Quarkonium production and TMDs in MadGraph5_aMC@NLO

Synergies between LHC and EIC for quarkonium physics

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Introduction: MadGraph5_aMC@NLO

MadGraph5_aMC@NLO = event / diagram generator (Python)
↪ compute matrix-element (helicity amplitude formalism)

- Total cross section
- Differential cross section
- Un-weighted event

⋆ Lagrangian
⇓
⋆Feynman Rules
⇓
⋆ Matrix Element
⇓
⋆ Parton Events
⇓
⋆Hadronise Events
⇓
⋆Detector Events

Master integral for symmetric collisions in MadGraph5:

$$\sigma(AA \to X) = \sum_{i,j} \int \mathrm{d}x_i \mathrm{d}x_j \mathrm{d}\Phi f_{i/A}(x_i) f_{j/A}(x_j) \hat{\sigma}_{(ab\to X)}(x_i, x_j, \mu_F, \mu_R)$$

How MadGraph5 works:

- Identify partonic processes and calculate partonic cross section
- Use PDFs (LHAPDF package)
- Do phase space integral and convolute with PDFs
- Generate events

Slide courtesy of L. Manna

NEW implementations:

- Asymmetric collisions (see talks by L. Manna and A. Safronov)
- This talk: **quarkonium production!**

There are many reasons why quarkonia are interesting:

- QCD studies!
- internal structure of nucleons
- gluon investigation (e.g. EIC) → TMDs!!
- ...

Focus on inclusive processes → factorisation:

$$\sigma(pp \to \mathcal{Q} + X) = \sum_{i,j,n} \int dx_1 dx_2 f_{i/p}(x_1) f_{j/p}(x_2) \times \hat{\sigma}(ij \to Q\bar{Q}[n] + X)\langle \mathcal{O}_n^{\mathcal{Q}} \rangle$$

$$n = {}^{2s+1}L_J^c$$

Models for quarkonium production: NRQCD, CSM, CEM, ...

Motivation: why automation of quarkonium cross sections?

Many reasons, including:

- global data-theory comparisons
- physics cases for future experimental facilities
- global NRQCD fits

Matrix element/event generators publicly available
($\hookrightarrow$ interfacing of e.g. HERWIG or PYTHIA with e.g. MG5_aMC)

$\downarrow$

facilitates complete computation

✓ versatility and enhanced physics simulation capabilities

✗ integration complexity, computational overhead, code compatibility and increased learning requirements

Why MadGraph5_aMC@NLO?

MadOnia:

- ✓ single quarkonium production phenomenology (only)
- ✗ (deprecated) module within MadGraph4

Helac-Onia:

- ✓ (S-wave/P-wave) multiple-quarkonium production based on tree-level helicity amplitudes
- ✗ limited to LO (not immediately extendable to NLO)

MadGraph5_aMC@NLO:

- ✓ flexibility to support SM, BSM and large number of particle physics models
- ? no quarkonia final states → (technical) complexities arise!

Goal: **automation of LO quarkonium with NLO in sight**

Objectives and plan of the project

☐ Implementation of quarkonia in MadGraph5_aMC@NLO at LO
↪ Single and multiple S-wave inclusive quarkonium production

- Colour projectors
- Spin projectors
- Interface
- Phase space adaptation

☐ Extensions to states with leading P-wave Fock states
LO implementation ⟶ ☐ NLO in the easiest way possible!

☐ TMD factorisation also to be implemented
↪ for example $gg \to$ di-J/ψ (... not only for quarkonia)

Status of the project

☐ Implementation of quarkonia in MadGraph5_aMC@NLO at LO
↪ Single and multiple S-wave inclusive quarkonium production

- Colour projectors → implemented ✓
- Spin projectors → implemented ✓
- Interface → implemented ✓
- Phase space adaptation

☐ Extensions to states with leading P-wave Fock states
LO implementation ⟶ ☐ NLO in the easiest way possible!

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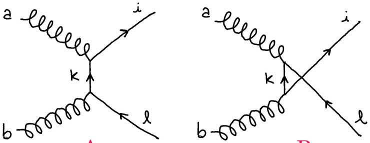
Colour projectors (1)

Quarkonium in the quantum state $n \rightarrow$ colour singlet or octet?

