Pushing the limits of the TMD evolution formalism to study gluon TMD distributions: J/ψ-pair production at LHC as a case study

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Outline

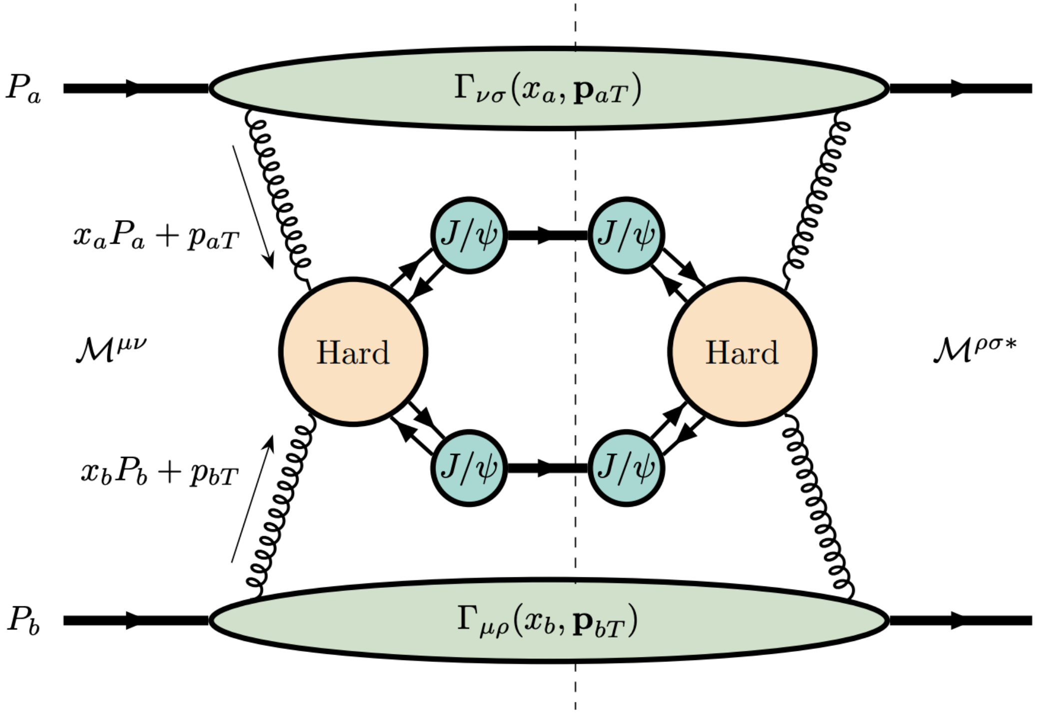
- Introduction

- TMD evolution in a nutshell
- A novel nonperturbative Sudakov factor
- Predictions of TM spectra and azimuthal modulations

- Conclusions

$p + p \to J/\psi + J/\psi + X$

- J/ψ-pair production gives via its q_T-spectrum and modulations access to the gluon TMDs *Lansberg et al. 2018, Scarpa et al. 2020*



$|\mathbf{q}_T| \ll \mu_H = M_{QQ}$

$x_{a,b} = \exp(\pm y_{QQ}) M_{QQ}/\sqrt{S}$

- Probe the transverse momentum of the partonic gluons via the observed quarkonia: $\boldsymbol{p}_{aT} + \boldsymbol{p}_{bT} = \boldsymbol{q}_T$
- The invariant mass M_{QQ} allows to study scale evolution of the TMDs

- Make use of CS-model in which TMD-factorization breaking effects are avoided (@ LO α_s^4)
- No TMDShF / smearing effects are expected for CS quarkonium at LO

- There are recent measurements of this process *LHCb 2023*

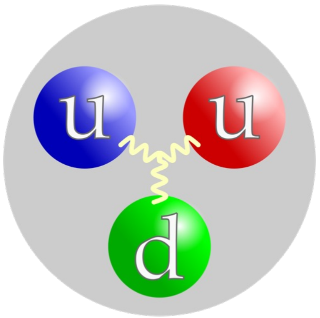
The Gluon TMD and the hadron correlator

- Unpolarized proton is parameterized by two functions at LO (twist ∼ 1/hard scale)
    - Unpolarized gluon distribution: f_1^g
    - Linearly polarized gluon distribution: $h_1^{\perp g}$

		Parent hadron polarisation		
		Unpolarised	Longitudinal	Transverse
Gluon polarisation	Unpolarised	f_1 (Number density)		$f_{1T}^{\perp}$ (Sivers)
	Circular		g_{1L} (Helicity)	g_{1T} (Worm-gear)
	Linear	$h_1^{\perp}$ (Boer-Mulders)	$h_{1L}^{\perp}$ (Worm-gear)	h_1 (Transversity) $h_{1T}^{\perp}$ (Pretzelosity)

$$\Gamma_U^{ij}(x,\mathbf{k}_T) = \frac{x}{2}\left\{ -g_T^{ij} \boxed{f_1(x,\mathbf{k}_T^2)} + \left(\frac{k_T^i k_T^j}{M_h^2} + g_T^{ij}\frac{\mathbf{k}_T^2}{2M_h^2}\right)\boxed{h_1^{\perp}(x,\mathbf{k}_T^2)}\right\}$$

Mulders and Rodrigues 2001



The differential cross section at LO

$$d\sigma^{gg}_{UU} \equiv \frac{d\sigma}{dM_{QQ}\, dy_{QQ}\, d^2\mathbf{q}_T\, d\cos\theta_{CS}\, d\phi_{CS}} = \frac{\sqrt{M^2_{QQ} - 4M^2_Q}}{(2\pi)^2 8S\, M^2_{QQ}} \Big\{ F_1 \times \mathcal{C}[f_1^g f_1^g] + F_2 \times \mathcal{C}[w_2 h_1^{\perp g} h_1^{\perp g}] + \{F_3 \times \mathcal{C}[w_3 f_1^g h_1^{\perp g}] + F_3' \times \mathcal{C}[w_3' h_1^{\perp g} f_1^g]\} \cos(2\phi_{CS}) + F_4 \times \mathcal{C}[w_4 h_1^{\perp g} h_1^{\perp g}] \cos(4\phi_{CS}) \Big\},$$

$$w_2 = \frac{1}{4M_h^4}\left[2(\mathbf{p}_{aT}\cdot\mathbf{p}_{bT})^2 - \mathbf{p}_{aT}^2\mathbf{p}_{bT}^2\right]$$

$$w_3 = \frac{1}{2M_h^2\mathbf{q}_T^2}[\mathbf{p}_{bT}^2\mathbf{q}_T^2 - 2(\mathbf{p}_{bT}\cdot\mathbf{q}_T)^2], \quad w_3' = \frac{1}{2M_h^2\mathbf{q}_T^2}[\mathbf{p}_{aT}^2\mathbf{q}_T^2 - 2(\mathbf{p}_{aT}\cdot\mathbf{q}_T)^2],$$

$$w_4 = 2\left(\frac{\mathbf{p}_{aT}\cdot\mathbf{p}_{bT}}{2M_h^2} - \frac{(\mathbf{p}_{aT}\cdot\mathbf{q}_T)(\mathbf{p}_{bT}\cdot\mathbf{q}_T)}{M_h^2\mathbf{q}_T^2}\right)^2 - \frac{\mathbf{p}_{aT}^2\mathbf{p}_{bT}^2}{4M_h^4}.$$

$$\langle\cos(2\phi_{CS})\rangle = \frac{1}{2}\frac{F_3\left(\mathcal{C}[w_3 f_1^g h_1^{\perp g}] + \mathcal{C}[w_3' h_1^{\perp g} f_1^g]\right)}{F_1\, \mathcal{C}[f_1^g f_1^g] + F_2\, \mathcal{C}[w_2 h_1^{\perp g} h_1^{\perp g}]},$$

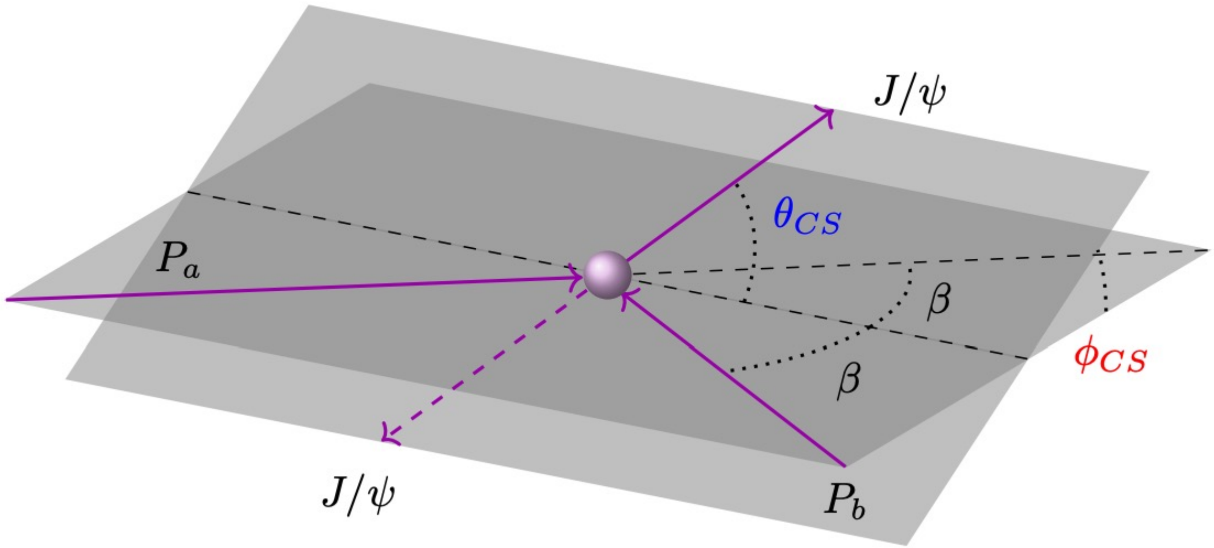
$$\langle\cos(4\phi_{CS})\rangle = \frac{1}{2}\frac{F_4\, \mathcal{C}[w_4 h_1^{\perp g} h_1^{\perp g}]}{F_1\, \mathcal{C}[f_1^g f_1^g] + F_2\, \mathcal{C}[w_2 h_1^{\perp g} h_1^{\perp g}]}.$$

$$\mathcal{C}[wfg] = \int d^2\mathbf{p}_{aT} \int d^2\mathbf{p}_{bT}\, \delta^2(\mathbf{p}_{aT} + \mathbf{p}_{bT} - \mathbf{q}_T)\, w(\mathbf{p}_{aT}, \mathbf{p}_{bT})\, f(x_a, \mathbf{p}_{aT}^2)\, g(x_b, \mathbf{p}_{bT}^2)$$

