

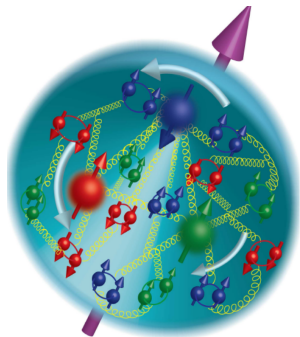
Quarkonium production and TMDs

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University and INFN Cagliari

Synergies between LHC and EIC
for quarkonium physics

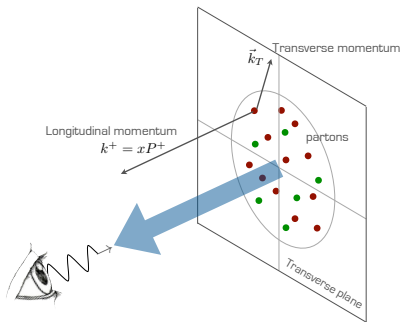
8 - 12 July 2024
ECT*, Trento (Italy)



TMDs: definition and properties

Transverse momentum dependent distributions (TMDs)

Three-dimensional distributions: provide information on the partonic longitudinal momentum and the two-dimensional transverse momentum

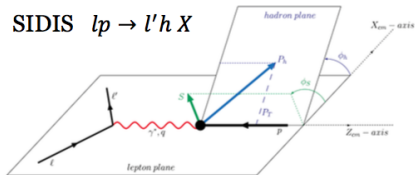


Renormalization scale μ and the Collins-Soper scale ζ not shown explicitly

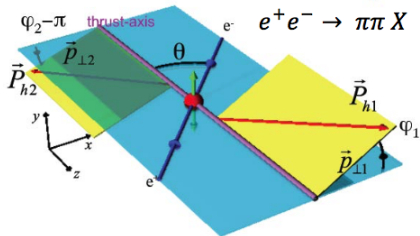
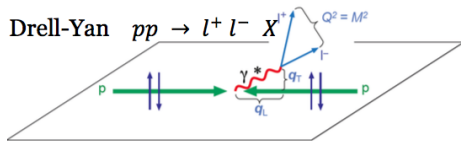
More detailed information on the proton's structure as compared to PDFs: 1D description is not always satisfactory, see i.e. spin effects

Two scale processes $Q^2 \gg q_T^2$

SIDIS $lp \rightarrow l'h X$



Drell-Yan $pp \rightarrow l^+ l^- X$



Factorization proven

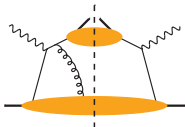
To all orders in α_s

Leading order in powers of $1/Q$ (twist)

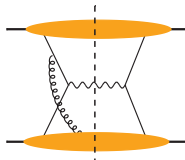
Collins, Cambridge University Press (2011)
Boussarie et al, TMD handbook 2304.03302

Gauge invariant definition of Φ (not unique)

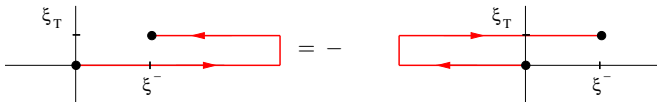
$$\Phi^{[M]} \propto \langle P, S | \bar{\psi}(0) \mathcal{U}_{[0, \xi]}^C \psi(\xi) | P, S \rangle$$



FSI in SIDIS



ISI in DY



Sign change of T -odd distributions: fundamental test, still under experimental scrutiny

ISI/FSI lead to process dependence of TMDs, could even break factorization

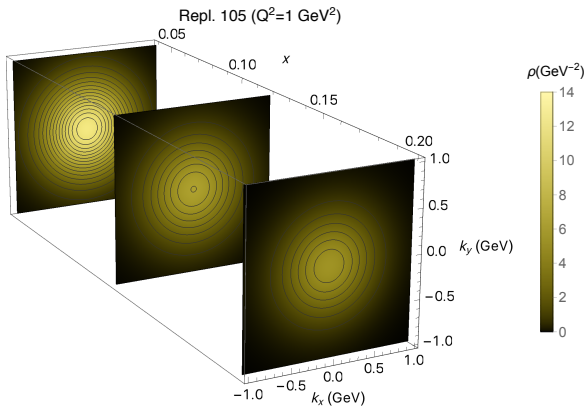
Collins, Qiu, PRD 75 (2007)

Collins, PRD 77 (2007)

Rogers, Mulders, PRD 81 (2010)