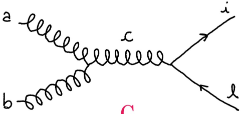
$$\mathcal{C}_1 = \delta_{ij}/\sqrt{N_c}$$
$$\mathcal{C}_8 = \sqrt{2}t^c_{ij}$$

- Example: $gg \rightarrow c\bar{c}$ **3 diagrams** at LO



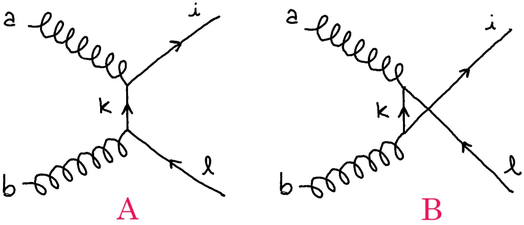
A B

Colour Singlet contribution



C

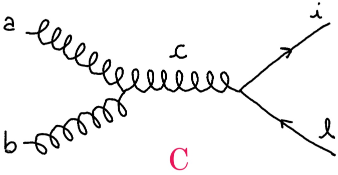
Colour Octet contribution



$(\mathrm{A}) \sim (t^a_{ik} t^b_{kl}) = (t^a t^b)_{il}$ → we set $(t^a t^b)_{il} = c1$

$(\mathrm{B}) \sim (t^b_{ik} t^a_{kl}) = (t^b t^a)_{il}$ → we set $(t^b t^a)_{il} = c2$

↪ open $c\bar{c}$ colour basis of dim $= 2$



$$(\mathrm{C}) \sim f^{abc}(t^c_{il}) = (t^a t^b)_{il} - (t^b t^a)_{il}$$

$$= \quad c1 \quad - \quad c2$$

$$c1c1^\dagger = (t^a t^b)_{il}(t^b t^a)_{il} = \mathrm{Tr}(t^a t^b t^b t^a) = \frac{16}{3}$$

$\vdots$

$$\longrightarrow \text{colour matrix: } \begin{pmatrix} c1c1^\dagger & c1c2^\dagger \\ c2c1^\dagger & c2c2^\dagger \end{pmatrix}$$

Apply colour projectors: $\mathcal{C}_1 = \delta^{il}$ and $\mathcal{C}_8 = t^c_{il}$

- Colour singlet:

$$(t^a t^b)_{il}\delta^{il} = \mathrm{Tr}(t^a t^b) \qquad \text{(A)}$$
$$(t^b t^a)_{il}\delta^{il} = \mathrm{Tr}(t^b t^a) \qquad \text{(B)}$$

- Colour octet:

$$\left.\begin{aligned}(t^a t^b)_{il} t^c_{il} &= \mathrm{Tr}(t^a t^b t^c) = \tfrac{1}{4}(d^{abc} + if^{abc}) \quad (1)\\ (t^b t^a)_{il} t^c_{il} &= \mathrm{Tr}(t^b t^a t^c) = \tfrac{1}{4}(d^{bac} + if^{bac}) \quad (2)\end{aligned}\right\} \quad (1) - (2): \quad \text{(C)}$$

The amplitude will be given by the sum of the three contributions

$$\mathcal{A} = A(\text{A}) + A(\text{B}) + A(\text{C})$$

Code: implementation of Colour Projectors

Colour projectors for *m* colour singlet and colour octet quarkonia production and associated production implemented ✓

Metacode: quarkonium formalism implemented via extension of python files which produce fortran code → numerical manipulations

In /mg5amcnlo/madgraph/**core**

- color_algebra.py
- color_amp.py
- helas_objects.py

Quarkonium in the quantum state $n \rightarrow$ spin singlet or triplet?

- Generic quarkonium spin projector: normalisation as arXiv:2402.19221

$$\frac{1}{2\sqrt{2m_Q m_{\bar{Q}}}}\bar{v}(p_2,\lambda_2)\Gamma_S u(p_1,\lambda_1) \quad (1)$$

- Generic fermion line: $\bar{u}(p_1,\lambda_1)\Gamma_1\cdots\Gamma_2 v(p_2,\lambda_2)$ (2)

Contracting (2) with (1): $S = 0, \gamma_5; 1, \not{\epsilon}(P)$

$$\boxed{\bar{v}(p_2,\lambda_2)\Gamma_S u(p_1,\lambda_1)}$$

Declaration of new effective spinors in:

- /mg5amcnlo/aloha/template_files/aloha_functions.f
- /mg5amcnlo/madgraph/core/helas_objects.py

Python: calls template_files → matrix_i.f files

Amplitudes organised into colour basis **JAMPs**

$$\mathcal{A} = \sum_i A_i \overset{\text{example}}{=} A(\text{A}) + A(\text{B}) + A(\text{C})$$

$A(\text{A}) = c1 A_1$ $\quad$ $A(\text{B}) = c2 A_2$ $\quad$ $A(\text{C}) = c1 A_{31} - c2 A_{32}$

JAMP decomposition

$$\begin{cases} \text{JAMP}_1 & = A_1 + A_{31} \propto c1 \\ \text{JAMP}_2 & = A_2 - A_{32} \propto c2 \end{cases}$$

$$|\mathcal{A}|^2 = \sum_{i,j=1,2} \text{JAMP}_i^* \langle c_i | c_j \rangle \text{JAMP}_j$$

(Depends on spin projectors - constructed from helas routines)
Efficiency: large number of Feynman diagrams possible...

