Spin asymmetries in semi-inclusive processes from the LHC to EIC

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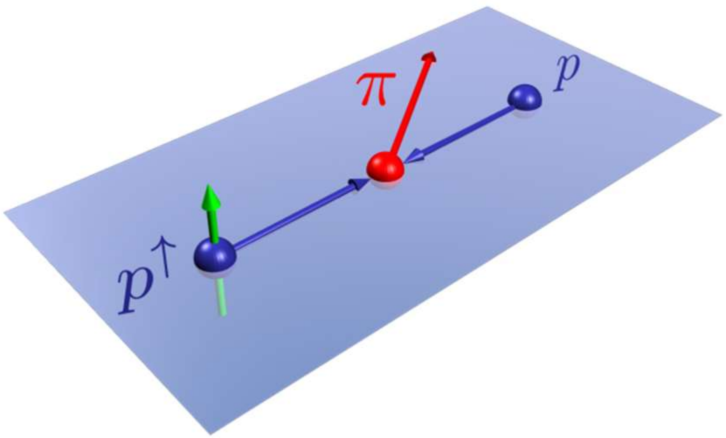
Synergies between LHC and EIC for quarkonium physics@ECT*, July 8-12

Large single spin asymmetry

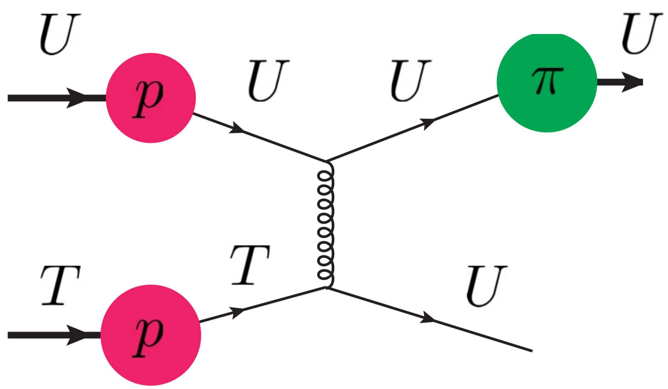
Single Spin Asymmetry (SSA)

$$A_N = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow}$$



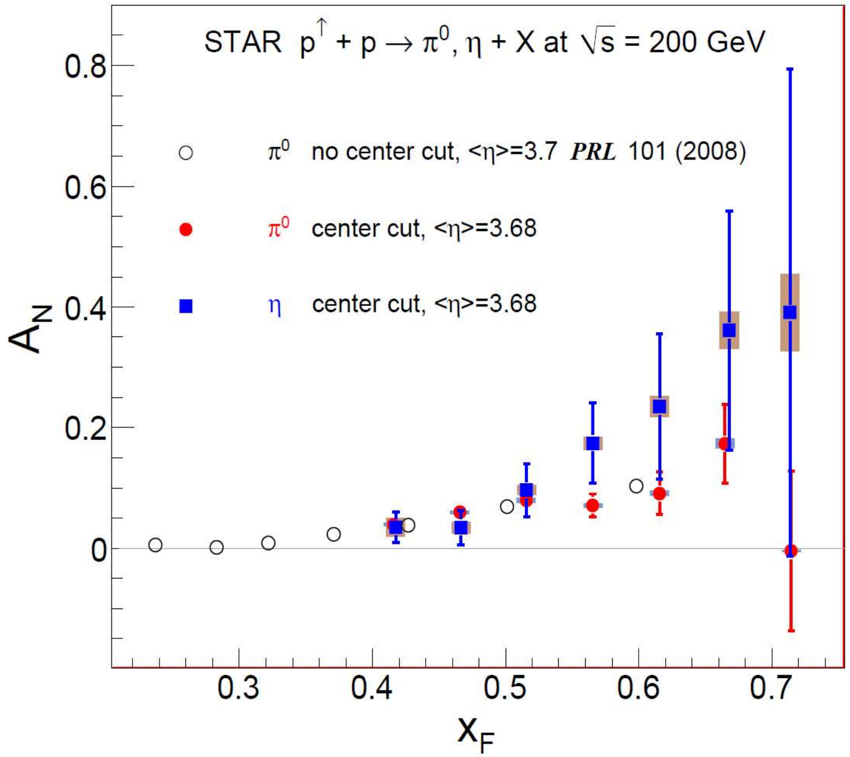
Spin polarizations between the parent hadron and a scattered parton are the same in the parton model

U: Unpolarized T: Transversely pol.



$$A_N \sim \alpha_s \frac{m_q}{P_T}$$

spin flip requires the quark mass



small quark mass makes the SSA negligible

apparent disagreement with the data

TMD and twist-3

Parton distribution function

$$\mathcal{F}.\mathcal{T}.\langle PS_\perp|\bar{\psi}_j(0)\psi_i(x^-)|PS_\perp\rangle = \underline{i(\gamma^5\sigma^{\alpha\beta}P_\beta)_{ij}} \quad \times \quad \underline{S_{\perp\alpha}} \quad \times \quad \delta q(x) + \cdots$$

chiral odd polarization

Proton is transversely polarized

TMD and twist-3

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chiral odd polarization

Proton is transversely polarized

Transverse-Momentum-Dependent(TMD) function

$$\mathcal{F}.\mathcal{T}.\langle PS_\perp|\bar{\psi}_j(0)\psi_i(x^-, x_T)|PS_\perp\rangle = \frac{1}{2M_N}\underline{(\gamma_\alpha)_{ij}}\epsilon^{\alpha\nu\rho\sigma}P_\nu k_{T\rho}\underline{S_{\perp\sigma}}f_{1T}^{\perp}(x, k_T) + \cdots$$

chiral even polarization

Proton is transversely polarized

TMD and twist-3

Parton distribution function

$$\mathcal{F.T.}\langle PS_\perp|\bar{\psi}_j(0)\psi_i(x^-)|PS_\perp\rangle = \underline{i(\gamma^5\sigma^{\alpha\beta}P_\beta)_{ij}} \quad \times \quad \underline{S_{\perp\alpha}} \quad \times \quad \delta q(x) + \cdots$$

chiral odd polarization — Proton is transversely polarized

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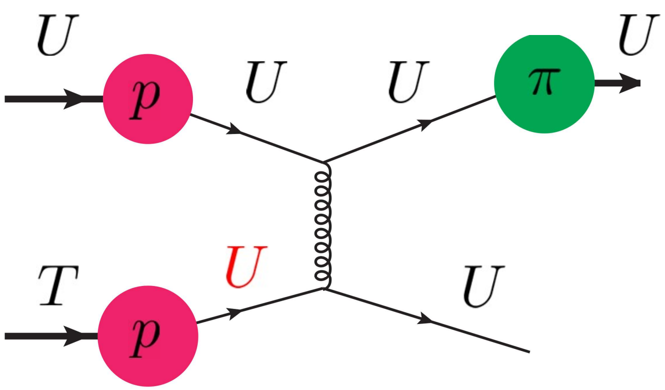
chiral even polarization — Proton is transversely polarized

Twist-3 function

$$\mathcal{F.T.}\langle PS_\perp|\bar{\psi}_j(0)gF^{\alpha+}(x_2^-)\psi_i(x_1^-)|PS_\perp\rangle = -\frac{M_N}{2}\epsilon^{\alpha\beta-+}\underline{(\not{P})_{ij}} \quad \times \quad \underline{S_{\perp\beta}} \quad \times \quad F_{FT}(x_1, x_2) + \cdots$$

chiral even polarization — Proton is transversely polarized



No spin flip!

possible to generate the large asymmetries

New pQCD frameworks for SSA

Transverse Momentum Dependent(TMD) factorization

· Applicable in small $P_T (Q \gg P_T \geq \Lambda_{QCD})$ region

cf. Sivers function $f_{1T}^{\perp}(x, k_{\perp})$

· Nonperturbative functions depend on the transverse momentum of partons

advantage: TMD functions have definite physical interpretation

disadvantage: limited applicable processes $\times\ pp \to \pi X$

Twist-3 in collinear factorization

twist-t is suppressed by $(1/Q)^{t-2}$

· Applicable in large $P_T (P_T \gg \Lambda_{QCD})$ region

· twist-3 multiparton correlation inside hadrons causes the large SSA

advantage: Applicable to many processes such as $pp \to \pi X$

disadvantage: Physical interpretation of the twist-3 functions is unclear

Overlapping region $\Lambda_{\mathrm{QCD}} \ll P_T \ll Q$

EIC is the first opportunity single experiment covers both regimes

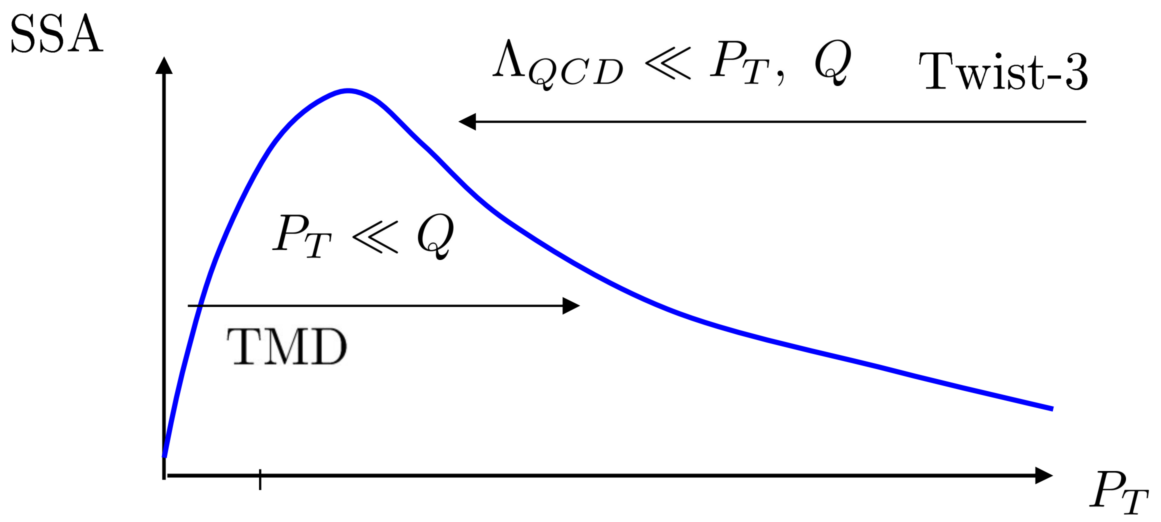
Relationship between TMD and twist-3

TMD and the collinear twist-3 could give the equivalent result in intermediate P_T