- Hard factors F_i

  Lansberg et al. 2018

- F_2 negligible when M_{QQ} is large or $|\cos\theta_{CS}| \leq 0.5$



$\mathcal{C}[f_1^g f_1^g]$

- $\mathcal{C}[f_1^g f_1^g]$ is a general quantity that determines the unpolarized differential cross section for any proton-proton process that are dominated by gluon-gluon fusion:
  - Higgs production *Sun et al. 2011, Boer et al. 2012*
  - $\eta_Q, \chi_{Q0}, \chi_{Q2}$ production *Boer and Pisano 2012*
  - Quarkonium + di-lepton production *Lansberg et al. 2017*

- Also, it appears next to quark-antiquark and quark-gluon contributions:
  - Higgs + jet production *Boer and Pisano 2014*
  - Di-jet production *Boer et al. 2009*
  - open heavy quark production *Boer et al. 2010, Pisano et al. 2013, Boer et al. 2016*

Introduction of evolution

- Beyond tree level, the TMDs and hard factor become scale dependent

  Collins and Soper 1981

- Implementing evolution is more easily done in impact parameter space, where convolutions become simple products

$$\frac{d\sigma}{d(\text{kinematic variables})\, d^2\mathbf{q}_T} = \int d^2\mathbf{b}_T\, e^{-i\mathbf{b}_T\cdot\mathbf{q}_T}\, \hat{W}(\mathbf{b}_T,\mu_H) + \mathcal{O}(\mathbf{q}_T^2/\mu_H^2)$$

$$\hat{W}(\mathbf{b}_T,\mu_H) = \hat{A}(\mathbf{b}_T;\zeta_A,\mu)\, \hat{B}(\mathbf{b}_T;\zeta_B,\mu)\, \mathcal{H}(\mu_H;\mu)$$

$$\mathcal{C}[f_1^g f_1^g] = \int_0^\infty \frac{db_T}{2\pi}\, b_T\, J_0(b_T q_T)\, \hat{f}_1^g(x_a,\mathbf{b}_T^2)\, \hat{f}_1^g(x_b,\mathbf{b}_T^2)$$

$$\hat{f}_1^g(x,\mathbf{b}_T^2) \equiv \int d^2\mathbf{p}_T\, e^{i\mathbf{b}_T\cdot\mathbf{p}_T}\, f_1^g(x,\mathbf{p}_T^2)$$

The Sudakov factor and scales

- CS Evolution: $\hat{f}(x, \mathbf{b}_T^2; \zeta, \mu) = e^{-S_A(b_T,\zeta,\zeta_0,\mu,\mu_0)}\, \hat{f}(x, \mathbf{b}_T^2; \zeta_0, \mu_0)$

$$S_A(b_T, \zeta, \zeta_0, \mu, \mu_0) = -\frac{1}{2}\hat{K}(b_T, \mu_0) \ln \frac{\zeta}{\zeta_0} - \int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma(\alpha_s(\mu'), 1) - \frac{1}{2}\gamma_K(\alpha_s(\mu')) \ln \frac{\zeta}{\mu'^2}\right]$$

- To avoid large logs in the hard factor $\mu \sim \mu_H$
- TMDs should be evaluated at their natural scale $\sqrt{\zeta_0} \sim \mu_0 \ll \sqrt{\zeta} \sim \mu$
- Instead of choosing a low, still perturbative scale, is common to take

$$\sqrt{\zeta_0} \sim \mu_0 \sim \boxed{\mu_b \equiv b_0/b_T} = 2e^{-\gamma_E}/|\mathbf{b}_T| \qquad \boxed{\mu_b \leq \mu_H}$$

- b_T must be constrained $\boxed{b_{T,\min} = b_0/M_{QQ} \leq b_T \leq b_{T,\max} = b_0/\mu_{NP}}$

- $b_{T,max}$ is the point where perturbation theory starts to fail: [0.5: 1.5] GeV^{-1}

b_T-domains and the nonperturbative Sudakov

1) $b_{T,\min} \leq b_T$

Boer and Den Dunnen 2014

$$\mu_b \to \mu_b' = \frac{b_0}{b_T + b_0/\mu_H}$$

Collins et al. 2016

$$\mu_b \to \tilde{\mu}_b' = \frac{b_0}{\sqrt{b_T^2 + b_0^2/\mu_H^2}}$$

2) $b_T \leq b_{T,\max}$

$$b_T^*(b_T) = \frac{b_T}{\sqrt{1 + (b_T/b_{T,\max})^2}}$$

Collins et al. 1982

$$\mu_b \to \tilde{\mu}_b' = \frac{b_0}{\sqrt{b_T^2 + b_0^2/M_{QQ}^2}} \to \tilde{\mu}_{b*}' = \frac{b_0}{\sqrt{b_T^{*2} + b_0^2/M_{QQ}^2}}$$

- For $b_T > b_{T,max}$:

$$\hat{W}(b_T, \mu_H) \equiv \hat{W}(b_T^*, \mu_H) e^{-S_{NP}(b_T;\mu_H,\mu_{NP})}$$

$$S_{NP}(b_T; \mu_H, \mu_{NP}) = \ln\left(\frac{\mu_H}{\mu_{NP}}\right) g_K(b_T) + g_{f_A}(b_T) + g_{f_B}(b_T)$$

The convolution(s)

$$\hat{f}_1^g(x, b_T^*; \tilde{\mu}'_{b*}) = f_1^g(x; \tilde{\mu}'_{b*}) + \mathcal{O}(\alpha_s) + \mathcal{O}(b_T \Lambda_{\rm QCD})$$

- Perturbative TMD tail

$$\hat{h}_1^{\perp g}(x, b_T^*; \tilde{\mu}'_{b*}) = -\frac{\alpha_s(\tilde{\mu}'_{b*})}{\pi} \int_x^1 \frac{dx'}{x'} \left(\frac{x'}{x} - 1\right) \left\{ C_A \, f_1^g(x'; \tilde{\mu}'_{b*}) + C_F \sum_{i=q,\bar{q}} f_1^i(x'; \tilde{\mu}'_{b*}) \right\} + \mathcal{O}(\alpha_s^2) + \mathcal{O}(b_T \Lambda_{\rm QCD})$$

Sun et al. 2011

- Suppressed by α_s

$$S_A(b_T^*; M_{QQ}, \tilde{\mu}'_{b*}) = \frac{1}{2}\frac{C_A}{\pi} \int_{\tilde{\mu}'^2_{b*}}^{M_{QQ}^2} \frac{d\mu'^2}{\mu'^2} \left[\alpha_s(\mu') + \frac{\alpha_s(\mu')^2}{4\pi} \left\{ \left(\frac{67}{9} - \frac{\pi^2}{3} \right) N_c - \frac{20}{9} T_R n_f \right) \right\} \right] \ln \frac{M_{QQ}^2}{\mu'^2} - \frac{1}{2}\frac{C_A}{\pi} \int_{\tilde{\mu}'^2_{b*}}^{M_{QQ}^2} \frac{d\mu'^2}{\mu'^2} \, \alpha_s(\mu') \, \beta_0$$

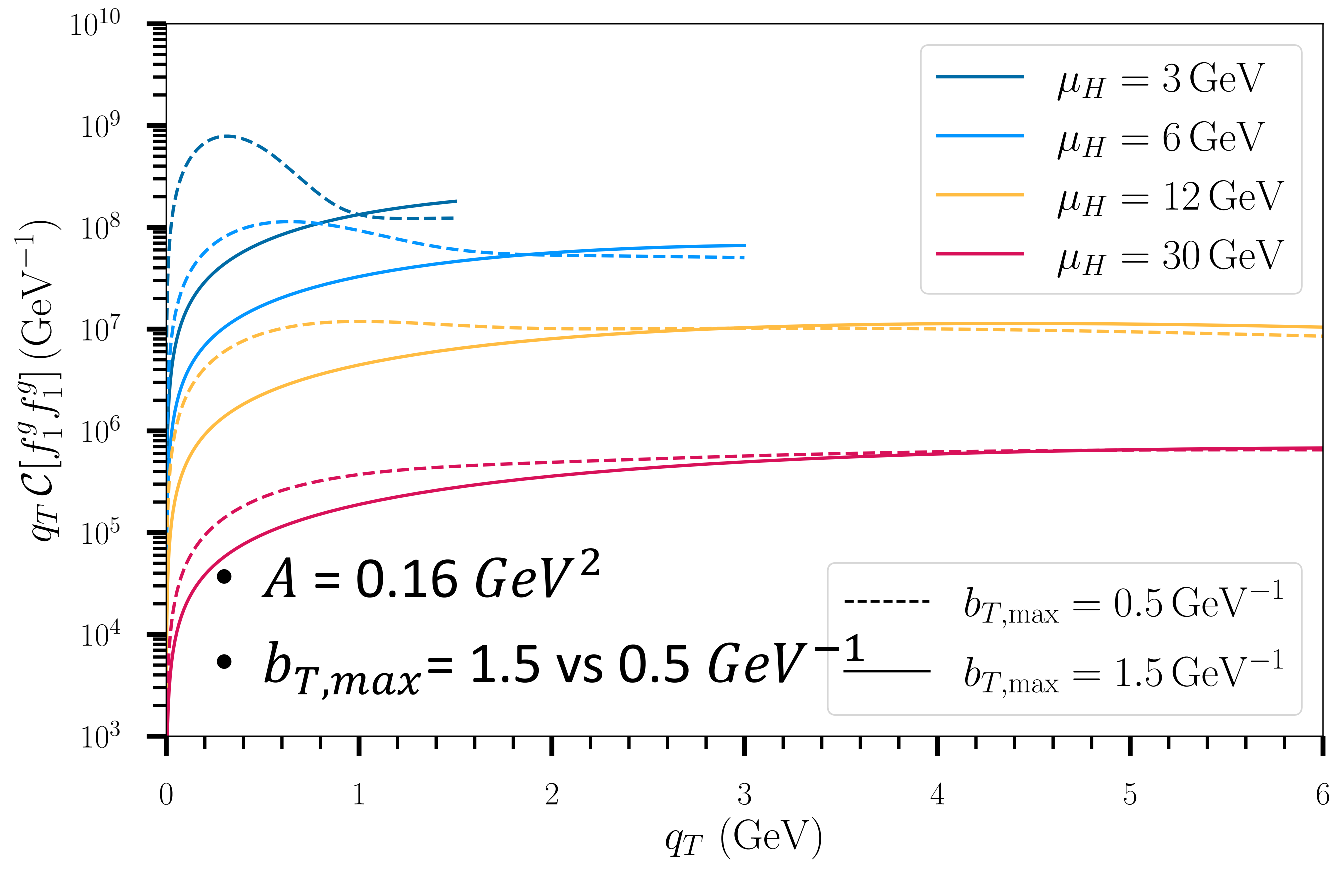
- NLL accuracy
- α_S 1-loop

$$\mathcal{C}[w_n f g](x_a, x_b, q_T; M_{QQ}) = \int_0^\infty \frac{db_T}{2\pi} \, b_T \, J_n(b_T \, q_T) \, e^{-2 S_A(b_T^*; M_{QQ}, \tilde{\mu}'_{b*})} e^{-S_{NP}(b_T; M_{QQ})} \times f(x_a, b_T^*; \tilde{\mu}'_{b*}) \, g(x_b, b_T^*; \tilde{\mu}'_{b*}) \, .$$