	Accuracy	SIDIS HERMES	SIDIS COMPASS	DY fixed target	DY collider	N of points	χ^2/N_{points}
Pavia 2017 arXiv:1703.10157	NLL	✓	✓	✓	✓	8059	1.55
SV 2019 arXiv:1912.06532	N^3LL^-	✓	✓	✓	✓	1039	1.06
MAP22 arXiv:2206.07598	N^3LL^-	✓	✓	✓	✓	2031	1.06
MAP24 arXiv:2405.13833	N^3LL^-	✓	✓	✓	✓	2031	1.08

Distribution of unpolarized quarks



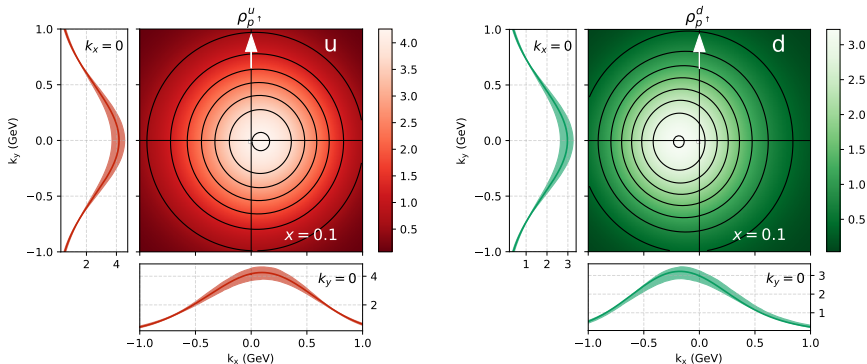
For unpolarized protons, the distribution of unp. quarks is cylindrically symmetric

What happens if the proton is transversely polarized?

Same formalism can be used to have a consistent picture (125 data points)

Distortion in the transverse plane of the TMD quark distribution in a p^\uparrow

$$\Phi_{q/p^\uparrow}^{[\gamma^+]}(x, k_x, k_y) = f_1^q(x, k_T^2) - \frac{k_x}{M} f_{1T}^{\perp q}(x, k_T^2) \quad [Q^2 = 4 \text{ GeV}^2]$$



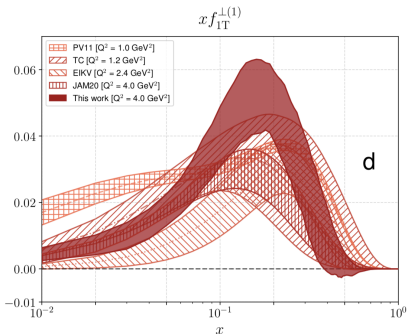
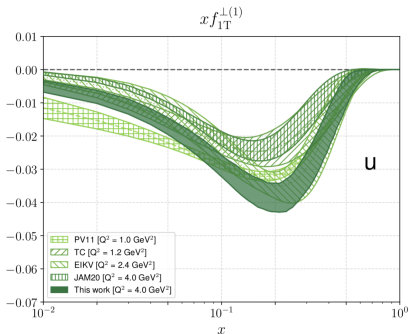
Bacchetta, Delcarro, Pisano, Radici, CP, PLB 827 (2022)

Non zero Siverts effect related to parton orbital angular momentum

The Sivvers function

First transverse moments

$$f_{1T}^{\perp(1)q}(x) = \int d^2\mathbf{k}_T \frac{k_T^2}{2M_p^2} f_{1T}^q(x, \mathbf{k}_T^2)$$

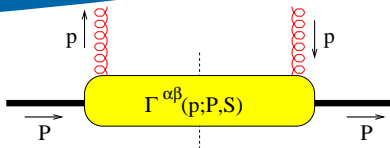


Bacchetta, Delcarro, Pisano, Radici, CP, PLB 827 (2022)

More data from CERN, JLab, EIC will help to reduce error bands and extend the ranges in x and Q^2

Gluon TMDs

The gluon-gluon correlation function



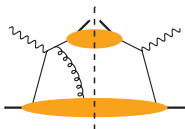
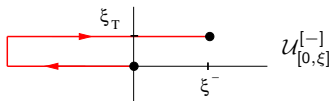
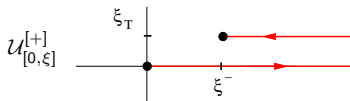
Gauge invariant definition of $\Gamma^{\mu\nu}$

$$\Gamma^{[\mathcal{U},\mathcal{U}']\mu\nu} \propto \langle P, S | \text{Tr}_c [F^{+\nu}(0) \mathcal{U}_{[0,\xi]}^C F^{+\mu}(\xi) \mathcal{U}_{[\xi,0]}^{C'}] | P, S \rangle$$