X. Ji, J.-W. Qiu, W.Vogelsang and F. Yuan, PRD73(2006) PLB638(2006)

A. Bacchetta, D. Boer, M. Diehl and P. J. Mulders, JHEP08 (2008)

F. Yuan and J. Zhou, PRL 103 (2009)



QS-function

$$\pi F_{FT}(x,x) = f_{1T}^{\perp(1)}(x)$$

k_T-moment of Sivers

Recent parameterizations rely on the relations between TMD and the collinear

L. Gamberg *et al.* [Jefferson Lab Angular Momentum (JAM) collaboration], Phys. Rev. D106 (2022)

The equivalence in the gluon distribution has been discussed recently

D. Boer, U. D'Alesio, F. Murgia, C. Pisano and P. Taels, JHEP09 (2020)

D. Boer, J. Bor, L. Maxia, C. Pisano and F. Yuan, JHEP08 (2023)

The equivalence in the gluon Sivers effect has not been discussed yet
because available results does not exist in the twist-3 side

Gluon Sivers effect

A lot of work have been done within TMD framework

· D-meson production

PRD 70, 074025 (2004) PRD 94, 114022 (2016) PRD 96, 036011 (2017)

PRD 97, 076001 (2018) PRD 99, 036013 (2019) PRD108, 034005 (2023)

· J/ψ production

PRD 91, 014005 (2015) PRD 96, 036011 (2017) EPJC 77, 854 (2017)

PRD 98, 014007 (2018) PRD 100, 014007 (2019) PRD 99, 036013 (2019) PRD 100, 094016 (2019)

EPJC 79, 1029 (2019) PRD 101, 054003 (2020) PRD 102, 094011 (2020) PRD 107, 014008 (2023)

CGI-GPM model is used for pp collision

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CGI-GPM model is used for pp collision

A few work have been done within the collinear twist-3

· D-meson production

PRD 78, 034005 (2008) PRD 78, 114013 (2008) PRD 82, 054005 (2010)
PRD 84, 014026 (2011)

· J/ψ production

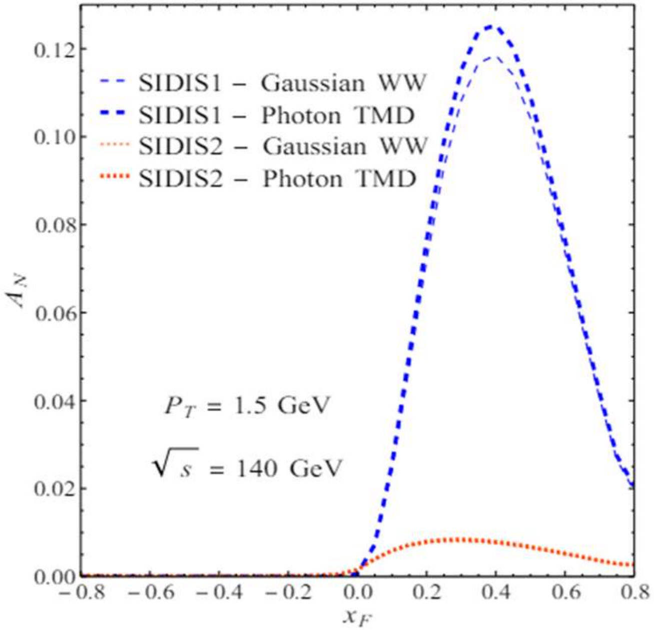
PRD 108, 054021 (2023)

Gluon Sivers in TMD

TMD factorization works in ep collision

- D-meson

R. M. Godbole, A. Kaushik and A. Misra, Phys. Rev. D97 (2018)

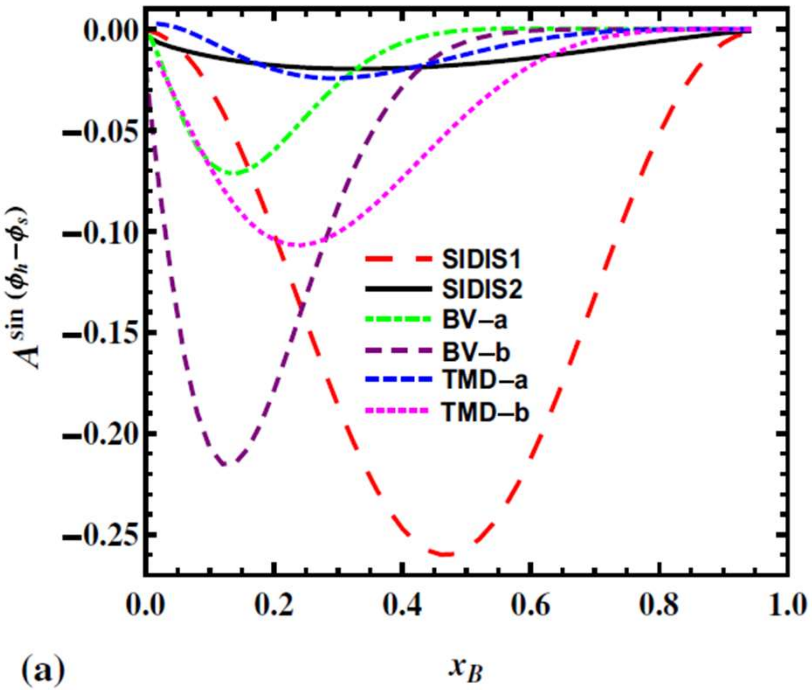


- D-meson+jet

PRD108, 034005 (2023)

- J/ψ(NRQCD)

A. Mukherjee and S. Rajesh, Eur.Phys.J.C 77 (2017)



- J/ψ+jet

PRD 100 094016 (2019), PRD 101 054003 (2020)

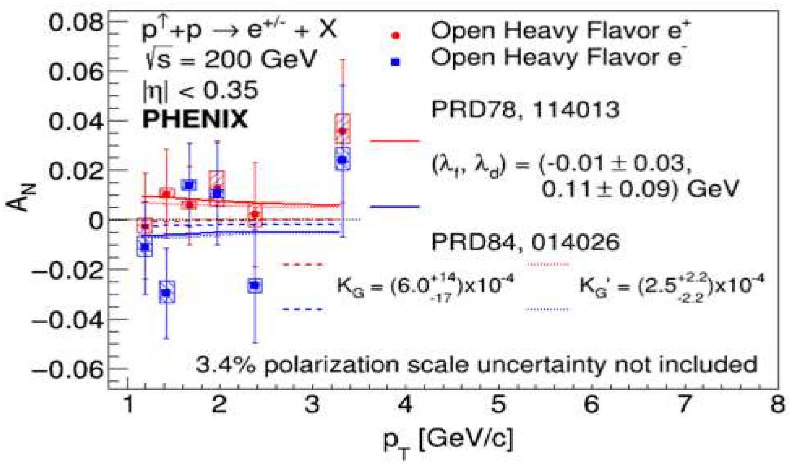
- J/ψ+photon

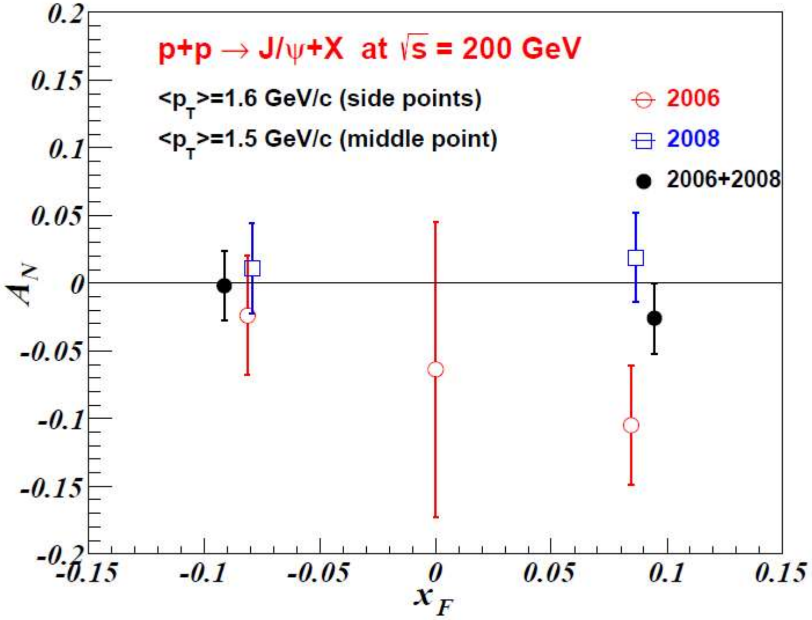
PRD 107, 014008 (2023)

