# DPS in quarkonium hadroproduction and photoproduction

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Introduction to double parton scattering (DPS) and hadronic Physics

Data and interpretation

DPS at the EIC?

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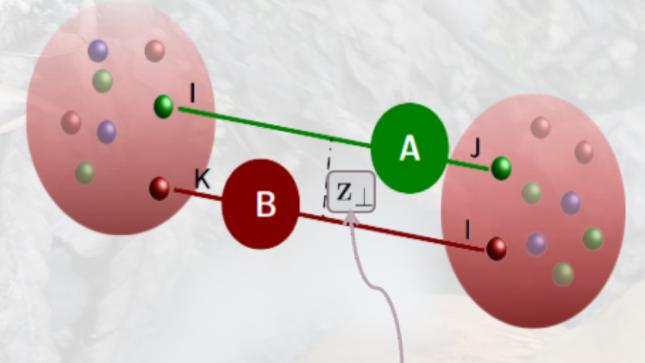
Introduction to double parton scattering (DPS) and hadronic Physics

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# Double Parton Scattering

Multiparton interaction (MPI) can contribute to the, pp, pA and AA cross section @ the LHC:



Transverse distance between two partons

$$d\sigma \propto \int d^2z_{\perp} F_{ij}(x_1, x_2, z_{\perp}, \mu_A, \mu_B) F_{kl}(x_3, x_4, z_{\perp}, \mu_A, \mu_B)$$

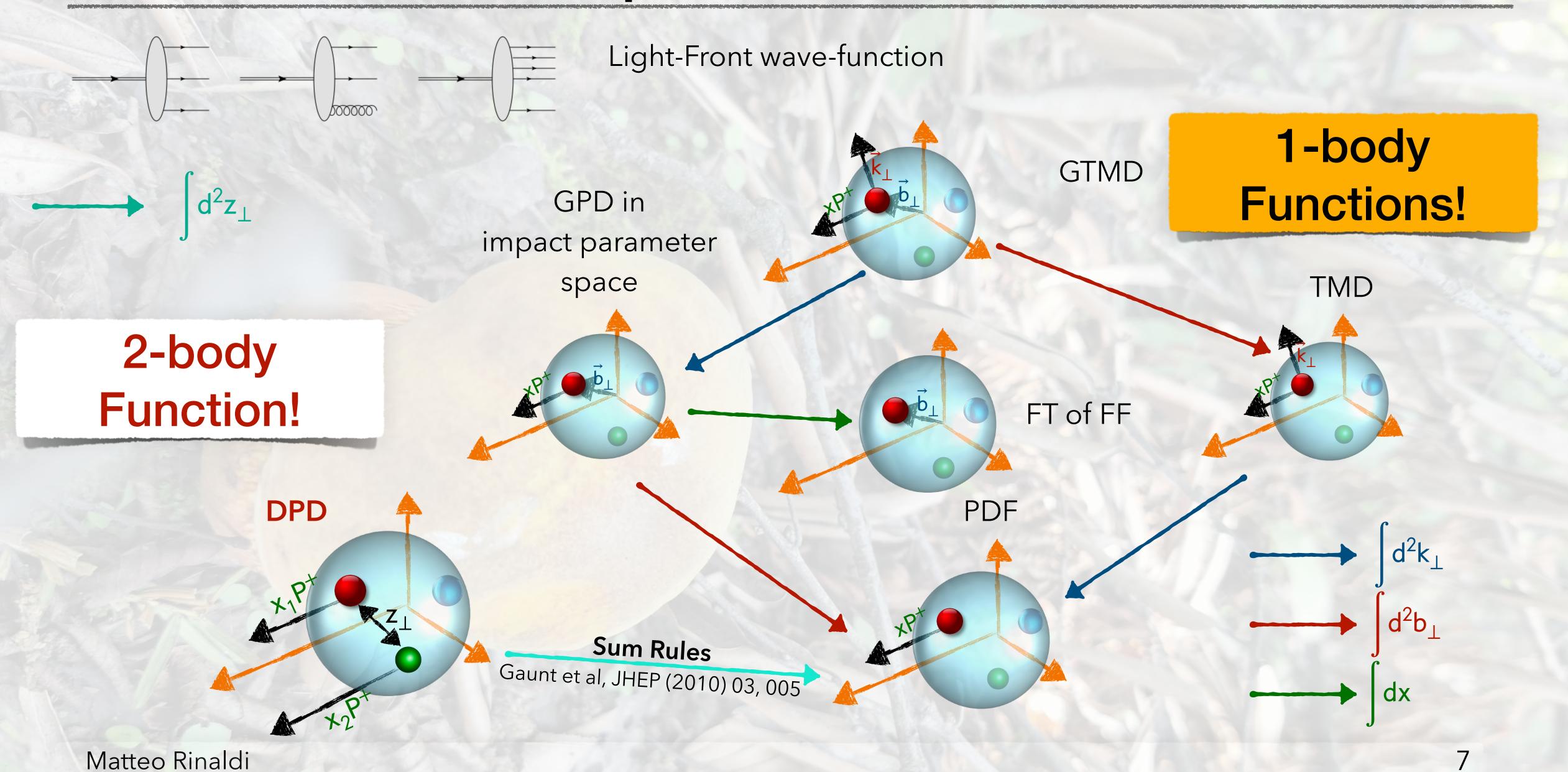
### Double Parton Distribution (DPD)

N. Paver and D. Treleani, Nuovo Cimento 70A, 215 (1982) Mekhfi, PRD 32 (1985) 2371 M. Diehl et al, JHEP 03 (2012) 089

$$\begin{split} F_{ij}^{\lambda_1,\lambda_2}(x_1,x_2,\vec{k}_\perp) &= (-8\pi P^+)\frac{1}{2}\sum_{\lambda}\int\! d\vec{z}_\perp\,e^{\mathrm{i}\vec{z}_\perp\cdot\vec{k}_\perp} \\ &\times \int \left[\prod_l^3 \frac{dz_l^-}{4\pi}\right] e^{ix_1P^+z_1^-/2} e^{ix_2P^+z_2^-/2} e^{-ix_1P^+z_3^-/2} \\ &\times \langle \lambda,\vec{P}=\vec{0}\big| \hat{\mathbb{O}}_i^1 \left(z_1^-\frac{\bar{n}}{2},z_3^-\frac{\bar{n}}{2}+\vec{z}_\perp\right) \hat{\mathbb{O}}_j^2 \left(z_2^-\frac{\bar{n}}{2}+\vec{z}_\perp,0\right) \big|\vec{P}=\vec{0},\lambda\rangle \end{split}$$

$$\hat{\mathcal{O}}_i^k(z,z') = \bar{q}_i(z)\hat{O}(\lambda_k)q_i(z')$$
 
$$\hat{O}(\lambda_k) = \frac{\bar{\eta}}{2}\frac{1+\lambda_k\gamma_5}{2} .$$

# Multidimensional picture of hadrons



 $F_{ik}(x_1, x_2, \overrightarrow{z}_{\perp})$  is unknown. For phenomenology @LHC kinematics (small x and many partons produced)

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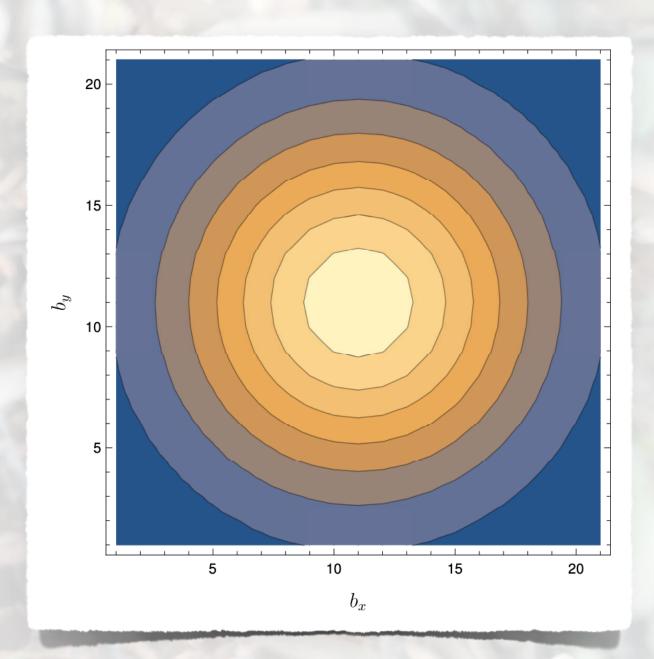
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Models can help to grasp general features

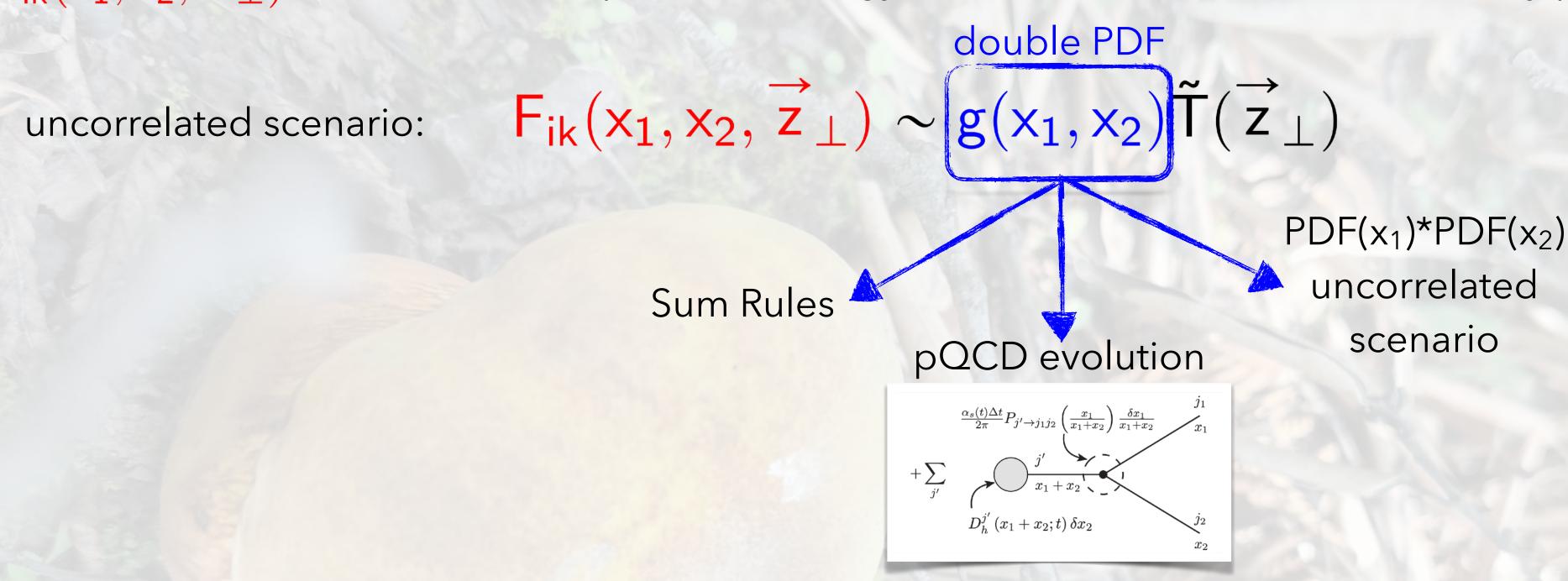
M.R., S. Scopetta et al, PRD 87 (2013) 114021 M.R., S, Scopetta et al, JHEP 12 (2014) 028 A. V. Manohar et al, PRD 87 (2013) 3, 034009

$$\left\langle b_{\perp}^{2}\right\rangle _{x_{1},x_{2}}^{ij}=\frac{\int d^{2}b_{\perp}b_{\perp}^{2}\tilde{F}_{ij}\left(x_{1},x_{2},b_{\perp},Q^{2}\right)}{\int d^{2}b_{\perp}\tilde{F}_{ij}\left(x_{1},x_{2},b_{\perp},Q^{2}\right)}$$



M.R. and F. A. Ceccopieri, JHEP 09 (2019) 097

 $F_{ik}(x_1, x_2, \overrightarrow{z}_{\perp})$  is unknown. For phenomenology @LHC kinematics (small x and many partons produced)



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 $F_{ik}(x_1, x_2, \vec{z}_{\perp})$  is unknown. For phenomenology @LHC kinematics (small x and many partons produced)

double PDF  $F_{ik}(x_1,x_2,\overrightarrow{z}_\perp) \sim g(x_1,x_2) \widetilde{T}(\overrightarrow{z}_\perp)$ uncorrelated scenario:  $PDF(x_1)*PDF(x_2)$ uncorrelated Sum Rules scenario pQCD evolution J. R. Gaunt et al, EPJC 69 (2010) 54-65 GS09=PDF\*PDF⊗pQCD O. Fedkevych, J. R. Gaunt, JHEP 02 (2023) 090 10 12 14 16 GS09: with  $1 \rightarrow 2$  / GS09: no  $1 \rightarrow 2$ CM energy (TeV)

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 $F_{ik}(x_1, x_2, \vec{z}_{\perp})$  is unknown. For phenomenology @LHC kinematics (small x and many partons produced)

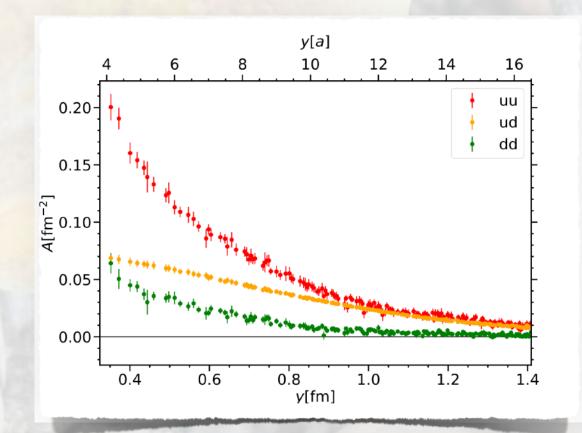
uncorrelated scenario:

$$F_{ik}(x_1, x_2, \overrightarrow{z}_{\perp}) \sim g(x_1, x_2) \widetilde{\Gamma}(\overrightarrow{z}_{\perp}) \longrightarrow$$

Probability distribution
of two partons at given
distance

Unknown Non perturbative object

Some information from lattice



G. S. Bali et al, JHEP 09 (2021) 121

 $F_{ik}(x_1, x_2, \vec{z}_{\perp})$  is unknown. For phenomenology @LHC kinematics (small x and many partons produced)

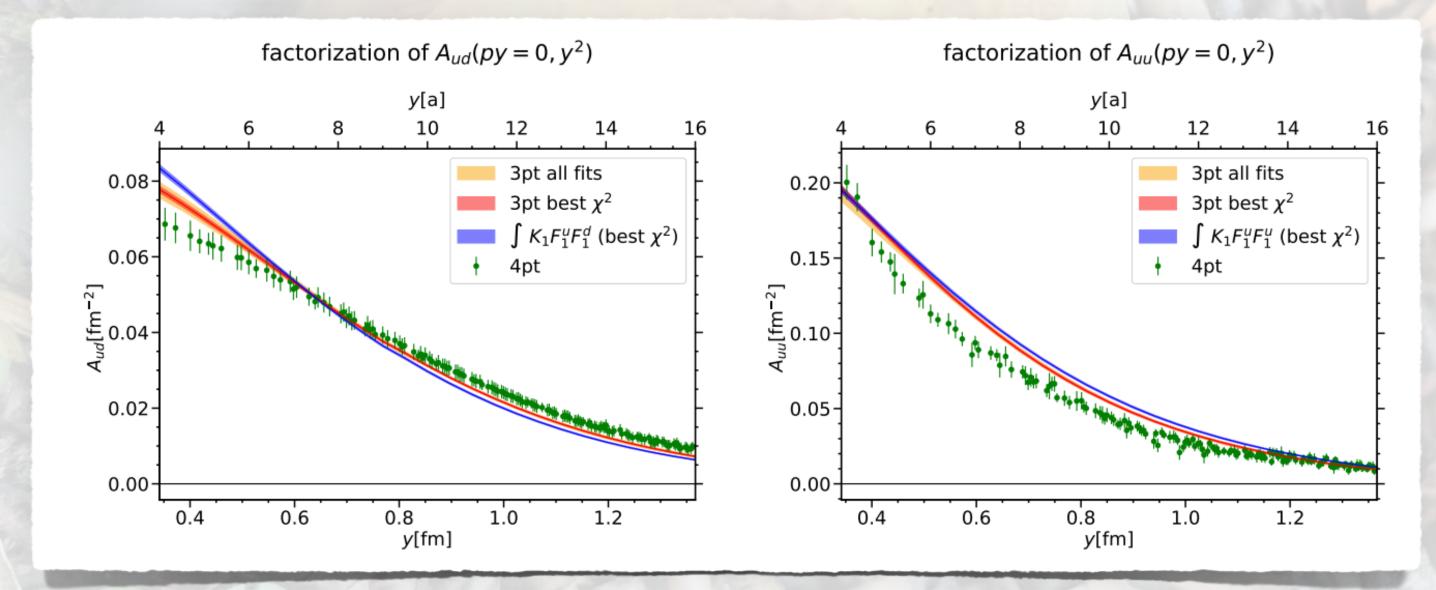
uncorrelated scenario:

$$F_{ik}(x_1,x_2,\overrightarrow{z}_\perp) \sim \underbrace{g(x_1,x_2)}_{\text{double PDF}} \widetilde{T}(\overrightarrow{z}_\perp) -$$

Probability distribution
of two partons at given
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Some information from lattice

Unknown Non perturbative object



Violation of the usual ansatz:

$$\tilde{T}(\vec{z}_{\perp}) \sim \int d^2b_1 f(b_1)f(b_1 - \vec{z}_{\perp})$$

Transverse distribution

DPD = GPD x GPD NOT WELL REPRODUCED!

G. S. Bali et al, JHEP 09, 106 (2021)

 $F_{ik}(x_1, x_2, \overrightarrow{z}_{\perp})$  is unknown. For phenomenology @LHC kinematics (small x and many partons produced)

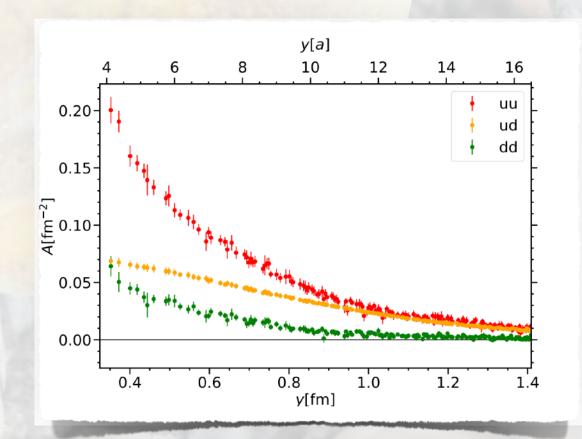
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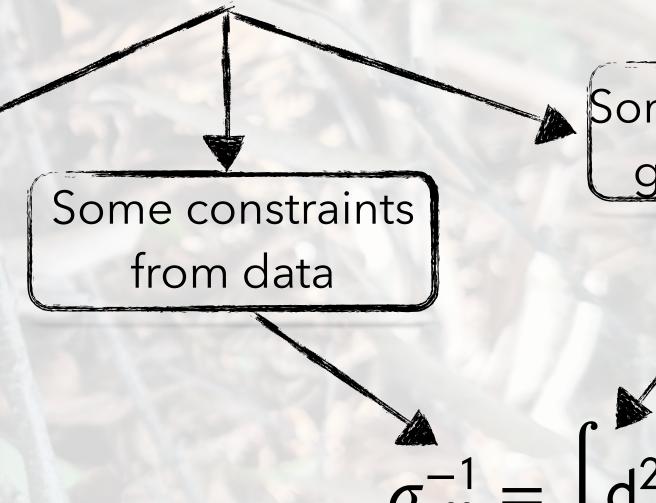
$$\mathsf{F}_{\mathsf{ik}}(\mathsf{x}_1,\mathsf{x}_2,\overrightarrow{\mathsf{z}}_\perp) \sim \underbrace{\mathsf{g}(\mathsf{x}_1,\mathsf{x}_2)}_{\mathsf{double PDF}} \widetilde{\mathsf{T}}(\overrightarrow{\mathsf{z}}_\perp) - \underbrace{\mathsf{g}(\mathsf{x}_1,\mathsf{x}_2)}_{\mathsf{T}} \widetilde{\mathsf{T}}(\overrightarrow{\mathsf{z}}_\perp)$$

Probability distribution
of two partons at given
distance

Unknown Non perturbative object

Some information from lattice





Some Constraints from general properties

 $= d^2 z_{\perp} \tilde{T}(z_{\perp})^2$ 

G. S. Bali et al, JHEP 09 (2021) 121

# Some Data and Effective Cross Section

$$\sigma_{\text{eff}}^{\text{pp}} = \frac{\text{m}}{2} \frac{\sigma_{\text{A}}^{\text{pp}} \sigma_{\text{B}}^{\text{pp}}}{\sigma_{\text{DPS}}^{\text{pp}}}$$

▶ Differential X-section single parton scattering for the process:  $pp \longrightarrow A(B) + X$ 

 $\rightarrow$  Differential X-section double parton scattering for the process:  $pp \longrightarrow A + B + X$ 

**POCKET FORMULA** 

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# Some Data and Effective Cross Section

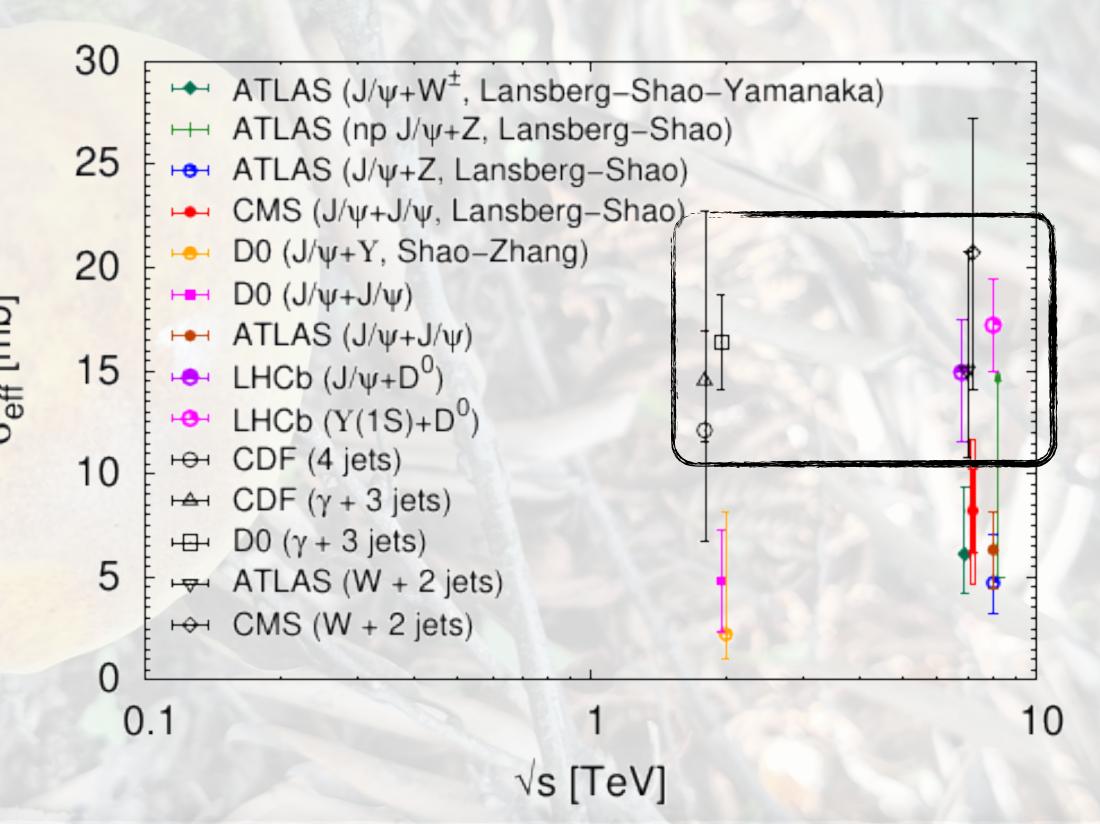
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### **POCKET FORMULA**

Results for W, Jet productions...

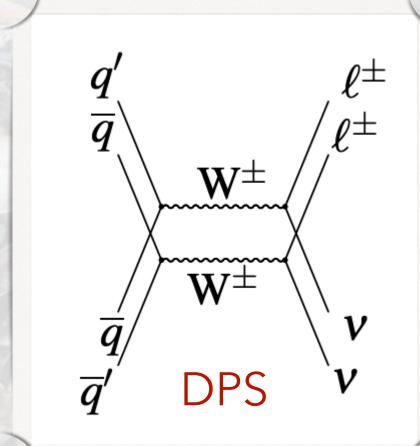


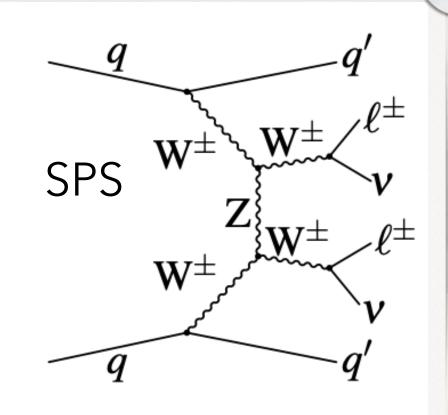
First observation of same sign WW via DPS:

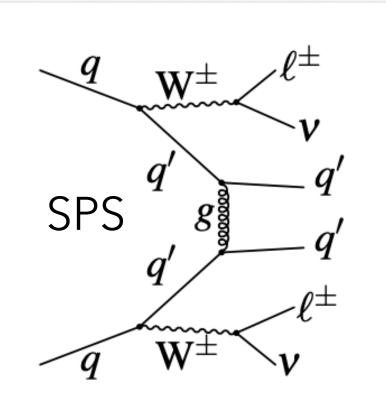
$$\sigma_{
m eff}=12.2^{+2.9}_{-2.2}~{
m mb}$$
 [CMS coll.], PRL 131 (2023) 091803

$$\sigma^{\rm DPS} \sim 6.28~{\rm fb}$$

### Some Data and Effective Cross Section - ssWW





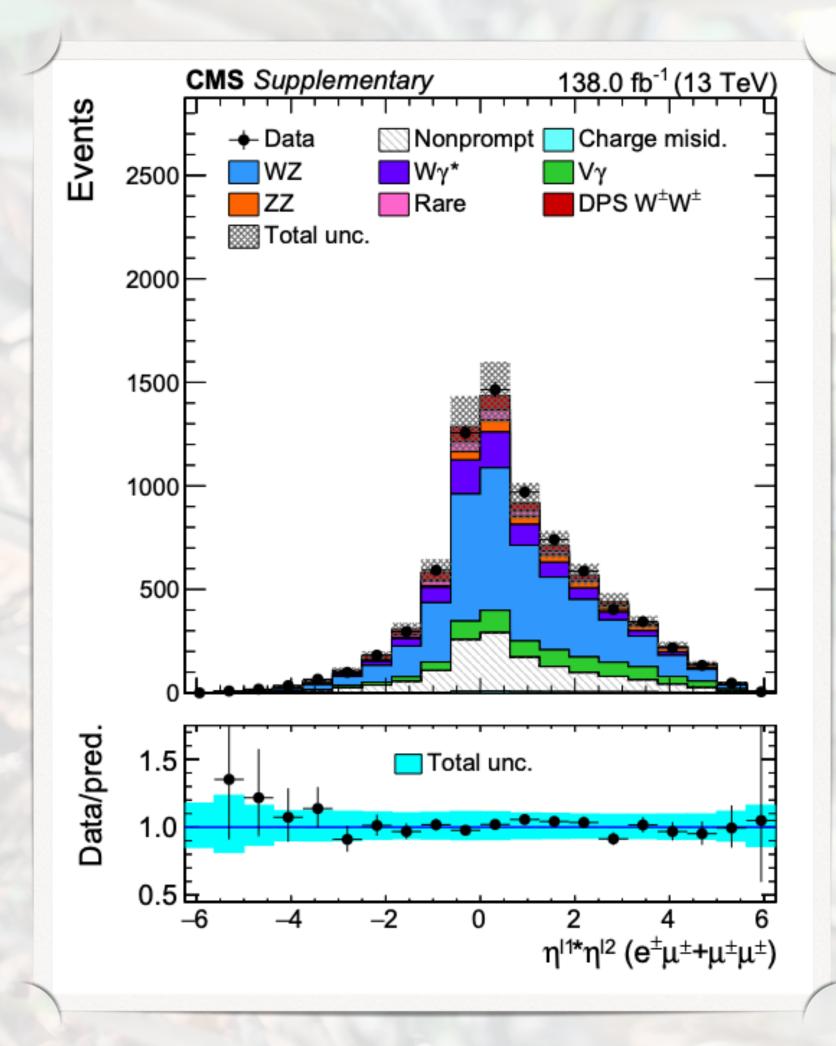


$$\sigma_{\text{eff}}^{\text{pp}} = \frac{m}{2} \frac{\sigma_{\text{A}}^{\text{pp}} \sigma_{\text{B}}^{\text{pp}}}{\sigma_{\text{DPS}}^{\text{pp}}}$$

