Exclusive J/ψ and Υ photoproduction at 1-loop with QCD Evolution and high-energy resummation

Synergies between LHC and EIC for quarkonium physics
Trento, Italy

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IJCLab

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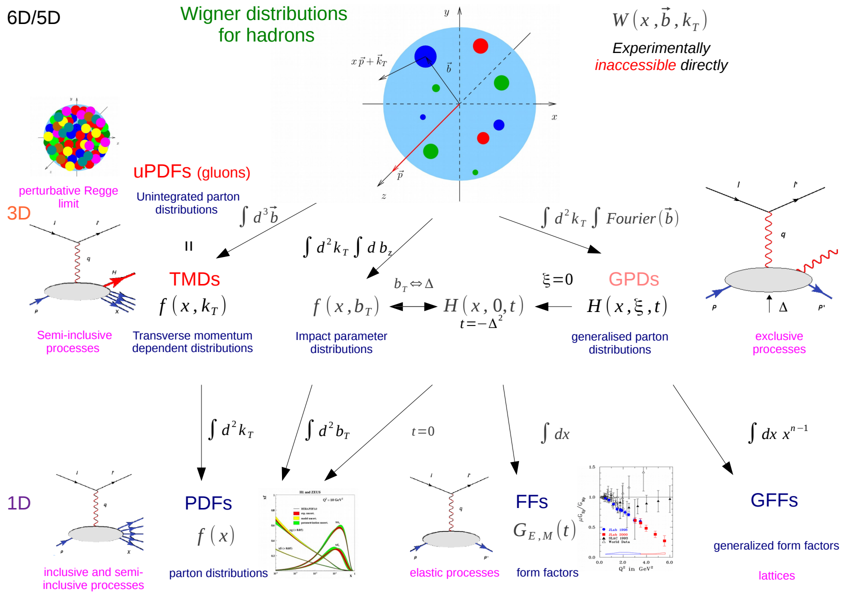
Based on work in progress with Chris Flett, Jean-Philippe Lansberg, Maxim Nefedov, Pawel Sznajder and Jakub Wagner

Introduction
From Wigner distributions to GPDs to PDFs

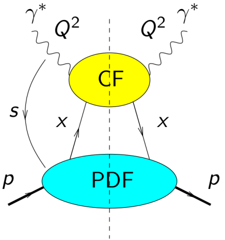


Deep Inelastic Scattering DIS: inclusive process

⇒ 1-dimensional structure

⇒ Collinear factorisation at the *cross section* level

Coefficient Function ⊗ Parton Distribution Function
(hard) (soft)

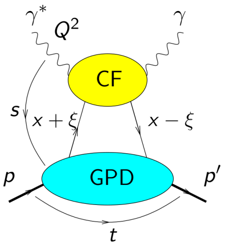


DVCS: exclusive process (non forward amplitude)

Fourier transf.: $t \leftrightarrow$ impact parameter

$\Rightarrow$ 3-dimensional structure

Collinear factorisation implies



Coefficient Function (hard) $\otimes$ Generalized Parton Distribution (soft)

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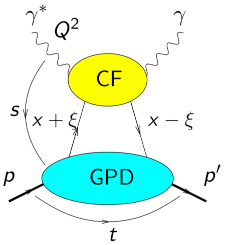


Coefficient Function (hard) $\otimes$ Generalized Parton Distribution (soft)

GPD $H(x, \xi, t)$:

x: *Average* momentum fraction of nucleon carried by the partons

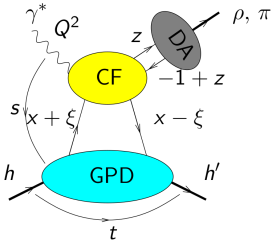
ξ: *Longitudinal* momentum fraction *transferred* to hard part

t: momentum difference squared of nucleons

[X. Ji: hep-ph/9609381], [A. Radyushkin: hep-ph/9604317, hep-ph/9704207]
[J. Collins, A. Freund: hep-ph/9801262], [D. Müller, D. Robaschik, B. Geyer, F.-M. Dittes, J. Horejsi: hep-ph/9812448]

DVMP: γ replaced by $\rho,\ \pi, \cdots$

GPD	$\otimes$	CF	$\otimes$	Distribution Amplitude
(soft)		(hard)		(soft)



[J. Collins, L. Frankfurt, M. Strikman: hep-ph/9611433]

[A. Radyushkin: hep-ph/9704207]

proofs valid only for some restricted cases

Quark GPDs at twist-2 [M. Diehl: hep-ph/0307382] (Note: $\Delta = p' - p$)

$$
\begin{aligned}
F^q &= \frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p'|\, \bar{q}(-\tfrac{1}{2}z)\, \gamma^+ q(\tfrac{1}{2}z)\, |p\rangle \Big|_{z^+=0,\, z_\perp=0} \\
&= \frac{1}{2P^+}\left[H^q(x,\xi,t)\, \bar{u}(p')\gamma^+ u(p) + E^q(x,\xi,t)\, \bar{u}(p')\frac{i\,\sigma^{+\alpha}\Delta_\alpha}{2m} u(p) \right].
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Forward limit: $H^q(x,\xi,t) \xrightarrow{\xi=0,t=0}$ PDF $q(x)$

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Gluon GPDs at twist 2 [M. Diehl: hep-ph/0307382]

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F^g = \frac{1}{P^+}\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p'| G^{+\mu}(-\frac{z}{2}) G^+_\mu(\frac{z}{2}) |p\rangle \Big|_{z^+=0, z_\perp=0}
$$

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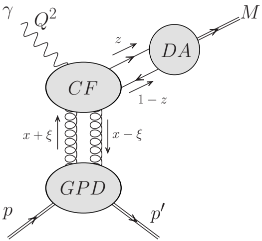
Exclusive quarkonium photoproduction and GPDs