...but colour basis much smaller!

Generate process: single, associated and multiple production
From the mg5amcnlo folder: type ./bin/mg5_aMC

```
MG_aMC >
```

- ⋆ Example: $pp \rightarrow \eta_c + c\bar{c}g$

```
MG_aMC > generate p p > c.c~(1S01) c c~ g
```

dot notation for onia with spectroscopic notation in brackets

or

```
MG_aMC > generate p p > etac c c~ g
```

Particle name directly in the process generation
New file containing onia information! onia_names_properties
To be added:

- MadGraph5 own pdg_code
- Principal quantum number

Implementation made in /mg5amcnlo/madgraph/interface/

Results: some checked processes

List of processes that have been checked:

- `g g > b.b~(1S01)`
- `g g > b.b~(1S08)` `single`
- `g g > b.b~(1S01) g` `single + elem.part.`
- `g g > b.b~(1S01) b.b~(1S01)`
- `g g > b.b~(1S01) c.c~(1S01)` `multiple`

Benchmarked our matrix elements squared against Helac-Onia:

$$\Rightarrow$$

- `g g > b.b~(1S01) g`

Phase-space point			
E	px	py	pz
0.5000000E+03	0.0000000E+00	0.0000000E+00	0.5000000E+03
0.5000000E+03	0.0000000E+00	0.0000000E+00	-0.5000000E+03
0.5000048E+03	0.1109232E+03	0.4448265E+03	-0.1995510E+03
0.4999952E+03	-0.1109232E+03	-0.4448265E+03	0.1995510E+03

Matrix element	1.4532913707599472E-005 GeV^0
Helac-Onia	1.45329122E-05 GeV^0
Ratio	1.0000001005670054

Some results: MadGraph5 vs Helac-Onia

- g g > b.b~(3S11) g

Matrix element	4.8065942918292797E-010 GeV^0
Helac-Onia	4.80663009E-10 GeV^0
Ratio	0.99999255151418964

- g g > c.c~(1S01) g

Matrix element	4.4080653118388797E-005 GeV^0
Helac-Onia	4.40806543E-05 GeV^0
Ratio	0.99999997242730454

- g g > b.b~(1S01) b.b~(1S01)

Matrix element	5.2953309332877083E-013 GeV^0
Helac-Onia	5.29533448E-13 GeV^0
Ratio	0.99999932957700877

Phase space intagration in MadGraph5: multi-channelling

↪ efficiency: parallel computation

Phase-space adaptation: onia in final state single particle, not two!

- Process like $gg \to c\bar{c}$ always interpreted as $2 \to 2$ process

Work in progress!

New parts in the code: search
ONIA

GitHub: release **onia** branch of MG5 version 3.x

Understanding the internal structure of the nucleon → TMDs
Correlations between k_T and the polarisation of the nucleon/parton
2 components ▸ collinear (x)
▸ transversal ($\vec{k}_\perp$) → generate q_T (final-state)

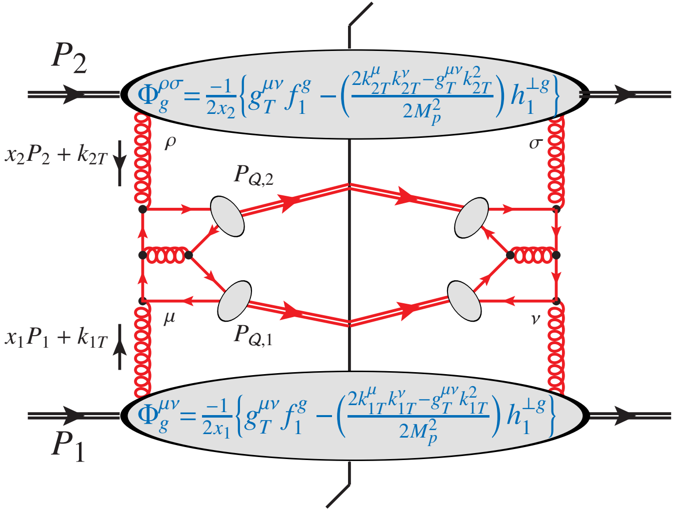
TMD factorisation ($q_T \ll Q$ hard scale)
General factorised cross section
↪ partonic scattering amplitude (*perturbative*)
↪ k_T-dependent correlators (*non-perturbative*)