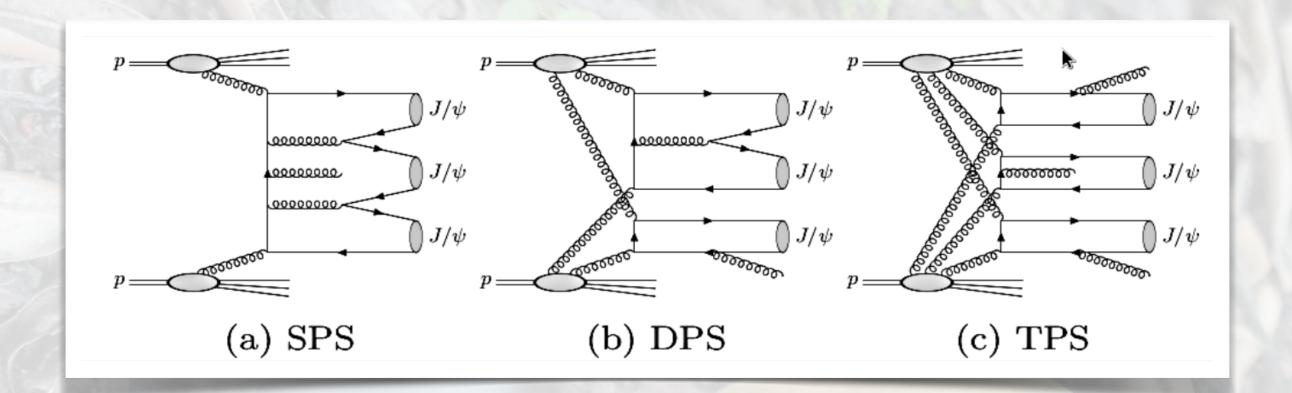
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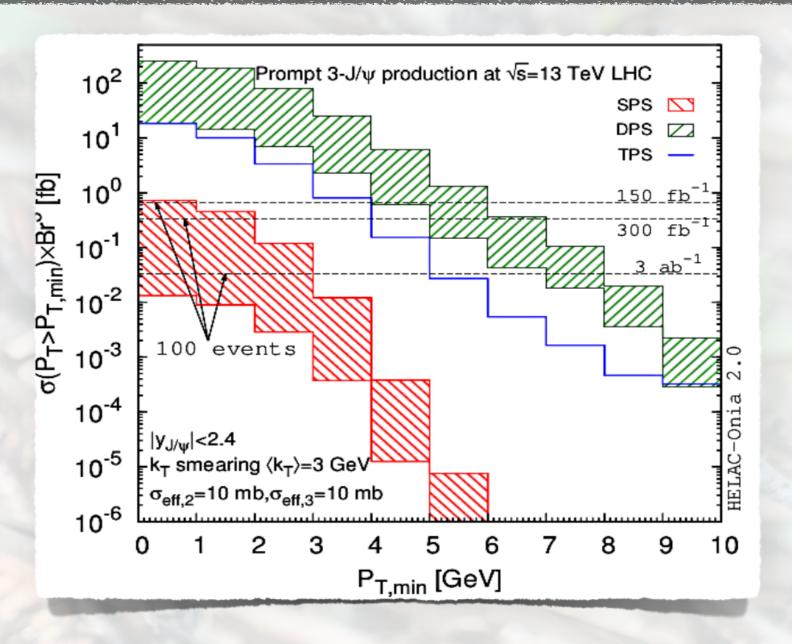
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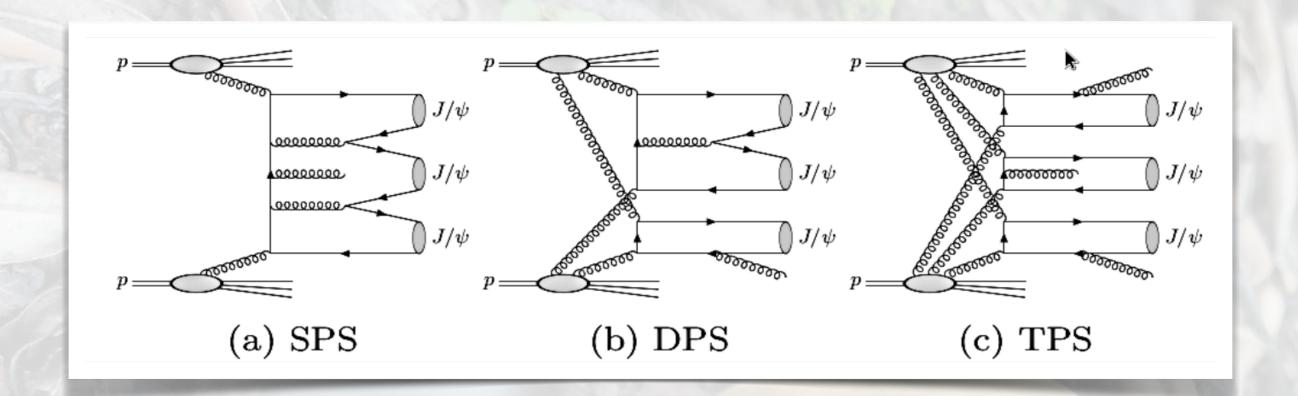
# Some Data and Effective Cross Section - $3 J/\psi$

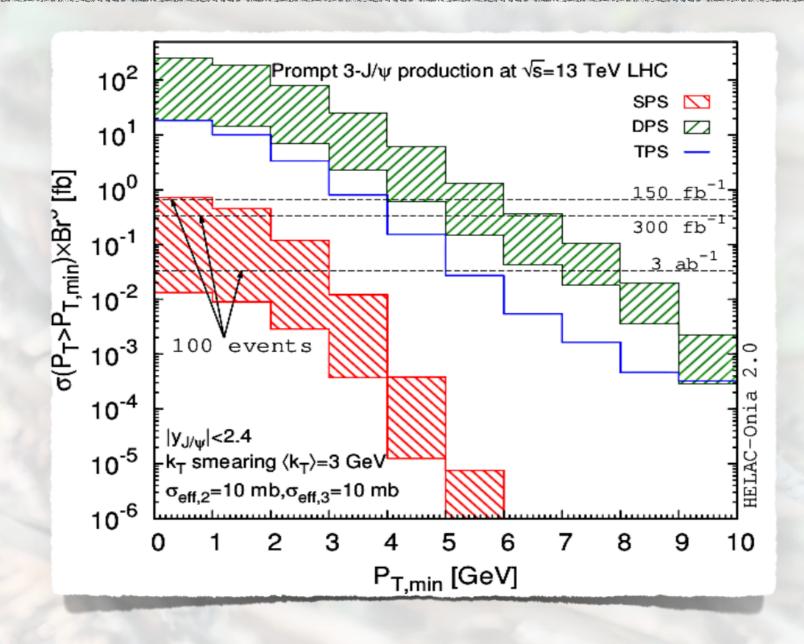


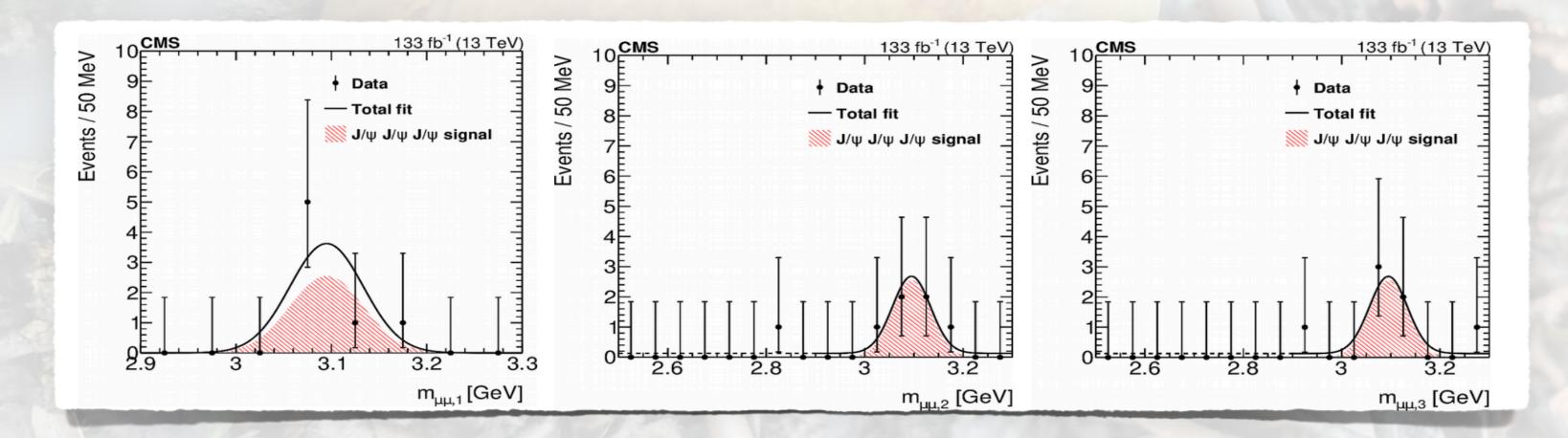


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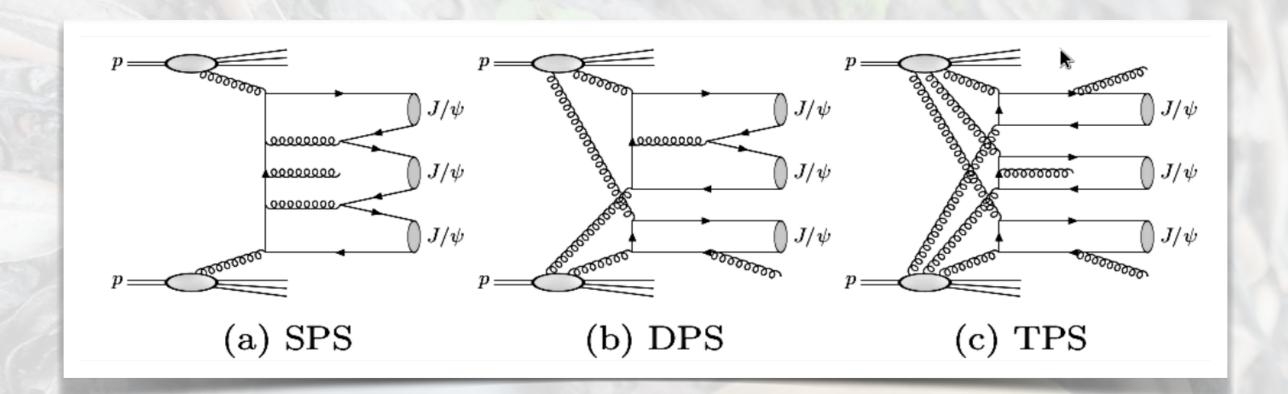


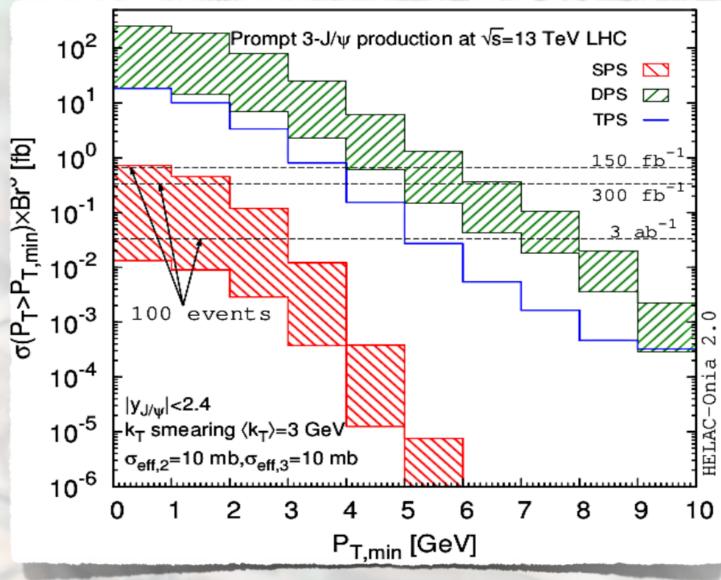




$$\sigma = 272^{+141}_{104} \text{ (stat)} \pm 17 \text{ (syst) fb}$$

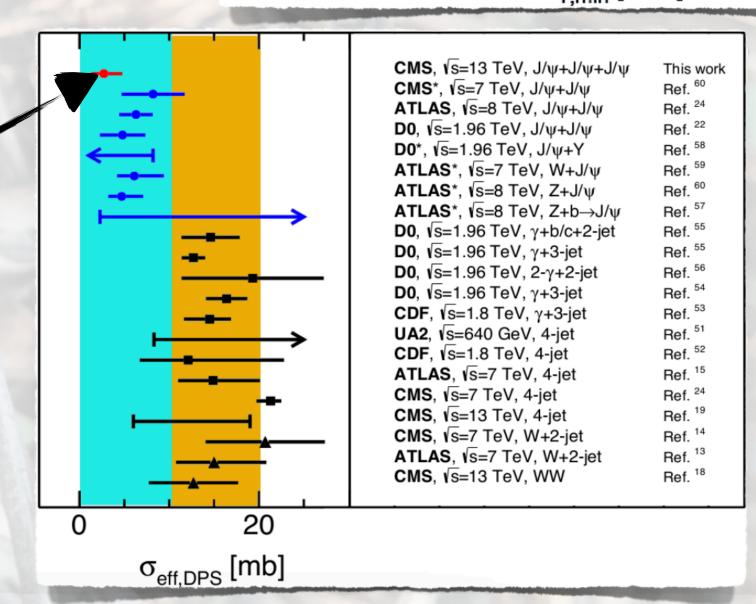
# Some Data and Effective Cross Section - $3 J/\psi$





Novel way to extract the DPS effective cross-section:

 $\sigma_{\text{eff,DPS}} = 2.7_{-1.0}^{-1.4} (\exp)_{-1.0}^{+1.5} (\text{theo}) \text{ mb}$ 



# Some Data and Effective Cross Section

$$\sigma_{\text{eff}}^{\text{pp}} = \frac{m}{2} \frac{\sigma_{\text{A}}^{\text{pp}} \sigma_{\text{B}}^{\text{pp}}}{\sigma_{\text{DPS}}^{\text{pp}}}$$

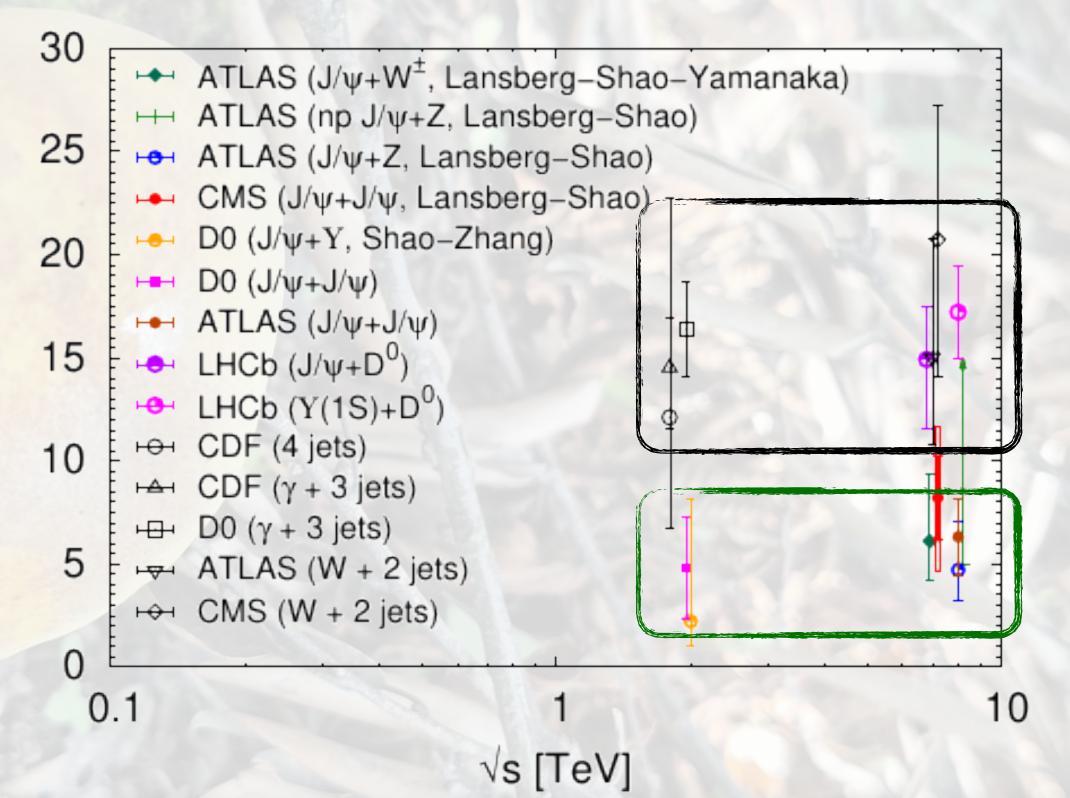
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### **POCKET FORMULA**

Results for W, Jet productions...

Results for quarkonium productions



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m eff} = 12.2^{+2.9}_{-2.2} \;\;{
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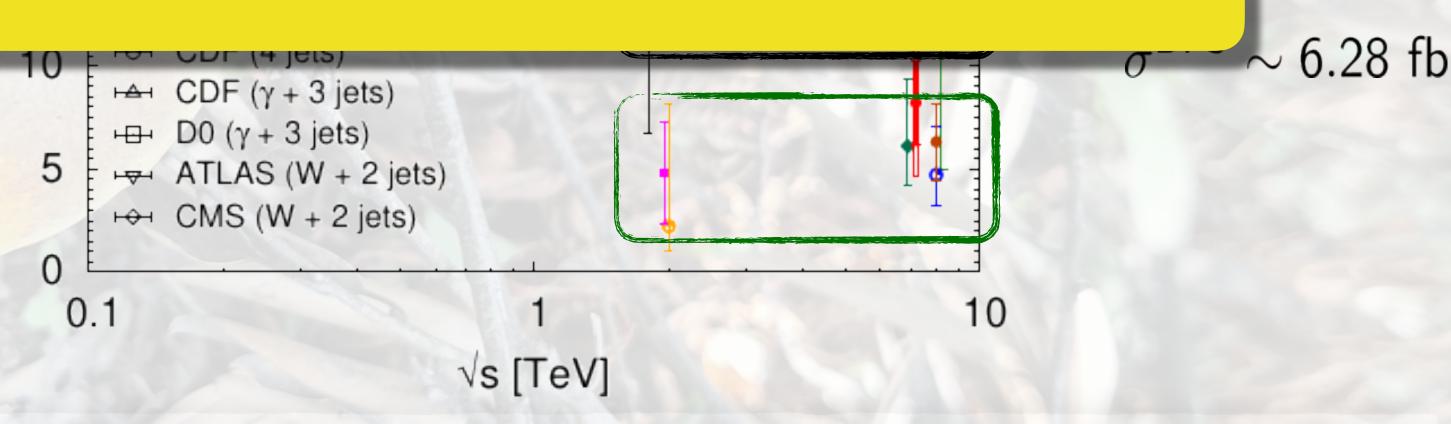
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**POCKET FORMULA** 

- 1) Process dependent?
- 2) Sensitive to correlations
- 3) Sensitive to the inner structure? predicted by all models!

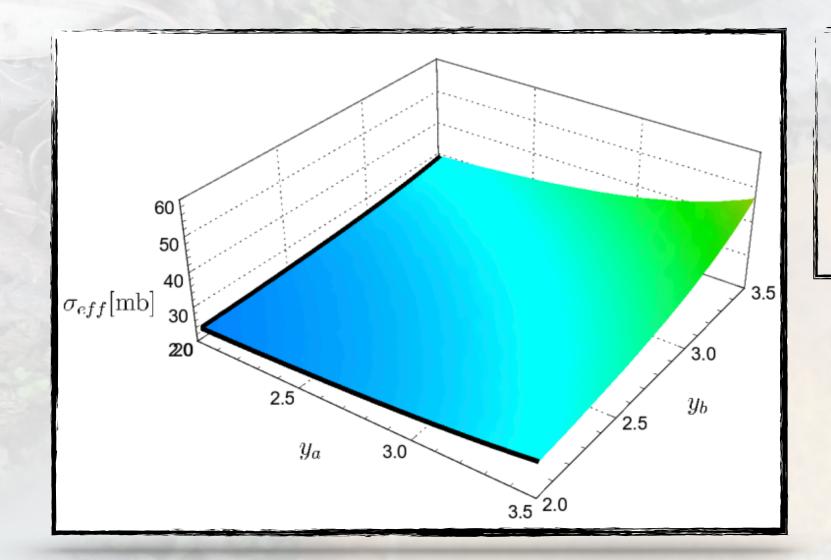
M.R. et al PLB 752,40 (2016) M. Traini, M. R. et al, PLB 768, 270 (2017) M. R. et al, Phys.Rev. D95 (2017) no.11, 114030 First observation of WW via DPS:

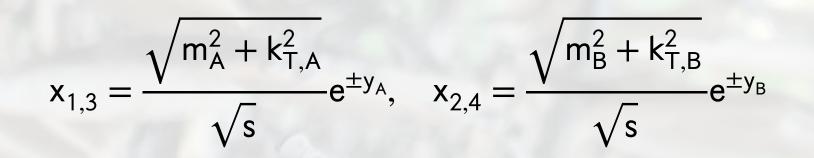
From the last MPI workshop <a href="https://indico.cern.ch/event/1281679/">https://indico.cern.ch/event/1281679/</a>
the idea of studying phenomenological implications of the dependence of  $\sigma_{\rm eff}$  on the kinematics came out!! PRL 131 (2023) 091803
We will work on that



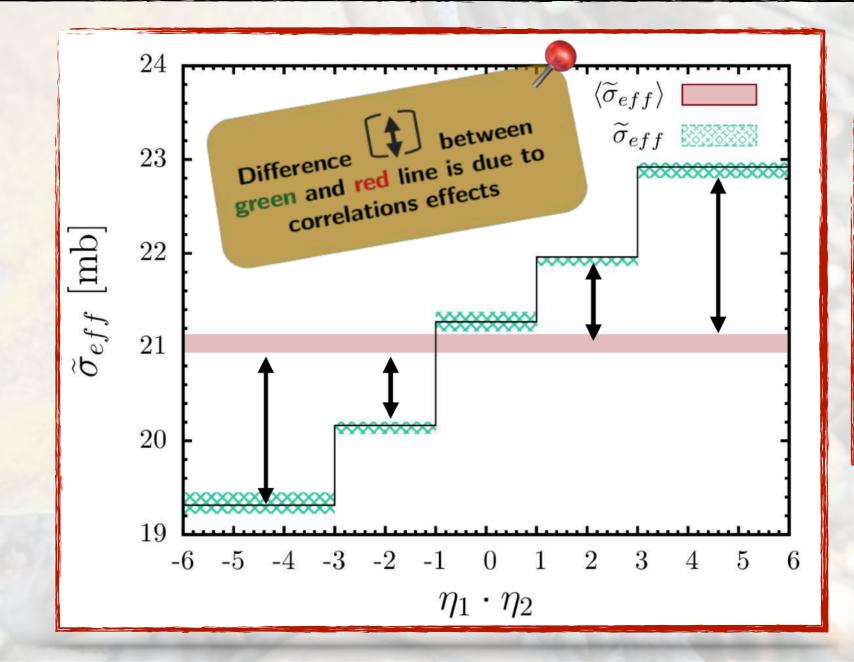
# Some Ideas on "non-constant" $\sigma_{\rm eff}$

$$\sigma_{\text{eff}} = \frac{\mathsf{m}}{2} \frac{\sigma_{\mathsf{A}} \sigma_{\mathsf{B}}}{\sigma_{\mathsf{A}+\mathsf{B}}^{\mathsf{DPS}}}$$





An example for the calculation of  $\sigma_{\rm eff}$  from gluon DPDs at high energy scales M. R. and F. A. Ceccopieri, JHEP 09 (2019) 12, 125003



Predictions from the calculation of same sign W's production at the LHC

F. A. Ceccopieri, M. R. and S. Scopetta, PRD 95 (2017), no.11, 114030

If DPDs factorize in terms of PDFs then

$$\sigma_{\text{eff}}^{-1} = \int d^2 z_{\perp} \ \tilde{T}(z_{\perp})^2 = \int \frac{d^2 k_{\perp}}{(2\pi)^2} \boxed{T(k_{\perp})^2}$$

As for the standard FF:

$$\langle z_{\perp}^2\rangle \propto \frac{d}{k_{\perp}dk_{\perp}} T(k_{\perp}) \bigg|_{k_{\perp}=0}$$

From the asymptotic behavior we got the following relation:

$$\frac{\sigma_{\rm eff}}{3\pi} \leq \langle {\rm z}_{\perp}^2 \rangle \leq \frac{\sigma_{\rm eff}}{\pi}$$

M. R. and F. A. Ceccopieri, PRD 97, no. 7, 071501 (2018)

Effective Form Factor (EFF) = FT of the probability distribution T  $T(k_{\perp}) \propto \int dx_1 dx_2 \ \tilde{F}(x_1, x_2, k_{\perp})$ 

First moment of DPD

If DPDs factorize in terms of PDFs then

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 Verified in all model calculations:

M. R. and F. A. Ceccopieri, PRD 97, no. 7, 071501 (2018)

 $DPD = GPD \otimes GPD$ 

Constituent quark models for:

proton M.R. and F. A. Ceccopieri, JHEP 09 (2019) 097

Pion

M.R. EPJC 80 (2020) 7, 678

W. Broniovski and E. R. Arriola PRD 101 (2020), 1, 014019

ρ M.R. EPJC 80 (2020) 7, 678

If DPDs factorize in terms of PDFs then

$$\sigma_{\text{eff}}^{-1} = \int d^2 \mathbf{z}_{\perp} \ \tilde{\mathbf{T}}(\mathbf{z}_{\perp})^2 = \int \frac{d^2 \mathbf{k}_{\perp}}{(2\pi)^2} \mathbf{T}(\mathbf{k}_{\perp})^2$$

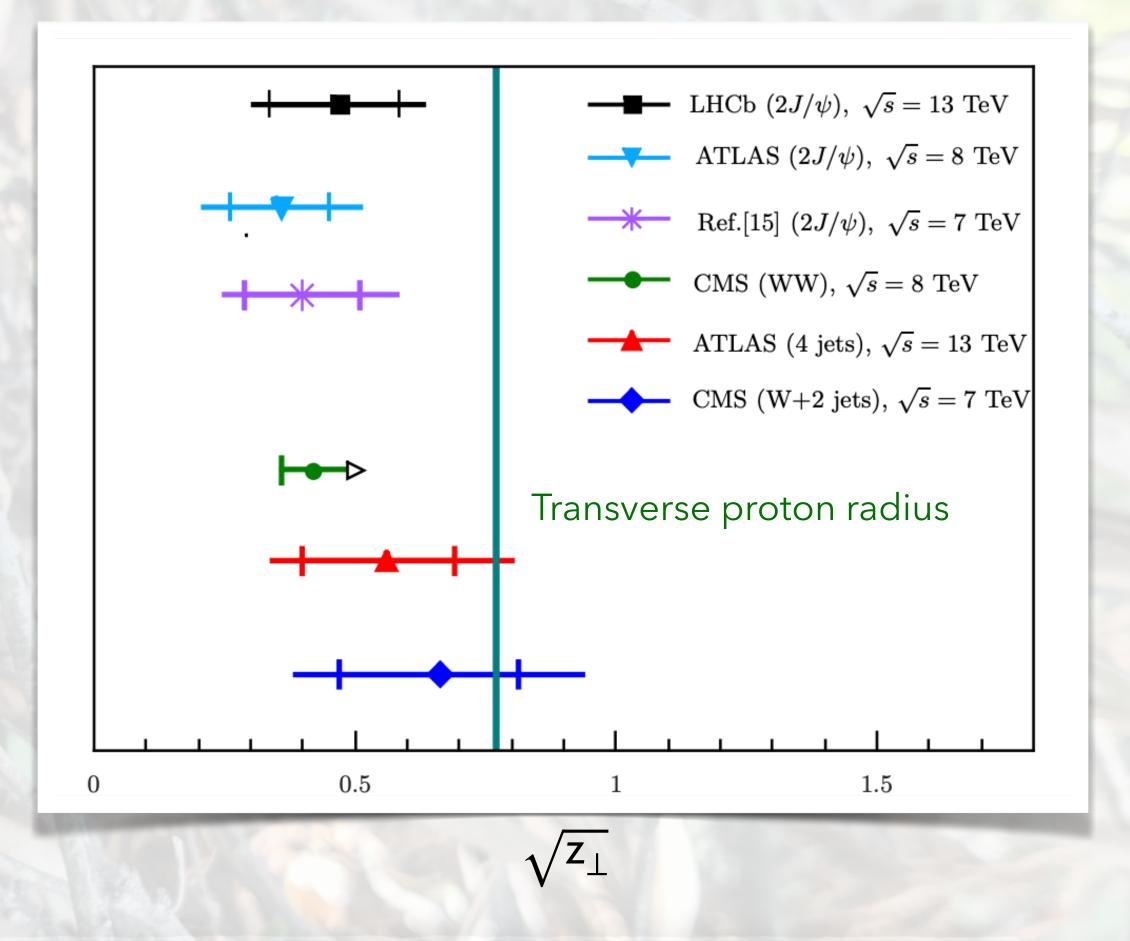
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M. R. and F. A. Ceccopieri, PRD 97, no. 7, 071501 (2018)



