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Synergies EIC-LHC for TMD studies with quarkonia

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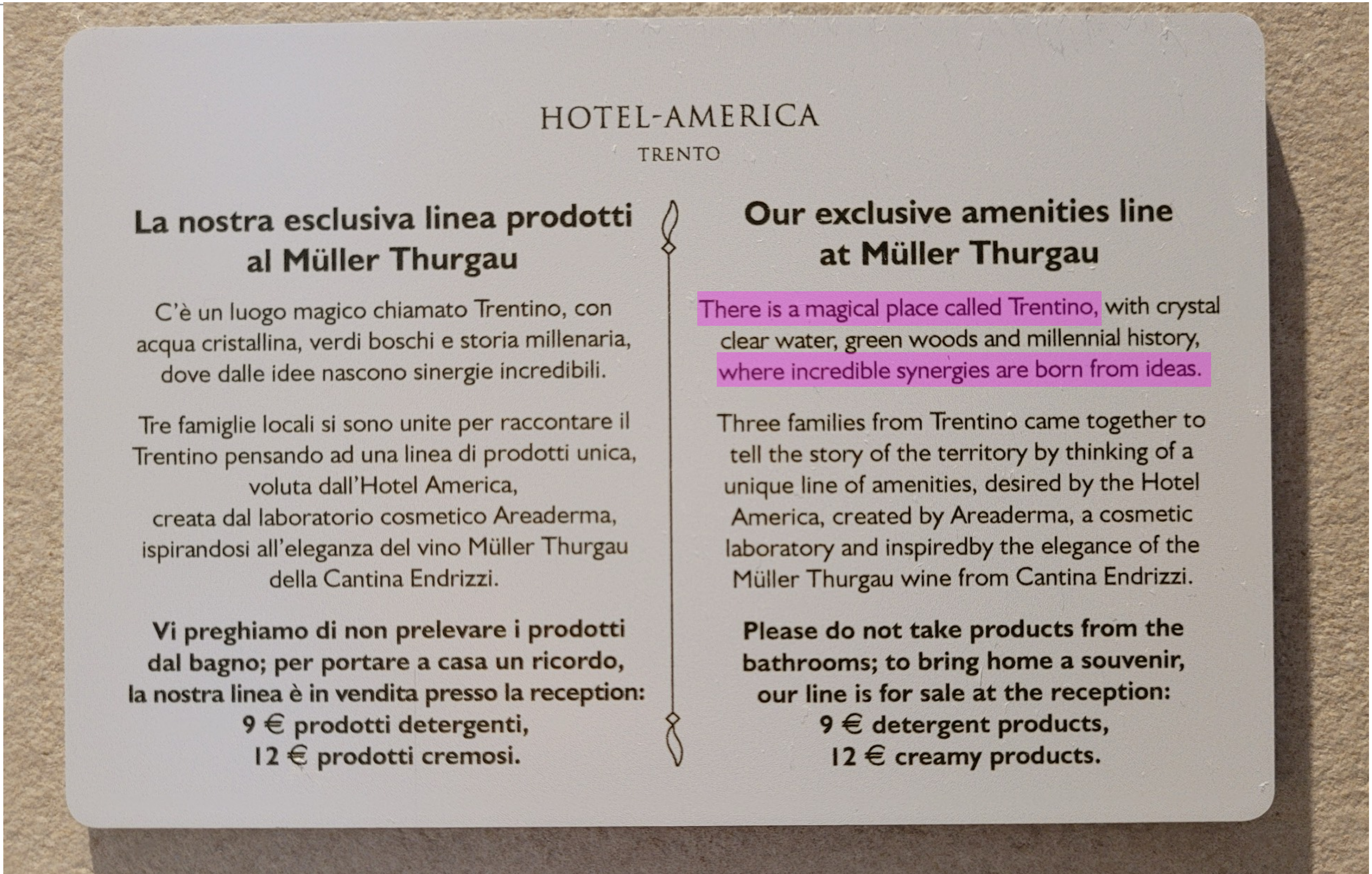
Synergies between LHC and EIC for quarkonium physics, ECT*, Trento, July 12, 2024

Synergies are everywhere in Trentino



Overview

- Quarkonium production at high versus low p_T (TMDs)
- LDMEs from TMD observables at EIC
- From LDMEs to Shape Functions
- Process dependence and factorization breaking
- TMD evolution studies

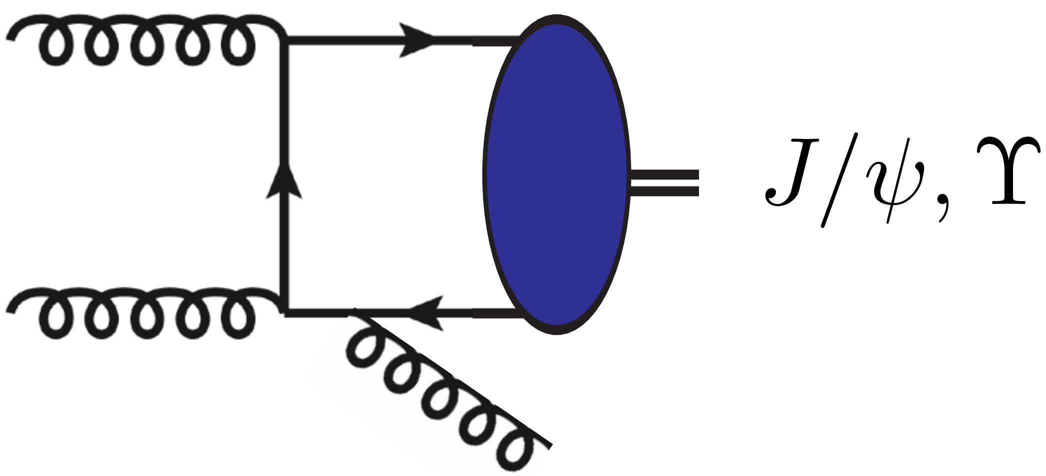
High vs low p_T

J/ψ production

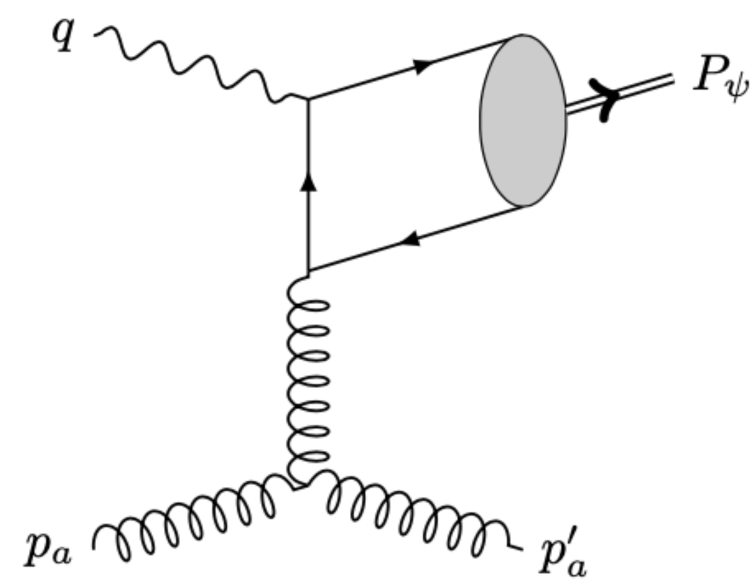
J/ψ production often considered in **NRQCD**

Bodwin, Braaten, Lepage, 1995; …

J/ψ production in electron-proton and proton-proton collisions considered at high p_T to ensure **collinear factorization**



Berger, Qiu, Wang, 2005;
Nayak, Qiu, Sterman, 2005;
Kang, Ma, Qiu, Sterman, 2014; …



$$d\sigma_{\mathrm{NRQCD}}(\mathcal{Q}+X) = \sum_n d\sigma(Q\bar{Q}[n]+X) \times \langle \mathcal{O}^{\mathcal{Q}}[n] \rangle$$

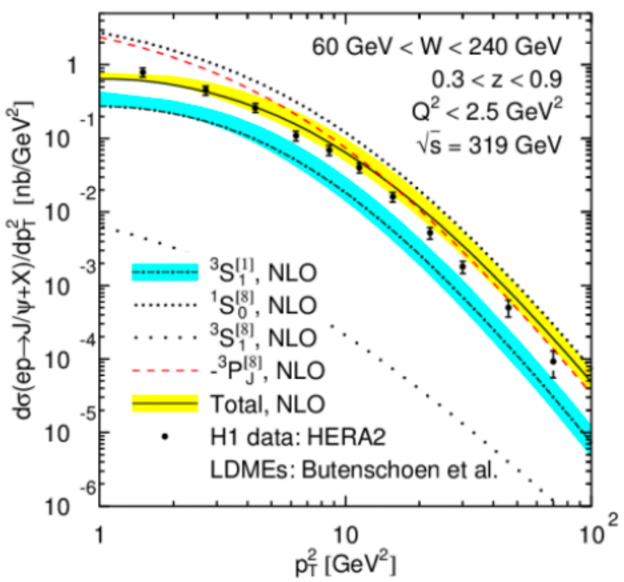
LDMEs

Hard to describe all HERA, Tevatron & LHC simultaneously

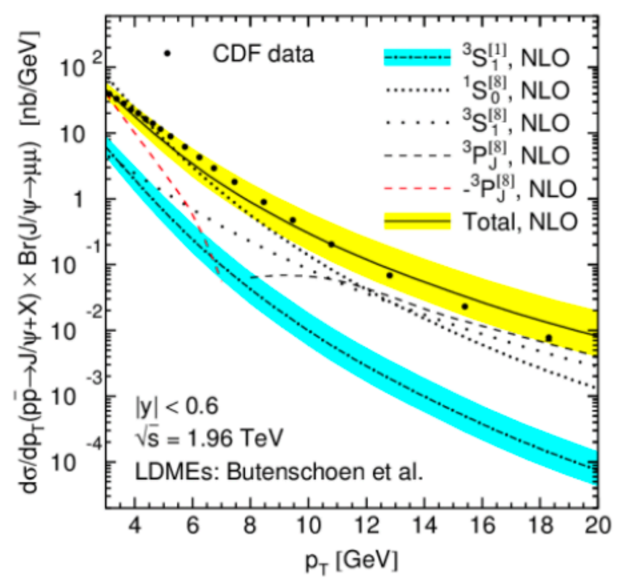
Extracted LDMEs vary considerably

Quarkonium production in NLO NRQCD

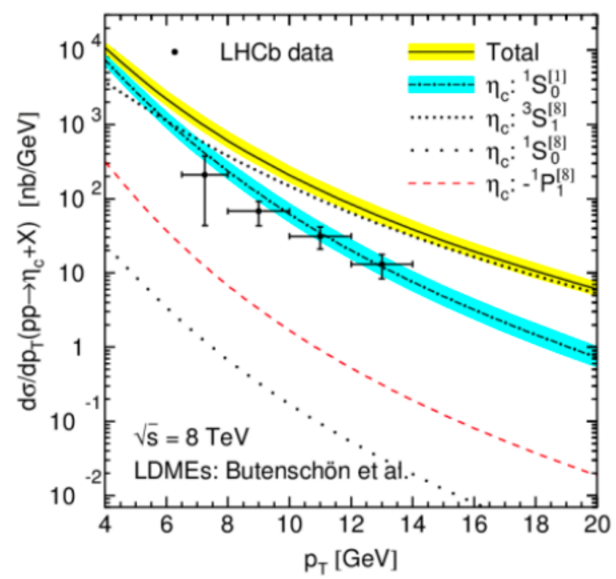
J/ψ Photoproduction



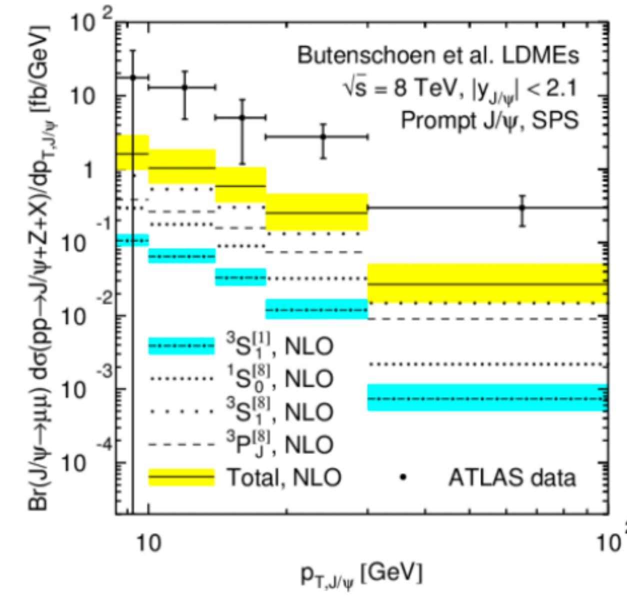
J/ψ Hadroproduction



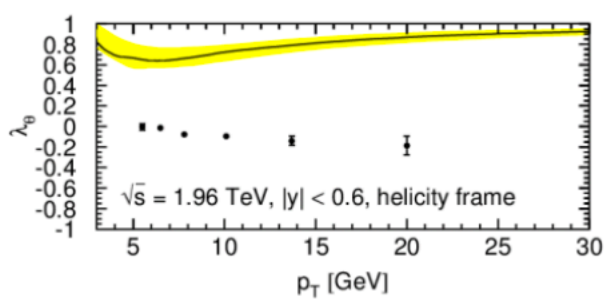
η_c Hadroproduction



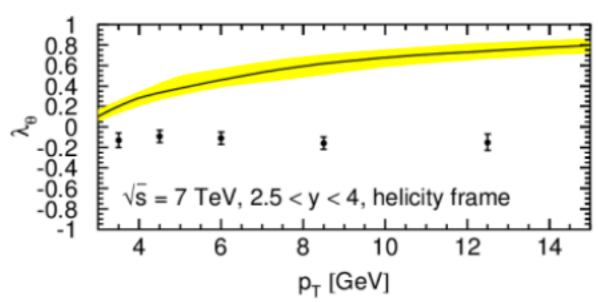
J/ψ + Z Hadroproduction



J/ψ Polarization (CDF)



J/ψ Polarization (LHCb)



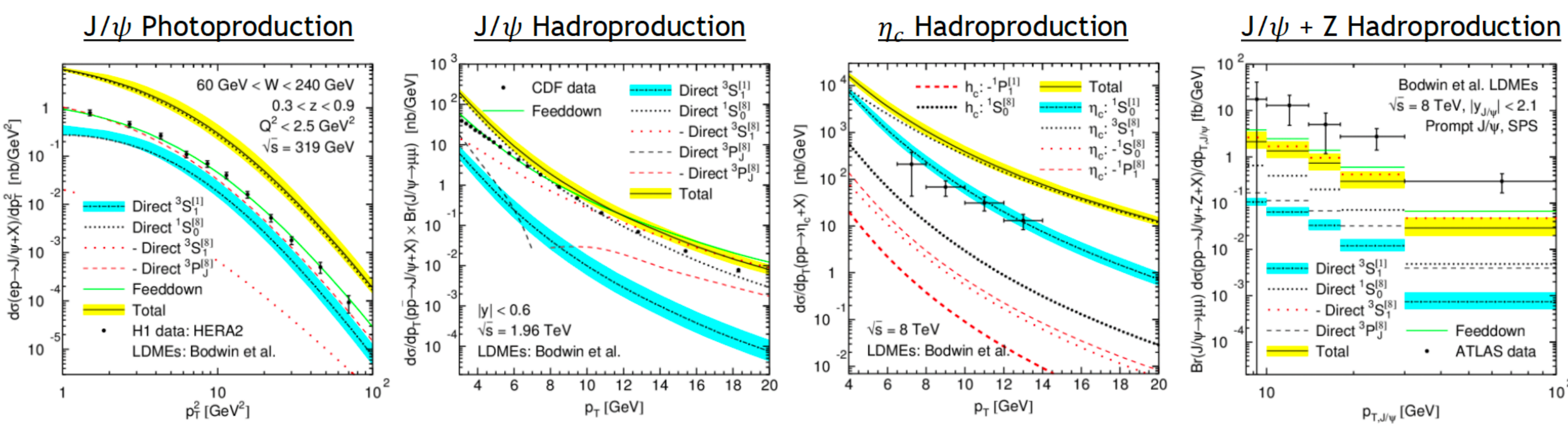
- Fit to 194 data points of J/ψ photo- and hadro-production, $\gamma\gamma$- and e^+e^- scattering
- $<O^{J/\psi}(1S08)>$=4.97±0.44, $<O^{J/\psi}(3S18)>$=0.22±0.06, $<O^{J/\psi}(3P08)>$=-1.61±0.20 [in 10^{-2} GeV^3 or 10^{-2} GeV^5]
- Ref.: [MB, Kniehl, PRD 84, 051501 (2011)]

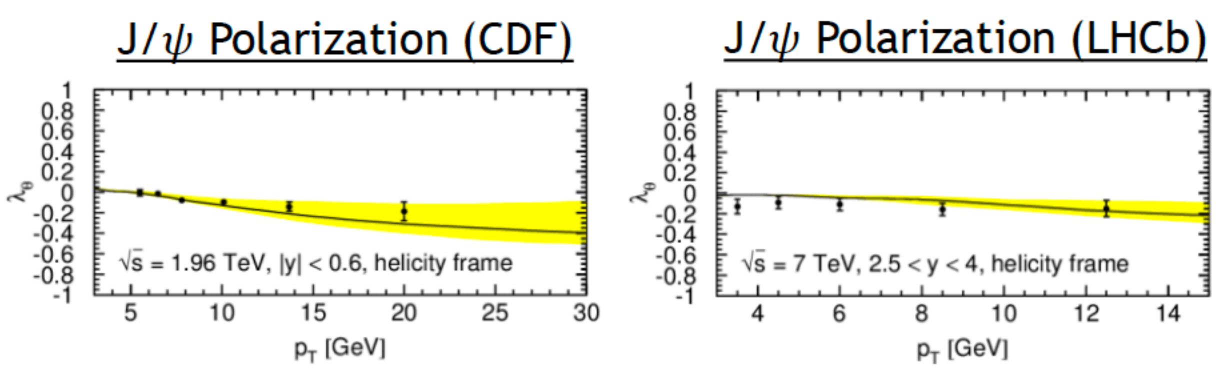
- Data fitted to is described within scale uncertainties, other observables not.