Factorisation at the *amplitude* level:

$$\mathcal{A} = \int_{-1}^{1} dx \int_{0}^{1} dz \, H(x) \phi(z) C(x,z)$$



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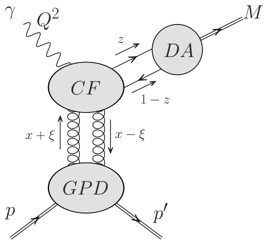

- No all-order proof of factorisation but *NLO* result indicates that it works

[D. Ivanov, A. Schafer, L. Szymanowski, G. Krasnikov: hep-ph/0401131]

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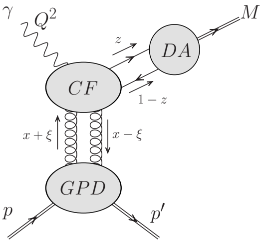

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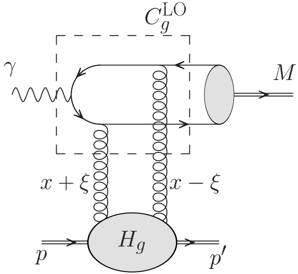
Generalised to electroproduction in [C. Flett, J. Gracey, S. Jones, T. Teubner: 2105.07657] *See also talk by Chris*

Leading order amplitude

- Exclusive J/ψ photoproduction probes gluon GPDs only at LO.
- Employ *static limit* (NRQCD): $\Longrightarrow \phi(z) \sim \delta(z - 1/2)$.



$$\mathcal{A} = \epsilon_\gamma^\mu \epsilon_M^{*\nu} \mathcal{T}^{\mu\nu}$$

$$\mathcal{T}_{\rm LO}^{\mu\nu} = -g_\perp^{\mu\nu} \int_{-1}^{1} \frac{dx}{x} \left[C_g^{\rm LO}\left(\frac{\xi}{x}\right) \frac{H_g(x,\xi,\mu_F)}{x} \right]$$

$$C_g^{\rm LO}\left(\frac{\xi}{x}\right) = \frac{F_{\rm LO}}{\left[1 + \frac{\xi}{x} - i\delta\,{\rm sgn}(x)\right]\left[1 - \frac{\xi}{x} + i\delta\,{\rm sgn}(x)\right]}$$

$$F_{\rm LO} = 4\pi\alpha_s e e_q \frac{2T_F}{N_c}\left(\frac{\left\langle \mathcal{O}\left[{}^3S_1^{[1]}\right]\right\rangle}{3m_c^3}\right)^{\frac{1}{2}}, \qquad \xi = \frac{M^2}{2W_{\gamma p}^2 - M^2} \sim \frac{M^2}{2W_{\gamma p}^2}$$

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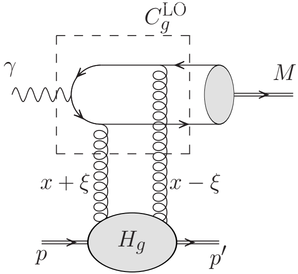

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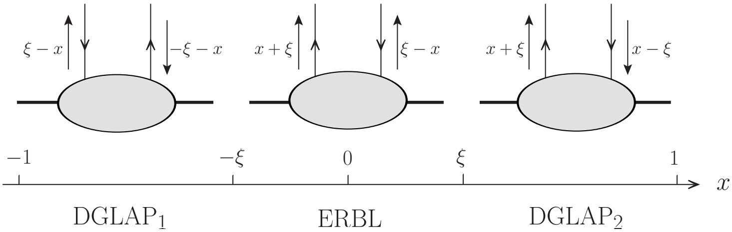
Large $W_{\gamma p}$ (small x in inclusive physics) $\leftrightarrow$ *small* ξ

Imaginary part of amplitude
DGLAP and ERBL regions



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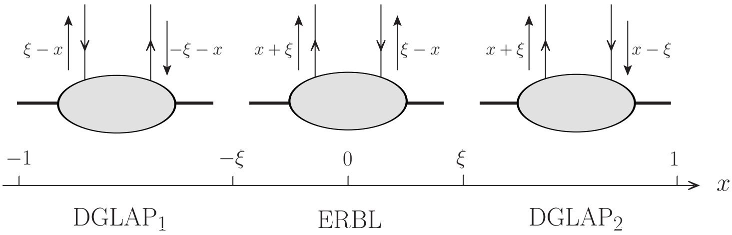

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For LO amplitude:

- Picks up *imaginary part* at $x = \pm\xi$.

$$\mathrm{Im}\, C_g^{\mathrm{LO}}\left(\frac{\xi}{x}\right) = -\pi \frac{F_{LO}}{2}\left[\delta\left(\frac{\xi}{x} - 1\right) + \delta\left(\frac{\xi}{x} + 1\right)\right]$$

$$\mathrm{Im}\mathcal{T}^{\mu\nu}_{\mathrm{LO}} = \pi \frac{g_\perp^{\mu\nu} F_{LO}}{\xi} H_g(\xi, \xi)$$

Modelling the GPDs
Double distributions

$$H_i(x,\xi) = \int_{-1}^{1} d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha\, \delta(\beta + \xi\alpha - x)\, d_i(\beta,\alpha)\,.$$

- Based on *double distributions (DDs)* [A. Radyushkin: hep-ph/9704207].

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- DDs $d_i(\beta,\alpha)$ even in α. $\Longrightarrow$ *polynomiality* property of GPDs

$$\int_{-1}^{1} dx\, x^n H_i(x,\xi,t) = \sum_{j=0,\,\mathrm{even}}^{n} (2\xi)^j A^i_{n+1,j}(t) + \mathrm{mod}(n,2)\,(2\xi)^{n+1} C^i_{n+1}(t).$$

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- Fix $t = t_{\mathrm{min}}$.
- Neglect D-terms [M. Polyakov, C. Weiss: hep-ph/990241].

