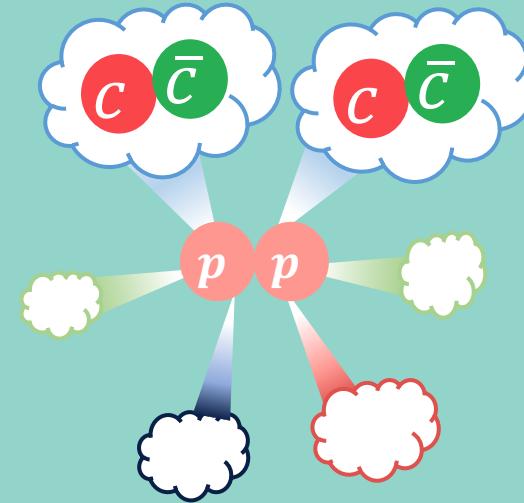


Experimental results on double J/ψ hadro-production



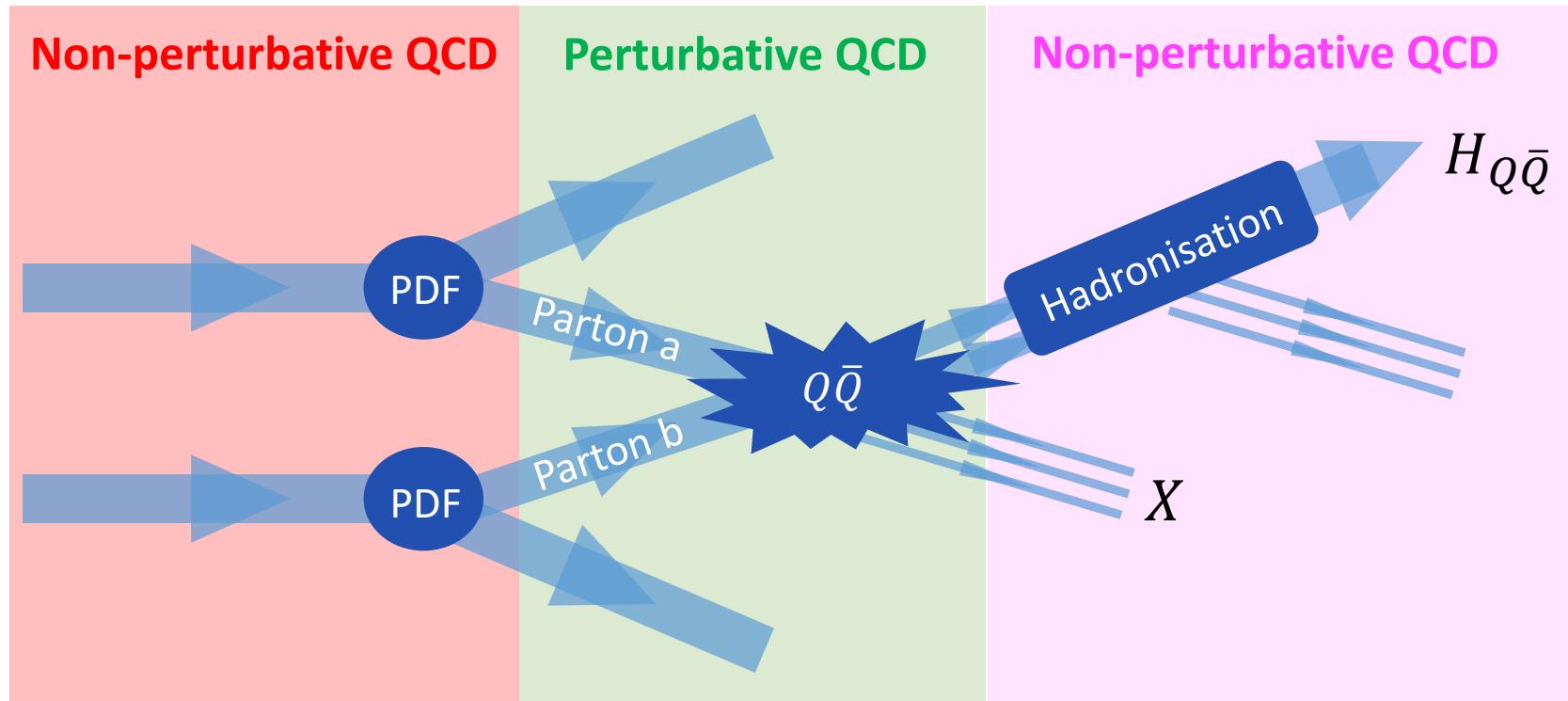
Liupan An

Peking University



Quarkonium production mechanism

- **Heavy quarkonium:** ideal system to study hadronization mechanism
- Non-relativistic QCD (NRQCD) provides the most successful description
- Yet not able to coherently describe prod.&pol. measurements in all collision systems

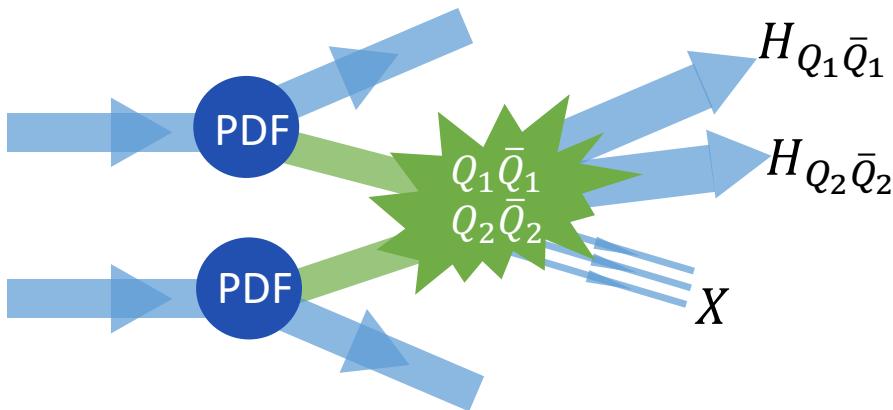


$$\sigma(H_{Q\bar{Q}}) = \sum_{a,b,n} \int dx_1 dx_2 f_{a/p}(x_1) f_{b/p}(x_2) |\mathcal{A}(ab \rightarrow Q\bar{Q}[n] + X)|^2 \times \langle \mathcal{O}^H(n) \rangle$$

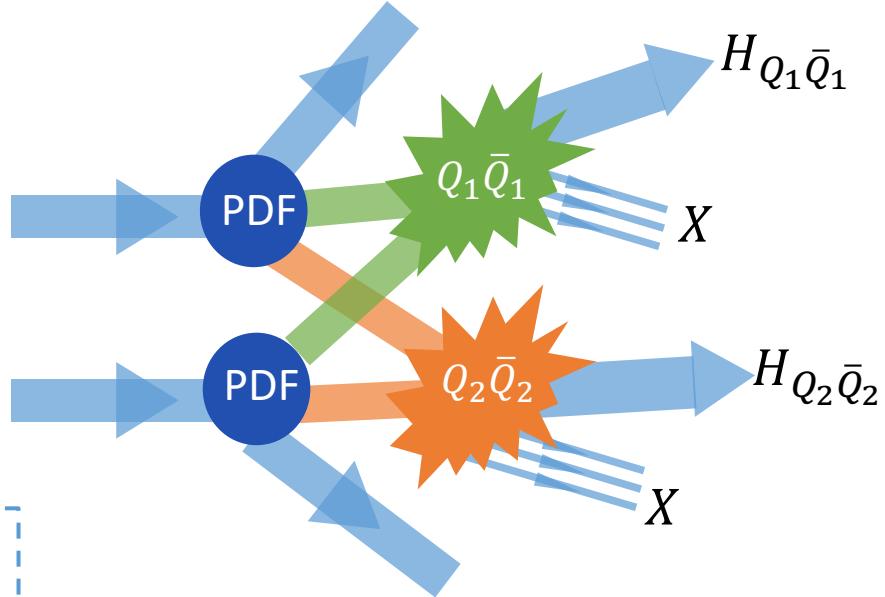
LDMEs: extracted from measurements
& process independent

Associated Quarkonium production

Single-parton scattering (SPS)



Double-parton scattering (DPS)



- ✓ To probe the quarkonium production mechanism puzzle
- ✓ Golden channel to probe gluon transverse momentum dependent (TMD) PDFs:
 - $h_1^{\perp g}(x, \mathbf{k}_T^2, \mu) \Rightarrow$ azimuthal asymmetry
 - $f_1^g(x, \mathbf{k}_T^2, \mu)$: affect p_T spectrum
[EPJC 80 (2020) 87]
- ✓ To search for fully heavy tetraquark states

- ✓ To provide information on parton transverse profile & correlations in colliding hadrons
- ✓ To understand multiparticle background ($Z + b\bar{b}$, W^+W^+ etc.) in both SM measurements and search for New Physics

Double Parton Scattering

$$\sigma_{Q_1 Q_2}^{\text{DPS}} = \frac{1}{1 + \delta_{Q_1 Q_2}} \sum_{i,j,k,l} \int dx_1 dx_2 dx'_1 dx'_2 d^2 \mathbf{b}_1 d^2 \mathbf{b}_2 d^2 \mathbf{b}$$

Generalized double parton PDF
SPS parton-level cross-section

$$\times \Gamma_{ij}(x_1, x_2, \mathbf{b}_1, \mathbf{b}_2) \times \hat{\sigma}_{ik}^{Q_1}(x_1, x'_1) \hat{\sigma}_{jl}^{Q_2}(x_2, x'_2) \times \Gamma_{kl}(x'_1, x'_2, \mathbf{b}_1 - \mathbf{b}, \mathbf{b}_2 - \mathbf{b})$$

Assuming:

- ✓ factorization of trans. & long. components

$$\Gamma_{ij}(x_1, x_2, \mathbf{b}_1, \mathbf{b}_2) = D_{ij}(x_1, x_2) T_{ij}(\mathbf{b}_1, \mathbf{b}_2)$$

- ✓ no correlation between two sets of partons

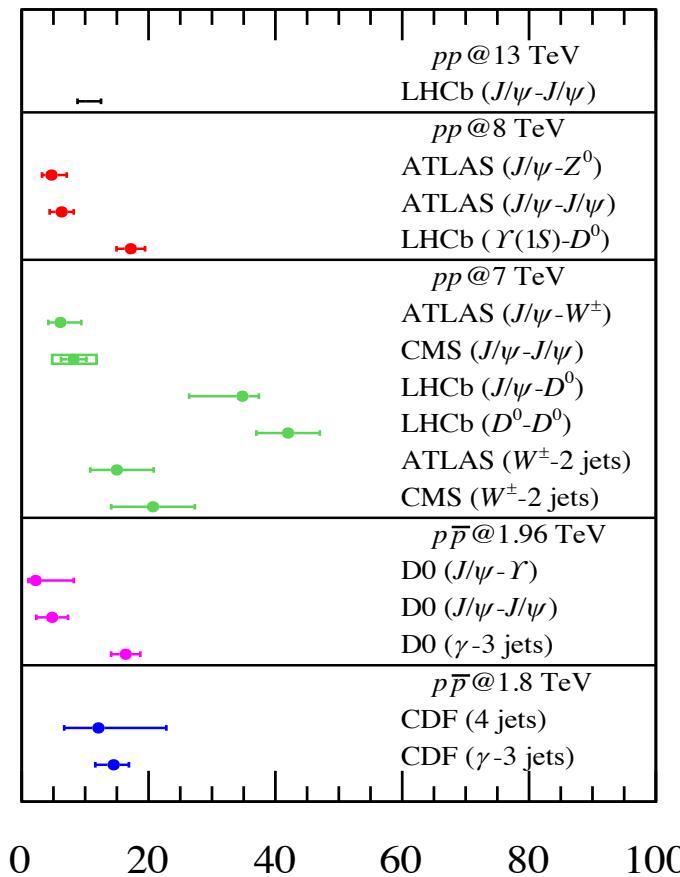
$$D_{ij}(x_1, x_2) = f_i(x_1) f_j(x_2), T_{ij}(\mathbf{b}_1, \mathbf{b}_2) = T_i(\mathbf{b}_1) T_j(\mathbf{b}_2)$$

⇒

$$\sigma_{Q_1 Q_2} = \frac{1}{1 + \delta_{Q_1 Q_2}} \frac{\sigma_{Q_1} \sigma_{Q_2}}{\sigma_{\text{eff}}}$$

$$\sigma_{\text{eff}} = 1 / \left[\int d^2 \mathbf{b} F(\mathbf{b})^2 \right], F(\mathbf{b}) = \int T(\mathbf{b}_i) T(\mathbf{b}_i - \mathbf{b}) d^2 \mathbf{b}_i$$