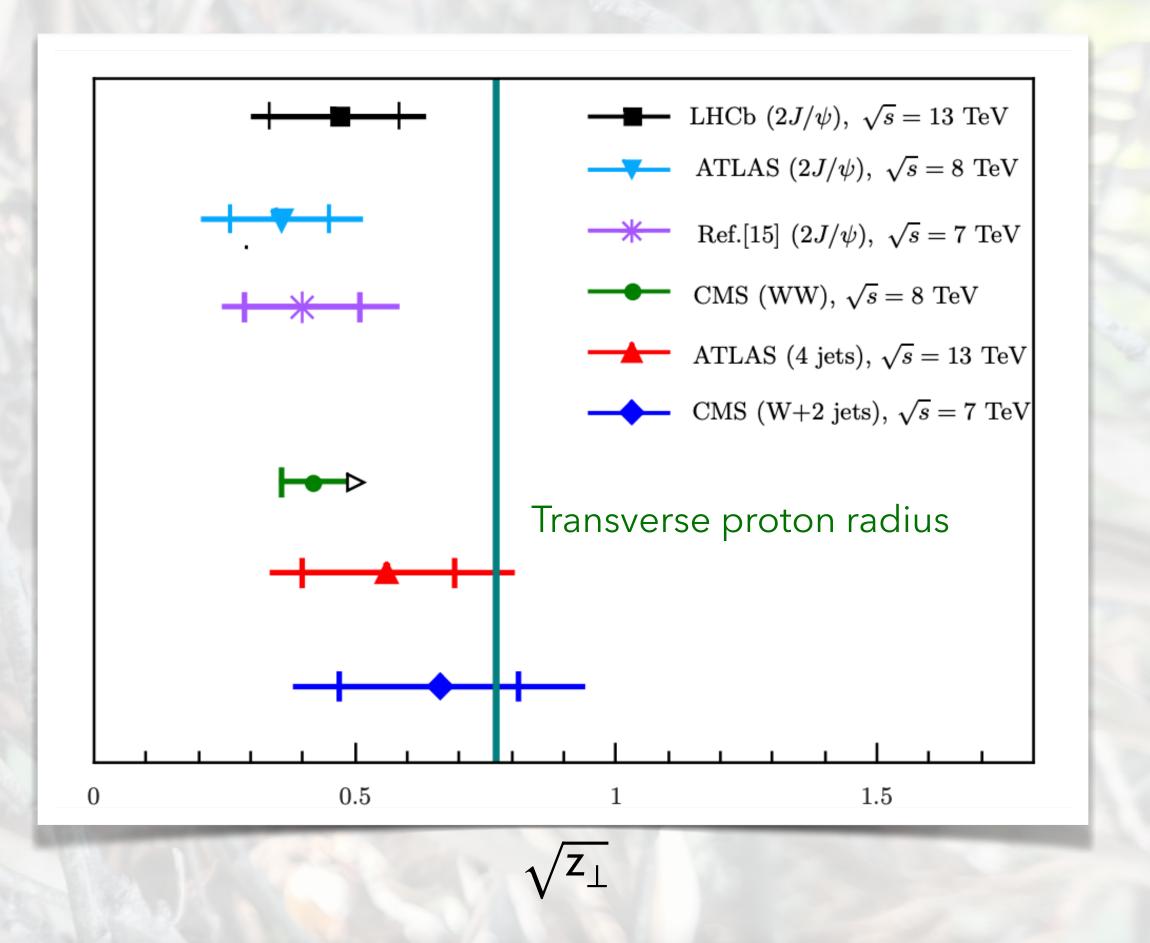
 $\int_{\text{eff}}^{-1} = \int d^2 z_{\perp} \tilde{T}(z_{\perp})^2 = \int \frac{d^2 k_{\perp}}{(2\pi)^2} \tilde{T}(k_{\perp})^2$ If DPDs factorize in term THE MEAN DISTANCE IS LOWER THEN

- THE PROTON RADIUS! in hadron-hadron collisions we do not
- access directly the distance! M.R. and F. A. Ceccopieri, JHEP 09 (2019) 097

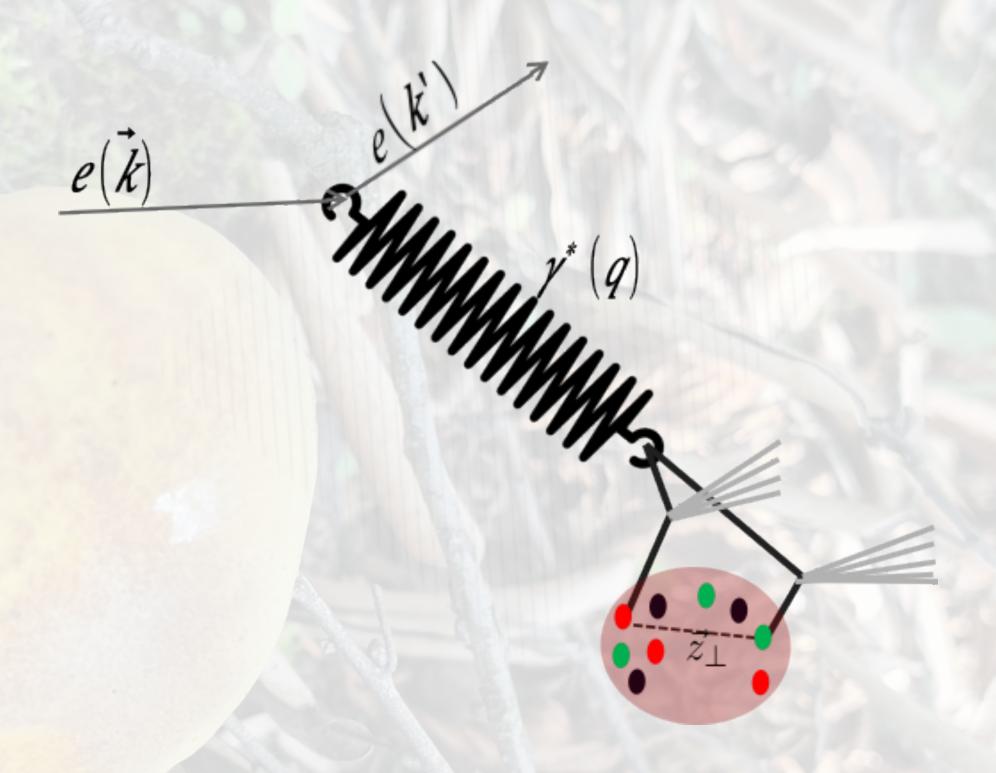
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$$\frac{\sigma_{\rm eff}}{3\pi} \leq \langle {\rm z}_{\perp}^2 \rangle \leq \frac{\sigma_{\rm eff}}{\pi}$$

M. R. and F. A. Ceccopieri, PRD 97, no. 7, 071501 (2018)



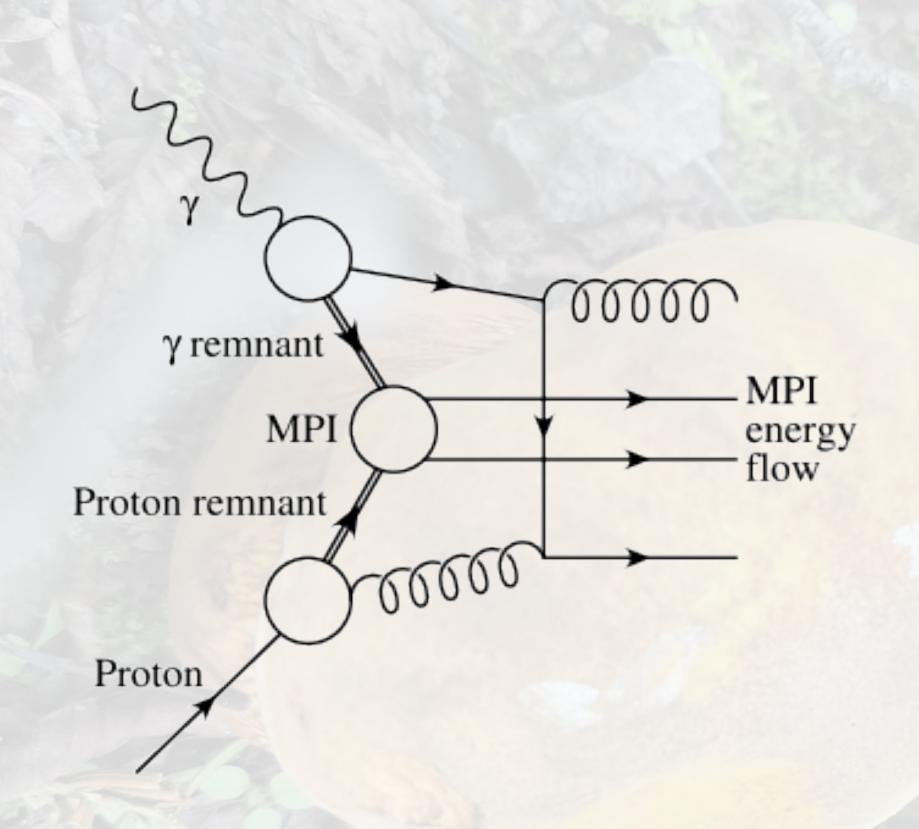
We consider the possibility offered by a DPS process involving a photon FLACTUATING in a quark-antiquark pair interacting with a proton:

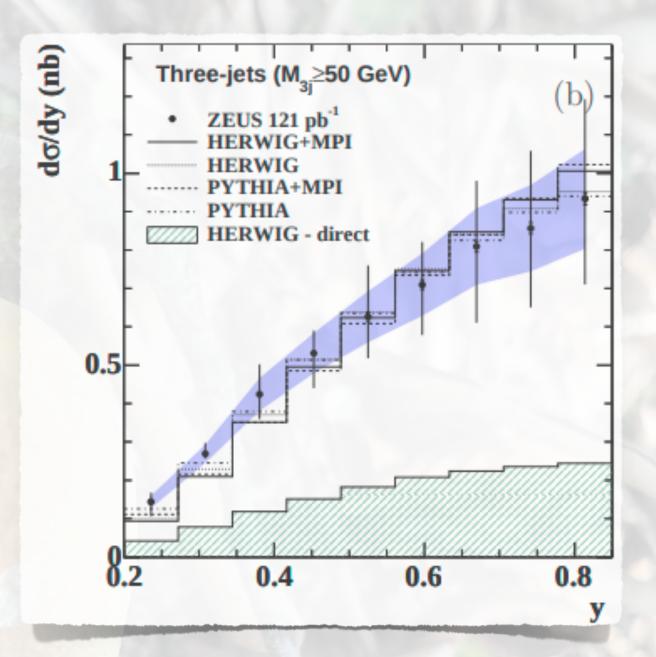


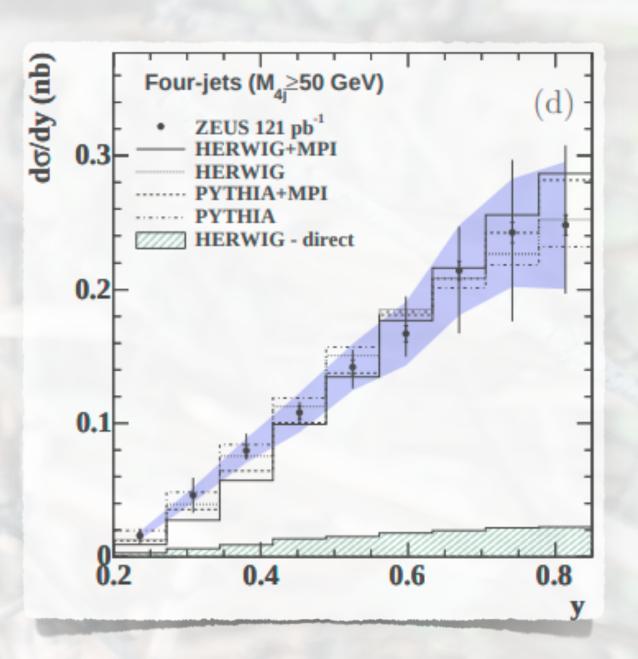
M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

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Already at HERA the importance of MPI for the 3,4 jets photo-production has been addressed:

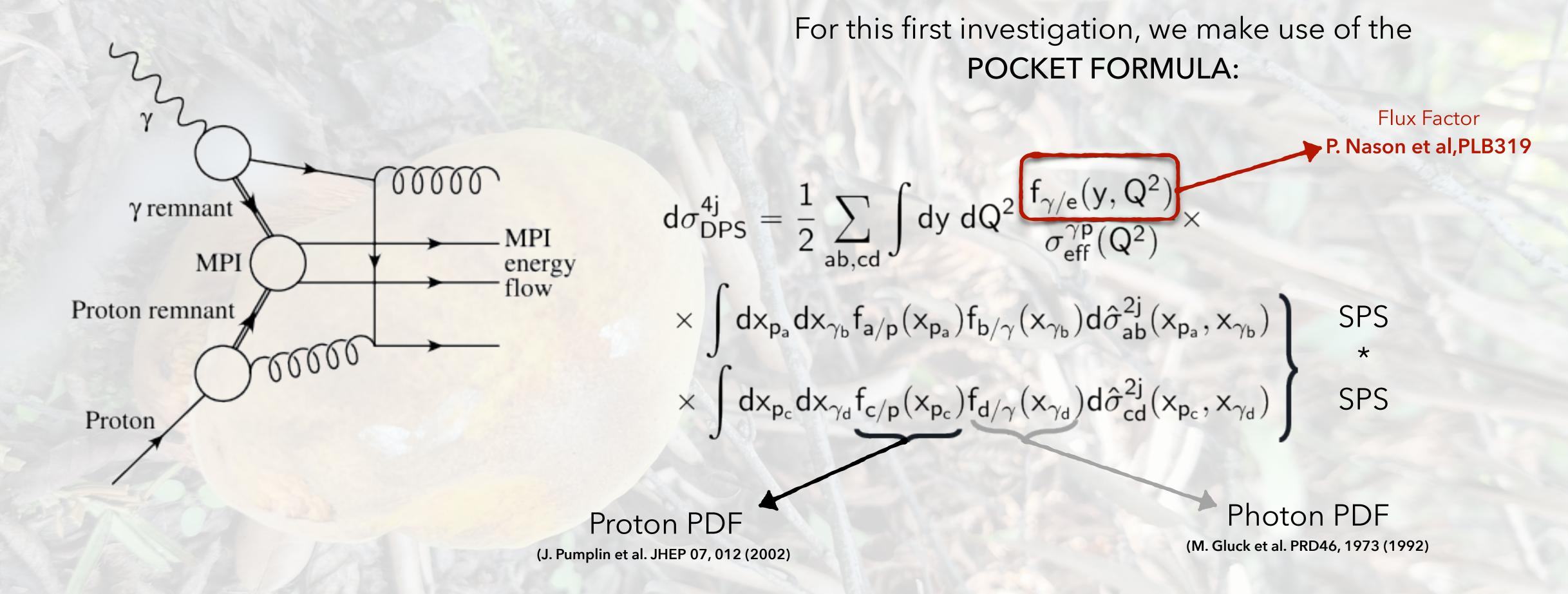




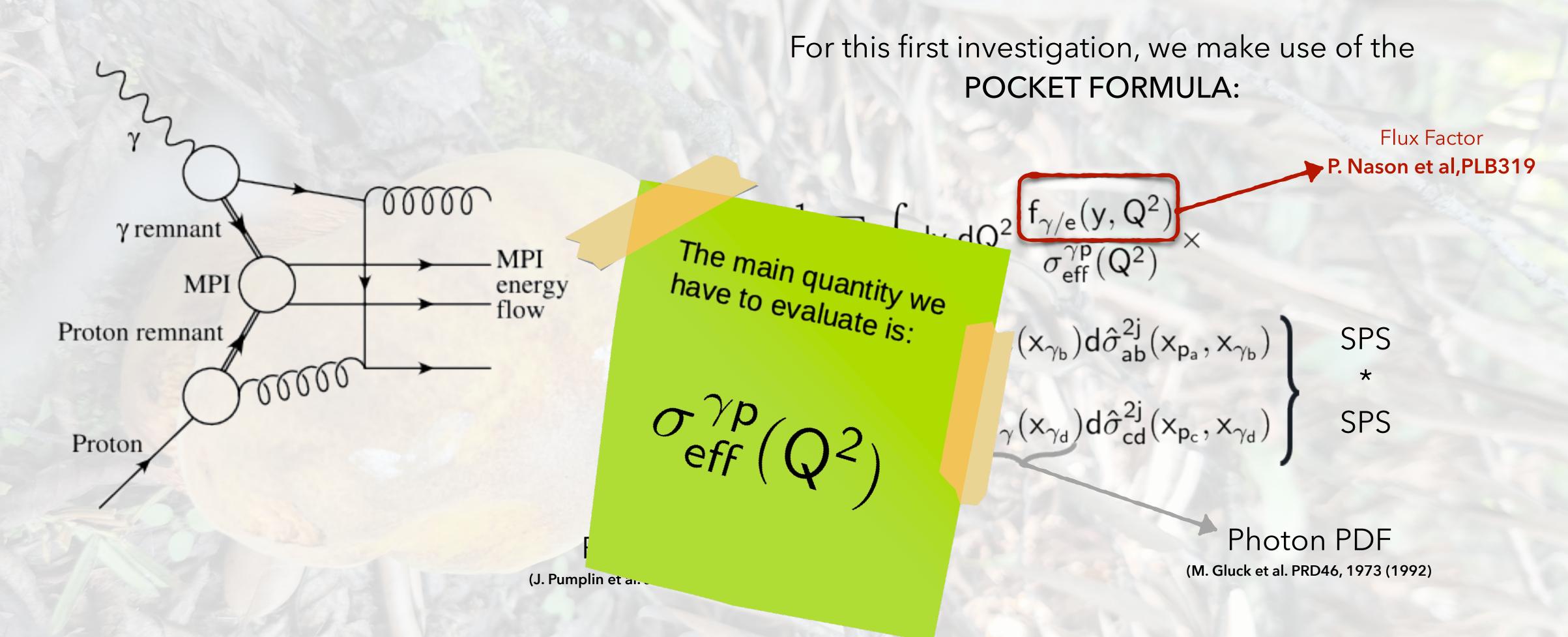


J. R. Forshaw et al, Z phys. C 72, 637 S. Chekanov et al [ZEUS coll.], Nucl. Phys B 792,1 (2008)

In order to study the impact of the DPS contribution to a process initiated via photon-proton interactions we evaluated the 4-JET photo-production at HERA (S. Checkanov et al. (ZEUS), Nucl. Phys B792, 1 (2008))



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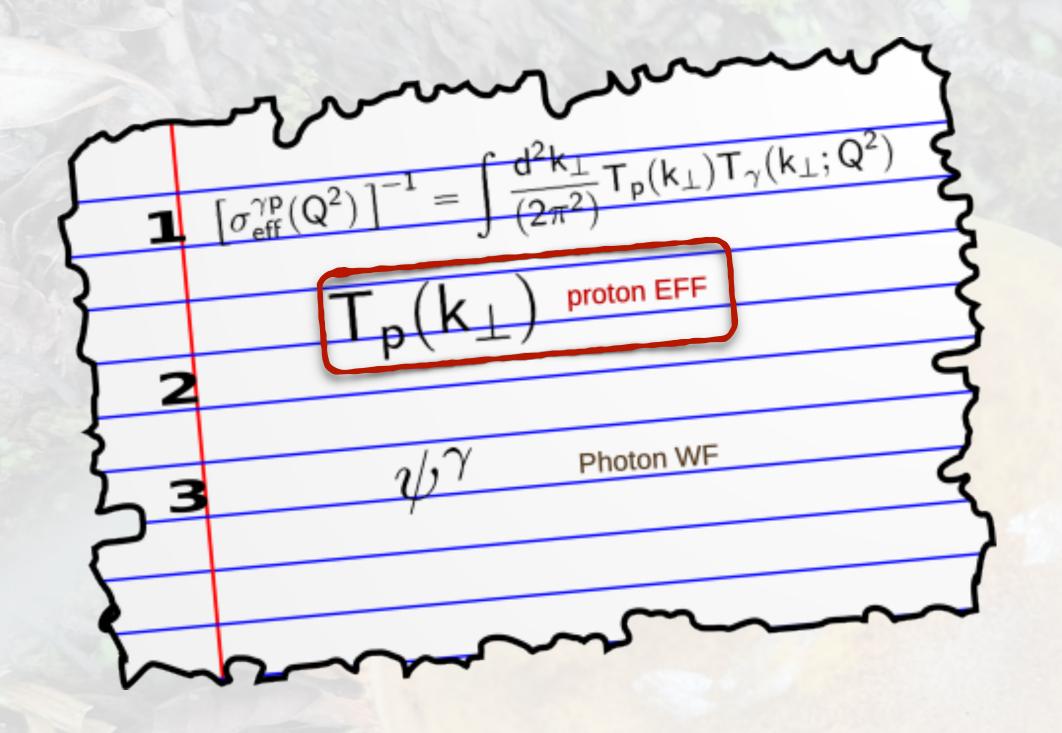
# The $\gamma$ – p effective cross-section

M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

The expression of this quantity is very similar to the proton-proton collision case and can be formally derived by comparing the product of SPS cross sections and the DPS one obtained in **Gaunt, JHEP 01, 042 (2013)** and describing a DPS from a vector bosons splitting with given Q<sup>2</sup> virtuality

The full DPS cross section depends on the amplitude of the splitting photon in a  $q - \bar{q}$  pair. The latter can be formally described within a Light-Front (LF) approach in terms of LF wave functions

The main ingredients of the calculations:



For the proton EFF use has been made of three choices:

$$e^{-\alpha_1 k_{\perp}^2}$$

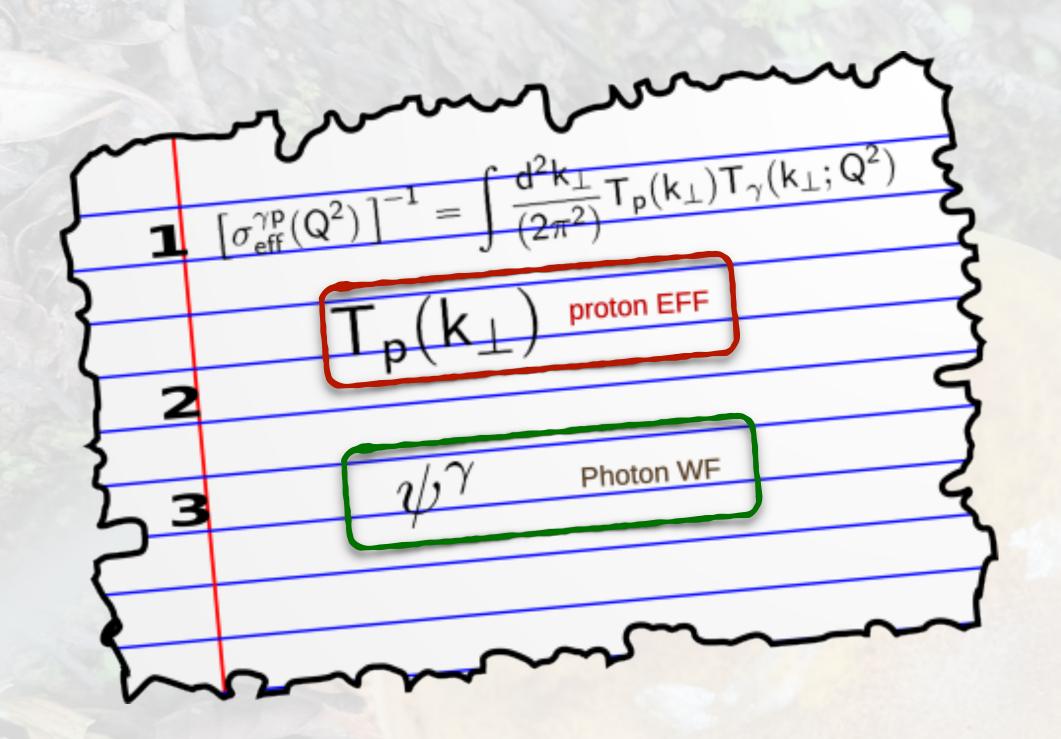
$$e^{-\alpha_1 k_{\perp}^2}$$
,  $\alpha_1 = 1.53 \text{ GeV}^{-2} \Longrightarrow \sigma_{\text{eff}}^{pp} = 15 \text{ mb}$ 

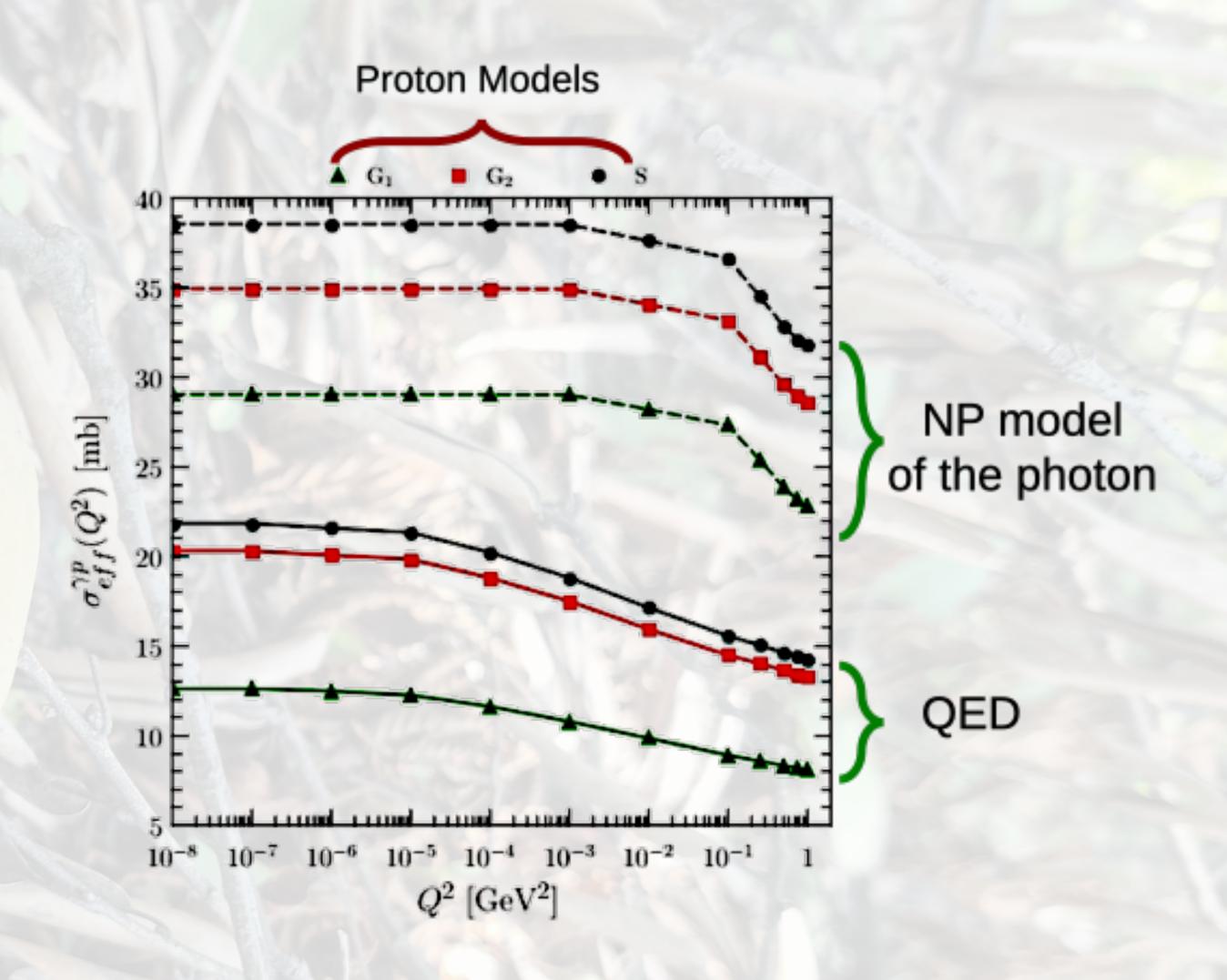
$$e^{-\alpha_2 k_\perp^2}$$

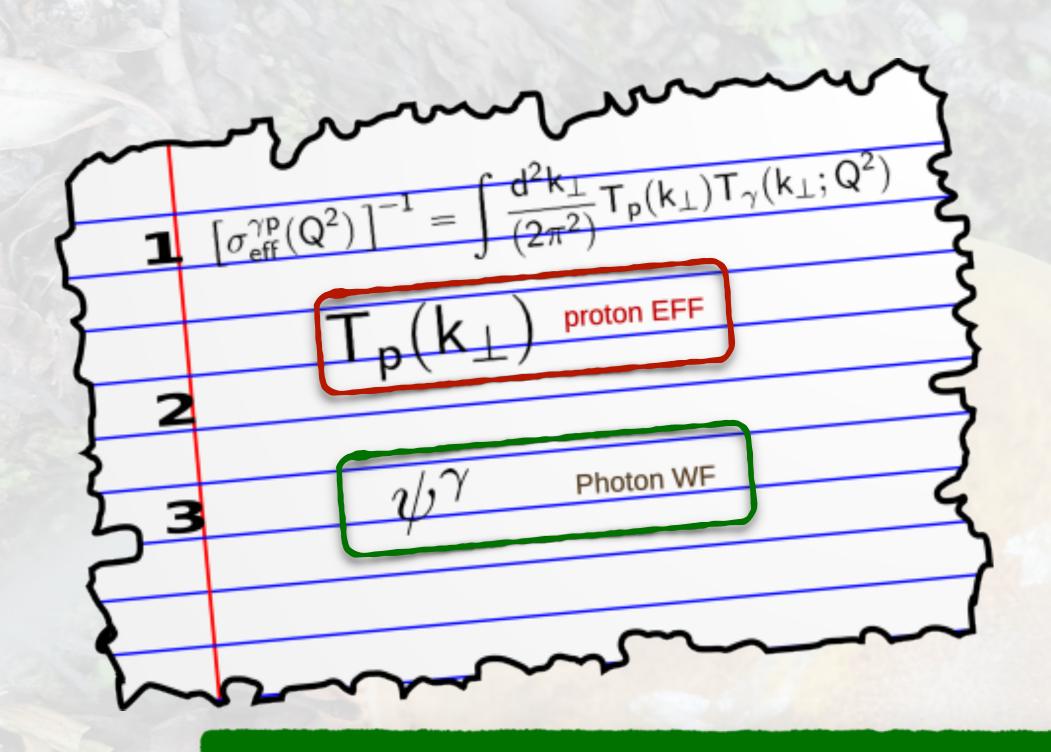
$$e^{-\alpha_2 k_\perp^2}$$
,  $\alpha_2 = 2.56 \text{ GeV}^{-2} \Longrightarrow \sigma_{\text{eff}}^{pp} = 25 \text{ mb}$ 

