“Non-J/ψ” photoproduction: theory overview

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This project is supported in parts by the European Union’s Marie Sklodowska Curie action “RadCor4HEF” (grant agreement No. 101065263) and by the European Union’s Horizon 2020 research and innovation programme under Grant agreement no. 824093

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[1] IJClab, Orsay

Motivations (I): understanding hadronisation

Description of production of any high-$p_T(\gg \Lambda_{\rm QCD})$ hadrons in QCD = (perturbative) production of quarks/gluons + *hadronisation.*

1. For light and heavy-light hadrons, hadronisation is studied phenomenologically:
   - Fragmantation Functions: based on factorisation theorems, fitted to describe data (first attempts to compute on the lattice)
   - Monte-Carlo models: hard to derive from QCD Lagrangian (string-based in Pythia, cluster hadronisation in Herwig,...)
2. Quarkonia – "Hydrogen atoms of QCD" ⇒ corrections to the "naive" quark model should be suppressed by powers of relative velocity (v) of heavy quarks in the bound state:

$$\begin{aligned} |J/\psi\rangle &= O(1)\left|c\bar{c}\left[{}^3S_1^{(1)}\right]\right\rangle + O(v)\left|c\bar{c}\left[{}^3P_J^{(8)}\right]+g\right\rangle \\ &+ O(v^{3/2})\left|c\bar{c}\left[{}^1S_0^{(8)}\right]+g\right\rangle + O(v^2)\left|c\bar{c}\left[{}^3S_1^{(8)}\right]+gg\right\rangle + \ldots, \end{aligned}$$

3. ⇒ let's try to use understand production of quarkonia. **This understanding will be a small-v limit for any future theory of hadronisation!**

Motivations (II): quarkonia as tools

If hadronisation mechanism **was well understood,** *then quarkonium production would be:*

1. An excellent tool to study gluon content of a proton/nucleus:
    - Small (or negligible) “valence” c and b content – production predominantly through coupling to gluons at high energies
    - Clean experimental signatures for J/ψ, $\Upsilon(nS)$, ...
    - relatively small $M_{J/\psi} \simeq 3GeV$ – *access to very small* $x \sim Me^{-y}/\sqrt{s} \sim 10^{-4} - 10^{-6}$ at the LHC.
2. A tool to study double/multiple parton scattering: due to significant cross sections of multiple/associated production and lower p_T/scales in comparison to vector bosons/jets
3. A probe for QGP: melting/recombination/parton energy loss could be studied
4. A tool to study of c-Higgs and b-Higgs couplings through associated production and Higgs decays
5. ...

Quarkonium production models

Unfortunately no existing model can describe all data on inclusive quarkonium hadro/photo/electro/e^+e^- production and polarisation observables.

Old ideas:

1. Colour Singlet Model: only **colour-singlet** $Q\bar{Q}$ pairs with the same orbital momentum/spin as corresponding potential-model state hadronise to the quarkonium.
2. NRQCD factorisation: based on the hierarchy of different colour/orbital momentum/spin states of the $Q\bar{Q}$-pair in the v-expansion for the quarkonium state
3. (Improved) Colour Evaporation Model assumes "democracy" of colour/orbital momentum/spin states of the $Q\bar{Q}$-pair

New ideas: Potential NRQCD, Soft-gluon factorisation, Shape-functions, ...

Motivation for new ideas:

- reduction of the numeber of free parameters
- improvement of perturbative convergence
- phenomenological problems

Non-relativistic QCD

The velocity-expansion for quarkonium eigenstate is a copy of corresponding arguments from atomic physics:

$$\begin{aligned} |J/\psi\rangle &= O(1)\left|c\bar{c}\left[{}^3S_1^{(1)}\right]\right\rangle + O(v)\left|c\bar{c}\left[{}^3P_J^{(8)}\right]+g\right\rangle \\ &+ O(v^{3/2})\left|c\bar{c}\left[{}^1S_0^{(8)}\right]+g\right\rangle + O(v^2)\left|c\bar{c}\left[{}^3S_1^{(8)}\right]+gg\right\rangle + \dots, \end{aligned}$$

for validity of this arguments, we should work in *non-relativistic EFT*, dynamics of which conserves number of heavy quarks. In such EFT, $Q\bar{Q}$-pair is produced in a point, by local operator:

$$\mathcal{A}_{\text{NRQCD}} = \langle J/\psi + X|\, \chi^\dagger(0)\kappa_n\psi(0)\, |0\rangle\,,$$

