Spin asymmetries in quarkonium production

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Based on: arXiv:2403.20017 in collaboration with Luca Maxia and Cristian Pisano

Synergies between LHC and EIC for quarkonium phyiscs
ECT*, Trento, July 8-12th, 2024

Outline

- Introduction
- Gluon TMDs
- C-even quarkonium production mechanism
- Results
- Summary

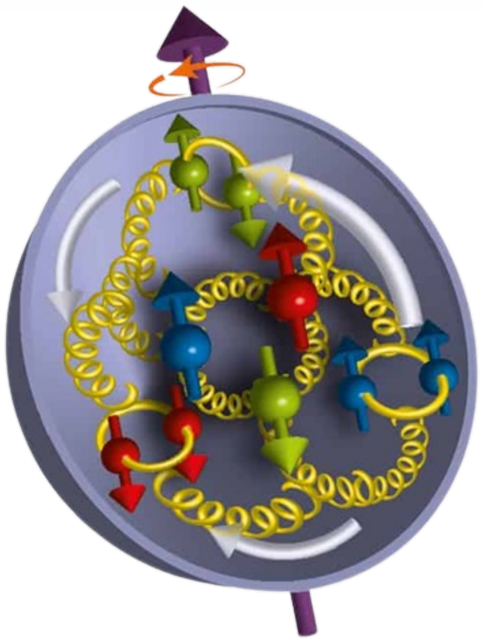
Introduction

- Studying the production of *C*-even quarkonia in *pp* collisions is a useful tool for probing gluon TMDs

- Gluon TMDs are still poorly known: since they encode the information on the intrinsic motion of the gluons inside hadrons, their knowledge is a key ingredient to understand polarization phenomena

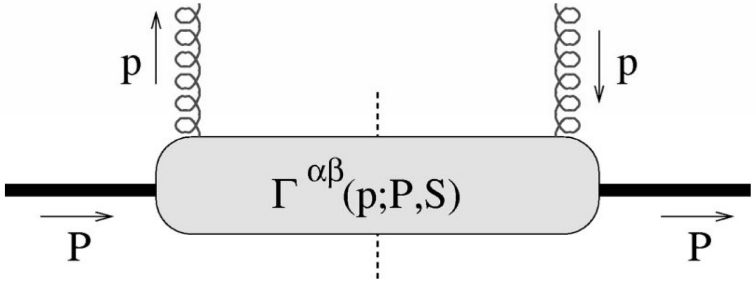
- TMD factorization + NRQCD

- Single spin asymmetries (SSAs)



Gluon TMDs

Gluon correlator



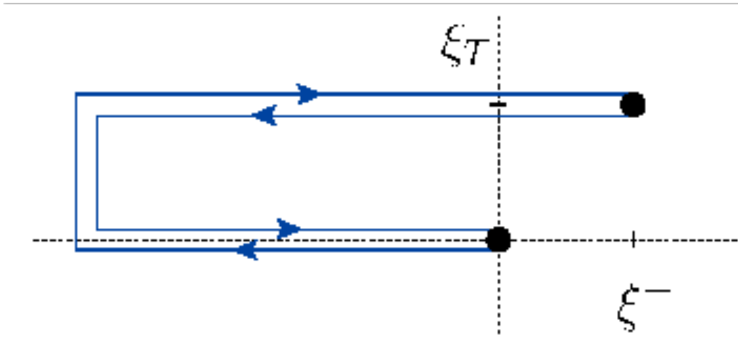
Mulders, Rodrigues, PRD 63 (2001)
Buffing, Mukherjee, Mulders, PRD 88 (2013)
Boer, Cotogno, Van Daal, Mulders, Signori, Zhou, JHEP 1610 (2016)

Gauge invariant definition of the gluon correlator

$$\Gamma_g^{[U,U']\alpha\beta}(x,\boldsymbol{p}_T) \propto \langle P,S|\ \mathrm{Tr}\left[\, F^{\alpha+}(0)\, U_{[0,\xi]} F^{\beta+}(\xi)\, U'_{[\xi,0]}\,\right] |P,S\rangle\, \big]_{\mathrm{LF}}$$

Gauge link:

$$\mathcal{U}^{\mathcal{C}}_{[0,\xi]} = \mathcal{P}\exp\Big(-ig\int_{\mathcal{C}[0,\xi]} ds_\mu A^\mu(s)\Big)$$



$\mathcal{U}^{[-]}_{[0,\xi]}$ gauge link

Gluon TMDs

GLUONS	*unpolarized*	*circular*	*linear*
U	f_1^g		$h_1^{\perp g}$
L		g_{1L}^g	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	g_{1T}^g	h_{1T}^g, $h_{1T}^{\perp g}$

T-even

T-odd

Angeles-Martinez et al., *Acta Phys., Pol B46 (2015)*

$h_1^{\perp g}$: linearly polarized gluon distribution in unpolarized hadron

$f_{1T}^{\perp g}$: gluon Sivers function in transversely polarized hadron

$h_{1T}^g, h_{1T}^{\perp g}$: helicity flip distributions

$h_1^g \equiv h_{1T}^g + \frac{\boldsymbol{p}_T^2}{2M_p^2} h_{1T}^{\perp g}$: vanish under p_T integration