$$S = \left(1 + \frac{k_\perp^2}{m_g^2}\right)^{-4}$$

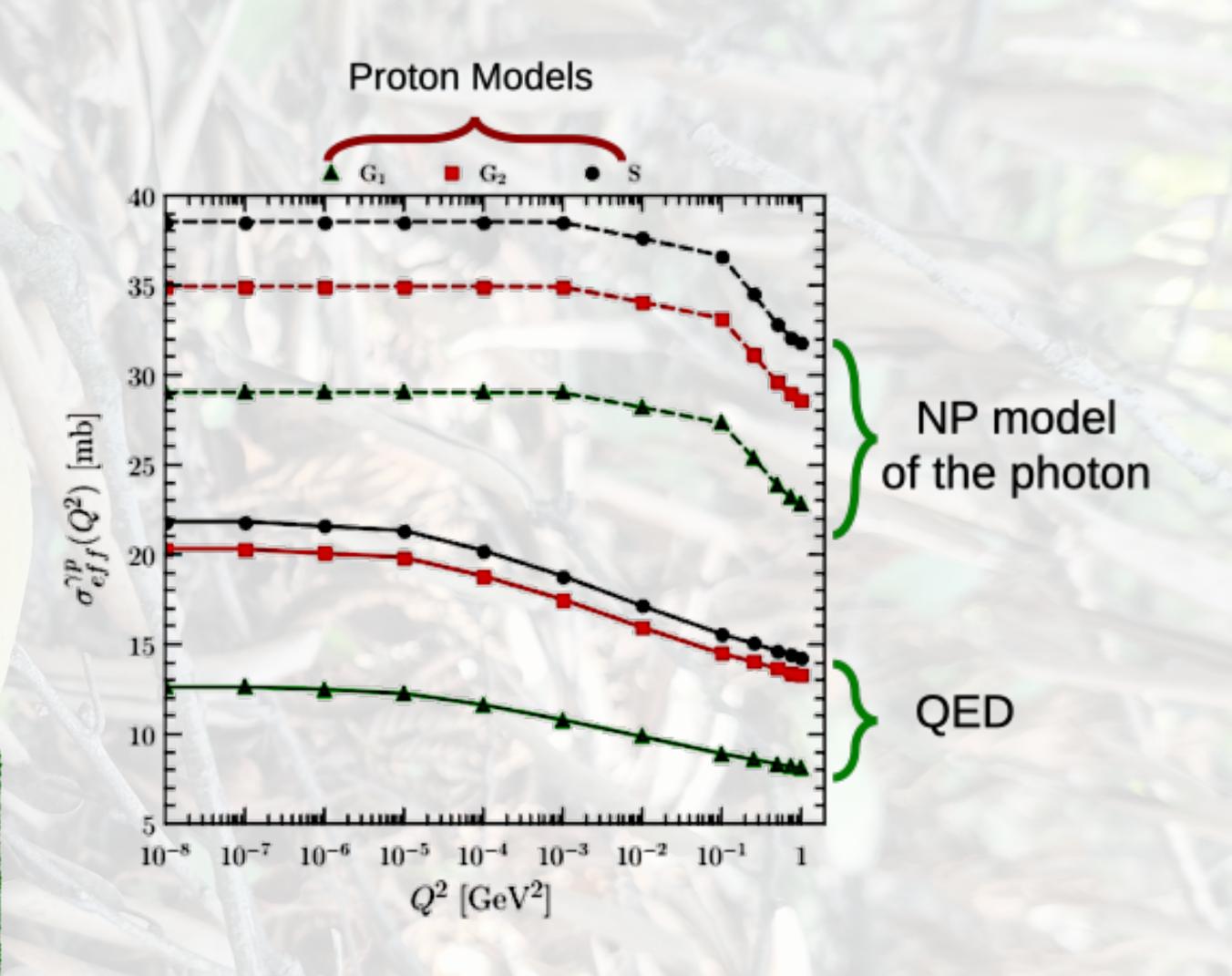
3) S 
$$\left(1 + \frac{k_\perp^2}{m_g^2}\right)^{-4}$$
,  $m_g^2 = 1.1 \text{ GeV}^2 \Longrightarrow \sigma_{\text{eff}}^{pp} = 30 \text{ mb}$ 







The effective cross-section depends on the photon virtuality! (NEW)



# The 4-jets DPS cross-section

$$\begin{split} d\sigma_{DPS}^{4j} &= \frac{1}{2} \sum_{ab,cd} \int dy \ dQ^2 \ \frac{f_{\gamma/e}(y,Q^2)}{\sigma_{eff}^{\gamma p}(Q^2)} \times \\ &\times \int dx_{p_a} dx_{\gamma_b} f_{a/p}(x_{p_a}) f_{b/\gamma}(x_{\gamma_b}) d\hat{\sigma}_{ab}^{2j}(x_{p_a},x_{\gamma_b}) \\ &\times \int dx_{p_c} dx_{\gamma_d} f_{c/p}(x_{p_c}) f_{d/\gamma}(x_{\gamma_d}) d\hat{\sigma}_{cd}^{2j}(x_{p_c},x_{\gamma_d}) \end{split}$$

### KINEMATICS:

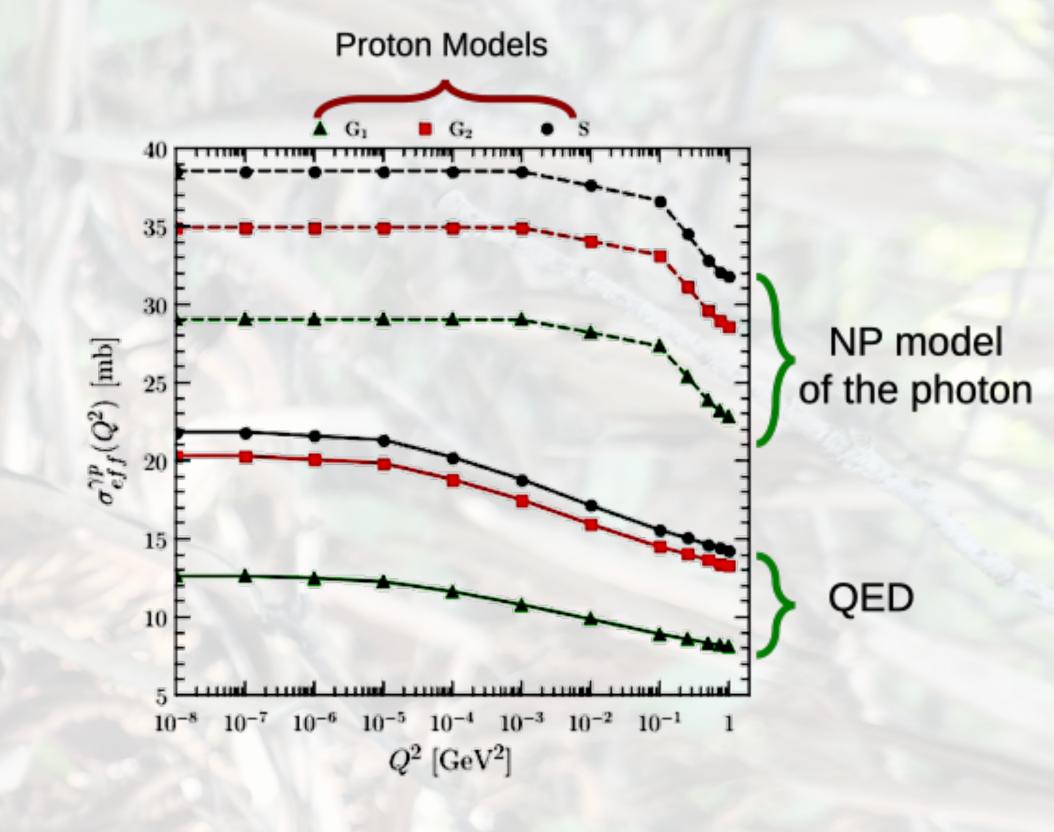
$$E_T^{jet} > 6 \text{ GeV}$$

$$|\eta_{
m jet}| < 2.4$$

$$Q^2 < 1 \; {\rm GeV}^2$$

$$0.2 \leqslant y \leqslant 0.85$$

The ZEUS collaboration quoted an integrated total 4-jet cross section of 136 pb S. Checkanov et al. (ZEUS), Nucl. Phys B792, 1 (2008)



**Proton Models** 

$d\sigma_{DPS}^{4j} = \frac{1}{2} \sum_{ab,cd} \int_{ab,cd} \int_{ab,cd$	
$ imes \int dx_{p_a} dx_{\gamma_b} f_{a/p}($	
$\times \int dx_{p_c} dx_{\gamma_d} f_{c/p}$	_

#### KINEMATICS:

$$E_T^{jet} > 6 \text{ GeV}$$

$$|\eta_{
m jet}| < 2.4$$

$$Q^2 < 1 \, \mathrm{GeV}^2$$

$$0.2 \leqslant y \leqslant 0.85$$

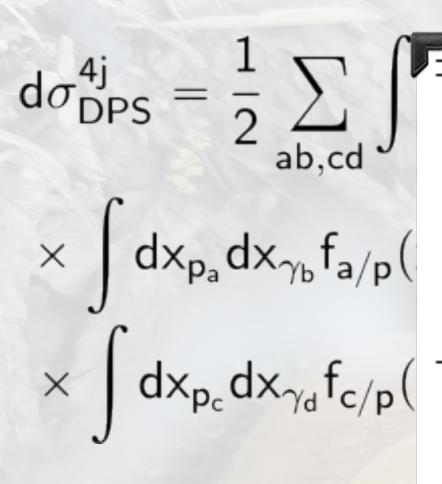
	, ,	2)			$G_1$ $G_2$	S S
			$\sigma_{DPS}$ [pb]			
Proton		$Q^2 \leq 10^{-2}$	$10^{-2} \le Q^2 \le 1$	$Q^2 \leq 1$	$\frac{\sigma_{DPS}}{\sigma_{tot}}$	NP model
Photon		$[GeV^2]$	$[GeV^2]$	$[GeV^2]$	[%]	of the photo
	$G_1$	35.1	18.6	53.7	40	
NP Na dal	$G_2$	29.1	15.2	44.3	33	
Model	$\mathbf{S}$	26.4	13.7	40.1	30	QED
	$G_1$	87.8	54.3	142.1	101	10 <sup>-2</sup> 10 <sup>-1</sup> 1
QED	$G_2$	54.3	33.4	87.7	65	
	S	50.5	31.1	81.6	60	
The ZEUS c	ollab	oration quoted	d an integrated tota	4-iet cros	s section	of 136 pb

S. Checkanov et al. (ZEUS), Nucl. Phys B792, 1 (2008)

# The 4-jets DPS cross-section

M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

**Proton Models** 



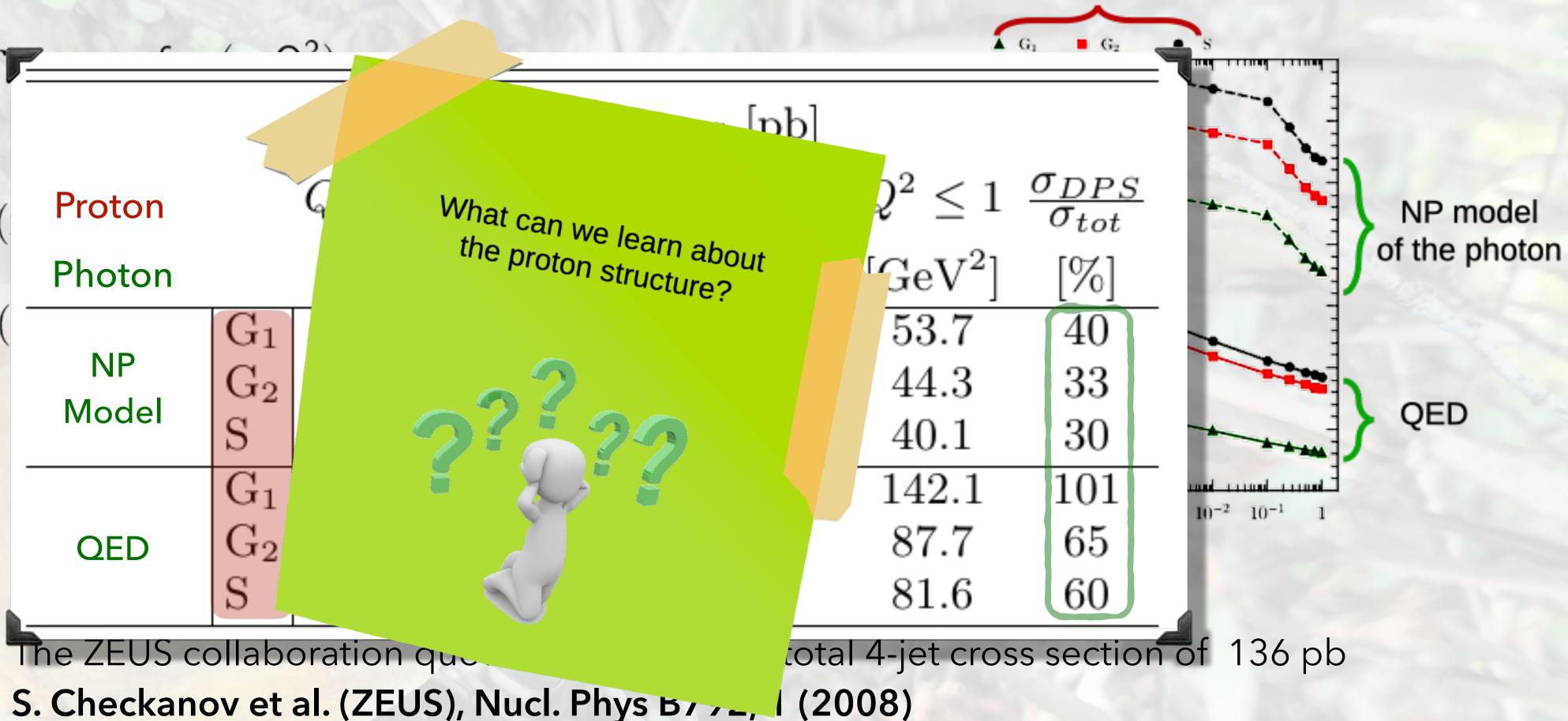
#### KINEMATICS:

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$$0.2 \leqslant y \leqslant 0.85$$



The effective cross section can be also written in terms of probability distribution:

$$\left[\sigma_{\rm eff}^{\gamma p}(Q^2)\right]^{-1} = \int d^2 z_{\perp} \ \tilde{F}_2^p(z_{\perp}) \tilde{F}_2^{\gamma}(z_{\perp}; Q^2)$$

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We can expand the distribution related to the photon:

$$\tilde{F}_2^\gamma(z_\perp;Q^2) = \sum_n C_n(Q^2) z_\perp^n$$

Coefficients determined in a given approach describing the photon structure

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Mean value of the transverse distance between two partons in the PROTON

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If we could measure  $\sigma_{
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The effective cross section can be also written in terms of probability distribution:



We can expend

 $\tilde{F}_2^{\gamma}(z)$  We estimated that with an integrated luminosity of 200 pb-1  $Q^2$  effects can be observed

$$\sigma_{\rm eff}^{\gamma p}(\zeta)$$

photon:

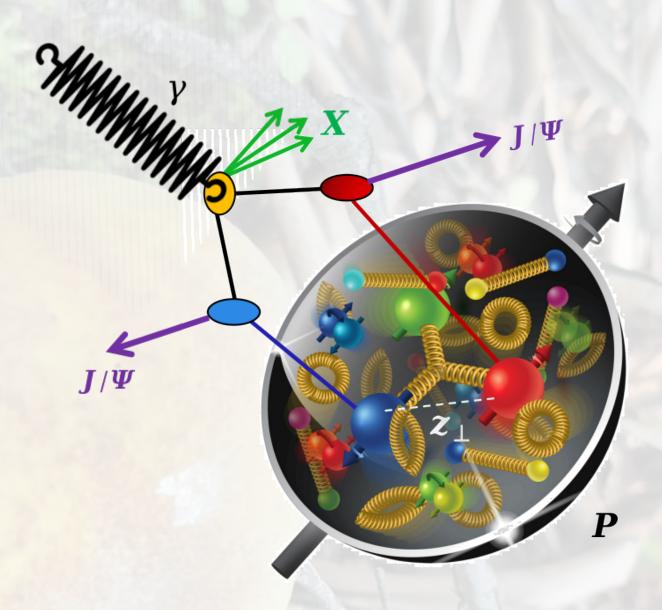
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Mean value of the transverse distance between two partons in the PROTON

If we could measure  $\sigma_{\rm eff}^{\gamma p}(Q^2)$  we could access NEW INFORMATION ON THE PROTON STRUCTURE

# Di J/\photo-production@EIC

Illustration of DPS for  $\gamma + p \rightarrow J/\psi + J/\psi + X$ 



We consider the possibility of **resolved** photon to estimate the DPS cross section in quarkonium-pair photoproduction at the EIC

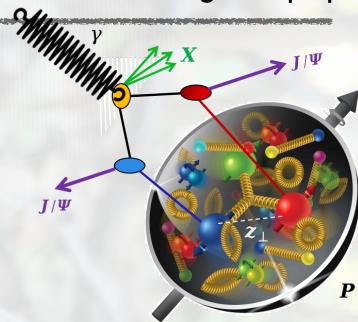
# Di J/w photo-production@EIC

F. A. Ceccopieri, H. S. Shao, J. P. Lansberg, M. R. and R. Sangem in prep.

\*Slide from R. Sangem

$$\sigma_{SPS}^{(J/\psi,J/\psi)} \propto \sum_{a=g,q} \int dx_{p_a} f_{a/p}(x_{p_a},\mu) d\hat{\sigma}^{\gamma a \to J/\psi + J/\psi + a}$$

unresolved/direct



# $\sigma_{SPS}^{(J/\psi,J/\psi)} \propto \sum_{a,b=g,q} \int dx_{\gamma_a} \, dx_{p_b} \underbrace{f_{a/\gamma}(x_{\gamma_a},\mu)}_{f_{b/p}(x_{p_b},\mu)} \underbrace{f_{b/p}(x_{p_b},\mu)}_{d\hat{\sigma}^{ab\to J/\psi+J/\psi}}$

resolved

$$\sigma_{DPS}^{(J/\psi,J/\psi)} \propto \frac{1}{2} \frac{1}{\sigma_{eff}^{\gamma p}} \sum_{a,b,c,d} \int dx_{\gamma_a} dx_{p_b} f_{a/\gamma}(x_{\gamma_a},\mu) f_{b/p}(x_{p_b},\mu) d\hat{\sigma}_{SPS}^{ab \to J/\psi}(x_{\gamma_a},x_{p_b})$$

$$\times dx_{\gamma_c} dx_{p_d} f_{c/\gamma}(x_{\gamma_c},\mu) f_{d/p}(x_{p_d},\mu) d\hat{\sigma}_{SPS}^{cd \to J/\psi}(x_{\gamma_c},x_{p_d})$$

Proton PDF

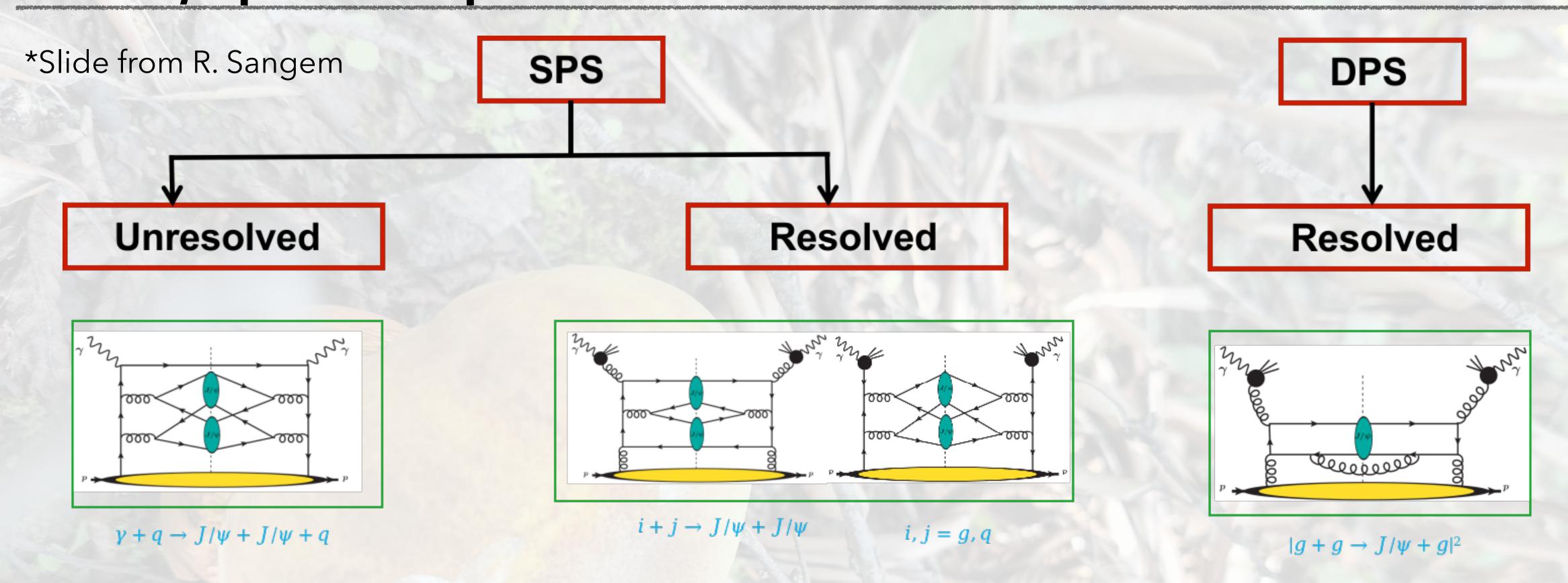
Photon PDF

Partonic x-sections

Single SPS resolved (namely same partonic cross section as hadroproduction)

### Di J/w photo-production@EIC

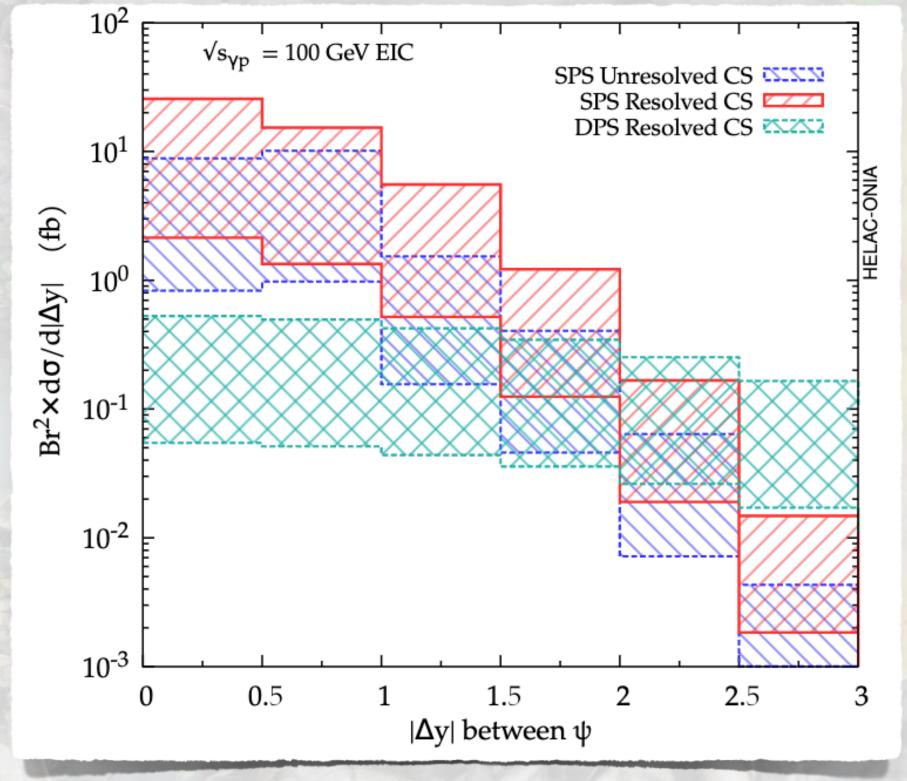
F. A. Ceccopieri, H. S. Shao, J. P. Lansberg, M. R. and R. Sangem in prep.