Slide by M. Butenschön

It turns out to be hard to describe all observables simultaneously

Quarkonium production in NLO NRQCD - 2





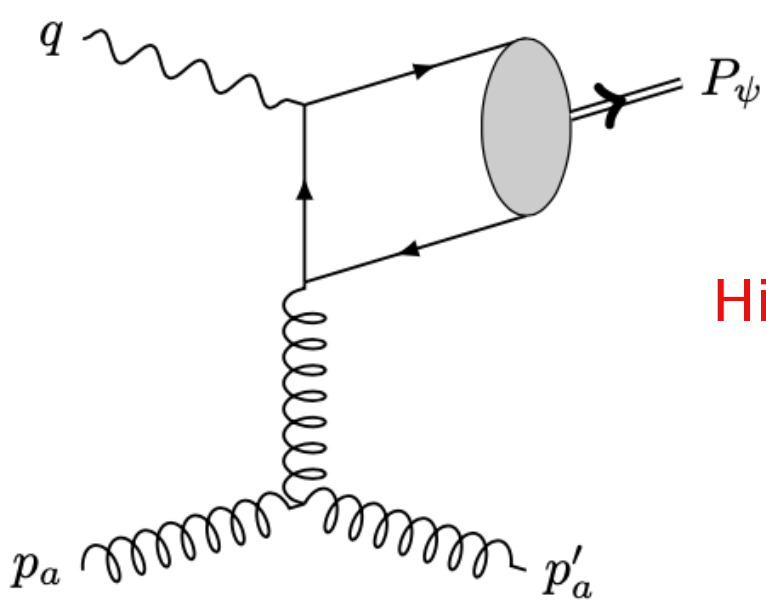
- Fit to $J/\psi, \psi(2S), \chi_{cJ}$ hadroproduction yield with $p_T > 3m_H$.
- Includes resummed FFs (we don't here → 2nd plot: small deviations)
- $<O^{J/\psi}$(1S08)>=11±1.4, $<O^{J/\psi}$(3S18)>=-0.71±0.36, $<O^{J/\psi}$(3P08)>=-0.70±0.34, $<O^{\psi(2S)}$(1S08)>=3.14±0.79, $<O^{\psi(2S)}$(3S18)>=-0.16 ±0.28, $<O^{\psi(2S)}$(3P08)>=-0.26±0.27, $<O^{\chi_{c0}}$(3P01)>=7.94 ±2.43, $<O^{\chi_{c0}}$(3S18)>=0.57±0.13 $[10^{-2}\ \mathrm{GeV}^{3\ \mathrm{or}\ 5}]$
- Ref.: [Bodwin, Chao, Chung, Kim, Lee, Ma, PRD 93, 034041 (2016)]

- Nontrivial outcome: Unpolarized J/ψ compatible with data.
  But: Small- and mid-p_T J/ψ hadro-; J/ψ photo-, η_c and $J/\psi + Z$ production not described.
  Also: **Direct $J/\psi + Z$** production unphysically **negative.**

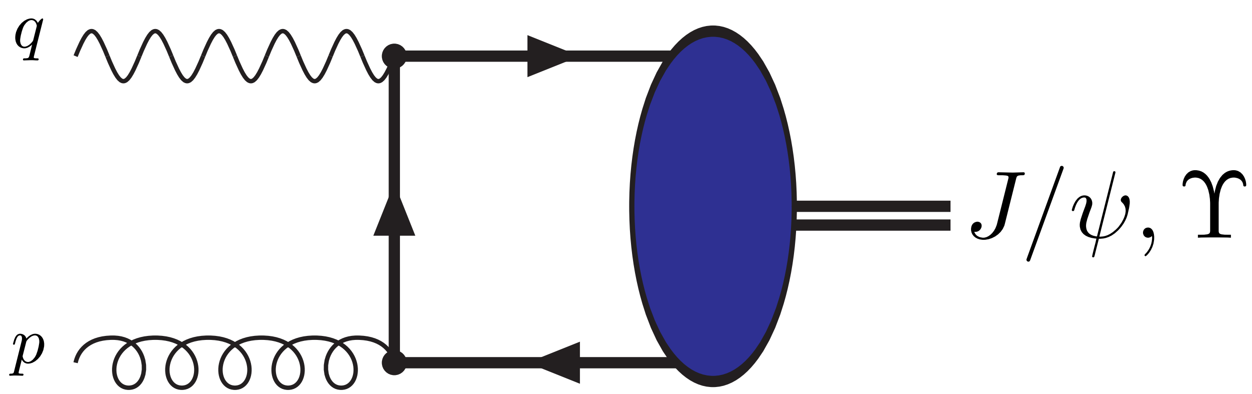
Slide by M. Butenschön

CDF, HERA, LHC all important in this endeavor - prime example of synergy

Including low p_T data

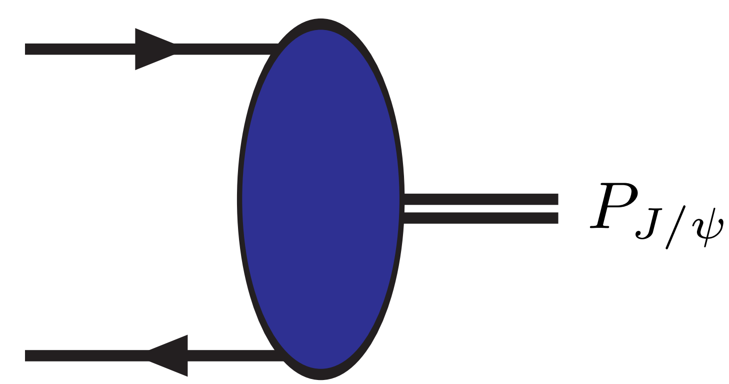


High p_T production: collinear factorization



Low p_T production: TMD factorization

One becomes sensitive to the transverse momentum distribution of the gluon inside the proton



Investigate gluon TMDs together with the LDMEs

Bacchetta, DB, Pisano, Taels, 2018; DB, Pisano, Taels, 2021

Gluon TMDs

Gluons TMDs

Gluon TMD correlator: $\Gamma_g^{\mu\nu}(x, p_T) \propto \langle P|F^{+\nu}(0)\,\mathcal{U}\,F^{+\mu}(\xi^-, \xi_T)\,\mathcal{U}'|P\rangle$

transverse momentum dependent (TMD)

For unpolarized protons:

$$\Gamma_U^{\mu\nu}(x, \boldsymbol{p}_T) = \frac{x}{2}\left\{ -g_T^{\mu\nu} f_1^g(x, \boldsymbol{p}_T^2) + \left(\frac{p_T^\mu p_T^\nu}{M_p^2} + g_T^{\mu\nu}\frac{\boldsymbol{p}_T^2}{2M_p^2}\right) h_1^{\perp\, g}(x, \boldsymbol{p}_T^2)\right\}$$

unpolarized gluon TMD

linearly polarized gluon TMD

Mulders, Rodrigues '01

Gluons inside *unpolarized* protons can be polarized!

For transversely polarized protons:

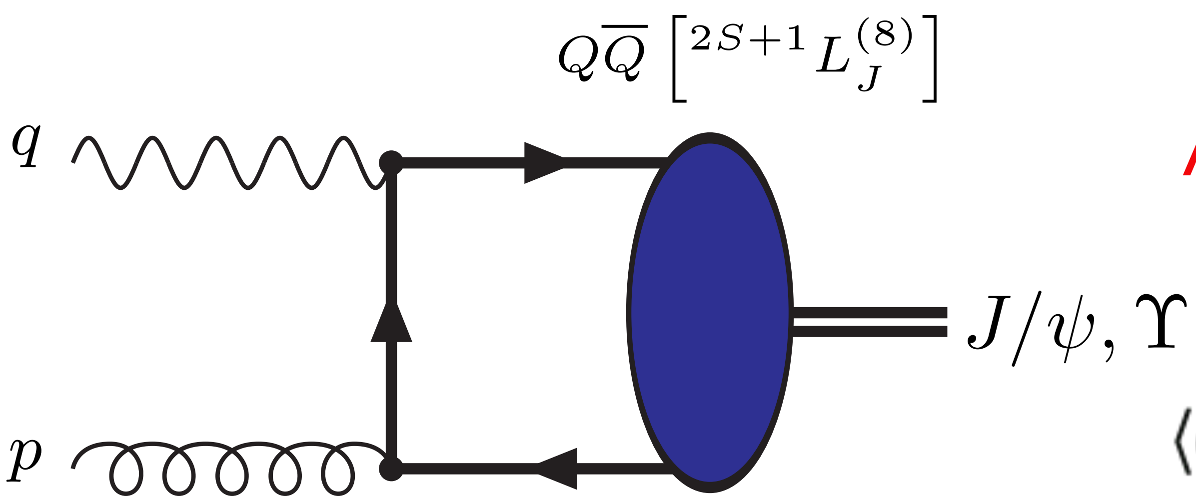
gluon Sivers TMD

$$\Gamma_T^{\mu\nu}(x, \boldsymbol{p}_T) = \frac{x}{2}\left\{ g_T^{\mu\nu}\frac{\epsilon_T^{\rho\sigma} p_{T\rho} S_{T\sigma}}{M_p} f_{1T}^{\perp\, g}(x, \boldsymbol{p}_T^2) + \ldots\right\}$$

Quarkonium production in ep

$e\,p \to e'\,\mathcal{Q}\,X$ with $\mathcal{Q}$ either a J/ψ or a Υ meson

Mukherjee, Rajesh, 2017; Sun, Zhang, 2017; Bacchetta, DB, Pisano, Taels, 2018;
Kishore, Mukherjee, 2018; Kishore, Mukherjee, Siddiqah, 2021; …



A $\cos(2\phi_T)$ asymmetry probes $h_1^{\perp\,g}$

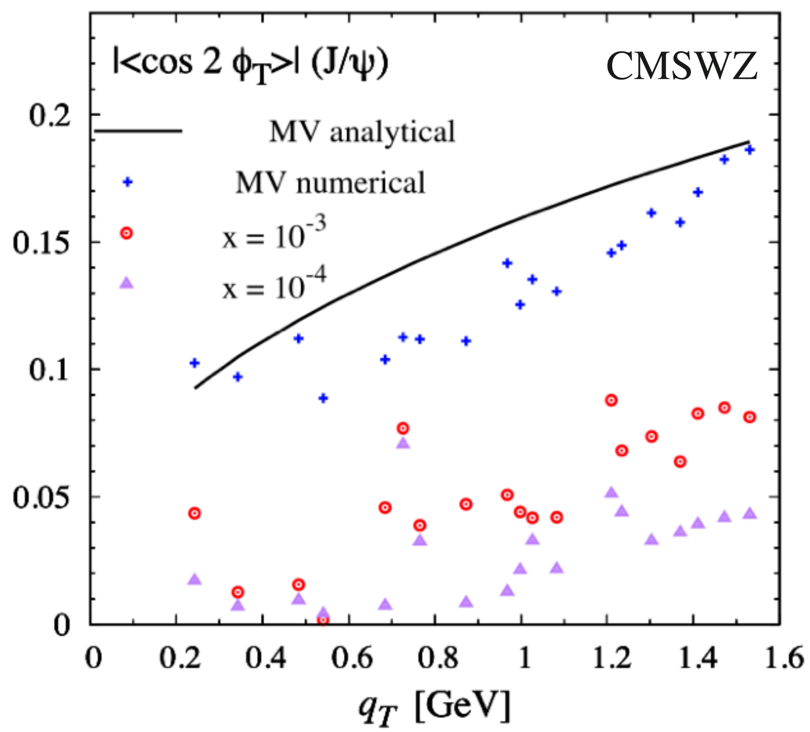
$$\langle \cos 2\phi_T \rangle = \frac{(1-y)\,\mathcal{B}_T^{\gamma^* g \to \mathcal{Q}}}{[1+(1-y)^2]\mathcal{A}_{U+L}^{\gamma^* g \to \mathcal{Q}} - y^2 \mathcal{A}_L^{\gamma^* g \to \mathcal{Q}}} \times \frac{\boldsymbol{q}_T^2}{2M_p^2}\,\frac{h_1^{\perp\,g}(x,\boldsymbol{q}_T^2)}{f_1^g(x,\boldsymbol{q}_T^2)}.$$

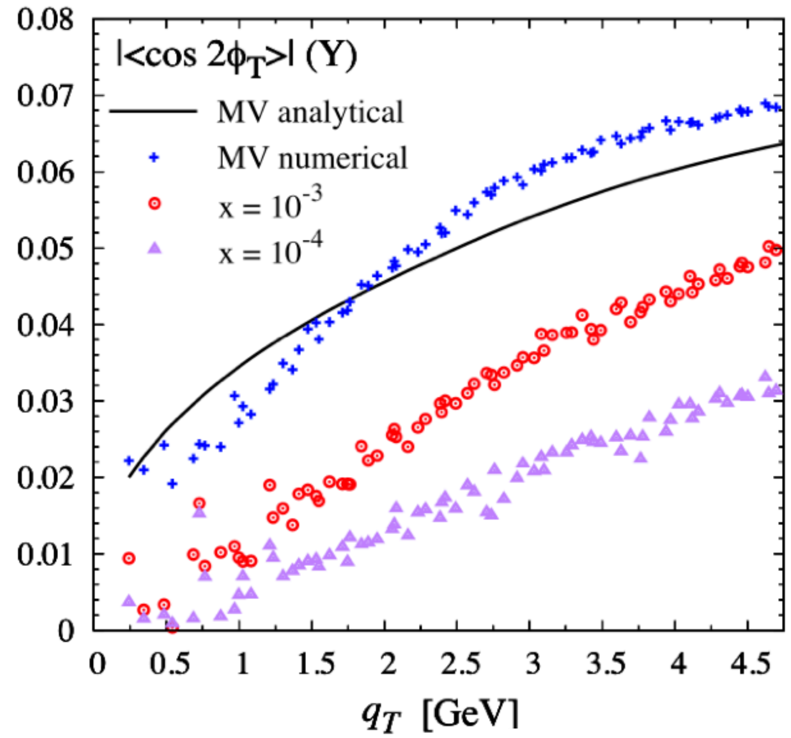
In LO NRQCD the prefactor of the asymmetry depends on y, Q, M_Q and on two quite uncertain Color Octet (CO) Long Distance Matrix Elements (LDMEs)

CS is α_s suppressed, but v^3 enhanced, higher minimum Q^2 and z cuts suppresses it

Bacchetta, DB, Pisano, Taels, 2018

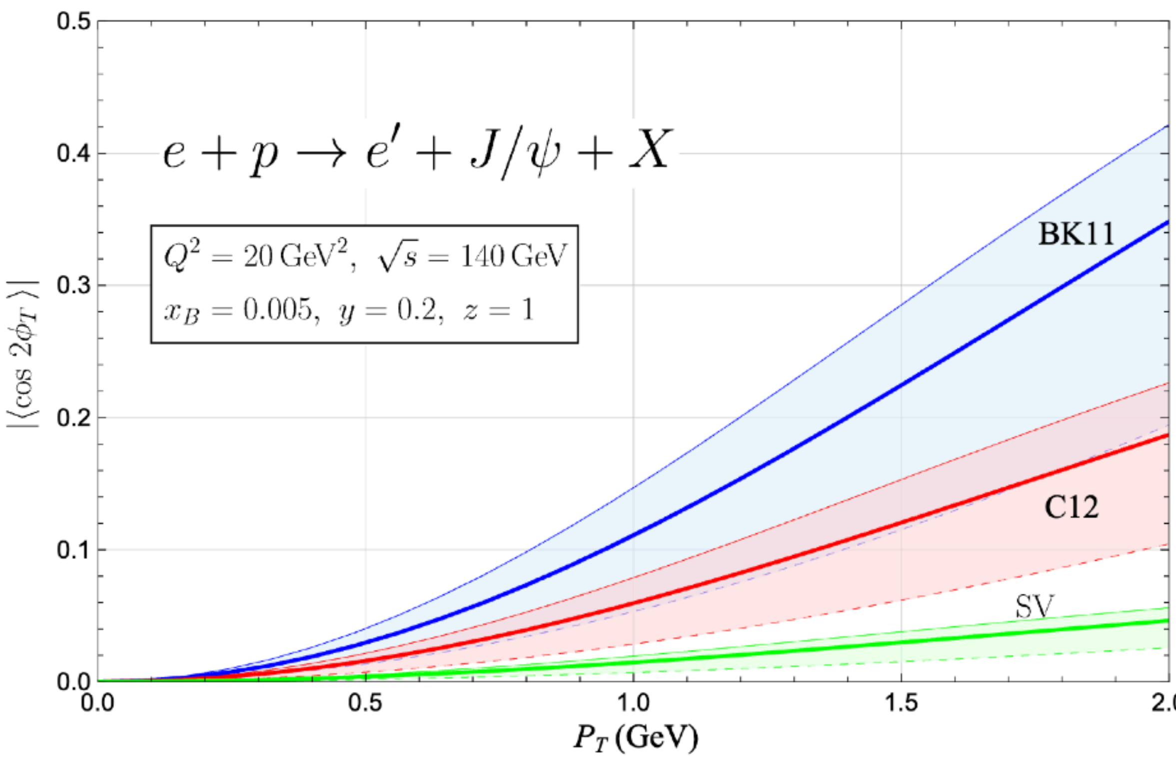
cos 2ϕ_T asymmetry - predictions



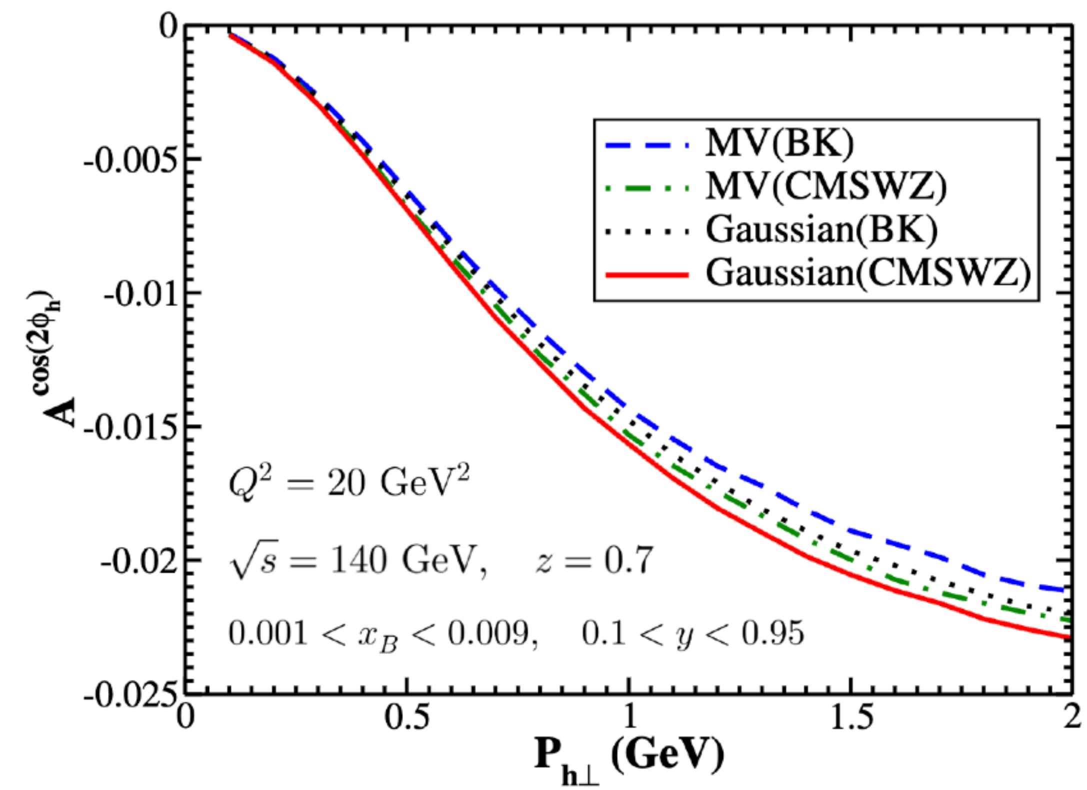


Asymmetries for $Q=M_Q$ & $y=0.1$ in the small-x MV model, including nonlinear evolution (numerical implementation on a 2D lattice)

Bacchetta, DB, Pisano, Taels, 2018



Bor, DB, 2022; EIC onium review, 2024



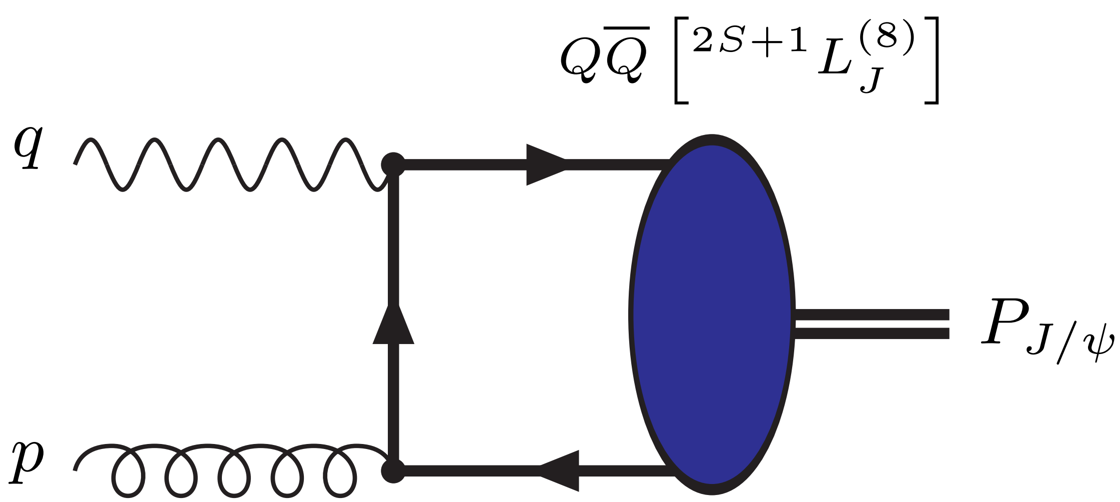
Kishore, Mukherjee, Pawar, Siddiqah, 2022

Despite the large uncertainties sizable cos2ϕ_T asymmetries are possible

Quarkonium production in ep

$$e\,p^{\uparrow} \to e'\,\mathcal{Q}\,X \text{ with } \mathcal{Q} \text{ either a } J/\psi \text{ or a } \Upsilon \text{ meson}$$

Godbole, Misra, Mukherjee, Rawoot, 2012/3; Godbole, Kaushik, Misra, Rawoot, 2015; Mukherjee, Rajesh, 2017; Rajesh, Kishore, Mukherjee, 2018; …