Gluon TMDs

Parametrization of correlators

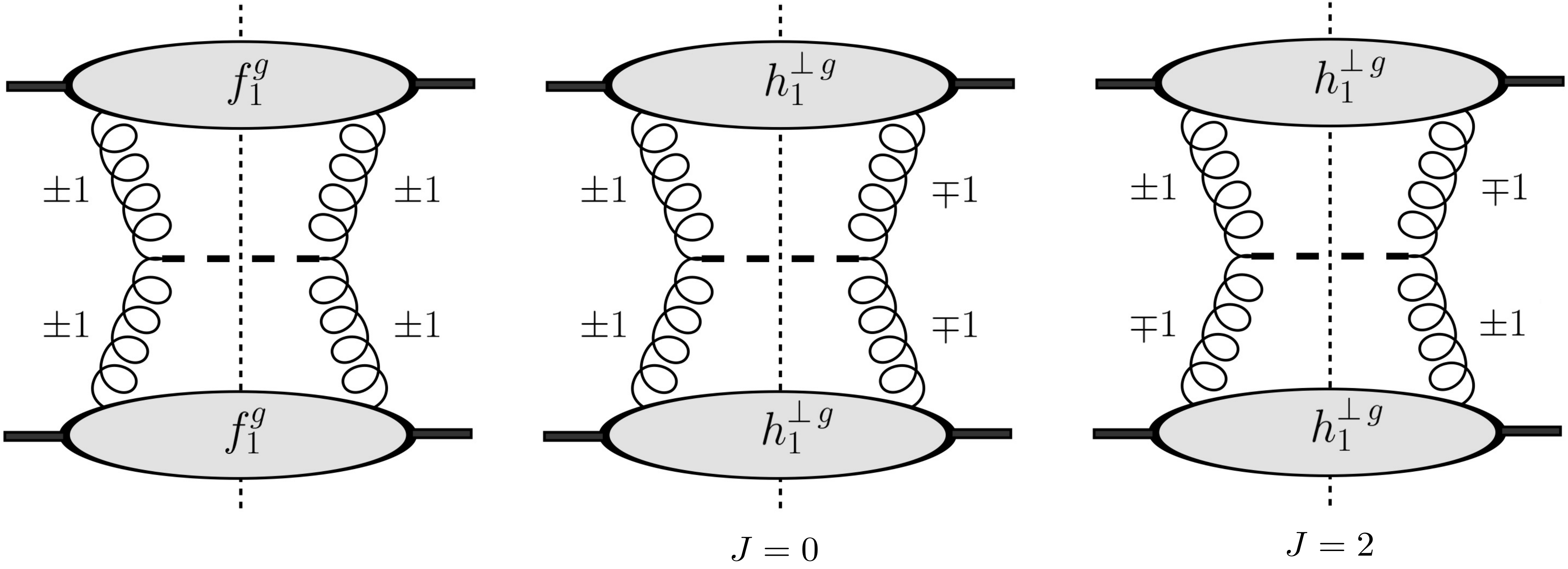
$$\Gamma_U^{\mu\nu}(x,\boldsymbol{p}_T) = \frac{1}{2x}\left\{ -\, g_T^{\mu\nu}\, f_1^g(x,\boldsymbol{p}_T^2) + \left(\frac{p_T^\mu p_T^\nu}{M_h^2} + g_T^{\mu\nu}\,\frac{\boldsymbol{p}_T^2}{2M_h^2}\right) h_1^{\perp\, g}(x,\boldsymbol{p}_T^2)\right\}$$

$$\Gamma_L^{\mu\nu}(x,\boldsymbol{p}_T) = \frac{1}{2x}\, S_L\, \left\{ i\epsilon_T^{\mu\nu}\, g_{1L}^g(x,\boldsymbol{p}_T^2) \,+\, \frac{\epsilon_T^{p_T\{\mu} p_T^{\nu\}}}{M_h^2}\, h_{1L}^{\perp\, g}(x,\boldsymbol{p}_T^2)\right\}$$

$$\Gamma_T^{\mu\nu}(x,\boldsymbol{p}_T) = \frac{1}{2x}\left\{ g_T^{\mu\nu}\,\frac{\epsilon_T^{p_T S_T}}{M_h}\, f_{1T}^{\perp\, g}(x,\boldsymbol{p}_T^2) \,+\, i\epsilon_T^{\mu\nu}\,\frac{p_T\cdot S_T}{M_h}\, g_{1T}^{\perp\, g}(x,\boldsymbol{p}_T^2) \,-\, \frac{\epsilon_T^{p_T\{\mu} S_T^{\nu\}} \,+\, \epsilon_T^{S_T\{\mu} p_T^{\nu\}}}{4M_h}\, h_1^g(x,\boldsymbol{p}_T^2) \right.$$
$$\left. +\, \frac{4\,(p_T\cdot S_T)\,\epsilon_T^{p_T\{\mu} p_T^{\nu\}} + \boldsymbol{p}_T^2\left[\epsilon_T^{p_T\{\mu} S_T^{\nu\}} + \epsilon_T^{S_T\{\mu} p_T^{\nu\}}\right]}{8M_h^3}\, h_{1T}^{\perp\, g}(x,\boldsymbol{p}_T^2)\right\}$$

