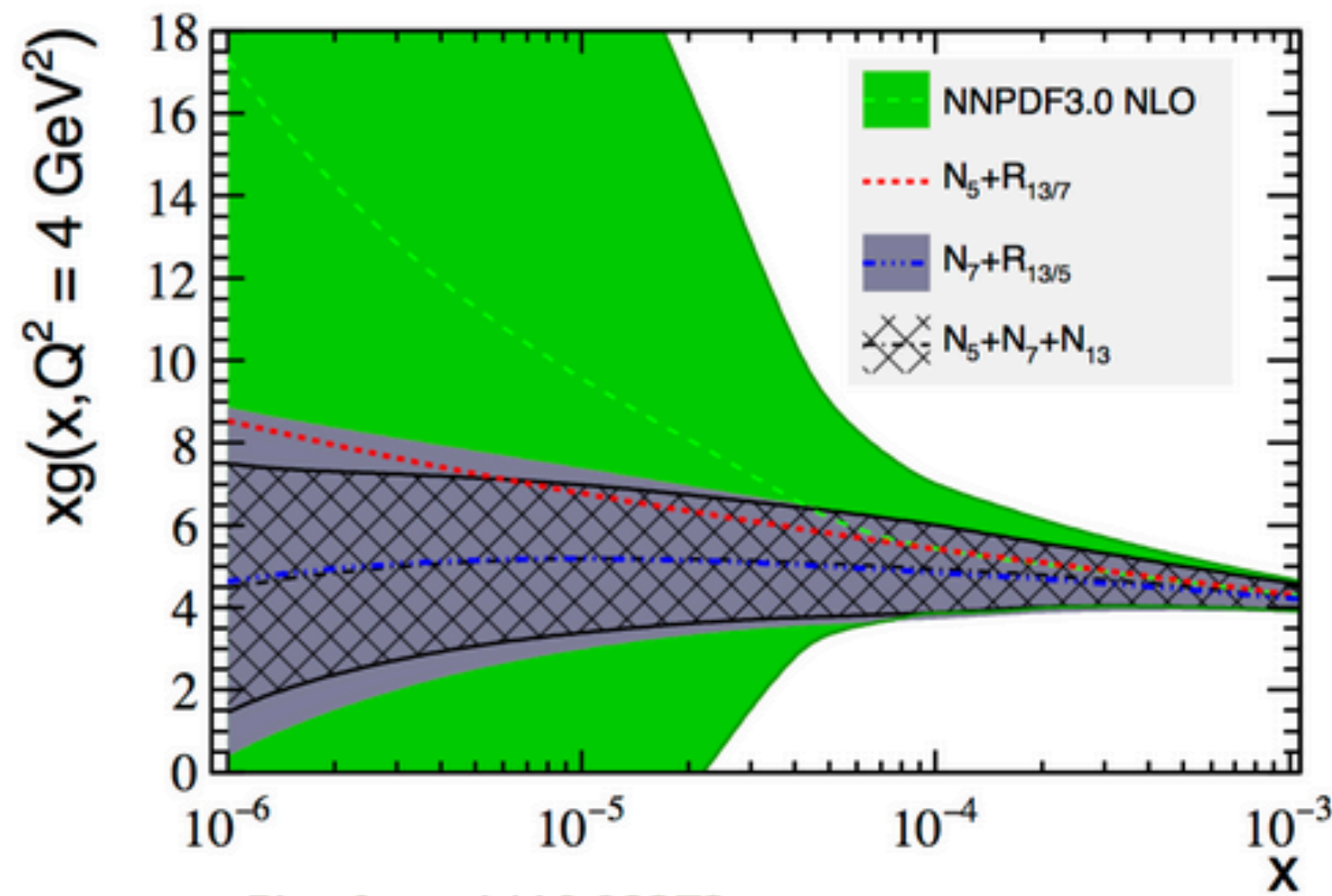
**Idea:** Construct ratios of observables in  $y$  and  $p_t$  bins to combat various uncertainties

$$N_X^{ij} = \frac{d^2\sigma(X \text{ TeV})}{dy_i^D d(p_T^D)_j} \bigg/ \frac{d^2\sigma(X \text{ TeV})}{dy_{\text{ref}}^D d(p_T^D)_j}$$