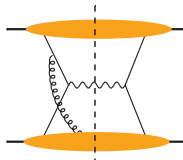
Mulders, Rodrigues, PRD 63 (2001)

Buffing, Mukherjee, Mulders, PRD 88 (2013)

Boer, Cotogno, Van Daal, Mulders, Signori, Zhou, JHEP 1610 (2016)



FSI in SIDIS



ISI in DY

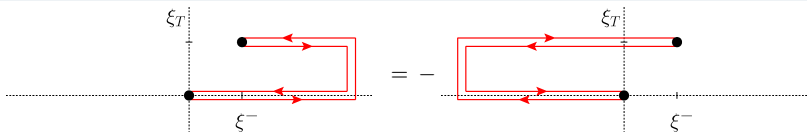
Related Processes

$ep^\dagger \rightarrow e' Q\bar{Q}X$, $ep^\dagger \rightarrow e'$ jet jet X probe GSF with $[++]$ gauge links (WW)

$p^\dagger p \rightarrow \gamma\gamma X$ (and/or other CS final state) probe GSF with $[--]$ gauge links

Analogue of the sign change of $f_{1T}^{\perp q}$ between SIDIS and DY (true also for h_1^g and $h_{1T}^{\perp g}$)

$$f_{1T}^{\perp g}[ep^\dagger \rightarrow e' Q\bar{Q}X] = -f_{1T}^{\perp g}[p^\dagger p \rightarrow \gamma\gamma X]$$



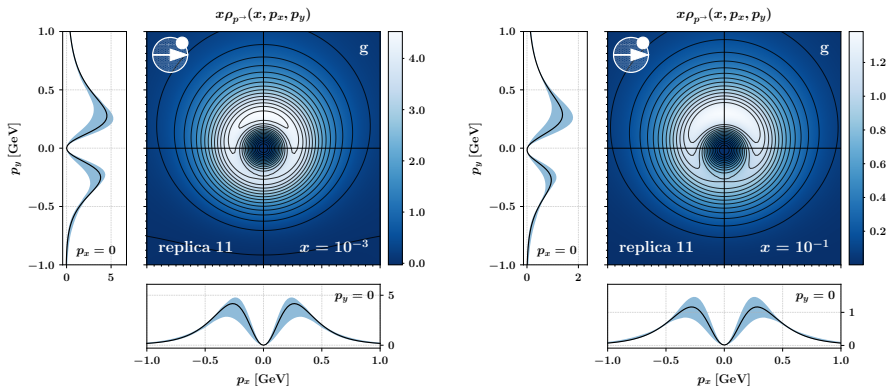
Boer, Mulders, CP, Zhou (2016)

Motivation to study gluon Sivers effects at both RHIC and the EIC

In contrast to quark TMDs, gluon TMDs are almost unknown, **however models exist:**

Bacchetta, Celiberto, Radici, Tels, EPJC 80 (2020)
 Chakrabarti, Choudhary, Gurjar, Kishore, Maji, Mondal, Mukherjee, PRD 108 (2023)
 Bacchetta, Celiberto, Radici, EPJC 84 (2024)

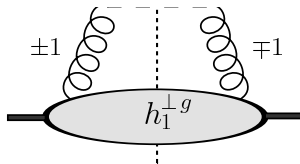
Distortion in the transverse plane expected also for the TMD gluon distribution in a $p \rightarrow$



Bacchetta, Celiberto, Radici, EPJC 84 (2024)

Gluons inside an unpolarized hadron can be linearly polarized

It requires nonzero transverse momentum



Interference between ± 1 gluon helicity states

Like the unpolarized gluon TMD, it is T -even and exists in different versions:

- ▶ $[++] = [--]$ (WW) (SIDIS and DY-like process)

Gluons can be probed in heavy quark production in both ep and pp scattering

Mukherjee, Rajesh, EPJC 77 (2017)
Lansberg, CP, Scarpa, Schlegel, PLB 784 (2018)
Rajesh, Kishore, Mukherjee, PRD 98 (2018)
Bacchetta, Boer, CP, Taels, EPJC 80 (2020)