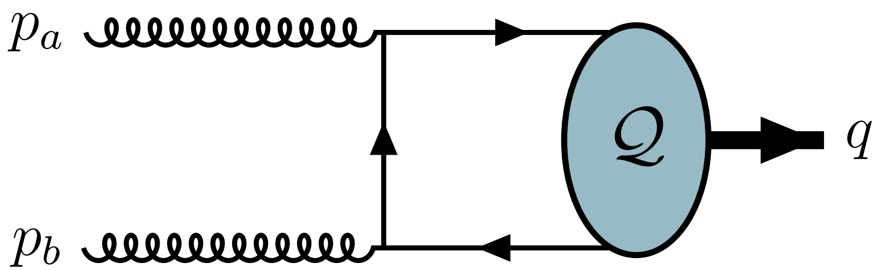
Introduction

Gluon helicities in the squared amplitude for J=0 and J=2 quarkonium production



Boer, Pisano, PRD 86 (2012)

NRQCD



Non Relativistic QCD (NRQCD)

- Double power series expansion → α_S, v
- Hard process calculated perturbatively
- Soft process given by LDMEs

$$v_c^2 \simeq 0.3$$
$$v_b^2 \simeq 0.1$$

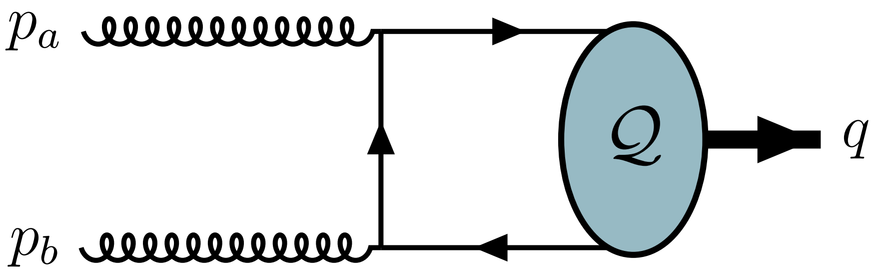
Bodwin, Braaten, Lepage, PRD 51 (1995)

C-even quarkonia

For this kind of quarkonium states Color-Singlet production mechanism dominates (CSM):

$$p(P_A, S_A) + p(P_B, S_B) \;\rightarrow\; Q\overline{Q}[^{2S+1}L_J^{(1)}](q) + X$$

Leading order diagram:



$$g(p_a) + g(p_b) \;\rightarrow\; Q\overline{Q}[^{2S+1}L_J^{(1)}](q)$$

$$d\sigma \propto \Gamma_g^{\mu\nu}(x_a, \boldsymbol{p}_{aT})\,\Gamma_g^{\rho\sigma}(x_b, \boldsymbol{p}_{bT})\,\mathcal{A}_{\mu\rho}\,(\mathcal{A}_{\nu\sigma})^*$$

Amplitudes using NRQCD

General expression for the amplitudes:

contains the LDME

$$\mathcal{A}(gg \to Q\bar{Q}[^{2S+1}L_J^{(1)}])(p_a, p_b, q) = \int \frac{d^4k}{(2\pi)^4} Tr[O(q,k)\,\phi(q,k)]$$

Baier, Ruckl, Z. Phys. C 19 (1983)
Boer, Pisano, PRD 86 (2012)

calculable perturbatively

$$\mathcal{A}[^1S_0^{(1)}] \propto R_0(0)\epsilon^{\mu\nu\rho\sigma} p_{a\rho} p_{b\sigma}$$

$$\mathcal{A}[^3P_0^{(1)}] \propto R_1'(0)\left[-3g^{\mu\nu} + \frac{2}{M^2} q^\mu p_a^\nu\right]$$

$$\mathcal{A}[^3P_2^{(1)}] \propto_s R_1'(0)\varepsilon_{J_z}^{\rho\sigma}(q)\left[\frac{4}{M^2} g^{\mu\nu} p_{a\rho} p_{a\sigma} - g_\rho^\mu g_\sigma^\nu - g_\rho^\nu g_\sigma^\mu\right]$$

$$\langle 0|\, \mathcal{O}_1^{\eta_Q}(^1S_0)\, |0\rangle = \frac{N_c}{2\pi}|R_0(0)|^2[1+\mathcal{O}(v^4)]$$

$$\langle 0|\, \mathcal{O}_1^{\chi_{QJ}}(^3P_J)\, |0\rangle = \frac{3N_c}{2\pi}(2J+1)|R_1'(0)|^2[1+\mathcal{O}(v^2)]$$

$$d\sigma^{pp\to Q\bar{Q}} = \sum_n d\hat{\sigma}[gg \to Q\bar{Q}]\, \langle 0|\, \mathcal{O}_n(^{2S+1}L_J^{(1)})\, |0\rangle$$