- GRV photon PDF is used PRD 46, 1973 (1992) , while CT18NLO PDF for proton T.J. Hou et al., PRD 103, 014013 (2021)
- HELAC-Onia latest version is used for generating matrix elements HS Shao, CPC 184, 2562 (2013), 198, 238 (2016)
  - CO LDMEs are taken from M. Butenschoen and B. A. Kniehl, PRD 84, 051501 (2011)
- We expect at least 600 four-muon events with 100 fb<sup>-1</sup> luminosity

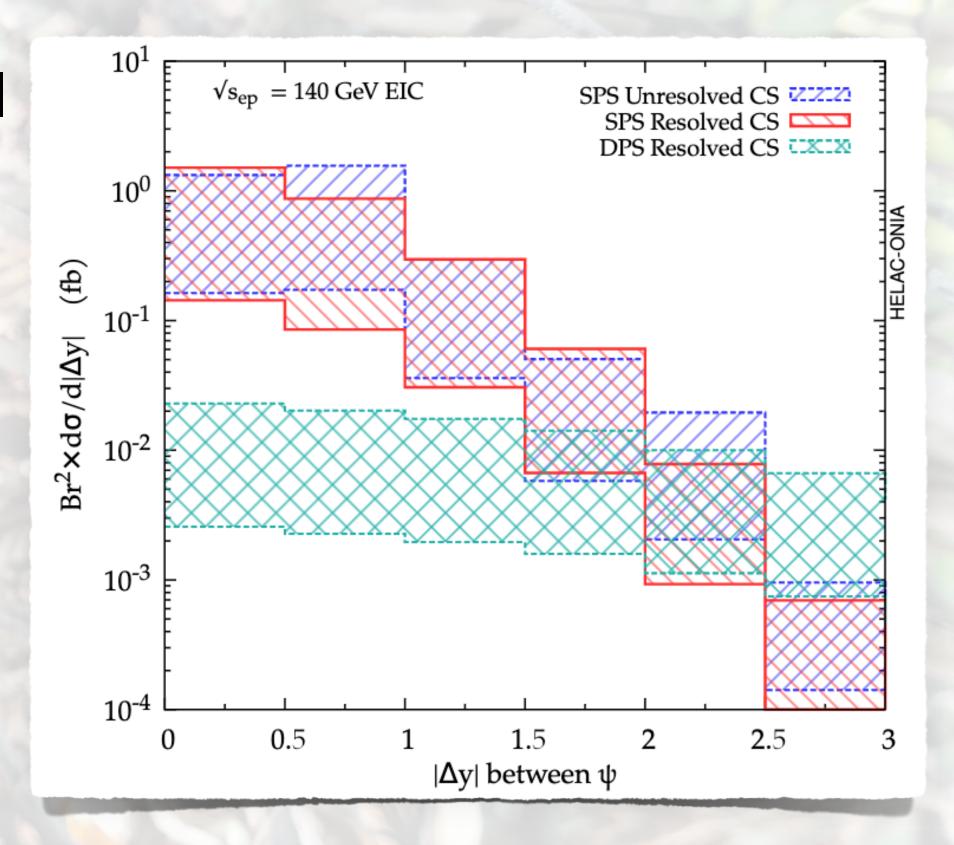
Absolute rapidity difference between the two  $J/\psi$ 

$$\sqrt{s_{\gamma p}} = 100 \text{ GeV}$$



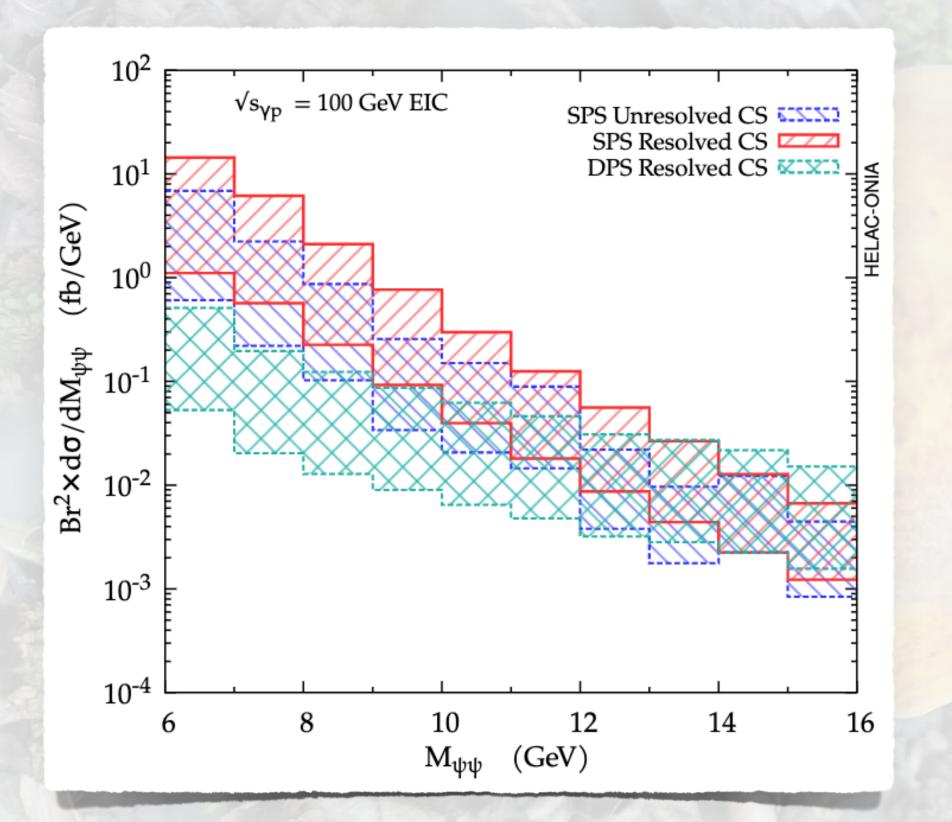
- DPS dominates at high  $|\Delta y|$
- DPS is suppressed at low  $|\Delta y|$



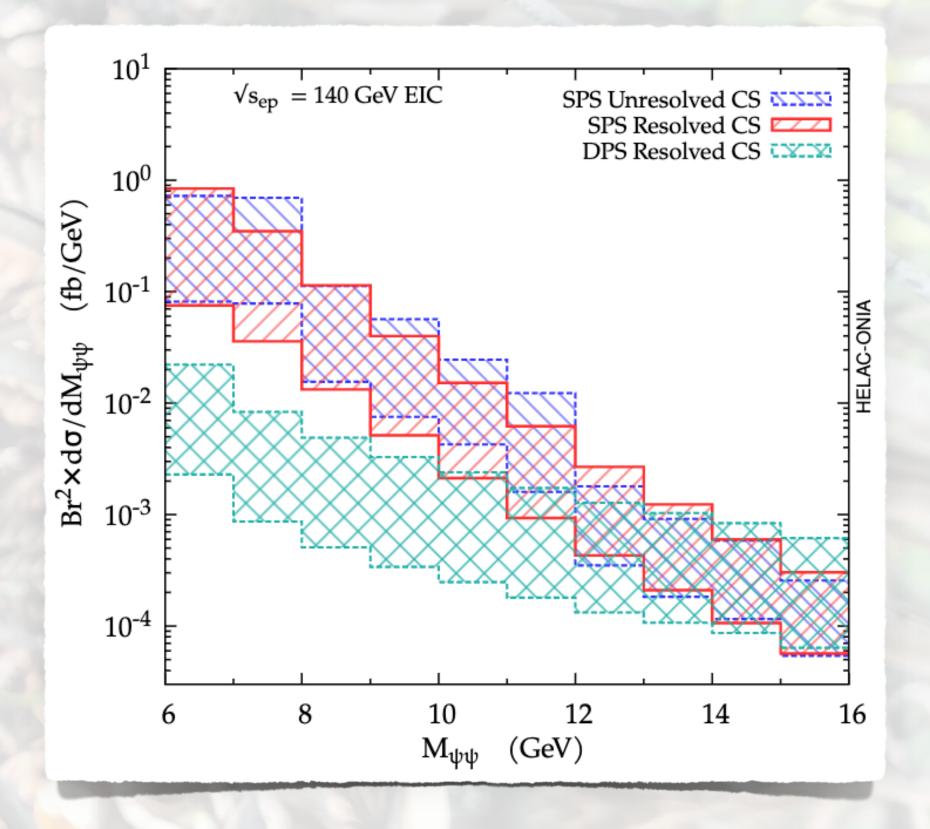


Invariant mass of the  $J/\psi$  pair

$$\sqrt{s_{\gamma p}} = 100 \text{ GeV}$$

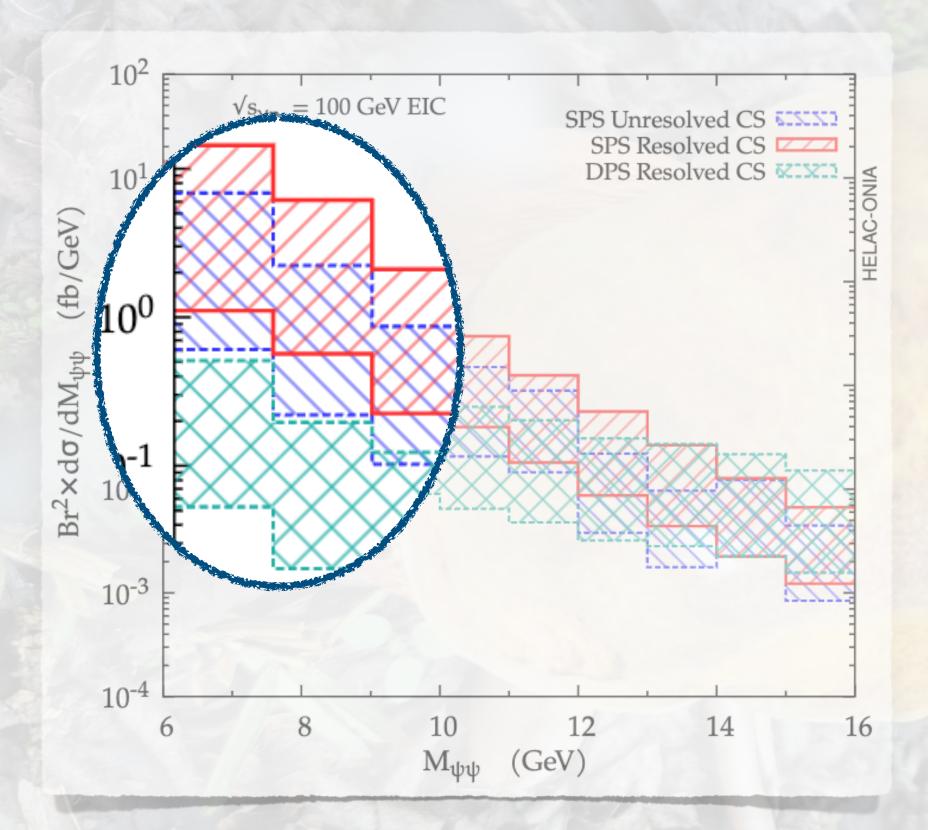






#### Invariant mass of the $J/\psi$ pair

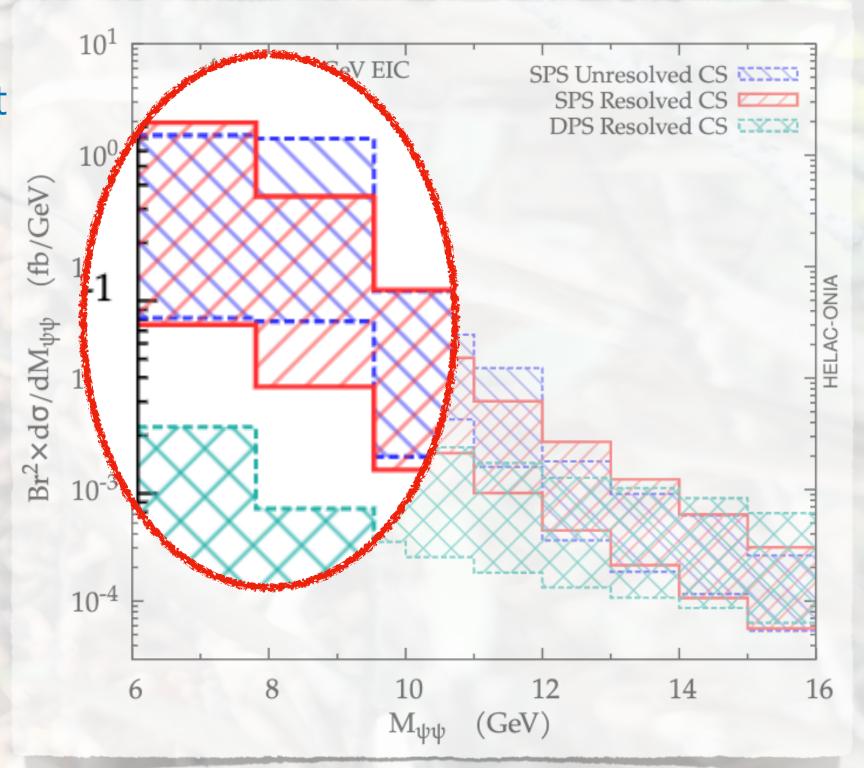
$$\sqrt{s_{\gamma p}} = 100 \text{ GeV}$$





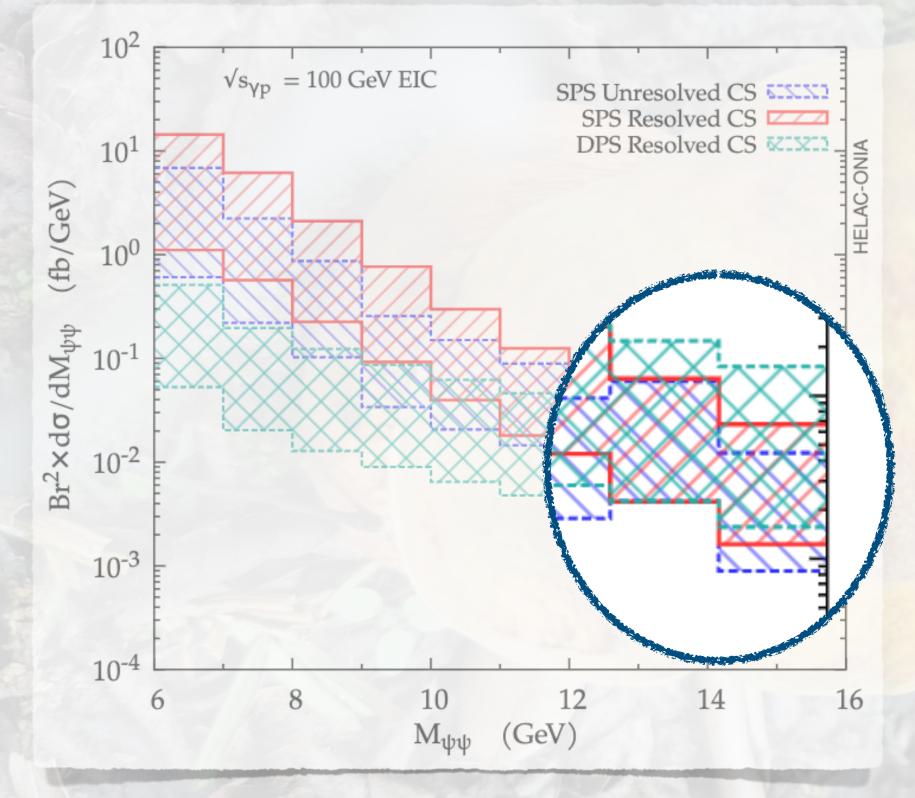
- DPS smaller then SPS, but not negligible
- DPS negligible





#### Invariant mass of the $J/\psi$ pair

$$\sqrt{s_{\gamma p}} = 100 \text{ GeV}$$



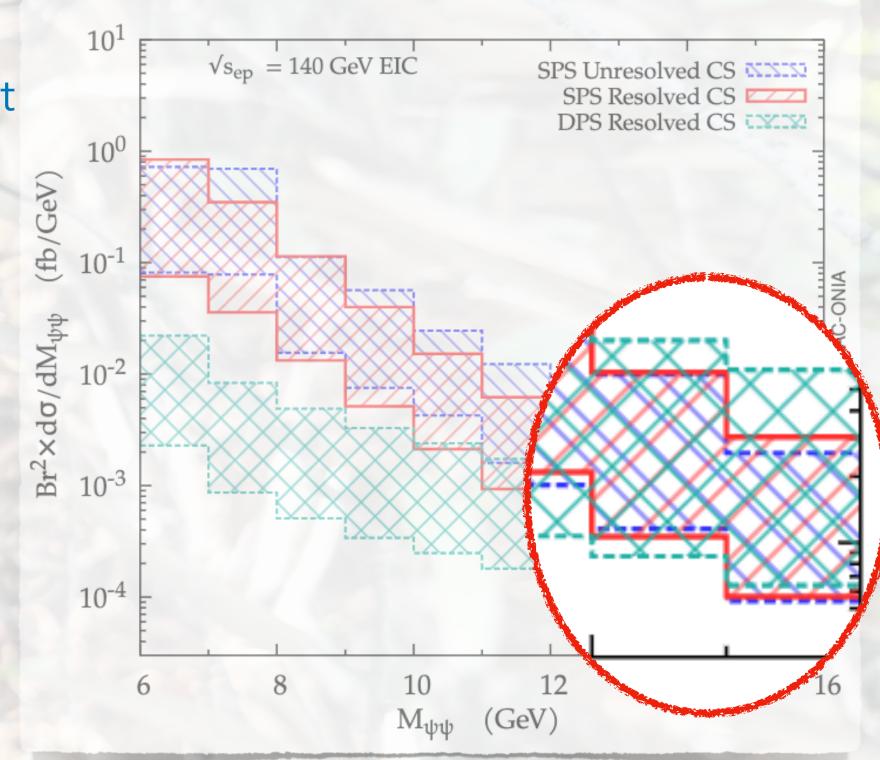
#### a) at low invariant mass:

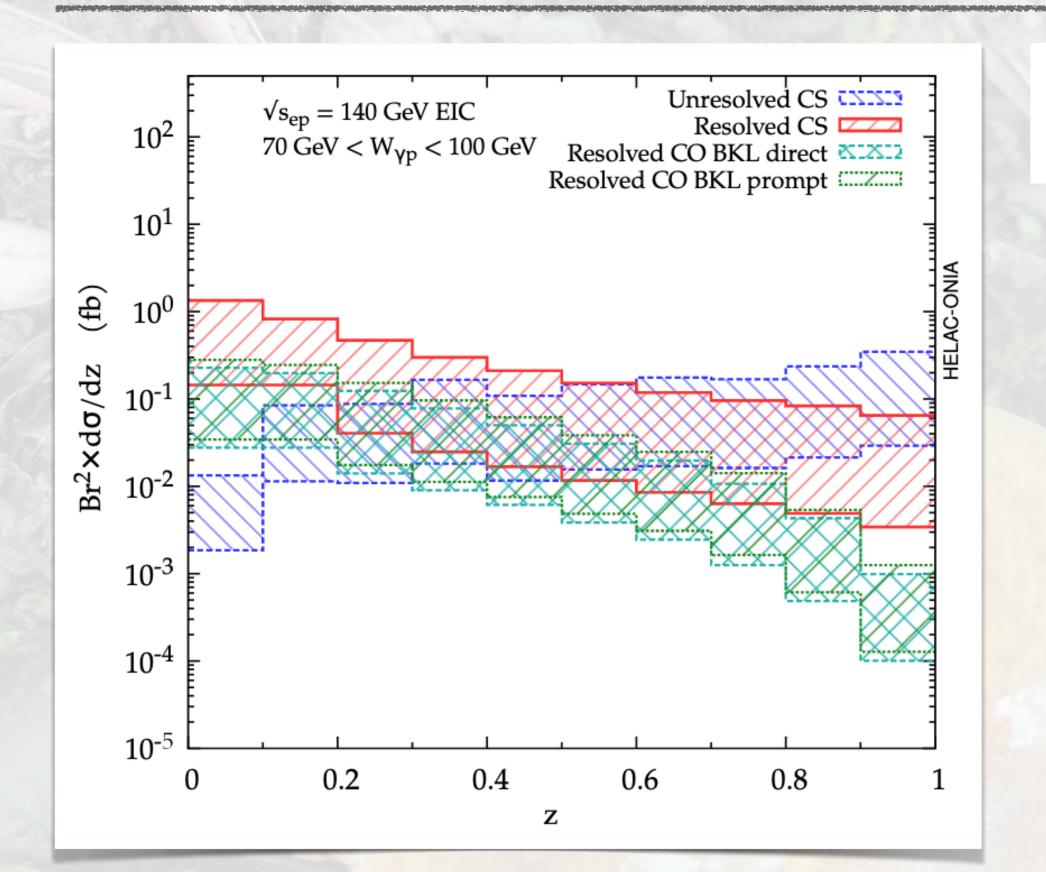
- DPS smaller then SPS, but not negligible
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#### b) at low invariant mass:

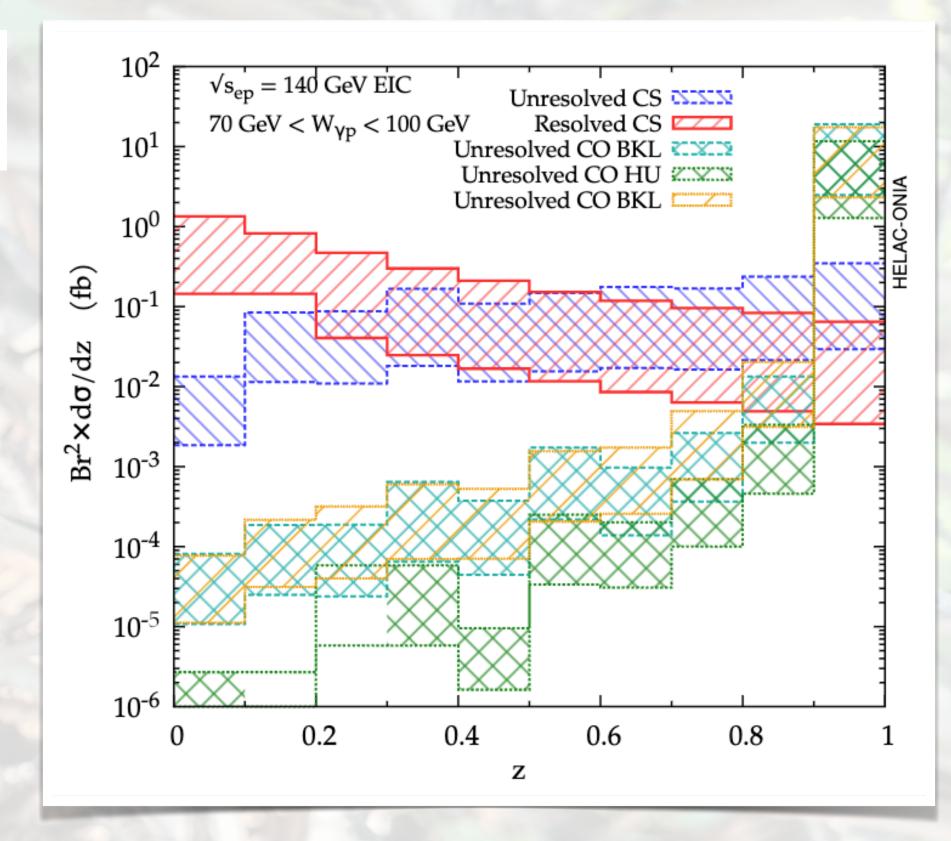
- DPS bigger then SPS
- DPS similar to SPS







$$\mu_0 = \frac{H_T}{2} = \frac{\sum_i \sqrt{p_{Ti}^2 + m_i^2}}{2}$$



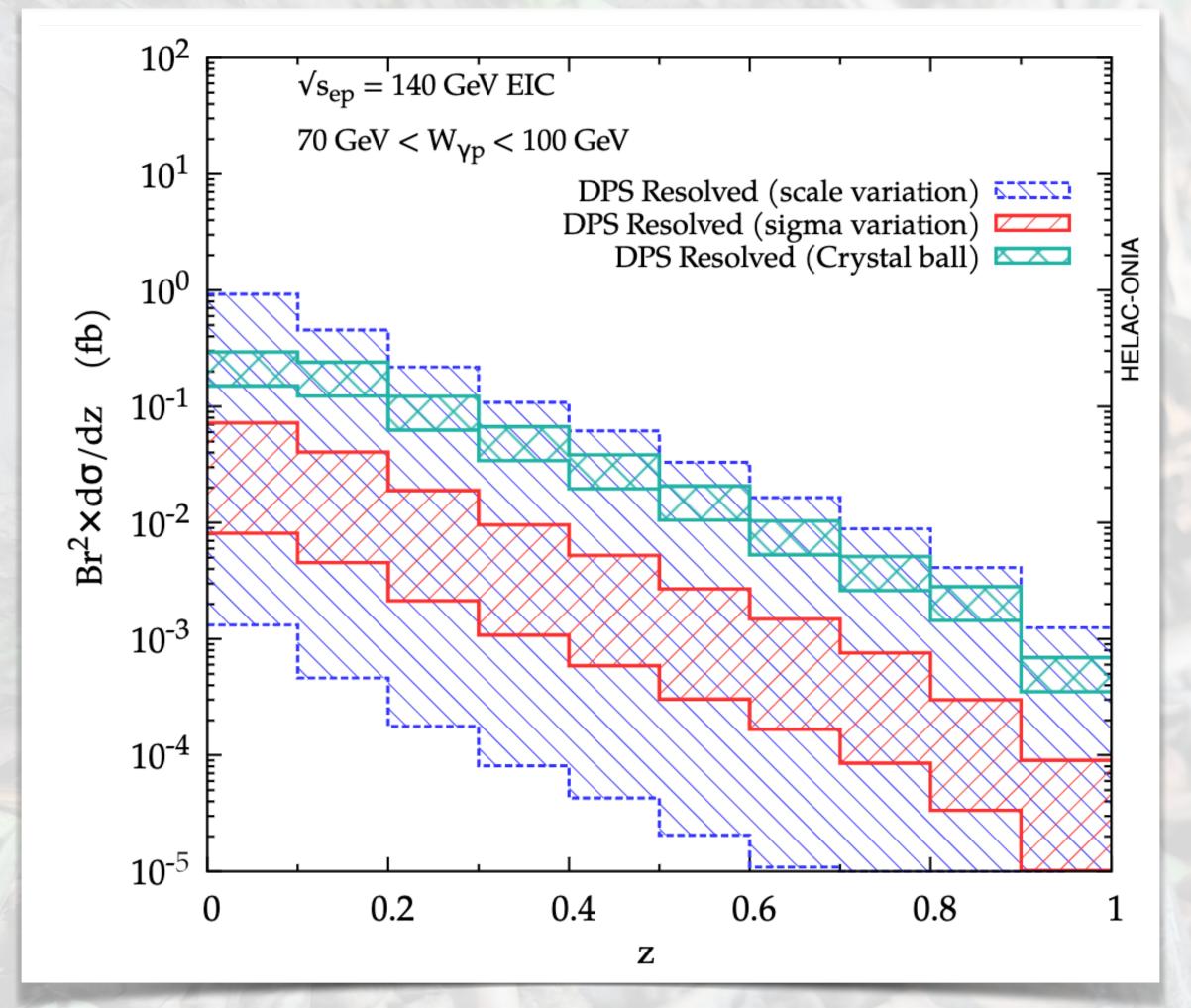
CO negligible if we cut z<0.9 (to be checked)

#### Numerical Results



F. A. Ceccopieri, H. S. Shao, J. P. Lansberg, M. R. and R. Sangem in prep.

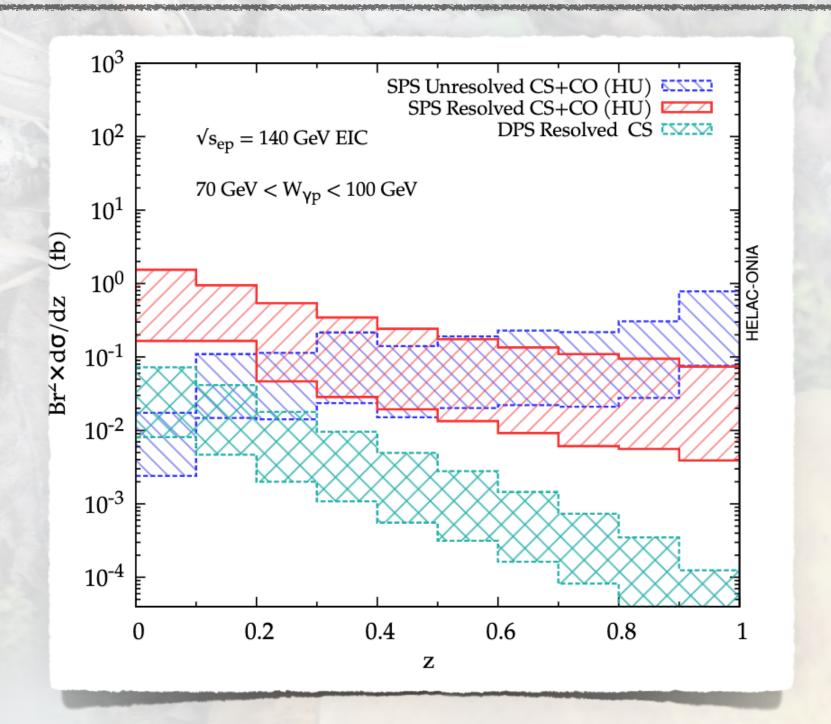
We also considered to use the Crystal Ball parametrization of the square of the amplitude  $gg \longrightarrow Q + X$ 

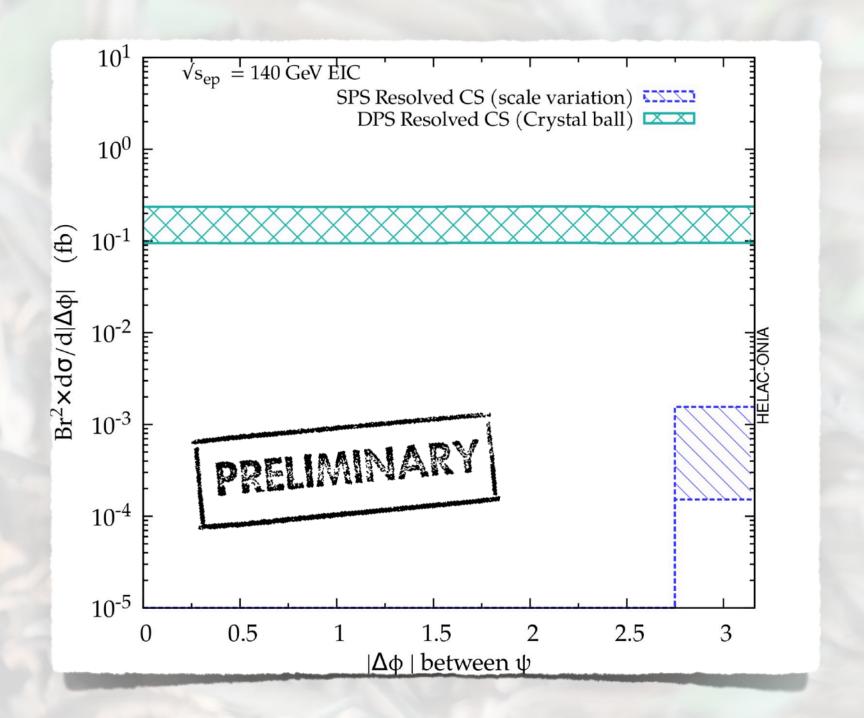


$$\frac{\lambda^{2} \kappa \hat{s}}{M_{\mathcal{Q}}^{2}} \exp(-\kappa \frac{P_{T}^{2}}{M_{\mathcal{Q}}^{2}}) \qquad \text{when } P_{T} \leq \langle P_{T} \rangle$$

$$\frac{\lambda^{2} \kappa \hat{s}}{M_{\mathcal{Q}}^{2}} \exp(-\kappa \frac{\langle P_{T} \rangle^{2}}{M_{\mathcal{Q}}^{2}}) \left(1 + \frac{\kappa}{n} \frac{P_{T}^{2} - \langle P_{T} \rangle^{2}}{M_{\mathcal{Q}}^{2}}\right)^{-n} \qquad \text{when } P_{T} > \langle P_{T} \rangle$$

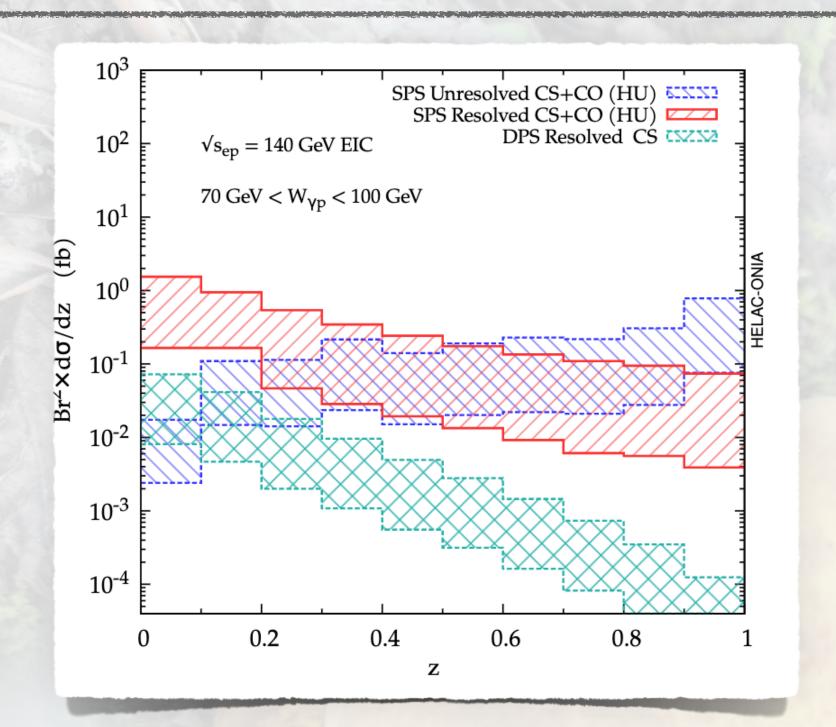
F. A. Ceccopieri, H. S. Shao, J. P. Lansberg, M. R. and R. Sangem in prep.