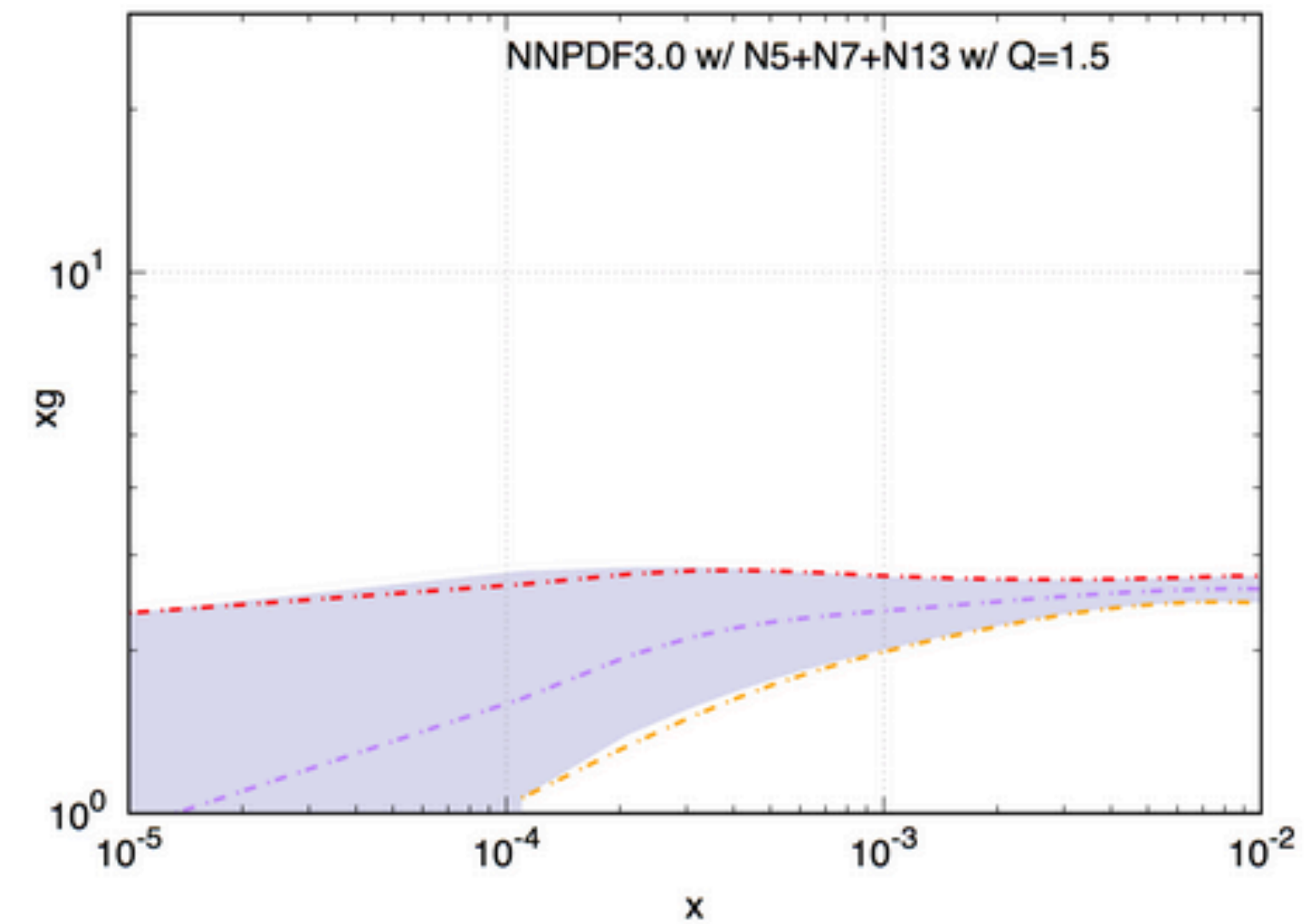
$$R_{13/X}^{ij} = \frac{d^2\sigma(13 \text{ TeV})}{dy_i^D d(p_T^D)_j} \bigg/ \frac{d^2\sigma(X \text{ TeV})}{dy_i^D d(p_T^D)_j}$$