Perturbative short-distance coefficients

Long distance matrix elements (LDME)

Cross sections

$$\frac{d\sigma(\eta_Q)}{dy d^2\boldsymbol{q}_T} \propto \mathcal{C}[f_1^g f_1^g][1 - R(\boldsymbol{q}_T^2)]$$

$$\frac{d\sigma(\chi_{Q0})}{dy d^2\boldsymbol{q}_T} \propto \mathcal{C}[f_1^g f_1^g][1 + R(\boldsymbol{q}_T^2)]$$

$$\frac{d\sigma(\chi_{Q2})}{dy d^2\boldsymbol{q}_T} \propto \mathcal{C}[f_1^g f_1^g]$$

$$R(\boldsymbol{q}_T^2) \equiv \frac{\mathcal{C}[w h_1^{\perp g} h_1^{\perp g}]}{\mathcal{C}[f_1^g f_1^g]}$$

Boer, Pisano, PRD 86 (2012)

The **convolution** is defined as:

$$\mathcal{C}[w\, F_1^g\, F_2^g] = \int d^2\boldsymbol{p}_{aT}\, d^2\boldsymbol{p}_{bT}\, w(\boldsymbol{p}_{aT}, \boldsymbol{p}_{bT})\, F_1^g(x_a, \boldsymbol{p}_{aT})\, F_2^g(x_b, \boldsymbol{p}_{bT})\, \delta^2(\boldsymbol{p}_{aT} + \boldsymbol{p}_{bT} - \boldsymbol{q}_T)$$

Cross sections

What's new??

$$\frac{\mathrm{d}\sigma[\mathcal{Q}]}{\mathrm{d}y\,\mathrm{d}^2\boldsymbol{q}_T} = F_{UU}^{\mathcal{Q}} + F_{UL}^{\mathcal{Q}}\,S_{BL} + F_{LU}^{\mathcal{Q}}\,S_{AL} + F_{UT}^{\mathcal{Q},\sin\phi_{S_B}}\,|\boldsymbol{S}_{BT}|\sin\phi_{S_B} + F_{TU}^{\mathcal{Q},\sin\phi_{S_A}}\,|\boldsymbol{S}_{AT}|\sin\phi_{S_A}$$
$$+ F_{LL}^{\mathcal{Q}}\,S_{AL}\,S_{BL} + F_{LT}^{\mathcal{Q},\cos\phi_{S_B}}\,S_{AL}\,|\boldsymbol{S}_{BT}|\cos\phi_{S_B} + F_{TL}^{\mathcal{Q},\cos\phi_{S_A}}\,|\boldsymbol{S}_{AT}|\,S_{BL}\cos\phi_{S_A}$$
$$+ |\boldsymbol{S}_{AT}|\,|\boldsymbol{S}_{BT}|\left(F_{TT}^{\mathcal{Q},\cos(\phi_{S_A}-\phi_{S_B})}\cos(\phi_{S_A}-\phi_{S_B}) + F_{TT}^{\mathcal{Q},\cos(\phi_{S_A}+\phi_{S_B})}\cos(\phi_{S_A}+\phi_{S_B})\right)$$

Kato, Maxia, Pisano, 2403.20017 (submitted to PRD)

Each structure function can be factorized:

$F^{\mathcal{Q}}$ →

$$H^{\mathcal{Q}} = \sum_{n=0}^{\infty}\left(\frac{\alpha_s}{\pi}\right)^n H_n^{\mathcal{Q}}$$

perturbative part

$$\mathcal{C}[w\,F_1^g\,F_2^g]$$

LDME

non-perturbative part

Structure functions

Unpolarized and single-transversely polarized structure functions

$$F_{UU}^{\eta_Q} \propto \Big(\mathcal{C}\big[f_1^g f_1^g\big] - \mathcal{C}\big[w_{UU}\, h_1^{\perp\, g} h_1^{\perp\, g}\big]\Big)$$

$$F_{UU}^{\chi_{Q0}} \propto \Big(\mathcal{C}\Big[f_1^g f_1^g\big] + \mathcal{C}\big[w_{UU}\, h_1^{\perp\, g} h_1^{\perp\, g}\big]\Big)$$

$$F_{UU}^{\chi_{Q2}} \propto \mathcal{C}\big[f_1^g f_1^g\big]$$

$$F_{UT}^{\eta_Q, \sin\phi_{S_B}} \propto \Big(- \mathcal{C}\big[w_{UT}^f\, f_1^g f_{1T}^{\perp\, g}\big] + \mathcal{C}\big[w_{UT}^h\, h_1^{\perp\, g} h_1^g\big] - \mathcal{C}\big[w_{UT}^{h^\perp}\, h_1^{\perp\, g} h_{1T}^{\perp\, g}\big]\Big)$$