A novel nonperturbative Sudakov ?

A simple Gaussian ansatz for S_{NP} has limitations

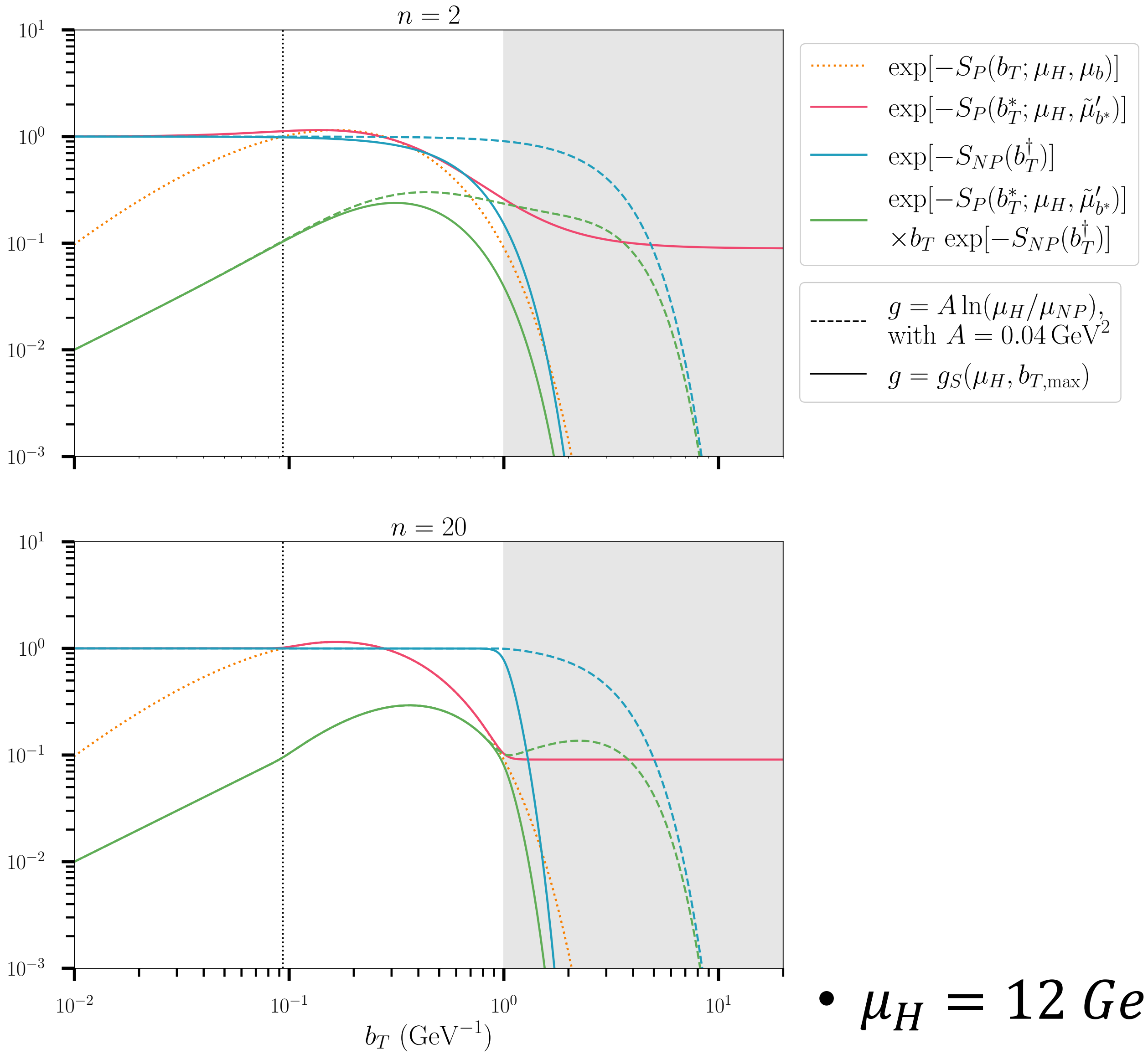
$$S_{NP}(b_T;\mu_H,\mu_{NP}) = A\ln\left(\frac{\mu_H}{\mu_{NP}}\right) b_T^2$$



- A = 0.16 GeV^2
- $b_{T,max}$= 1.5 vs 0.5 GeV^{-1}

- Generates upward bump for small $b_{T,max}$ and μ_H due to large contributions of the integrand at large b_T
- TMD and x independent
- Does not provide $b_{T,max}$-invariance
- Particularly relevant for quarkonia

Another problem identified



- $\mu_H = 12\ GeV$

- We want to trust perturbative physics when we can, to study S_{NP}

$$b_T^* = \frac{b_T}{\left(1 + (b_T/b_{T,\max})^n\right)^{1/n}}, \quad b'_T = \left(b_T^n + b_{T,\min}^n\right)^{1/n}$$

- Remove the ‘kink’ at the same order n

$$S_{NP}(b_T) = g\, b_T^{\dagger\,2} \quad b_T^{\dagger\,2} = \left(b_T^n + b_{T,\max}^n\right)^{2/n} - b_{T,\max}^2$$

$$g_S(\mu_H, b_{T,\max}) = \frac{\left|\frac{\partial}{\partial b_T} S_A(b_T; \mu_H, \mu_b)\Big|_{b_T = b_{T,\max}}\right|}{2^{2/n}\, b_{T,\max}}$$

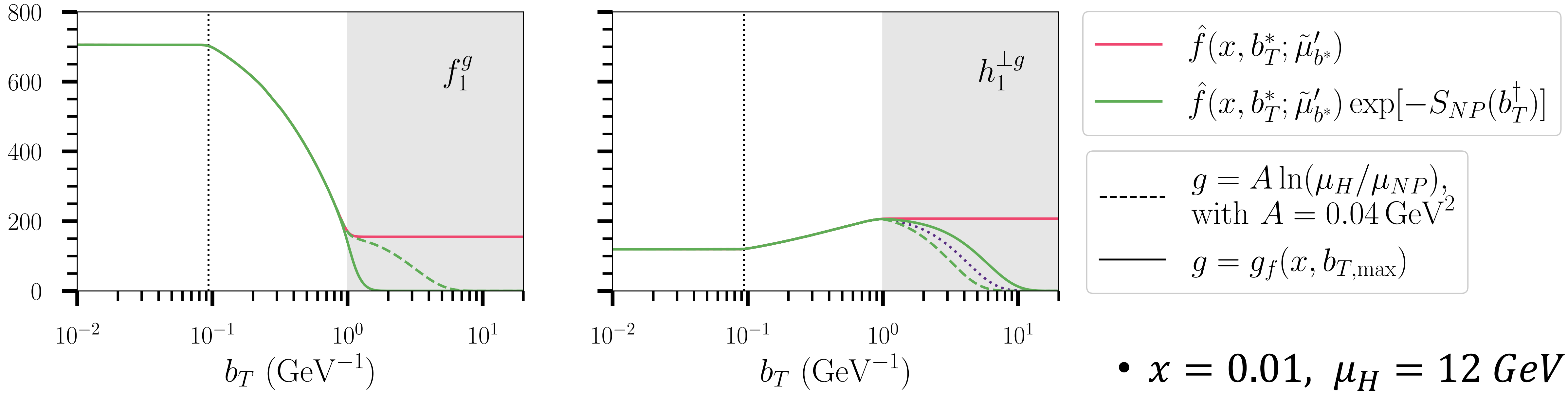
- Nonperturbative physics is dependent on perturbative physics!