Forward limits of GPDs: factorisation of the double distributions:

$$d_i(\beta, \alpha) = f_i(\beta) \times h_i(\beta, \alpha)$$

such that the profile function $h_i(\beta, \alpha)$ satisfies

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For the profile function [A. Radyushkin: hep-ph/9805342, hep-ph/9810466]

$$h_i(\beta,\alpha) = \frac{\Gamma(2n_i+2)}{2^{2n_i+1}\Gamma^2(n_i+1)} \frac{\left((1-|\beta|)^2-\alpha^2\right)^{n_i}}{(1-|\beta|)^{2n_i+1}}\,.$$

$n_i \leftrightarrow$ width of the profile function (generates *skewness*):

$n_i \to \infty \implies$ no ξ dependence in GPDs

Our approach inspired from Goloskokov-Kroll (GK) model [S. Goloskokov, P. Kroll: hep-ph/0611290]

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GK-inspired model

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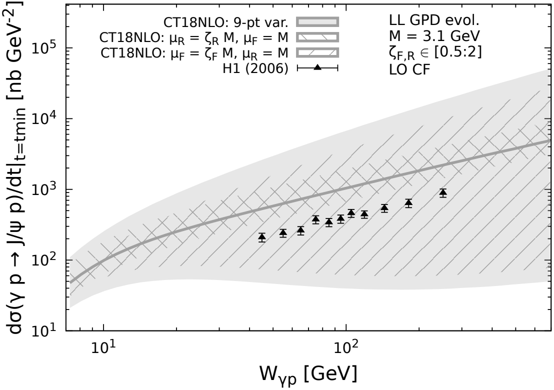
Full LL GPD evolution performed, using APFEL++ [V. Bertone, H. Dutrieux, C. Mezrag, J. M. Morgado: 2206.01412]

LO cross section



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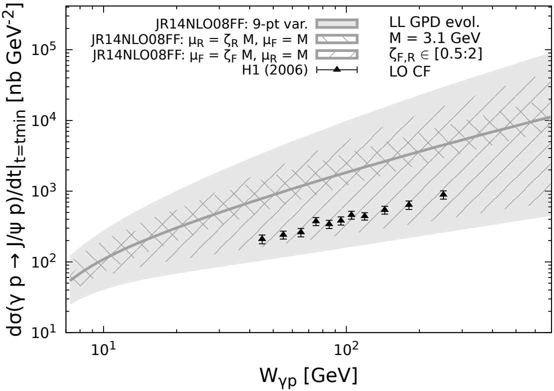
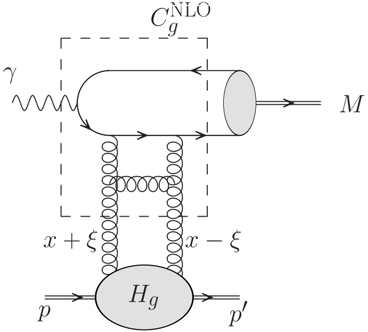


For small values of μ_F, cross section does not increase with energy.

$\Longrightarrow$ CT18NLO PDF set used to construct the GPDs has a local maximum at small x...

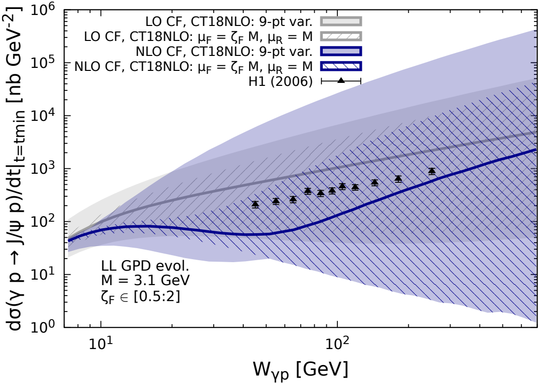
NLO amplitude

NLO amplitude has contributions from *both* quark and gluon GPDs:



Imaginary part comes fully from the *DGLAP region* ($\xi \leq |x| \leq 1$)

NLO cross section



NLO prediction has *huge uncertainties* at high energies.

Already observed in the original paper [D. Ivanov, A. Schafer, L. Szymanowski, G. Krasnikov: hep-ph/0401131].

Origin of problem for NLO cross section

$$\mathcal{T}^{\mu\nu}_{\text{NLO}} \supset i\pi \frac{g_\perp^{\mu\nu} F_{LO}}{\xi} \left[H_g(\xi,\xi) + \frac{\alpha_s(\mu_R) C_A}{\pi} \ln\left(\frac{M^2}{4\mu_F^2}\right) \int_\xi^1 \frac{dx}{x} H_g(x,\xi) \right.$$
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$H_g(x,\xi) \sim \mathrm{const}$, as $x \to \xi$ for small ξ
$\Longrightarrow$ appearance of $\ln \xi$ (high-energy logs).

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Opposite sign to LO for $\mu_F > M/2$.

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In the DGLAP evolution of low ξ GPDs, the probability of emitting a new gluon is *strongly enhanced* by the large value of $\ln \xi$.

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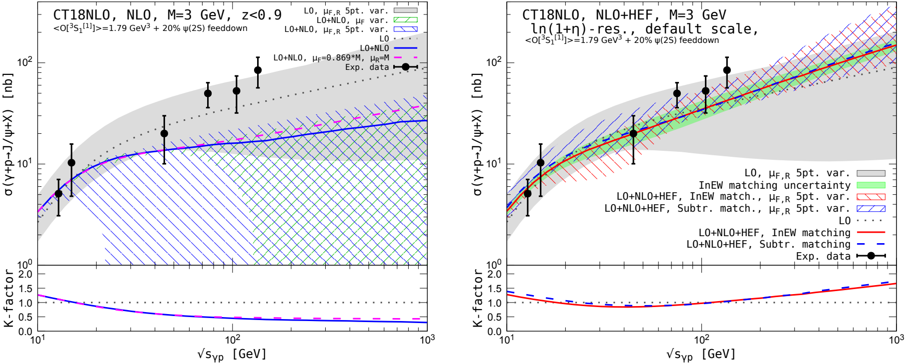
$\Longrightarrow$ Hints towards a solution through *resummation* of these logarithms...