Using LO NRQCD the Sivers asymmetry is:

$$A^{\sin(\phi_S-\phi_T)} = \frac{|\boldsymbol{q}_T|}{M_p}\,\frac{f_{1T}^{\perp\,g}(x,\boldsymbol{q}_T^2)}{f_1^g(x,\boldsymbol{q}_T^2)}$$

$$\frac{A^{\cos 2\phi_T}}{A^{\sin(\phi_S+\phi_T)}} = \frac{\boldsymbol{q}_T^2}{M_p^2}\,\frac{h_1^{\perp\,g}(x,\boldsymbol{q}_T^2)}{h_1^g(x,\boldsymbol{q}_T^2)}$$

$$\frac{A^{\cos 2\phi_T}}{A^{\sin(\phi_S-3\phi_T)}} = -\frac{1}{2}\,\frac{h_1^{\perp\,g}(x,\boldsymbol{q}_T^2)}{h_{1T}^{\perp\,g}(x,\boldsymbol{q}_T^2)}$$

$$\frac{A^{\sin(\phi_S-3\phi_T)}}{A^{\sin(\phi_S+\phi_T)}} = -\frac{\boldsymbol{q}_T^2}{2M_p^2}\,\frac{h_{1T}^{\perp\,g}(x,\boldsymbol{q}_T^2)}{h_1^g(x,\boldsymbol{q}_T^2)}$$

CO NRQCD LDMEs cancel out in ratios of asymmetries at LO

Bacchetta, DB, Pisano, Taels, 2018

Higher order corrections
may complicate this simple picture

CO LDMEs

Color Octet LDMEs from EIC

One can also consider ratios where the TMDs cancel out

Allows to obtain new experimental information on the poorly known CO LDMEs

and to learn more about the quarkonium production mechanism

One way is to exploit the linear polarization of the gluon and/or the polarization state of the produced quarkonium (P=L or T):

$$\int \mathrm{d}\phi_T \, \frac{\mathrm{d}\sigma^{UP}}{\mathrm{d}y \, \mathrm{d}x_B \, \mathrm{d}^2\boldsymbol{q}_T} = 2\pi \mathcal{N} A^{UP} f_1^g(x, \boldsymbol{q}_T^2) \,,$$

$$\int \mathrm{d}\phi_T \cos 2\phi_T \, \frac{\mathrm{d}\sigma^{UP}}{\mathrm{d}y \, \mathrm{d}x_B \, \mathrm{d}^2\boldsymbol{q}_T} = \pi \mathcal{N} B^{UP} \frac{\boldsymbol{q}_T^2}{M_p^2} h_1^{\perp g}(x, \boldsymbol{q}_T^2)$$

At LO the ratios A^{UL}/A^{UU}, A^{UT}/A^{UU}, B^{UL}/B^{UU}, B^{UT}/B^{UU} only depend on y, Q and the two CO LDMEs, for example for $Q^2=4M_Q^2$:

$$\frac{A_{UL}}{A_{UU}} = \frac{(1+(1-y)^2)\,\mathcal{O}_8^S/3 + (6-6y+y^2)\,\mathcal{O}_8^P/M_Q^2}{(1+(1-y)^2)\,\mathcal{O}_8^S + (10-10y+3y^2)\,\mathcal{O}_8^P/M_Q^2}$$

$$\frac{B_{UL}}{B_{UU}} = \frac{\mathcal{O}_8^S/3 - \mathcal{O}_8^P/M_Q^2}{\mathcal{O}_8^S - \mathcal{O}_8^P/M_Q^2} \,.$$

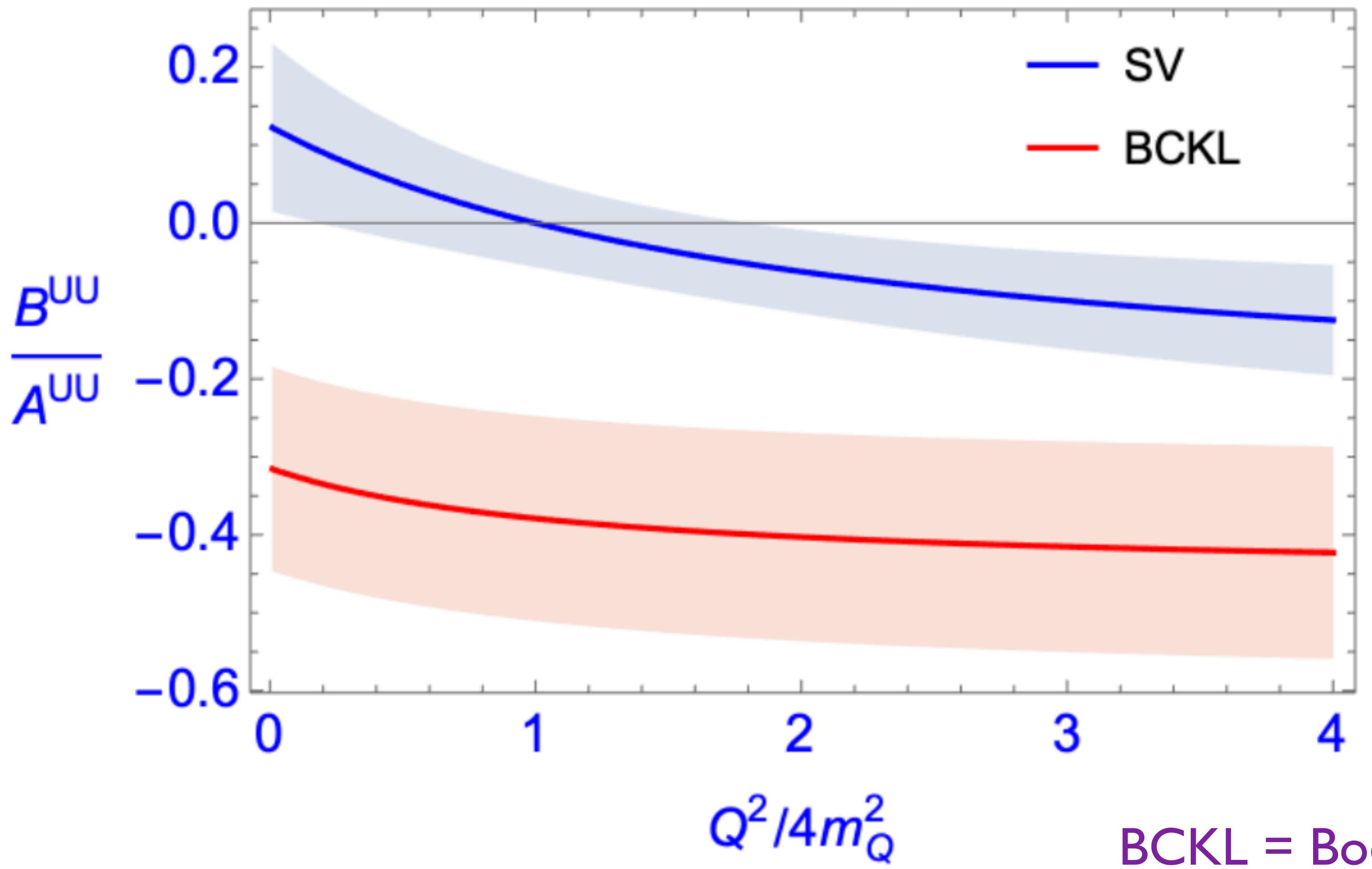
$$\mathcal{O}_8^S \equiv \langle 0|\mathcal{O}_8^{\mathcal{Q}}(^1S_0)|0\rangle$$

$$\mathcal{O}_8^P \equiv \langle 0|\mathcal{O}_8^{\mathcal{Q}}(^3P_0)|0\rangle$$

DB, Pisano, Taels, 2021

Azimuthal Asymmetry

Ratio for unpolarized quarkonium production involves the same two unknowns:



$$\int \mathrm{d}\phi_T \, \frac{\mathrm{d}\sigma^{UP}}{\mathrm{d}y\, \mathrm{d}x_B\, \mathrm{d}^2\boldsymbol{q}_T} = 2\pi \mathcal{N}\, A^{UP}\, f_1^g(x, \boldsymbol{q}_T^2)\,,$$

$$\int \mathrm{d}\phi_T \cos 2\phi_T \, \frac{\mathrm{d}\sigma^{UP}}{\mathrm{d}y\, \mathrm{d}x_B\, \mathrm{d}^2\boldsymbol{q}_T} = \pi \mathcal{N}\, B^{UP}\, \frac{\boldsymbol{q}_T^2}{M_p^2}\, h_1^{\perp g}(x, \boldsymbol{q}_T^2)$$

Different LDME extractions differ considerably, most have $O_8^P \ll O_8^S$ (e.g. BCKL), but one extraction has $O_8^P = O_8^S$ (SV)

$$\mathcal{O}_8^S \equiv \langle 0|\mathcal{O}_8^{\mathcal{Q}}(^1S_0)|0\rangle$$

$$\mathcal{O}_8^P \equiv \langle 0|\mathcal{O}_8^{\mathcal{Q}}(^3P_0)|0\rangle$$

BCKL = Bodwin et al., 2014
SV= Sharma & Vitev, 2013

Overconstrained system allows to cross check the extraction

Bacchetta, DB, Pisano, Taels, 2018

Caveat: LDMEs extracted at NLO, hence proper extraction requires a NLO analysis

Color Octet LDMEs from EIC

One can also consider ratios to open heavy quark production to cancel out TMDs

This requires a comparison of $e\, p \to e'\, \mathcal{Q}\, X$ and $ep \to e' Q\bar{Q} X$

$$\mathcal{R} = \frac{\int d\phi_T\, d\sigma^{\mathcal{Q}}(\phi_S, \phi_T)}{\int d\phi_T\, d\phi_\perp\, d\sigma^{Q\overline{Q}}(\phi_S, \phi_T, \phi_\perp)}$$

$$\mathcal{R}^{\cos 2\phi} = \frac{\int d\phi_T \cos 2\phi_T\, d\sigma^{\mathcal{Q}}(\phi_S, \phi_T)}{\int d\phi_T\, d\phi_\perp \cos 2\phi_T\, d\sigma^{Q\overline{Q}}(\phi_S, \phi_T, \phi_\perp)}$$

Two observables depending on the same two unknowns:

$$\mathcal{O}_8^S \equiv \langle 0|\mathcal{O}_8^{\mathcal{Q}}(^1S_0)|0\rangle$$

$$\mathcal{O}_8^P \equiv \langle 0|\mathcal{O}_8^{\mathcal{Q}}(^3P_0)|0\rangle$$

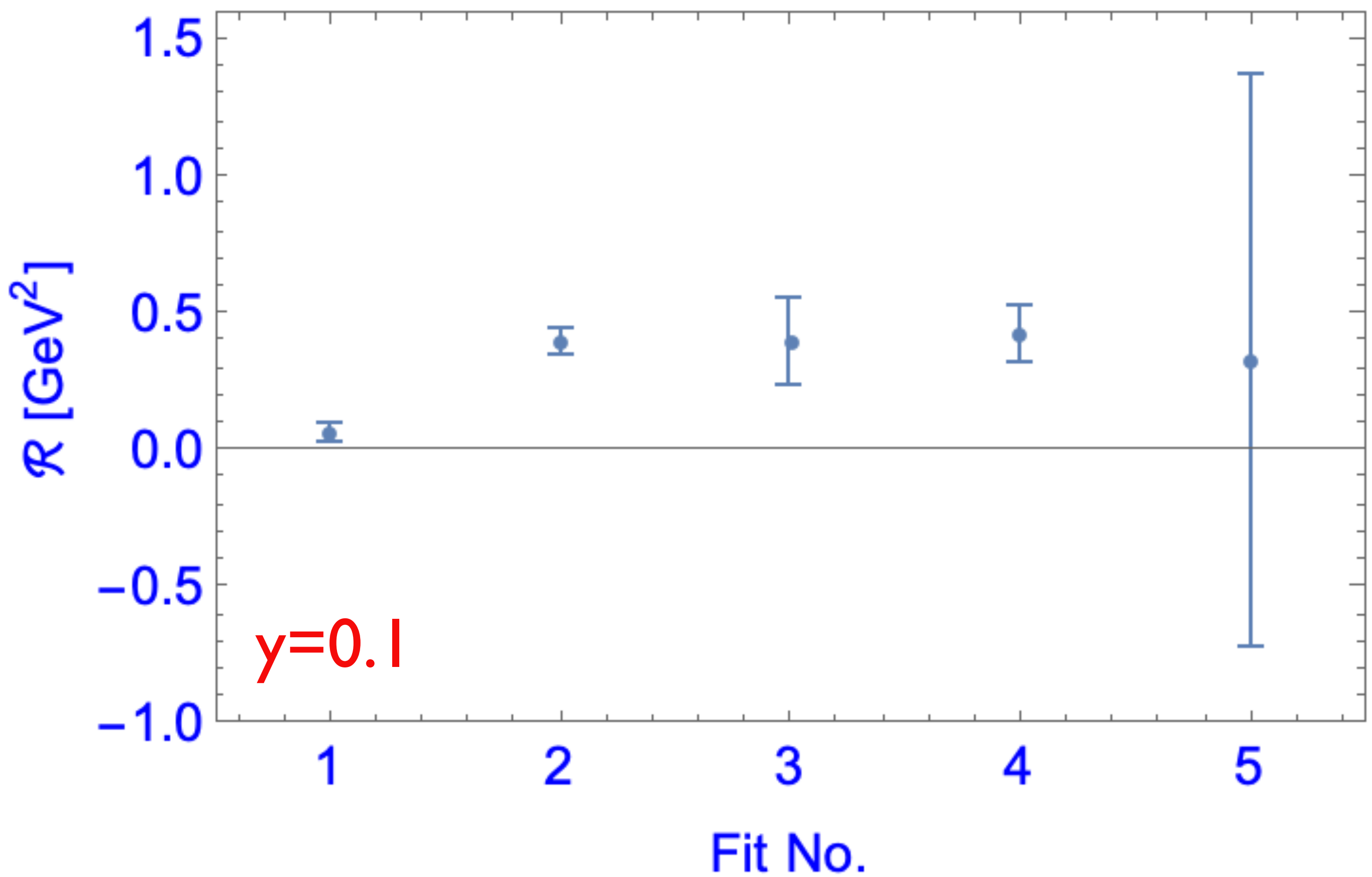
$$\mathcal{R} = \frac{27\,\pi^2}{4}\,\frac{1}{M_Q}\,\frac{[1+(1-y)^2]\,\mathcal{O}_8^S + (10-10y+3y^2)\,\mathcal{O}_8^P/M_Q^2}{26-26y+9y^2}$$

$z = 1/2$

$$\mathcal{R}^{\cos 2\phi_T} = \frac{27\pi^2}{4}\,\frac{1}{M_Q}\left[\mathcal{O}_8^S - \frac{1}{M_Q^2}\,\mathcal{O}_8^P\right]$$

To avoid evolution we choose $K\perp = Q = 2M_Q$

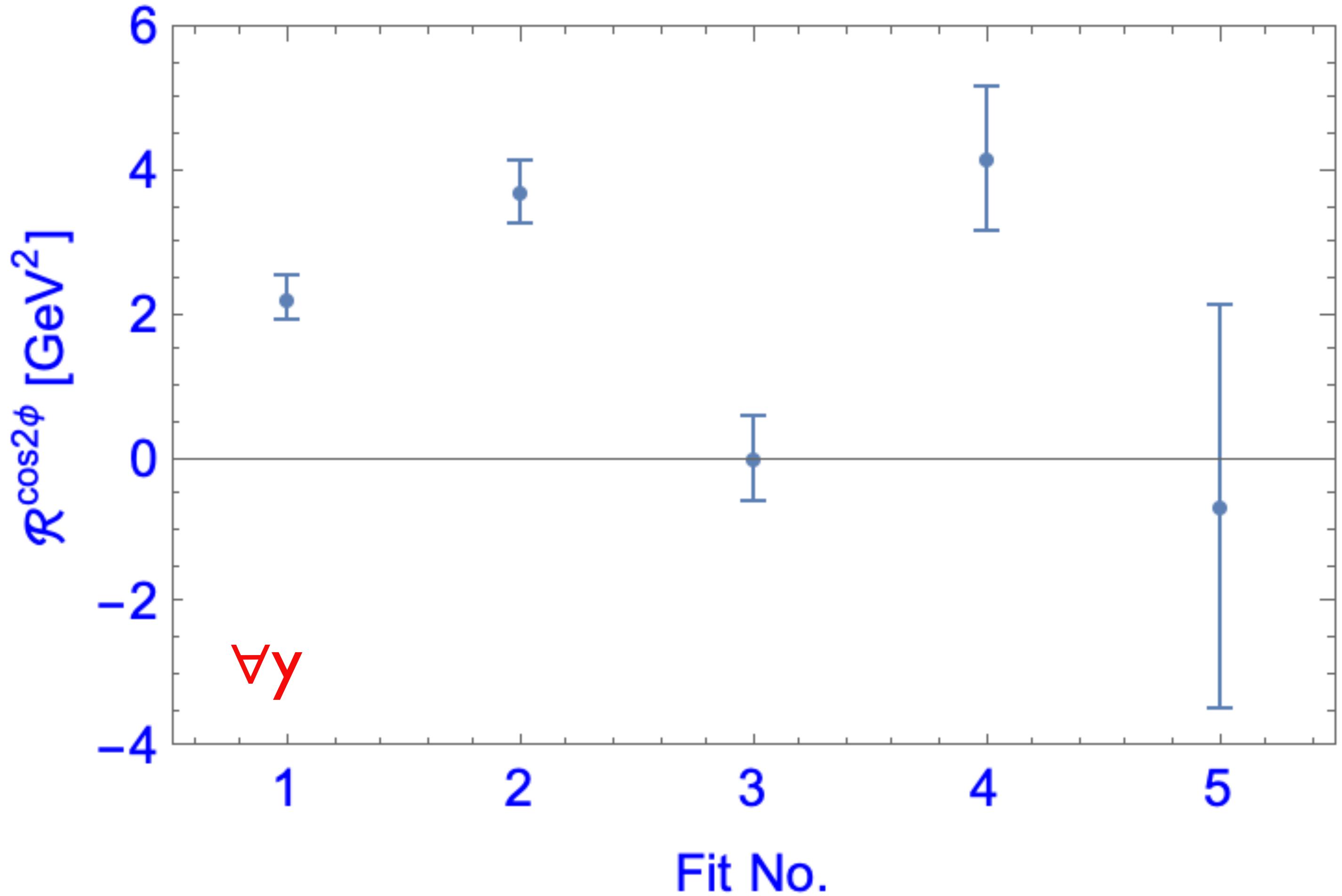
Bacchetta, DB, Pisano, Taels, 2018



Ratios not normalized to

[0,1] for $\mathcal{R}$ or [-1,1] for $\mathcal{R}^{\cos(2\phi)}$

$$\frac{\mathcal{R}^{\cos 2\phi_T}}{\mathcal{R}} = \frac{\langle \cos 2\phi_T \rangle_{\mathcal{Q}}}{\langle \cos 2\phi_T \rangle_{Q\overline{Q}}}$$