No available data from ep

CGI-GPM in *pp* collision

some data from *pp* collision





It is not easy to justify the application of TMD to *pp* collision

CGI-GPM in *pp* collision

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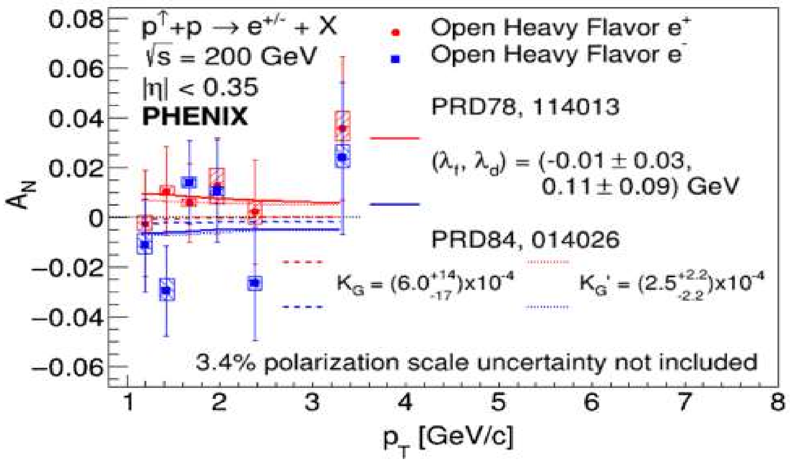
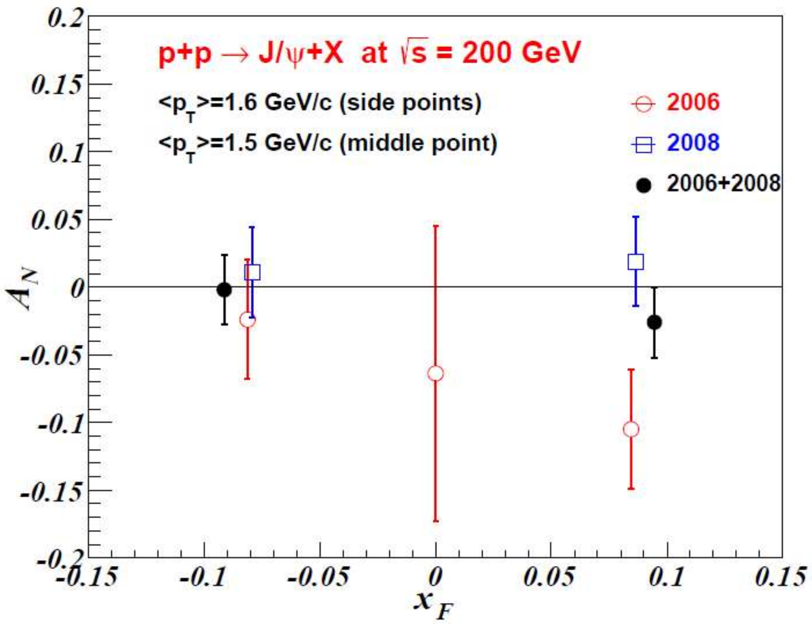




It is not easy to justify the application of TMD to *pp* collision

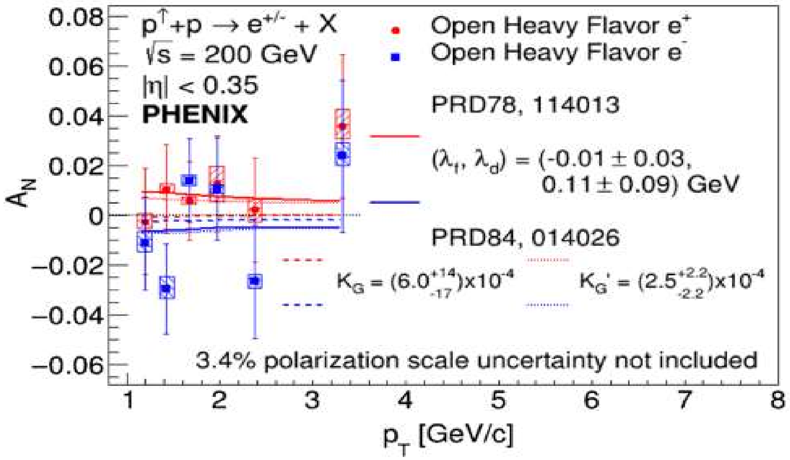
Generalized parton model(GPM) uses a phenomenology based ansatz

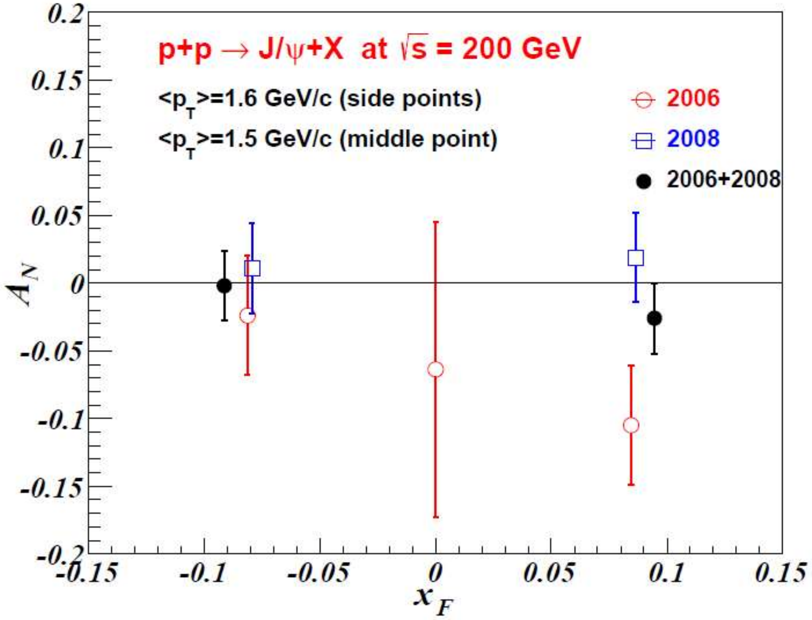
M. Anselmino et al., Phys. Rev. D73 (2006)

$$E_h \frac{d\Delta\sigma}{d^3 P_h} = \frac{\alpha_s^2}{S} \sum_{a,b,c} \int \frac{dx_a}{x_a} d^2 k_{aT} f_{1T}^{\perp a}(x_a, k_{aT}^2) \frac{\epsilon^{k_{aT} S_A n \bar{n}}}{M} \int \frac{dx_b}{x_b} d^2 k_{bT} f_{b/B}(x_b, k_{bT}^2) \int \frac{dz_c}{z_c^2} D_{h/c}(z_c) H^U_{ab\to c}(\hat{s}, \hat{t}, \hat{u}) \delta(\hat{s} + \hat{t} + \hat{u})$$

CGI-GPM in *pp* collision

some data from *pp* collision





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CGI-GPM in *pp* collision

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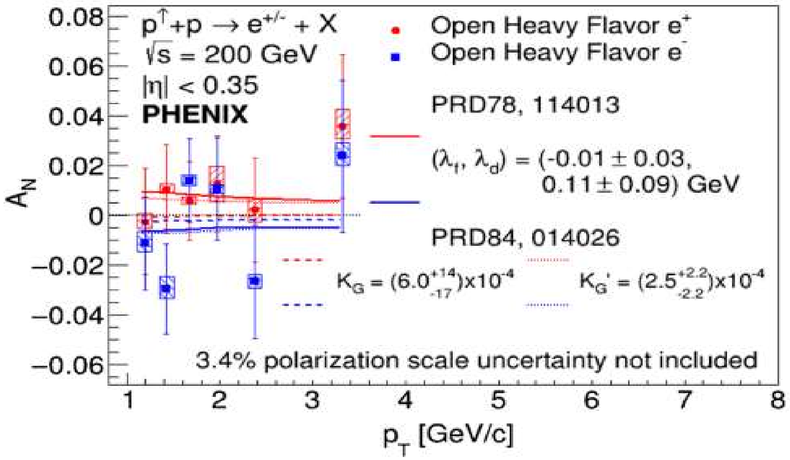

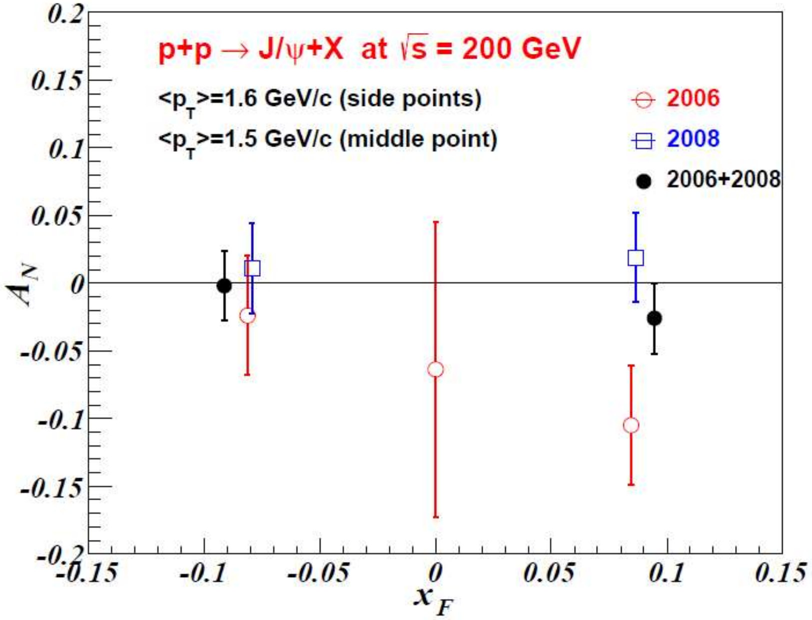

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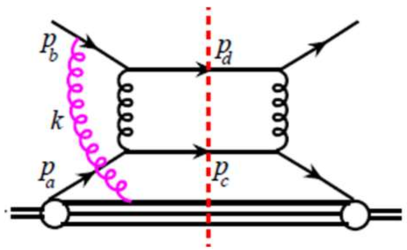
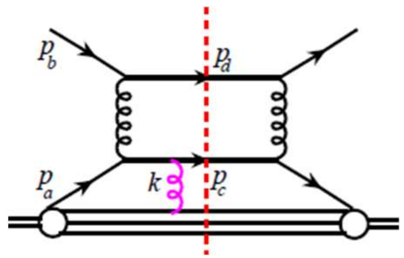
Construction of the Wilson line(Color Gauge Invariance) is considered under the eikonal approximation