Different operators "couple" to different Fock states:

$$\chi^\dagger(0)\psi(0) \leftrightarrow \left|c\bar{c}\left[{}^1S_0^{(1)}\right]\right\rangle,\ \chi^\dagger(0)\sigma_i\psi(0) \leftrightarrow \left|c\bar{c}\left[{}^3S_1^{(1)}\right]\right\rangle,$$

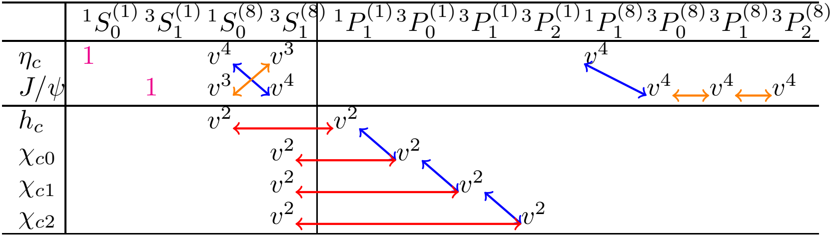
$$\chi^\dagger(0)\sigma_i T^a\psi(0) \leftrightarrow \left|c\bar{c}\left[{}^3S_1^{(8)}\right]\right\rangle,\ \chi^\dagger(0)D_i\psi(0) \leftrightarrow \left|c\bar{c}\left[{}^1P_1^{(8)}\right]\right\rangle, \dots$$

squared NRQCD amplitude (=LDME):

$$\sum_X |\mathcal{A}|^2 = \langle 0|\, \underbrace{\psi^\dagger\kappa_n^\dagger\chi a^\dagger_{J/\psi} a_{J/\psi}\chi^\dagger\kappa_n\psi}_{\mathcal{O}_n^{J/\psi}}\, |0\rangle = \left\langle \mathcal{O}_n^{J/\psi} \right\rangle,$$

Non-relativistic QCD

Velocity-scaling of LDMEs follows from velocity-scaling of corresponding Fock states and of operators $\chi^\dagger \kappa_n \psi$:



	${}^1S_0^{(1)}$	${}^3S_1^{(1)}$	${}^1S_0^{(8)}$	${}^3S_1^{(8)}$	${}^1P_1^{(1)}$	${}^3P_0^{(1)}$	${}^3P_1^{(1)}$	${}^3P_2^{(1)}$	${}^1P_1^{(8)}$	${}^3P_0^{(8)}$	${}^3P_1^{(8)}$	${}^3P_2^{(8)}$
η_c	1		v^4	v^3					v^4			
J/ψ		1	v^3	v^4						v^4	v^4	v^4
h_c			v^2		v^2							
χ_{c0}				v^2		v^2						
χ_{c1}				v^2			v^2					
χ_{c2}				v^2				v^2				

Note that:

- Colour-singlet LDMEs are LO in v for S-wave states ⇒ *Colour-Singlet Model*
- For P-wave states the CS and CO LDMEs are of the same order ⇒ *mixing*
- Connection between LDMEs for η_c and J/ψ through *Heavy-Quark Spin Symmetry*

Matching procedure between QCD and NRQCD:

$$v \ll 1 : \mathcal{A}_{\rm QCD}(gg \to Y_{Q\bar{Q}(v)}) = \sum_n f_n \left\langle Y_{Q\bar{Q}(v)} \right| \chi^\dagger(0)\kappa_n\psi(0) \left|0\right\rangle + O(v^{\#}),$$

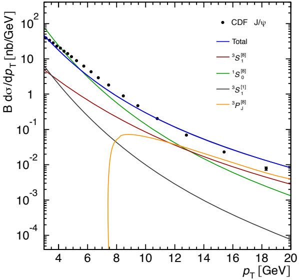
⇒ NRQCD factorization formula ("theorem") [Bodwin, Braaten, Lepage 95'] :

$$\sigma(gg \to \mathcal{H} + X) = \sum_n \sigma(gg \to Q\bar{Q}[n] + X) \left\langle \mathcal{O}_n^{\mathcal{H}} \right\rangle .$$