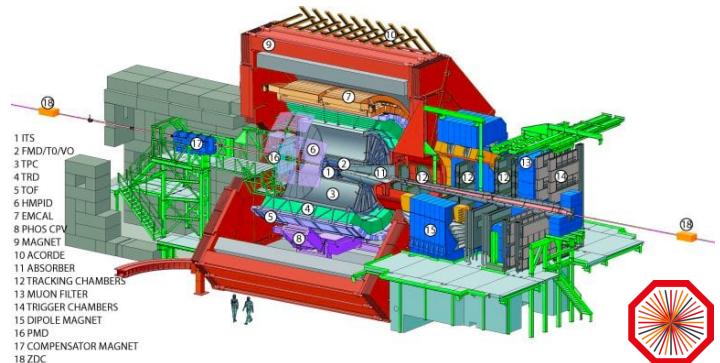
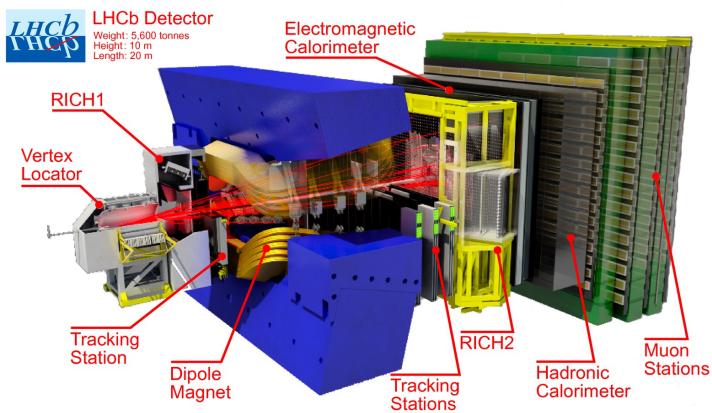
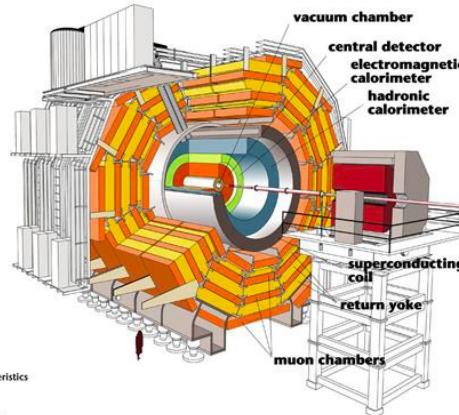
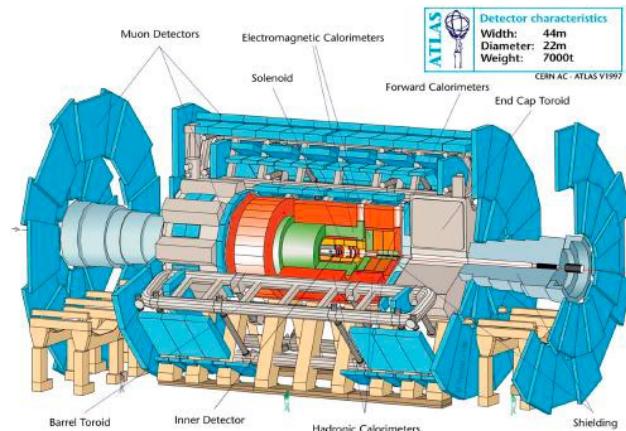
expected to be universal under the given assumptions



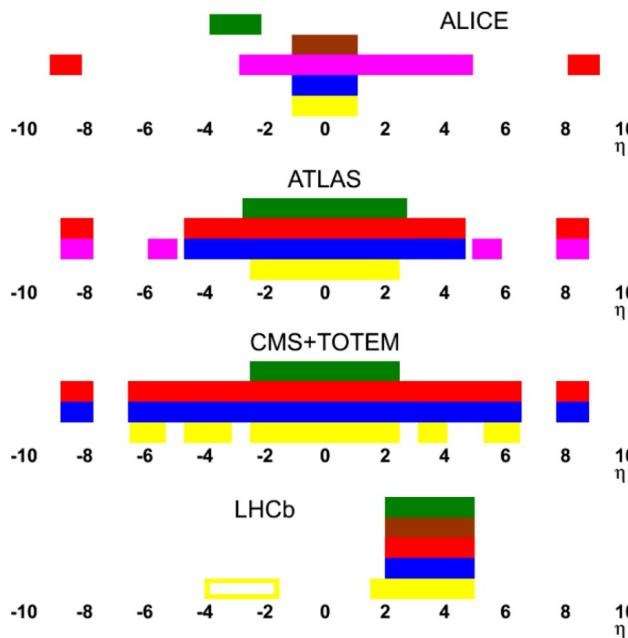
[PoS (LHC2020) 172;
arXiv: 2009.12555]

σ_{eff} [mb]
4/29

LHC experiments



ALICE



✓ ATLAS & CMS

- central
- muon mode

✓ LHCb

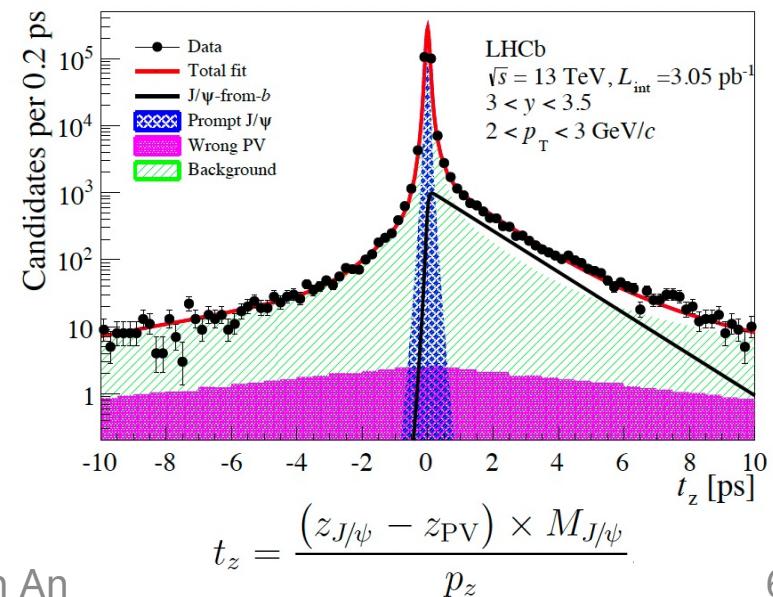
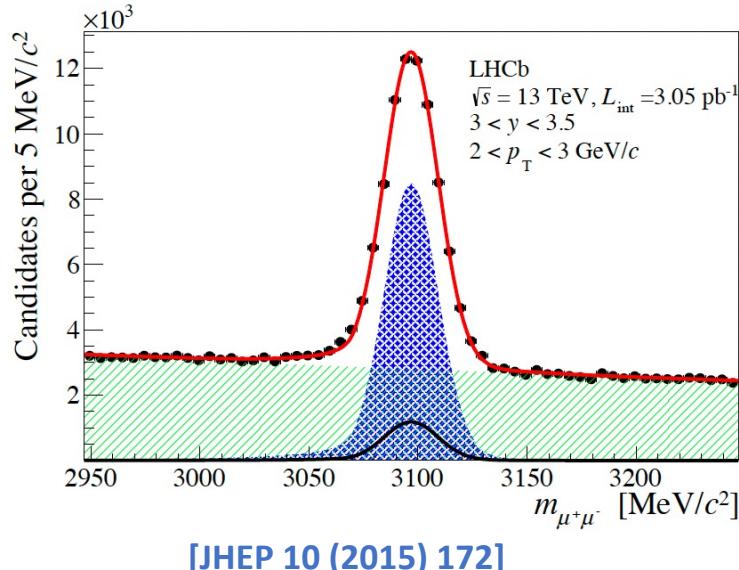
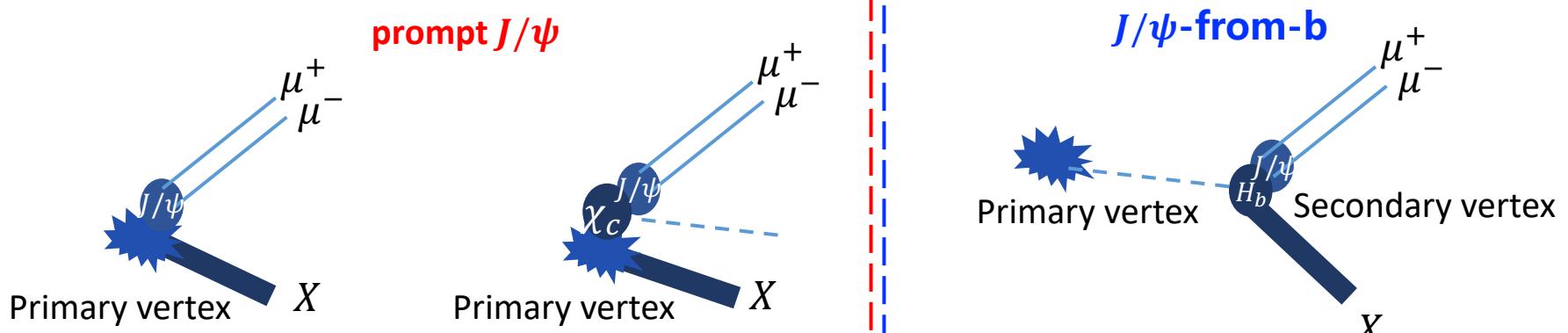
- forward
- excellent PID

✓ ALICE

- central+forward
- muon coverage

Analysis strategy

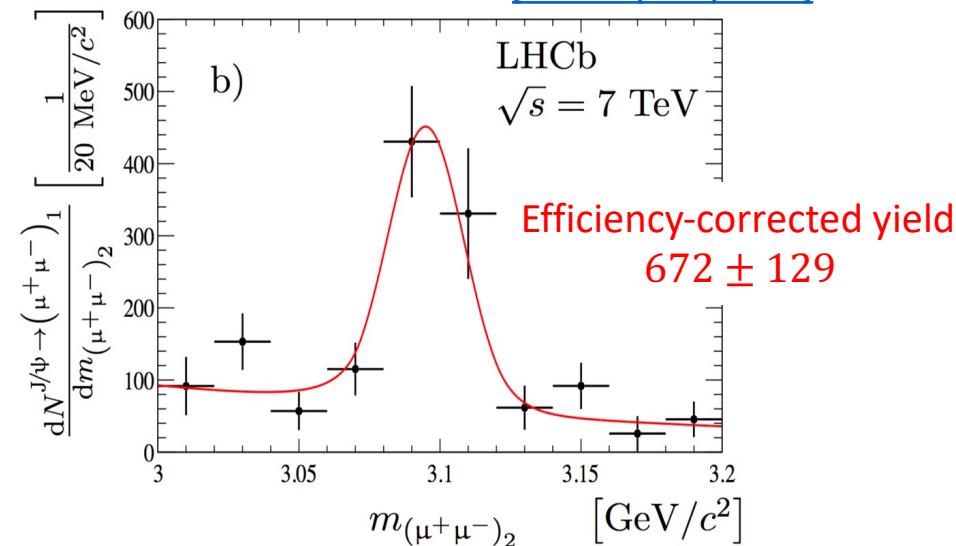
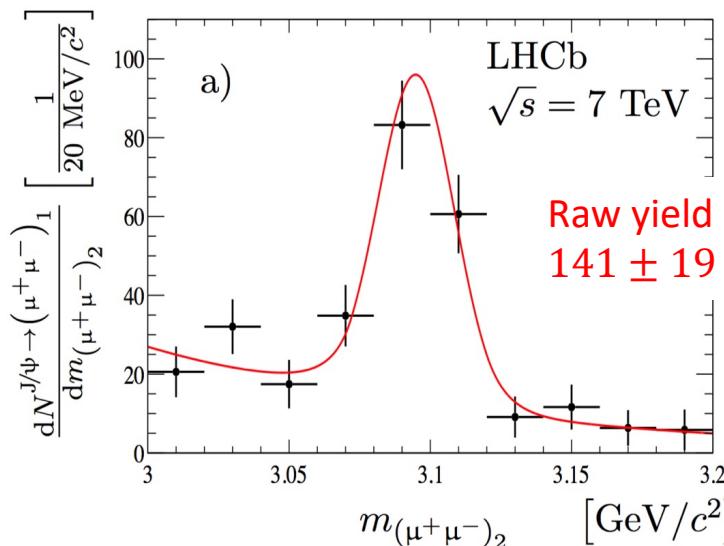
$$\frac{d\sigma}{dv} = \frac{N}{\mathcal{L} \times \varepsilon \times \mathcal{B}(J/\psi \rightarrow \mu^+ \mu^-) \times \Delta v}$$



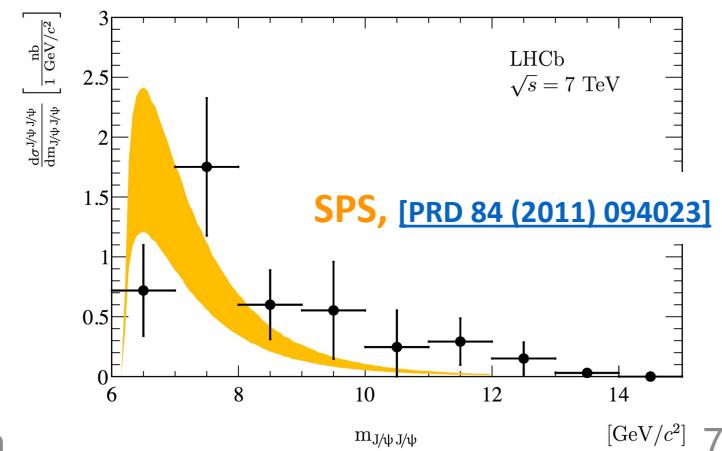
di- J/ψ @ 7 TeV

- Using 37.5 pb^{-1} data at $\sqrt{s} = 7 \text{ TeV}$
- Fiducial region: $2 < y^{J/\psi} < 4.5, p_T^{J/\psi} < 10 \text{ GeV}/c$
- Observed with significance $> 6\sigma$ (from- b contribution negligible)

[\[PLB 707 \(2012\) 52-59\]](#)