find decreasing gluon at the lowest  $x$  they may probe



Plot from 1610.09373





## Sensitivity to the $\overline{\text{MS}}$ gluon PDF

- Remain in  $\overline{\text{MS}}$  scheme with  $Q_0$  subtracted coefficient functions to NLO accuracy
- Subtraction does not affect IR or UV divergence renormalisation procedures
- Soft singularity at  $l=0$  is removed after subtracting off the LO part of the NLO coefficient function before integral over loop momentum from 0 to  $Q_0$  is performed

$$\Delta \text{Im} \mathcal{M}^q = \frac{\alpha_s^2}{2\pi} \int_{\xi}^1 dx (F_q(x, \xi, m_c) - F_q(-x, \xi, m_c)) \left( \int_0^{Q_0^2} (M_a^q + M_b^q) \frac{2\pi m_c^4}{\hat{s}^2} dl^2 \right)$$

- Precisely this **FINITE** contribution that is subtracted from full  $\overline{\text{MS}}$  coefficient functions to avoid double counting inherent within  $\overline{\text{MS}}$  scheme (subtraction fundamentally ubiquitous but numerically relevant for low scale processes only\*)

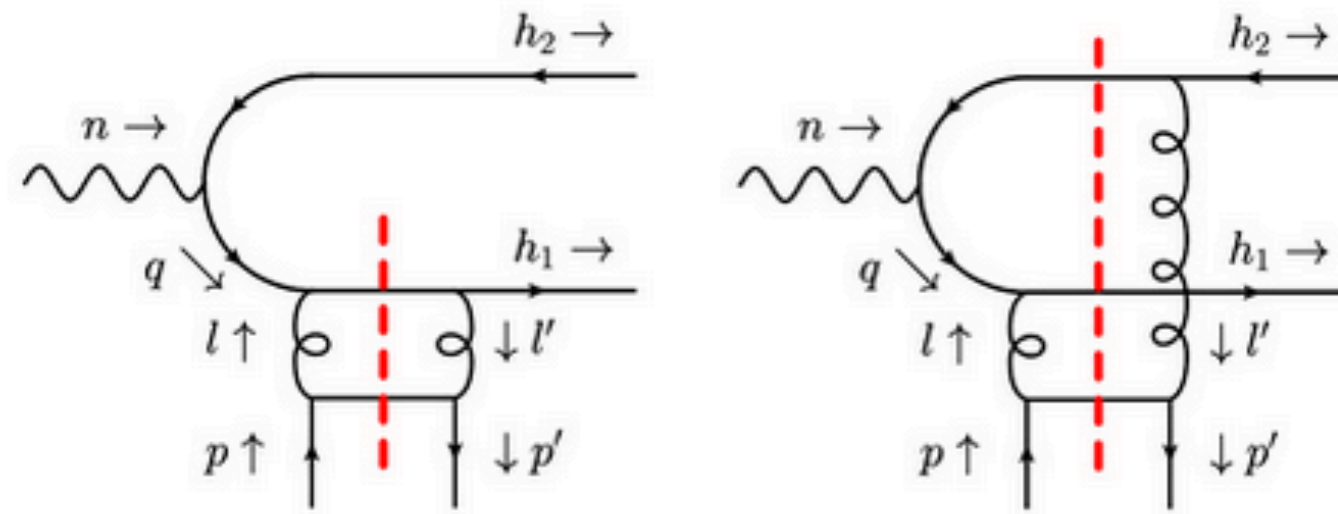
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\*see 1912.09304 for procedure applied to inclusive DIS and Drell-Yan production