$$d\sigma = \int dx_1 dx_2 d^2\vec{k}_{T1} d^2\vec{k}_{T2} \delta^{(2)}(\vec{k}_{T1} + \vec{k}_{T2} - \vec{q}_T)$$

$$\times \Phi_g^{\mu\nu}(x_1, \vec{k}_{T1}) \Phi_g^{\rho\sigma}(x_2, \vec{k}_{T2}) \left[\hat{\mathcal{M}}_{\mu\rho} \, \hat{\mathcal{M}}^{*}_{\nu\sigma} \right]\Big|_{\substack{k_1 = x_1 P_1 \\ k_2 = x_2 P_2}} + \mathcal{O}\left(\frac{q_T^2}{Q^2}\right)$$

LO Feynman diagram for $p(P_1) + p(P_2) \to \mathcal{Q}(P_{Q,1}) + \mathcal{Q}(P_{Q,1}) + X$

F. Scarpa, D. Boer, M.G. Echevarria, J.-P. Lansberg, M. Schlegel, EPJC (2020) 80:87



J.-P. Lansberg, C. Pisano, F. Scarpa, M. Schlegel, PLB 784(2018) 217

[see talk by J. Bor]

The general formula for the cross section of gluon fusion is:

$$
\begin{aligned}
d\sigma^{gg} \propto\; & F_1 \times \mathcal{C}[f_1^g f_1^g] \\
& + F_2 \times \mathcal{C}[w_2 h_1^{\perp g} h_1^{\perp g}] \\
& + (F_3 \times \mathcal{C}[w_3 f_1^g h_1^{\perp g}] + F_3' \times \mathcal{C}[w_3' h_1^{\perp g} f_1^g]) \cos(2\Phi_{CS}) \\
& + (F_4 \times \mathcal{C}[w_4 h_1^{\perp g} h_1^{\perp g}]) \cos(4\Phi_{CS})
\end{aligned}
$$

- F_i: hard scattering coefficients
- w_i: transverse weights
- f_1^g and $h_1^{\perp g}$: unpolarised and linearly polarised gluon TMDs
- Φ_{CS}: Collins-Soper azimuthal angle

Work in terms of helicities!

J.-P. Lansberg, C. Pisano, M. Schlegel, Nucl.Phys.B 920 (2017) 192-210

$$\rightarrow d\sigma = \frac{(2\pi)^4}{S^2} dPS_n \sum_{\lambda_a,\bar{\lambda}_a,\lambda_b,\bar{\lambda}_b=\pm 1} \frac{1}{(N_c^2-1)^2}\times$$

$$\sum_{a,b;I} \mathcal{A}^{ab}_{\lambda_a,\lambda_b;I}(\bar{k}_a, \bar{k}_b; \{P_i\}) \mathcal{A}^{ab*}_{\bar{\lambda}_a,\bar{\lambda}_b;I}(\bar{k}_a, \bar{k}_b; \{P_i\})\times$$

$$\int d^2k_{aT} \int d^2k_{bT} \delta^{(2)}(d^2k_{aT} + d^2k_{bT} - d^2q_T)\times$$

$$\Phi_{\bar{\lambda}_a,\lambda_a}(x_a, d^2k_{aT}, \zeta_a, \mu)\Phi_{\bar{\lambda}_b,\lambda_b}(x_b, d^2k_{bT}, \zeta_b, \mu) + \mathcal{O}(q_T/Q)$$

$$\Phi_{\lambda_a,\bar{\lambda}_a} = \frac{1}{2x_a}\left(\delta_{\lambda_a,\bar{\lambda}_a}\; f_1^g + \frac{k_{ax}^2 - k_{ay}^2 - 2i\lambda_a k_{ax}k_{ay}}{2M^2}\delta_{\lambda_a,-\bar{\lambda}_a} h_1^{\perp g}\right)$$

Implementation of quarkonia in MG5: in progress!

- ⋆ **Colour projectors** implemented
- ⋆ **Spin-projectors** implemented
- ⋆ **Interface** implemented
- ⋆ Benchmarked our matrix elements squared against Helac-Onia
- **Phase-space adaptation** to be implemented next

outlook

- **P-wave** extension
- From LO to **NLO**! see H-S. Shao, A. Hamed, L. Simon arXiv:2402.19221
- **TMD factorisation**