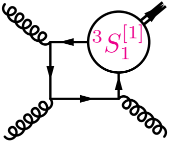
NRQCD factorisation: p_T-behaviour in pp

$$\frac{d\sigma}{dp_T^2}(pp \to \mathcal{H} + X) = \sum_n \frac{d\sigma}{dp_T^2}(pp \to Q\bar{Q}[n] + X)\,\langle \mathcal{O}_n^{\mathcal{H}} \rangle\,.$$

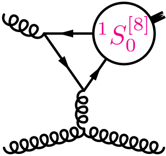
NLO, plot from hep-ph/1403.3970 :



At **LO**:



$\sim p_T^{-8}$



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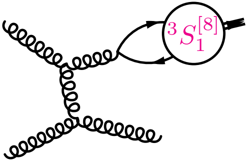
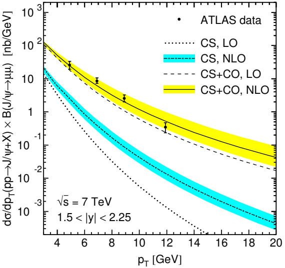


$\sim p_T^{-4}$

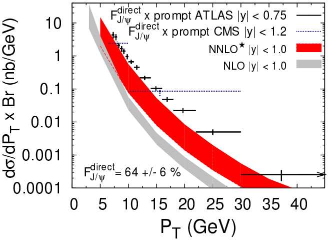
NRQCD factorisation: what does work?

- *Un-polarized* p_T distributions of J/ψ in hadro-, -photoproduction, e^+e^- annihilation; as well as hadroproduction of χ_{cJ} and $\psi(2S)$ can be described. The same is true for $\Upsilon(nS)$, $\chi_{bJ}(nS)$.
- Solves the problem of non-cancelling IR divergence at NLO in CSM for P-wave states production and decay through mixing with ${}^3S_1^{(8)}$ or ${}^1S_0^{(8)}$ states at $O(v^2)$.
- Covers the gap between CSM (@LO and NLO) and data at high-p_T in hadroproduction, due to contribution of CO states. If NNLO corrections in CS are as large as needed to close this gap, then perturbative expansion is just useless and we should stop doing quarkonia.

NLO NRQCD, [Butenschön, Kniehl, '11]



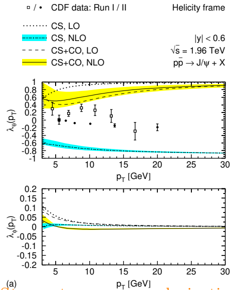
NLO and NNLO* CSM [Lansberg '11]



Problems: Polarisation

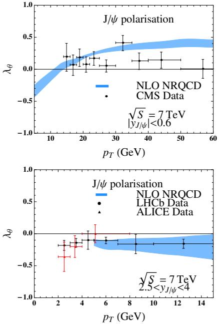
LDME fit	J/ψ hadropr.	J/ψ photopr.	J/ψ polar.	η_c hadropr.
Butenschön et al.	✓($p_T > 3$ GeV)	✓	✗	✗
Chao et al. + η_c	✓($p_T > 6.5$ GeV)	✗	✓	✓
Zhang et al.	✓($p_T > 6.5$ GeV)	✗	✓	✓
Gong et al.	✓($p_T > 7$ GeV)	✗	✓	✗
Chao et al.	✓($p_T > 7$ GeV)	✗	✓	✗
Bodwin et al.	✓($p_T > 10$ GeV)	✗	✓	✗

Global fit [Butenschön, Kniehl, '12]



Strong transverse polarisation due to $^3S_1^{[8]}$ and $^3P_J^{[8]}$ states at high p_T

Example hadroproduction dominated fit [Chao et.al., '14]



Problems: HQSS and photoproduction

LDME fit	J/ψ hadropr.	J/ψ photopr.	J/ψ polar.	η_c hadropr.
Butenschön et al.	✓($p_T > 3$ GeV)	✓	✗	✗
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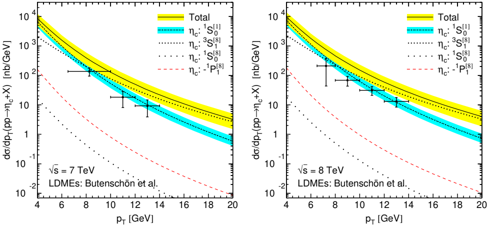
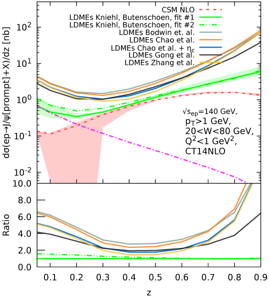
Gong et al.	✓($p_T > 7$ GeV)	✗	✓	✗
Chao et al.	✓($p_T > 7$ GeV)	✗	✓	✗
Bodwin et al.	✓($p_T > 10$ GeV)	✗	✓	✗