Similarly for the perturbative tails

- Extra factor g_f needed to remove other 'kinks'
- Exception of absolute value when $g_f < \Lambda_{QCD}^2$:
  $\Longrightarrow g_f = \Lambda_{QCD}^2$

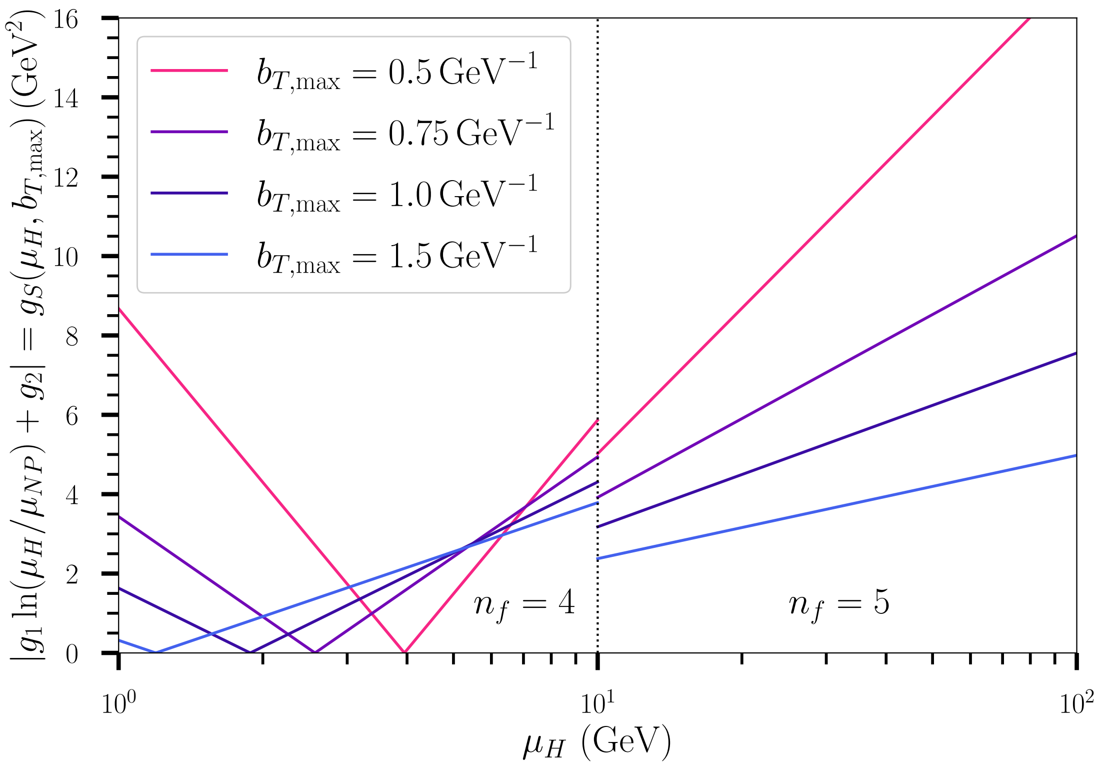
$$g_f(x, b_{T,\max}) = \frac{\left| \frac{\partial}{\partial b_T} \ln f(x, b_T; \mu_b) \big|_{b_T = b_{T,\max}} \right|}{2^{2/n}\, b_{T,\max}}$$

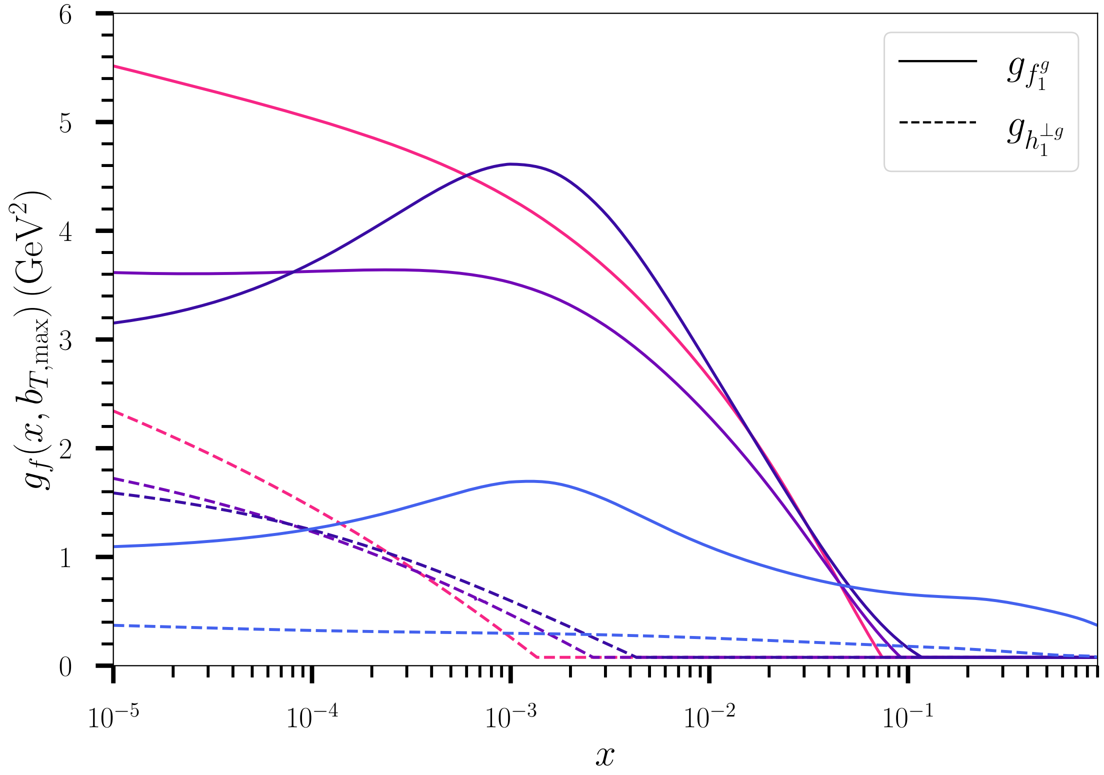
- MHST20lo_as130



- $x = 0.01,\ \mu_H = 12\ GeV$

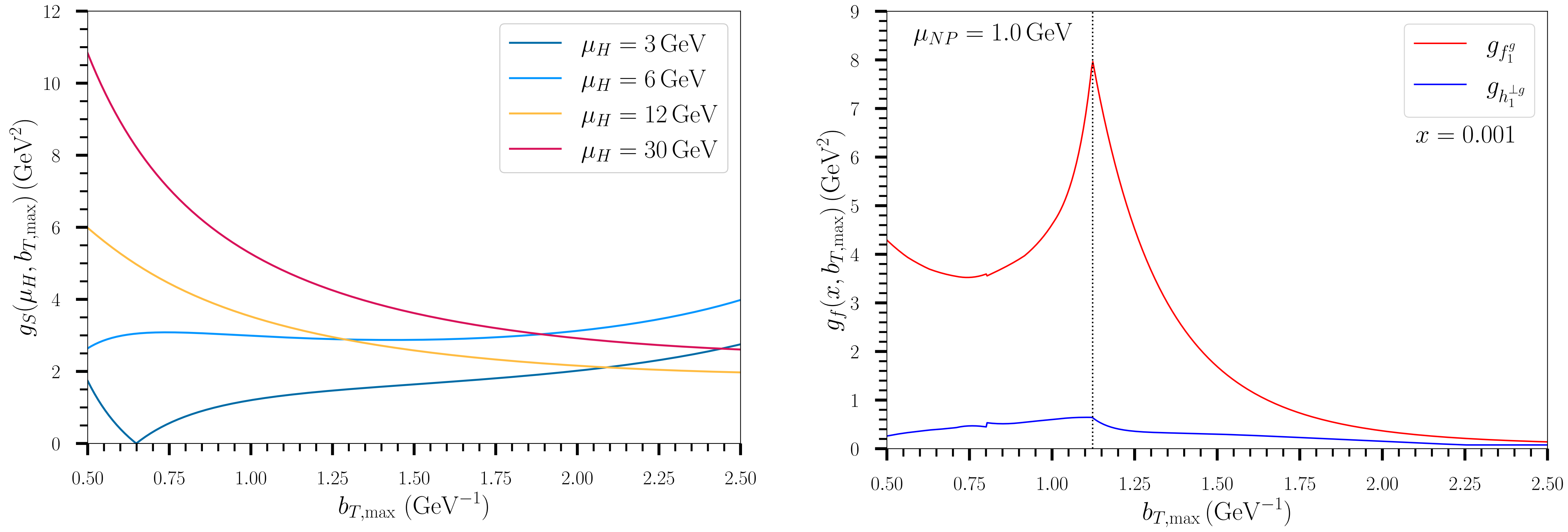
The behaviour of g's





- g_S follows theoretical trend – extra term also employed in other studies *Collins 2012*
- g_f difficult to compare with literature: can be of same order depending on x and the kind of TMD tail

$b_{T,max}$ dependence



- Flip $\Longrightarrow$ It is not recommended to use too small $b_{T,max}$ for low μ_H
- One expects smooth behaviour as a function of $b_{T,max}$: discontinuities due to PDF set $\Longrightarrow$ It is not recommended to use $b_{T,max} > b_0$ for this method

The novel nonperturbative Sudakov

$$S_{NP}(b_T; x_a, x_b, \mu_H, b_{T,\max}) = \left(g_S(\mu_H, b_{T,\max}) + g_f(x_a, b_{T,\max}) + g_f(x_b, b_{T,\max})\right) b_T^{\dagger\,2} = g\, b_T^{\dagger\,2}$$

- Larger values g reasonable because of larger n (and smaller $b_{T,max}$)
- g can be taken lager than the found value by matching, to suppress nonperturbative physics more, but not smaller (gives back 'kink')

- Solves strange behaviour for small $b_{T,max}$ and μ_H
- Takes into account x and TMD dependence