$$F_{UT}^{\chi_{Q0}, \sin\phi_{S_B}} \propto \Big(- \mathcal{C}\big[w_{UT}^f\, f_1^g f_{1T}^{\perp\, g}\big] - \mathcal{C}\big[w_{UT}^h\, h_1^{\perp\, g} h_1^g\big] + \mathcal{C}\big[w_{UT}^{h^\perp}\, h_1^{\perp\, g} h_{1T}^{\perp\, g}\big]\Big)$$

$$F_{UT}^{\chi_{Q2}, \sin\phi_{S_B}} \propto -\mathcal{C}\big[w_{UT}^f\, f_1^g f_{1T}^{\perp\, g}\big]$$

$$w_{UU} = \frac{\boldsymbol{p}_{aT}^2 \boldsymbol{p}_{bT}^2}{4M_p^4} \cos\big[2(\phi_a - \phi_b)\big]$$

$$w_{UT}^f = \frac{|\boldsymbol{p}_{bT}|}{M_p} \cos\phi_b$$

$$w_{UT}^h = \frac{\boldsymbol{p}_{aT}^2\, |\boldsymbol{p}_{bT}|}{4M_p^3} \cos(\phi_b - 2\phi_a)$$

$$w_{UT}^{h^\perp} = \frac{\boldsymbol{p}_{aT}^2\, |\boldsymbol{p}_{bT}|^3}{8M_p^5} \cos(3\phi_b - 2\phi_a)$$

Observables (in principle) measurable at LHCSpin

Single Spin Asymmetries (SSAs)

Proton A unpolarized
Proton B **transversely polarized**

$$A_N^{\mathcal{Q},\sin\phi_S} = 2\frac{\int d\phi_S \sin\phi_S [d\sigma(\phi_S) - d\sigma(\phi_S+\pi)]}{\int d\phi_S [d\sigma(\phi_S) + d\sigma(\phi_S+\pi)]} = \frac{F_{UT}^{\mathcal{Q},\sin\phi_S}}{F_{UU}^{\mathcal{Q}}}$$

Upper bounds for SSAs using **gaussian parameterization**

$$f_1^g(x, \boldsymbol{p}_T^2) = \frac{f_1^g(x)}{\pi\langle p_T^2\rangle} \exp\left[-\frac{\boldsymbol{p}_T^2}{\langle p_T^2\rangle}\right]$$