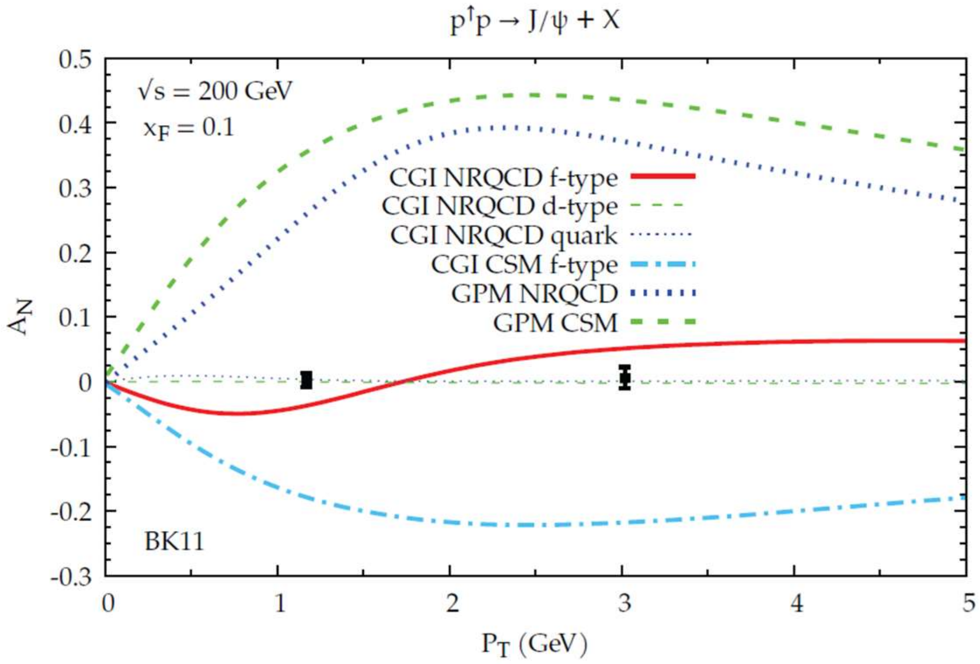
L. Gamberg and Z. B. Kang, Phys. Lett. B696 (2011)

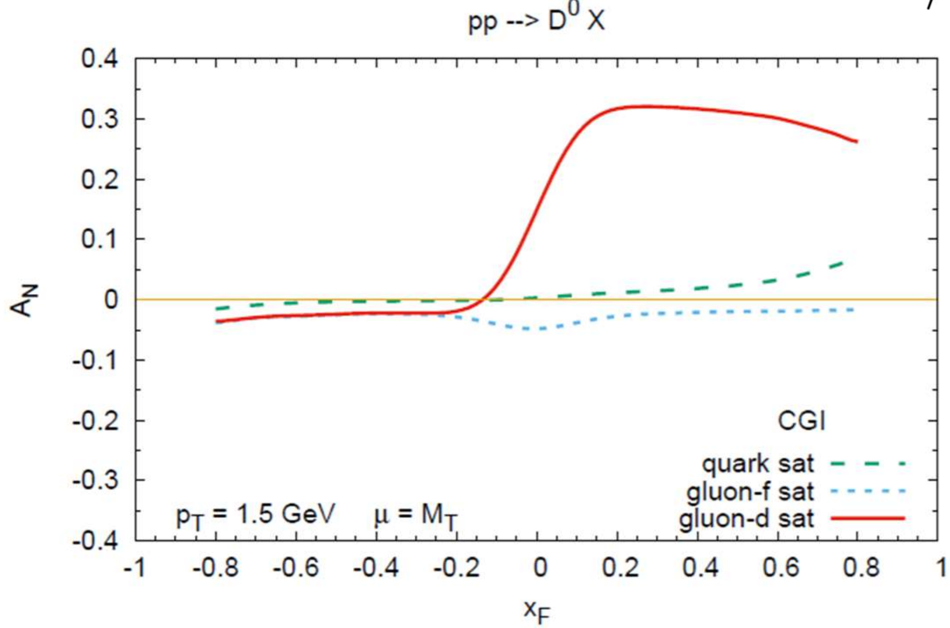


$$\left[\frac{-g}{-k^+ - i\epsilon}T^a\right] u(p_b)$$

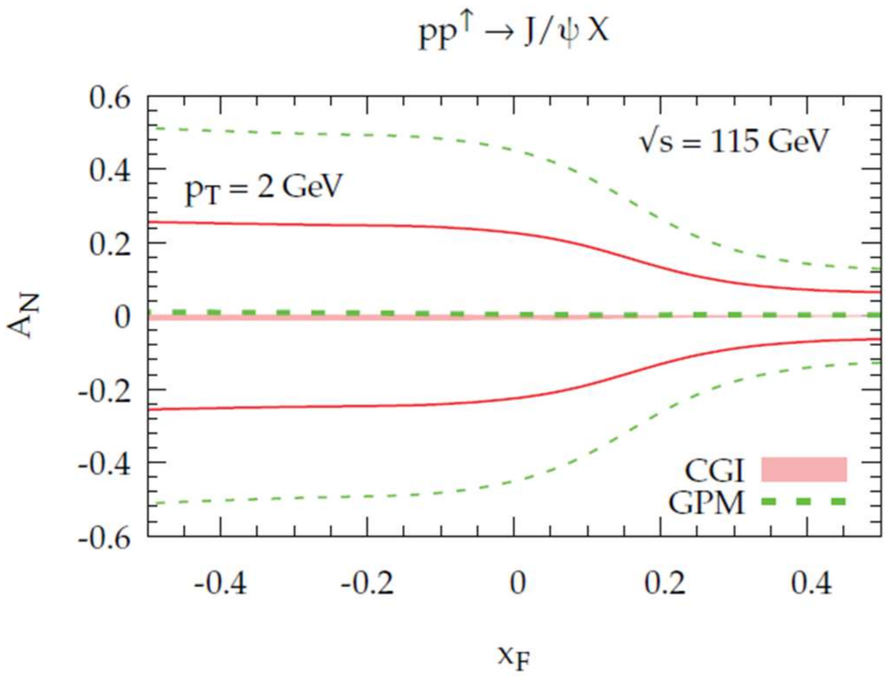


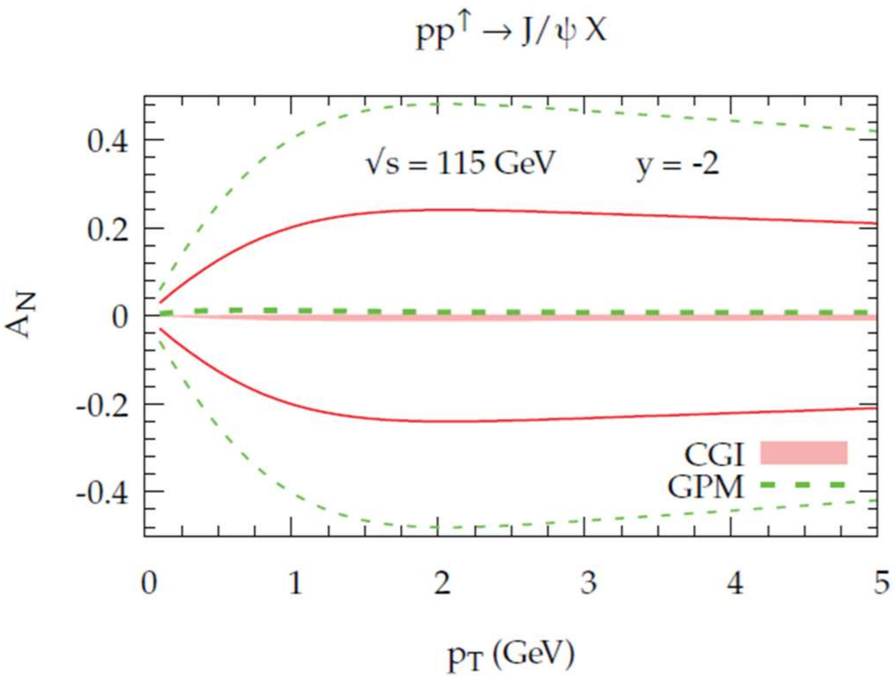
$$\bar{u}(p_c)\left[\frac{g}{-k^+ + i\epsilon}T^a\right]$$