Global fit [Butenschön, Kniehl, '12]



J/ψ-photoproduction at the EIC vs $z = (p_{J/\psi}P)/(qP)$, using NLO calculation data by M. Butenschön



Why to look at other states at the EIC?

It seems that the consistent set of NRQCD LDMEs, which describes *hadroproduction* data can be found. Maybe LDMEs are just non-universal across collision systems? Then it should be possible to describe the J/ψ, χ_c, η_c and $\psi(2S)$ production in *ep*-collisions using one set of LDMEs!

- The χ_c-production has CS and CO contributions at the same order in v^2, CO is unavoidable at NLO for χ_c. The NLO calculation for *photoproduction* exists and finite-Q^2 production at NLO is within reach.
- The η_c-production is a test of HQSS relations between LDMEs. Is CO=0 for η_c or more complicated picture with cancellations between different LDME channels is possible for photo/electro production as it is the case for hadroproduction?
- The $\psi(2S)$-production is essentially free from feeddown from other charmonia, but has the same LDMEs as J/ψ so $\psi(2S)$ is much "cleaner" phenomenologically than J/ψ.

Both (p_T and z-)differential distributions at $Q^2 \simeq 0$ and $Q^2 \neq 0$ as well as **polarisation observables** should be measured to get a complete picture.

χ_c production in *ep*

Channels:

- Direct photoproduction at the LO in α_s:

$$\gamma^{(*)} + g \to c\bar{c}\left[{}^3P^{[1]}_{0,1,2}, {}^3S^{[8]}_1\right] + g,$$

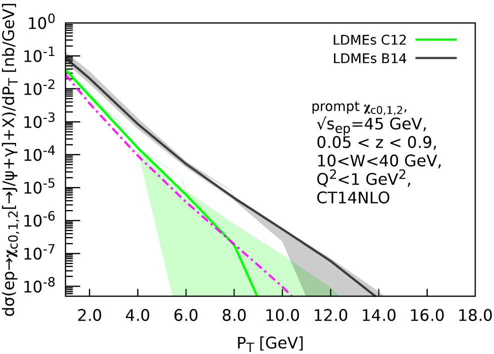
- Resolved-photon channels:

$$(\gamma \to) g/q + g \to c\bar{c}\left[{}^3P^{[1]}_{0,1,2}, {}^3S^{[8]}_1\right] + g/q,$$

- feeddown from $\psi(2S)$

χ_c photoproduction predictions

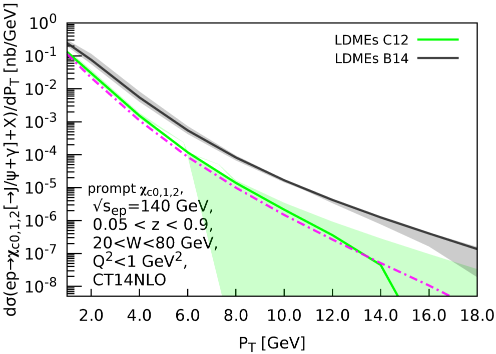
p_T-spectrum [M. Butenschön] , $\sqrt{s_{ep}} = 45$ GeV:



- For most of the LDME sets, both direct and resolved-photon contributions are **negative**
- The LDME sets of Bodwin et. al and Butenschön et. al (the latter one uses χ_c LDMEs from Ma et. al) give positive cross section
- The resolved-photon contribution (dash-dotted line) dominates

χ_c photoproduction predictions

p_T-spectrum [M. Butenschön] , $\sqrt{s_{ep}} = 140$ GeV:



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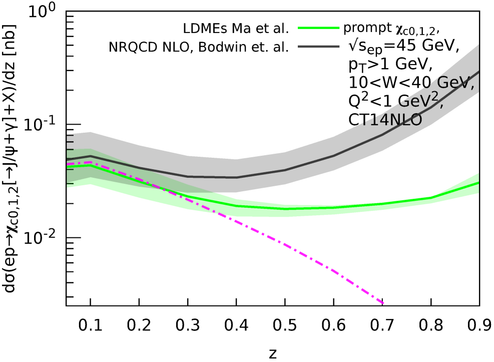
χ_c photoproduction predictions

$z = (p_{J/\psi}P)/(qP) = p^+_{J/\psi}/q^+$-spectrum [M. Butenschön] , $\sqrt{s_{ep}} = 45$ GeV:



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χ_c photoproduction predictions

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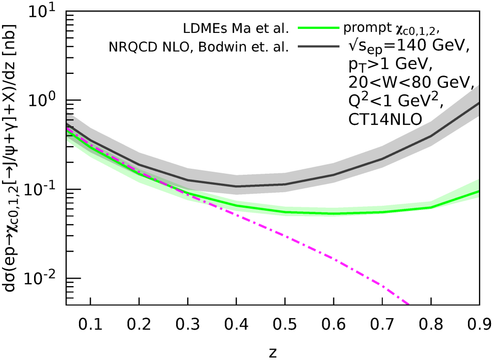

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η_c-photoproduction

- Direct-photon channels [H-F Zhang *et. al*, 2019] , LO in α_s:

$$\boxed{\gamma + g \to c\bar{c}\left[{}^1S_0^{[1]}\right] + g + g,}$$

$$\gamma + g \to c\bar{c}\left[{}^1S_0^{[8]}\right] + g,$$

$$\gamma + q(\bar{q}) \to c\bar{c}\left[{}^1S_0^{[8]}\right] + q(\bar{q}),$$

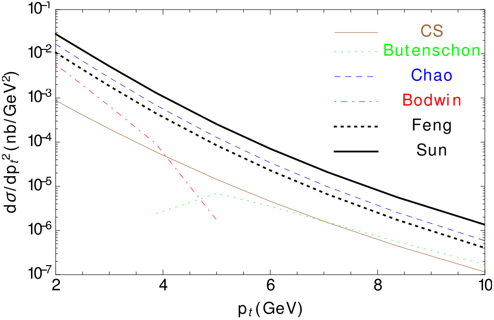
$$\gamma + g \to c\bar{c}\left[{}^3S_1^{[8]}\right] + g,$$

$$\gamma + g \to c\bar{c}\left[{}^1P_1^{[8]}\right] + g,$$

- $+$ resolved-photon channels

η_c-photoproduction at HERA

$E_e = 27.5$ GeV, $E_p = 920$ GeV, $60 < W_{\gamma p} < 240$ GeV, $0.3 < z < 0.6$, p_T-distribution [H-F Zhang *et. al*, 2019] :



- LO computation with NLO LDMEs, take wit a grain of salt!
- Some LDME sets lead to negative cross sections
- CO contributions are large, pure CSM is ~ 10 times below

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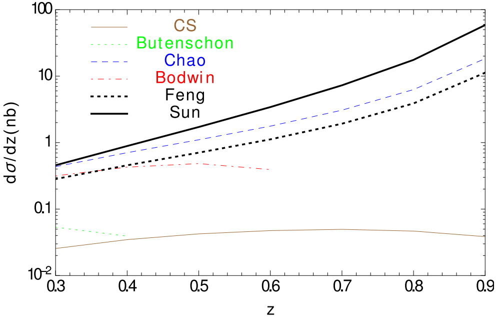
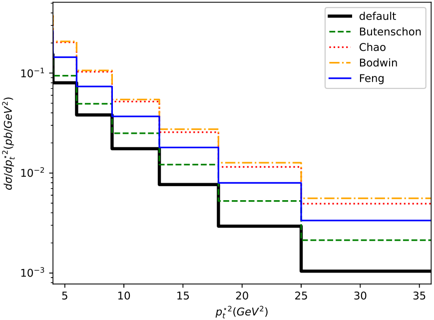


- LO computation with NLO LDMEs, take wit a grain of salt!
- Some LDME sets lead to negative cross sections
- CO contributions are large, pure CSM is ∼ 10 times below
- No resolved contribution! Expect a peak at $z \to 0$