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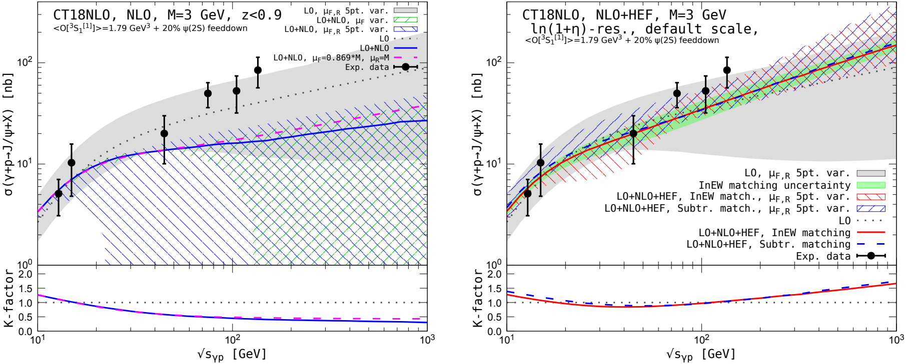
Instabilities in the inclusive case

Solution through resummation



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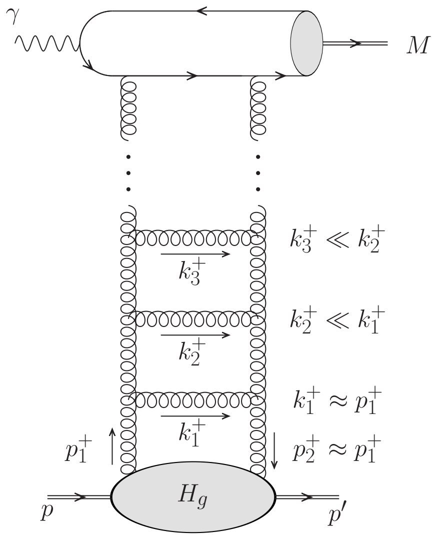
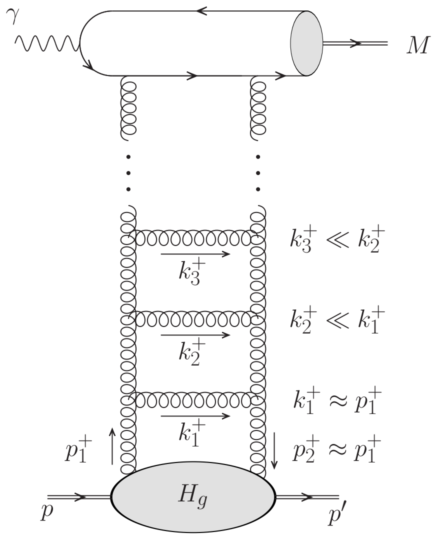
Uncertainties even lead to negative cross-sections!

Multiple gluon emissions:
BFKL ladder and resummation



- Logarithms are generated by emission of gluons, with strong ordering in + lightcone momentum.
- They become large at high energies, and need to be *resummed*.



- Logarithms are generated by emission of gluons, with strong ordering in $+$ lightcone momentum.
- They become large at high energies, and need to be *resummed*.
- We implement a *resummation* of these BFKL-type logs, consistent with fixed-order evolution of GPD:
  $\Longrightarrow$ *Doubly-logarithmic approximation (DLA)*

Implementation of high-energy resummation

HEF resummation of $\sim \hat{\alpha}_s^n \ln^{n-1}(\frac{x}{\xi})$ *at integrand level* ($\hat{\alpha}_s = \frac{\alpha_s C_A}{\pi}$) to the imaginary part of the $C_g(\frac{\xi}{x})$:

$$C_g^{\mathrm{HEF}}\left(\frac{\xi}{x}\right) = \frac{-i\pi}{2}\frac{F_{\mathrm{LO}}}{\left(\frac{\xi}{x}\right)}\int_0^\infty d\mathbf{q}_T^2\, \mathcal{C}_{gi}\left(\frac{\xi}{x}, \mathbf{q}_T^2, \mu_F, \mu_R\right) h(\mathbf{q}_T^2),$$

$$h(\mathbf{q}_T^2) = \frac{M^2}{M^2 + 4\mathbf{q}_T^2}.$$

Implementation of high-energy resummation

Resummation factor, $\mathcal{C}_{gi}\left(\frac{\xi}{x}, \mathbf{q}_T^2, \mu_F, \mu_R\right)$ in the *Doubly-Logarithmic Approximation* (DLA) (in order to be consistent with fixed-order evolution of GPD) is given by the Blümlein-Collins-Ellis formula [hep-ph/9506403]

$$\mathcal{C}_{gg}^{(\mathrm{DL})}\left(\frac{\xi}{x}, \mathbf{q}_T^2, \mu_F^2, \mu_R^2\right) = \frac{\hat{\alpha}_s}{\mathbf{q}_T^2} \begin{cases} J_0\left(2\sqrt{\hat{\alpha}_s \ln\left(\frac{x}{\xi}\right) \ln\left(\frac{\mu_F^2}{\mathbf{q}_T^2}\right)}\right) & \text{if } \mathbf{q}_T^2 < \mu_F^2, \\ I_0\left(2\sqrt{\hat{\alpha}_s \ln\left(\frac{x}{\xi}\right) \ln\left(\frac{\mathbf{q}_T^2}{\mu_F^2}\right)}\right) & \text{if } \mathbf{q}_T^2 > \mu_F^2. \end{cases}$$

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$\Longrightarrow$ resums terms scaling like $\left(\hat{\alpha}_s \ln(x/\xi) \ln(\mu_F^2/\mathbf{q}_T^2)\right)^n$ to all orders in perturbation theory.