Quarkonium production at the LHC

C = +1 quarkonium production

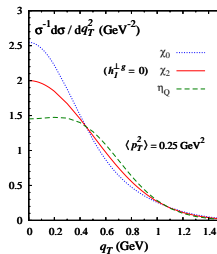
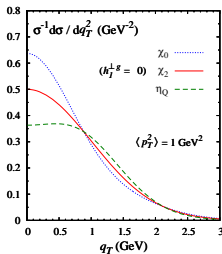
q_T -distribution of η_Q and χ_{QJ} ($Q = c, b$) in the kinematic region $q_T \ll 2M_Q$

$$\frac{1}{\sigma(\eta_Q)} \frac{d\sigma(\eta_Q)}{dq_T^2} \propto f_1^g \otimes f_1^g [1 - R(q_T^2)] \quad [\text{pseudoscalar}] \quad R(q_T^2) = \frac{h_1^{\perp g} \otimes h_1^{\perp g}}{f_1^g \otimes f_1^g}$$

$$\frac{1}{\sigma(\chi_{Q0})} \frac{d\sigma(\chi_{Q0})}{dq_T^2} \propto f_1^g \otimes f_1^g [1 + R(q_T^2)] \quad [\text{scalar}]$$

$$\frac{1}{\sigma(\chi_{Q2})} \frac{d\sigma(\chi_{Q2})}{dq_T^2} \propto f_1^g \otimes f_1^g$$

Boer, CP, PRD 86 (2012)



Proof of factorization at NLO for $pp \rightarrow \eta_Q X$ in the Color Singlet Model (CSM)

Ma, Wang, Zhao, PRD 88 (2013); PLB 737 (2014)

Echevarria, JHEP 1910 (2019)

Future (polarized) fixed target experiments at LHC

Talk by N. Kato

$$\frac{d\sigma}{dQdYd^2q_Td\Omega} \approx A f_1^g \otimes f_1^g + B f_1^g \otimes h_1^{\perp g} \cos(2\phi_{CS}) + C h_1^{\perp g} \otimes h_1^{\perp g} \cos(4\phi_{CS})$$

den Dunnen, Lansberg, CP, Schlegel, PRL 112 (2014)

- ▶ valid up to corrections $\mathcal{O}(q_T/Q)$
- ▶ Y : rapidity of the J/ψ -pair, along the beam in the hadronic c.m. frame
- ▶ $d\Omega = d\cos\theta_{CS} d\phi_{CS}$: solid angle for J/ψ -pair in the Collins-Soper frame

Analysis similar to the one for $pp \rightarrow J/\psi \gamma^* X$ and $pp \rightarrow J/\psi J/\psi X$

Lansberg, CP, Schlegel, NPB 920 (2017)

Lansberg, CP, Scarpa, Schlegel, PLB 784 (2018)

The three contributions can be disentangled by defining the transverse moments

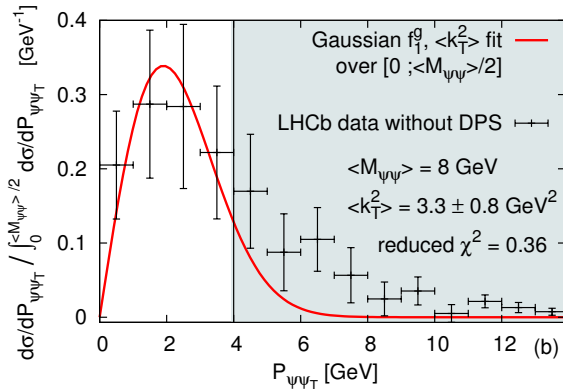
$$\langle \cos n\phi_{CS} \rangle \equiv \frac{\int_0^{2\pi} d\phi_{CS} \cos(n\phi_{CS}) \frac{d\sigma}{dQdYd^2q_Td\Omega}}{\int_0^{2\pi} d\phi_{CS} \frac{d\sigma}{dQdYd^2q_Td\Omega}} \quad (n = 2, 4)$$

$$\int d\phi_{CS} d\sigma \implies f_1^g \otimes f_1^g$$

$$\langle \cos 2\phi_{CS} \rangle \implies f_1^g \otimes h_1^{\perp g}$$

$$\langle \cos 4\phi_{CS} \rangle \implies h_1^{\perp g} \otimes h_1^{\perp g}$$

We consider $q_T = P_T^{\psi\psi} \leq M_{\psi\psi}/2$ in order to have two different scales

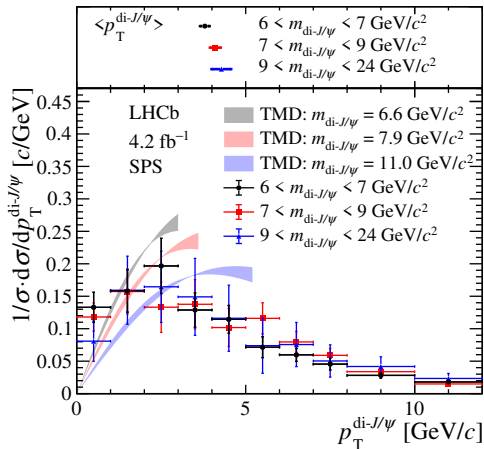


Lansberg, CP, Scarpa, Schlegel, PLB 784 (2018)
LHCb Coll., JHEP 06 (2017)