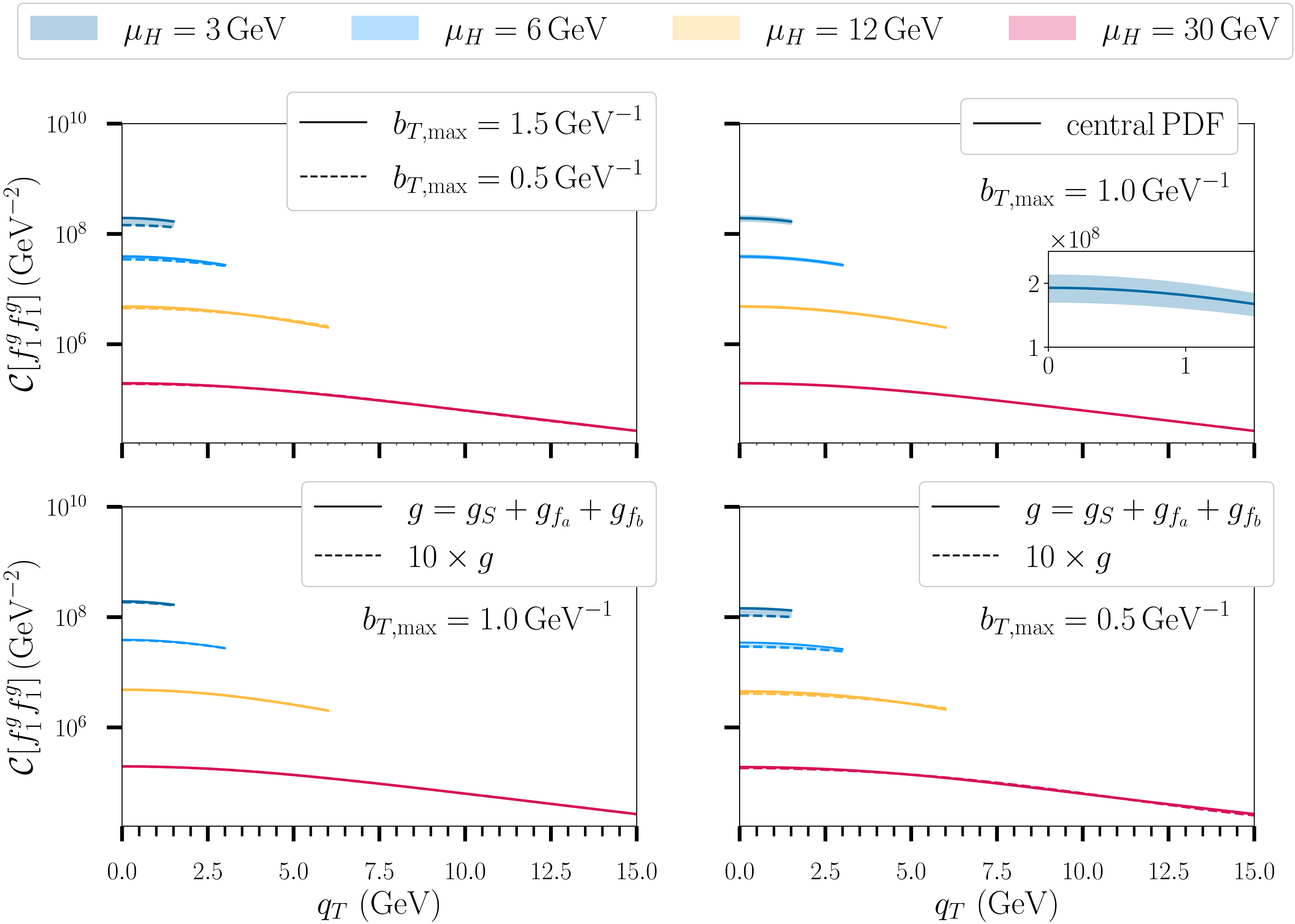
- $b_{T,max}$-invariance of $\widehat{W}$ not directly observable and hard to obtain with our robust method. However, now it does take $b_{T,max}$ systematically into account.

Nonperturbative uncertainties

- The PDF set uncertainty (Hessian):

$$(\Delta O)_+ = \sqrt{\sum_{j=1}^{k} \left\{ \max\left[O(S_j^+) - O(S_0), O(S_j^-) - O(S_0), 0\right] \right\}^2} \quad (\Delta O)_- = \sqrt{\sum_{j=1}^{k} \left\{ \max\left[O(S_0) - O(S_j^+), O(S_0) - O(S_j^-), 0\right] \right\}^2}$$

- S_{NP} uncertainties:
  - $b_{T,max}$ variation; $[0.5{:}\ 1.5]\ GeV^{-1} \rightarrow\ \mu_{NP} = [2.25{:}\ 0.75]\ GeV$
  - g increasement; f.e. $g \rightarrow 10g$



$\mu_H = 3\,\mathrm{GeV}$
$\mu_H = 6\,\mathrm{GeV}$
$\mu_H = 12\,\mathrm{GeV}$
$\mu_H = 30\,\mathrm{GeV}$
$b_{T,\max} = 1.5\,\mathrm{GeV}^{-1}$
$b_{T,\max} = 0.5\,\mathrm{GeV}^{-1}$
central PDF
$b_{T,\max} = 1.0\,\mathrm{GeV}^{-1}$
$\times 10^8$
$g = g_S + g_{f_a} + g_{f_b}$
$10 \times g$
$b_{T,\max} = 1.0\,\mathrm{GeV}^{-1}$
$g = g_S + g_{f_a} + g_{f_b}$
$10 \times g$
$b_{T,\max} = 0.5\,\mathrm{GeV}^{-1}$
$\mathcal{C}[f_1^g f_1^g]\ (\mathrm{GeV}^{-2})$
10^{10}
10^8
10^6
q_T (GeV)
0.0
2.5
5.0
7.5
10.0
12.5
15.0

$q_{T,max} = M_{QQ}/2$

Perturbative uncertainties

- Scale variation, $\mu \to C\mu$ with C $= [1/2 : 2]$
  - C_1 times $\mu_b = b_0/b_T$ and C_2 times $\mu_H = M_{QQ} = Q$
  - C_3 times μ_b in the perturbative TMD tails *Melis et al. 2015*

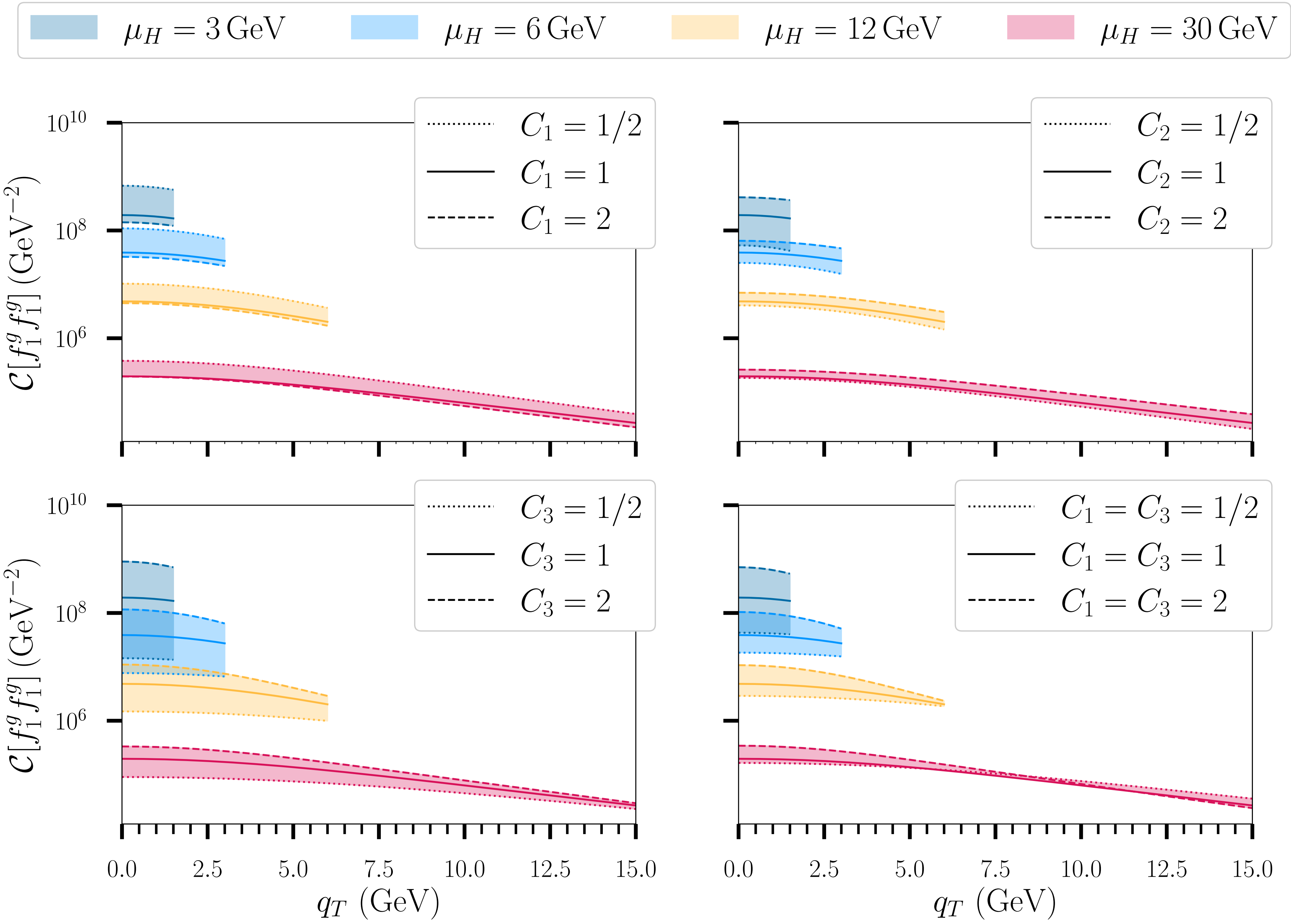
    Note:
    - C_1 and C_3 contain in practice the b_T-expressions (so also C_2)
    - Scale variation alters also g_S and g_f

$$S_A(b_T^*; C_2\,Q, C_1\,\tilde{\mu}'_{b*}) = \frac{1}{2}\frac{C_A}{\pi}\int_{C_1^2\,\tilde{\mu}'^2_{b*}}^{C_2^2\,Q^2}\frac{d\mu'^2}{\mu'^2}\left(\left\{\alpha_s(\mu') + \frac{\alpha_s(\mu')^2}{4\pi}\left[\left(\frac{67}{9}-\frac{\pi^2}{3}\right)N_c - \frac{20}{9}T_R n_f + 2\beta_0\ln C_1\right]\right\}\ln\frac{C_2^2\,Q^2}{\mu'^2} - \alpha_s(\mu')\left[\frac{\beta_0}{6} + 2\ln\frac{C_2}{C_1}\right]\right),$$

$$\hat{f}_1^g(x, b_T^*; \tilde{\mu}'_{b*}) = f_1^g(x; \tilde{\mu}'_{b*}) + \mathcal{O}(\alpha_s) + \mathcal{O}(b_T\Lambda_{\mathrm{QCD}})$$