η_c-electroproduction at the EIC

$20 < W_{\gamma p} < 80$ GeV, $4 < Q^2 < 36$ GeV^2, $0 < z < 0.6$, $4 < (p_T^*)^2 < 100$ GeV^2, p_T^*-distribution [H-F Zhang, X-M Mo, 2021] :



Default = fit of Zhang et. al

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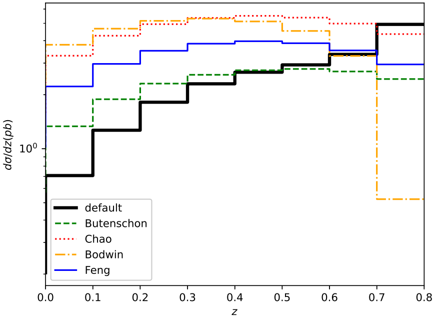

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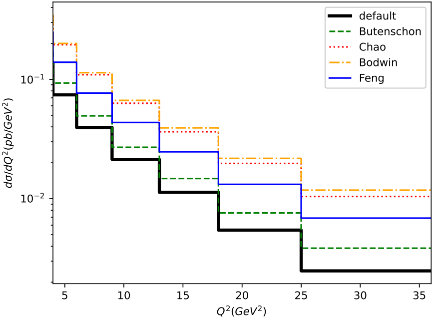
η_c-electroproduction at the EIC

$20 < W_{\gamma p} < 80$ GeV, $4 < Q^2 < 36$ GeV^2, $0 < z < 0.6$, $4 < (p_T^*)^2 < 100$ GeV^2, Q^2-distribution [H-F Zhang, X-M Mo, 2021] :



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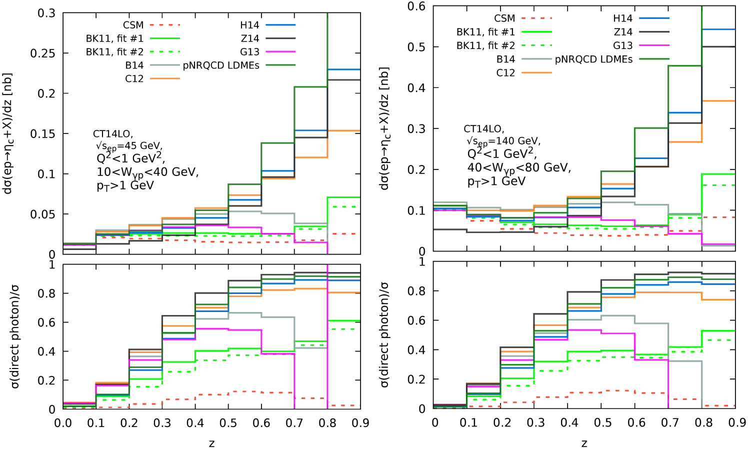
η_c-photoproduction at the EIC

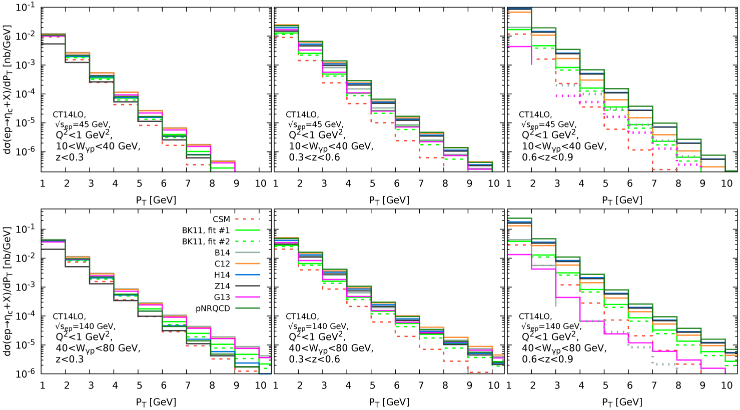
z-distributions, $\sqrt{s_{ep}} = 45$ and 140 GeV (calculation by H-F Zhang):



Most of LDME sets have $z \to 1$ peak but some don't and are close to CSM prediction. Again, this is the LO computation with NLO LDMEs, be careful interpreting it!