- $\sigma^{J/\psi J/\psi} = 5.1 \pm 1.0 \pm 1.1 \text{ nb}$
 - ✓ $\sigma_{\text{SPS}} = 4.1 \pm 1.2 \text{ nb}$
 - ✓ $\sigma_{\text{DPS}} \approx 2.5 \text{ nb}$



di- J/ψ @ 13 TeV

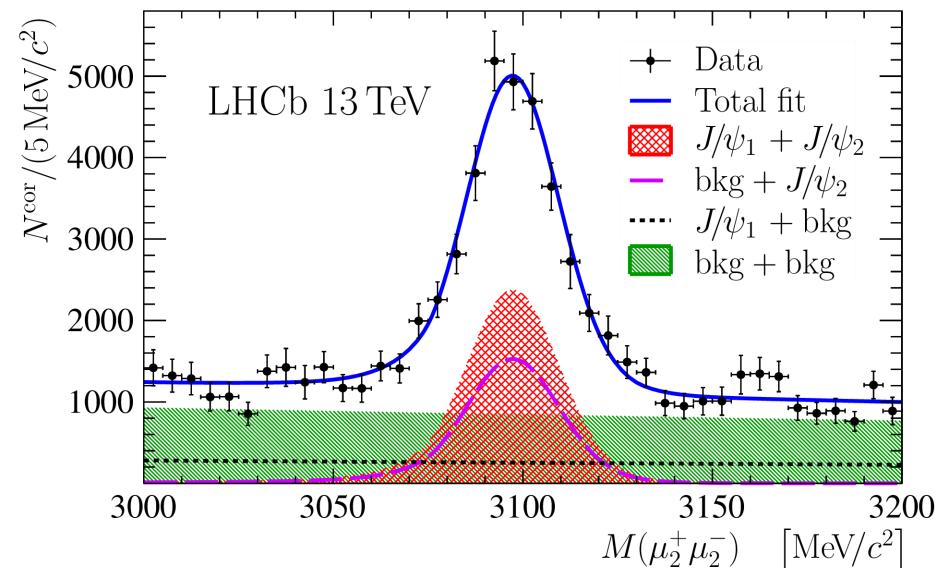
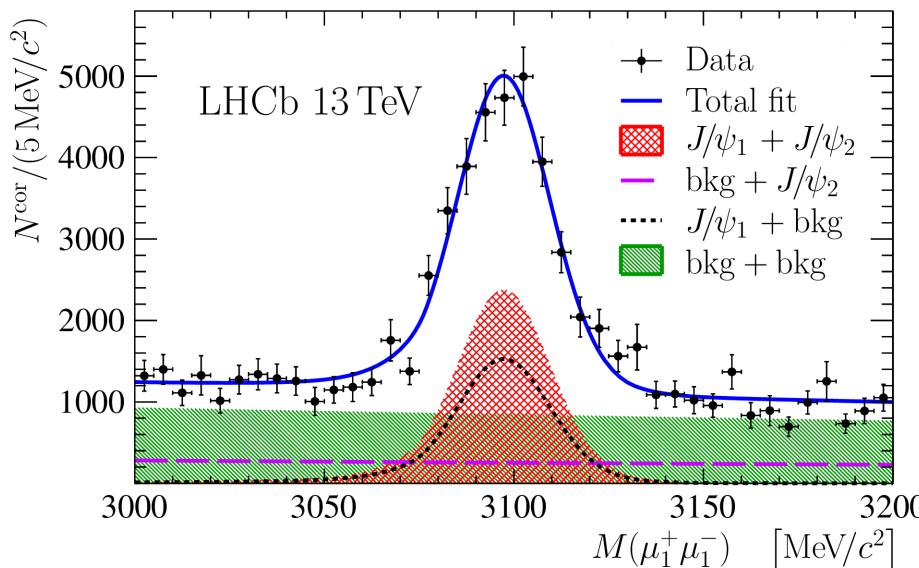
[JHEP 06 (2017) 047]

➤ $\mathcal{L} = 279 \text{ pb}^{-1}$, pp collisions @ $\sqrt{s} = 13 \text{ TeV}$

➤ Kinematic range of J/ψ :
 $p_T < 10 \text{ GeV}/c$ for $2.0 < y < 4.5$

➤ Signal yield determination

- ✓ Residual from- b component determined using simulation together with $\sigma(pp \rightarrow b\bar{b})$ and $\sigma(\text{prompt } J/\psi)$



➤ $\sigma(J/\psi J/\psi) = 15.2 \pm 1.0(\text{stat}) \pm 0.9(\text{syst}) \text{ nb}$

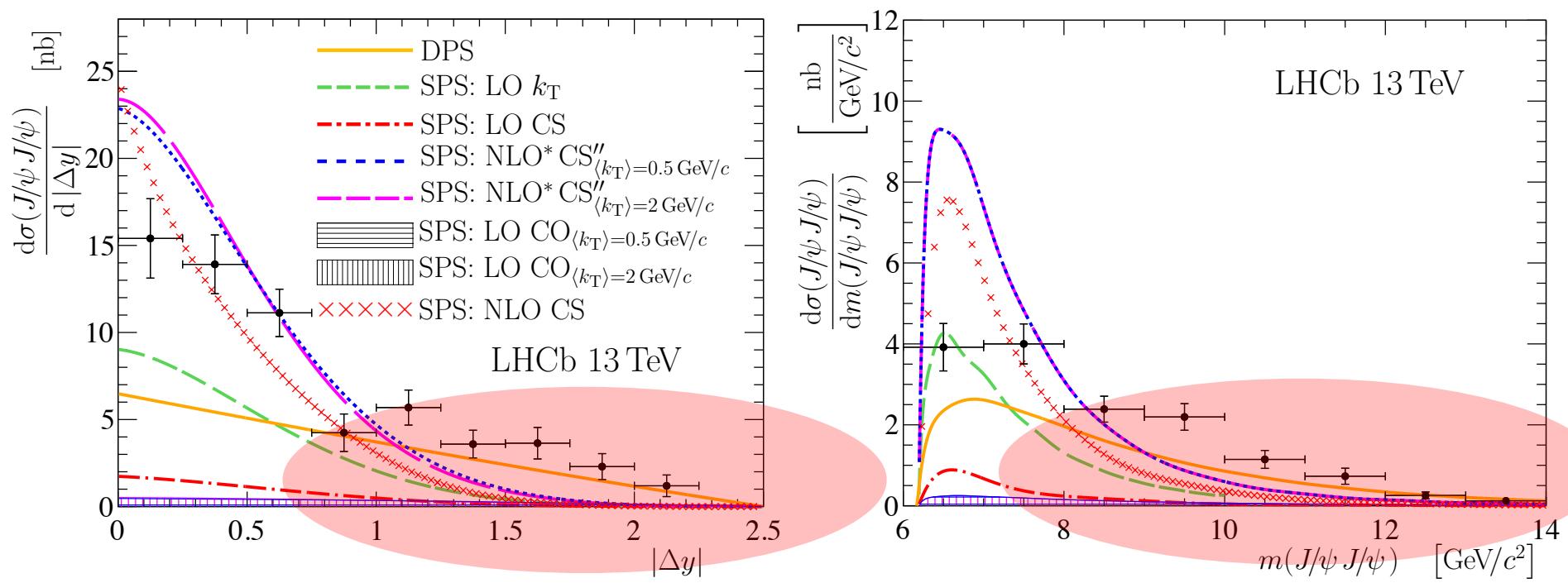
Comparison to theory

		$\sigma(J/\psi J/\psi)$ [nb]	[JHEP 06 (2017) 047]
	no p_T cut	$p_T > 1 \text{ GeV}/c$	$p_T > 3 \text{ GeV}/c$
SPS	LO Colour-singlet	$1.3 \pm 0.1^{+3.2}_{-0.1}$	—
	LO Colour-octet	$0.45 \pm 0.09^{+1.42+0.25}_{-0.36-0.34}$	—
	LO k_T	$6.3^{+3.8+3.8}_{-1.6-2.6}$	$5.7^{+3.4+3.2}_{-1.5-2.1}$
	NLO* Colour-singlet'	—	$4.3 \pm 0.1^{+9.9}_{-0.9}$
	NLO* Colour-singlet''	$15.4 \pm 2.2^{+51}_{-12}$	$14.8 \pm 1.7^{+53}_{-12}$
	NLO Colour-singlet	$11.9^{+4.6}_{-3.2}$	—
	DPS	$8.1 \pm 0.9^{+1.6}_{-1.3}$	$7.5 \pm 0.8^{+1.5}_{-1.2}$
LHCb result		$15.2 \pm 1.0 \pm 0.9$	$13.5 \pm 0.9 \pm 0.9$
DPS: assuming $\sigma_{\text{eff}} = 14.5 \pm 1.7^{+1.7}_{-2.3} \text{ mb}$		[PRD 56 (1997) 38111]	

- LO Colour-octet : contribution very small
- LO Colour-singlet/ NLO* Colour-singlet' and LO k_T : need DPS contribution
- NLO* Colour-singlet'' and NLO Colour-singlet : consistent with our measurement by itself; overestimated if there is DPS contribution

Differential cross-sections

- Differential cross-sections of different variables compared to theory predictions
 - ✓ Most significant indication of DPS comes from $|\Delta y|$
 - ✓ DPS contribution essential for the region $|\Delta y| > 1.5$
 - ✓ Also clear indication from $m(J/\psi J/\psi)$



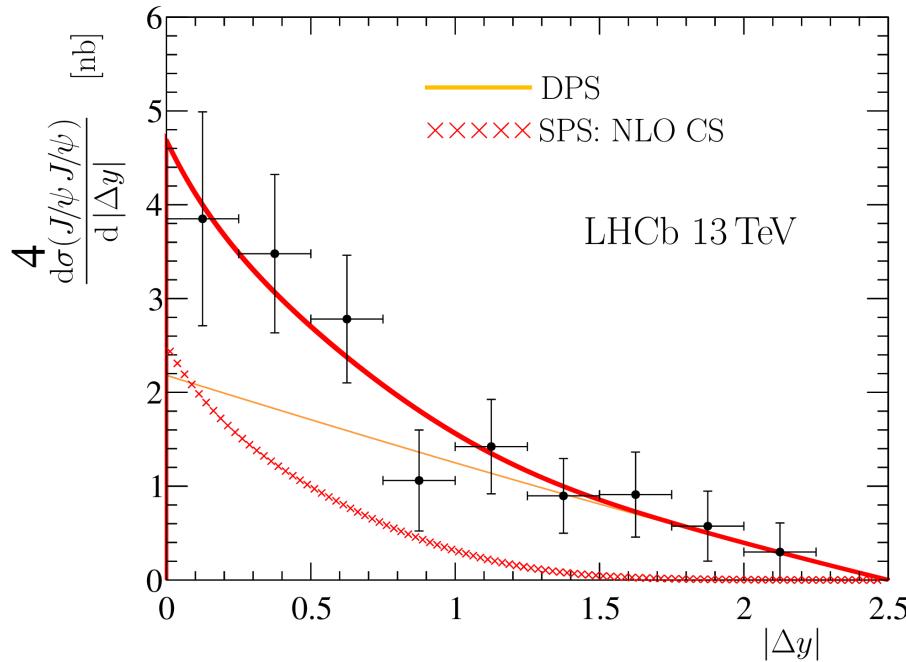
[\[JHEP 06 \(2017\) 047\]](#)

DPS extraction

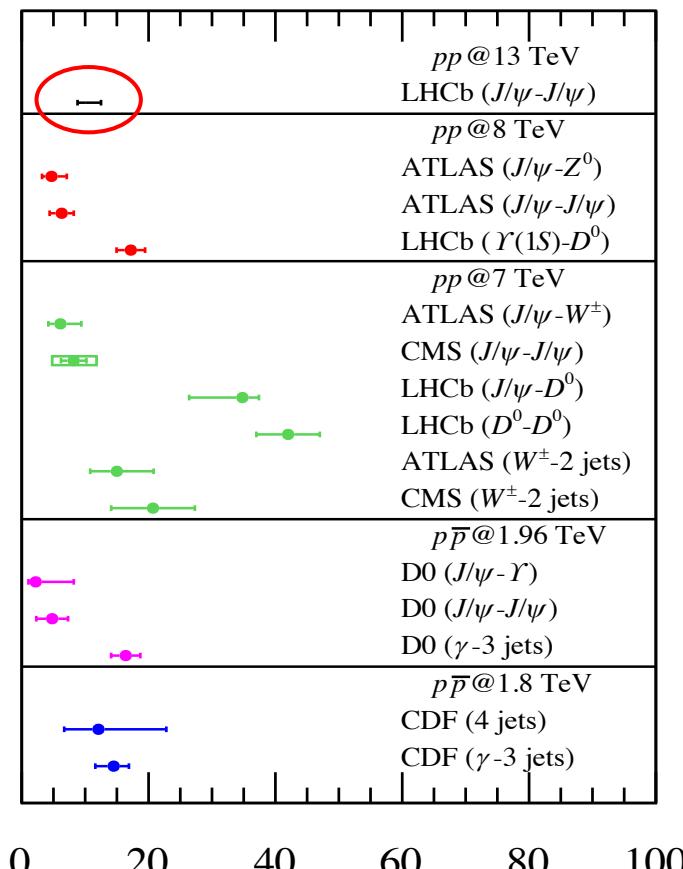
[\[JHEP 06 \(2017\) 047\]](https://doi.org/10.1007/JHEP06(2017)047)

➤ Template DPS+SPS fits performed for different variables using various models

$$\frac{d\sigma}{dv} = \sigma_{\text{DPS}} F_{\text{DPS}}(v) + \sigma_{\text{SPS}} F_{\text{SPS}}(v)$$