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For the quark channel, the resummation factor is given in the DLA by:

$$\mathcal{C}_{gq}\left(\frac{\xi}{x}, \mathbf{q}_T^2, \mu_F^2, \mu_R^2\right) = \frac{C_F}{C_A}\left[\mathcal{C}_{gg}\left(\frac{\xi}{x}, \mathbf{q}_T^2, \mu_F^2, \mu_R^2\right) - \delta\left(1 - \frac{\xi}{x}\right)\delta(\mathbf{q}_T^2)\right].$$

Implementation of high-energy resummation

Useful representation in Mellin space:

$$\mathcal{C}_{gg}^{(\mathrm{DL})}(N, \mathbf{q}_T^2, \mu_F^2, \mu_R^2) \quad = \quad R(\gamma_{gg}) \, \frac{\gamma_{gg}}{\mathbf{q}_T^2} \left(\frac{\mathbf{q}_T^2}{\mu_F^2} \right)^{\gamma_{gg}} .$$

γ_{gg} is the solution to the equation

$$\frac{\hat{\alpha}_s}{N} \chi(\gamma_{gg}) = 1, \quad \chi(\gamma) = 2\varphi(1) - \varphi(\gamma) - \varphi(1-\gamma), \quad \varphi(\gamma) = \frac{d \ln \Gamma(\gamma)}{d\gamma}$$

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Mellin transform maps logarithms $\ln\left(\frac{x}{\xi}\right)$ to the poles at $N = 0$:

$$\frac{x}{\xi}\ln^{k-1}\left(\frac{x}{\xi}\right) \leftrightarrow \frac{(k-1)!}{N^k}.$$

Implementation of resummation: $C^{\mathrm{CF}} \to C^{\mathrm{HEF}}$

$$C_g^{\mathrm{HEF}}\left(\frac{\xi}{x}\right) = \frac{-i\pi\hat{\alpha}_s F_{\mathrm{LO}}}{2\left|\frac{\xi}{x}\right|}\sqrt{\frac{L_\mu}{L_x}}\left\{I_1\left(2\sqrt{L_x L_\mu}\right) - 2\sum_{k=1}^{\infty}\mathrm{Li}_{2k}(-1)\left(\frac{L_x}{L_\mu}\right)^k I_{2k-1}\left(2\sqrt{L_x L_\mu}\right)\right\},$$

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This yields, when expanded in α_s,

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- First two terms in α_s match the fixed-order CF computation at small x.

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- First two terms in α_s match the fixed-order CF computation at small x.
- *Cannot fix the scale at NNLO to get rid of all the $1/\xi$-enhanced contributions*

Quark coefficient function:

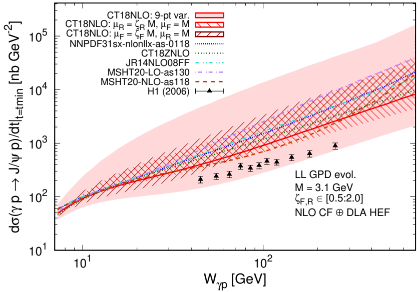
$$C_q^{\text{HEF}}\left(\frac{\xi}{x}\right) = \frac{2C_F}{C_A}C_g^{\text{HEF}}\left(\frac{\xi}{x}\right),$$

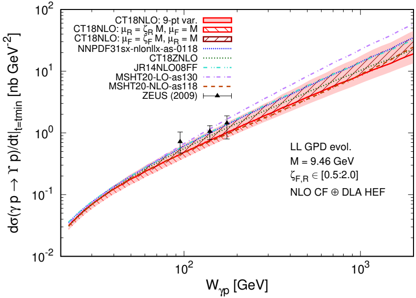
Matching

We use *subtractive matching*:

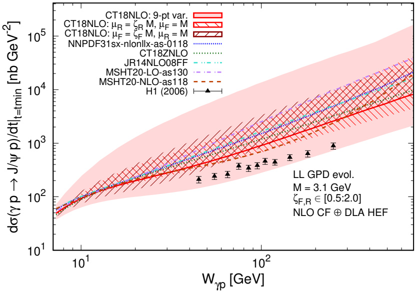
$$
\begin{aligned}
C_{g,q}^{\text{match.}}\left(\frac{\xi}{x}\right) &= C_{g,q}^{\text{NLO CF}}\left(\frac{\xi}{x}\right) - C_{g,q}^{\text{asy.}}\left(\frac{\xi}{x}\right) + C_{g,q}^{\text{HEF}}\left(\frac{\xi}{x}\right), \\
C_{g}^{\text{asy.}}\left(\frac{\xi}{x}\right) &= \frac{C_A}{2C_F} C_{q}^{\text{asy.}}\left(\frac{\xi}{x}\right) \\
&= \frac{-i\pi F_{\mathrm{LO}}}{2}\left[\delta\left(\left|\frac{\xi}{x}\right| - 1\right) + \frac{\hat{\alpha}_s}{\left|\frac{\xi}{x}\right|}\ln\left(\frac{M^2}{4\mu_F^2}\right)\right].
\end{aligned}
$$

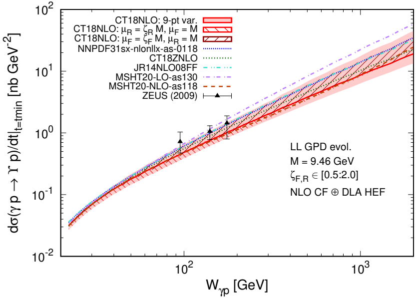
- $C_{g}^{\text{asy.}}\left(\frac{\xi}{x}\right)$: first two terms in the α_s expansion of $C_{g}^{\text{HEF}}\left(\frac{\xi}{x}\right)$.
- Matching performed before x-integration.