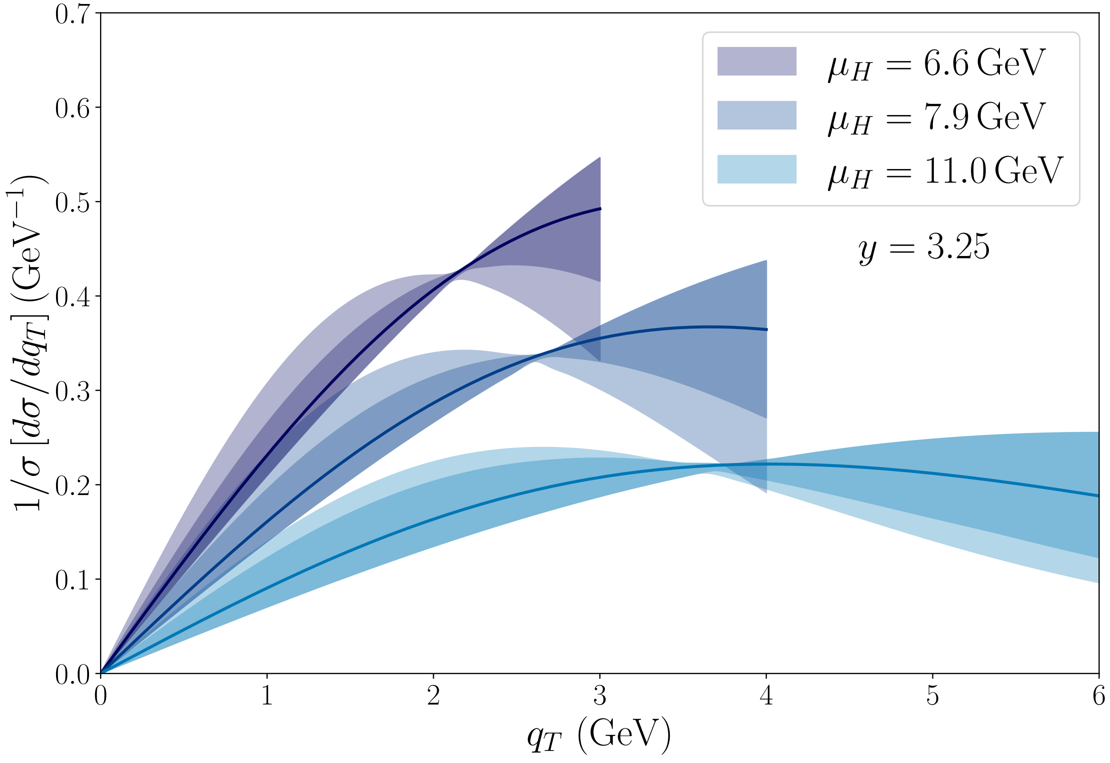
- $C_3 = C_1/C_2$

Collins et al. 1984



$b_{T,max} = 1.0\ GeV^{-1}$

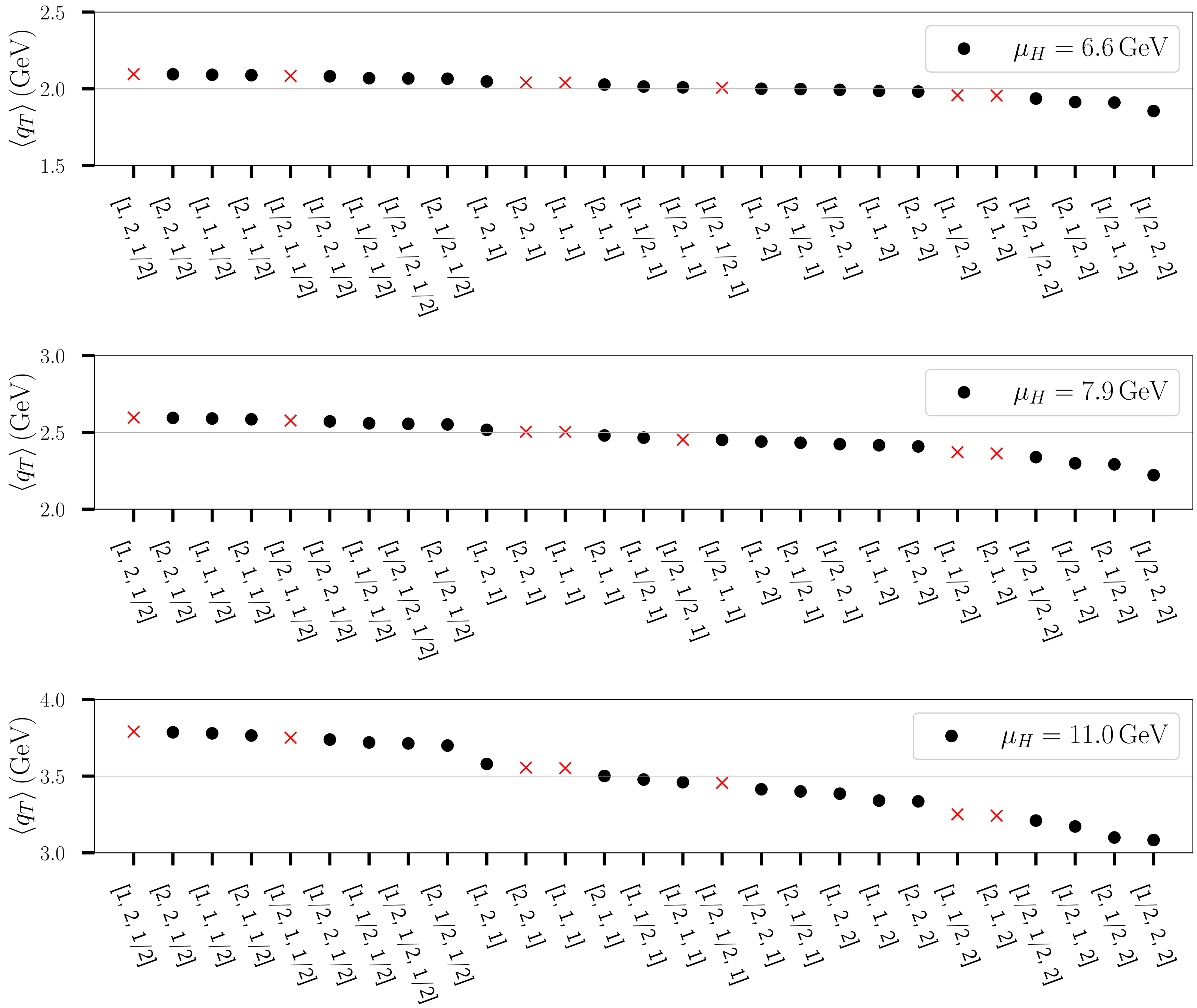
Comparison with the LHCb data



- Envelope from all 27 scale combinations
- Collider mode: $\sqrt{s} = 13\,TeV$

$$d\sigma/dq_T \propto q_T\,\mathcal{C}[f_1^g f_1^g]$$

$$\frac{1}{\sigma}\left(\frac{d\sigma}{dq_T}\right) = \frac{d\sigma/dq_T}{\int_0^{q_{T,\max}} dq_T\,(d\sigma/dq_T)} \approx \frac{q_T\,\mathcal{C}[f_1^g f_1^g]}{\int_0^{q_{T,\max}} dq_T\,(q_T\,\mathcal{C}[f_1^g f_1^g])}$$