σ_{eff} : 8.8~12.5 mb

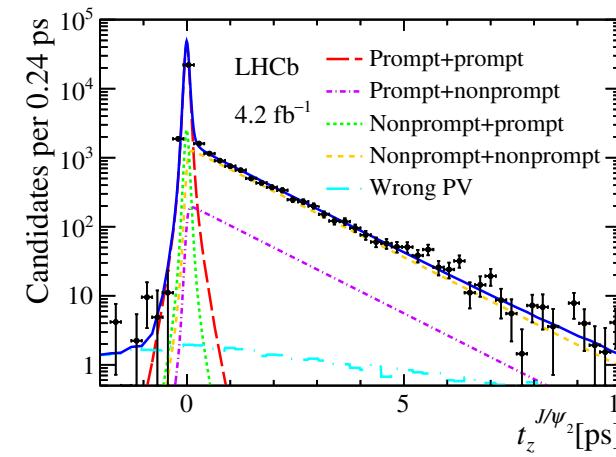
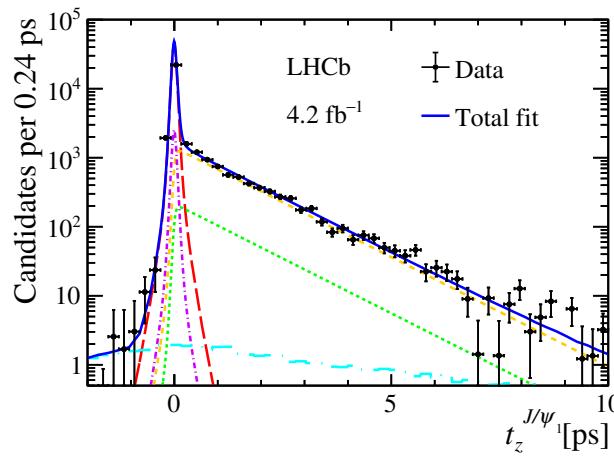
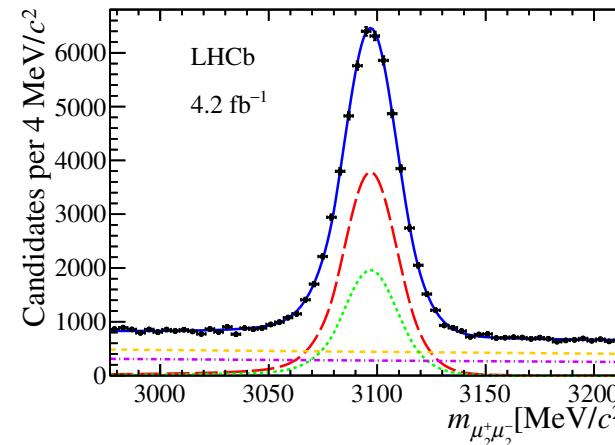
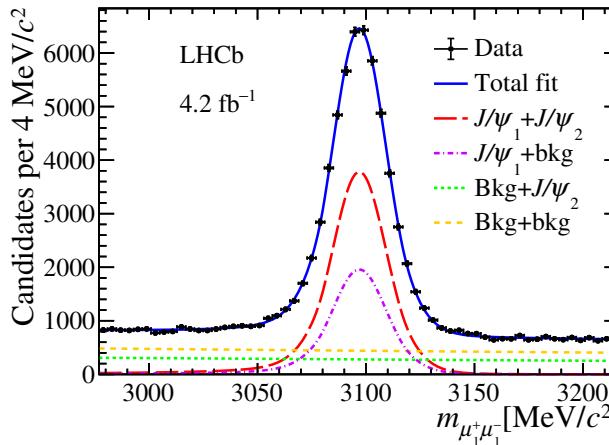


Variable	LO k_T	$\langle k_T \rangle = 2 \text{ GeV}/c$	$\langle k_T \rangle = 0.5 \text{ GeV}/c$	NLO CS
$p_T(J/\psi J/\psi)$	9.7 ± 0.5	8.8 ± 5.6	9.3 ± 1.0	—
$y(J/\psi J/\psi)$	—	11.9 ± 7.5	10.0 ± 5.0	—
$m(J/\psi J/\psi)$	10.6 ± 1.1	10.2 ± 1.0	—	10.4 ± 1.0
$ \Delta y $	12.5 ± 4.1	12.2 ± 3.7	12.4 ± 3.9	11.2 ± 2.9

di- J/ψ @ 13 TeV update

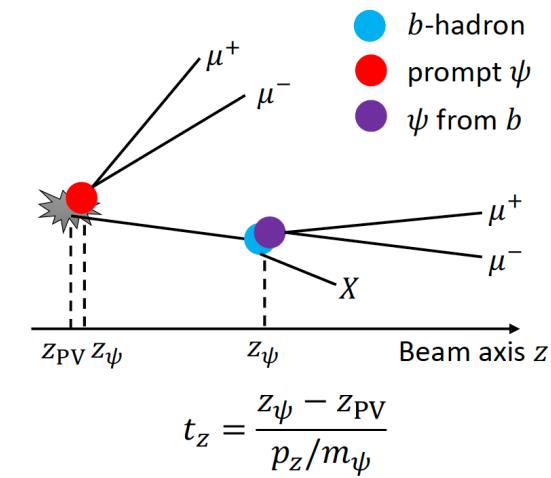
[JHEP 03 (2024) 088]

➤ Fiducial region: $2 < y(J/\psi) < 4.5, p_T(J/\psi) < 14 \text{ GeV}$



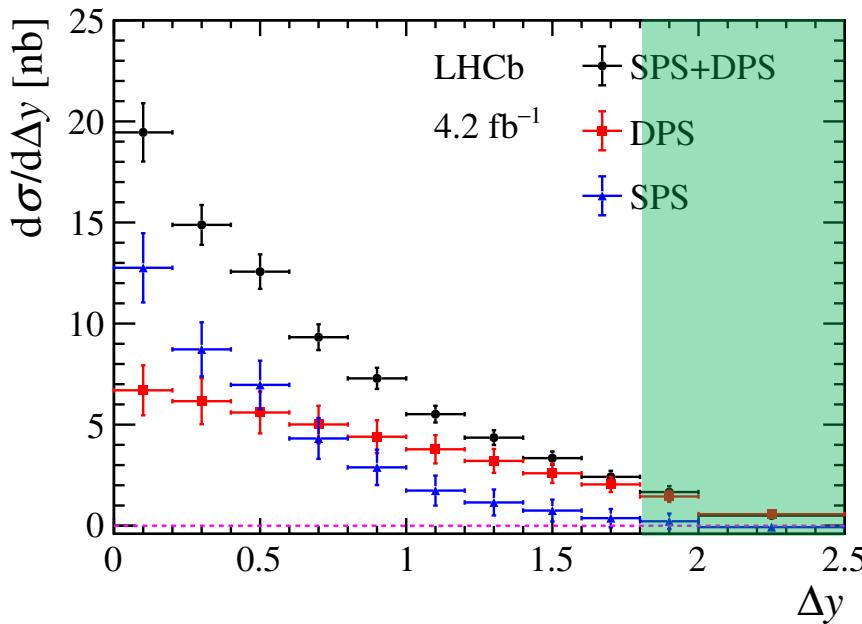
$$N(J/\psi - J/\psi)_{\text{prompt}} = (2.187 \pm 0.020) \times 10^4$$

$$\sigma(J/\psi - J/\psi) = 16.36 \pm 0.28(\text{stat}) \pm 0.88(\text{syst}) \text{ nb}$$

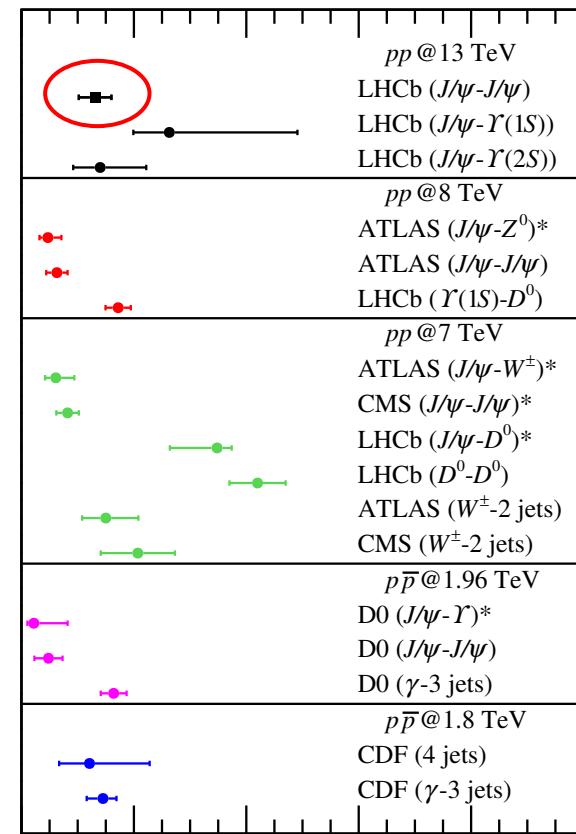


SPS and DPS separation

[JHEP 03 (2024) 088]



➤ SPS & DPS separated assuming negligible SPS contribution in $1.8 < \Delta y < 2.5$ according to NRQCD predictions



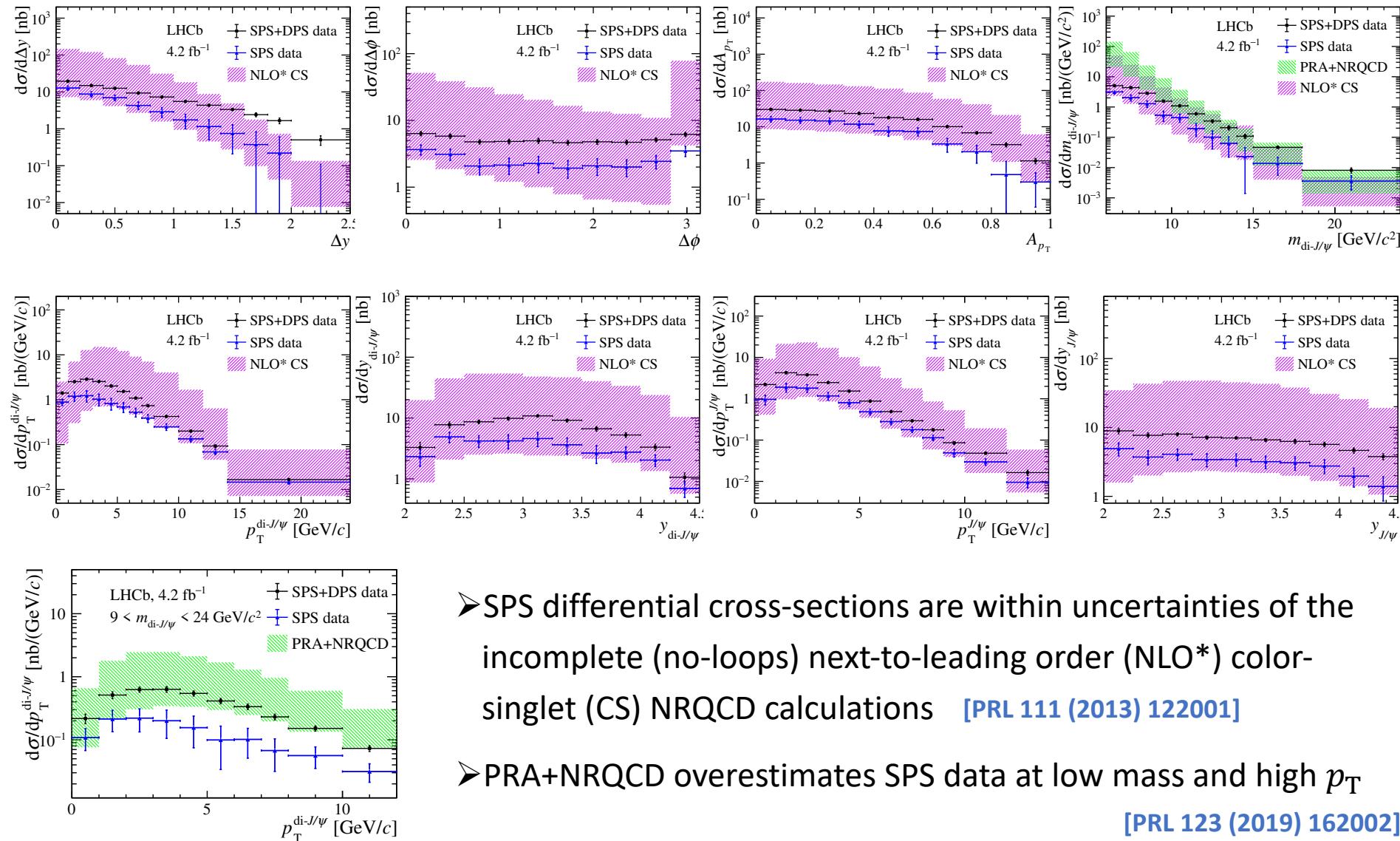
$$\sigma(J/\psi - J/\psi)_{\text{DPS}} = 8.6 \pm 1.2(\text{stat}) \pm 1.0(\text{syst}) \text{ nb}$$

$$\sigma(J/\psi - J/\psi)_{\text{SPS}} = 7.9 \pm 1.2(\text{stat}) \pm 1.1(\text{syst}) \text{ nb}$$

$$\sigma_{\text{eff}} = 13.1 \pm 1.8(\text{stat}) \pm 2.3(\text{syst}) \text{ mb}$$

Differential $J/\psi - J/\psi$ cross-section

[JHEP 03 (2024) 088]



➤ SPS differential cross-sections are within uncertainties of the incomplete (no-loops) next-to-leading order (NLO*) color-singlet (CS) NRQCD calculations [PRL 111 (2013) 122001]

➤ PRA+NRQCD overestimates SPS data at low mass and high p_T

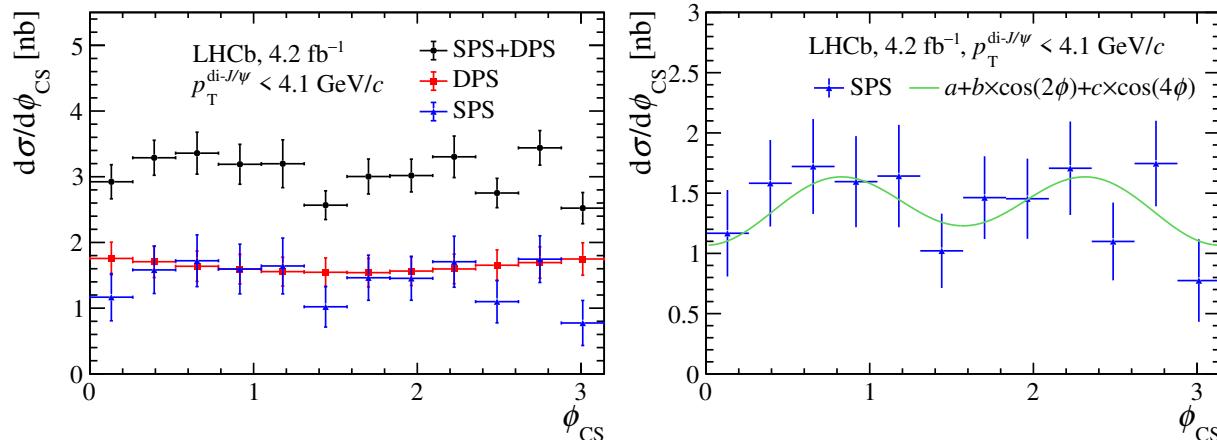
[PRL 123 (2019) 162002]