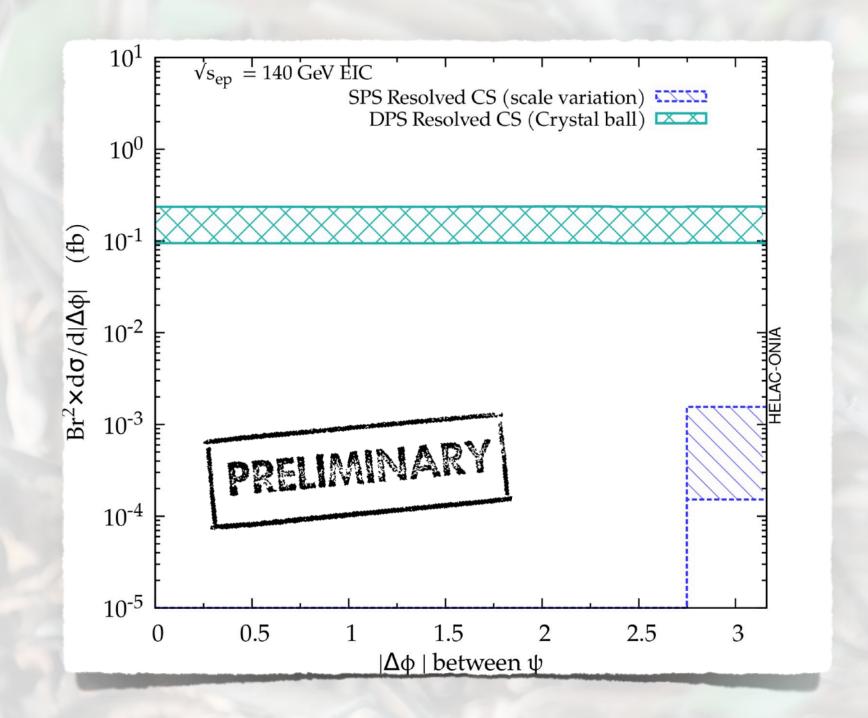




\* for z<0.1, SPS resolved dominates — unique opportunity to investigate the PHOTON structure

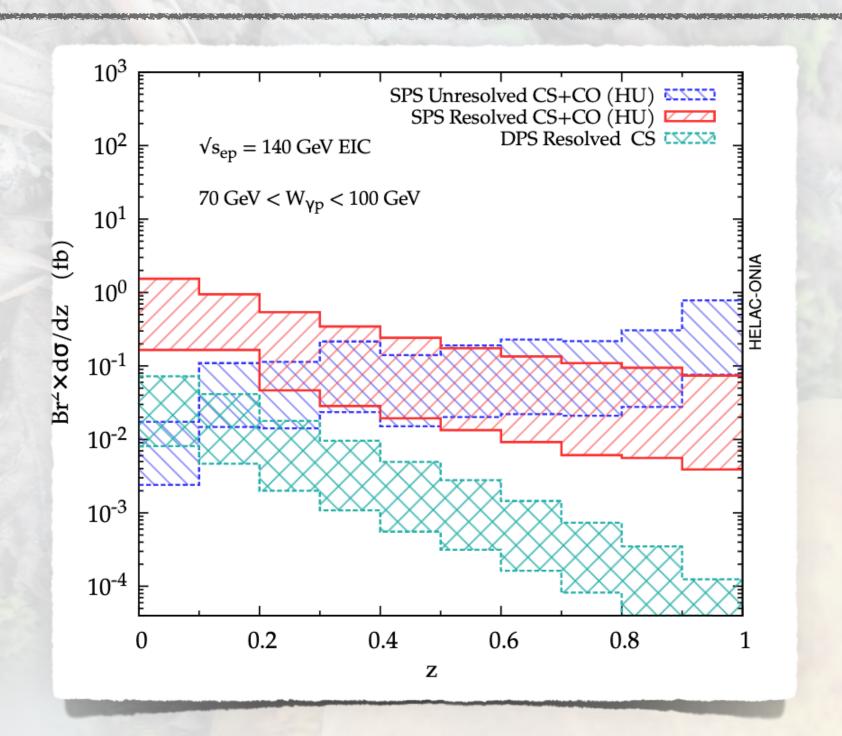
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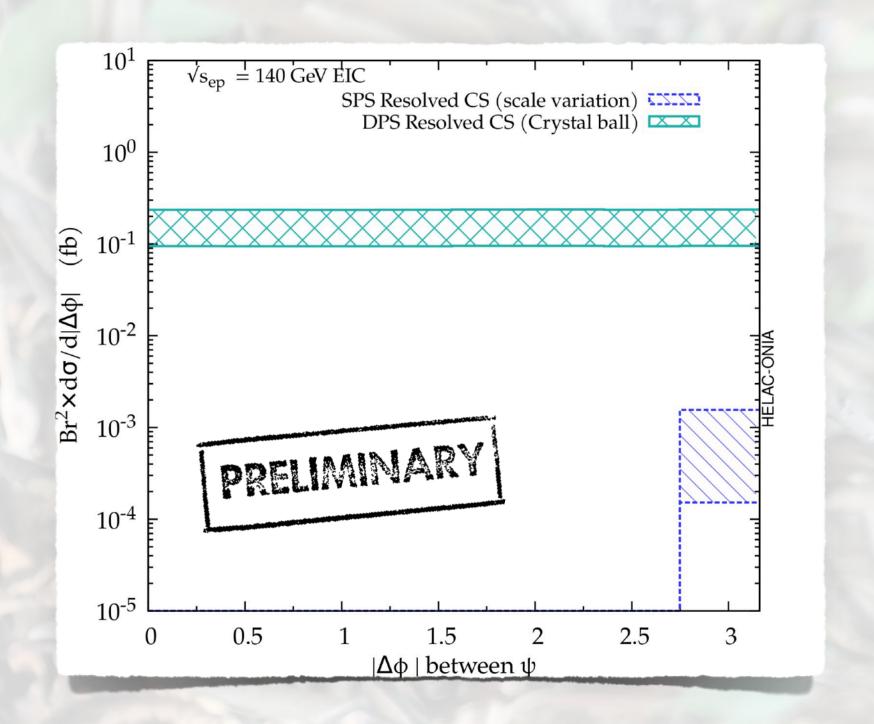




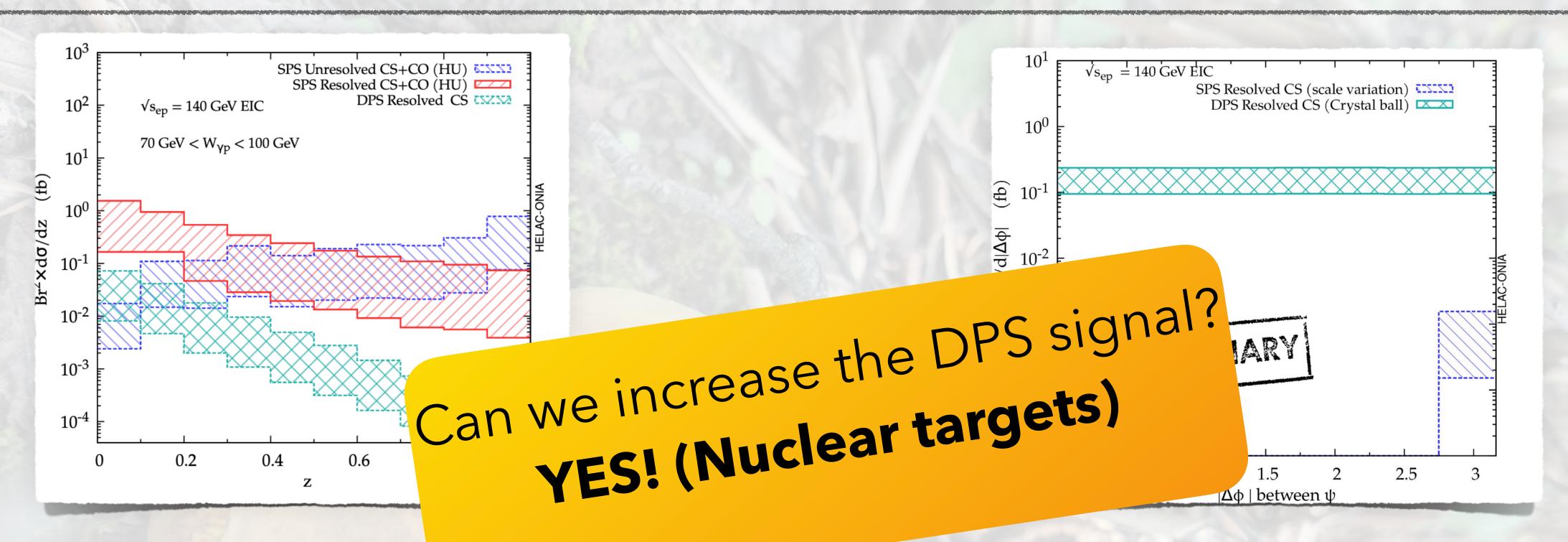
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F. A. Ceccopieri, H. S. Shao, J. P. Lansberg, M. R. and R. Sangem in prep.





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- \* as for DPS studies @LHC, the cross-section dependence on the relative azimuthal angle is relevant to access the DPS contribution!



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For DPS in pA and AA collisions the following references were missing:

- 1)Same-sign WW production in proton-nucleus collisions at the LHC as a signal for double parton scattering D. d'E. & A. Snigirev, PLB 718 (2013) 1395-1400
- 2)Enhanced J/ΨJ/\PsiJ/Ψ-pair production from double parton scatterings in nucleus-nucleus collisions at the Large Hadron Collider D. d'E. & A. Snigirev, PLB 727 (2013) 157-162
- 3)Pair production of quarkonia and electroweak bosons from double-parton scatterings in nuclear collisions at the LHC D. d'E. & A. Snigirev, Nucl. Phys. A 931 (2014) 303-308

and for TPS:

Triple-parton scatterings in proton-nucleus collisions at high energies

D. d'E. & A. Snigirev, EPJC 78 (2018) 5, 359

$$egin{aligned} \mathsf{F}_{\mathsf{a}_1\mathsf{a}_2}(\mathsf{x}_1,\mathsf{x}_2,\mathsf{f y}_\perp) &= 2p^+ \int rac{dz_1^-}{2\pi} rac{dz_2^-}{2\pi} dy^- e^{i\left(x_1z_1^- + x_2z_2^-
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angle \end{aligned}$$

In this case we have two mechanisms that contribute:

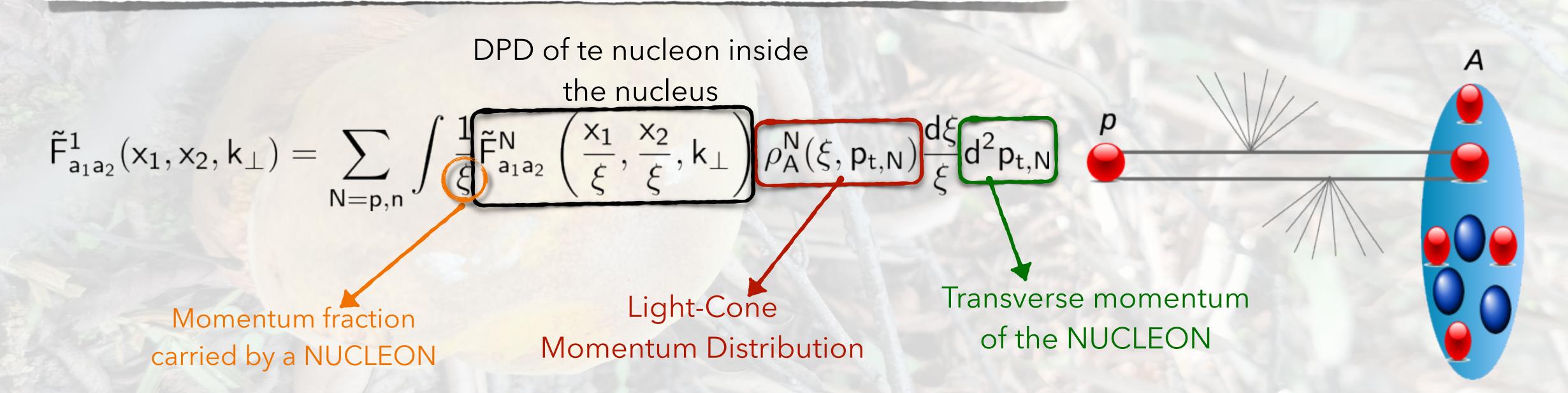
Matteo Rinaldi

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B. Blok et al, EPJC (2013) 73:2422

**DPS 1**: The two partons belong to the SAME nucleon in the nucleus!

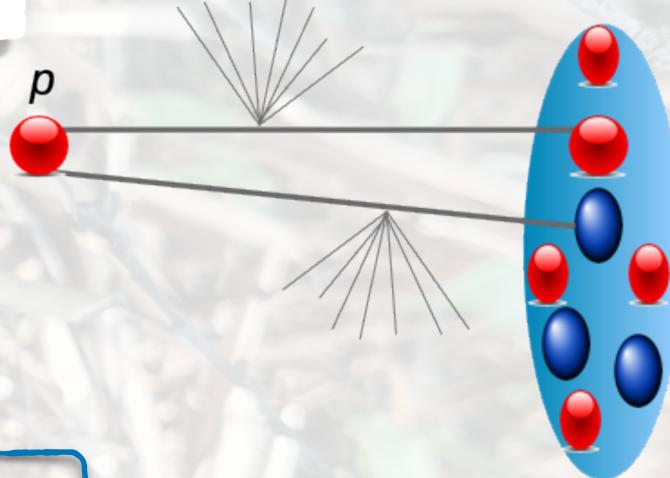


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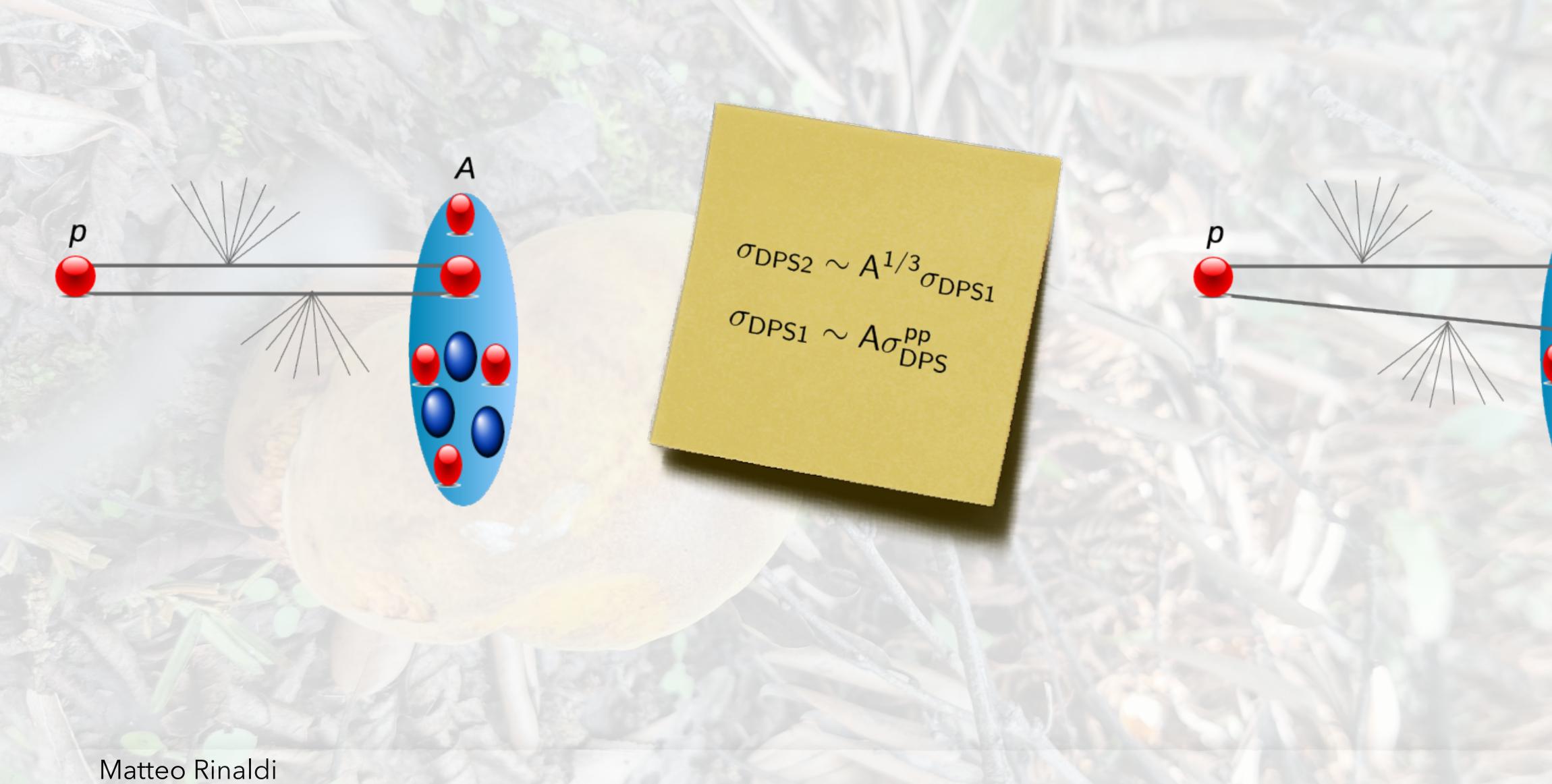
**DPS 2**: The two partons belong to the DIFFERENT nucleons in the nucleus!



$$\begin{split} \tilde{\textbf{F}}_{a_{1}a_{2}}^{2}(\textbf{x}_{1},\textbf{x}_{2},\vec{\textbf{k}}_{\perp}) \propto & \int \frac{1}{\xi_{1}\xi_{2}} \prod_{i=1}^{i=A} \frac{d\xi_{i}d^{2}p_{ti}}{\xi_{i}} \delta \Bigg( \sum_{i} \xi_{i} - A \Bigg) \delta^{(2)} \Bigg( \sum_{i} \mathbf{p}_{ti} \Bigg) \psi_{A}^{*}(\xi_{1},\xi_{2},p_{t1},p_{t2},\ldots) \\ & \times \psi_{A} \Big( \xi_{1},\xi_{2},p_{t1} + \vec{\textbf{k}}_{\perp},p_{t2} - \vec{\textbf{k}}_{\perp},\ldots \Big) G_{a_{1}}^{N_{1}} \Big( \textbf{x}_{1}/\xi_{1},|\vec{\textbf{k}}_{\perp}| \Big) G_{a_{2}}^{N_{2}} \Big( \textbf{x}_{2}/\xi_{2},|\vec{\textbf{k}}_{\perp}| \Big) \end{split}$$

Nucleus wf

Nucleon GPD



61

# DPS in $\gamma$ A collisions with light nuclei?

M.R. in progress

In p-Pb collisions there are some difficulties (personal view):

- 1) both cross-sections (DPS1 and DPS2) depends on proton DPD (still almost unknown) therefore both mechanisms are very important could be difficult to extract some information on the proton DPD
- 2) for heavy nuclei is difficult to perform calculation with wave-function obtained from realistic potentials

Matteo Rinaldi

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Matteo Rinaldi

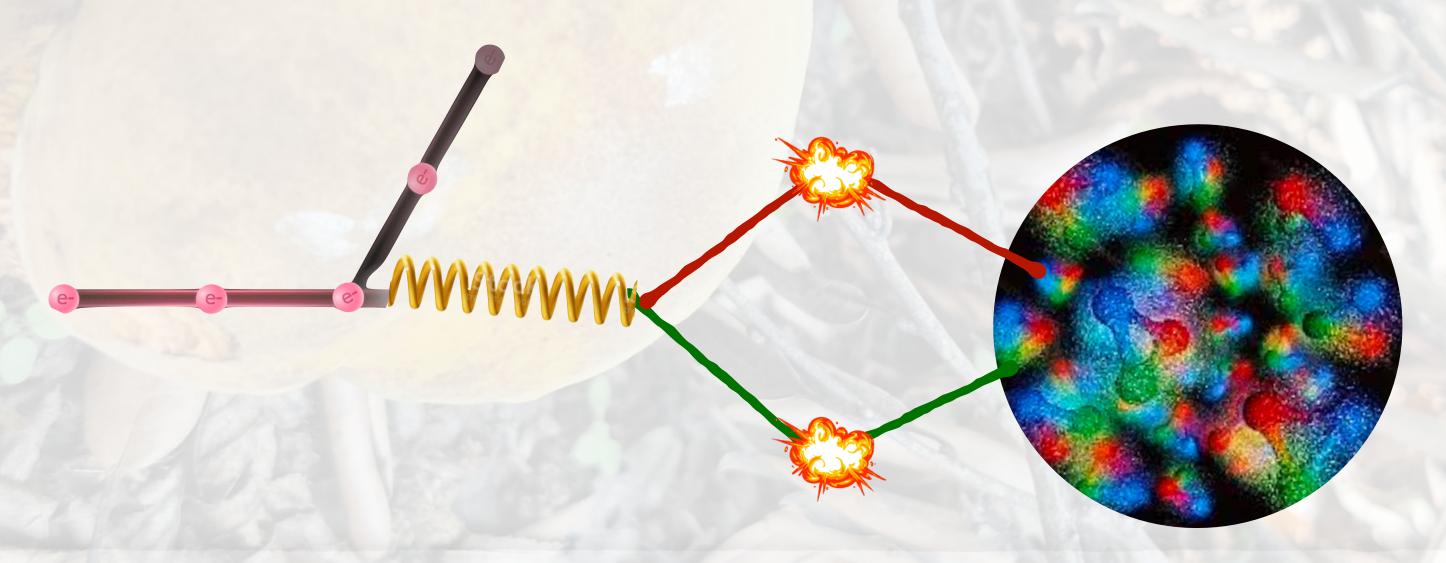
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#### **POSSIBLE SOLUTION?**



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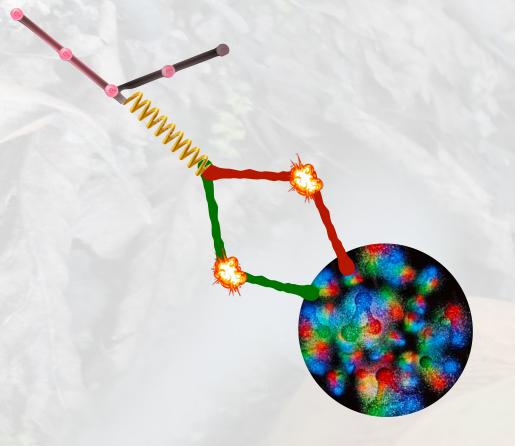
- 1) both cross-sections (DPS1 and DPS2) depends on proton DPD (still almost unknown) therefore both mechanisms are very important could be difficult to extract some information on the proton DPD
- 2) for heavy nuclei is difficult to perform calculation with wave-function obtained from realistic potentials

#### **POSSIBLE SOLUTION?**

- 1) In γA the DPS2 will not contain any DPD of the proton this mechanism can now be viewed as a background that can be evaluated if we properly treat the photon (as previously discussed) and the Nuclear geometry
- 2) For light nuclei these calculations can be done starting from realistic wave-function (Av18 or chiral potential)!

Could we access the DPD of bound nucleons? Double EMC effect?

For example in DPS1:



$$\tilde{F}_{a_{1}a_{2}}^{1}(x_{1},x_{2},k_{\perp}) = \sum_{N=p,n} \int \frac{1}{\xi} \tilde{F}_{a_{1}a_{2}}^{N} \left( \frac{x_{1}}{\xi}, \frac{x_{2}}{\xi}, k_{\perp} \right) \boxed{\rho_{A}^{N}(\xi,p_{t,N})} \frac{d\xi}{\xi} d^{2}p_{t,N}$$

The nuclear light-cone distribution can be evaluated with realistic wave-function (from Av18 +UIV potential) in a fully relativistic and Poincaré covariant approach for:

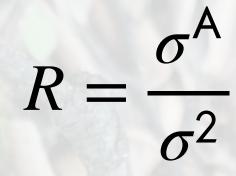
- 1) H<sup>2</sup> in E. Pace and G. Salmé, TNPI2000 (2001), arXiv:nucl-th/0106004
- 2) He³ in e.g. A. Del Dotto et al, PRC 95, 014001 (2017), M.R. et al, PLB 839 (2023), 137810
- 3) He4 from F. Fornetti, E. Pace, M. R., G. Salmé, S. Scopetta and M. Viviani, PLB 851 (2024) 138587

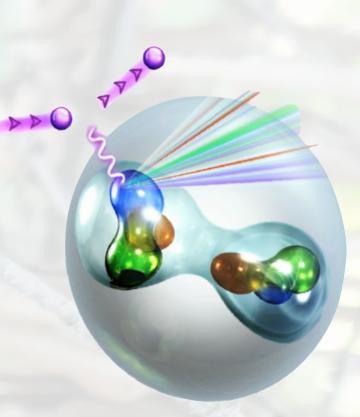
Let us consider DIS processes of nuclei at high energies and evaluate the ratio of cross-sections of different nuclei (A = generic nucleus with A nucleons, 2 = deuteron):

$$R = \frac{\sigma^{\mathsf{A}}}{\sigma^2}$$

since the energy binding per nucleon is few MeV

Let us consider DIS processes of nuclei at high energies and evaluate the ratio of cross-sections of different nuclei (A = generic nucleus with A nucleons, 2 = deuteron):





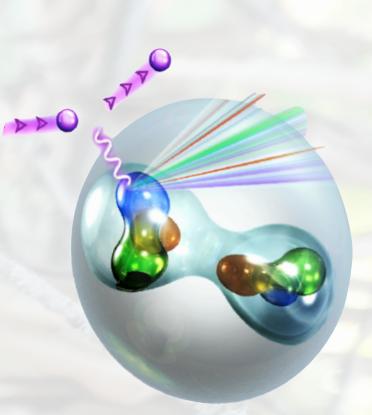
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$$R \sim 1$$

Let us consider DIS processes of nuclei at high energies and evaluate the ratio of cross-sections of different nuclei (A = generic nucleus with A nucleons, 2 = deuteron):

 $R = \frac{\sigma^{A}}{\sigma^{2}} \sim \frac{\mathsf{F}_{2}^{A}(\mathsf{x})}{\mathsf{F}_{2}^{2}(\mathsf{x})}$ 

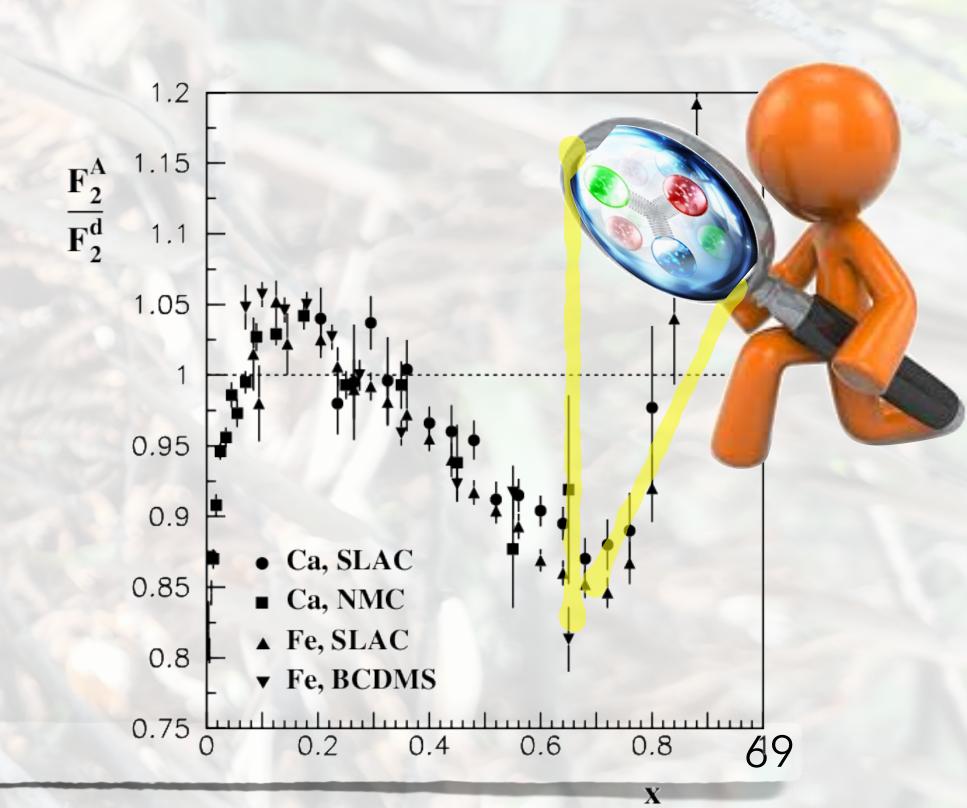
Nucleare Structure Function



since the energy binding per nucleon is few MeV

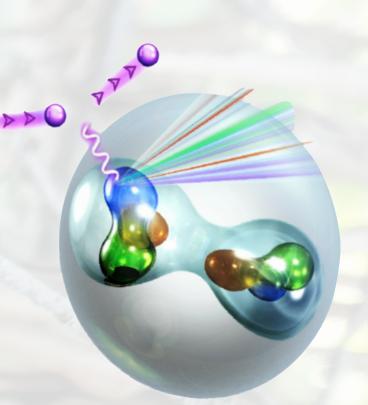
 $R \sim 1$ 

We found a 10% effect! Why?