## Sensitivity to the $\overline{\text{MS}}$ gluon PDF

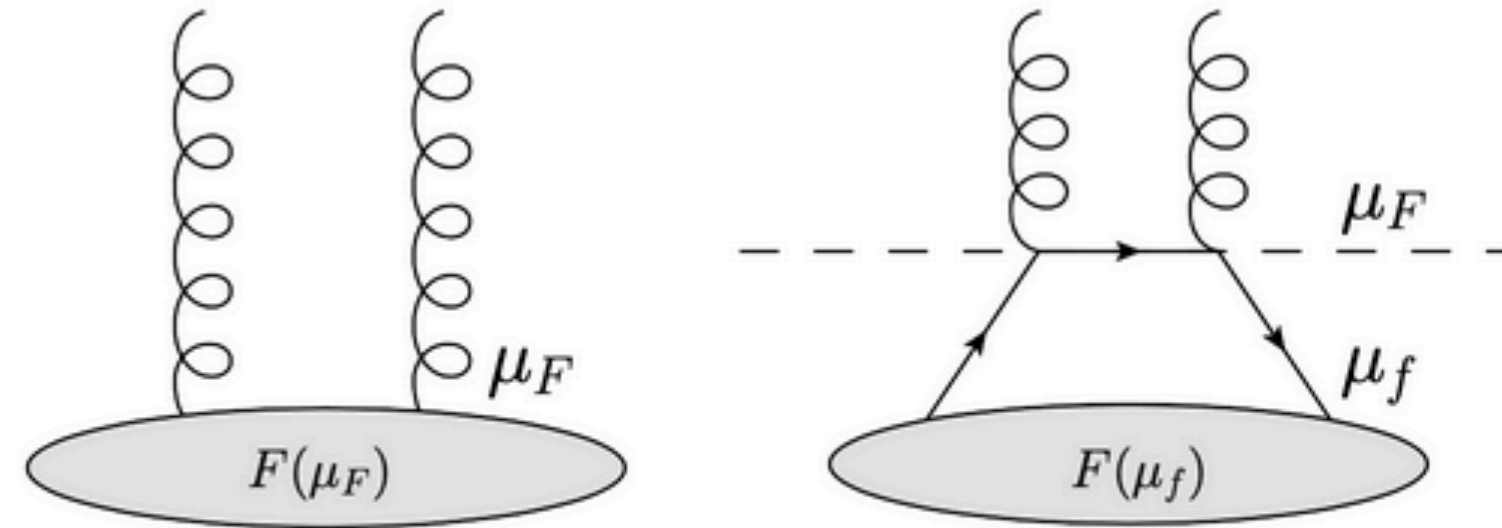
$$\Delta \text{Im} \mathcal{M}^q = \frac{\alpha_s^2}{2\pi} \int_{\xi}^1 dx (F_q(x, \xi, m_c) - F_q(-x, \xi, m_c)) \left( \int_0^{Q_0^2} (M_a^q + M_b^q) \frac{2\pi m_c^4}{\hat{s}^2} dl^2 \right)$$

- Precisely this **FINITE** contribution that is subtracted from full  $\overline{\text{MS}}$  coefficient functions to avoid double counting inherent within  $\overline{\text{MS}}$  scheme (subtraction fundamentally ubiquitous but numerically relevant for low scale processes only)



- NLO diagrams for quark and gluon channel considered. Contain both LO and NLO contributions. Subtract off LO contribution (part given by LO (generalised) DGLAP evolution  $P_{LO} \times C^0$ , see previous) before integration over  $l$  is performed, cancelling soft singularity  $dl^2/l^2$ .

## Treatment of double logarithmic contribution



**Ideology:** Use scale shifting to find optimal scale that removes the largest contribution from the NLO correction \*

At fact. scale.  $\mu_f$ , quark contribution is part of NLO hard matrix element

At fact. scale  $\mu_F$ , absorbed quark contribution into LO result

Effect of scale change driven by (generalised, skewed) DGLAP evolution:

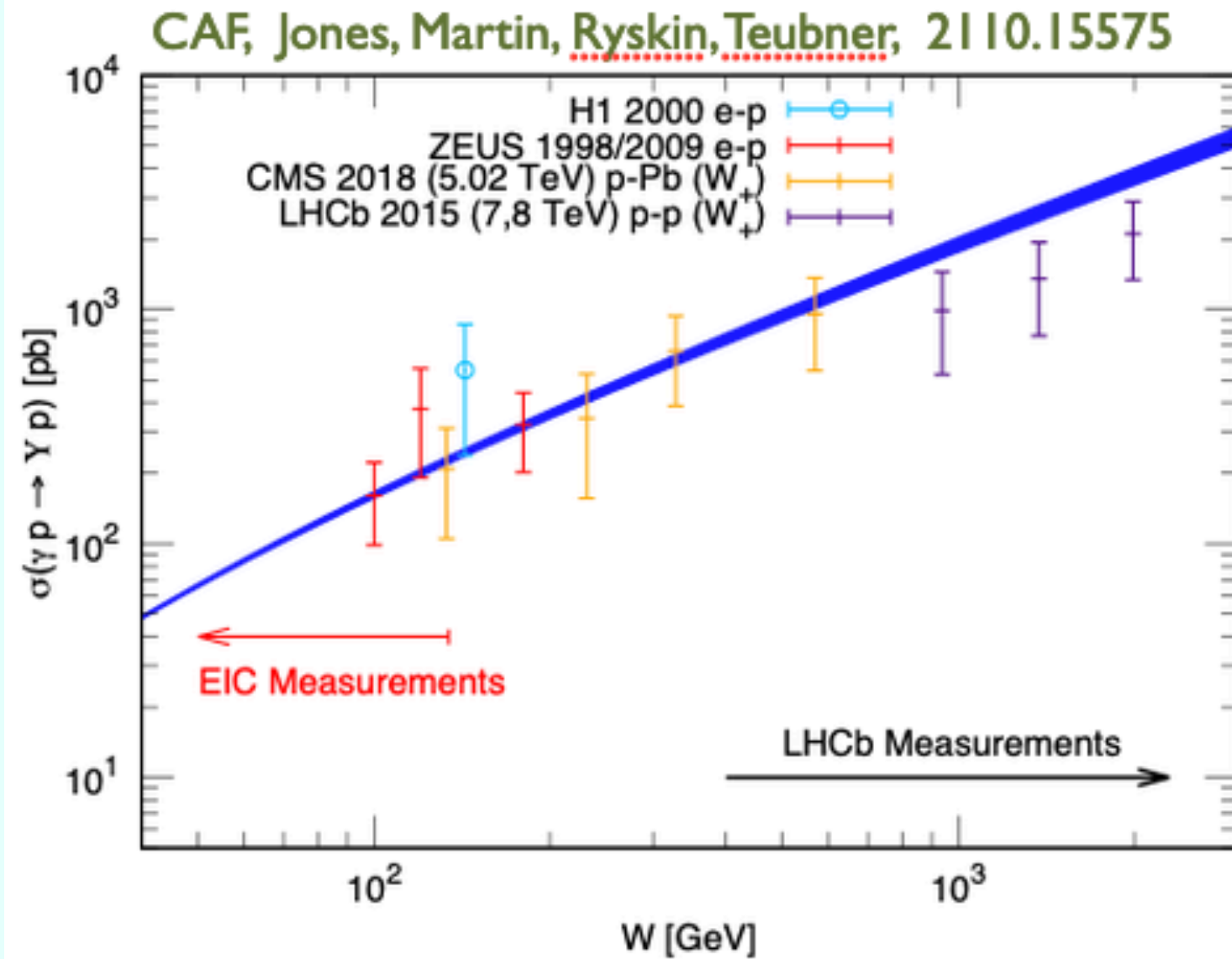
$$A^{(0)}(\mu_f) = \left( C^{(0)} + \frac{\alpha_s}{2\pi} \ln \left( \frac{\mu_f^2}{\mu_F^2} \right) C^{(0)} \otimes V \right) \otimes F(\mu_F)$$