[Comput. Phys. Commun. 184 (2013) 2562] [Comput. Phys. Commun. 198 (2016) 238]

Gluon TMD PDFs study

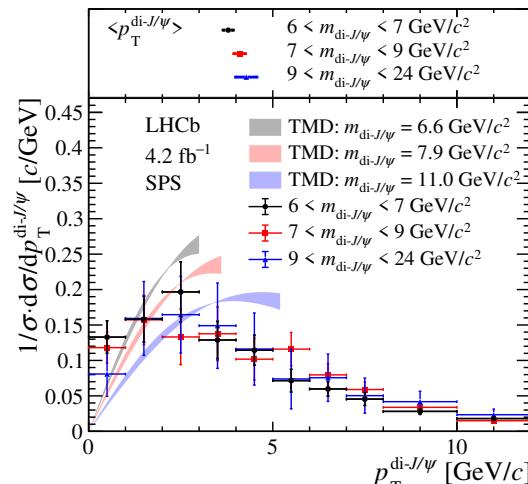
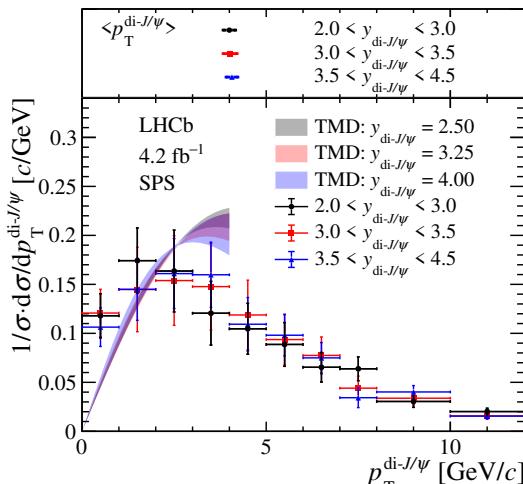
[JHEP 03 (2024) 088]

- $h_1^{\perp g}(x, k_T^2, \mu) \Rightarrow$ azimuthal asymmetry
 $d\sigma/d\phi_{CS} = a + b \times \cos(2\phi_{CS}) + c \times \cos(4\phi_{CS})$



- $f_1^g(x, k_T^2, \mu)$: affect p_T spectrum
 - ✓ p_T shape shows no dependence on y

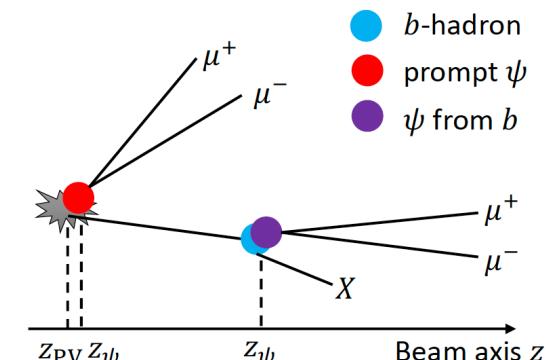
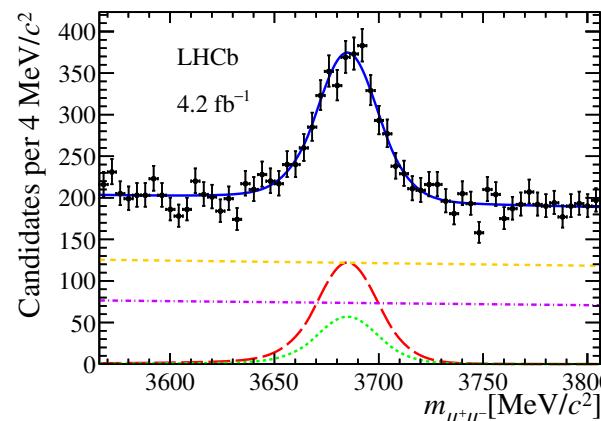
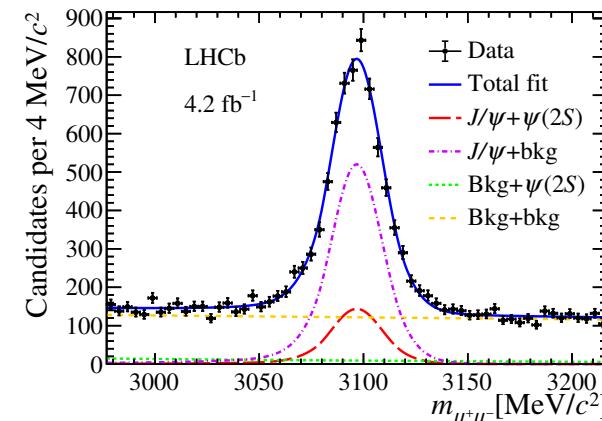
[EPJC 80 (2020) 87]
✓ No obvious broadening of p_T spectrum wrt increasing m given large uncertainties



$J/\psi - \psi(2S)$ @ 13 TeV

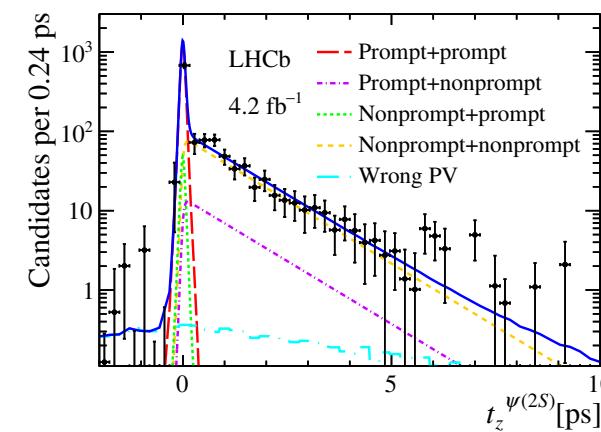
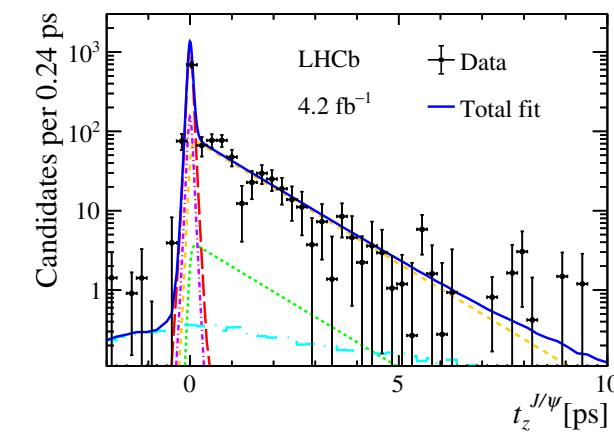
[JHEP 05 (2024) 259]

➤ Fiducial region: $2 < y(\psi) < 4.5, p_T(\psi) < 14$ GeV



$$t_z = \frac{z_\psi - z_{PV}}{p_z/m_\psi}$$

$$N(J/\psi - \psi(2S))_{\text{prompt}} = 629 \pm 50$$

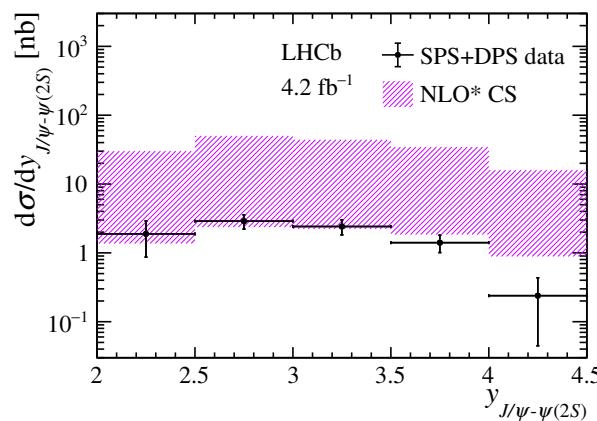
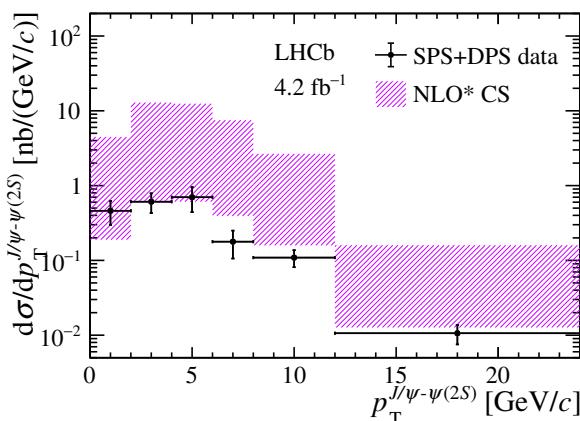
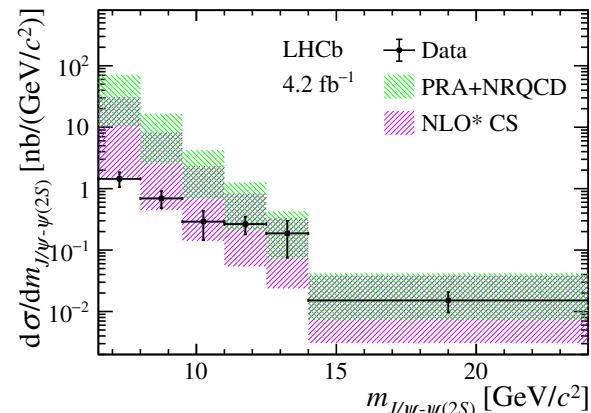
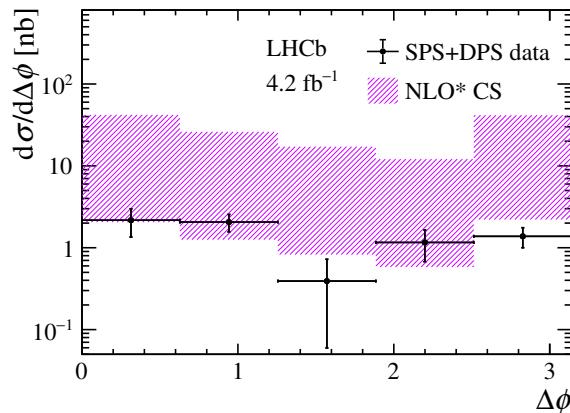
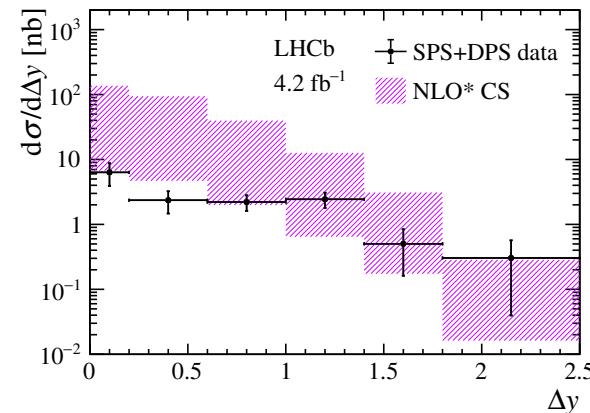


$$\sigma(J/\psi - \psi(2S)) = 4.49 \pm 0.71(\text{stat}) \pm 0.26(\text{syst}) \text{ nb}$$

$$\sigma_{\text{eff}}(\text{lower limit}) = \frac{\sigma(J/\psi)\sigma(\psi(2S))}{\sigma(J/\psi - \psi(2S))} = 7.1 \pm 1.1(\text{stat}) \pm 0.8(\text{syst}) \text{ mb}$$

Differential $J/\psi - \psi(2S)$ cross-section

[JHEP 05 (2024) 259]



- Results consistent with NLO* CS NRQCD calculations albeit DPS is not subtracted
[\[PRL 111 \(2013\) 122001\]](#) [\[Comput. Phys. Commun. 184 \(2013\) 2562\]](#) [\[Comput. Phys. Commun. 198 \(2016\) 238\]](#)
- PRA+NRQCD overestimates SPS data at low mass
[\[PRL 123 \(2019\) 162002\]](#)

$J/\psi - \psi(2S)$ vs. $J/\psi - J/\psi$

[JHEP 05 (2024) 259]

➤ Predictions on the ratio between $\sigma(J/\psi - \psi(2S))$ and $\sigma(J/\psi - J/\psi)$ give

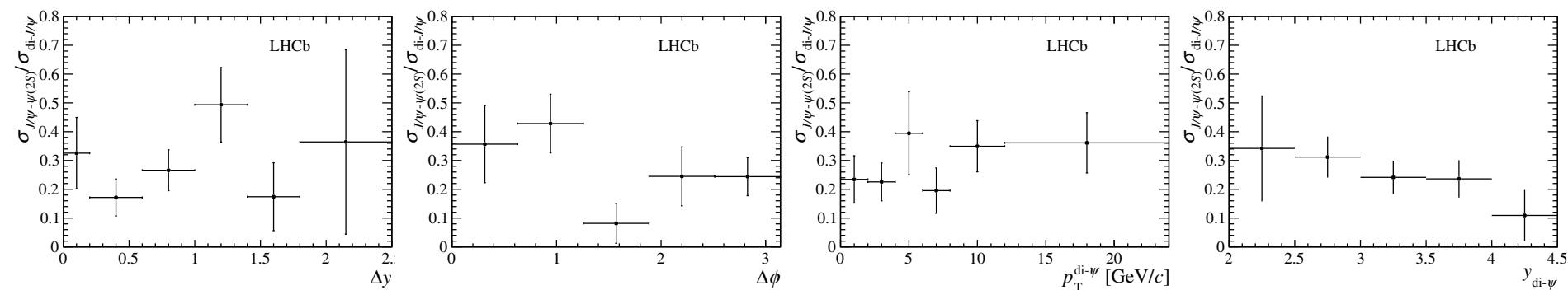
✓ SPS: 0.94 ± 0.030 [PLB 751 (2015) 479]

✓ DPS: 0.282 ± 0.027 [JHEP 10 (2015) 172] [EPJC 80 (2020) 185]

$$\frac{\sigma(J/\psi - \psi(2S))}{\sigma(J/\psi - J/\psi)} = 0.274 \pm 0.044(\text{stat}) \pm 0.008(\text{syst})$$