Gaussian model:

$$f_1^g(x, k_T^2) = \frac{f_1^g(x)}{\pi \langle k_T^2 \rangle} \exp\left(-\frac{k_T^2}{\langle k_T^2 \rangle}\right)$$

No obvious broadening can be seen due to the large uncertainties



LHCb Coll., 2311.14085

Scarpa, Boer, Echevarria, Lansberg, CP, Schlegel EPJC 80 (2020)

Talk by J. Bor

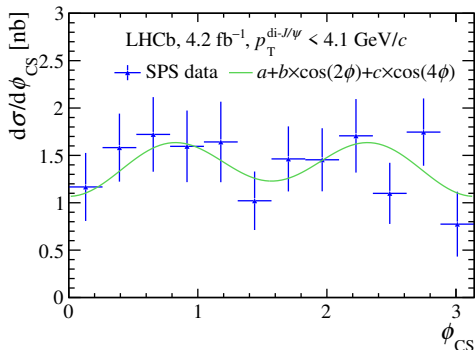
The average values of the p_T distributions slightly increase with mass

$$\langle \cos 2\phi \rangle = -0.029 \pm 0.050 \text{ (stat)} \pm 0.009 \text{ (syst)}$$

$$\langle \cos 4\phi \rangle = -0.087 \pm 0.052 \text{ (stat)} \pm 0.013 \text{ (syst)}$$

Theoretical predictions consistent with measurements

Scarpa, Boer, Echevarria, Lansberg, CP, Schlegel EPJC 80 (2020)



LHCb Coll., 2311.14085

The results are consistent with zero, but the presence of an azimuthal asymmetry at a few percent level is allowed

Quarkonium production at the EIC

$e p \rightarrow e J/\psi X$ (with the inclusion of TMD shape functions)

Mukherjee, Rajesh, EPJ.C 77 (2017)

Kishore, Mukherjee, PRD 99 (2019)

Bacchetta, Boer, CP, Taels, EPJ.C 80 (2020)

Boer, Bor, Maxia, CP, Yuan, JHEP 08 (2023)

$e p \rightarrow e J/\psi \text{jet} X$

D'Alesio, Murgia, CP, Taels, PRD 100 (2019)

Kishore, Mukherjee, Pawar, Siddiqah, PRD 106 (2022)

Maxia, Yuan, 2403.02097

Talk by L. Maxia

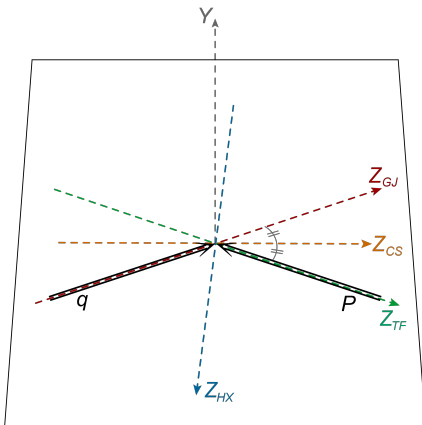
$e p \rightarrow e J/\psi \gamma X$

Chakrabarti, Kishore, Mukherjee, Rajesh, PRD 107 (2023)

$e p \rightarrow e J/\psi \pi X$

Banu, Mukherjee, Pawar, Rajesh, 2406.00271

We study $\gamma^*(q) + p(P) \rightarrow J/\psi(P_\psi) + X$ in the J/ψ rest frame



HX: Helicity

TF: Target

CS: Collins-Soper

GJ: Gottfried-Jackson

The frames are related to each other by a rotation around the Y axis

Model-independent arguments (gauge invariance, hermiticity, parity conservation) lead to eight independent helicity structure functions:

Lam, Tung, PRD 18 (1978)
Boer, Vogelsang, PRD 74 (2006)

$$\mathcal{W}_T^{\mathcal{P}} \equiv \mathcal{W}_{11}^{\mathcal{P}} = \mathcal{W}_{-1-1}^{\mathcal{P}}$$

$$\mathcal{W}_L^{\mathcal{P}} \equiv \mathcal{W}_{00}^{\mathcal{P}}$$

$$\mathcal{W}_{\Delta}^{\mathcal{P}} \equiv \sqrt{2} \operatorname{Re} \mathcal{W}_{10}^{\mathcal{P}}$$

$$\mathcal{W}_{\Delta\Delta}^{\mathcal{P}} \equiv \mathcal{W}_{1-1}^{\mathcal{P}} = \mathcal{W}_{-11}^{\mathcal{P}}$$