Results stable.

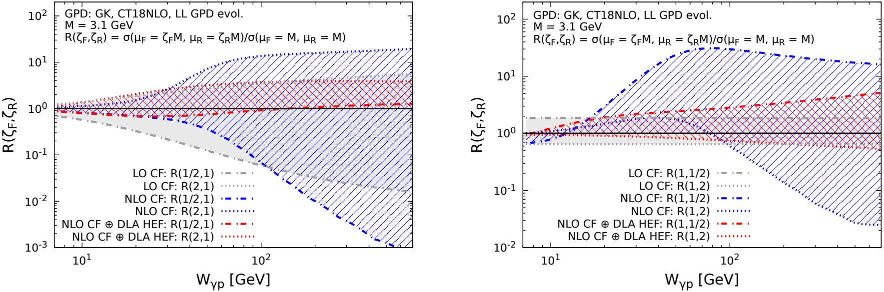




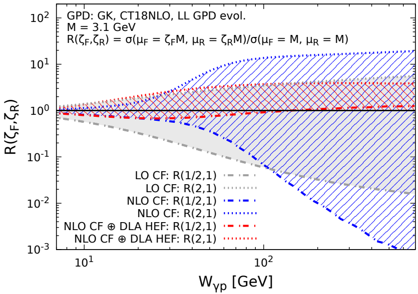
Results stable.

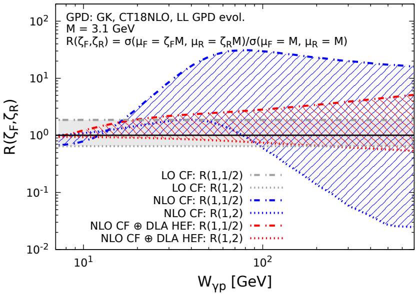
To improve accuracy in J/ψ, probably need to consider *higher twist* or *relativistic corrections*.

Stabilisation after resummation: J/ψ



Left: μ_F *uncertainty smaller* for resummed result.





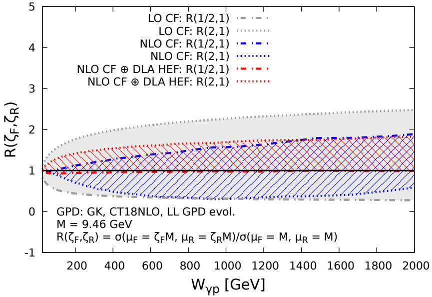
Left: μ_F *uncertainty smaller* for resummed result.

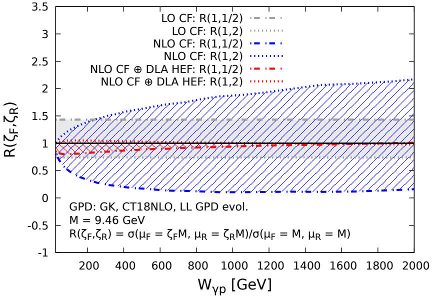
Right: μ_R *uncertainty smaller*, but grows at high energy (still less than fixed order computation).

$\implies$ Due to $\sigma \sim W_{\gamma p}^{\alpha_s(\mu_R)}$ at large $W_{\gamma p}$ (hard pomeron contribution).

Results

Stabilisation after resummation: Υ





Uncertainties smaller for resummed case.

Also smaller compared to J/ψ case.

Shuvaev transform

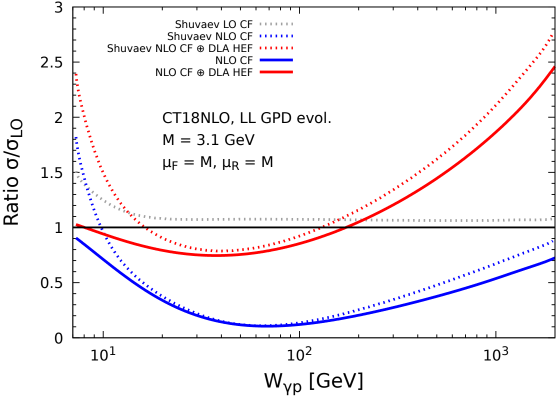
At small ξ, it was shown that the leading log evolution of GPD reduce to *DGLAP* [A. Shuvaev: hep-ph/990218, A. Shuvaev, K. Golec-Biernat, A. Martin, M. Ryskin: hep-ph/9902410]

GPDs can be related to PDFs through [A. Martin, C. Nockles, M. Ryskin, A. Shuvaev, T. Teubner: 0812.3558]

$$H_q(x,\xi,\mu_F) = \int_{-1}^{1} \mathrm{d}x' \left[\frac{2}{\pi}\,\mathrm{Im}\int_0^1 \frac{\mathrm{d}s}{y(s)\,\sqrt{1-y(s)x'}}\right] \frac{\mathrm{d}}{\mathrm{d}x'}\left(\frac{q(x',\mu_F)}{|x'|}\right),$$

$$H_g(x,\xi,\mu_F) = \int_{-1}^{1} \mathrm{d}x' \left[\frac{2}{\pi}\,\mathrm{Im}\int_0^1 \frac{\mathrm{d}s(x+\xi(1-2s))}{y(s)\,\sqrt{1-y(s)x'}}\right] \frac{\mathrm{d}}{\mathrm{d}x'}\left(\frac{g(x',\mu_F)}{|x'|}\right),$$