unpolarized TMD

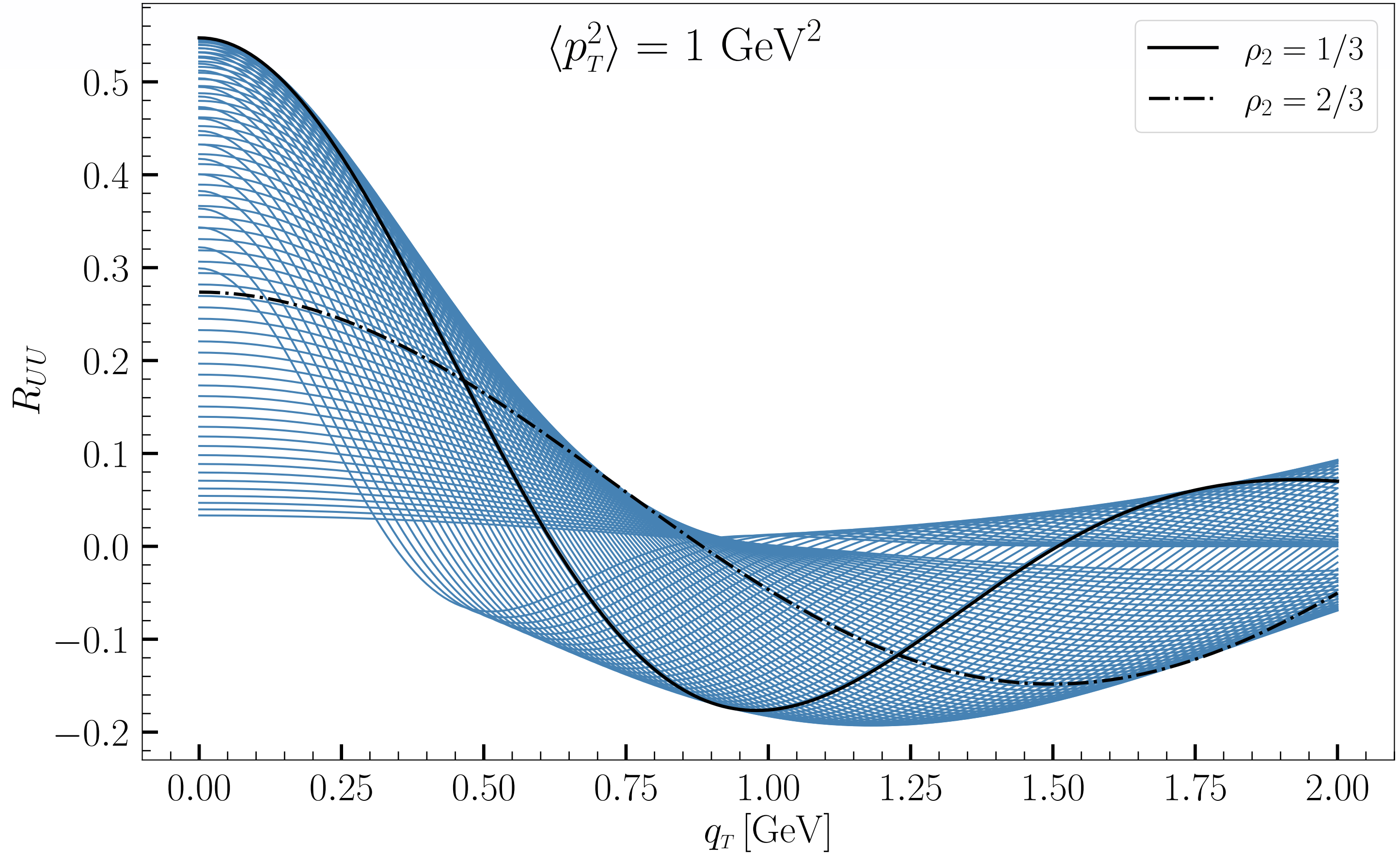
Positivity bounds

$$\left|f_{1T}^{\perp\,g}(x,\boldsymbol{p}_T^2)\right|,\ \left|h_1^g(x,\boldsymbol{p}_T^2)\right| \le \frac{M_p}{|\boldsymbol{p}_T|} f_1^g(x,\boldsymbol{p}_T^2)\,,$$

$$\frac{1}{2}\left|h_1^{\perp\,g}(x,\boldsymbol{p}_T^2)\right| \le \frac{M_p^2}{\boldsymbol{p}_T^2} f_1^g(x,\boldsymbol{p}_T^2)\,,$$

$$\frac{1}{2}\left|h_{1T}^{\perp\,g}(x,\boldsymbol{p}_T^2)\right| \le \frac{M_p^3}{|\boldsymbol{p}_T|^3} f_1^g(x,\boldsymbol{p}_T^2)\,.$$

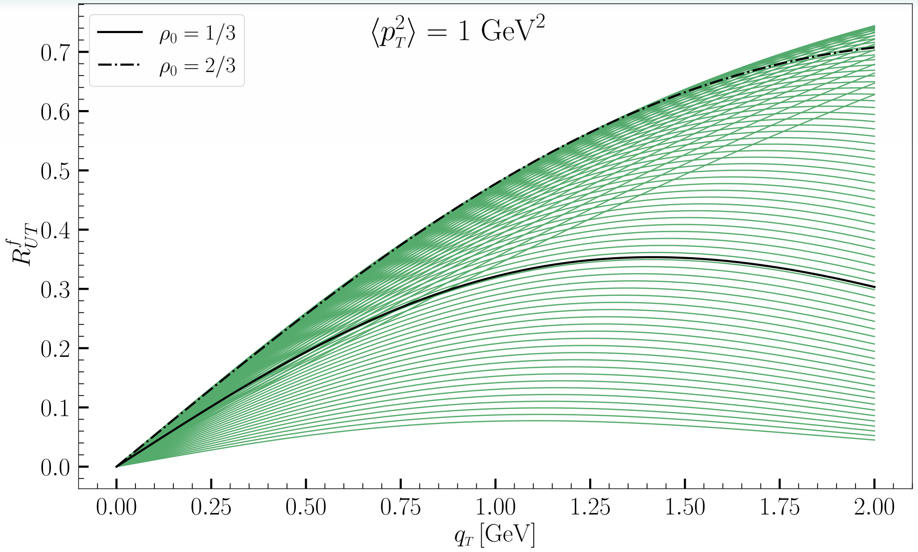
Results



$$R_{UU} = \frac{\mathcal{C}\left[w_{UU}^h \, h_1^{\perp\, g} \, h_1^{\perp\, g}\right]}{\mathcal{C}\left[f_1^g \, f_1^g\right]}$$