- ▶ $\mathcal{P} = \perp, \parallel$: γ^* polarization (w.r.t. P, q)
- ▶ $\Lambda = T, L, \Delta, \Delta\Delta$: J/ψ helicity

However, by looking at the angular dependence of the decaying leptons only four linear combinations can be disentangled

$$\mathcal{W}_{\Lambda} \equiv \left[1 + (1 - y)^2\right] \mathcal{W}_{\Lambda}^{\perp} + (1 - y) \mathcal{W}_{\Lambda}^{\parallel} \quad \text{with} \quad \Lambda = T, L, \Delta, \Delta\Delta$$

Usual SIDIS variables:

$$Q^2 = -q^2, \quad x_B = \frac{Q^2}{2P \cdot q}, \quad y = \frac{P \cdot q}{P \cdot \ell}, \quad z = \frac{P \cdot P_\psi}{P \cdot q}$$

Cross section differential in $\Omega = (\theta, \varphi)$, solid angle of the decaying lepton ℓ^+

$$d\sigma \equiv \frac{d\sigma}{dx_B dy d^4P_\psi d\Omega}$$

$$d\sigma \propto \frac{\alpha^2}{yQ^2} \left[\mathcal{W}_T(1 + \cos^2 \theta) + \mathcal{W}_L(1 - \cos^2 \theta) + \mathcal{W}_\Delta \sin 2\theta \cos \varphi + \mathcal{W}_{\Delta\Delta} \sin^2 \theta \cos 2\varphi \right]$$

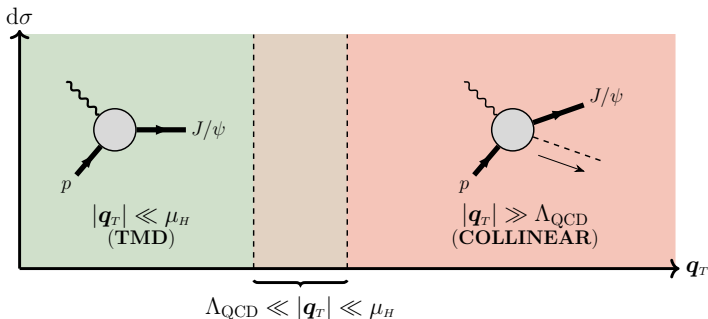
Alternatively, in terms of the polarization parameters λ, μ, ν :

$$d\sigma \propto \frac{\alpha^2}{yQ^2} (\mathcal{W}_T + \mathcal{W}_L) \left[1 + \lambda \cos^2 \theta + \mu \sin 2\theta \cos \varphi + \frac{1}{2} \nu \sin^2 \theta \cos 2\varphi \right]$$

$$\lambda = \frac{\mathcal{W}_T - \mathcal{W}_L}{\mathcal{W}_T + \mathcal{W}_L}, \quad \mu = \frac{\mathcal{W}_\Delta}{\mathcal{W}_T + \mathcal{W}_L}, \quad \nu = \frac{2\mathcal{W}_{\Delta\Delta}}{\mathcal{W}_T + \mathcal{W}_L}$$

Three physical scales, two theoretical tools

Bacchetta, Boer, Diehl, Mulders, JHEP 08 (2008)
Boer, D'Alesio, Murgia, CP, Tael, JHEP 09 (2020)
D'Alesio, Maxia, Murgia, CP, Rajesh, JHEP 037 (2022)
Boer, Bor, Maxia, CP, Yuan, JHEP 08 (2023)

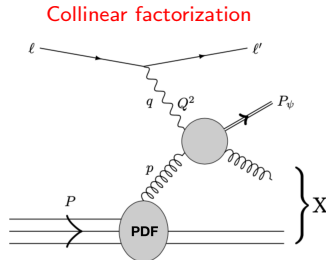


TMD factorization proven only for light hadron production in SIDIS

Matching in the intermediate region: a test of TMD factorization

NRQCD is applicable at high $P_{\psi T} \geq 2M_c$, unmodified by soft gluon emission

$P_{\psi T}$ generated in the hard part through recoil of unobserved particles



Infrared divergences are parametrized in terms of universal LDMEs and PDFs

$$\sigma = \sum_n H^{[n]} \otimes f_1(x) \otimes \langle \mathcal{O}^{[n]} \rangle$$