η_c-photoproduction at the EIC

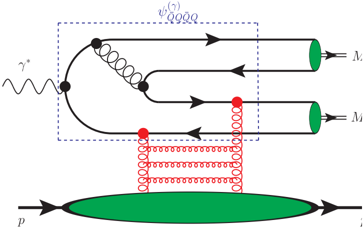
p_T-distributions, $\sqrt{s_{ep}} = 45$ and 140 GeV (calculation by H-F Zhang):

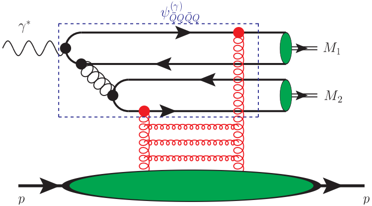


Exclusive photoproduction of opposite C-parity quarkonium pairs: dipole picture (low-x)

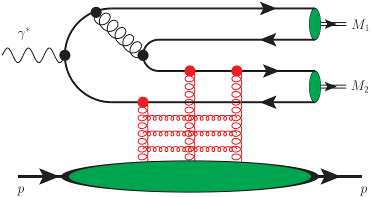
[Andrade, Siddikov, Schmidt, 2022; Siddikov, Schmidt, 2023]

- Opposite C-parity pair, e.g. $J/\psi + \eta_c$:



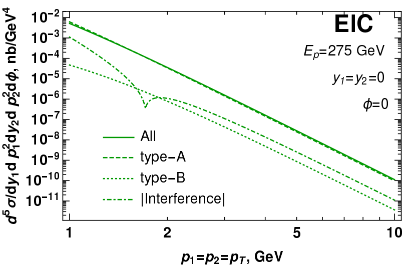


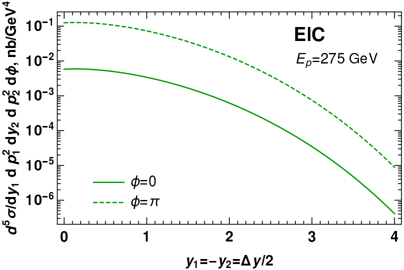
- Same C-parity pair, e.g. $J/\psi + J/\psi$:

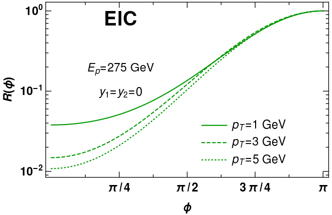


Exclusive photoproduction of opposite C-parity quarkonium pairs: dipole picture (low-x)

The “bCGC”-parametrisation [Kowalski, L. Motyka, G. Watt, 2006] of the dipole cross section was used for numerical estimates in [Andrade, Siddikov, Schmidt, 2022] .







Exclusive photoproduction of opposite C-parity quarkonium pairs: collinear factorisation

$$\sum_{\text{spins}} \left|\mathcal{A}^{(\mathfrak{a})}_{\gamma p \to M_1 M_2 p}\right|^2 = \frac{1}{\left(2-x_B\right)^2}\Big[4\left(1-x_B\right)\left(\mathcal{H}_{\mathfrak{a}}\mathcal{H}^*_{\mathfrak{a}} + \tilde{\mathcal{H}}_{\mathfrak{a}}\tilde{\mathcal{H}}^*_{\mathfrak{a}}\right) \tag{1}$$

$$- x_B^2\left(\mathcal{H}_{\mathfrak{a}}\mathcal{E}^*_{\mathfrak{a}} + \mathcal{E}_{\mathfrak{a}}\mathcal{H}^*_{\mathfrak{a}} + \tilde{\mathcal{H}}_{\mathfrak{a}}\tilde{\mathcal{E}}^*_{\mathfrak{a}} + \tilde{\mathcal{E}}_{\mathfrak{a}}\tilde{\mathcal{H}}^*_{\mathfrak{a}}\right)$$

$$- \left(x_B^2 + \left(2-x_B\right)^2\frac{t}{4m_N^2}\right)\mathcal{E}_{\mathfrak{a}}\mathcal{E}^*_{\mathfrak{a}} - x_B^2\frac{t}{4m_N^2}\tilde{\mathcal{E}}_{\mathfrak{a}}\tilde{\mathcal{E}}^*_{\mathfrak{a}}\Big]\,, \qquad \mathfrak{a} = L, T$$