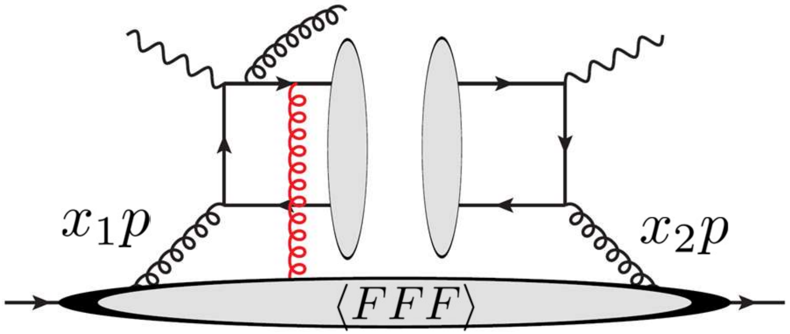


predictions for LHC PoS DIS2019 (2019) 230





Gluon Sivers in twist-3

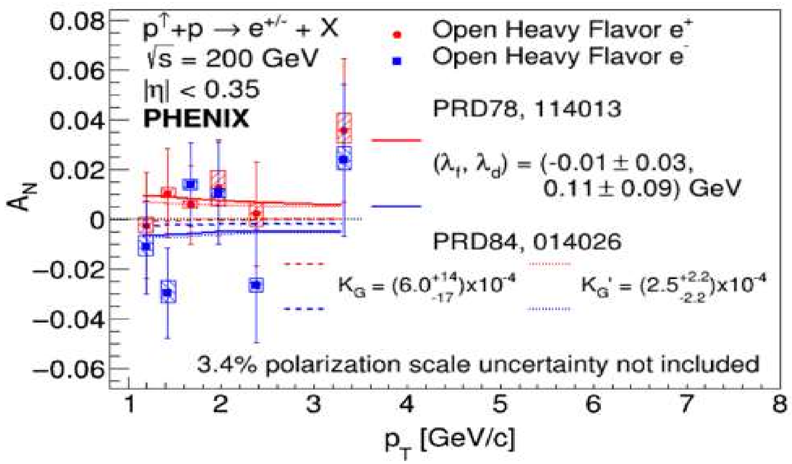


H. Beppu, Y. Koike, K. Tanaka and SY, Phys. Rev. D82 (2010)

$$\int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \langle PS_\perp | \left\{ \begin{array}{c} d_{bca} \\ i f_{bca} \end{array} \right\} (-i) F_b^{\beta n}(0)[0,\mu n] i F_c^{\gamma n}(\mu n)[\mu n, \lambda n] i F_a^{\alpha n}(\lambda n) | PS_\perp \rangle$$

$$= 2iM_N [g^{\alpha\beta} \epsilon^{\gamma p n S} \left\{ \begin{array}{c} O(x_1,x_2) \\ N(x_1,x_2) \end{array} \right\} + g^{\beta\gamma} \epsilon^{\alpha p n S} \left\{ \begin{array}{c} O(x_2,x_2-x_1) \\ N(x_2,x_2-x_1) \end{array} \right\} + g^{\gamma\alpha} \epsilon^{\beta p n S} \left\{ \begin{array}{c} O(x_1,x_1-x_2) \\ N(x_1,x_1-x_2) \end{array} \right\}$$

Separation of them is always a problem



N. J. Abdulameer et al. [PHENIX], Phys. Rev. D107 (2023)

$$O(x,x) = N(x,x) = K_G G(x) \qquad K_G = (6.0^{+14}_{-17}) \times 10^{-4}$$

Upper bound of the functions

Twist-3 calculation

SY and D. Zheng, Phys. Rev. D106 (2022)

Hadronic tensor

$$w_{\rho\sigma}\left(p, q, \frac{P_h}{z}\right) = \omega^{\mu}_{\alpha}\omega^{\nu}_{\beta}\int \frac{dx}{x^2}\Phi^{\alpha\beta}(x)S_{\mu\nu,\rho\sigma}(xp) + \omega^{\mu}_{\alpha}\omega^{\nu}_{\beta}\omega^{\lambda}_{\gamma}\int \frac{dx}{x^2}\Phi^{\alpha\beta\gamma}_{\partial}(x)\frac{\partial}{\partial k^{\lambda}}S_{\mu\nu,\rho\sigma}(k)|_{k=xp}$$
$$-\frac{1}{2}\omega^{\mu}_{\alpha}\omega^{\nu}_{\beta}\omega^{\lambda}_{\gamma}\int dx_1\int dx_2\left(\frac{-if^{abc}}{N_c(N_c^2-1)}N^{\alpha\beta\gamma}(x_1,x_2) + \frac{N_c d^{abc}}{(N_c^2-4)(N_c^2-1)}O^{\alpha\beta\gamma}(x_1,x_2)\right)$$
$$\times\frac{1}{x_1-i\epsilon}\frac{1}{x_2+i\epsilon}\frac{1}{x_2-x_1-i\epsilon}S^{abc}_{\mu\nu\lambda,\rho\sigma}(x_1p, x_2p),$$

Twist-3 calculation

Hadronic tensor

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Twist-3 calculation

SY and D. Zheng, Phys. Rev. D106 (2022)

combinations of
$N(x,x),\ N(x,0)$

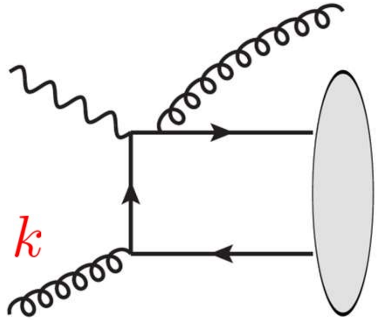
Hadronic tensor

$$
\begin{aligned}
w_{\rho\sigma}\left(p,q,\frac{P_h}{z}\right) &= \cancel{\omega^\mu_\alpha\omega^\nu_\beta\int\frac{dx}{x^2}\Phi^{\alpha\beta}(x)S_{\mu\nu,\rho\sigma}(xp)} + \omega^\mu_\alpha\omega^\nu_\beta\omega^\lambda_\gamma\int\frac{dx}{x^2}\boxed{\Phi^{\alpha\beta\gamma}_\partial(x)}\frac{\partial}{\partial k^\lambda}S_{\mu\nu,\rho\sigma}(k)|_{k=xp} \\
&\quad -\frac{1}{2}\omega^\mu_\alpha\omega^\nu_\beta\omega^\lambda_\gamma\int dx_1\int dx_2\left(\frac{-if^{abc}}{N_c(N_c^2-1)}N^{\alpha\beta\gamma}(x_1,x_2)+\frac{N_c d^{abc}}{(N_c^2-4)(N_c^2-1)}O^{\alpha\beta\gamma}(x_1,x_2)\right) \\
&\quad \times\frac{1}{x_1-i\epsilon}\frac{1}{x_2+i\epsilon}\frac{1}{x_2-x_1-i\epsilon}S^{abc}_{\mu\nu\lambda,\rho\sigma}(x_1p,x_2p),
\end{aligned}
$$

Twist-3 calculation

SY and D. Zheng, Phys. Rev. D106 (2022)

combinations of $N(x,x),\ N(x,0)$

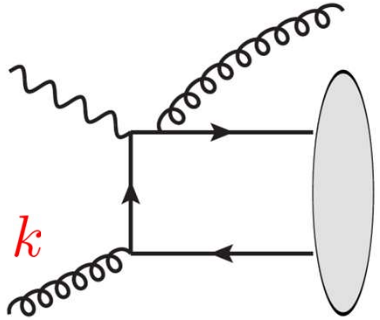
$\frac{\partial}{\partial k^{\lambda}}\Big|$  $\Big|^2$

Hadronic tensor

$$
\begin{aligned}
w_{\rho\sigma}\left(p,q,\frac{P_h}{z}\right) = {} & \cancel{\omega^{\mu}_{\alpha}\omega^{\nu}_{\beta}\int\frac{dx}{x^2}\Phi^{\alpha\beta}(x)S_{\mu\nu,\rho\sigma}(xp)} + \omega^{\mu}_{\alpha}\omega^{\nu}_{\beta}\omega^{\lambda}_{\gamma}\int\frac{dx}{x^2}\boxed{\Phi^{\alpha\beta\gamma}_{\partial}(x)}\boxed{\frac{\partial}{\partial k^{\lambda}}S_{\mu\nu,\rho\sigma}(k)|_{k=xp}} \\
& -\frac{1}{2}\omega^{\mu}_{\alpha}\omega^{\nu}_{\beta}\omega^{\lambda}_{\gamma}\int dx_1\int dx_2\left(\frac{-if^{abc}}{N_c(N_c^2-1)}N^{\alpha\beta\gamma}(x_1,x_2)+\frac{N_c d^{abc}}{(N_c^2-4)(N_c^2-1)}O^{\alpha\beta\gamma}(x_1,x_2)\right) \\
& \times\frac{1}{x_1-i\epsilon}\frac{1}{x_2+i\epsilon}\frac{1}{x_2-x_1-i\epsilon}S^{abc}_{\mu\nu\lambda,\rho\sigma}(x_1p,x_2p),
\end{aligned}
$$

Twist-3 calculation

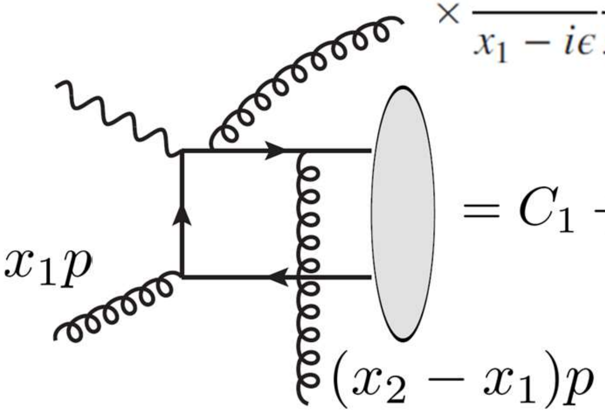
SY and D. Zheng, Phys. Rev. D106 (2022)