Let us consider DIS processes of nuclei at high energies and evaluate the ratio of cross-sections of different nuclei (A = generic nucleus with A nucleons, 2 = deuteron):

Nucleare Structure Function



since the energy binding per nucleon is few MeV

 $R \sim 1$ 

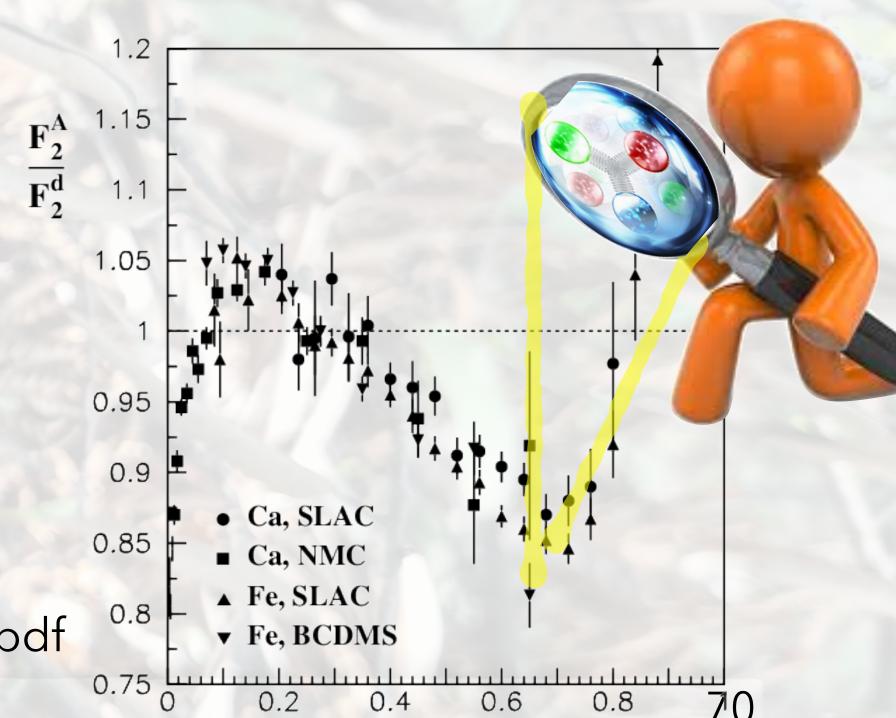
We found a 10% effect! Why?

#### **EMC** = European Muon Collaboration **Everyone's Model is Cool**

that "EMC means Everyone's Model is Cool". It is interesting to note that none of the earliest models were that concerned with the role of two-nucleon correlations, except in relation to six-quark bags.

https://cds.cern.ch/record/1734943/files/vol53-issue4-p035-e.pdf

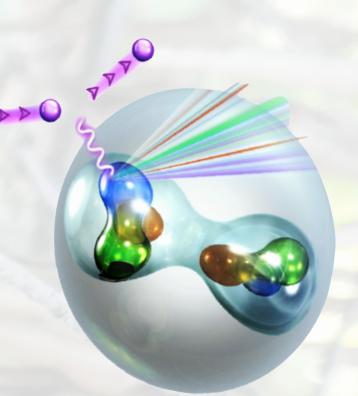
Matteo Rinaldi



Let us consider DIS processes of nuclei at high energies and evaluate the ratio of cross-sections of different nuclei (A = generic nucleus with A nucleons, 2 = deuteron):

DPS can help!!

Nucleare Structure **Function** 



since the energy binding per nucleon is few Me

 $R \sim 1$ 

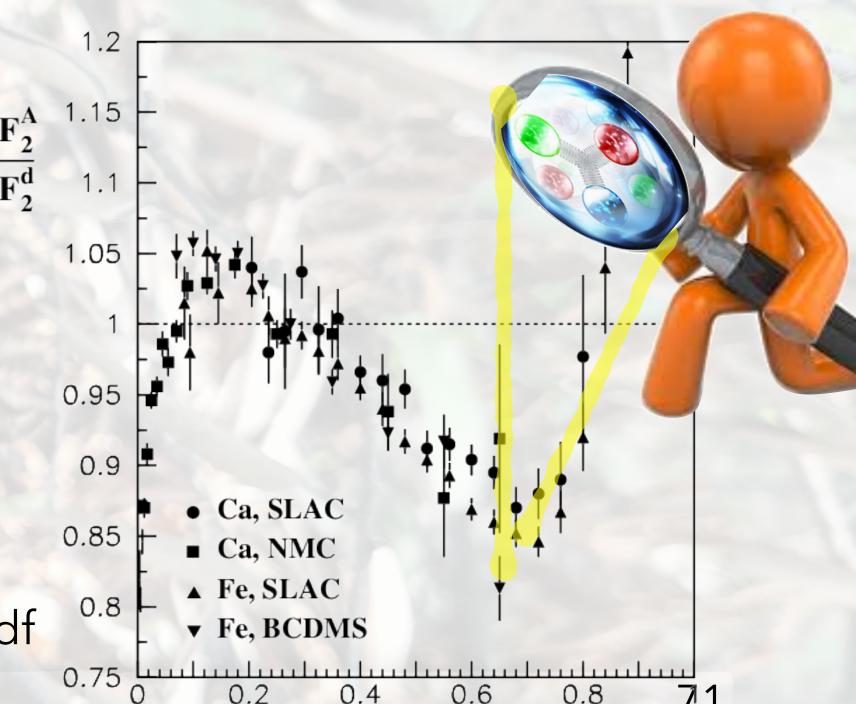
We found a 10% effect! Why?

Matteo Rinaldi

**EMC** = **European Muon Everyone's Model is Cool** 

that "EMC means Everyone's Model is Cool". It is interesting to note that none of the earliest models were that concerned with the role of two-nucleon correlations, except in relation to six-quark bags.

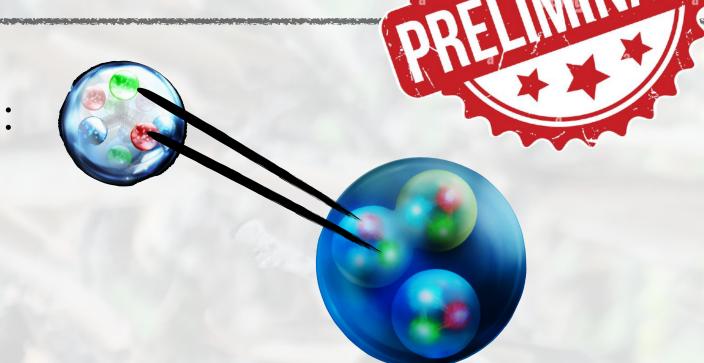
https://cds.cern.ch/record/1734943/files/vol53-issue4-p035-e.pdf



#### DPS1 and double EMC effect

Nuclear DPD (in momentum space) corresponding to the DPS1 mechanism:

$$\tilde{F}^1_{a_1a_2}(x_1,x_2,k_\perp) = \sum_{N=p,n} \int \frac{1}{\xi} \tilde{F}^N_{a_1a_2} \left( \frac{x_1}{\xi}, \frac{x_2}{\xi}, k_\perp \right) \boxed{\rho^N_A(\xi,p_{t,N})} \frac{d\xi}{\xi} d^2 p_{t,N}$$



The nuclear light-cone distribution can be evaluated with realistic wave-function (from Av18 +UIV potential) for light nuclei and modeled for heavy ions.

We can define the double structure functions (dSF):

$$F^{2,A}(x_1, x_2) \equiv \sum_{ij} e_i^2 e_j^2 x_1 x_2 \tilde{F}_{ij}^1(x_1, x_2, 0)$$

We can generalize the EMC ratio: 
$$R_{EMC}^{A}(x) = \frac{F_2^{A}(x)}{A} \frac{2}{F_2^{2}(x)}$$
  $R_{2EMC}^{A}(x_1, x_2) = \frac{F^{2,A}(x_1, x_2)}{A} \frac{2}{F^{2,2}(x_1, x_2)}$ 

 $^{2}H$ 

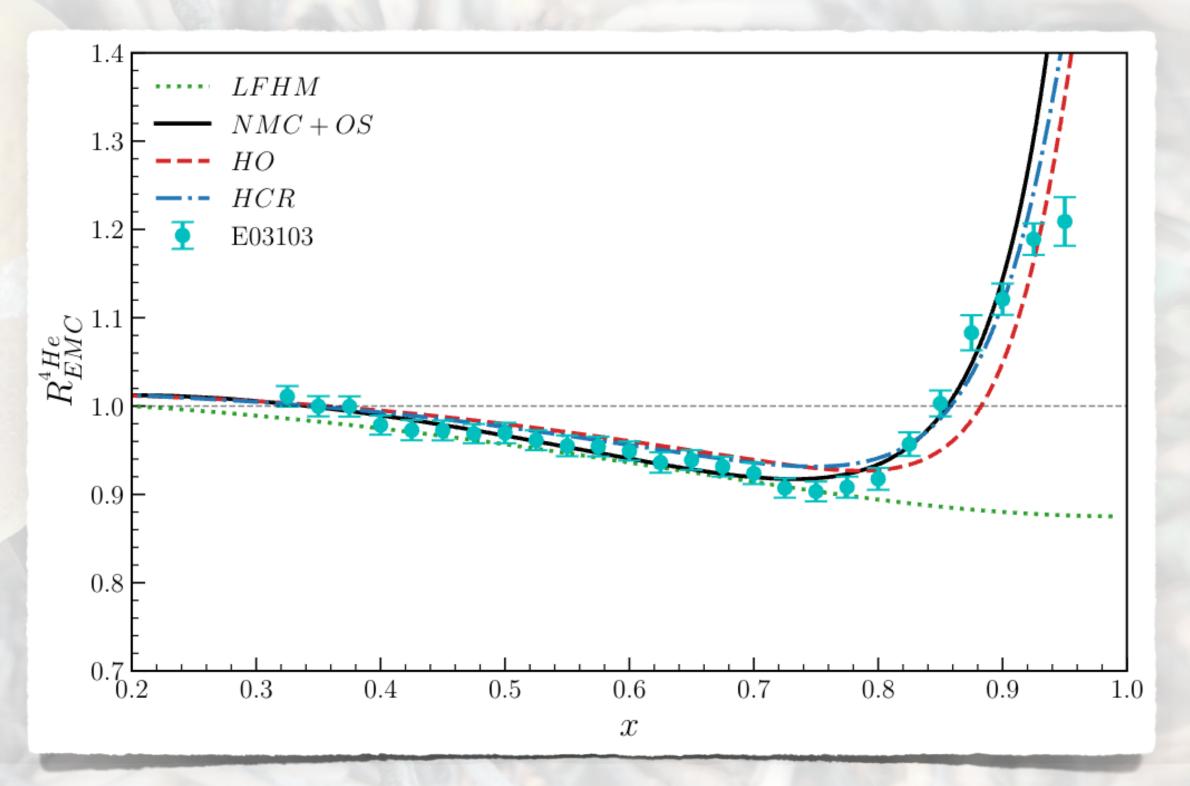
Nuclear DPD (in momentum space) corresponding to the DPS1 mechanism:

$$\tilde{F}_{a_1a_2}^1(x_1,x_2,k_\perp) = \sum_{N=p,n} \int \frac{1}{\xi} \tilde{F}_{a_1a_2}^N \left(\frac{x_1}{\xi},\frac{x_2}{\xi},k_\perp\right) \boxed{\rho_A^N(\xi,p_{t,N})} \frac{d\xi}{\xi} d^2p_{t,N}$$



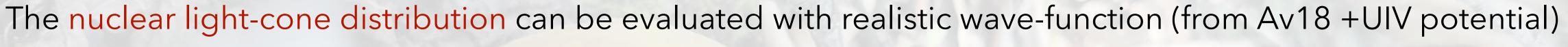
for light nuclei and modeled for heavy ions.

$$R_{EMC}^{A}(x) = \frac{F_2^{A}(x)}{A} \frac{2}{F_2^{2}(x)}$$



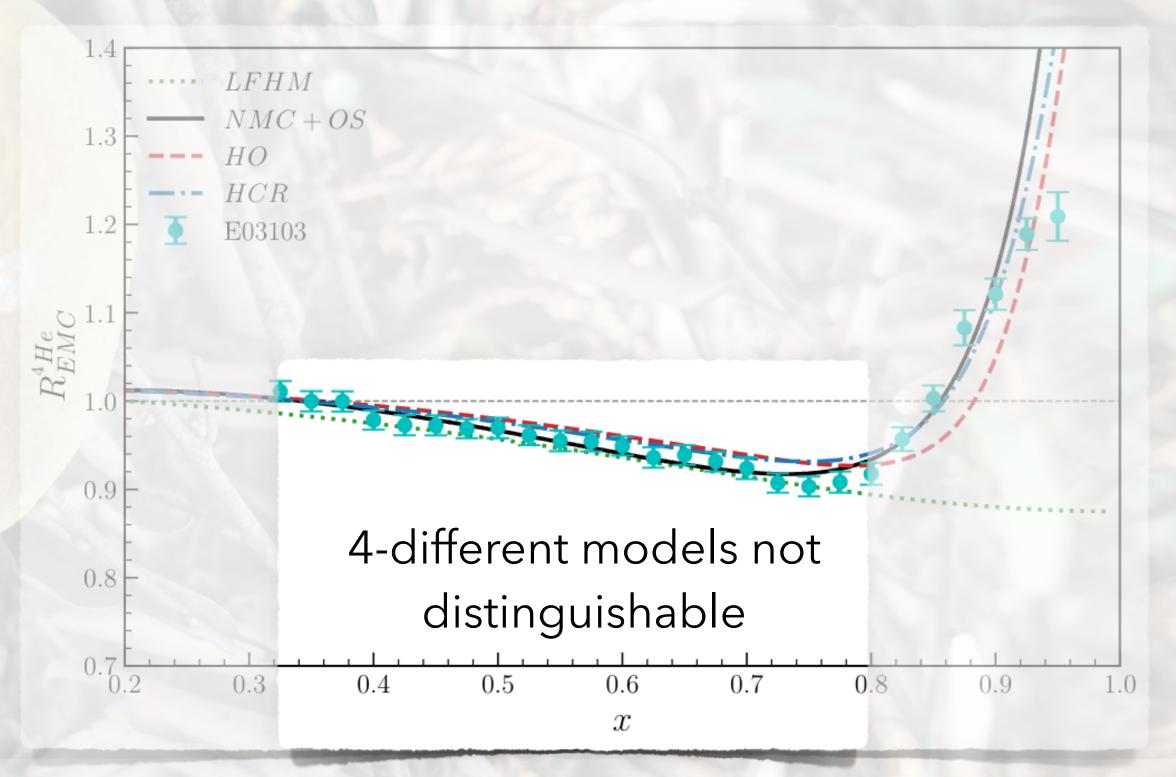
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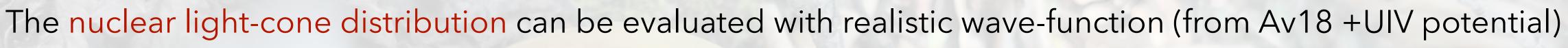
for light nuclei and modeled for heavy ions.

$$R_{EMC}^{A}(x) = \frac{F_2^{A}(x)}{A} \frac{2}{F_2^{2}(x)}$$



Nuclear DPD (in momentum space) corresponding to the DPS1 mechanism:

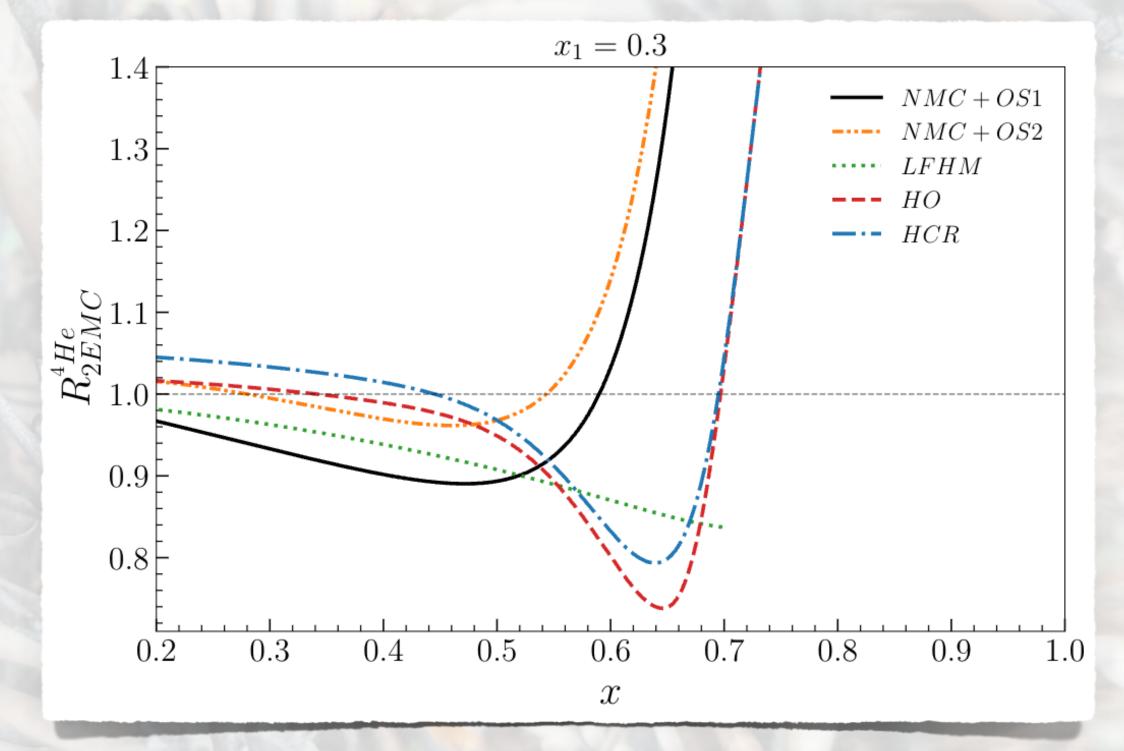
$$\tilde{F}_{a_1a_2}^1(x_1,x_2,k_\perp) = \sum_{N=p,n} \int \frac{1}{\xi} \tilde{F}_{a_1a_2}^N \left(\frac{x_1}{\xi},\frac{x_2}{\xi},k_\perp\right) \boxed{\rho_A^N(\xi,p_{t,N})} \frac{d\xi}{\xi} d^2p_{t,N}$$



for light nuclei and modeled for heavy ions.

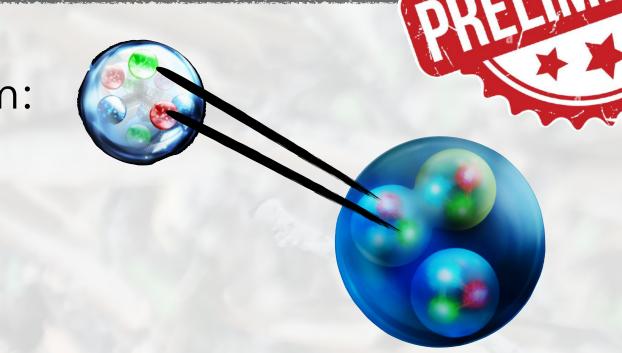
$$R_{2EMC}^{A}(x_1, x_2) = \frac{F^{2,A}(x_1, x_2)}{A} \frac{2}{F^{2,2}(x_1, x_2)}$$

4-different models leading different results!!



Nuclear DPD (in momentum space) corresponding to the DPS1 mechanism:

$$\tilde{F}_{a_1a_2}^1(x_1,x_2,k_\perp) = \sum_{N=p,n} \int \frac{1}{\xi} \tilde{F}_{a_1a_2}^N \left(\frac{x_1}{\xi},\frac{x_2}{\xi},k_\perp\right) \boxed{\rho_A^N(\xi,p_{t,N})} \frac{d\xi}{\xi} d^2p_{t,N}$$



The nuclear light-cone distribution can be evaluated with realistic wave-function (from Av18 +UIV potential) for light nuclei and modeled for heavy ions.

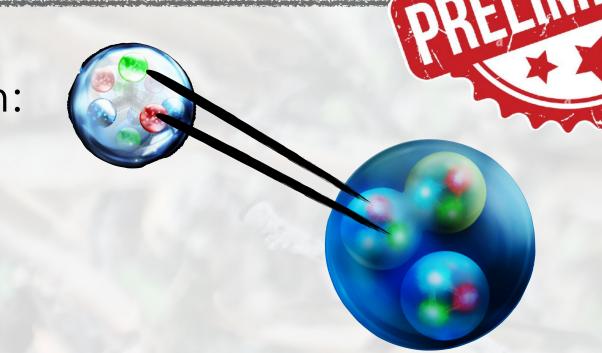
#### Deep inside the nucleon DPDs

Usually, DPDs are studied for Physics of the low x region, i.e.:

$$\tilde{F}_{ij}(x_1, x_2, 0) \sim f_i(x)f_j(x_2)$$

Nuclear DPD (in momentum space) corresponding to the DPS1 mechanism:

$$\tilde{F}_{a_{1}a_{2}}^{1}(x_{1},x_{2},k_{\perp}) = \sum_{N=p,n} \int \frac{1}{\xi} \tilde{F}_{a_{1}a_{2}}^{N} \left( \frac{x_{1}}{\xi}, \frac{x_{2}}{\xi}, k_{\perp} \right) \boxed{\rho_{A}^{N}(\xi,p_{t,N})} \frac{d\xi}{\xi} d^{2}p_{t,N}$$



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#### Deep inside the nucleon DPDs

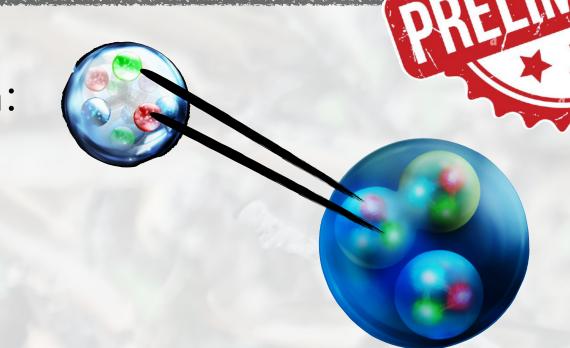
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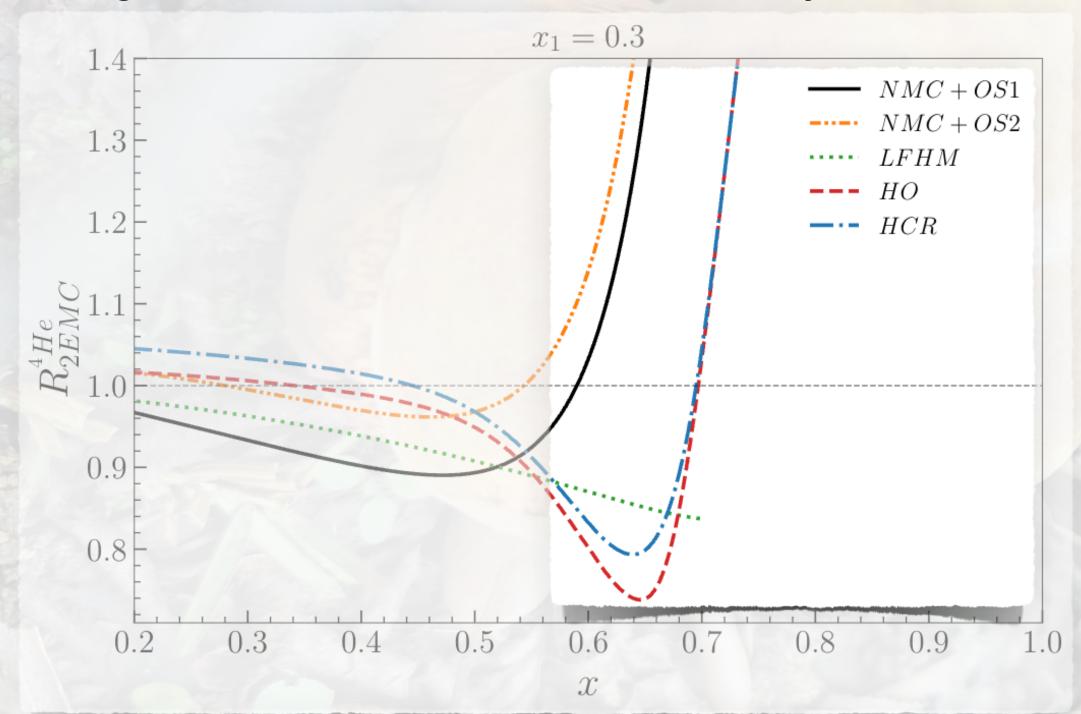
what happens when  $x_1 + x_2 \sim 1$ ? How fast DPDs go to zero? We do not know!

Nuclear DPD (in momentum space) corresponding to the DPS1 mechanism:

$$\tilde{F}_{a_1a_2}^1(x_1,x_2,k_\perp) = \sum_{N=p,n} \int \frac{1}{\xi} \tilde{F}_{a_1a_2}^N \left(\frac{x_1}{\xi},\frac{x_2}{\xi},k_\perp\right) \boxed{\rho_A^N(\xi,p_{t,N})} \frac{d\xi}{\xi} d^2p_{t,N}$$



The nuclear light-cone distribution can be evaluated with realistic wave-function (from Av18 +UIV potential) for light nuclei and modeled for heavy ions.



#### Deep inside the nucleon DPDs

Usually, DPDs are studied for Physics of the low x region, i.e.:

$$\tilde{F}_{ij}(x_1, x_2, 0) \sim f_i(x) f_j(x_2)$$

what happens when  $x_1 + x_2 \sim 1$ ? How fast DPDs go to zero?