Based on fits the ratio of asymmetries could be anywhere between 0 and ∞

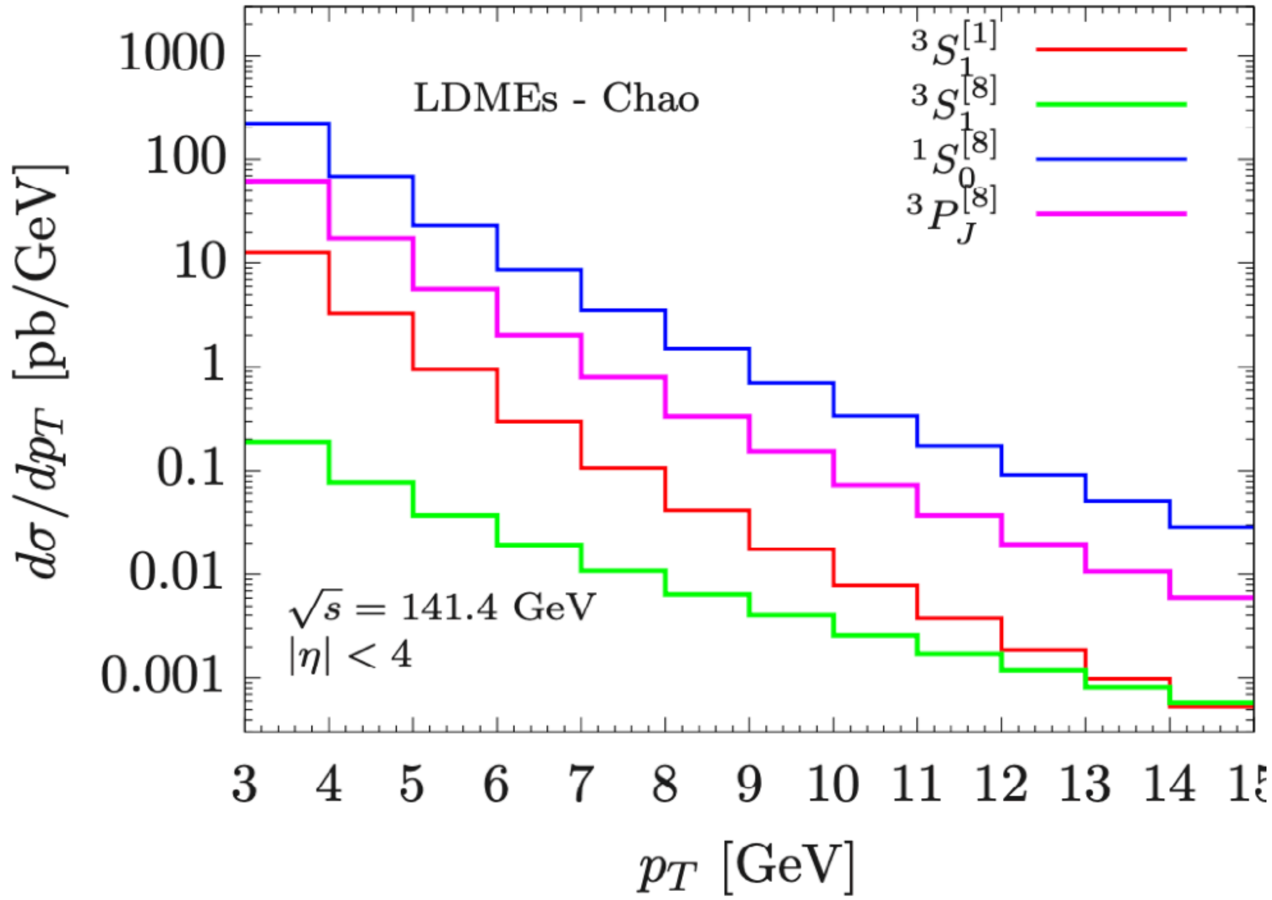
But rough average of the fits would indicate that the $\cos(2\phi_T)$ asymmetry in open HQ production could be of $\mathcal{O}(10)$ times larger in J/ψ production

Polarized quarkonia at EIC

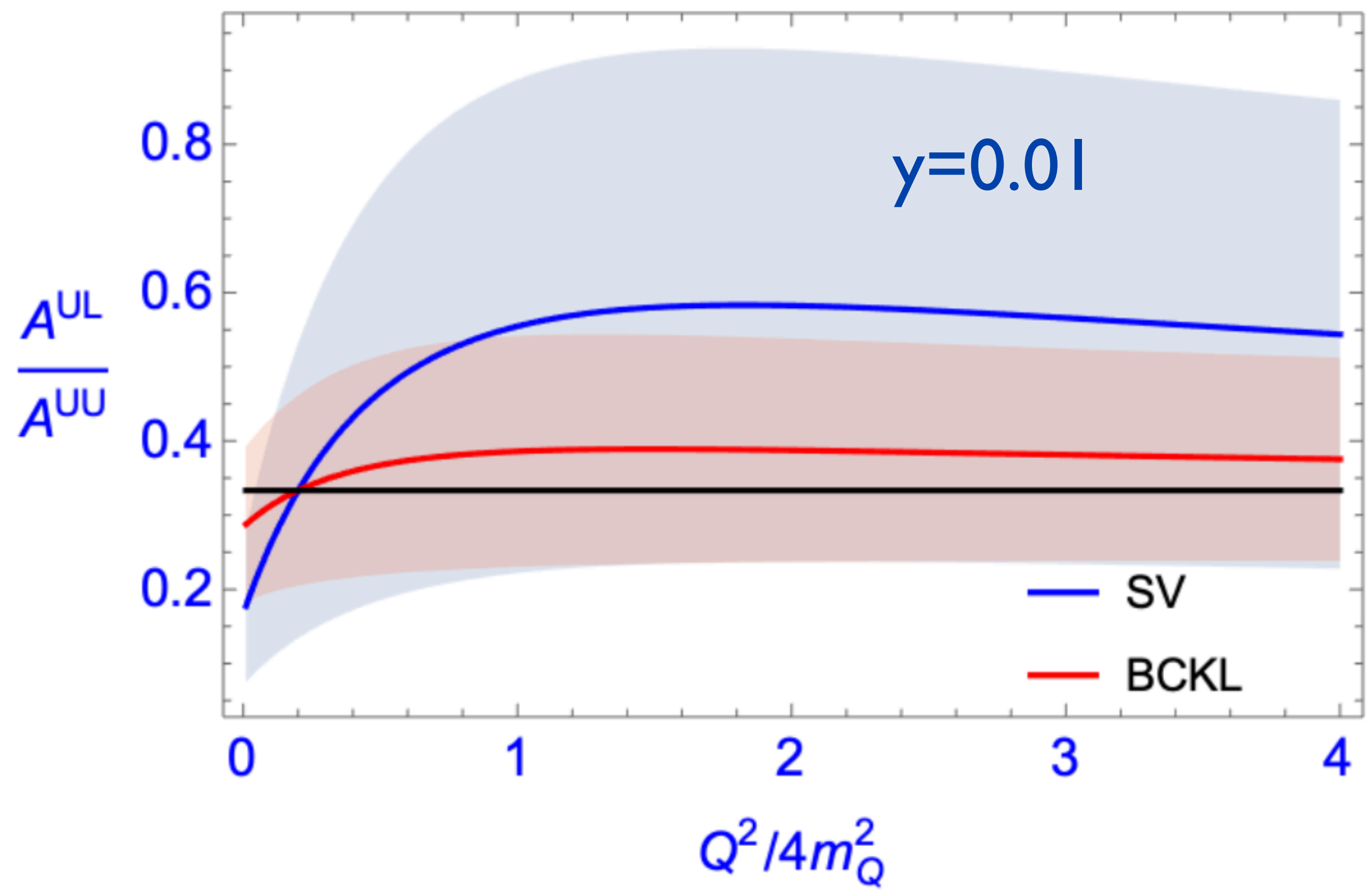
Will the quarkonium state be produced polarized at EIC?

In $e\,p \to \mathcal{Q}\,X$ (untagged e', dominated by $Q^2 \approx 0$) in collinear factorization at high p_T the $^1S_0$ state dominates ≈ unpolarized

In LO NRQCD study in the TMD regime A^{UL}/A^{UU} can be far from 1/3



Qiu, Wang, Qing, 2020



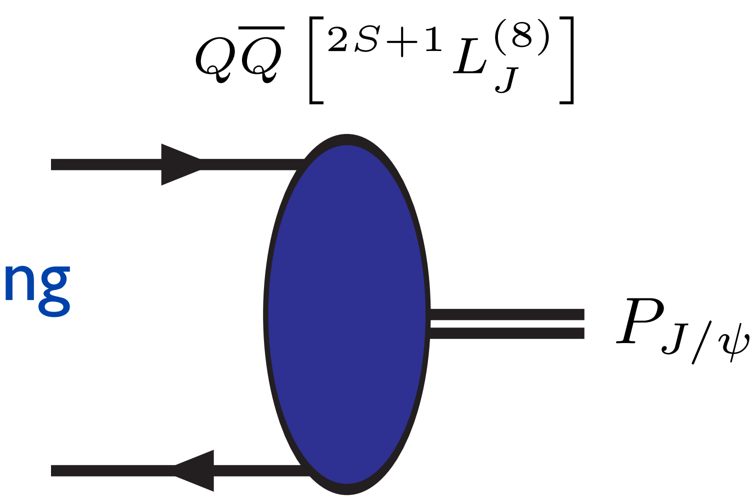
DB, Pisano, Taels, 2021

$$\left.\frac{A_{UL}}{A_{UU}}\right|_{\mathcal{O}_8^P \ll \mathcal{O}_8^S} \approx \frac{1}{3}$$

can be used to demonstrate the relevance of $^3P_0$

Shape functions

Effect of smearing



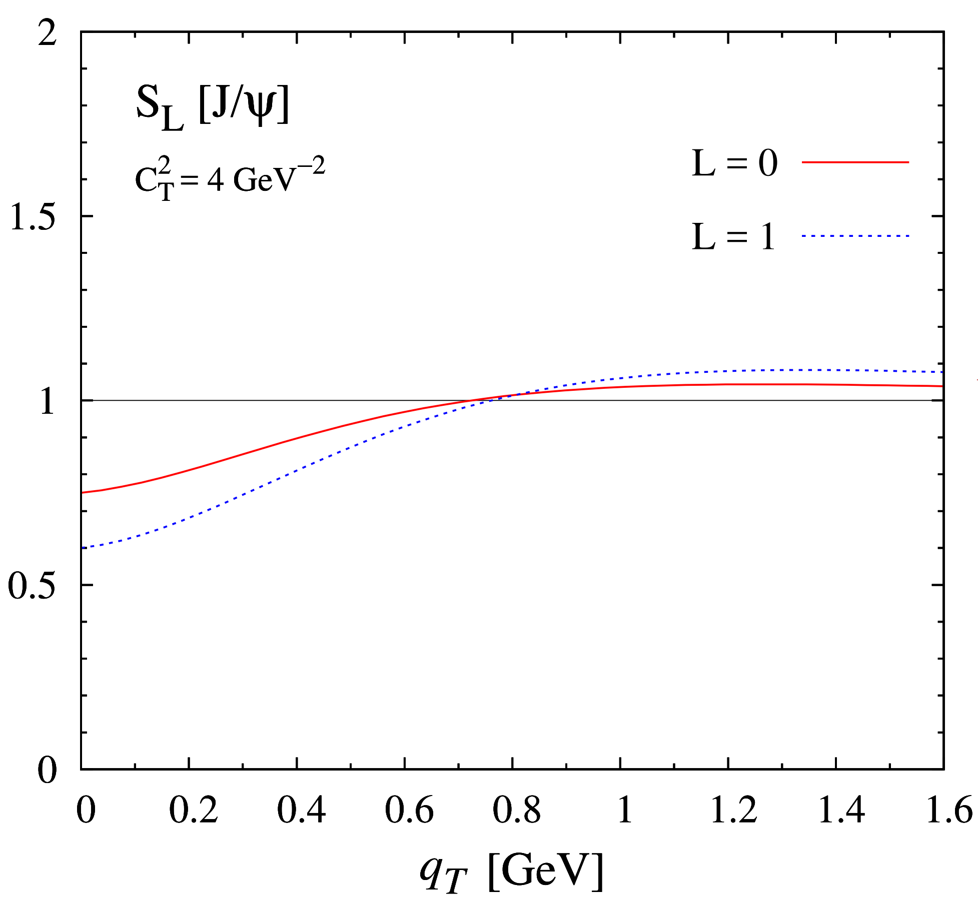
In reality the process of $Q\bar{Q} \to J/\Psi$ involves some k_T-smearing

TMD factorization requires still unknown "shape functions"

Echevarria, 2019; Fleming, Makris & Mehen, 2019

If L dependent this smearing would affect the extraction of CO LDMEs:

$$\mathcal{R} = \frac{27\,\pi^2}{4}\,\frac{1}{M_Q}\,\frac{[1+(1-y)^2]\,\mathcal{O}_8^S\,S_0(x,\boldsymbol{q}_T^2) + (10-10y+3y^2)\,\mathcal{O}_8^P/M_Q^2\,S_1(x,\boldsymbol{q}_T^2)}{26-26y+9y^2}$$



$$S_L(x,\boldsymbol{q}_T^2) = \frac{\mathcal{C}[f_1^g\,\Delta_L](x,\boldsymbol{q}_T^2)}{f_1^g(x,\boldsymbol{q}_T^2)}$$

← Example study using positronium-like functions

Bacchetta, DB, Pisano, Taels, 2018

However these are *not* the wave functions of the quarkonia ($\Delta \neq \Psi$)

Shape function vs LDMEs

$$\frac{d\sigma^{UP}}{dy\,dx_B\,d^2\boldsymbol{P}_T^*} = \mathcal{N}\left[\sum_n A_{UP}^{[n]} C\left[f_1^g\,\Delta^{[n]}\right] + \sum_n B_{UP}^{[n]} C\left[w h_1^{\perp g}\,\Delta_h^{[n]}\right]\cos 2\phi_T^*\right]$$

$$C\left[w h_1^{\perp g}\,\Delta_h^{[n]}\right](\boldsymbol{q}_T) \equiv \int d^2\boldsymbol{p}_T \int d^2\boldsymbol{k}_T\,\delta^2(\boldsymbol{p}_T+\boldsymbol{k}_T-\boldsymbol{q}_T)\,w(\boldsymbol{p}_T,\boldsymbol{q}_T)\,h_1^{\perp g}(x,\boldsymbol{p}_T^2)\,\Delta_h^{[n]}(\boldsymbol{k}_T^2)$$

$$\langle\cos 2\phi_T^*\rangle \equiv \frac{\int d\phi_T^*\cos 2\phi_T^*\,\frac{d\sigma^{UU}}{dy\,dx_B\,d^2\boldsymbol{P}_T^*}}{\int d\phi_T^*\,\frac{d\sigma^{UU}}{dy\,dx_B\,d^2\boldsymbol{P}_T^*}} = \frac{1}{2}\frac{\sum_n B_{UU}^{[n]}\,C\left[w h_1^{\perp g}\,\Delta_h^{[n]}\right]}{\sum_n A_{UU}^{[n]}\,C\left[f_1^g\,\Delta^{[n]}\right]}$$

$$\Delta^{[n]}(\boldsymbol{k}_T^2;\mu^2) \simeq \langle\mathcal{O}^{\mathcal{Q}}[n]\rangle\,\Delta(\boldsymbol{k}_T^2;\mu^2)$$

Assumption: shape functions ∝ LDMEs

Idem for Δ_h

$$A_{UU} = \sum_n A_{UU}^{[n]}\,\langle\mathcal{O}^{\mathcal{Q}}[n]\rangle$$

$$B_{UU} = \sum_n B_{UU}^{[n]}\,\langle\mathcal{O}^{\mathcal{Q}}[n]\rangle.$$

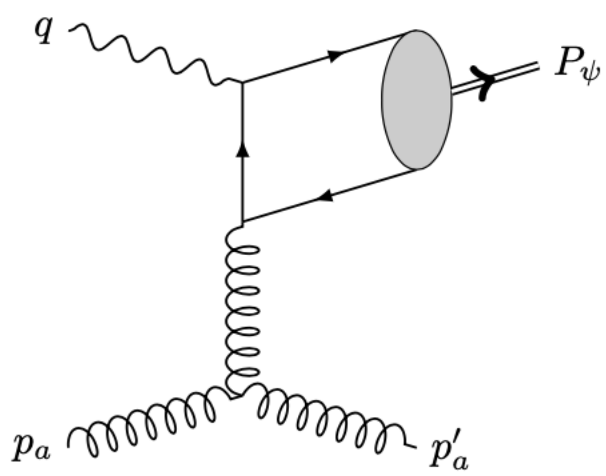
$$\langle\cos 2\phi_T^*\rangle = \frac{1}{2}\frac{B_{UU}}{A_{UU}}\frac{C\left[w h_1^{\perp g}\,\Delta_h\right]}{C\left[f_1^g\,\Delta\right]}$$

q_T dependent

B^{UU}/A^{UU} same as before

Consequently, the TMD evolved expressions will also be proportional to the LO expressions in terms of LDMEs

Matching high and low transverse momentum



Calculate the high p_T contributions and consider their low p_T limit

This should match with the high p_T limit of the TMD contribution, but that can only work out after including a shape function Δ:

$$\mathcal{F}_{UU,T} = \frac{2\pi^2 \alpha_s e_c^2}{M_\psi (M_\psi^2 + Q^2)} \left[\langle 0 | \mathcal{O}(^1S_0^{[8]}) | 0 \rangle \, + \, 4 \, \frac{(7M_\psi^4 + 2M_\psi^2 Q^2 + 3Q^4)}{M_\psi^2 (M_\psi^2 + Q^2)^2} \langle 0 | \mathcal{O}(^3P_0^{[8]}) | 0 \rangle \right] \times f_1^g(x, p_T^2) \Big|_{\boldsymbol{p}_T = \boldsymbol{q}_T},$$

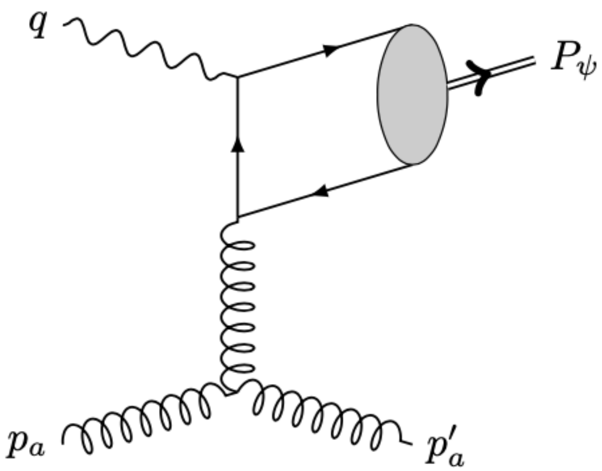
$$\int \mathrm{d}^2\boldsymbol{p}_T \int \mathrm{d}^2\boldsymbol{k}_T \, \delta^2(\boldsymbol{q}_T - \boldsymbol{p}_T - \boldsymbol{k}_T) \, f_1^g(x, \boldsymbol{p}_T^2; \mu^2) \, \Delta^{[n]}(\boldsymbol{k}_T^2, \mu^2) \; = \; \mathcal{C}[f_1^g \, \Delta_L](x, \boldsymbol{q}_T^2)$$

It turns out that the perturbative tail is L (or more generally) n independent:

$$\Delta^{[n]}(\boldsymbol{k}_T^2; \bar{\mu}^2) \; \simeq \; \langle \mathcal{O}^{\mathcal{Q}}[n] \rangle \, \Delta(\boldsymbol{k}_T^2; \bar{\mu}^2)$$

Effect of smearing & matching

LO NRQCD: $\Delta^{[n]}(\boldsymbol{k}_T^2;\mu^2) = \langle 0|\mathcal{O}(n)\,|0\rangle\,\delta^2(\boldsymbol{k}_T)$



Taking into account unexpected non-continuous behavior:

$$\Delta^{[n]}(z,\boldsymbol{k}_T^2;\widetilde{Q}^2) = -\frac{\alpha_s}{2\pi^2\boldsymbol{k}_T^2}C_A\left(1+\log\frac{M_\psi^2}{M_\psi^2+Q^2}\right)\langle\mathcal{O}[n]\rangle\delta(1-z)$$

$$\widetilde{Q}^2 = M_\psi^2 + Q^2$$

DB, Bor, Maxia, Pisano, Yuan, 2023;
DB, D'Alesio, Murgia, Pisano, Taels, 2020

$\Longrightarrow \Delta^{[n]}(\boldsymbol{k}_T^2;\bar{\mu}^2) \simeq \langle \mathcal{O}^{\mathcal{Q}}[n]\rangle\,\Delta(\boldsymbol{k}_T^2;\bar{\mu}^2)$ holds at $\mathcal{O}(\alpha_s)$, at least at large $k_T \gg \Lambda_{QCD}$

One might think the log is of the form $\log\frac{M_\psi^2}{\mu_H^2}$, but it is not, rather it is $\log\frac{M_\psi^2\mu_H^2}{\tilde{Q}^4}$

Shape functions can be different for different quarkonia, but there is also process dependence, like in open heavy quark production!