$\frac{\partial}{\partial k^{\lambda}} \Bigg|$  $\Bigg|^2$

combinations of
$N(x,x),\ N(x,0)$

Hadronic tensor

$$w_{\rho\sigma}\left(p,q,\frac{P_h}{z}\right) = \cancel{\omega^{\mu}_{\alpha}\omega^{\nu}_{\beta}\int\frac{dx}{x^2}\Phi^{\alpha\beta}(x)S_{\mu\nu,\rho\sigma}(xp)} + \omega^{\mu}_{\alpha}\omega^{\nu}_{\beta}\omega^{\lambda}_{\gamma}\int\frac{dx}{x^2}\boxed{\Phi^{\alpha\beta\gamma}_{\partial}(x)}\boxed{\frac{\partial}{\partial k^{\lambda}}S_{\mu\nu,\rho\sigma}(k)|_{k=xp}}$$
$$-\frac{1}{2}\omega^{\mu}_{\alpha}\omega^{\nu}_{\beta}\omega^{\lambda}_{\gamma}\int dx_1\int dx_2\left(\frac{-if^{abc}}{N_c(N_c^2-1)}N^{\alpha\beta\gamma}(x_1,x_2)+\frac{N_c d^{abc}}{(N_c^2-4)(N_c^2-1)}O^{\alpha\beta\gamma}(x_1,x_2)\right)$$
$$\times\frac{1}{x_1-i\epsilon}\frac{1}{x_2+i\epsilon}\frac{1}{x_2-x_1-i\epsilon}\boxed{S^{abc}_{\mu\nu\lambda,\rho\sigma}(x_1p,x_2p),}$$

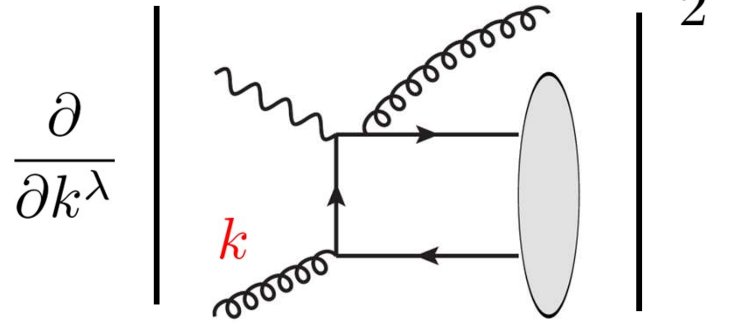


$$= C_1 + \frac{1}{x_2-x_1-i\epsilon}C_2 + \frac{1}{x_1-i\epsilon}C_3 + \frac{1}{x_1-Ax+i\epsilon}$$

$A \neq 0$

Twist-3 calculation

SY and D. Zheng, Phys. Rev. D106 (2022)



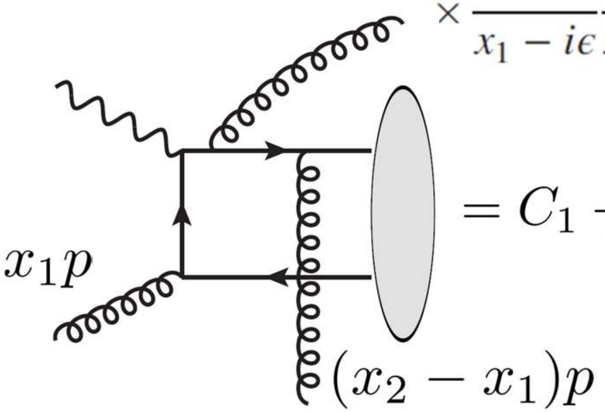
combinations of
$N(x,x),\ N(x,0)$

Hadronic tensor

$$w_{\rho\sigma}\left(p,q,\frac{P_h}{z}\right)=\cancel{\omega^{\mu}_{\alpha}\omega^{\nu}_{\beta}\int\frac{dx}{x^2}\Phi^{\alpha\beta}(x)S_{\mu\nu,\rho\sigma}(xp)}+\omega^{\mu}_{\alpha}\omega^{\nu}_{\beta}\omega^{\lambda}_{\gamma}\int\frac{dx}{x^2}\boxed{\Phi^{\alpha\beta\gamma}_{\partial}(x)}\boxed{\frac{\partial}{\partial k^{\lambda}}S_{\mu\nu,\rho\sigma}(k)|_{k=xp}}$$

$$-\frac{1}{2}\omega^{\mu}_{\alpha}\omega^{\nu}_{\beta}\omega^{\lambda}_{\gamma}\int dx_1\int dx_2\left(\frac{-if^{abc}}{N_c(N_c^2-1)}N^{\alpha\beta\gamma}(x_1,x_2)+\frac{N_c d^{abc}}{(N_c^2-4)(N_c^2-1)}O^{\alpha\beta\gamma}(x_1,x_2)\right)$$

$$\times\frac{1}{x_1-i\epsilon}\frac{1}{x_2+i\epsilon}\frac{1}{x_2-x_1-i\epsilon}\boxed{S^{abc}_{\mu\nu\lambda,\rho\sigma}(x_1p,x_2p),}$$



$$=C_1+\frac{1}{x_2-x_1-i\epsilon}C_2+\frac{1}{x_1-i\epsilon}C_3+\frac{1}{x_1-Ax+i\epsilon}$$

$$A\neq 0$$

Combined with the complex conjugate contribution,

$$\frac{1}{x'-Ax-i\epsilon}-\frac{1}{x'-Ax+i\epsilon}=2\pi i\delta(x'-Ax)\qquad\frac{1}{(x'-x+i\epsilon)^2}-\frac{1}{(x'-x-i\epsilon)^2}=2\pi i\frac{\partial}{\partial x'}\delta(x'-x)$$

$$\frac{d}{dx}N(x,x),\ \frac{d}{dx}N(x,0),N(x,x),\ \ N(x,0),N(x,Ax),\ \ N(x,(1-A)x),\ \ N(Ax,-(1-A)x)$$

Cross section formula

Longjie Chen, Hongxi Xing and SY, Phys. Rev. D108 (2023)

$$\text{SIDIS } e^-(\ell) + p^\uparrow(p) \to e^-(\ell') + J/\psi(P_{J/\psi}) + X$$

$$S_{ep} = (p+\ell)^2 \qquad x_B = \frac{Q^2}{2p\cdot q} \qquad Q^2 = -(\ell - \ell')^2 \qquad z_f = \frac{p\cdot P_{J/\psi}}{p\cdot q}$$

$$\frac{d^6\Delta\sigma}{dx_B dQ^2 dz_f dP_T^2 d\phi d\chi} = \frac{\alpha_{em}^2\alpha_s^2 e_c^2(2\pi M_N)}{4\pi S_{ep}^2 x_B^2 Q^2}\left(\mathcal{N}\langle\mathcal{O}^{J/\psi}({}^3S_1^{[1]})\rangle\right)\sum_{i=1,\ldots,4,8,9}\mathcal{A}_i(\phi-\chi)\mathcal{S}_i(\Phi_S-\chi)$$
$$\times\int\frac{dx}{x^2}\delta\left[\frac{P_T^2}{Q^2} - \left(1-\frac{1}{\hat{x}}+\frac{m_{J/\psi}^2}{z_f Q^2}\right)\left(1-\frac{1}{z_f}\right)\right]\left[N(x,x)\sigma_i^{N1} + N(x,0)\sigma_i^{N2} + N(x,Ax)\sigma_i^{N3}\right.$$
$$\left.+ N(x,(1-A)x)\sigma_i^{N4} + N(Ax,-(1-A)x)\sigma_i^{N5}\right]$$

- C-odd function $O(x_1, x_2)$ is canceled

- Derivative terms $\frac{d}{dx}N(x,x)$, $\frac{d}{dx}N(x,0)$ are canceled,
  while nonderivative functions $N(x,x), N(x,0)$ survive

  contradiction to Feng Yuan, Phys. Rev. D78 (2008)

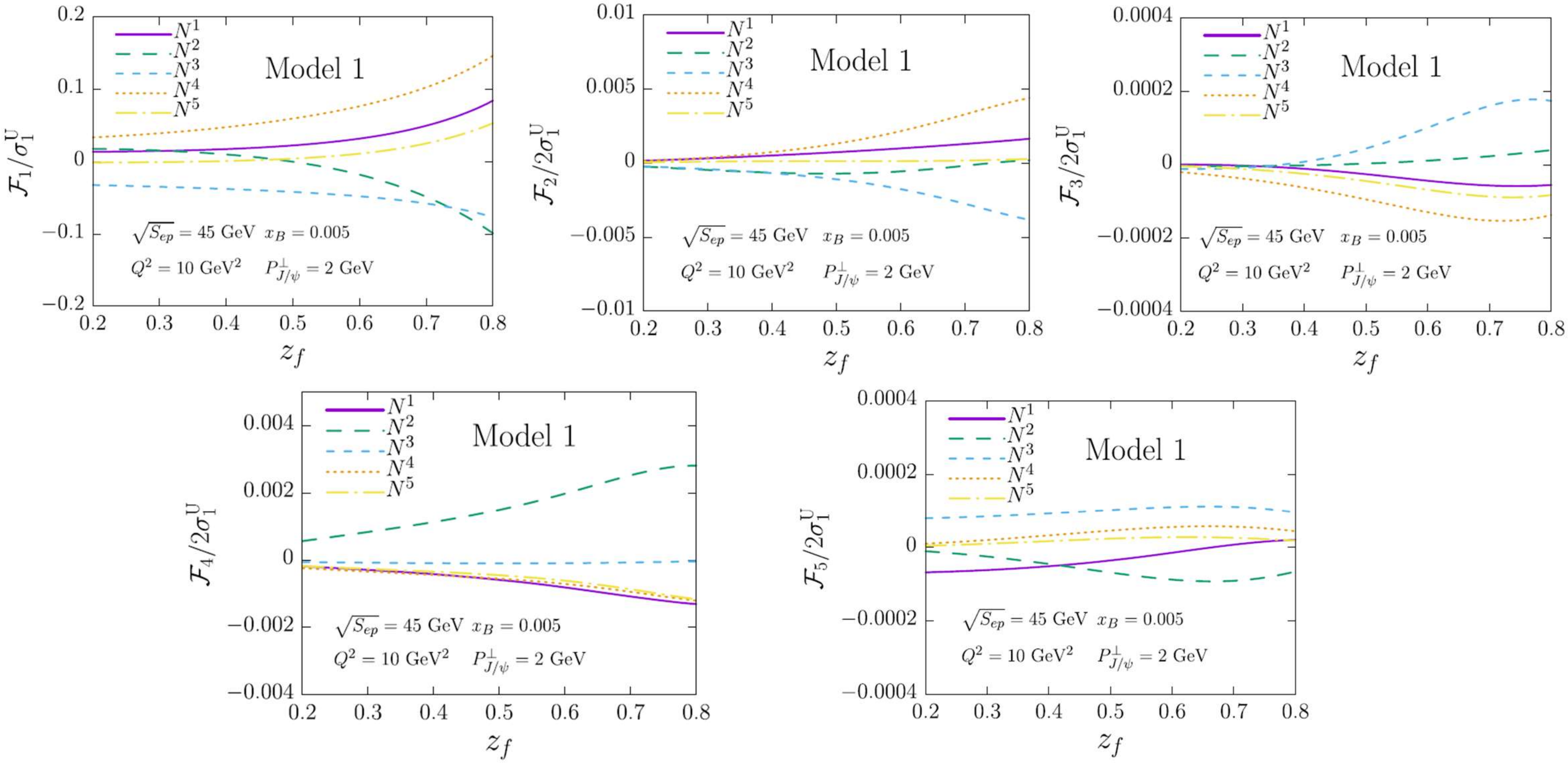
- Hard-pole contribution exist in J/ψ production