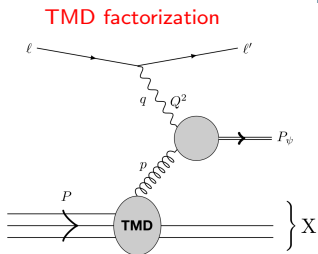
NRQCD: double expansion in α_s and v

At small $P_{\psi T}$, soft gluons cannot be factorized, TMD factorization needed

LDMEs promoted to TMD shape functions: $\langle \mathcal{O}^{[n]} \rangle \longrightarrow \Delta^{[n]}(k_T)$, which encode two soft mechanisms: formation of the bound state + soft gluon radiation

Echevarria, JHEP 10 (2019)

Fleming, Makris, Mehen, JHEP 04 (2020)



TMD PDFs depend on a factorization and a rapidity scale and are not universal

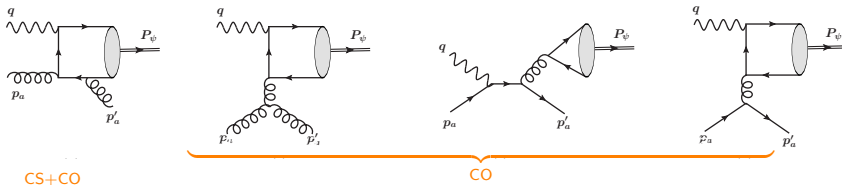
$$d\sigma = \sum_n H^{[n]} \otimes \mathcal{C}[f_1 \Delta^{[n]}](x, q_T)$$

\mathcal{C} : convolution in momentum space

The helicity structure functions can be calculated within the NRQCD framework

Contributing partonic subprocesses at the orders α_s^2 and v^4

$$\gamma^*(q) + a(p_a) \rightarrow J/\psi(P_\psi) + a(p'_a) \quad a = g, q, \bar{q}$$



Fock states included in the calculation: $3S_1^{[1]}$, $1S_0^{[8]}$, $3S_1^{[8]}$, $3P_0^{[8]}$

($Q^2 = 0$) Beneke, Kramer, Vanttinen, PRD 57 (1998)
 (W_T, W_L) Yuan, Chao, PRD 63 (2001)
 (unpolarized) Kniehl, Zwirner, NPB 621 (2002)

q_T : transverse momentum of the photon w.r.t. P_ψ, P

Frame-independent leading power behavior of the structure functions up to corrections of $\mathcal{O}(\Lambda_{\text{QCD}}/|\mathbf{q}_T|)$, $\mathcal{O}(|\mathbf{q}_T|/Q)$ in the region $\Lambda_{\text{QCD}}^2 \ll \mathbf{q}_T^2 \ll Q^2$

$$\mathcal{W}_T^\perp = \widehat{w}_T^\perp \frac{1}{\mathbf{q}_T^2} \left[L \left(\frac{Q^2 + M_\psi^2}{\mathbf{q}_T^2} \right) f_1^g(x, \mu^2) + (P_{gg} \otimes f_1^g + P_{gi} \otimes f_1^i)(x, \mu^2) \right]$$

$$\mathcal{W}_L^\perp = \widehat{w}_L^\perp \frac{1}{\mathbf{q}_T^2} \left[L \left(\frac{Q^2 + M_\psi^2}{\mathbf{q}_T^2} \right) f_1^g(x, \mu^2) + (P_{gg} \otimes f_1^g + P_{gi} \otimes f_1^i)(x, \mu^2) \right]$$

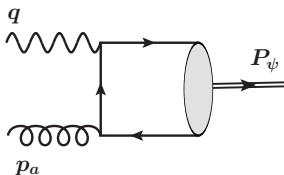
$$\mathcal{W}_L^\parallel = \widehat{w}_L^\parallel \frac{1}{\mathbf{q}_T^2} \left[L \left(\frac{Q^2 + M_\psi^2}{\mathbf{q}_T^2} \right) f_1^g(x, \mu^2) + (P_{gg} \otimes f_1^g + P_{gi} \otimes f_1^i)(x, \mu^2) \right]$$

$$\mathcal{W}_{\Delta\Delta}^\perp = \widehat{w}_{\Delta\Delta}^\perp \frac{1}{\mathbf{q}_T^2} \left(\delta P_{gg} \otimes f_1^g + \delta P_{gi} \otimes f_1^i \right)(x, \mu^2)$$