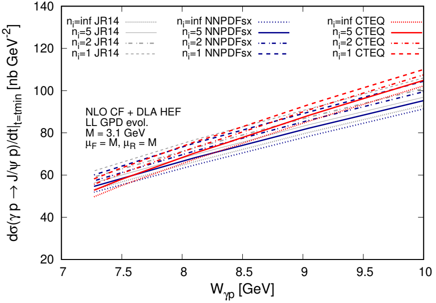
$$y(s) = \frac{4s(1-s)}{x+\xi(1-2s)}.$$

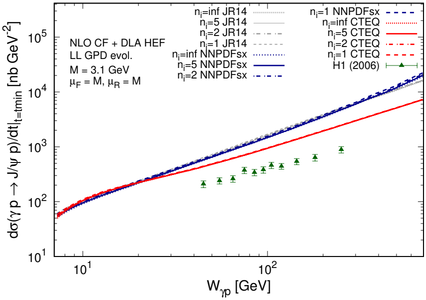


Constant shift between cross-sections:

$\Longrightarrow$ Consequence of *difference between GPDs* using Shuvaev and the full GPD determination using DDs.

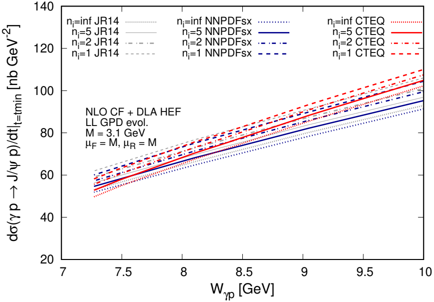
Dependence on n_i parameter: J/ψ

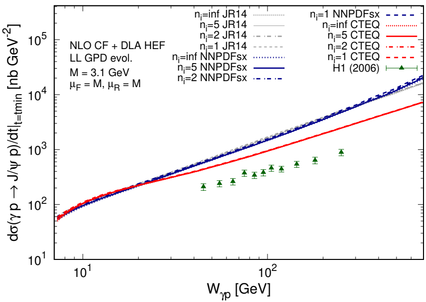




Profile function:

$$h_i(\beta,\alpha) = \frac{\Gamma(2n_i+2)}{2^{2n_i+1}\Gamma^2(n_i+1)} \frac{\left((1-|\beta|)^2-\alpha^2\right)^{n_i}}{(1-|\beta|)^{2n_i+1}} .$$





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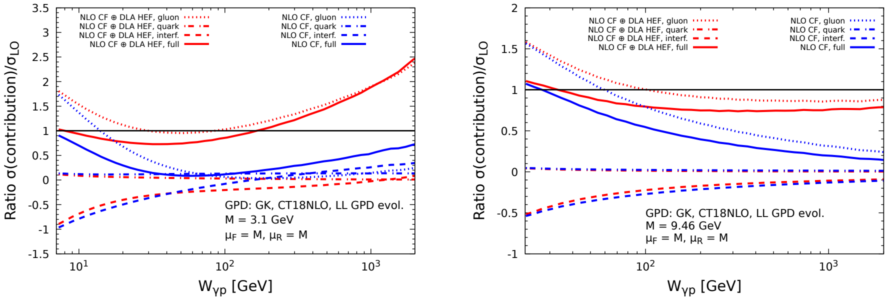
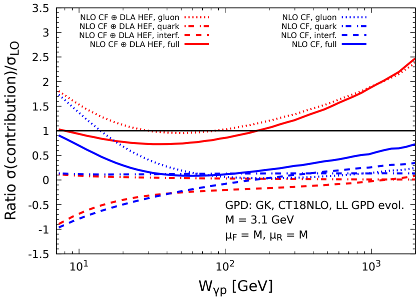
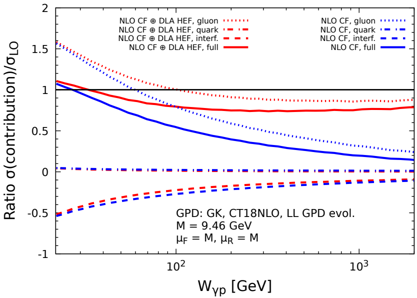
Caveat: Changing n_i represents a (small) subset of potential GPDs...



Fixed Order computation: unreliable at high energies.





Fixed Order computation: unreliable at high energies.

For the resummed predictions, the gluon GPD contribution dominates at *high energies*.

Conclusion

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- Future: Investigate more flexible GPD modelling/how to fit GPD from such exclusive J/ψ and Υ photoproduction data.

BACKUP SLIDES

Scale fixing?

$$\mathcal{T}^{\mu\nu}_{\text{NLO}} \supset i\pi \frac{g_\perp^{\mu\nu} F_{LO}}{\xi} \left[H_g(\xi,\xi) + \hat{\alpha}_s \ln\left(\frac{M^2}{4\mu_F^2}\right) \int_\xi^1 \frac{dx}{x} H_g(x,\xi) \right.$$
$$\left. + \hat{\alpha}_s \frac{C_F}{C_A} \ln\left(\frac{M^2}{4\mu_F^2}\right) \int_\xi^1 dx\, (H_q(x,\xi) - H_q(-x,\xi)) \right]$$

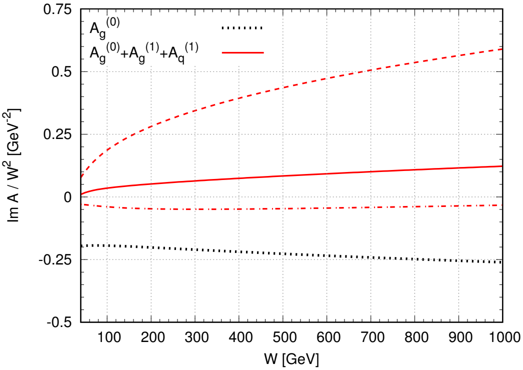
Choose $\mu_F = m_c$. $\implies$ Large $\ln\xi$ terms cancel [S. Jones, A. Martin, M. Ryskin, T. Teubner: 1507.06942].