2.5
2.0
1.5
⟨q_T⟩ (GeV)
μ_H = 6.6 GeV
[1, 2, 1/2]
[2, 2, 1/2]
[1, 1, 1/2]
[2, 1, 1/2]
[1/2, 1, 1/2]
[1/2, 2, 1/2]
[1, 1/2, 1/2]
[1/2, 1/2, 1/2]
[2, 1/2, 1/2]
[1, 2, 1]
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[1, 1/2, 1]
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[1, 2, 2]
[2, 1/2, 1]
[1/2, 2, 1]
[1, 1, 2]
[2, 2, 2]
[1, 1/2, 2]
[2, 1, 2]
[1/2, 1/2, 2]
[2, 1/2, 2]
[1/2, 1, 2]
[1/2, 2, 2]
3.0
2.5
2.0
⟨q_T⟩ (GeV)
μ_H = 7.9 GeV
[1, 2, 1/2]
[2, 2, 1/2]
[1, 1, 1/2]
[2, 1, 1/2]
[1/2, 1, 1/2]
[1/2, 2, 1/2]
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[2, 1, 2]
[1/2, 1/2, 2]
[1/2, 1, 2]
[2, 1/2, 2]
[1/2, 2, 2]
4.0
3.5
3.0
⟨q_T⟩ (GeV)
μ_H = 11.0 GeV
[1, 2, 1/2]
[2, 2, 1/2]
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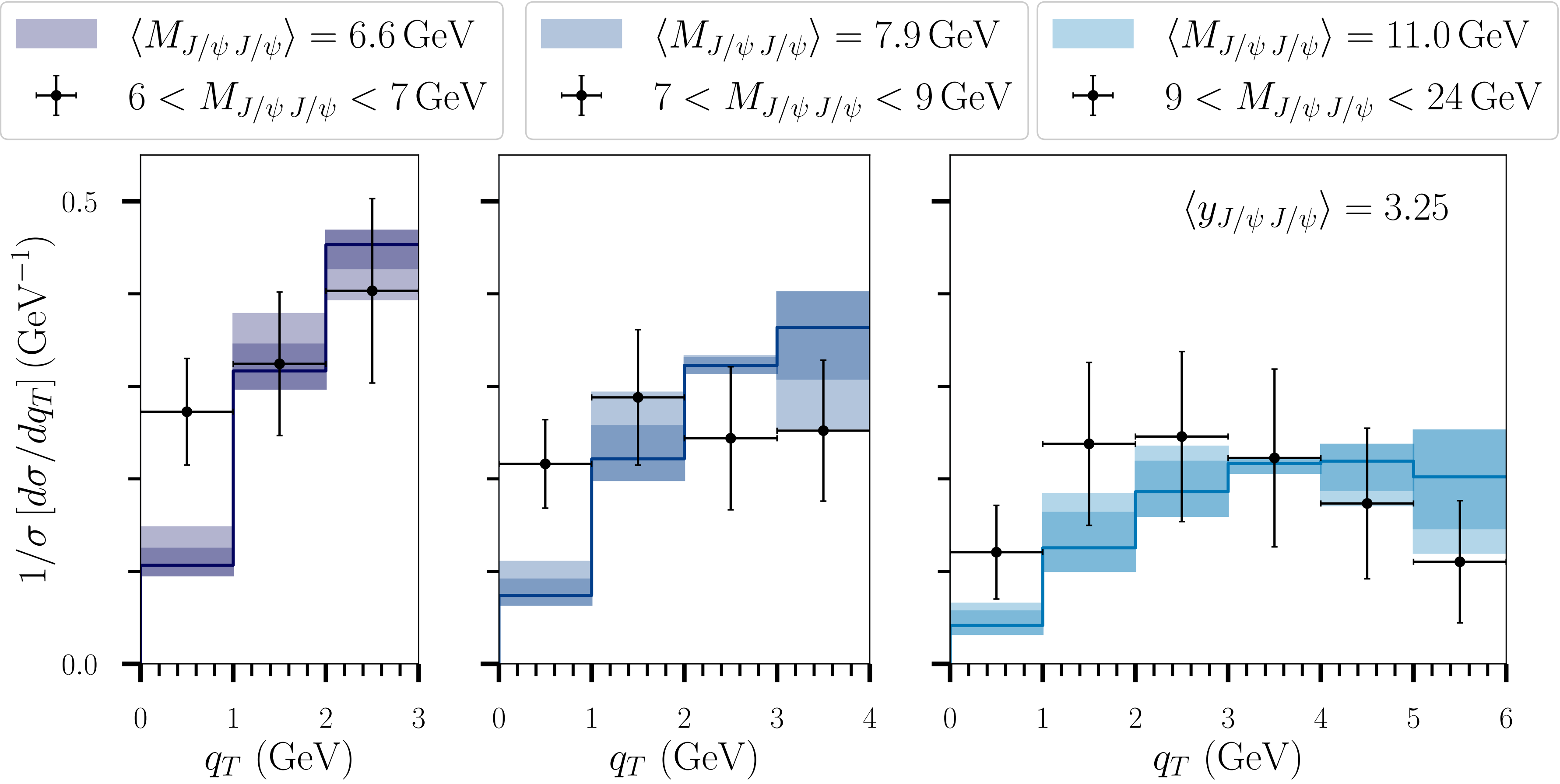
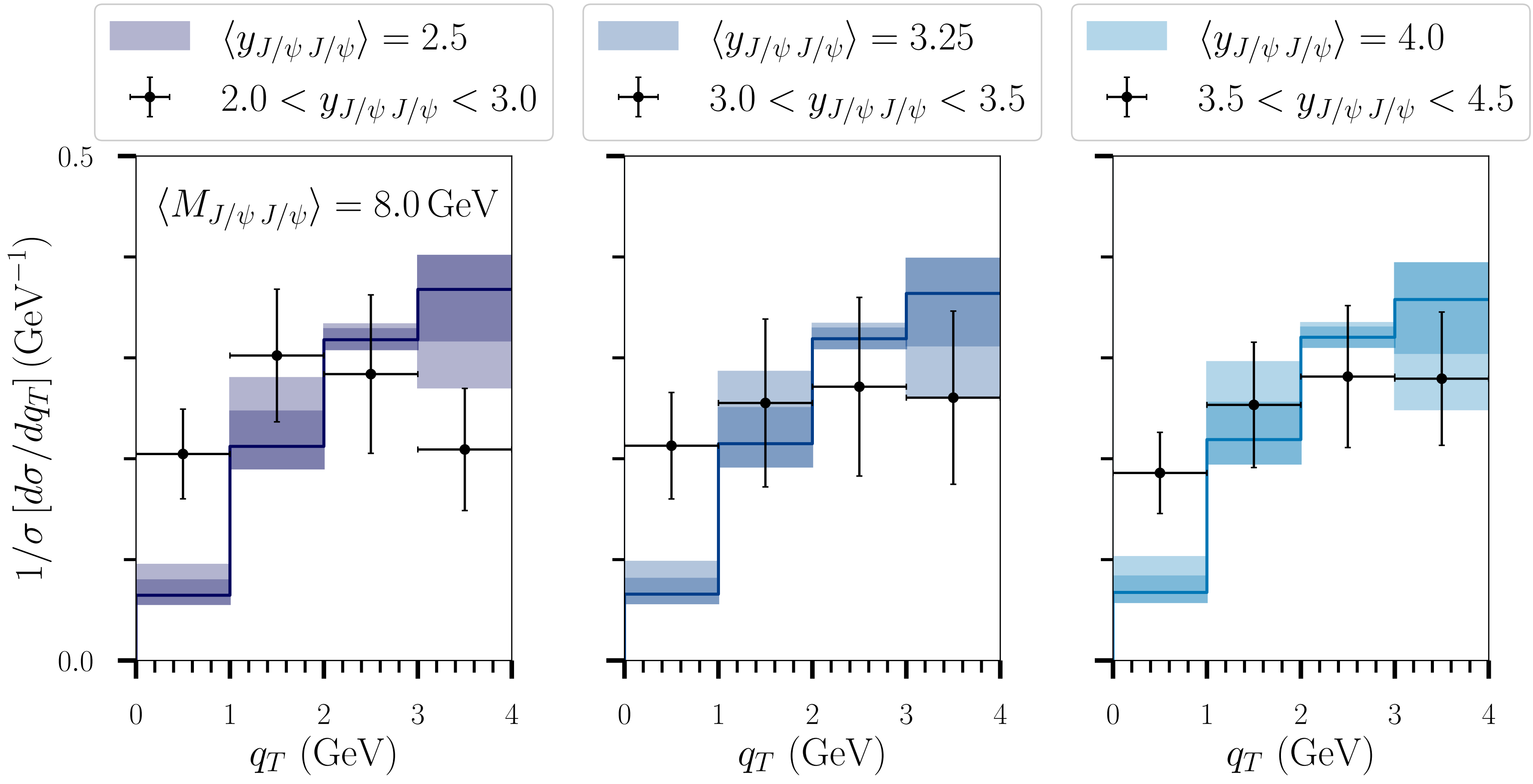
[1, 2, 2]
[1, 1, 2]
[2, 2, 2]
[1, 1/2, 2]
[2, 1, 2]
[1/2, 1/2, 2]
[1/2, 1, 2]
[2, 1/2, 2]
[1/2, 2, 2]

Comparison with the LHCb data (1)



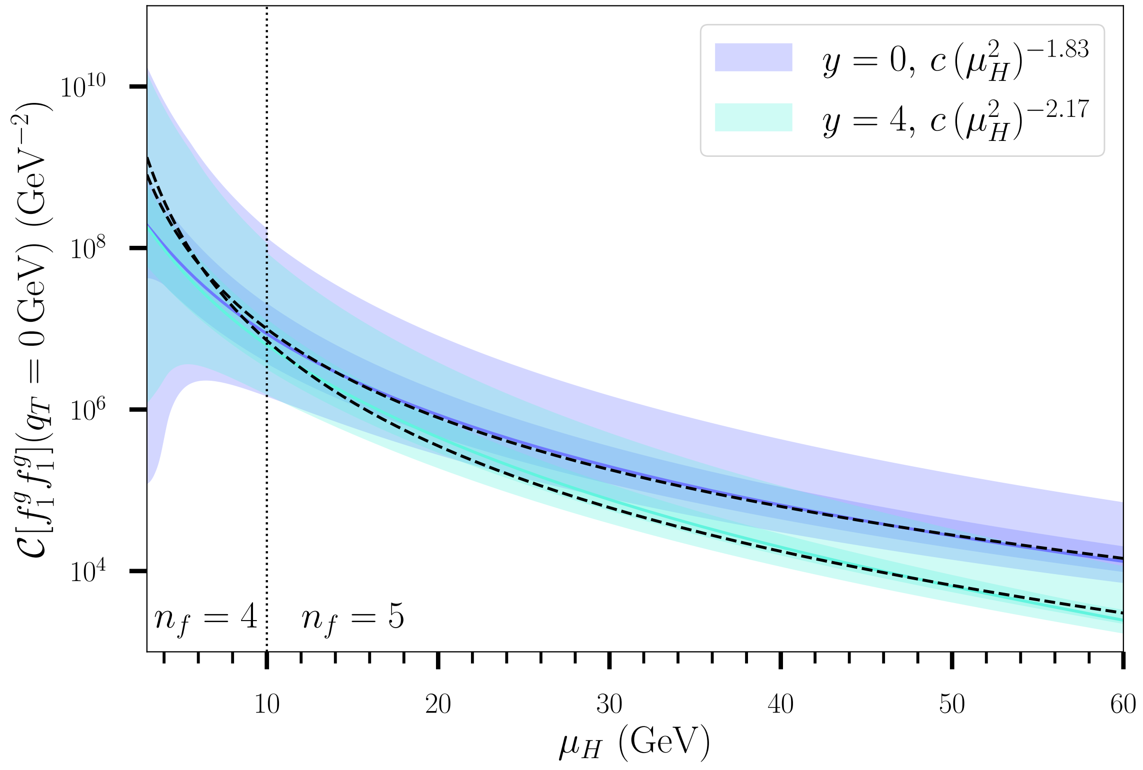
- DPS subtracted data
- $C_3 \neq C_1/C_2$ provides better agreement with data

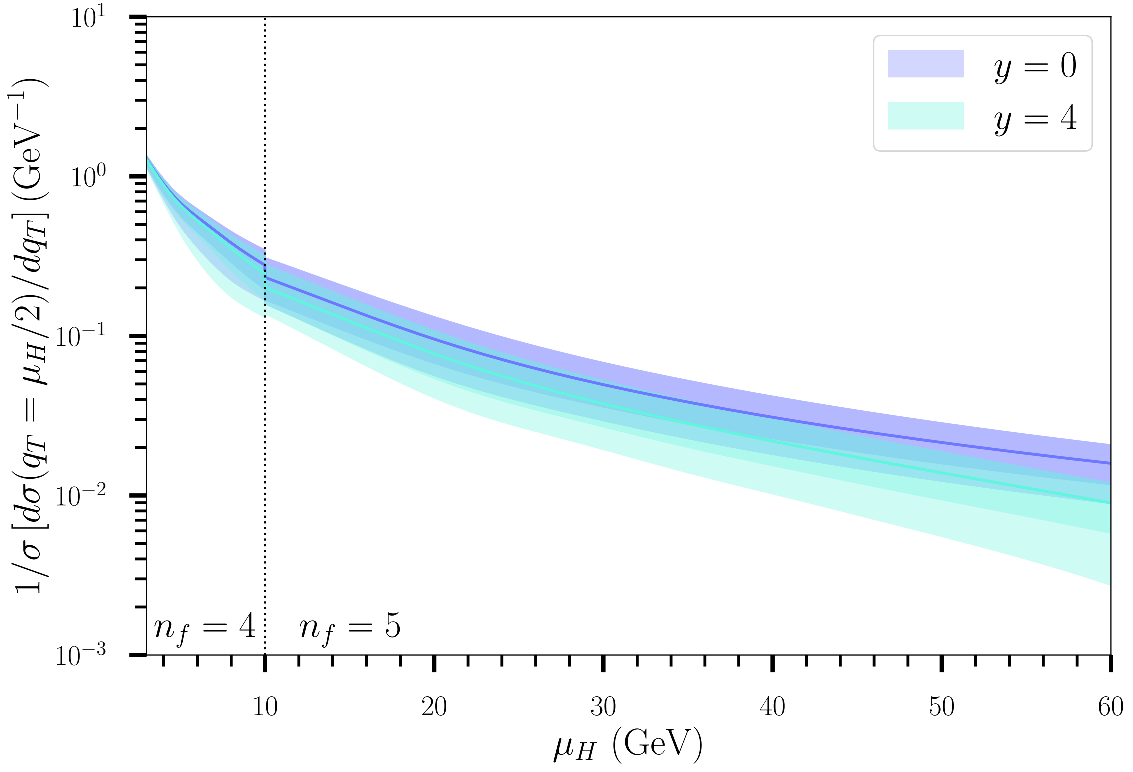
Comparison with the LHCb data (2)