Catani, Grazzini, Torre, 2015

This may yield results that are very different from NRQCD analyses

Shape function vs LDMEs

Perturbative tail of the shape functions in b space:

$$S_{[n]\to J/\psi}(b_T;\mu,\zeta_B) = \sum_{[m]} C_{[m]}^{[n]}(b_T;\mu,\zeta_B) \times \frac{\langle \mathcal{O}^{[m]}\rangle}{N_{pol}^{(J)}} + \mathcal{O}(b_T\Lambda_{\rm QCD})$$

${}^1S_0^{[8]}$ channel:

$$C^S_{{}^1S_0^{[8]}}(b_T;\mu,\zeta_B) = 1 + \frac{\alpha_s C_A}{2\pi} L_T\,(1-\ln\zeta_B)$$

$$C^S_{{}^1P_1^{[1]}}(b_T;\mu) = -\frac{\alpha_s}{2\pi}\frac{8\,C_F}{3m_c^2} L_T$$

$$C^S_{{}^1P_1^{[8]}}(b_T;\mu) = -\frac{\alpha_s}{2\pi}\frac{8\,B_F}{3m_c^2} L_T$$

${}^3P_{J=0,1,2}^{[8]}$ channel

$$C^P_{{}^3P_J^{[8]}}(b_T;\mu,\zeta_B) = 1 + \frac{\alpha_s C_A}{2\pi} L_T\,(1-\ln\zeta_B)$$

$$C^P_{{}^3D_{J+1}^{[1]}}(b_T;\mu) = -\frac{\alpha_s}{2\pi}\frac{8\,C_F}{3m_c^2} L_T$$

$$C^P_{{}^3D_{J+1}^{[8]}}(b_T;\mu) = -\frac{\alpha_s}{2\pi}\frac{8\,B_F}{3m_c^2} L_T$$

Echevarria, Romera, Taels, 2407.04793

with $L_T = \ln\left(\mu^2 b_T^2 e^{2\gamma_E}/4\right)$ and $N_{pol}^{(J)} = 2J+1$

Includes v-suppressed off-diagonal terms

$$\sum_{bc} d^{abc} d^{ebc} = 4 B_F \delta^{ae}$$

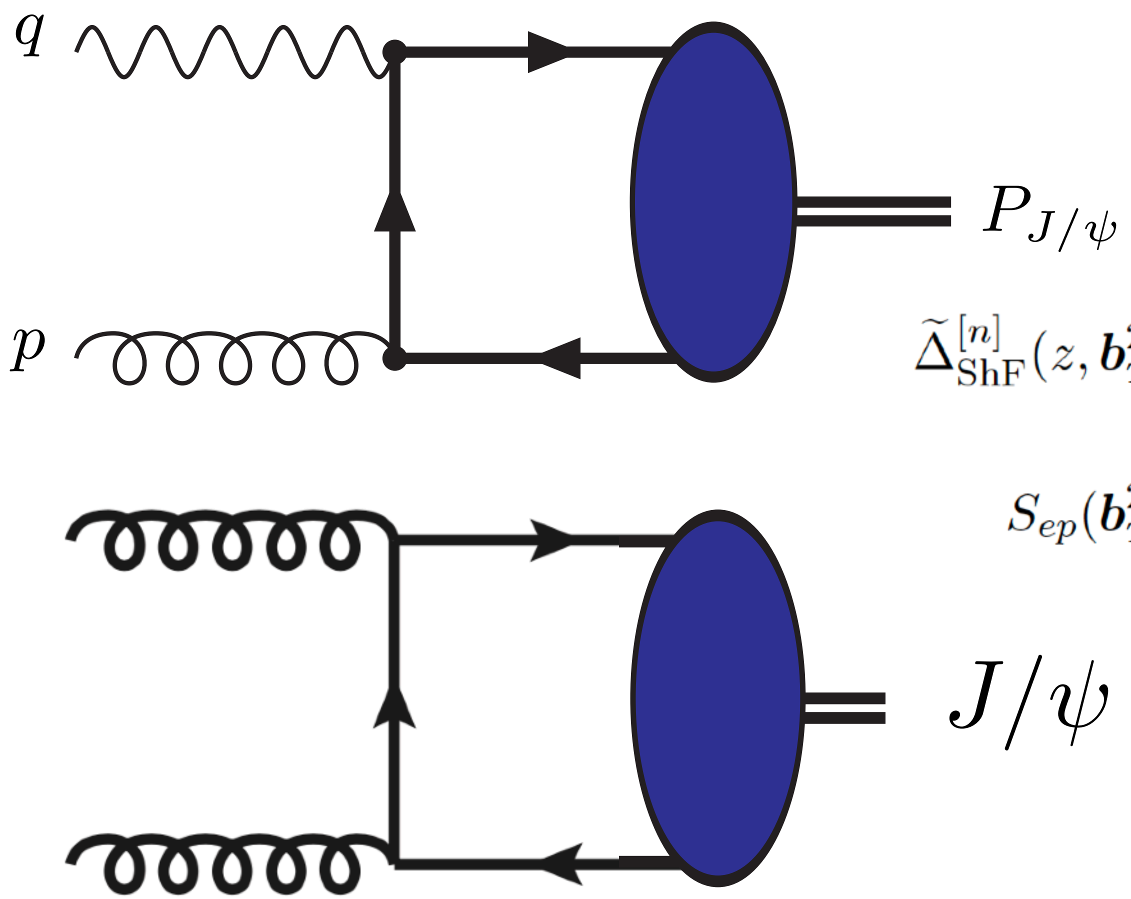
$$B_F = (N_c^2-4)/4N_c$$

$$C_A = N_c \qquad C_F = (N_c^2-1)/2N_c$$

Confirms: $\Delta^{[n]}(\boldsymbol{k}_T^2;\mu^2) \simeq \langle \mathcal{O}^{\mathcal{Q}}[n]\rangle\, \Delta(\boldsymbol{k}_T^2;\mu^2)$

$$\zeta_B \stackrel{?}{=} \frac{M_\psi^2 \mu_H^2}{\tilde{Q}^4} \qquad \tilde{Q}^2 = M_\psi^2 + Q^2$$

Generalized factorization expressions



Sun, C.-P. Yuan & F. Yuan, 2013

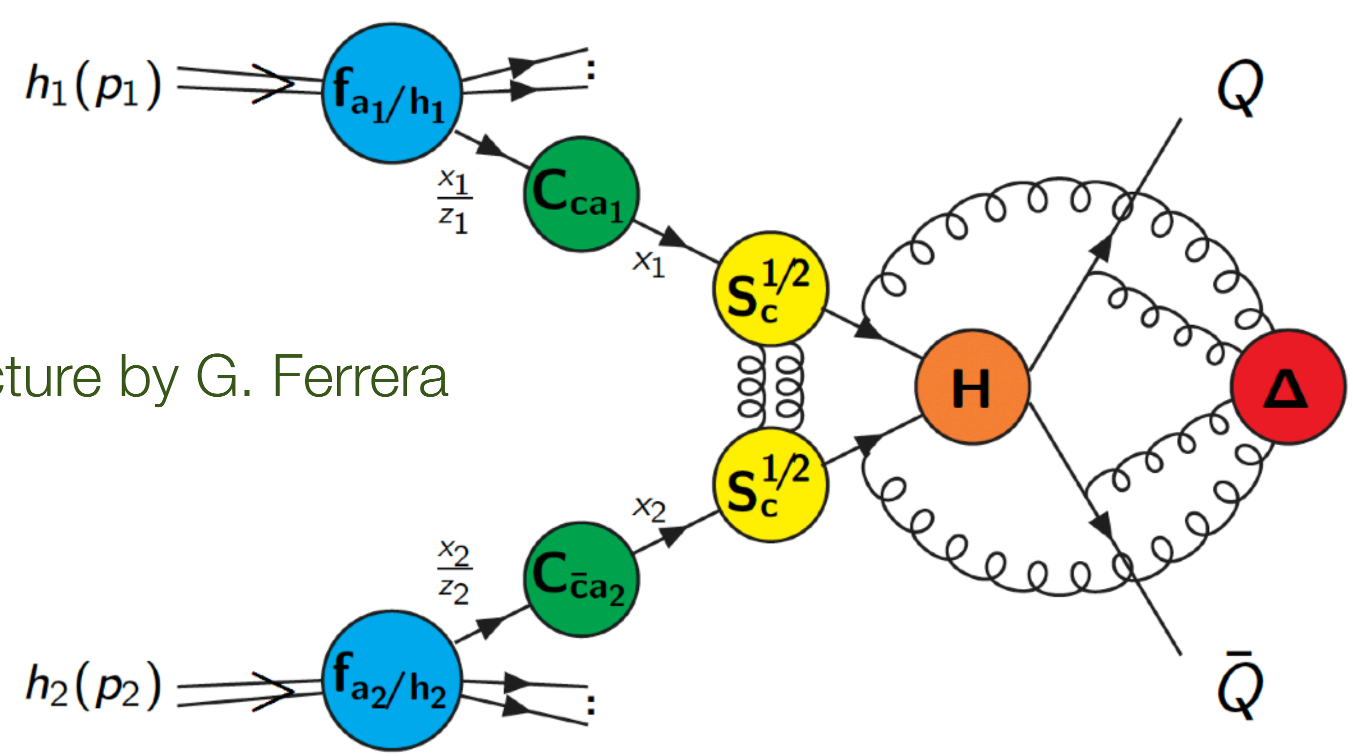
$$\Delta^{[n]}_{ep}(\mu_H^2) = \Delta^{[n]}_{\rm ShF}(\mu_H^2) \times S_{ep}(\mu_H^2)$$

$$\widetilde{\Delta}^{[n]}_{\rm ShF}(z, \boldsymbol{b}_T^2; \mu_H^2) = \frac{1}{2\pi}\left[1 + \frac{\alpha_s}{2\pi} C_A \left(1 + \log\frac{M_\psi^2}{\mu_H^2}\right)\log\frac{\mu_H^2}{\mu_b^2}\right]\langle\mathcal{O}[n]\rangle\delta(1-z)$$

$$S_{ep}(\boldsymbol{b}_T^2; \mu_H^2) = 1 + \frac{\alpha_s}{2\pi} C_A \left(2\log\frac{\mu_H^2}{M_\psi^2 + Q^2}\right)\log\frac{\mu_H^2}{\mu_b^2} .$$

$$\Delta^{[n]}_{pp}(\mu_H^2) = \Delta^{[n]}_{\rm ShF}(\mu_H^2) \times S_{pp}(\mu_H^2)$$

$$S_{pp}(\mu_H^2) = 1 + \frac{\alpha_s}{2\pi} C_A \left(3\log\frac{\mu_H^2}{M_\psi^2}\right)\log\frac{\mu^2}{\mu_b^2}$$



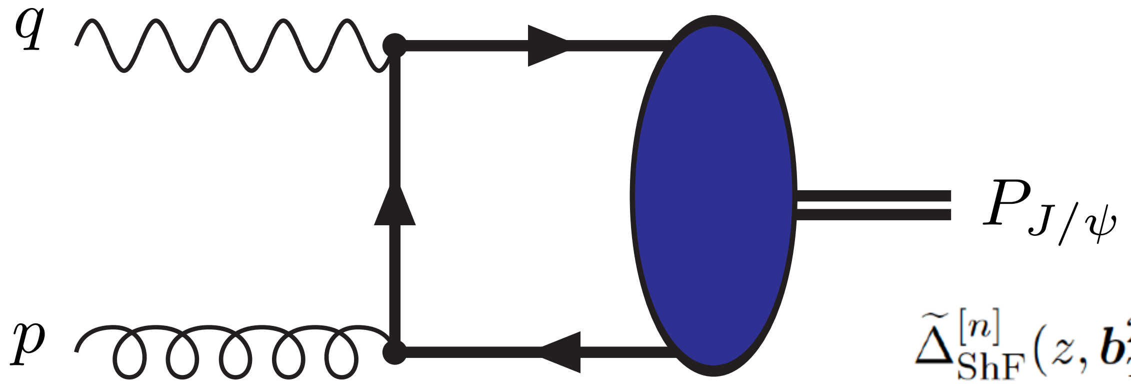
Picture by G. Ferrera

Additional process dependent factor
Similar to open heavy quark pair production

Catani, Grazzini, Torre, 2014

ep case: Zhu, Sun, Fuan, 2013; del Castillo, Echevarria, Makris, Scimemi, 2020

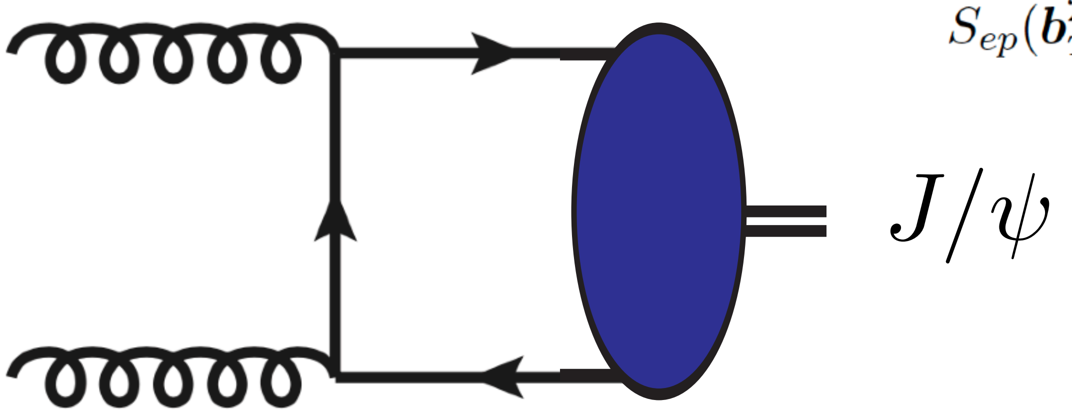
Generalized factorization expressions



$$\Delta^{[n]}_{ep}(\mu_H^2) = \Delta^{[n]}_{\rm ShF}(\mu_H^2) \times S_{ep}(\mu_H^2)$$

$$\widetilde{\Delta}^{[n]}_{\rm ShF}(z, \boldsymbol{b}_T^2; \mu_H^2) = \frac{1}{2\pi}\left[1 + \frac{\alpha_s}{2\pi} C_A \left(1 + \log \frac{M_\psi^2}{\mu_H^2}\right) \log \frac{\mu_H^2}{\mu_b^2}\right] \langle \mathcal{O}[n] \rangle \delta(1-z)$$

$$S_{ep}(\boldsymbol{b}_T^2; \mu_H^2) = 1 + \frac{\alpha_s}{2\pi} C_A \left(2 \log \frac{\mu_H^2}{M_\psi^2 + Q^2}\right) \log \frac{\mu_H^2}{\mu_b^2} \, .$$



$$\Delta^{[n]}_{pp}(\mu_H^2) = \Delta^{[n]}_{\rm ShF}(\mu_H^2) \times S_{pp}(\mu_H^2)$$

$$S_{pp}(\mu_H^2) = 1 + \frac{\alpha_s}{2\pi} C_A \left(3 \log \frac{\mu_H^2}{M_\psi^2}\right) \log \frac{\mu^2}{\mu_b^2}$$

Recipe to test the process dependence:

Determine $\Delta^{[n]}_{pp}(M_\psi^2) = \Delta^{[n]}_{\rm ShF}(M_\psi^2) (\neq \Delta^{[n]}_{ep}(M_\psi^2))$

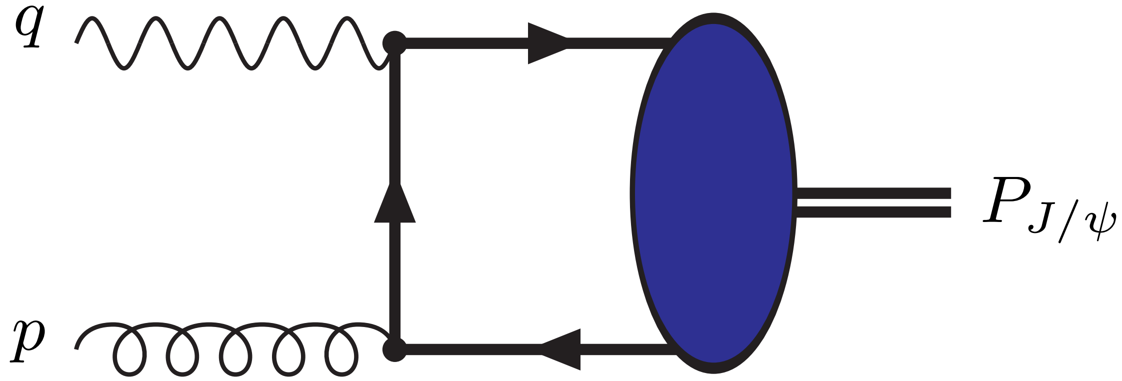
Evolve $\Delta^{[n]}_{\rm ShF}(M_\psi^2)$ to $\Delta^{[n]}_{\rm ShF}(\widetilde{Q}^2)$ = $\Delta^{[n]}_{ep}(\widetilde{Q}^2)$ $\quad \widetilde{Q}^2 = M_\psi^2 + Q^2$

Compare to experimental determination of $\Delta^{[n]}_{ep}(\widetilde{Q}^2)$

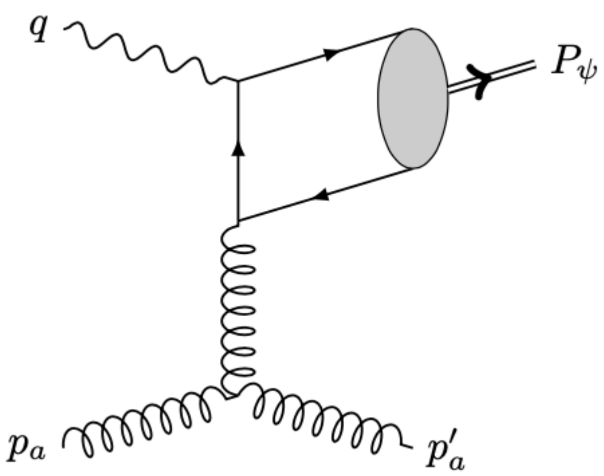
DB, Bor, Maxia, Pisano, Yuan, 2023

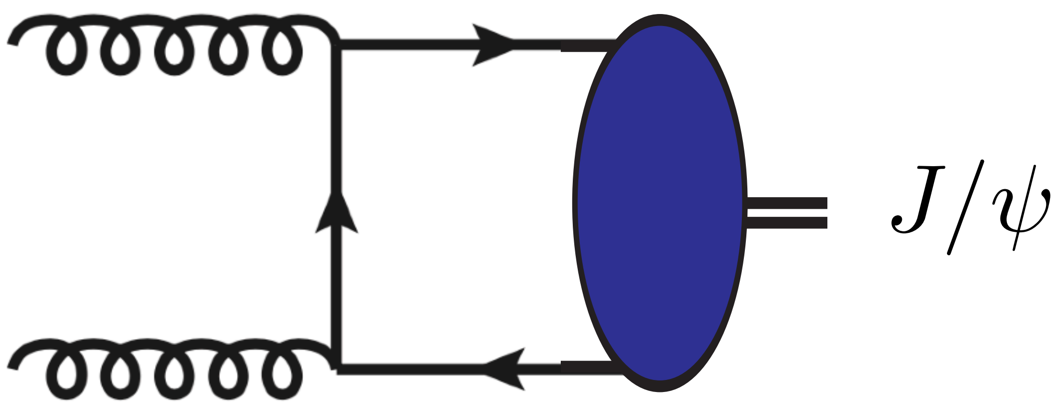
The challenge will be to do this test for CS and CO shape functions for the same quarkonium state

Factorization breaking



CO shape function





CO involves complicated gauge link structure in gluonic correlators & possibly factorization breaking

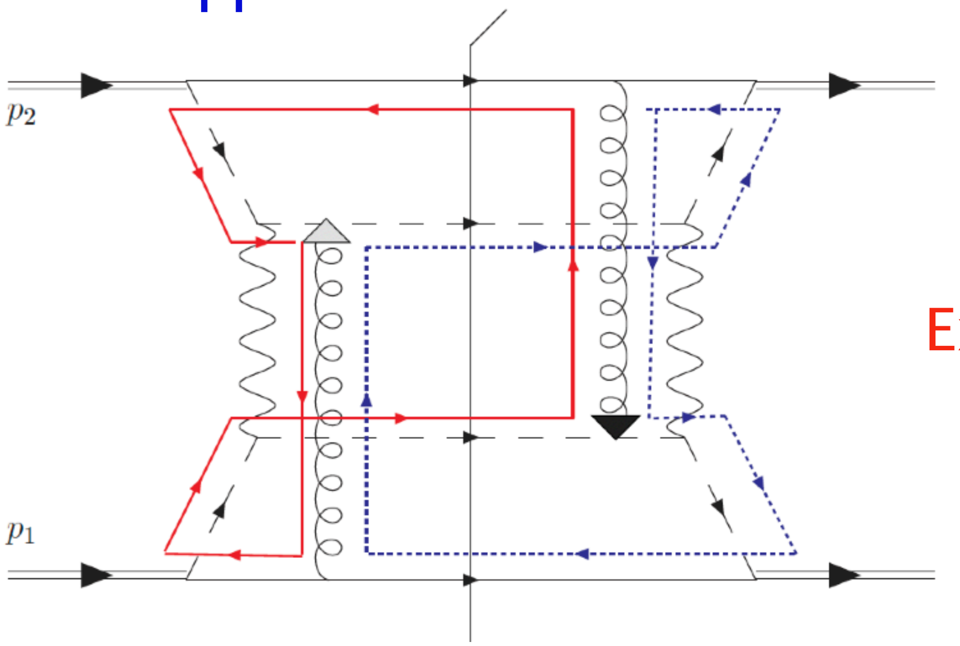
Single hard scale process (no Q^2), therefore, process dependence not easily recognized, but Sudakov factor different from CS quarkonium production