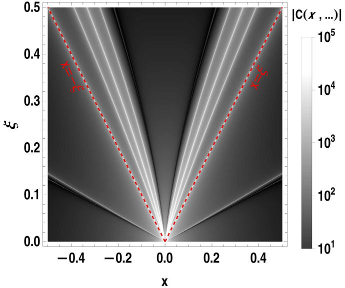
where

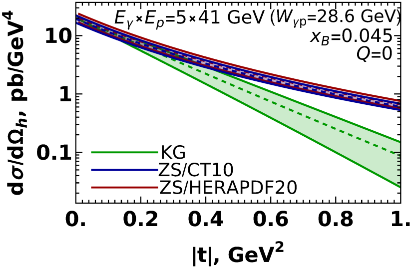
$$\mathcal{H}_{\mathfrak{a}} = \int_{-1}^{1} dx\, c_{\mathfrak{a}}\left(x,\, y_1,\, y_2\right) H_g\left(x, \xi, t\right), \quad \mathcal{E}_{\mathfrak{a}} = \int_{-1}^{1} dx\, c_{\mathfrak{a}}\left(x,\, y_1,\, y_2\right) E_g\left(x, \xi, t\right), \tag{2}$$

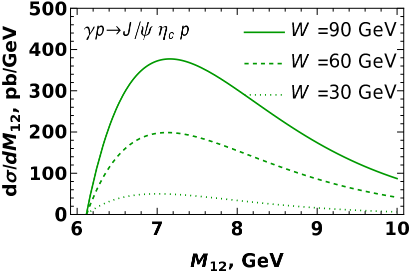
$$\tilde{\mathcal{H}}_{\mathfrak{a}} = \int_{-1}^{1} dx\, \tilde{c}_{\mathfrak{a}}\left(x,\, y_1,\, y_2\right) \tilde{H}_g\left(x, \xi, t\right), \quad \tilde{\mathcal{E}}_{\mathfrak{a}} = \int_{-1}^{1} dx\, \tilde{c}_{\mathfrak{a}}\left(x,\, y_1,\, y_2\right) \tilde{E}_g\left(x, \xi, t\right), \tag{3}$$

Exclusive photoproduction of $J/\psi + \eta_c$: collinear factorisation

[Siddikov, Schmidt, 2023]







Conclusions and outlook

- For the understanding of quarkonium production mechanism it is important to study photo/electro production of states different from J/ψ
- Existing LDME fits predict, that photoproduction of χ_c, surprisingly, seems to be dominated by resolved-photon. Reality may be different.
- Photo/electro-production of η_c will allow to test HQSS relation between LDMEs in the ep-environment. *Howewer full-NLO computation is needed.*
- Exclusive photoproduction of quarkonium pairs may put important constraints on gluon GPDs

Thank you for your attention!

LDME fits

LDME fit	J/ψ hadropr.	J/ψ photopr.	J/ψ polar.	η_c hadropr.	$J/\psi + Z$
Butenschön et al.	✓($p_T > 3$ GeV)	✓	✗	✗	✗
Chao et al. + η_c	✓($p_T > 6.5$ GeV)	✗	✓	✓	✗
Zhang et al.	✓($p_T > 6.5$ GeV)	✗	✓	✓	✗
Gong et al.	✓($p_T > 7$ GeV)	✗	✓	✗	✗
Chao et al.	✓($p_T > 7$ GeV)	✗	✓	✗	✗
Bodwin et al.	✓($p_T > 10$ GeV)	✗	✓	✗	✗
Brambilla et al.	✓($p_T > 9$ GeV)	✗	✓	(✗✓)	✓

Quarkonium in the potential model

Cornell potential:

$$V(r) = -C_F \frac{\alpha_s(1/r)}{r} + \sigma r,$$

neglect linear part, because quarkonium is “small” (~ 0.3 fm) → Coulomb wavefunction (for effective mass $\frac{m_1 m_2}{m_1+m_2} = \frac{m_Q}{2}$):

$$R(r) = \frac{\sqrt{m_Q^3 \alpha_s^3 C_F^3}}{2} e^{-\frac{\alpha_s C_F}{2} m_Q r}$$

$$\langle v^2 \rangle = \frac{C_F^2 \alpha_s^2}{2}, \langle r \rangle = \frac{3}{2C_F} \frac{1}{m_Q v}$$

$$\Rightarrow \boxed{\alpha_s^2(m_Q v) \simeq v^2}$$