- Rapidity dependence is small

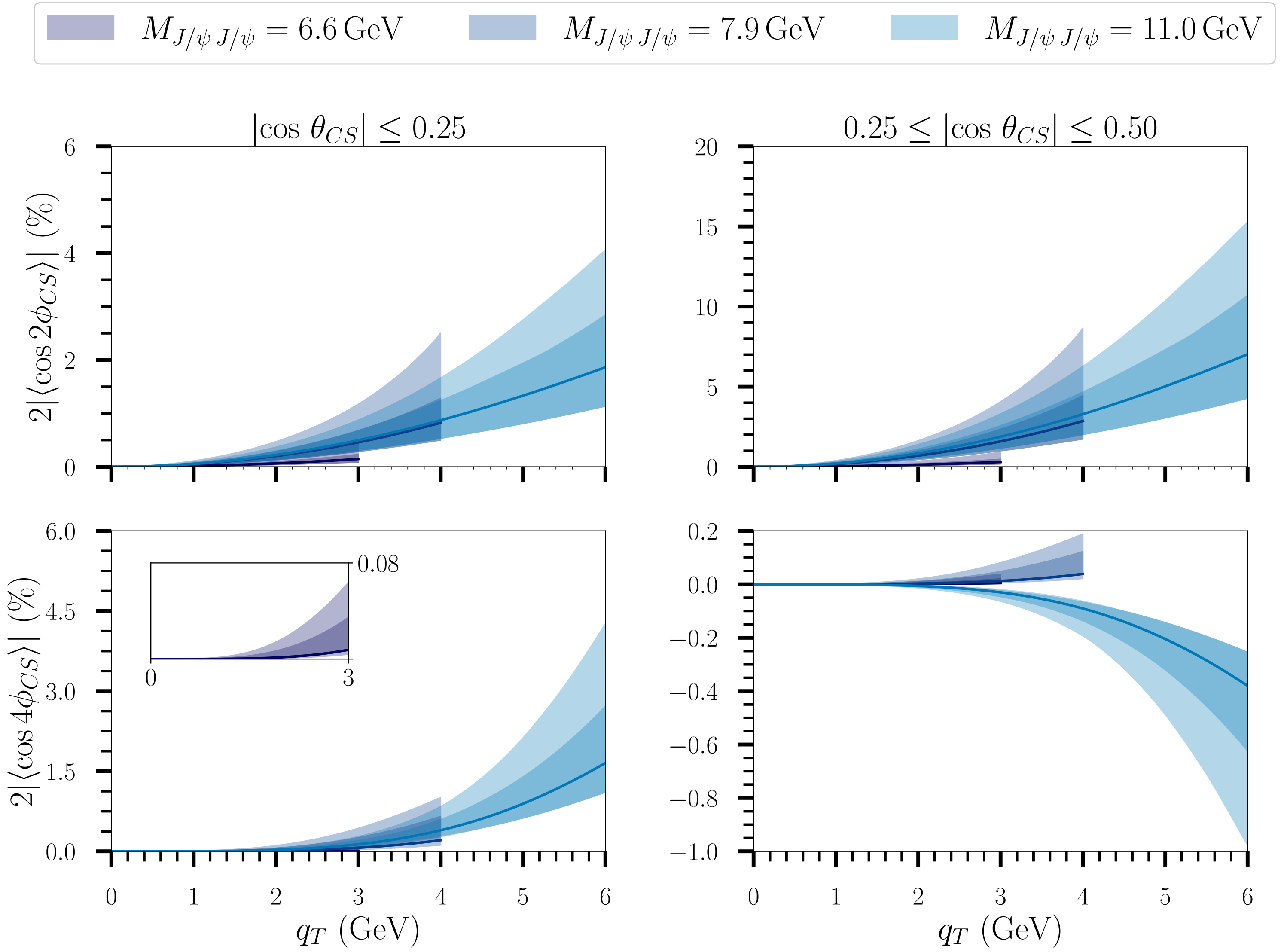
Power law behaviour of the hard scale





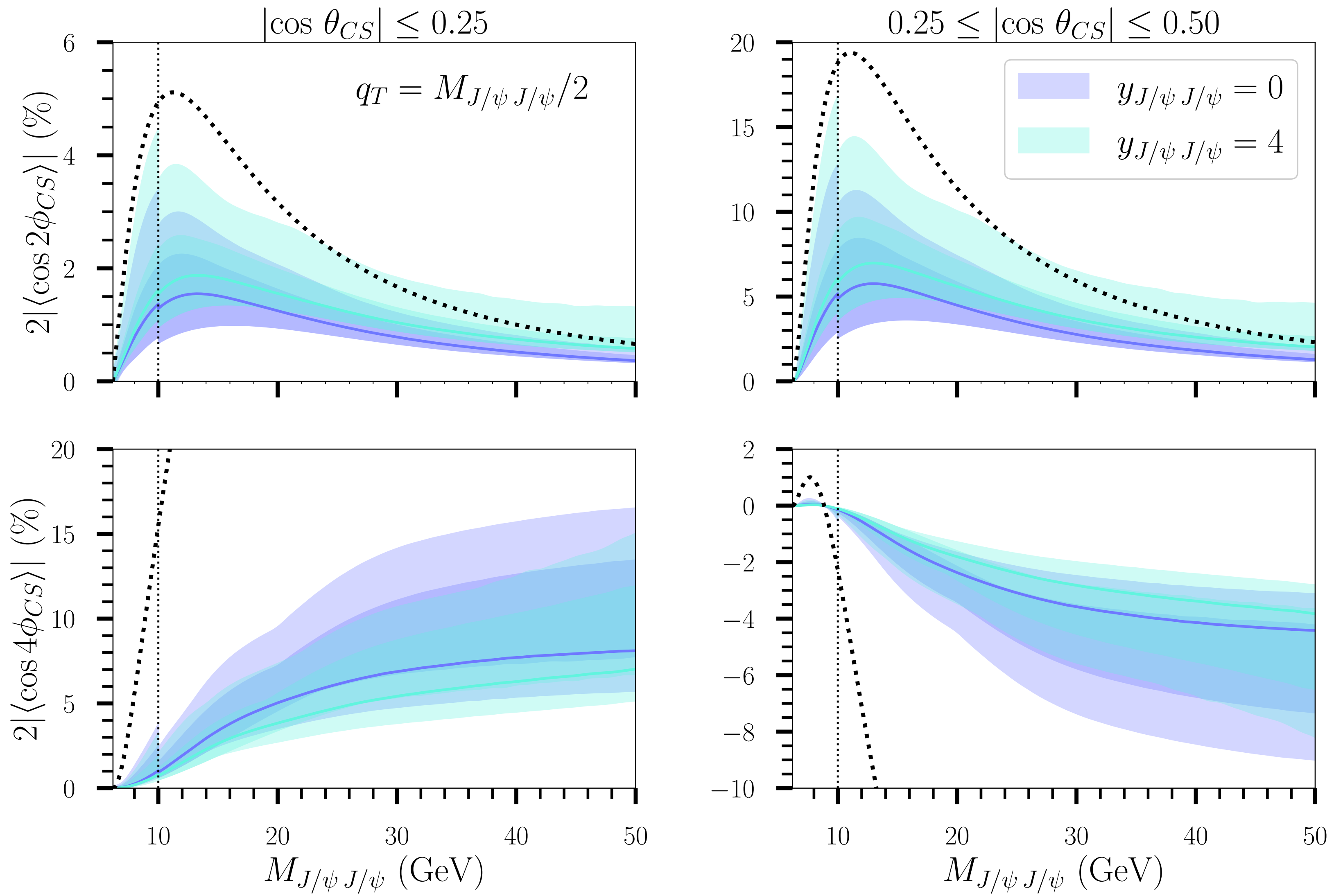
- Saddle point approximation for perturbative Sudakov suppression $\sim\mu_H^{-1.53}$: PDFs and nonperturbative Sudakov
- For larger rapidity they decreases faster

Azimuthal modulations



- The $\cos 2\phi_{CS}$-modulation provides a way to determine the sign of $h_1^{\perp g}$
- The $\cos 4\phi_{CS}$-modulation can be used to extract $h_1^{\perp g}$ independently from f_1^g (when $C[f_1^g f_1^g]$ is known)
- Sign flip due to F_4

Azimuthal modulations



- The uncertainties for the $\cos 2\phi_{CS}$-modulations **always** become larger than the upper bounds (that follows from the positivity bound of the TMDs) at some point

- The $\cos 4\phi_{CS}$-modulations including uncertainties are much smaller than their bound: due to smaller $\mathcal{C}\left[w_4 h_1^{\perp g} h_1^{\perp g}\right]$

Conclusions

- We have investigated the interplay between the perturbative and nonperturbative regions that endorsed a novel nonperturbative Sudakov factor that solves problems that can arise with a simple Gaussian ansatz.

- Perturbative uncertainties are much larger than nonperturbative uncertainties: higher order corrections should be taken into account for more precise predictions.
- Our predictions including scale uncertainties are agreeable with data: especially when $C_3 \neq C_1/C_2$.

- It might be suitable to probe the TMD evolution formalism as well by extracting the power law behaviour of the hard scale e.g. of the normalized cross section at a specific TM

$\Longrightarrow$ *Bor, Boer, Colpani Serri and Lansberg [in progress … 2024]*