⇒ it confirms a prominent DPS contribution to $J/\psi - J/\psi$ production in a novel way, independent of the kinematic correlation of two J/ψ mesons

➤ Differential cross-section ratios are also measured, but more statistics needed



di- J/ψ @ 7 TeV

[JHEP 1409 (2014) 094]

➤ $\mathcal{L} = 4.7 \text{ fb}^{-1}$, pp collisions @ $\sqrt{s} = 7 \text{ TeV}$

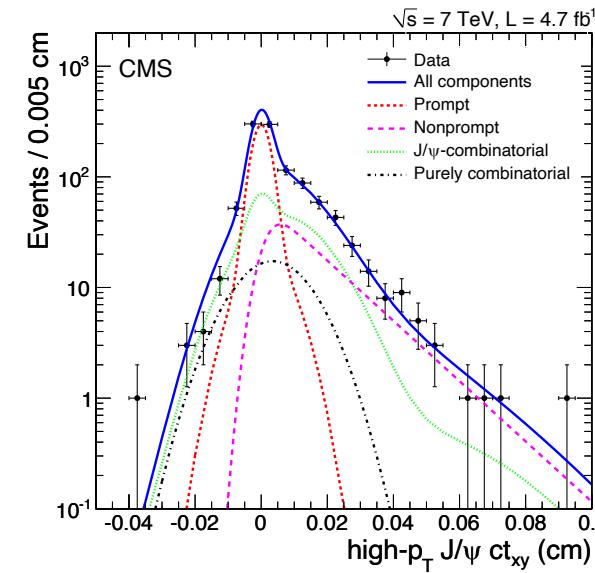
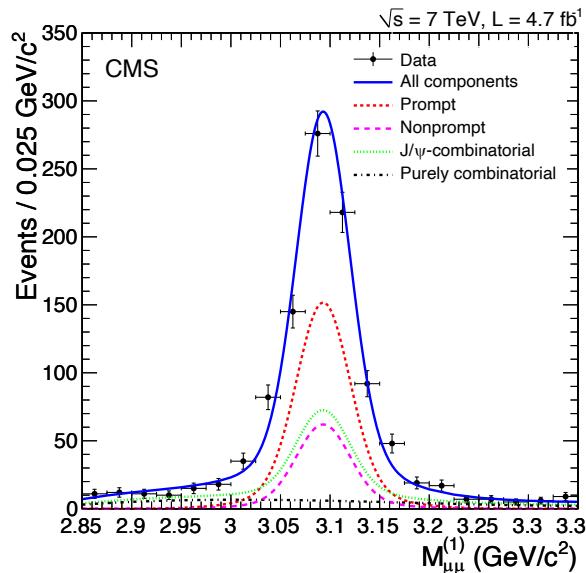
➤ Kinematic range of J/ψ :

$p_T > 6.5 \text{ GeV}/c$ for $|y| < 1.2$;

$p_T > 6.5 - 4.5 \text{ GeV}/c$ for $1.2 < |y| < 1.43$;

$p_T > 4.5 \text{ GeV}/c$ for $1.43 < |y| < 2.2$

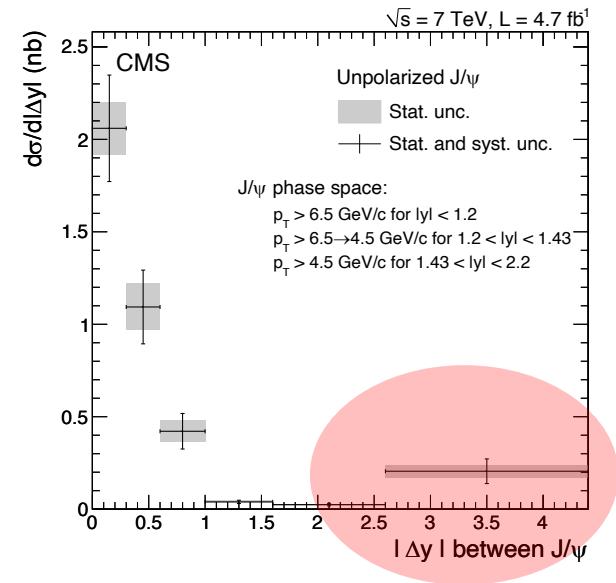
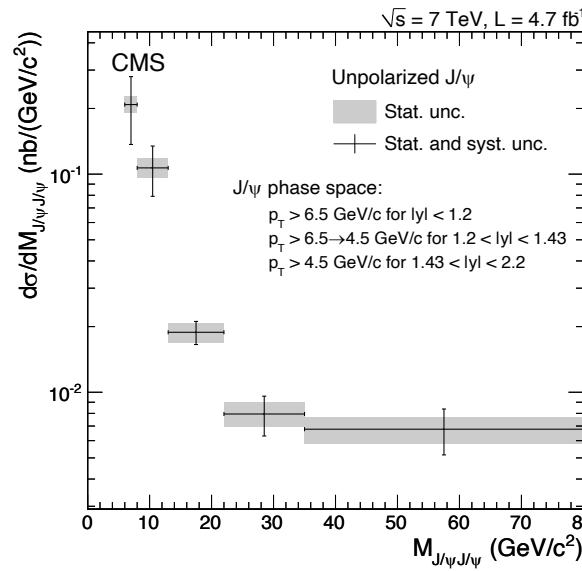
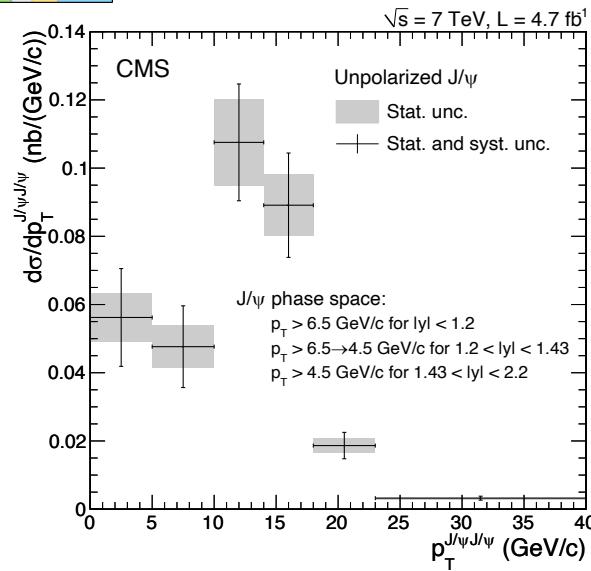
➤ Signal yield determination



➤ $\sigma(J/\psi J/\psi) = 1.49 \pm 0.07(\text{stat}) \pm 0.13(\text{syst}) \text{ nb}$

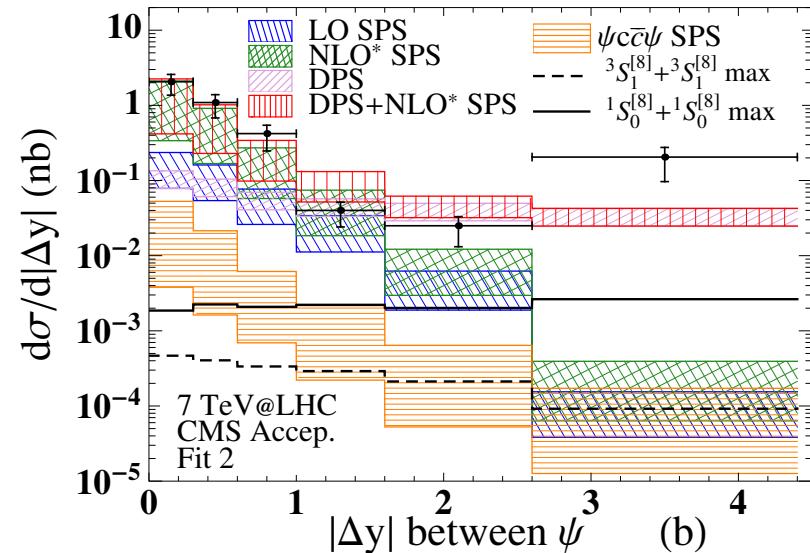
Differential cross-sections

[JHEP 1409 (2014) 094]



- SPS and DPS contributions determined by performing SPS+DPS templated fits to these differential cross-sections:
- $\sigma_{\text{eff}} = 8.2 \pm 2.0(\text{stat}) \pm 2.9(\text{syst}) \text{ mb}$

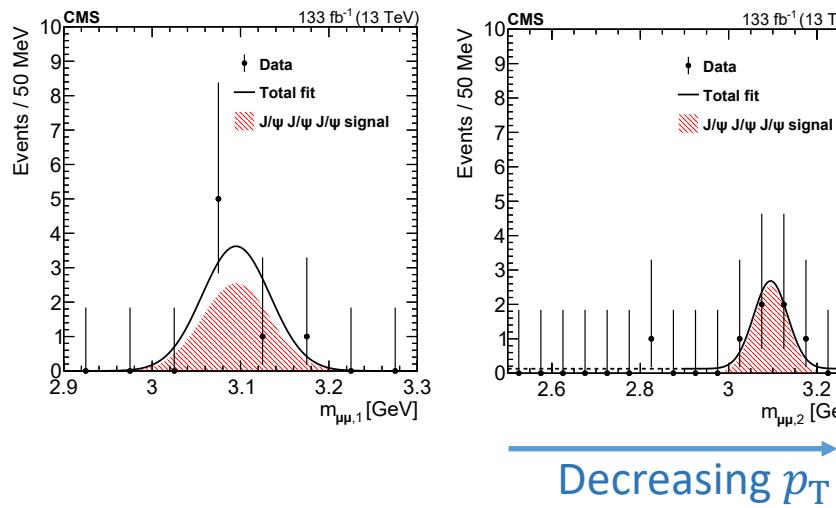
J. P. Lansberg, H. S. Shao: [PLB 751 (2015) 479]



tri- J/ψ @ 13 TeV

[Nat. Phys. 19 (2023) 338]

➤ First observation of triple J/ψ production

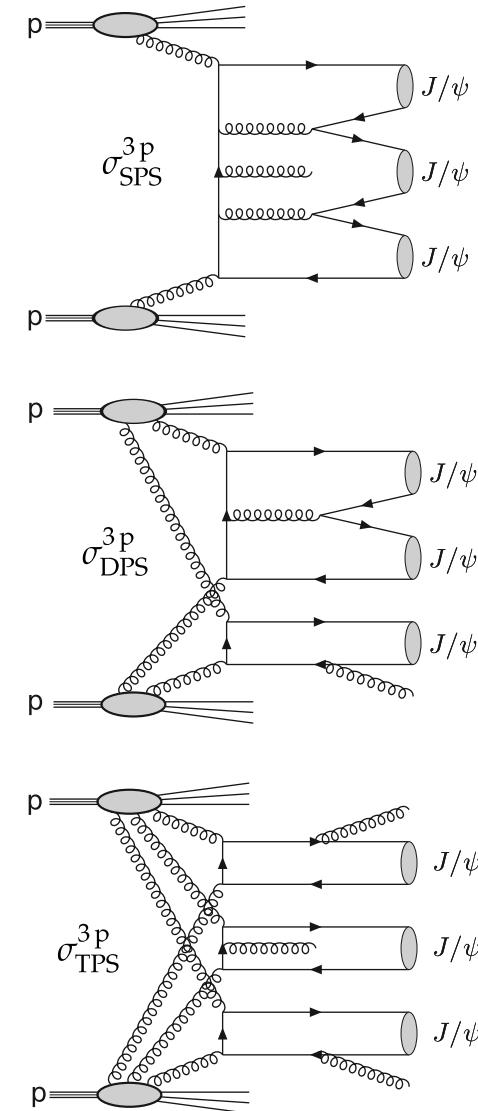


- 1 event → “3 non-prompt”
- 2 events → “2 non-prompt + 1 prompt”
- 1 event → “1 non-prompt + 2 prompt”
- 1 event → “3 prompt”

✓ Inclusive fiducial cross-section

$$\sigma(pp \rightarrow J/\psi J/\psi J/\psi + X) = 272^{+141}_{-104} \text{ (stat.)} \pm 17 \text{ (syst.) fb}$$

Pure prompt production:



σ_{eff} determination

[Nat. Phys. 19 (2023) 338]

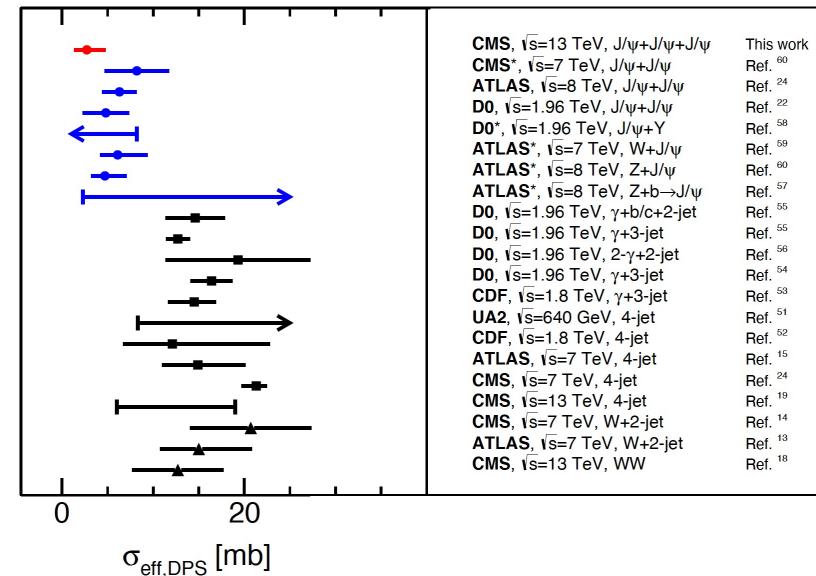
$$\begin{aligned}\sigma_{\text{tot}}^{3J/\psi} &= \sigma_{\text{SPS}}^{3J/\psi} + \sigma_{\text{DPS}}^{3J/\psi} + \sigma_{\text{TPS}}^{3J/\psi} \\ &= \left(\sigma_{\text{SPS}}^{3p} + \sigma_{\text{SPS}}^{2p1np} + \sigma_{\text{SPS}}^{1p2np} + \sigma_{\text{SPS}}^{3np} \right) \\ &\quad + \left(\sigma_{\text{DPS}}^{3p} + \sigma_{\text{DPS}}^{2p1np} + \sigma_{\text{DPS}}^{1p2np} + \sigma_{\text{DPS}}^{3np} \right) + \left(\sigma_{\text{TPS}}^{3p} + \sigma_{\text{TPS}}^{2p1np} + \sigma_{\text{TPS}}^{1p2np} + \sigma_{\text{TPS}}^{3np} \right)\end{aligned}$$