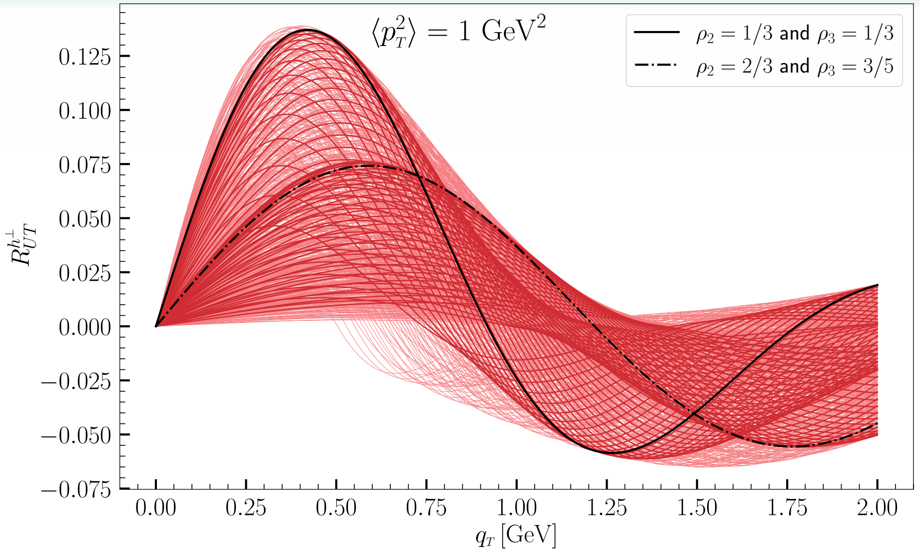
$$0.1 < \rho_2 < 0.9$$

Results



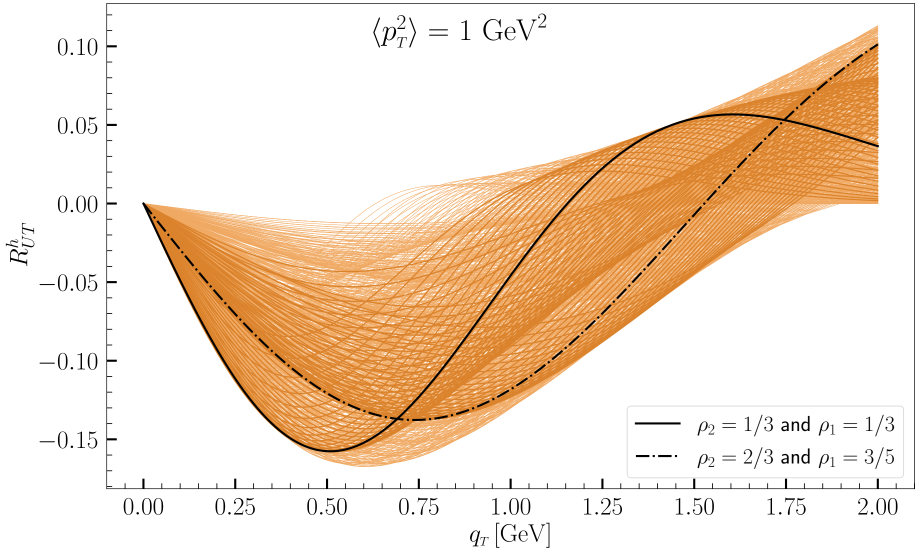
$$R_{UT}^{f} = \frac{\mathcal{C}\left[w_{UT}^{f}\, f_1^{g}\, f_{1T}^{\perp g}\right]}{\mathcal{C}\left[f_1^{g}\, f_1^{g}\right]}$$

$0.1 < \rho_0 < 0.9$



$$R_{UT}^{h} = \frac{\mathcal{C}\left[w_{UT}^{h}\, h_1^{\perp g}\, h_1^{g}\right]}{\mathcal{C}\left[f_1^{g}\, f_1^{g}\right]}$$