However, impossible to move all enhanced by powers of $\ln\xi$ contributions from the coefficient function into the GPD (through μ_F evolution)

Big part of NLO correction from the hard coefficient eliminated, *but not from higher order contributions*.

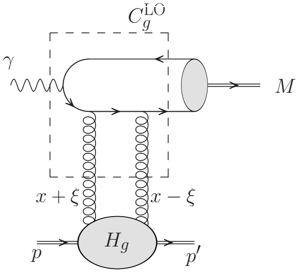
Result after scale-fixing procedure

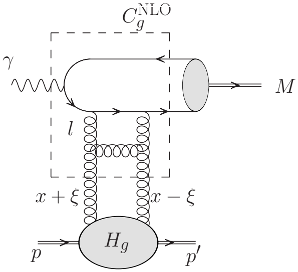
Plot from S. Jones, A. Martin, M. Ryskin, T. Teubner [1610.02272]



Q_0 subtraction procedure

S. Jones, A. Martin, M. Ryskin, T. Teubner [1610.02272]





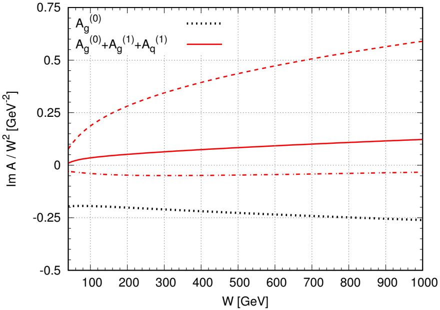
To avoid double counting, exclude the $|l^2| < Q_0^2$ domain whose contribution is already included in the LO term using the input gluon GPD.

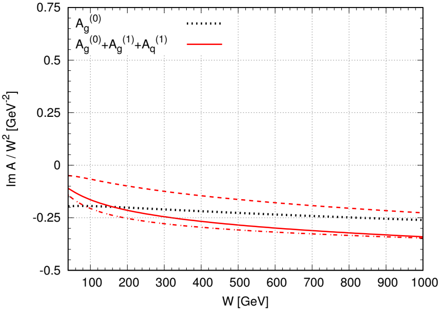
$\Longrightarrow$ Subtract the NLO DGLAP contribution $|l^2| < Q_0^2$ from the NLO $\overline{\mathrm{MS}}$ CF to *avoid double counting* with input GPD at scale Q_0

Typically power suppressed, but sizeable here: $\mathcal{O}(\frac{Q_0^2}{M^2})$

Result after Q_0 subtraction

Plots from S. Jones, A. Martin, M. Ryskin, T. Teubner [1610.02272]





Left: Scale-fixing procedure only

Right: Scale-fixing and Q_0 subtraction

Process-dependent procedure!!

High-energy limit of NLO calculation

At high energies, it is possible to relate the real part of the amplitude to the imaginary part through the *high-energy Regge dispersion relation*:

$$\frac{\mathrm{ReA}}{\mathrm{ImA}} = \frac{\pi}{2}\left(\frac{\partial \ln \mathrm{ImA}}{\partial \ln(1/\xi)}\right)$$

Note: $\mathrm{A} = \varepsilon_\mu \varepsilon_\nu \mathcal{T}^{\mu\nu}$.

Used in C. Flett, S. Jones, A. Martin, M. Ryskin, T. Teubner [1908.08398] and C. Flett, A. Martin, M. Ryskin, T. Teubner [2006.13857] to determine gluon PDFs at low x using exclusive J/ψ photoproduction.

We tested the validity of the above by performing an explicit comparison between the two ways of obtaining the real part.

Shuvaev Transform

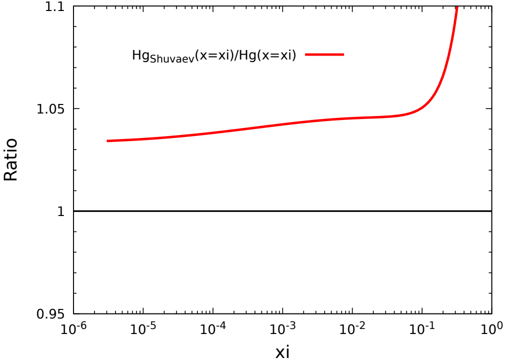
Conformal moments of the GPDs:

$$H^N(\xi) = \int_{-1}^{1} dx\, R_N(x) H(x,\xi)$$

Conformal moments are polynomials in *even* powers of ξ

$$H^N(\xi) = \sum_{k=0}^{\lfloor (N+1)/2 \rfloor} c_k^N \xi^{2k} = c_0^N + c_1^N \xi^2 + \ldots$$

Leading term c_0^N is the Mellin moment of the PDF.



Systematic shift between GPD generated from Shuvaev transform and GPD from double distribution.

Resummation $\implies \hat{\alpha}_s \ln \left|\frac{x}{\xi}\right| \to \left|\frac{x}{\xi}\right|^{\hat{\alpha}_s}$

At small ξ, gluon GPD obeys power law: $H_g(x, \xi) \sim x^{-\beta}$.

Then, the resummed amplitude:

$$\mathcal{A} \sim \int_{\xi}^{1} \frac{dx}{x} x^{-\beta} \left(\frac{x}{\xi}\right)^{\hat{\alpha}_s} = \frac{\xi^{-\hat{\alpha}_s} - \xi^{-\beta}}{\hat{\alpha}_s - \beta} \sim \xi^{-\max(\hat{\alpha}_s, \beta)}$$

When $\hat{\alpha}_s$ is large enough, changing the renormalisation scale μ_R directly affects the slope of cross-section in ξ.