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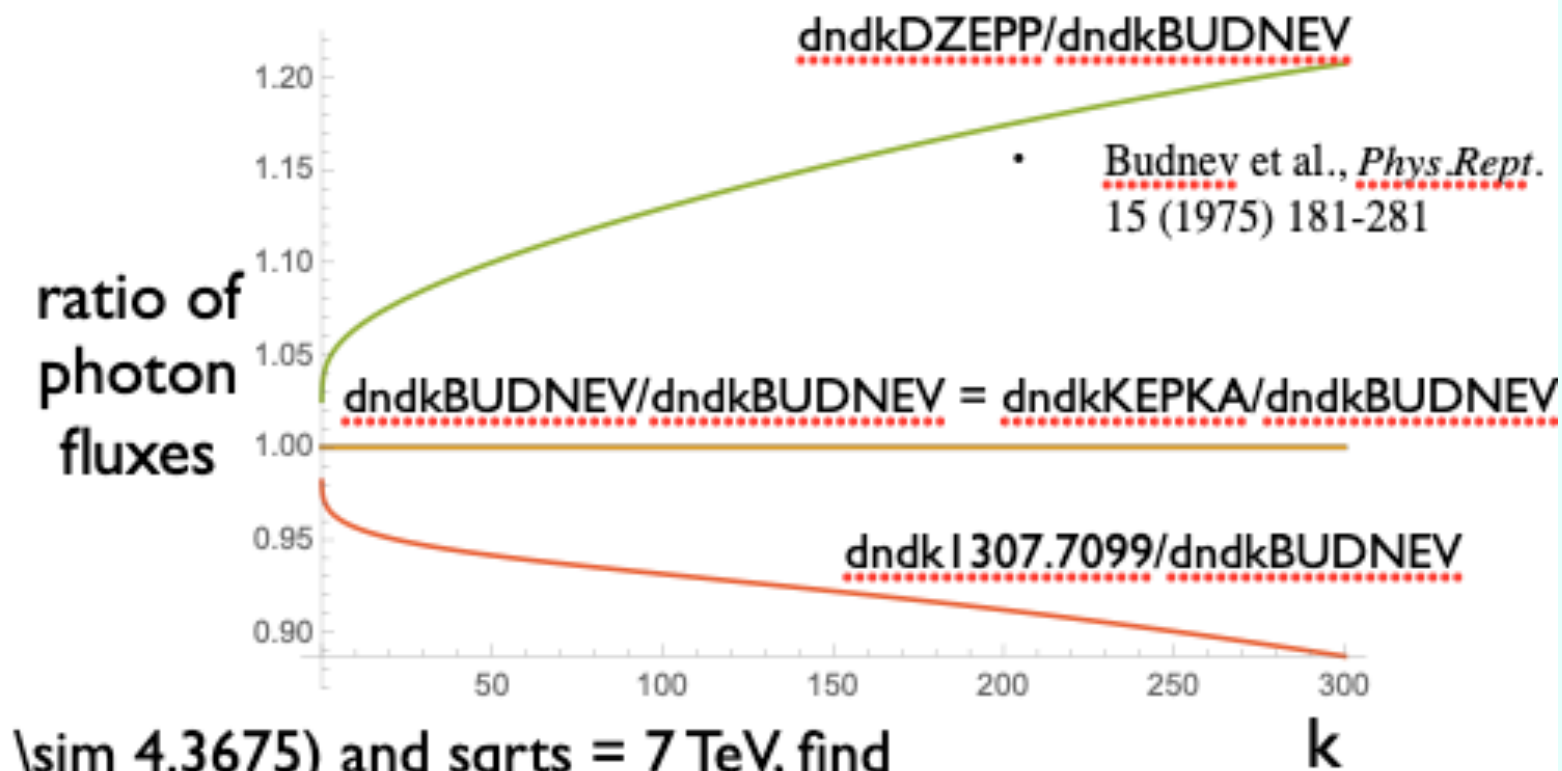
\* At small  $x_i$ , this is the double logarithmic contribution  $\sim \ln(1/x_i) \ln(\mu_F^2/mc^2)$



# Other results in UPC: Photon flux in Upsilon photoprod. in pp



-DGLAP evolve gluon PDF obtained from fit to J/psi data to scale of Upsilon photoproduction and use as input to make cross-section prediction (blue band)



For J/psi rapidity at border of LHCb acceptance ( $y \sim 4.3675$ ) and  $\sqrt{s} = 7$  TeV, find  
 $(\text{ssI307.7099} * \text{fluxI307.7099}) / (\text{ssBudnev} * \text{fluxBudnev}) = 0.94901$   
 ~ 5% effect

For J/psi rapidity outside border of LHCb acceptance ( $y \sim 5.125$ ) and  $\sqrt{s} = 7$  TeV, find  
 $(\text{ssI307.7099} * \text{fluxI307.7099}) / (\text{ssBudnev} * \text{fluxBudnev}) = 1.24832$   
 ~ 25% effect

Upsilon photoproduction photon energies will be larger so discrepancy between fluxes (and survival factors) will be larger and we enter the region where the approximation of I307.7099 flux breaks down at much lower rapidities and, importantly, within the acceptance of LHCb

=> use Budnev flux (without negligence of  $O(x)$  terms)

=> large W unfolded photoproduction LHCb data should be shifted upwards