- ✓ SPS cross-sections taken from HELAC-ONIA at LO or NLO* (**prompt**) & MADGRAPH5_aMC@NLO + Pythia8 (**non-prompt**)
- ✓ Taking $\sigma_{\text{eff,TPS}} = (0.82 \pm 0.11) \sigma_{\text{eff,DPS}}$

$$\Rightarrow \sigma_{\text{eff,DPS}} = 2.7^{+1.4}_{-1.0} (\text{exp.})^{+1.5}_{-1.0} (\text{th.}) \text{ mb}$$

- ✓ DPS and TPS dominate

Process:	3 prompt	2 prompt+1 nonprompt	1 prompt+2 nonprompt	3 nonprompt	Total
$\sigma_{\text{SPS}}^{3J/\psi}$ (fb)	<0.005	5.7	0.014	12	18
$N_{\text{SPS}}^{3J/\psi}$	0.0	0.10	0.0	0.22	0.32
$\sigma_{\text{DPS}}^{3J/\psi}$ (fb)	8.4	8.9	90	95	202
$N_{\text{DPS}}^{3J/\psi}$	0.15	0.16	1.65	1.75	3.7
$\sigma_{\text{TPS}}^{3J/\psi}$ (fb)	6.1	19.4	20.4	7.2	53
$N_{\text{TPS}}^{3J/\psi}$	0.11	0.36	0.38	0.13	1.0
$\sigma_{\text{tot}}^{3J/\psi}$ (fb)	15	34	110	114	272
$N_{\text{tot}}^{3J/\psi}$	0.3	0.6	2.0	2.1	5.0

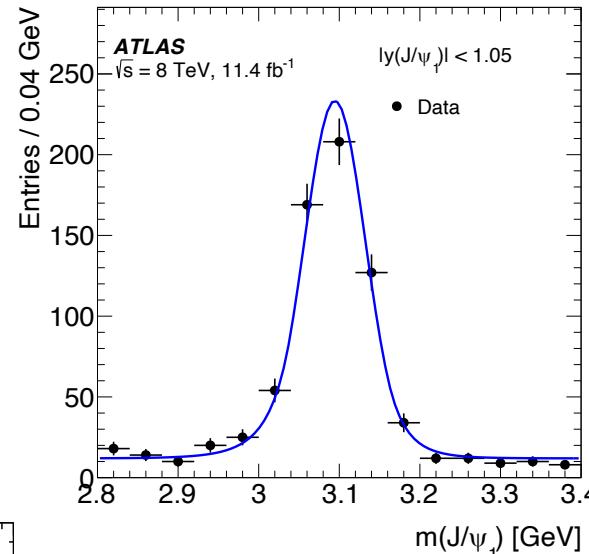


di- J/ψ @ 8 TeV

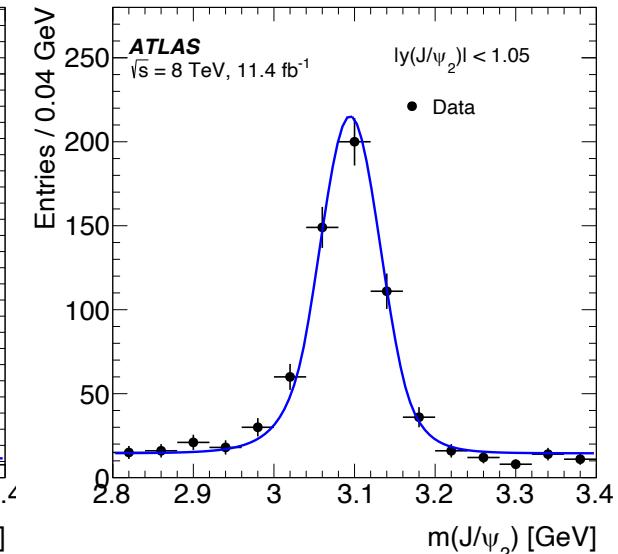
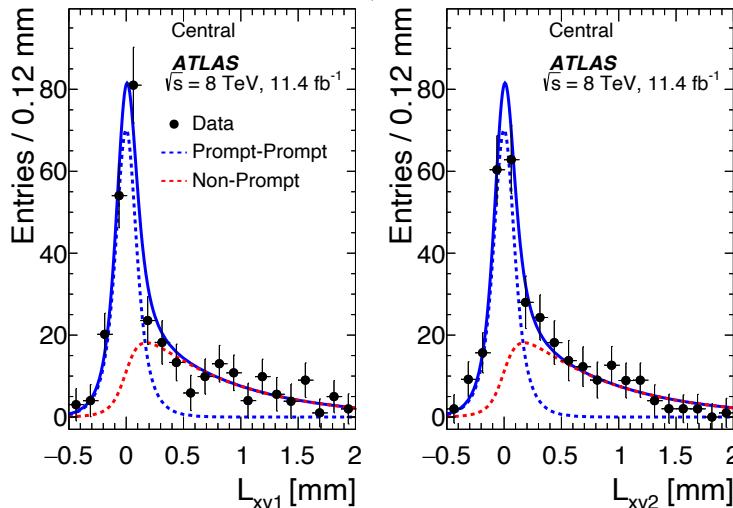
- $\mathcal{L} = 11.4 \text{ fb}^{-1}$, pp collisions @ $\sqrt{s} = 8 \text{ TeV}$
- Kinematic range of J/ψ : $p_T > 8.5 \text{ GeV}/c$ for $|y| < 2.1$
- Signal extraction

[EPJC 77 (2017) 76]

Non- J/ψ background

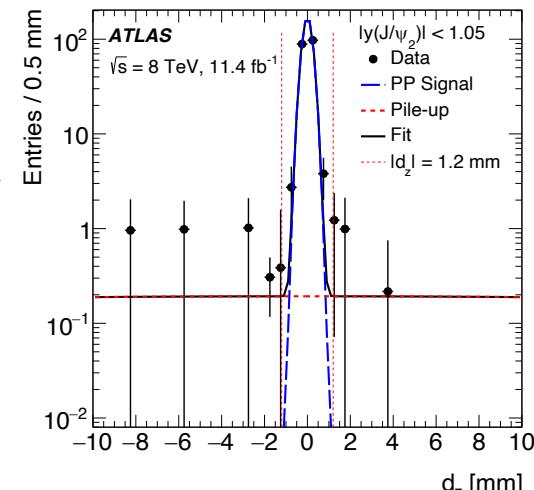


Non-prompt background



d_z : distance between two J/ψ vertices along z direction

Pile-up



Cross-sections

[EPJC 77 (2017) 76]

- Fiducial cross-section measured in two $y(J/\psi)$ regions due to different mass resolutions
- Within the muon kinematic acceptance: $p_T^\mu > 2.5 \text{ GeV}/c, |\eta^\mu| < 2.3$

$15.6 \pm 1.3 \text{ (stat)} \pm 1.2 \text{ (syst)} \pm 0.2 \text{ (BF)} \pm 0.3 \text{ (lumi)} \text{ pb, for } |y| < 1.05,$ $13.5 \pm 1.3 \text{ (stat)} \pm 1.1 \text{ (syst)} \pm 0.2 \text{ (BF)} \pm 0.3 \text{ (lumi)} \text{ pb, for } 1.05 \leq |y| < 2.1$
- Extrapolating to the full fiducial region assuming zero polarisation

$82.2 \pm 8.3 \text{ (stat)} \pm 6.3 \text{ (syst)} \pm 0.9 \text{ (BF)} \pm 1.6 \text{ (lumi)} \text{ pb, for } |y| < 1.05,$ $78.3 \pm 9.2 \text{ (stat)} \pm 6.6 \text{ (syst)} \pm 0.9 \text{ (BF)} \pm 1.5 \text{ (lumi)} \text{ pb, for } 1.05 \leq |y| < 2.1$

DPS extraction

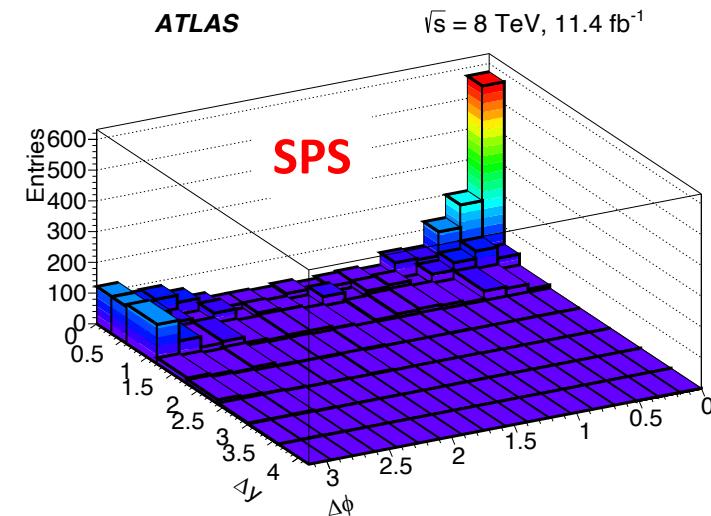
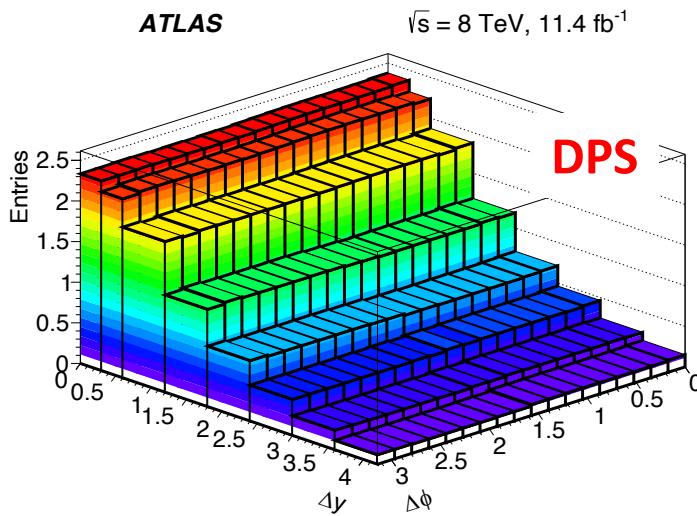
[EPJC 77 (2017) 76]

➤ Data-driven DPS template

- ✓ J/ψ mesons taken from two random events of the di- J/ψ data sample
- ✓ Normalized to the data in the region $\Delta y > 1.8$ and $\Delta\phi \leq \pi/2$, where there is negligible SPS contribution

➤ Data-driven SPS template

- ✓ Subtracting the DPS template from the $(\Delta y, \Delta\phi)$ distribution of the background-subtracted data



➤ Each candidate is assigned w_{DPS} and w_{SPS}

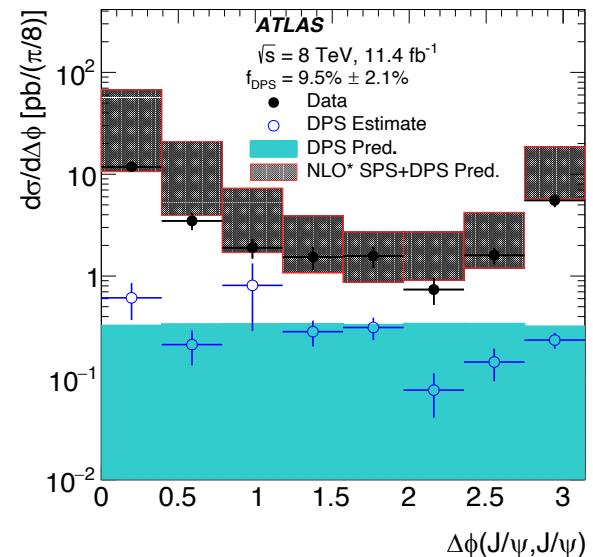
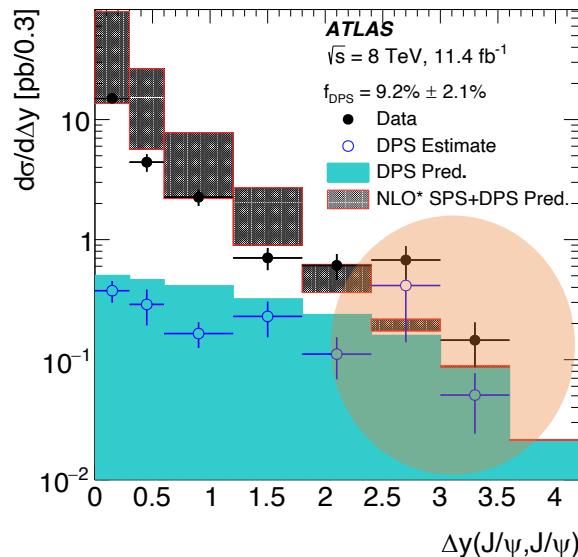
$$\textcolor{red}{f_{\text{DPS}} = (9.2 \pm 2.1 \pm 0.5)\%}$$

$$\sigma_{\text{eff}} = 6.3 \pm 1.6(\text{stat}) \pm 1.0(\text{syst}) \pm 0.1(\text{BF}) \pm 0.1(\text{lumi}) \text{ mb}$$