$\widehat{w}_\Lambda^{\mathcal{P}}$: partonic structure functions for $\gamma^* \mathcal{P} g \rightarrow J/\psi \Lambda$: depend on NRQCD LDMEs

$$L \left(\frac{Q^2 + M_\psi^2}{\mathbf{q}_T^2} \right) \equiv C_A \left[\ln \frac{Q^2 + M_\psi^2}{\mathbf{q}_T^2} - 1 - \ln \frac{M_\psi^2}{M_\psi^2 + Q^2} \right] - \frac{11C_A - 4n_f T_R}{6}$$

When $q_T^2 \ll Q^2$ at $\mathcal{O}(\alpha_s)$ only color-octet (CO) production channels dominate



Neglecting smearing effects in quarkonium formation:

$$\mathcal{W}_T^\perp = \hat{w}_T^\perp f_1^g(x, \mathbf{q}_T^2) \quad \mathcal{W}_T^\parallel = \hat{w}_T^\parallel f_1^g(x, \mathbf{q}_T^2) \quad \mathcal{W}_L^\parallel = \hat{w}_L^\parallel f_1^g(x, \mathbf{q}_T^2)$$

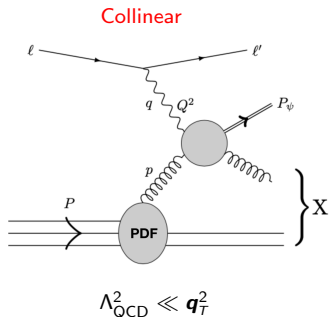
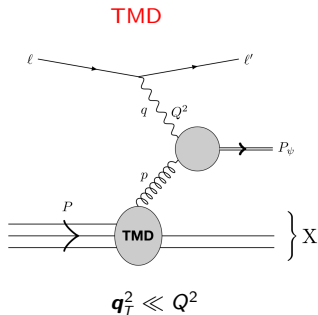
$$\mathcal{W}_{\Delta\Delta}^\perp = \hat{w}_{\Delta\Delta}^\perp h_1^{\perp g}(x, \mathbf{q}_T^2)$$

$\mathcal{W}_{\Delta\Delta}^\perp$ gives access to $h_1^{\perp g}$ and to the poorly known 3P_0 LDME

Smearing effects encoded in $\Delta^{[n]}$ need to be included to match the result in the intermediate overlapping region $\Lambda_{\text{QCD}}^2 \ll \mathbf{q}_T^2 \ll Q^2$

Imposing the matching of the TMD and collinear results in the overlapping region

$$\Lambda_{\text{QCD}}^2 \ll \mathbf{q}_T^2 \ll Q^2: \quad f_1^{\mathcal{G}} \rightarrow \mathcal{C}[f_1^{\mathcal{G}} \Delta^{[n]}]$$



Boer, D'Alesio, Murgia, CP, Taels, JHEP 09 (2020)
 D'Alesio, Maxia, Murgia, CP, Rajesh, JHEP 037 (2022)
 Boer, Bor, Maxia, CP, Yuan, JHEP 08 (2023)

Knowing the perturbative tail of the gluon TMD, we determine the one of $\Delta^{[n]}$

Factorization scale fixed to be: $\mu^2 = M_\psi^2 + Q^2$

$$\Delta^{[n]}(k_T^2, \mu^2) = -\frac{\alpha_S}{2\pi^2 k_T^2} C_A \left(1 + \log \frac{M_\psi^2}{M_\psi^2 + Q^2} \right) \langle \mathcal{O}^{[n]} \rangle \quad \text{for } k_T \gg \Lambda_{\text{QCD}}$$

Boer, Bor, Maxia, CP, Yuan, JHEP 08 (2023)

Less divergent than fragmentation functions of light quarks $\propto \log Q^2/k_T^2$

Independent of J/ψ polarization and CO quantum numbers

It should not depend on Q^2 : hint of process dependence (photoproduction result is obtained by imposing $Q^2 = 0$)

- ▶ Unpolarized quark TMDs of the proton are quite well-known, theoretical analysis can be improved (by looking at Y -term, flavor dependence, ...)
- ▶ Azimuthal asymmetries in J/ψ production in SIDIS could probe WW-type gluon TMDs (similar to SIDIS for quark TMDs)
- ▶ Quarkonia as well are good probes for gluon TMDs: first extraction of unpolarized gluon TMD from LHC data on di- J/ψ production
- ▶ Different behavior of WW and dipole gluon TMDs accessible at RHIC, LHCspin and at EIC, overlap of both *spin* and *small-x* programs

Talk by D. Boer