  A. Schafer and J. Zhou, Phys. Rev. D88 (2013)

We proceed with the calculation of color-octet contributions

$\longrightarrow$ matching between TMD and twist-3

Numerical simulation



$$N^{1,2,3,4,5} = \{N(x,x),\ N(x,0),\ N(x,Ax),\ N(x,(1-A)x),\ N(Ax,-(1-A)x)\} = \ 0.002xG(x)$$

EIC energy: $\sqrt{S_{ep}} = 45$ GeV $\quad x_B = 0.005 \quad Q^2 = 10$ GeV$^2 \quad P_{J/\psi}^{\perp} = 2$ GeV

Extension to pp collision

Longjie Chen, Hongxi Xing and SY, in progress

SIDIS is much easier because the pole structure is simple

(Only the final state interactions)

$$\frac{1}{x_1 - i\epsilon}\frac{1}{x_2 - x_1 - i\epsilon}\frac{1}{x_2 + i\epsilon}S_{\mu\nu\lambda}(x_1 p, x_2 p)$$

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In pp, all possible signs of $i\epsilon$ appear in the cross section

$$\frac{1}{x_1 - i\epsilon}\frac{1}{x_2 - x_1 - i\epsilon}\frac{1}{x_2 + i\epsilon}S^1_{\mu\nu\lambda}(x_1 p, x_2 p) + \frac{1}{x_1 - i\epsilon}\frac{1}{x_2 - x_1 + i\epsilon}\frac{1}{x_2 + i\epsilon}S^2_{\mu\nu\lambda}(x_1 p, x_2 p)$$

$$+\frac{1}{x_1 + i\epsilon}\frac{1}{x_2 - x_1 - i\epsilon}\frac{1}{x_2 + i\epsilon}S^3_{\mu\nu\lambda}(x_1 p, x_2 p) + \frac{1}{x_1 + i\epsilon}\frac{1}{x_2 - x_1 + i\epsilon}\frac{1}{x_2 + i\epsilon}S^4_{\mu\nu\lambda}(x_1 p, x_2 p)$$

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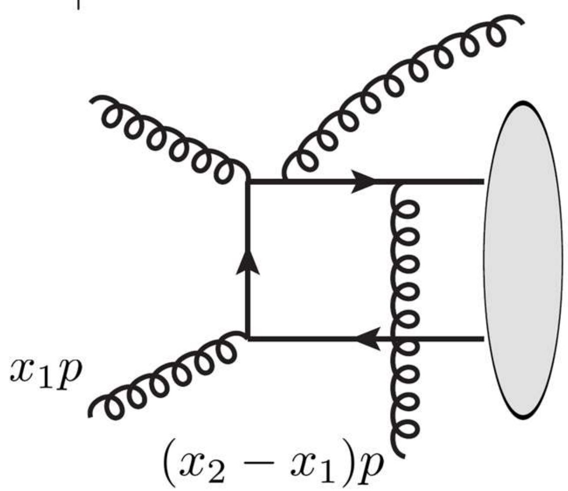

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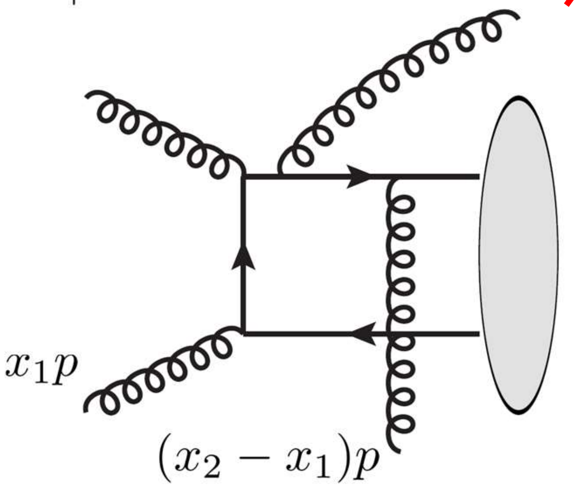

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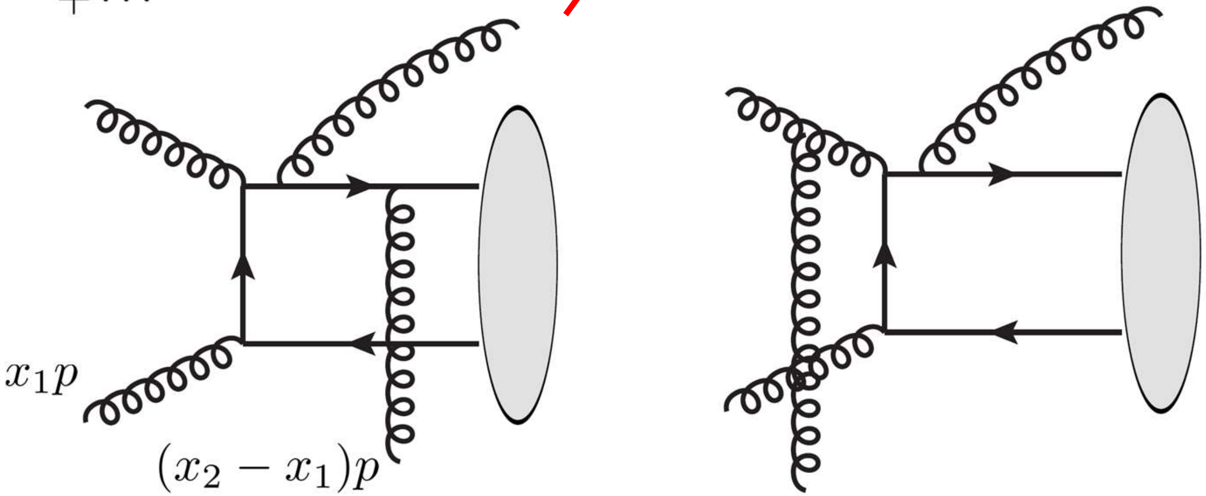

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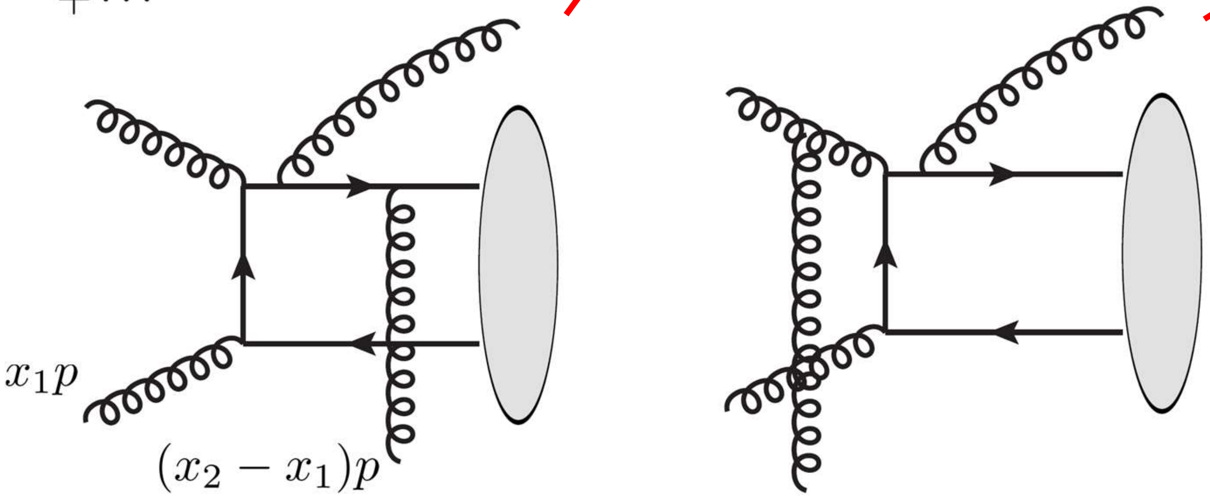