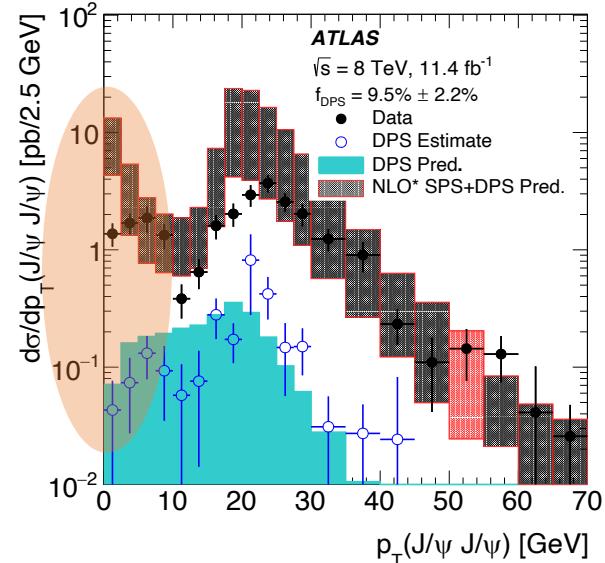
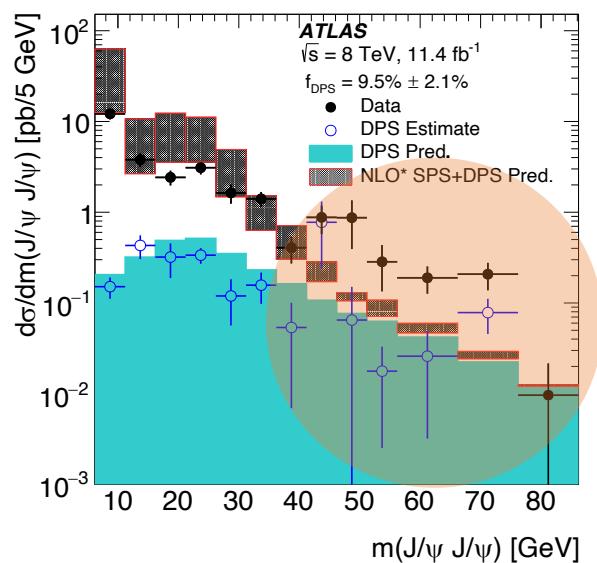
Comparison to theory (I)

[EPJC 77 (2017) 76]

- Compared to DPS + NLO* SPS predictions
- DPS fraction fixed to measured f_{DPS}



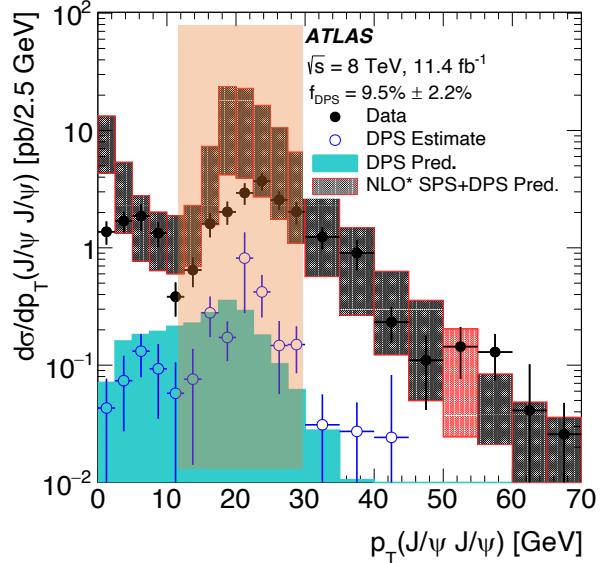
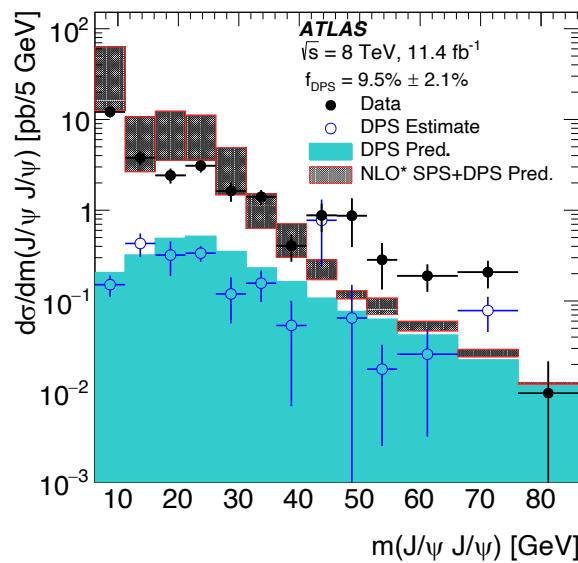
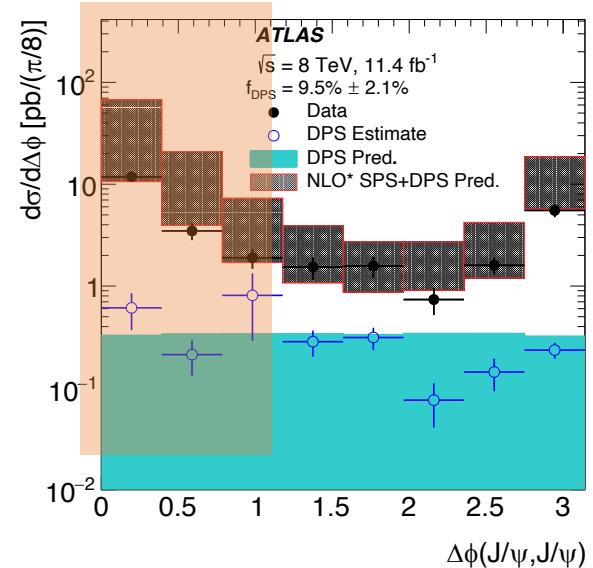
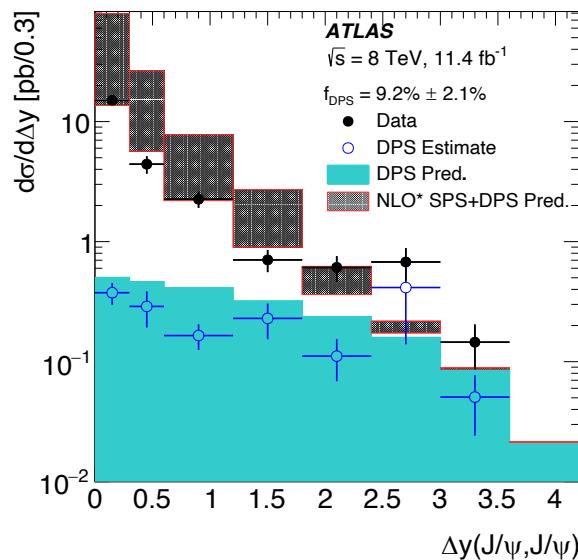
- ✓ Some discrepancy for the total cross-section in regions of away topology
- ✓ Maybe due to the constant factor for feed-down in NLO* SPS



Comparison to theory (II)

[EPJC 77 (2017) 76]

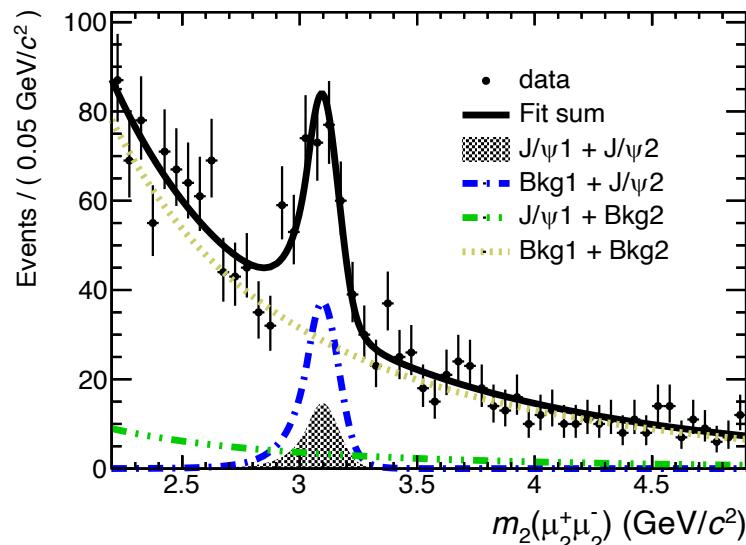
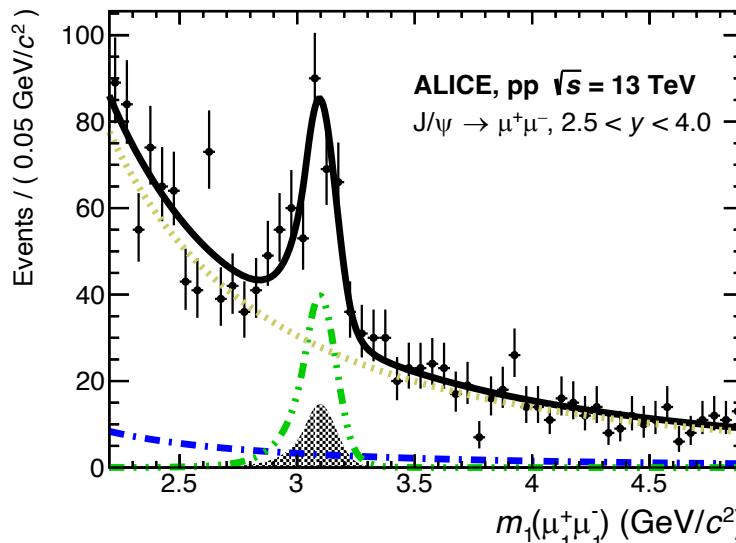
- Compared to DPS + NLO* SPS predictions
- DPS fraction fixed to measured f_{DPS}



di- J/ψ @ 13 TeV

[PRC 108 (2023) 045203]

- $\mathcal{L} = 24.11 \text{ pb}^{-1}$, pp collisions @ $\sqrt{s} = 13 \text{ TeV}$
- Kinematic range of J/ψ : $p_T > 0 \text{ GeV}/c$ for $2.5 < y < 4$



- ✓ $\sigma(J/\psi - J/\psi) = 10.3 \pm 2.3(\text{stat.}) \pm 1.3(\text{syst.}) \text{ nb}$
- ✓ $\sigma_{\text{prompt}}(J/\psi - J/\psi) = 7.3 \pm 1.7(\text{stat.})^{+1.9}_{-2.1}(\text{syst.}) \text{ nb}$
- ✓ Assuming only the DPS process contributes:
- ✓ $\sigma_{\text{eff}}(J/\psi - J/\psi) = \frac{1}{2} \frac{\sigma_{\text{prompt}}(J/\psi)^2}{\sigma_{\text{prompt}}(J/\psi - J/\psi)} = 6.7 \pm 1.6(\text{stat.}) \pm 2.7(\text{syst.}) \text{ mb}$

Summary and prospects

◆ Double J/ψ production measured by all four LHC experiments

- ✓ LHCb: 7&13 TeV
- ✓ CMS: 7 TeV
- ✓ ATLAS: 8 TeV
- ✓ ALICE: 13 TeV

◆ What we have learned on DPS?

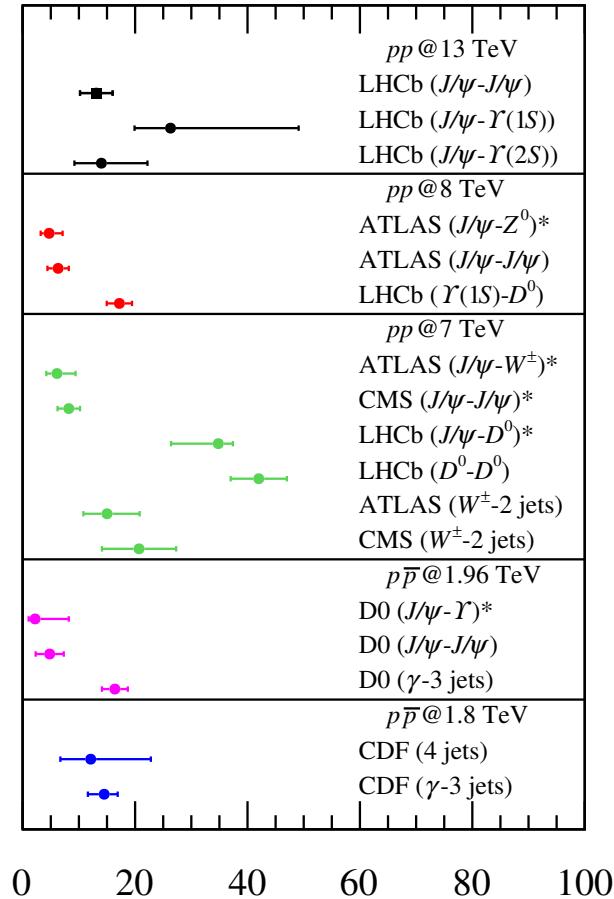
- ✓ Similar-level σ_{eff} : a good starting point
- ✓ How to further investigate the discrepancies?
- ✓ A unified way to separate SPS and DPS?

◆ What else we can learn from di-quarkonium prod.?

- ✓ gluon TMD
- ✓ ...

◆ Other di-quarkonium modes of interest?

- ✓ $J/\psi + \chi_c, J/\psi + \eta_c \dots$



[PoS (LHCPheno2020) 172; σ_{eff} [mb]
arXiv: 2009.12555]

Back up

Leading twist TMD PDFs

[PR12-09-014]

Nucleon Quark	Unpol.	Long.	Trans.
Unpol.	$f_1 = \bullet$		$f_{1T}^\perp = \bullet \uparrow - \bullet \downarrow$
Long.		$g_{1L} = \bullet \rightarrow - \bullet \rightarrow$	$g_{1T} = \bullet \uparrow - \bullet \uparrow$
Trans.	$h_1^\perp = \bullet \downarrow - \bullet \downarrow$	$h_{1L}^\perp = \bullet \rightarrow - \bullet \rightarrow$	$h_{1T}^\perp = \bullet \uparrow \uparrow - \bullet \uparrow \uparrow$

Sketch of CS frame