Sun, C.-P. Yuan & F. Yuan, 2013

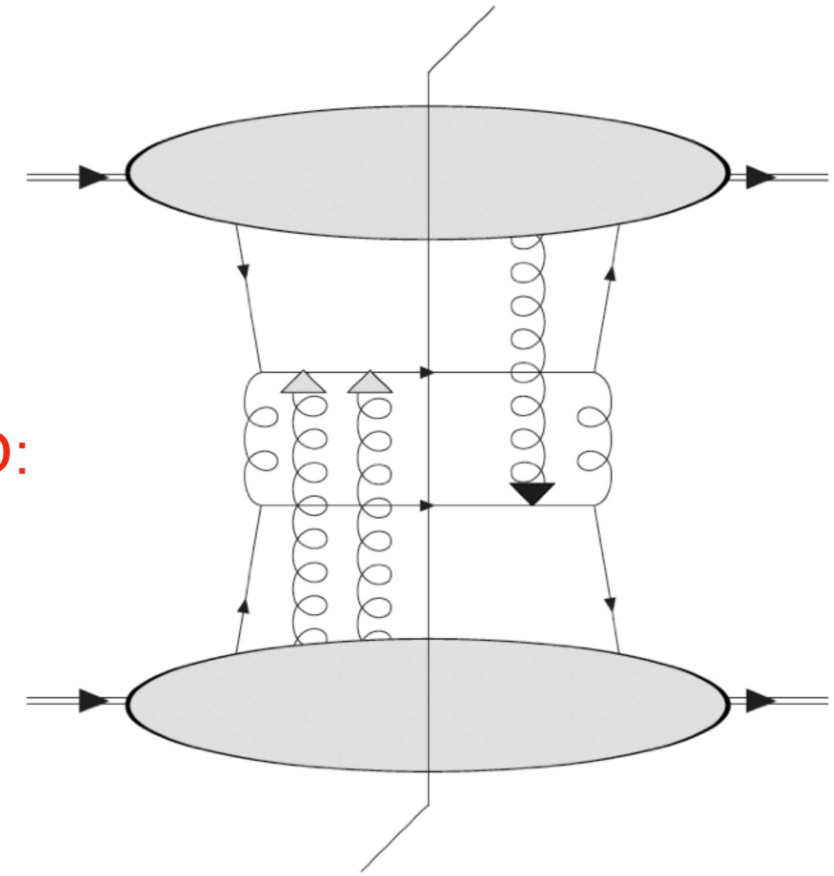
ep and pp collisions both needed to determine importance of factorization breaking effects and of process dependence

Factorization breaking

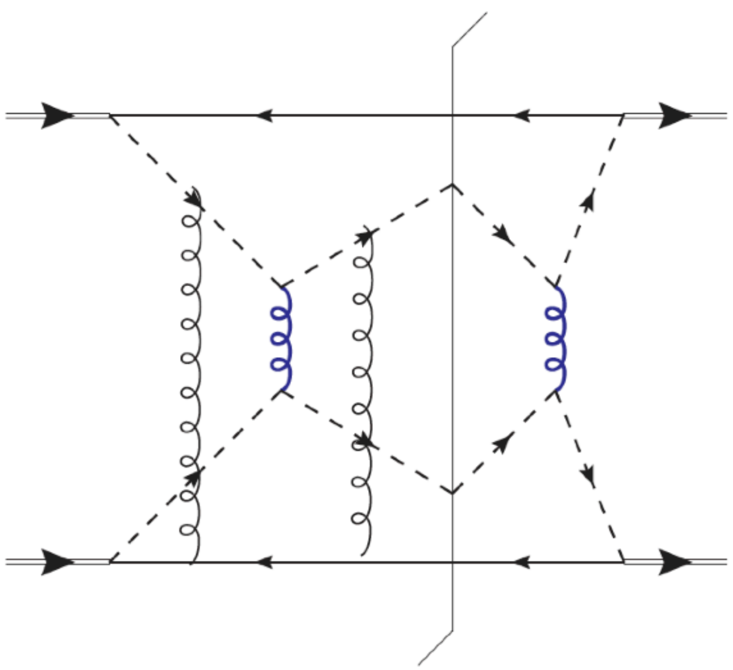
Nonfactorizing diagrams in pp→jet jet X due to gauge links
Also for pp→h h’ X and for CO-CO in di-J/ψ production

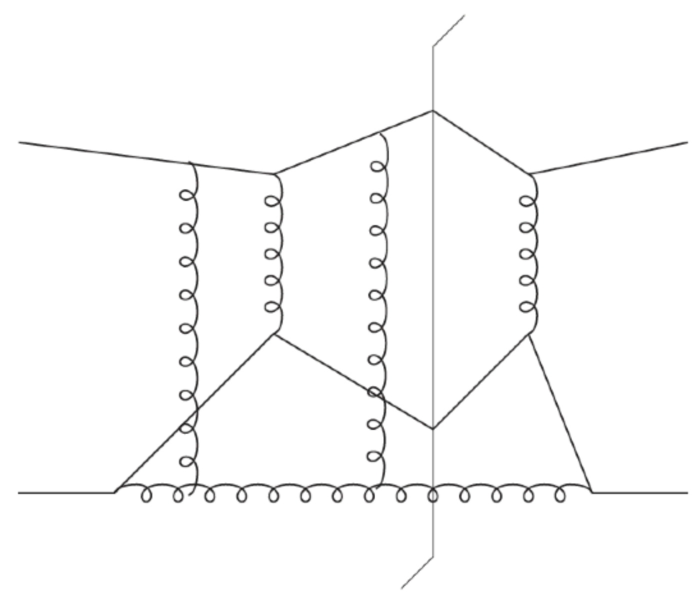


Example diagram in QCD:



Rogers, Mulders, 2010

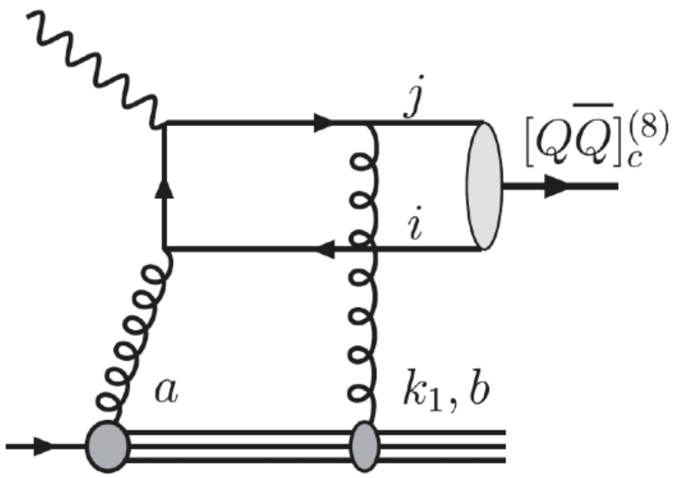


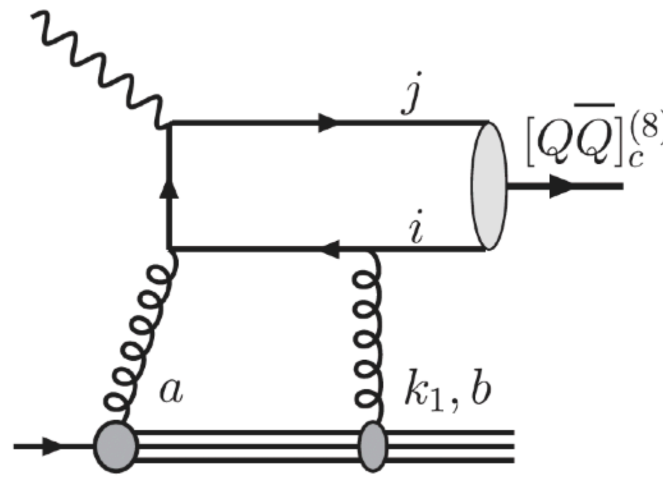


Collins, Qiu, 2007
Collins, 2007

No proof of factorization breaking for the sum over diagrams of this order in QCD
Even if it leads to factorization breaking, the importance/magnitude of it is unknown
Experimental data must be used to show this (like done for diffractive PDFs)

Effects of ISI & FSI



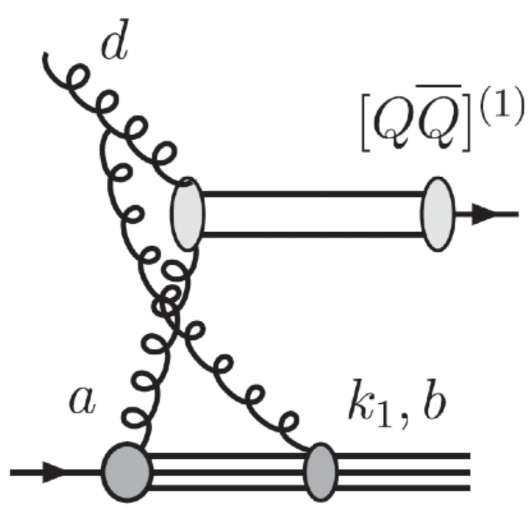


SSA from gluon Sivers

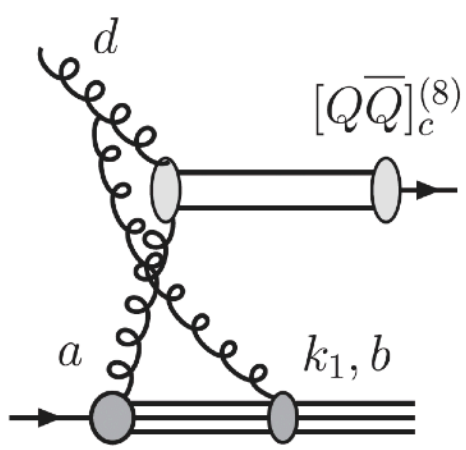
Nonzero SSA only for CO in ep

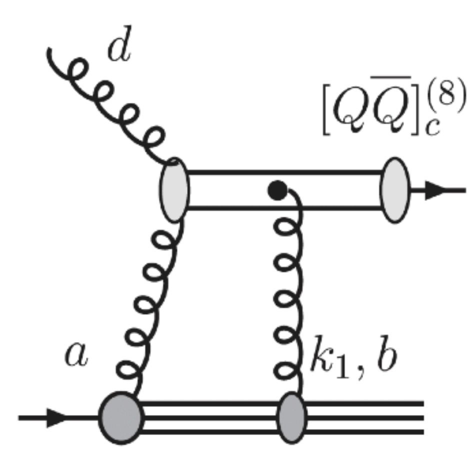
But not at twist-3 (large p_T)
cf. Shinsuke Yoshida's talk

F. Yuan, 2008



Nonzero SSA only for CS in pp





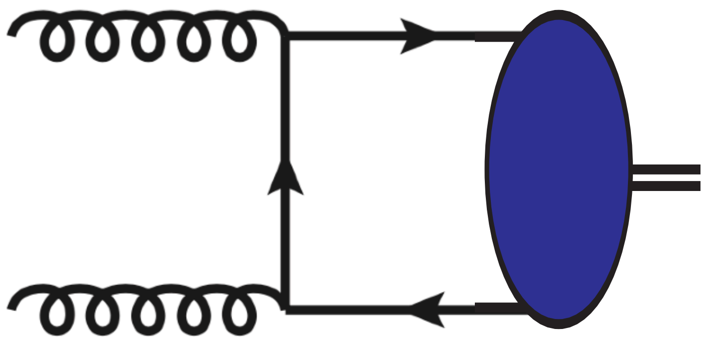
Zero SSA for CO in pp

Interesting to check to what extent this type of process dependence is correct: synergy with polarized FT experiments at LHC

TMD evolution studies

$\chi_{c,b}$ production

$pp \to \chi_Q X$



$\chi_{c0}\,,\,\chi_{b0}$

In LO NRQCD the differential cross sections in pp and pA are:

DB, Pisano, 2012

$$\frac{d\sigma(\eta_Q)}{dy\, d^2\boldsymbol{q}_T} = \frac{2}{9}\,\frac{\pi^3\alpha_s^2}{M^3\, s}\,\langle 0|\mathcal{O}_1^{\eta_Q}({}^1S_0)|0\rangle\,\mathcal{C}\,[f_1^g\, f_1^g]\,\left[1 - R(\boldsymbol{q}_T^2)\right]$$

$$\frac{d\sigma(\chi_{Q0})}{dy\, d^2\boldsymbol{q}_T} = \frac{8}{3}\,\frac{\pi^3\alpha_s^2}{M^5\, s}\,\langle 0|\mathcal{O}_1^{\chi_{Q0}}({}^3P_0)|0\rangle\,\mathcal{C}\,[f_1^g\, f_1^g]\,\left[1 + R(\boldsymbol{q}_T^2)\right]$$

$$\frac{d\sigma(\chi_{Q2})}{dy\, d^2\boldsymbol{q}_T} = \frac{32}{9}\,\frac{\pi^3\alpha_s^2}{M^5\, s}\,\langle 0|\mathcal{O}_1^{\chi_{Q2}}({}^3P_2)|0\rangle\,\mathcal{C}\,[f_1^g\, f_1^g]$$

$$R(\boldsymbol{q}_T^2) \equiv \frac{\mathcal{C}[w h_1^{\perp g} h_1^{\perp g}]}{\mathcal{C}[f_1^g f_1^g]}$$

χ_{QJ} LDMEs are order v^2 w.r.t. η_Q

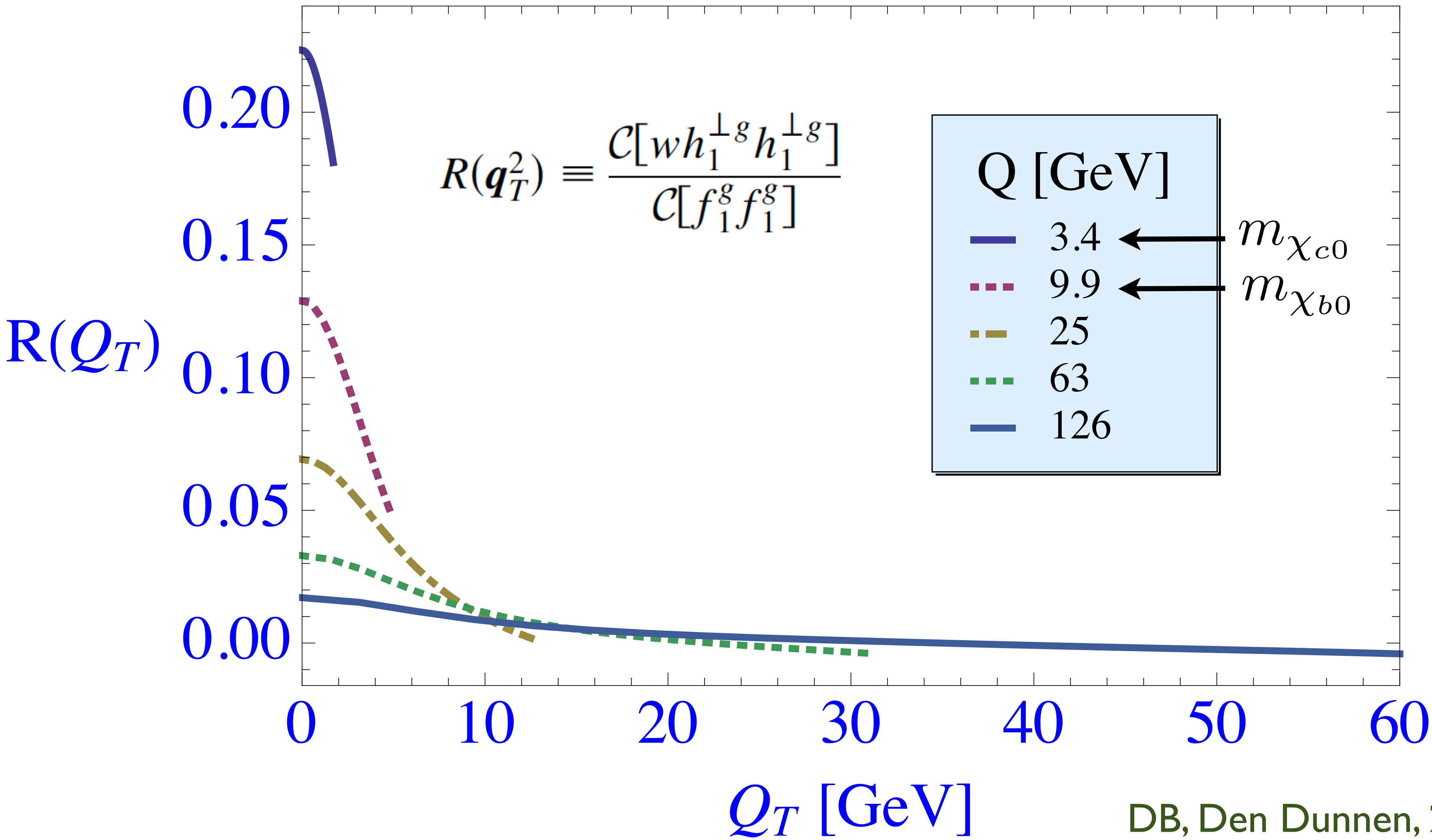
For χ_{Q1} there is no contribution due to Landau-Yang theorem

Comparing $pp \to H\, X$, where $H = \chi_{c0}$, χ_{b0} or Higgs allows to test TMD evolution

Effect of TMD evolution

Comparing $pp \to H\,X$, where $H = \chi_{c0}$, χ_{b0} or Higgs allows to test TMD evolution

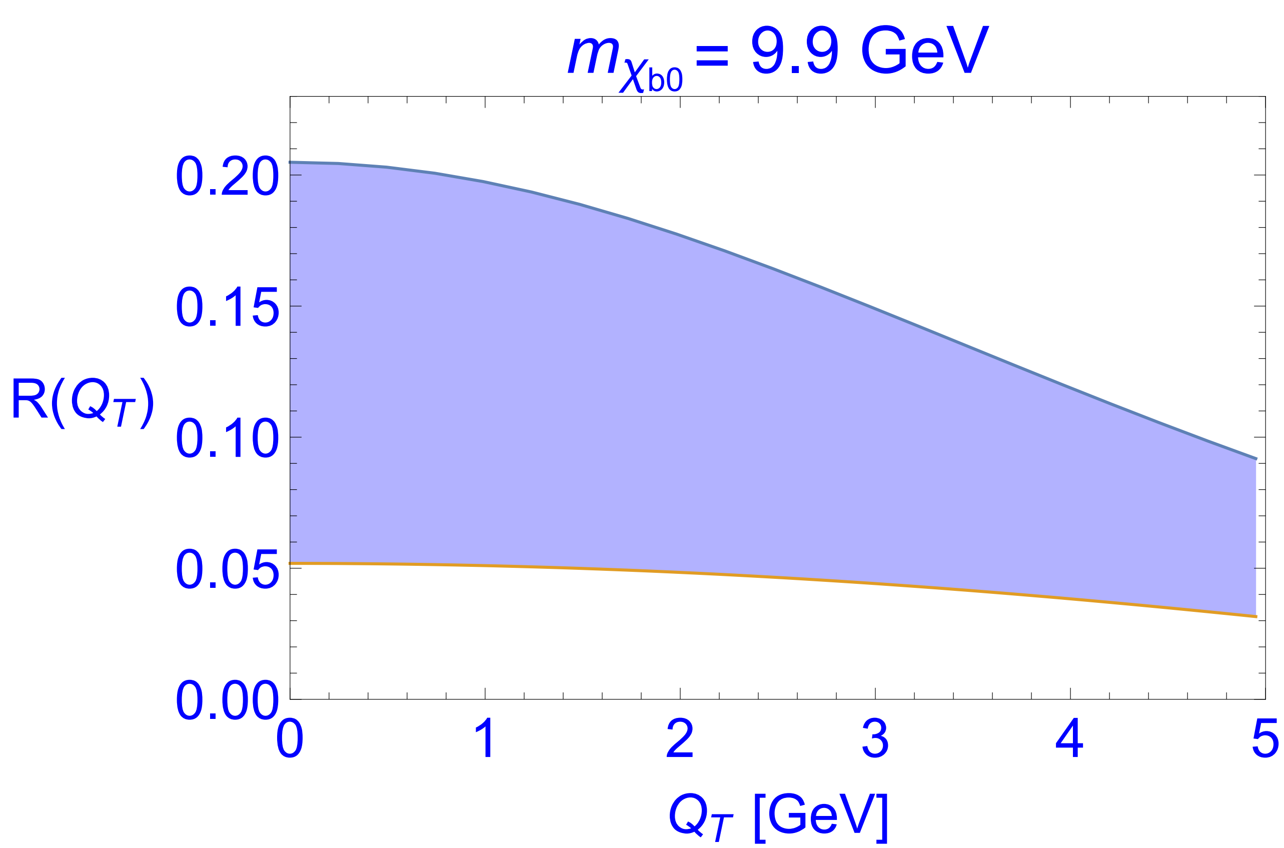
The relative contribution from linearly polarized gluons w.r.t. unpolarized gluons decreases with increasing mass of the produced state (which sets the hard scale):