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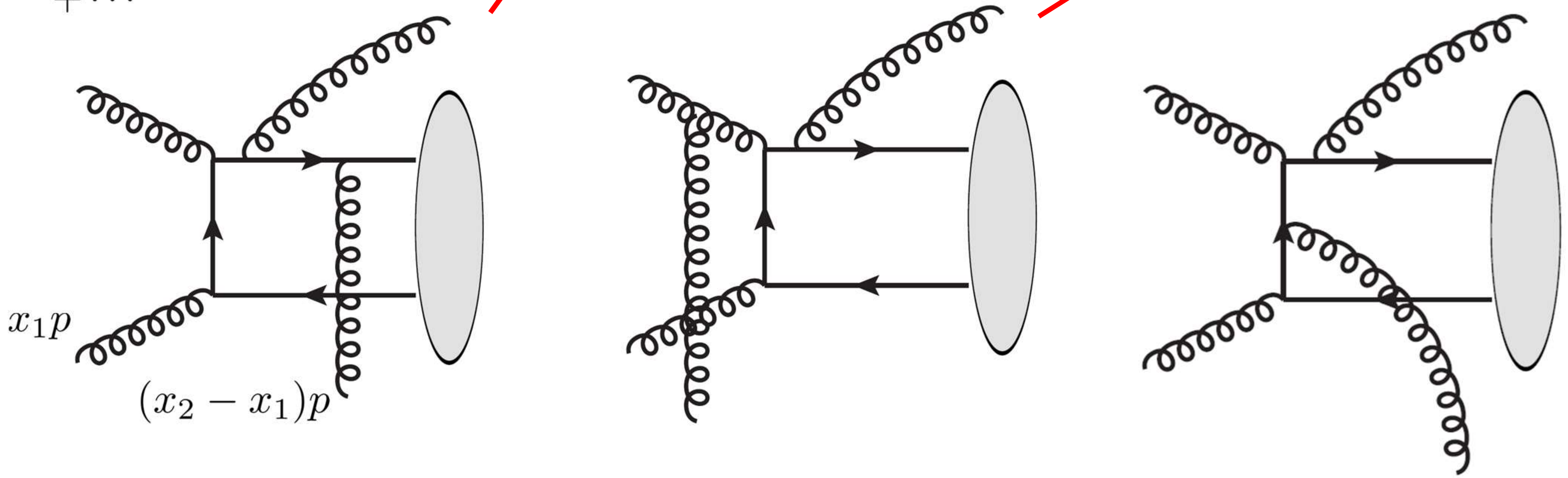

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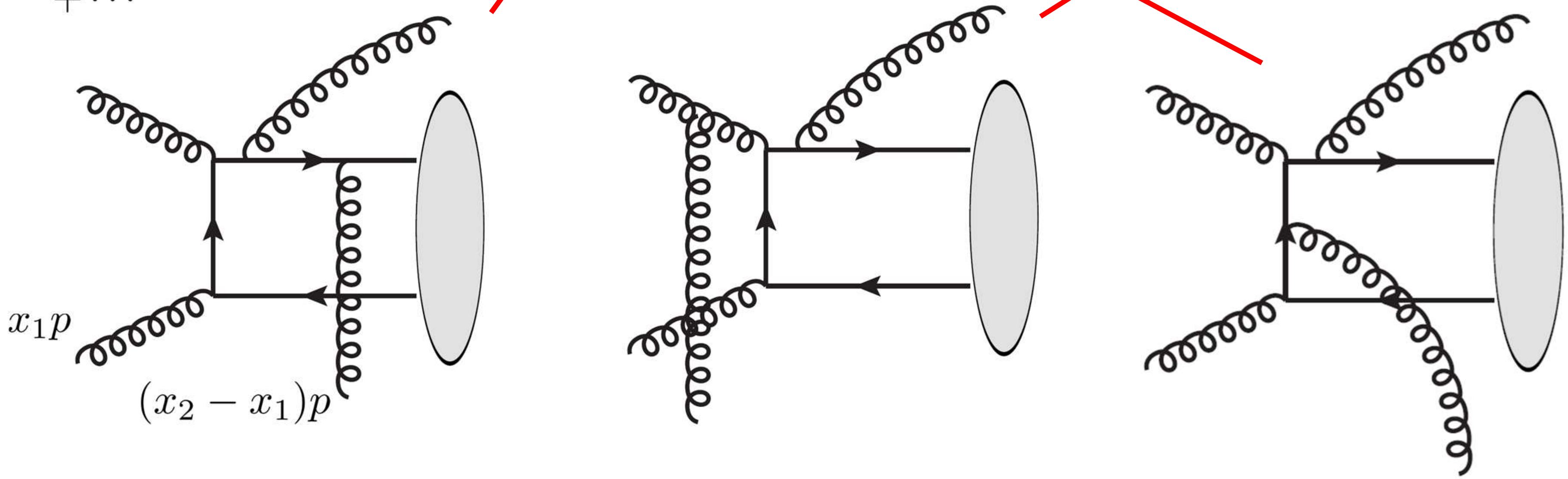

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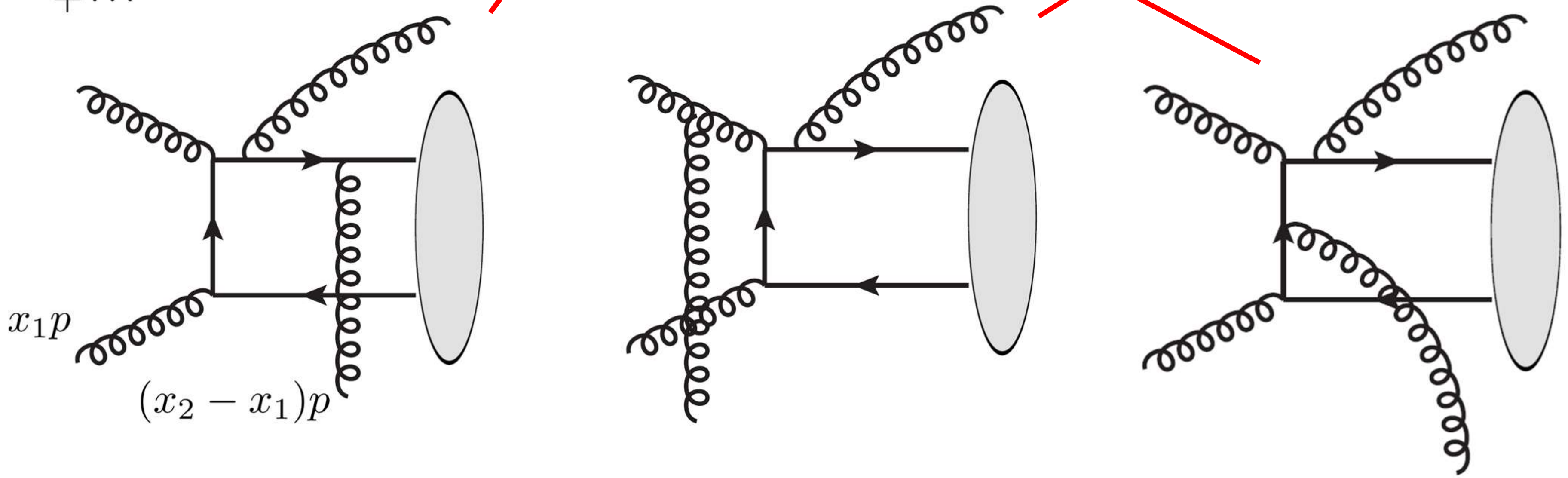

J/ψ SSA in hadronic reactions can be treated in a rigorous way

Summary

- The SSAs in the heavy meson productions are ideal observables to investigate the gluon Sivers effect.

- The data from *ep* at the future EIC experiment will contribute to the understanding of the gluon Sivers effect and the relation between two frameworks.

- The gluon contribution is more accessible in *pp*. The spin experiment at LHC experiment play a special role.

- TMD framework has been well developed in the past several years. Just waiting for new data.

- On the other hand, more theoretical effort is needed in the colinear twist-3.

Backup

The cross section has five types azimuthal dependences

$$\boxed{\frac{d^6\Delta\sigma}{dx_B dQ^2 dz_f dP_T^2 d\phi d\chi} = \sin(\phi_h - \phi_S)(\mathcal{F}_1 + \mathcal{F}_2\cos\phi_h + \mathcal{F}_3\cos 2\phi_h) + \cos(\phi_h - \phi_S)(\mathcal{F}_4\sin\phi_h + \mathcal{F}_5\sin 2\phi_h)}$$

We perform numerical calculations for five normalized structure functions

$$\frac{\mathcal{F}_1}{\sigma_1^{\rm U}},\quad \frac{\mathcal{F}_2}{2\sigma_1^{\rm U}},\quad \frac{\mathcal{F}_3}{2\sigma_1^{\rm U}},\quad \frac{\mathcal{F}_4}{2\sigma_1^{\rm U}},\quad \frac{\mathcal{F}_5}{2\sigma_1^{\rm U}}$$

$\sigma_1^{\rm U}$: unpolarized cross section

$$\frac{\mathcal{F}_1}{\sigma_1^{\rm U}} = \frac{2\pi M_N}{[\frac{4}{y^2}(1-y+\frac{y^2}{2})\hat{\sigma}_1 - 2\hat{\sigma}_2]\bar{x}G(\bar{x})}\Big[\frac{4}{y^2}(1-y+\frac{y^2}{2})\Big(\sum_{i=1}^{5} N^i(\bar{x})\sigma_1^{Ni}\Big) - 2\Big(\sum_{i=1}^{5} N^i(\bar{x})\sigma_2^{Ni}\Big)\Big] \qquad y = \frac{Q^2}{x_B S_{ep}}$$

LDME is canceled in the ratio

$$N^{1,2,3,4,5}(x) = \{N(x,x),\ N(x,0),\ N(x,Ax),\ N(x,(1-A)x),\ N(Ax,-(1-A)x)\}$$

We use two models
$$\begin{cases} N(x,x) = 0.002xG(x) \\ N(x,x) = 0.0005x^{\frac{1}{2}}G(x) \end{cases}$$
Upper bound of the experimental data

Z. B. Kang, J. W. Qiu, W. Vogelsang and F. Yuan, Phys. Rev. D78 (2008)

Y. Koike and SY, Phys. Rev. D84 (2011)