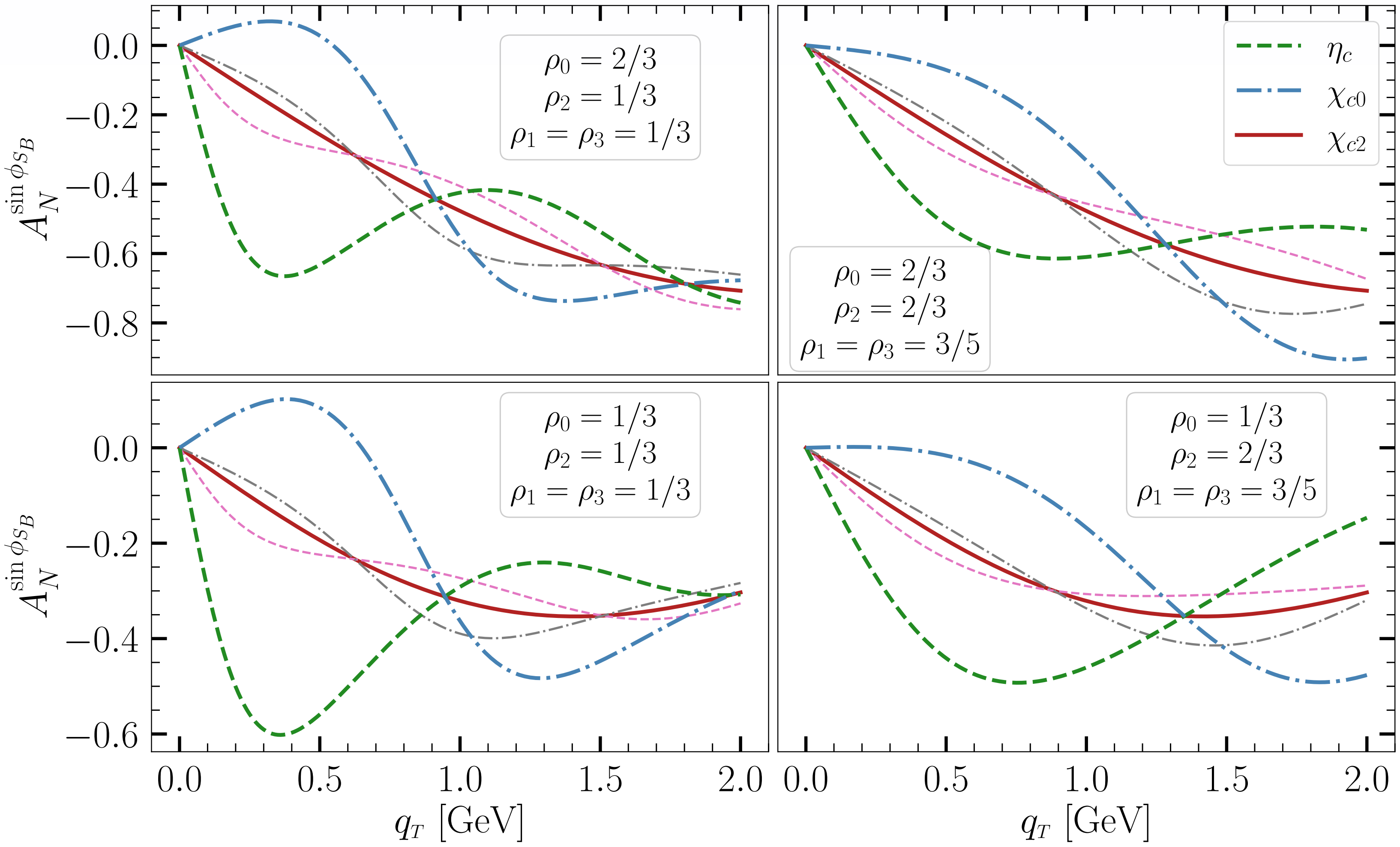
$0.1 < \rho_3 < 0.9$ for thicker lines



$$R_{UT}^{h\perp} = \frac{\mathcal{C}\left[w_{UT}^{h\perp}\, h_1^{\perp g}\, h_{1T}^{\perp g}\right]}{\mathcal{C}\left[f_1^{g}\, f_1^{g}\right]}$$

$0.1 < \rho_1 < 0.9$ for thicker lines

C-even quarkonia: upper bounds of SSAs



$$A_N^{\eta_Q, \sin \phi_{S_B}} = \frac{-R_{UT}^f + R_{UT}^h - R_{UT}^{h^\perp}}{1 - R_{UU}}$$

$$A_N^{\chi_{Q0}, \sin \phi_{S_B}} = \frac{-R_{UT}^f - R_{UT}^h + R_{UT}^{h^\perp}}{1 + R_{UU}}$$

$$A_N^{\chi_{Q2}, \sin \phi_{S_B}} = -R_{UT}^f$$

Summary of the talk

- *C*-even quarkonium production in *p*-*p* collisions
- Used NRQCD and CS mechanism
- Max values of transverse SSAs for different quarkonium states
- Asymmetries depend on the parametrization of the gluon TMDs but are independent of the LDMEs
- Observables measurable at LHCSpin
- Transverse and longitudinal double-spin asymmetries

Thank you for your attention!

Backup slides

Gaussian parametrization of the gluon TMDs

$$f_{1T}^{\perp\, g}(x, \boldsymbol{p}_T^2) = \mathcal{N}_0(x)\, \frac{f_1^g(x)}{\pi \langle p_T^2 \rangle^{3/2}}\, M_p \sqrt{\frac{2(1-\rho_0)}{\rho_0}}\, \exp\left[\frac{1}{2} - \frac{1}{\rho_0}\, \frac{\boldsymbol{p}_T^2}{\langle p_T^2 \rangle}\right]$$

$$h_1^g(x, \boldsymbol{p}_T^2) = \mathcal{N}_1(x)\, \frac{f_1^g(x)}{\pi \langle p_T^2 \rangle^{3/2}}\, M_p \sqrt{\frac{2(1-\rho_1)}{\rho_1}}\, \exp\left[\frac{1}{2} - \frac{1}{\rho_1}\, \frac{\boldsymbol{p}_T^2}{\langle p_T^2 \rangle}\right]$$

$$h_1^{\perp\, g}(x, \boldsymbol{p}_T^2) = 2\,\mathcal{N}_2(x)\, \frac{f_1^g(x)}{\pi \langle p_T^2 \rangle^{2}}\, M_p^2\, \frac{(1-\rho_2)}{\rho_2}\, \exp\left[1 - \frac{1}{\rho_2}\, \frac{\boldsymbol{p}_T^2}{\langle p_T^2 \rangle}\right]$$

$$h_{1T}^{\perp\, g}(x, \boldsymbol{p}_T^2) = 2\,\mathcal{N}_3(x)\, \frac{f_1^g(x)}{\pi \langle p_T^2 \rangle^{5/2}}\, M_p^3 \left[\frac{2(1-\rho_3)}{3\rho_3}\right]^{3/2} \exp\left[\frac{3}{2} - \frac{1}{\rho_3}\, \frac{\boldsymbol{p}_T^2}{\langle p_T^2 \rangle}\right]$$

$$0 < \rho_i < 1$$