DB, Den Dunnen, 2014

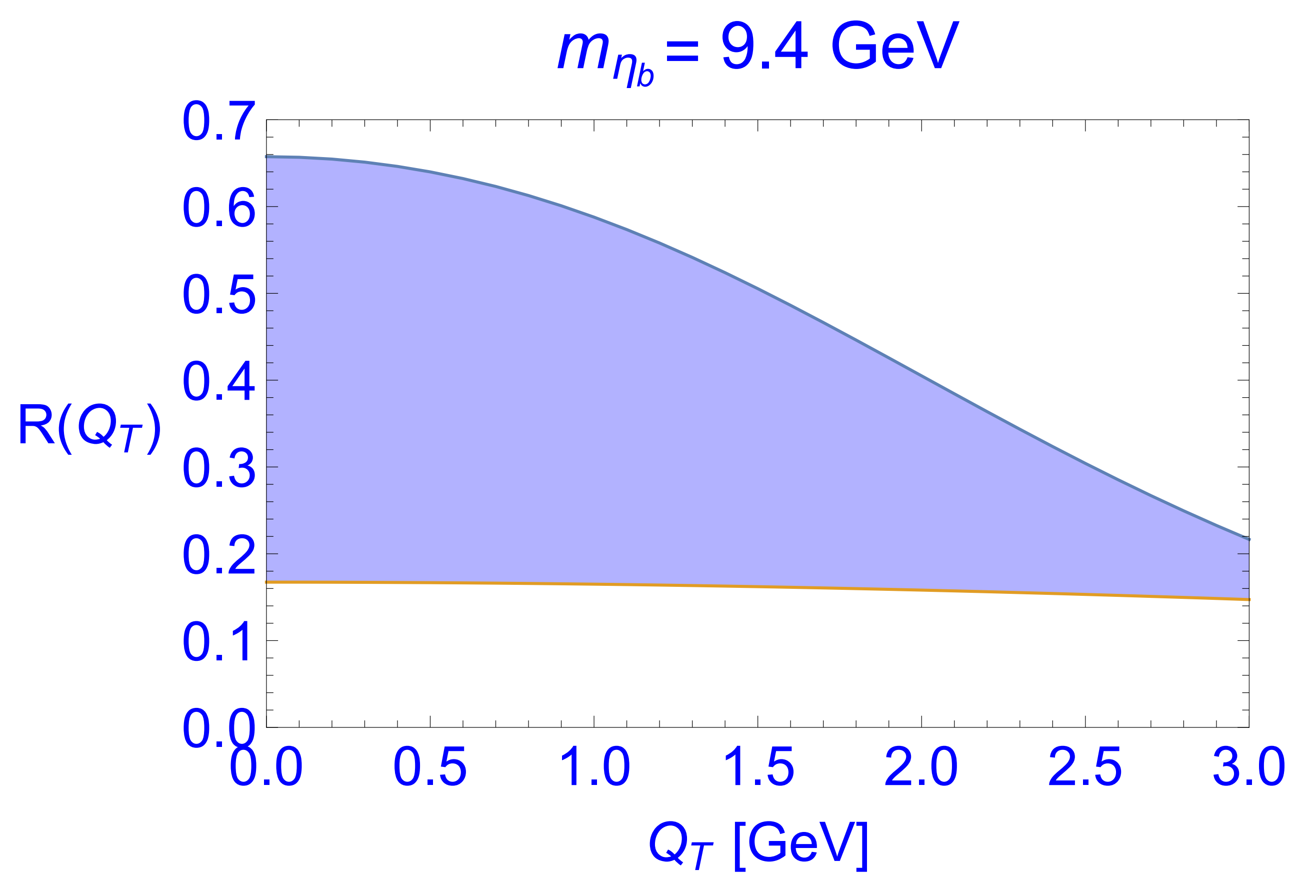
Bottomonium production in pp

The range of predictions for C-even (pseudo-)scalar bottomonium production:



DB, Den Dunnen, 2014

Variation of nonperturbative input for the TMDs and treatment of the very small b region (b<1/Q)

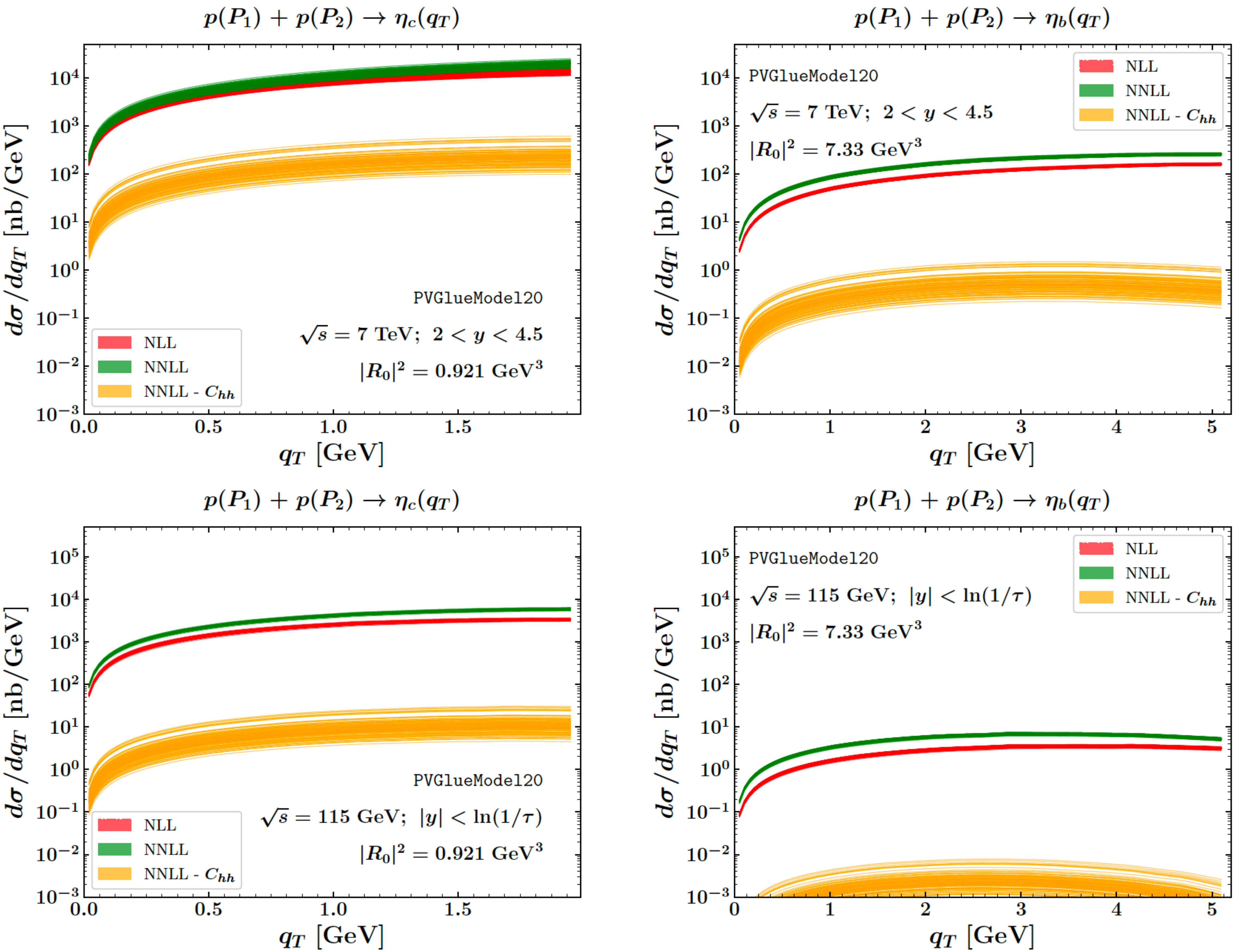


Echevarria, Kasemets, Mulders, Pisano, 2015

Variation of the nonperturbative Sudakov factor and the renormalization scale

Very large theoretical uncertainties, even more so for charmonium production, but contribution of 20% or more can be expected

$pp \to \eta_{c,b}\, X$



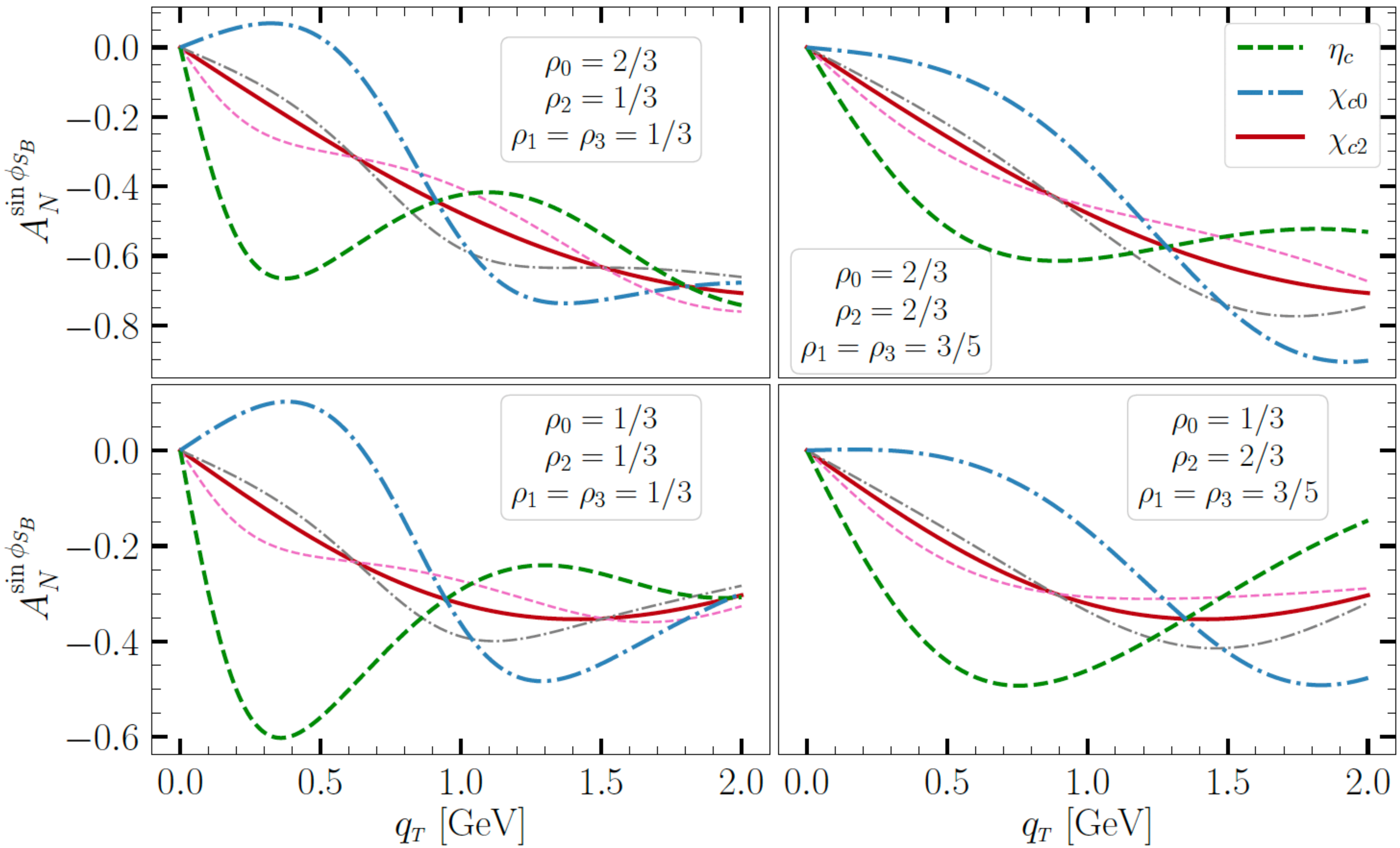
Bacchetta, Celiberto, Radici, Signori, 2022

$p^{\uparrow}p \to \eta_{c,b}\, X$

$$F_{TU}^{\eta_Q,\sin\phi_{S_A}} = H^{\eta_Q}\left(\mathcal{C}\left[w_{TU}^{f}\, f_1^{g} f_{1T}^{\perp g}\right] - \mathcal{C}\left[w_{TU}^{h}\, h_1^{\perp g} h_1^{g}\right] + \mathcal{C}\left[w_{TU}^{h^\perp}\, h_1^{\perp g} h_{1T}^{\perp g}\right]\right) \langle 0|\mathcal{O}_1^{\eta_Q}({}^1S_0)|0\rangle\,,$$

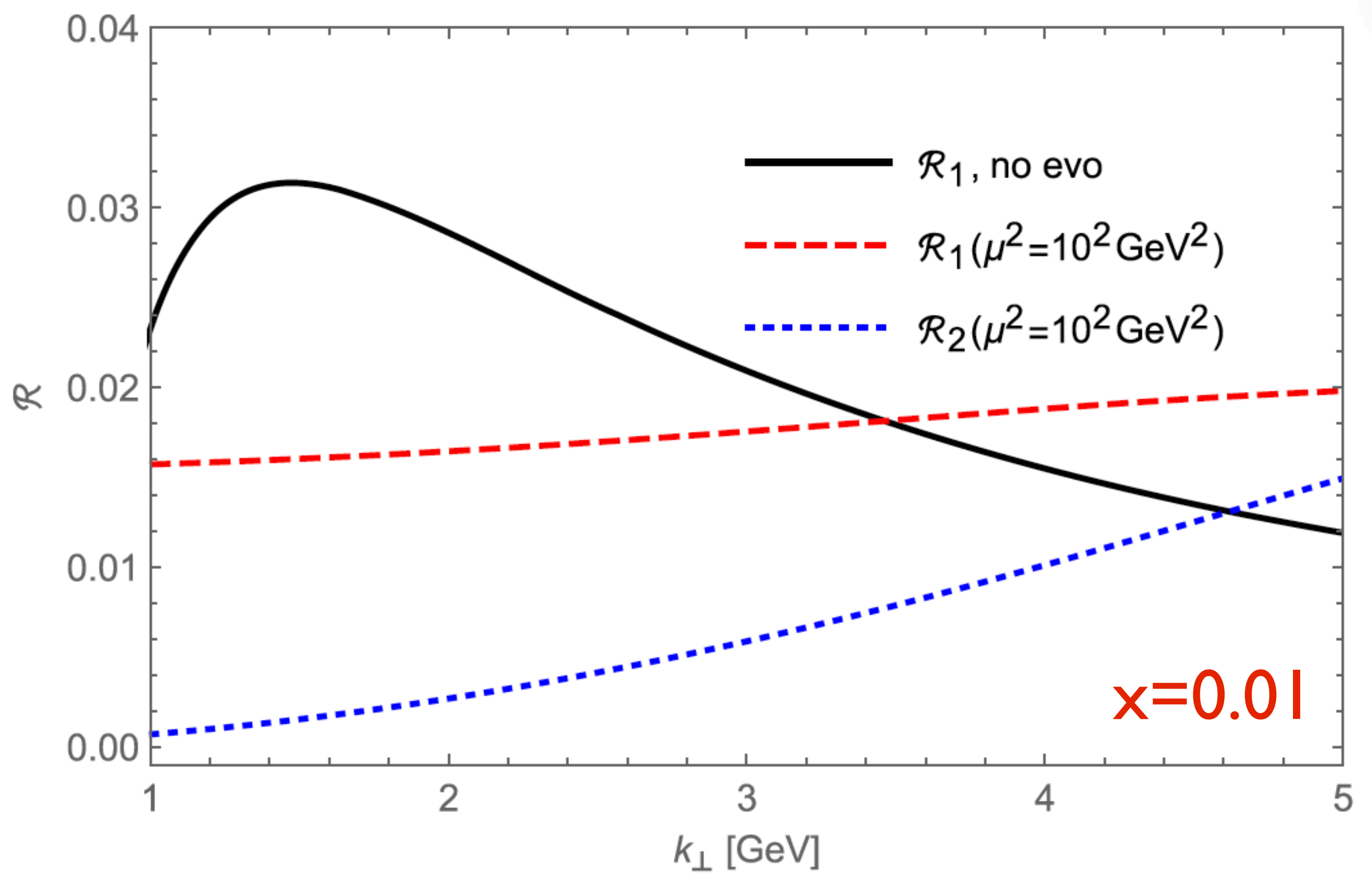
$$F_{TU}^{\chi_{Q0},\sin\phi_{S_A}} = H^{\chi_{Q0}}\left(\mathcal{C}\left[w_{TU}^{f}\, f_1^{g} f_{1T}^{\perp g}\right] + \mathcal{C}\left[w_{TU}^{h}\, h_1^{\perp g} h_1^{g}\right] - \mathcal{C}\left[w_{TU}^{h^\perp}\, h_1^{\perp g} h_{1T}^{\perp g}\right]\right) \langle 0|\mathcal{O}_1^{\chi_{Q0}}({}^3P_0)|0\rangle\,,$$

$$F_{TU}^{\chi_{Q2},\sin\phi_{S_A}} = H^{\chi_{Q2}}\,\mathcal{C}\left[w_{TU}^{f}\, f_1^{g} f_{1T}^{\perp g}\right] \langle 0|\mathcal{O}_1^{\chi_{Q2}}({}^3P_2)|0\rangle\,,$$



Kato, Maxia, Pisano, 2024

T-odd gluon TMDs at small x - scale evolution



$$\mathcal{R}_1(\mu^2) = \frac{x f_{1T}^{\perp g}(\mu^2)}{x f_1^g(\mu^2)}$$

$$\mathcal{R}_2(\mu^2) = \frac{-\frac{\mathbf{k}_\perp^2}{2M^2} x h_{1T}^{\perp g}(\mu^2)}{x f_1^g(\mu^2)}$$

Initial scale 1.6 GeV

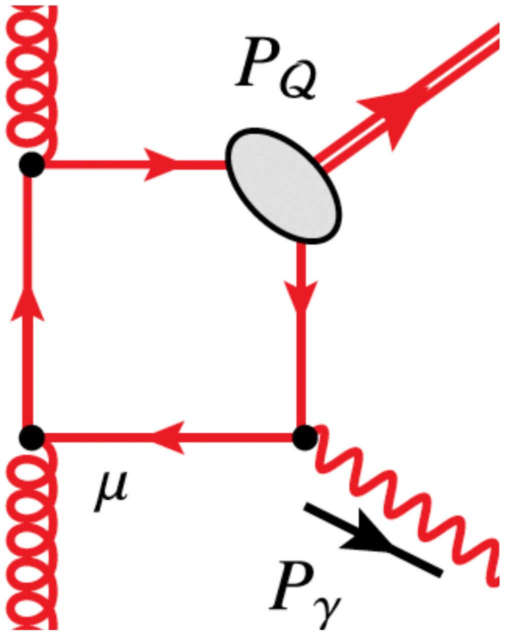
DB, Hagiwara, Jian Zhou & Ya-Jin Zhou, 2022

Scale evolution does not preserve the small-x equality of T-odd dipole gluon TMDs

Comparison of $p^\uparrow p \rightarrow \eta_c X$ and $p^\uparrow p \rightarrow \eta_b X$ can test this (for WW gluons TMDs)

At polarized FT experiment at LHC?

TMD regime in pp collisions

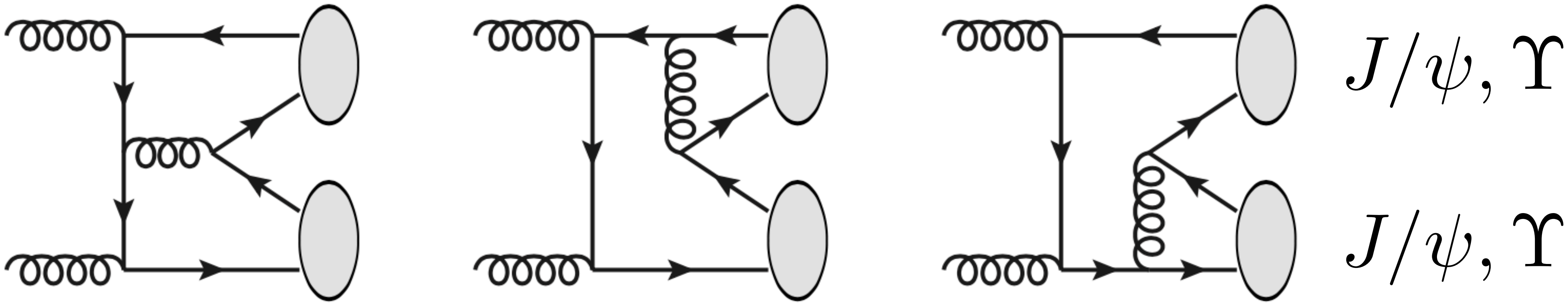


Associated production of J/ψ with a photon, both with large pT, but their sum needs to be small

A good probe of the gluon TMD and its evolution

Den Dunnen, Lansberg, Pisano, Schlegel, 2014

Works in double J/ψ production too:

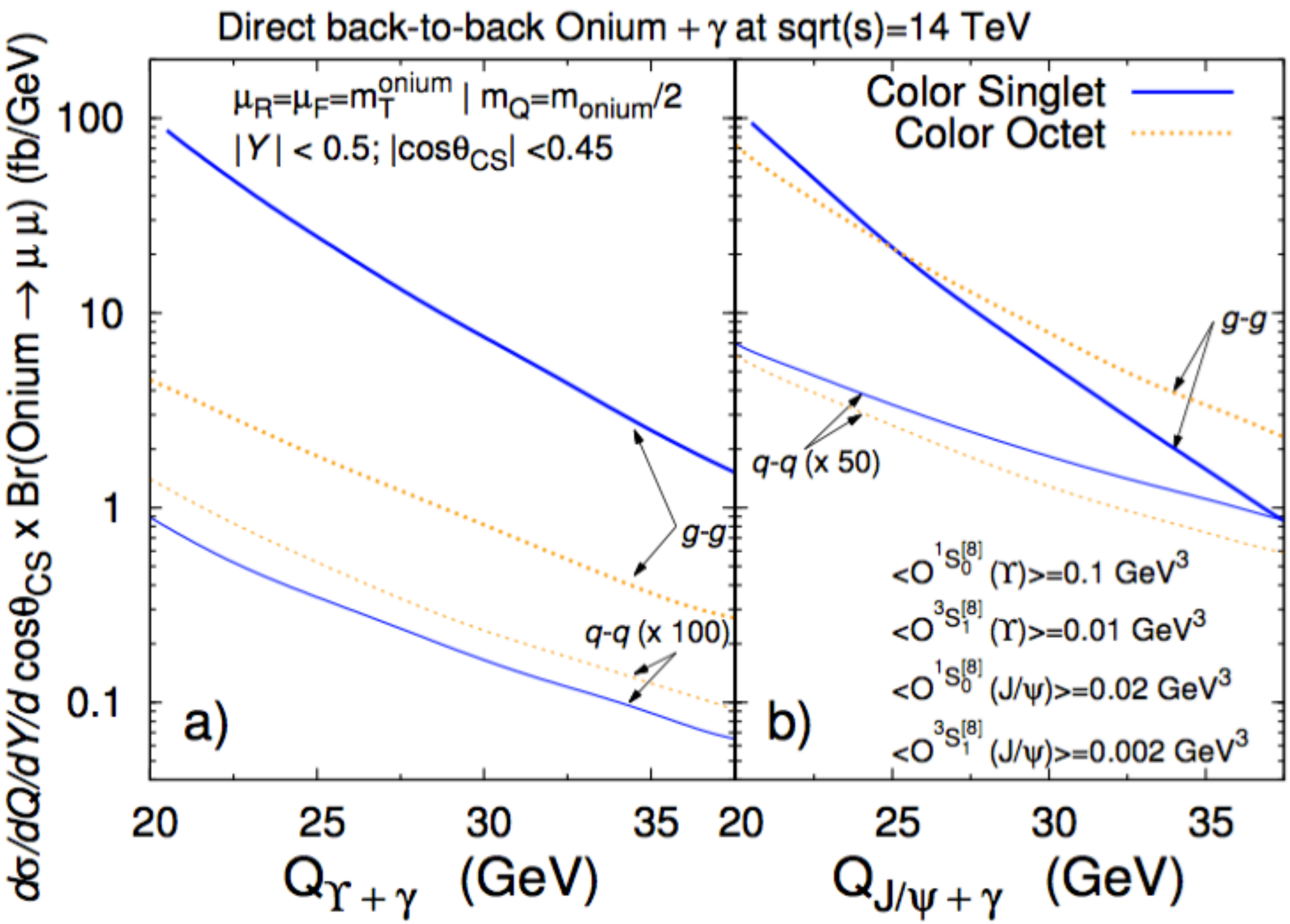


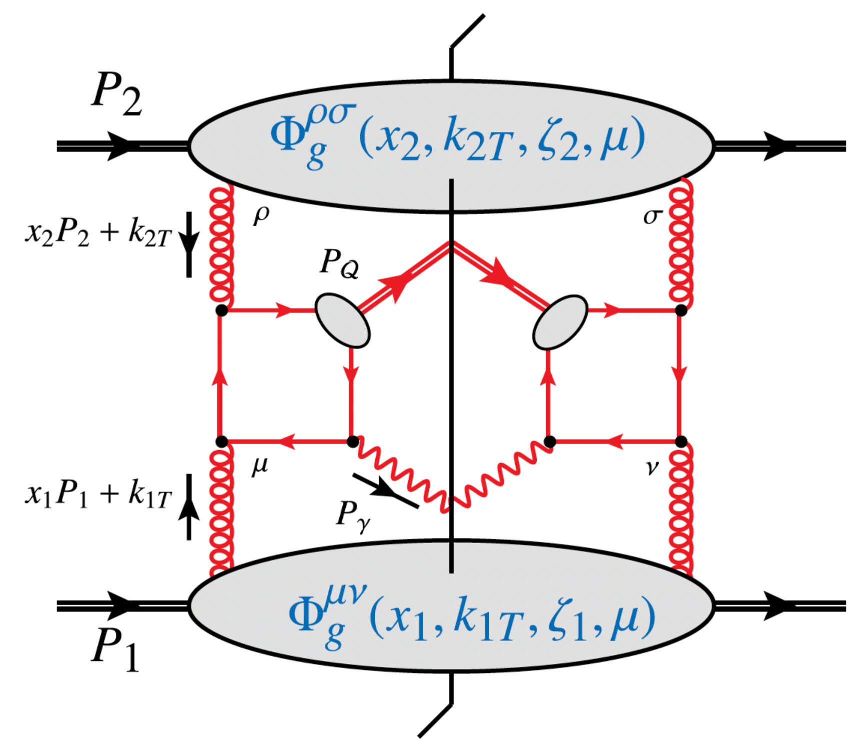
CS–CS ≫ CO–CO

Lansberg, Pisano, Scarpa, Schlegel, 2018

Associated J/ψ production

$p\,p \to \mathcal{Q}\,\gamma\,X$ could be a good process to extract $f_1^g(x, p_T^2)$ at LHC

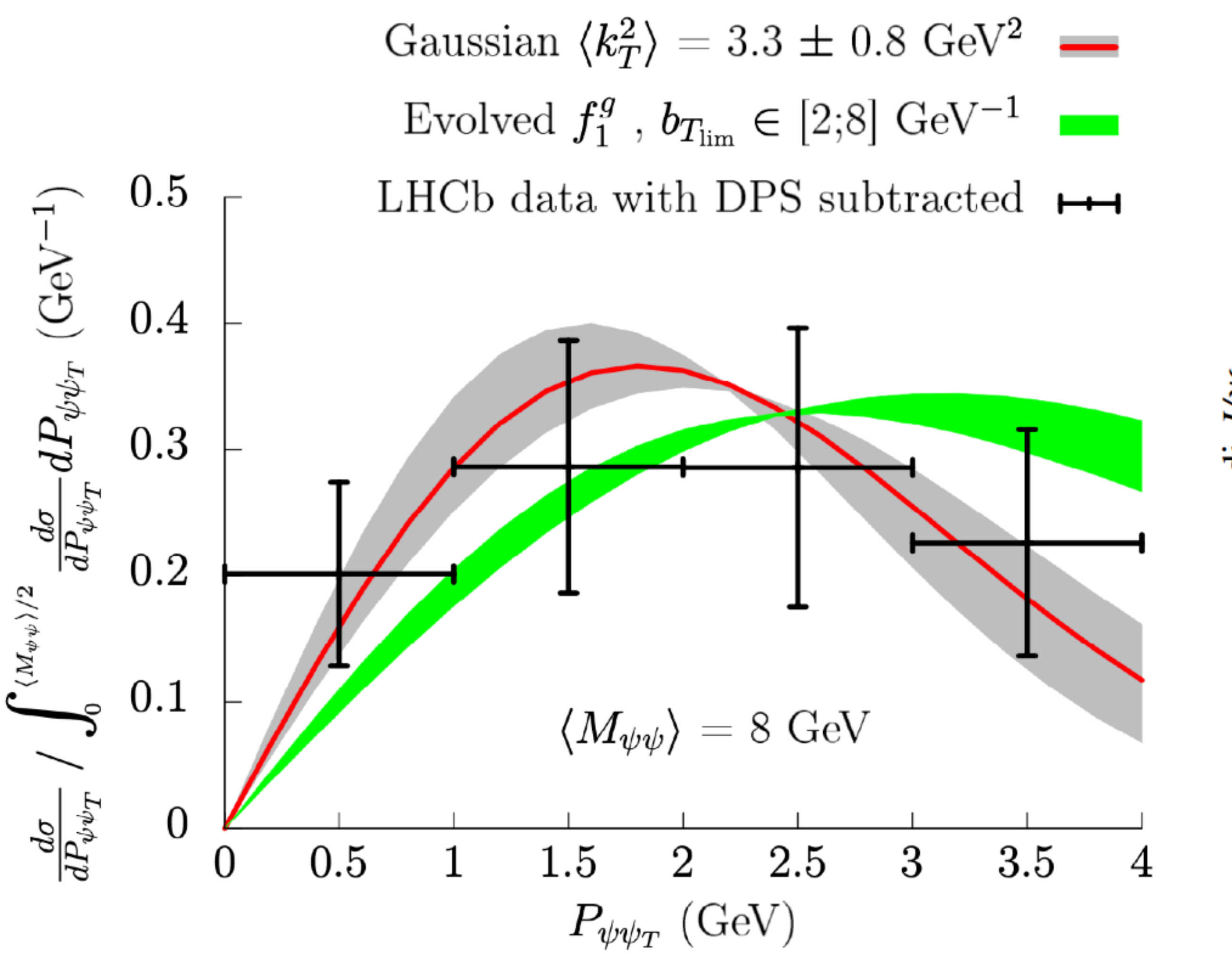




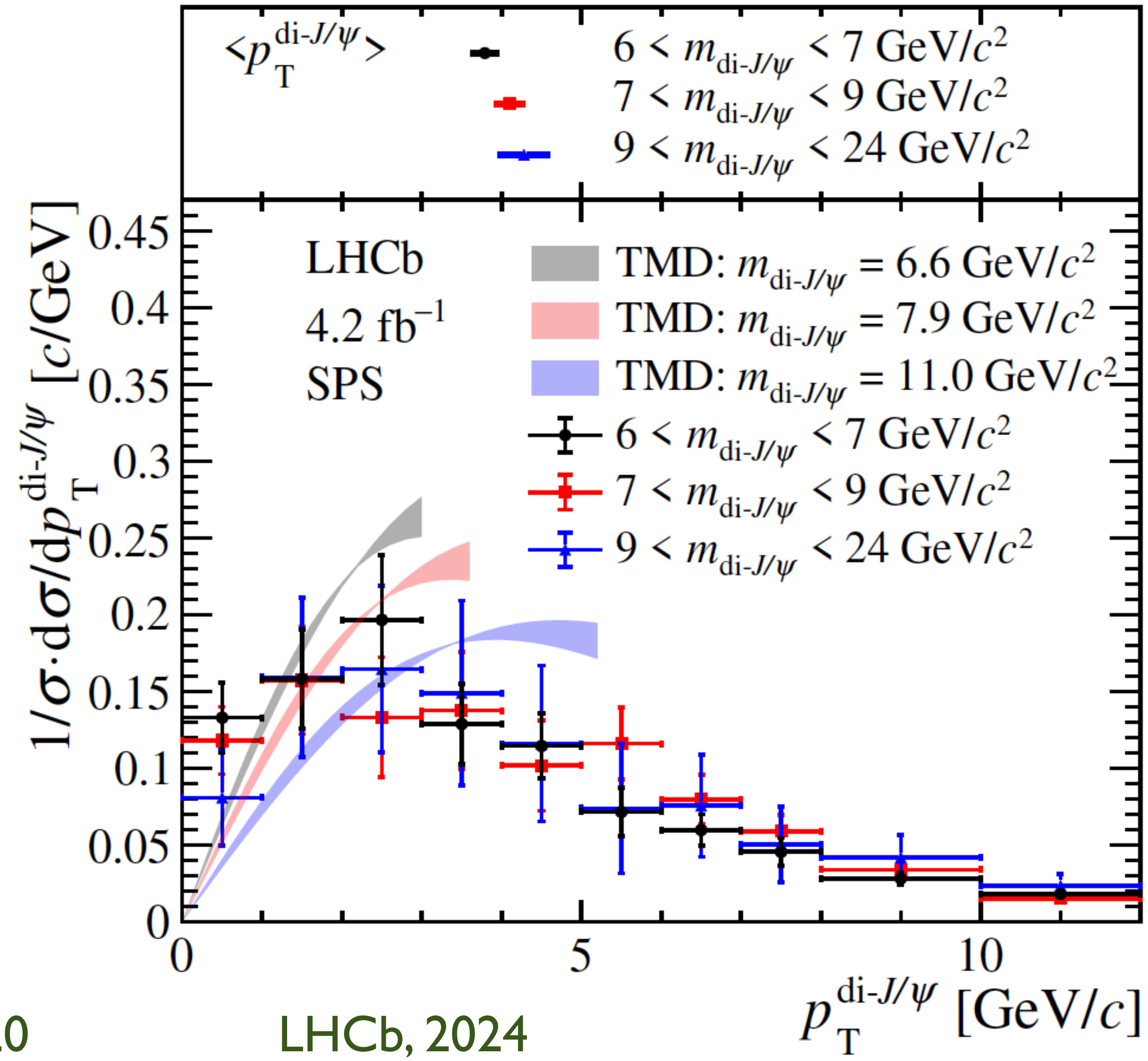
Den Dunnen, Lansberg, Pisano, Schlegel, 2014

The CS contribution dominates in ϒ+γ production and for lower invariant mass of the pair also in J/ψ+γ production

J/ψ pair production



Scarpa, DB, Echevarria, Lansberg, Pisano, Schlegel, 2020



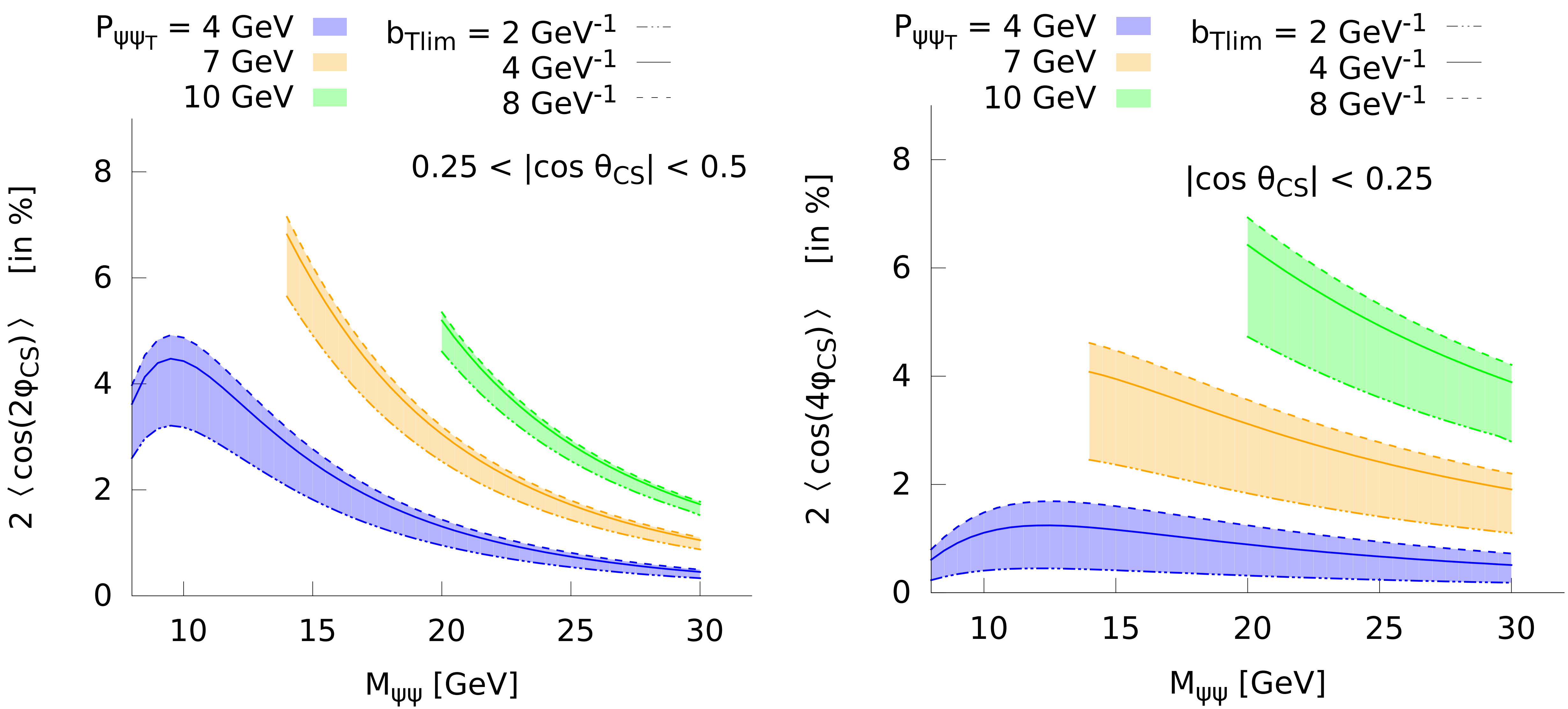
LHCb, 2024

J/ψ pair invariant mass allows to study TMD evolution

The shape of this normalized (DPS subtracted) distribution and its scale evolution is not fully described by the TMD description (also not within uncertainties from nonperturbative physics) [soon to be updated, see Jelle Bor's talk]

Linear gluon polarization in di-J/Ψ production

$h_1^{\perp g}$ can be probed through angular modulations in $p\,p \to J/\psi\, J/\psi\, X$



Estimated to lead to 1-5% level azimuthal modulations at LHC (incl. TMD evolution)

Scarpa, DB, Echevarria, Lansberg, Pisano, Schlegel, 2019

CO contributions estimated to be below the percent level, except at large Δy

Conclusions

Conclusions

- Quarkonium production in both pp and ep collisions are well suited to study gluon TMDs, but can also offer new opportunities to learn about the quarkonium production mechanism itself

- In the TMD regime CO LDMEs can be extracted by exploiting the quarkonium polarization, linear gluon polarization or open heavy quark pair production

- In TMD regime polarization of quarkonia produced in ep collisions not firmly predicted

- Generally, predictions are still quite uncertain because of many factors: unknown nonperturbative Sudakov factors, scale uncertainty, poorly known LDMEs or unknown shape functions

- Process dependence, factorization breaking, and TMD evolution all need to be studied more extensively by comparing ep and pp observables

- Lots of synergies of EIC & LHC regarding TMD studies, include SSAs in a polarized FT experiment at LHC