This information could be extracted from this ratio! Its rise depends on how proton and neutron DPDs go to zero

Nuclear DPD (in momentum space) corresponding to the DPS1 mechanism:

$$\tilde{F}^1_{a_1 a_2}(x_1, x_2, k_\perp) = \sum_{N=p,n} \int \frac{1}{\xi} \tilde{F}^N_{a_1 a_2} \left( \frac{x_1}{\xi}, \frac{x_2}{\xi}, k_\perp \right) \boxed{\rho_A^N(\xi, p_{t,N})} \frac{d\xi}{\xi} d^2 p_{t,N}$$

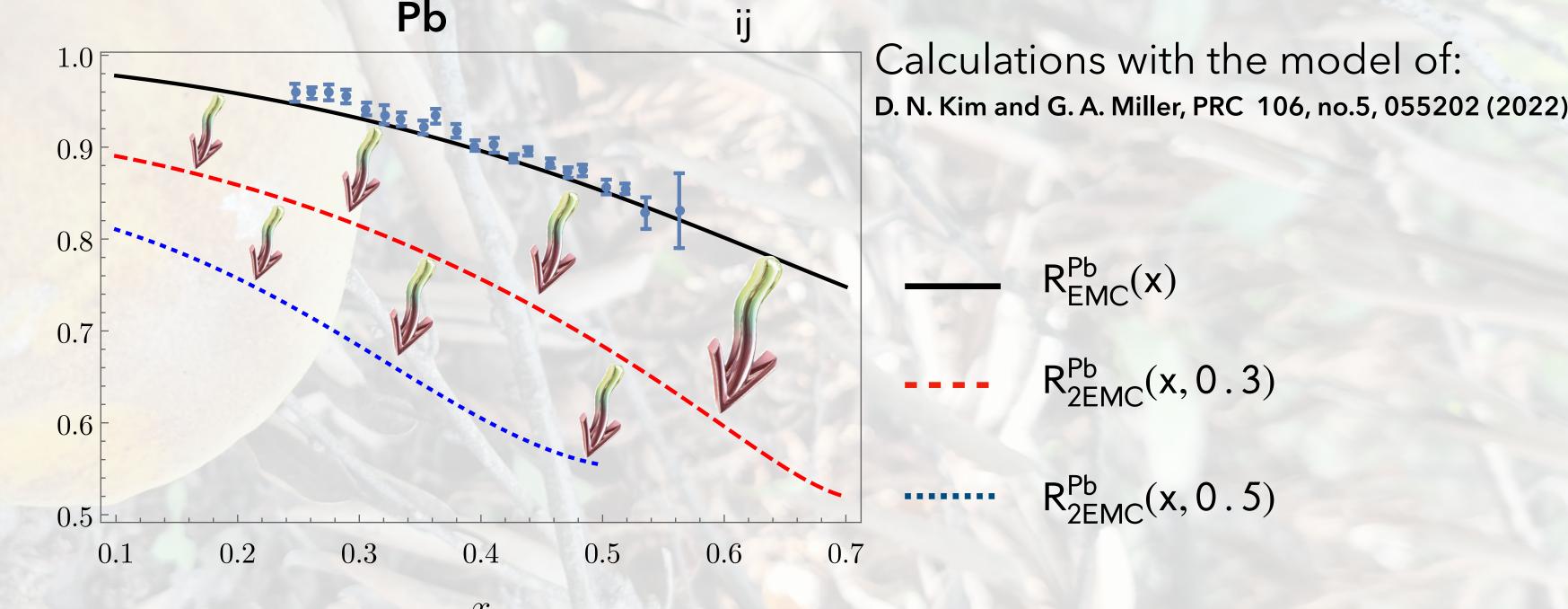


We can generalize the EMC ratio:

We can define the double structure functions (dSF):

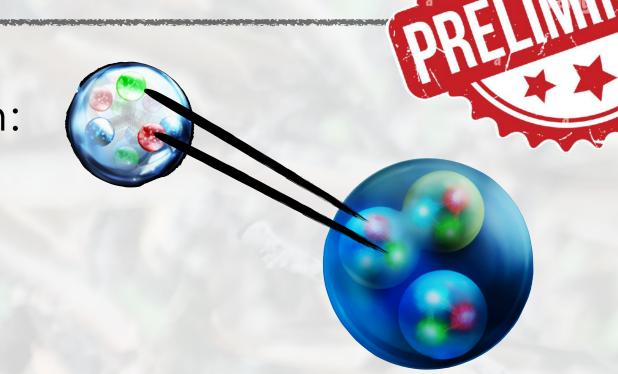
$$R_{EMC}^{A}(x) = \frac{F_2^{A}(x)}{A} \frac{2}{F_2^{2}(x)}$$

$$R_{2EMC}^{A}(x_1, x_2) = \frac{F^{2,A}(x_1, x_2)}{A} \frac{2}{F^{2,2}(x_1, x_2)}$$



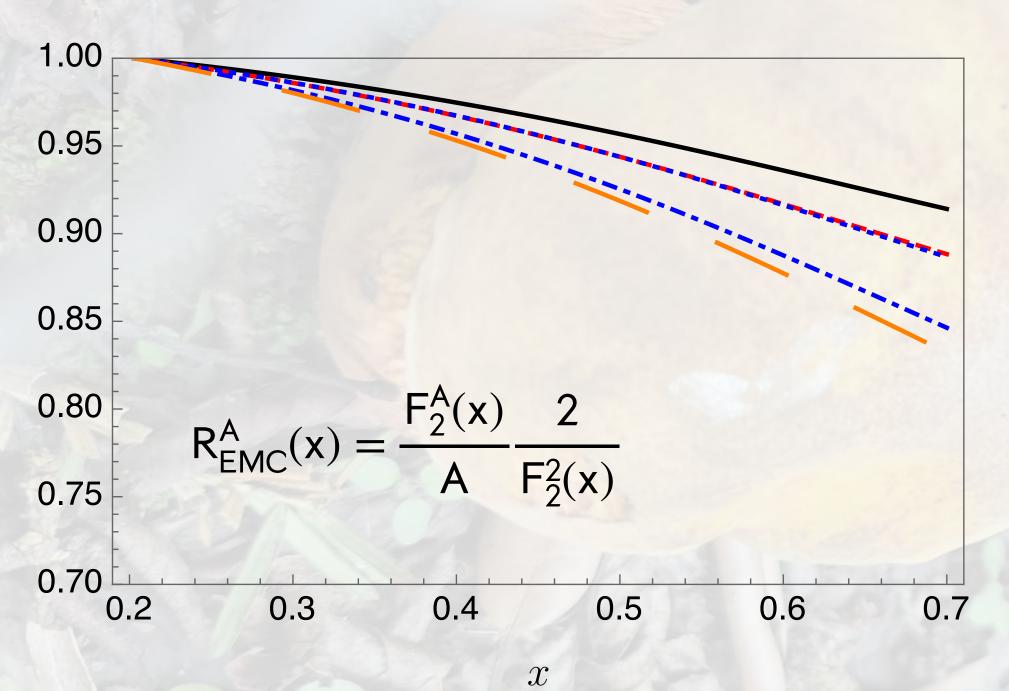
Nuclear DPD (in momentum space) corresponding to the DPS1 mechanism:

$$\tilde{F}_{a_1a_2}^1(x_1,x_2,k_\perp) = \sum_{N=p,n} \int \frac{1}{\xi} \tilde{F}_{a_1a_2}^N \left(\frac{x_1}{\xi},\frac{x_2}{\xi},k_\perp\right) \boxed{\rho_A^N(\xi,p_{t,N})} \frac{d\xi}{\xi} d^2p_{t,N}$$

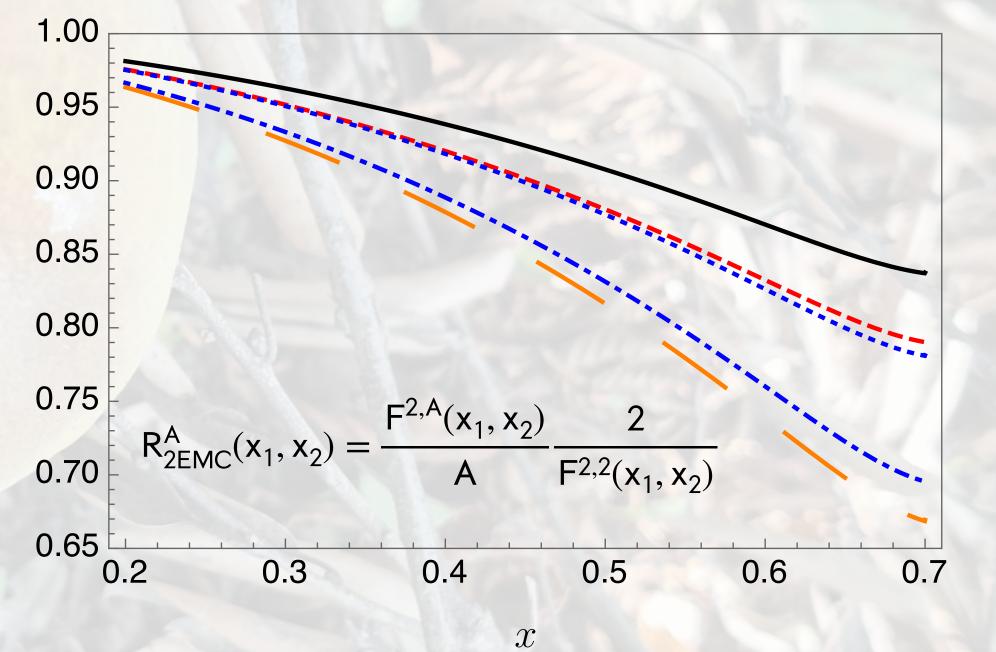


Calculations with the model of:

D. N. Kim and G. A. Miller, PRC 106, no.5, 055202 (2022)



#### Stronger dependence on A in DPS!



<sup>4</sup>He

12**C** 

63**C**u

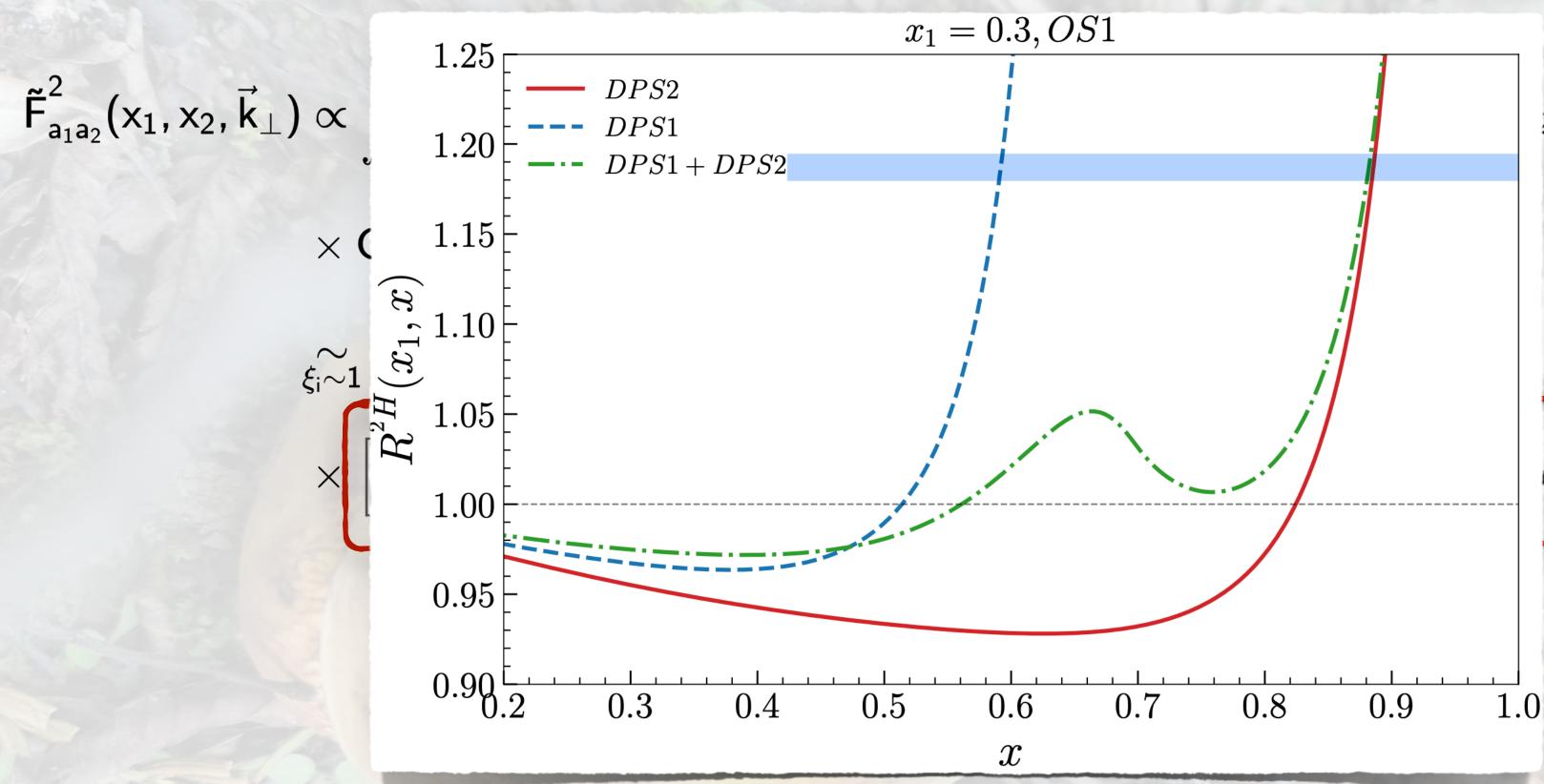
197**A**u

208Pb

Matteo Rinaldi

For example in DPS2:

For example in DPS2:



$$\psi_{\mathsf{A}}\left(\xi_{\mathsf{1}},\xi_{\mathsf{2}},\mathsf{p}_{\mathsf{t}1}+\vec{\mathsf{k}}_{\perp},\mathsf{p}_{\mathsf{t}2}-\vec{\mathsf{k}}_{\perp}\right)$$

$$\psi_{\mathsf{t2}})\psi_{\mathsf{A}}\!\left(\xi_{\mathsf{1}},\xi_{\mathsf{2}},\mathsf{p}_{\mathsf{t1}}+\vec{\mathsf{k}}_{\perp},\mathsf{p}_{\mathsf{t2}}-\vec{\mathsf{k}}_{\perp}
ight)\!\right]$$

For example in DPS2:

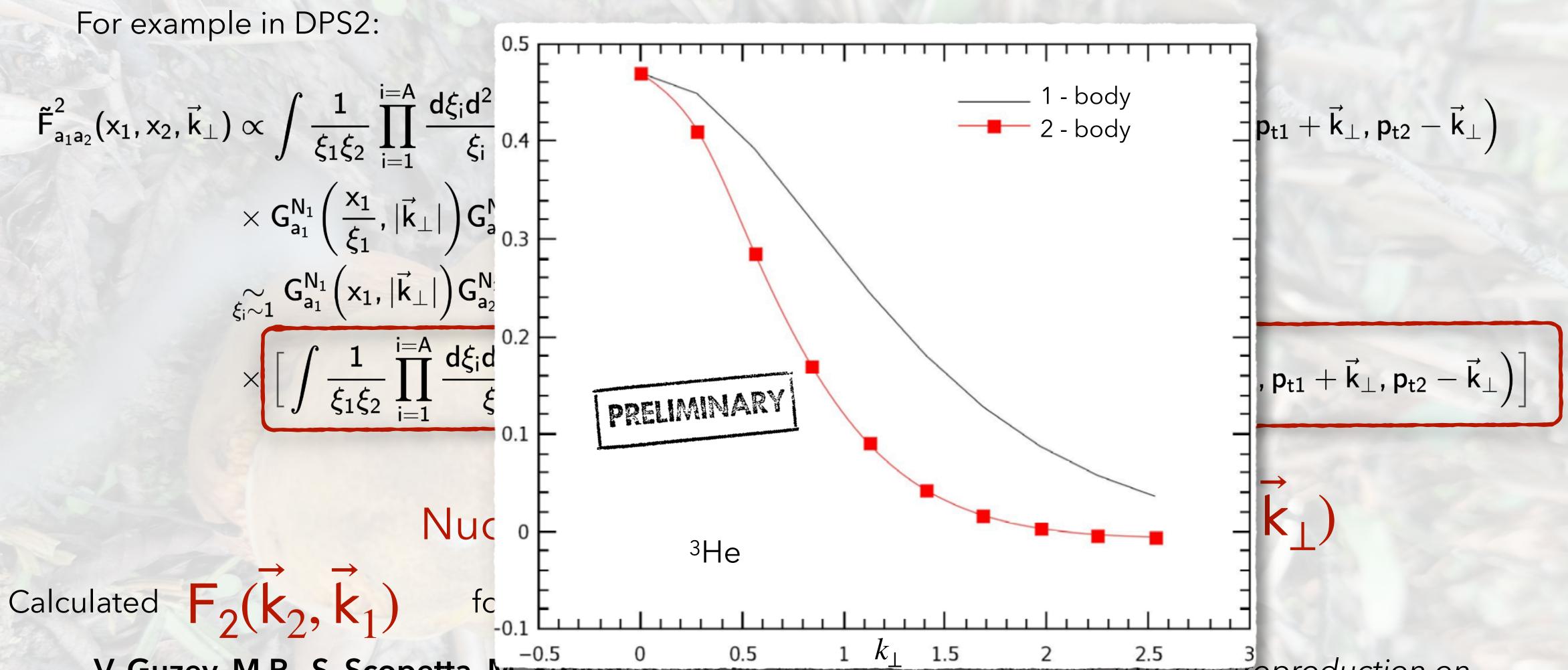
Nuclear 2-body form factor

$$F_2(\vec{k}_\perp, -\vec{k}_\perp)$$

Calculated  $F_2(k_2, k_1)$ 

for <sup>3</sup>He and <sup>4</sup>He in:

V. Guzey, M.R., S. Scopetta, M. Strikman and M. Viviani et al, "Coherent J/ $\Psi$  electroproduction on He4 and He3 at the EIC: probing Nuclear shadowing one nucleon at a time", PRL 129 (2022) 24, 242503



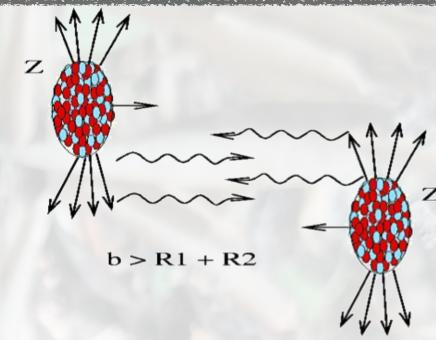
V. Guzey, M.R., S. Scopetta, M. Sunkman and W. viviani et al., Conference  $^{2.5}$  electroproduction on He4 and He3 at the EIC: probing Nuclear shadowing one nucleon at a time", PRL 129 (2022) 24, 242503

Matteo Rinaldi

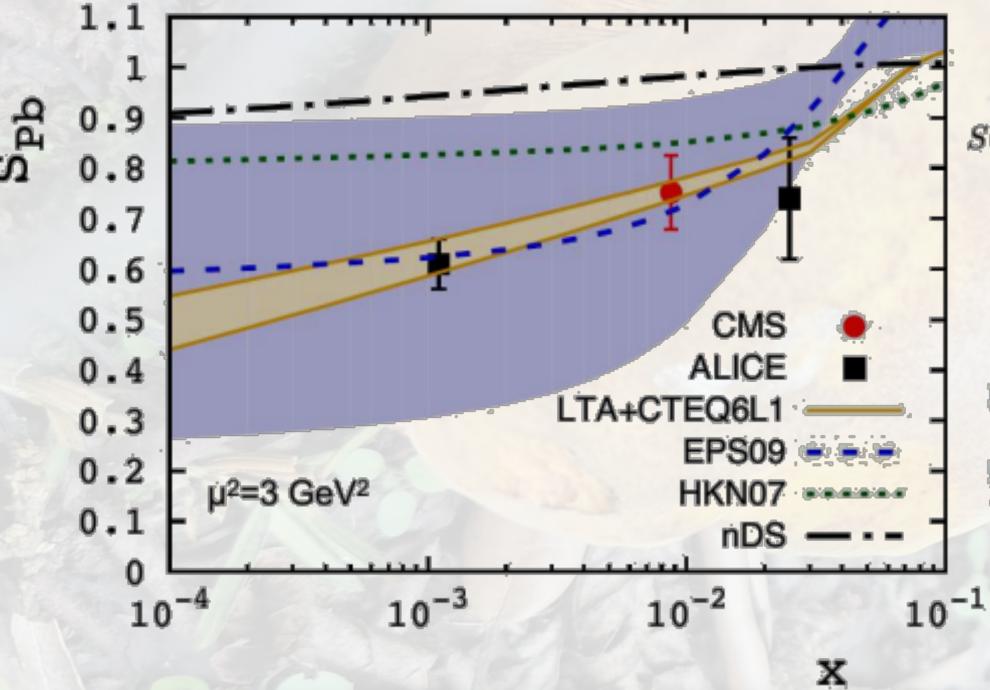
## J/w electroproduction on light-nuclei

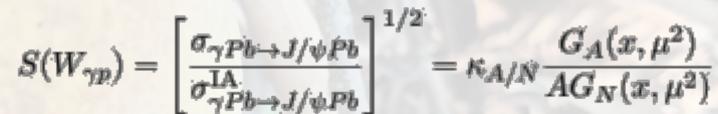
Gluon shadowing in UPC collisions @ LHC

Large (up to 40%) Leading twist (LT) shadowing in:  $\gamma + Pb/Au \rightarrow \rho(J/\Psi) + Pb/Au$  Explained/predicted (Frankfurt, Guzey, Strikman Phys. Rep. 512 (2012) 255)









LTA: Guzey, Zhalov JHEP 1310 (2013) 207. EPS09: Eskola, Paukkunen, Salgado, JHEP 0904 (2009) 065

HKN07: Hirai, Kumano, Nagai, PRC 76 (2007)

nDS: de Florian, Sassot, PRD 69 (2004) 074028

Introduction. Studies of nuclear shadowing have a long history [1–5]. In quantum mechanics and in the eikonal limit, it is manifested in the total hadron-nucleus cross section being smaller than the sum of individual hadron-nucleon cross sections. In essence, this is due to simultaneous interactions of the projectile with  $k \geq 2$  nucleons of the nuclear target, leading to a reduction (shadowing) of the total cross section. In this frame-

## Learning from light nuclei - I

- O Problem:
  - @ EIC/LHC it is challenging to measure coherent scattering at  $t \neq 0$  for A  $\approx 200$ ; Large coherence length: information on interactions with many nucleons, in average
- Solution:
  use the lightest nuclei, especially ³He and ⁴He, to study coherent effects for interactions with exactly 2 nucleons in the range of 0 < -t < 0.5 GeV2.

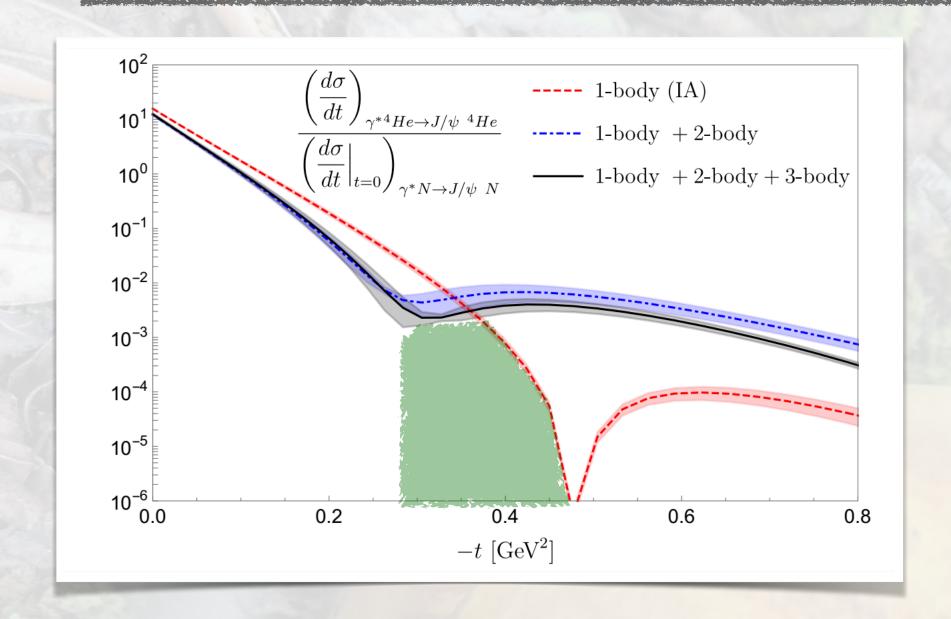
#### Complementary measurements with light ion beams @ the EIC:

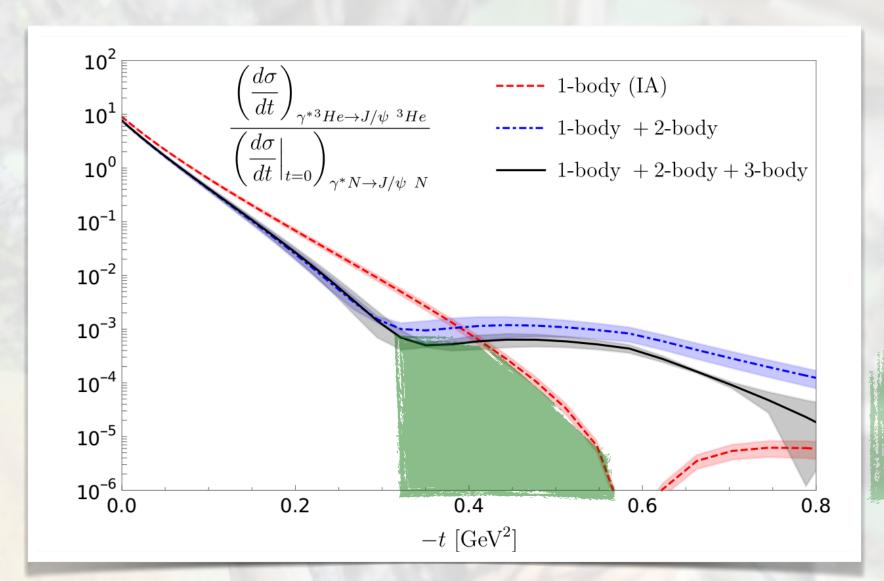
- Scattering off 2 and 3 nucleons can be separately probed
- no excited states -> easy to select coherent events

#### Here:

Results on J/Ψ diffractive electro-production off <sup>3</sup>He – <sup>4</sup>He V. Guzey, M. R., S. Scopetta, M. Strikman and M. Viviani, PRL 129 (2022) 24, 24503

#### Results for J/ $\Psi$ exclusive production @EIC: xB $\approx$ 10-3





Error bars account:

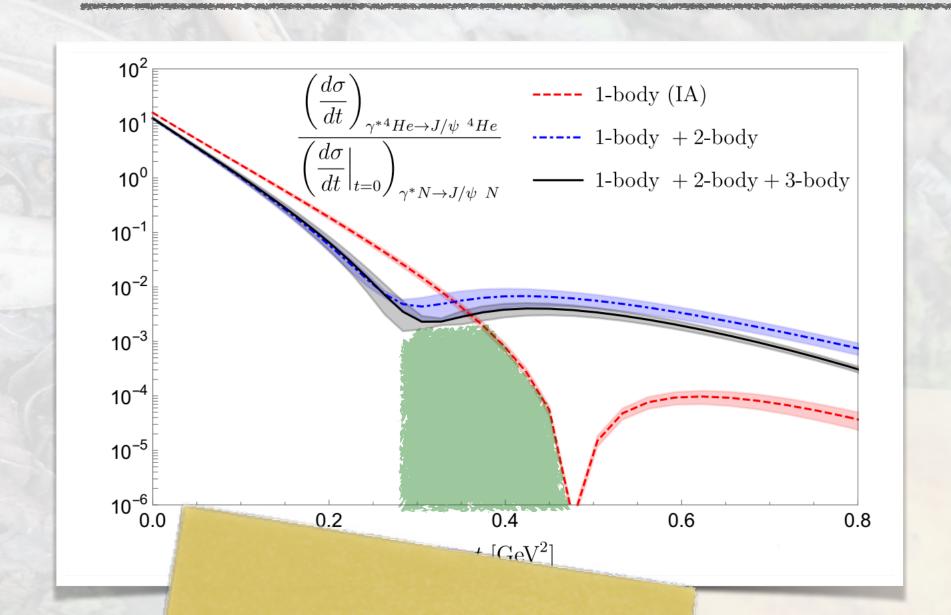
-10% of variation for Bo

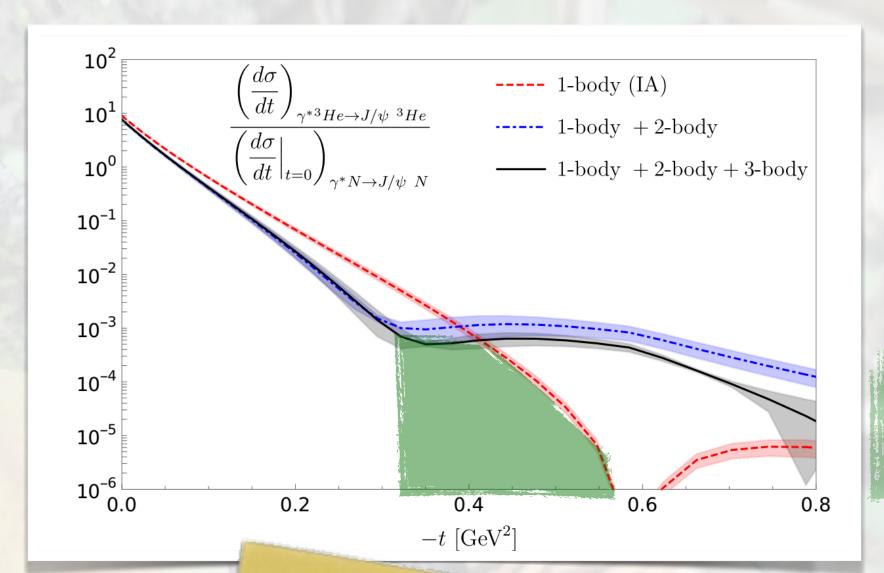
-15 of variation in  $< \sigma^2 >$ 

=Multi parton interactions effects

- ✓ 1-body + 2-body re-scatterings dominate the cross-sections shift of the minimum due to 2-body dynamics
- ✓ 1-body dynamics under theoretical control: very good chances to disentangle
- 2-body dynamics (LT gluon shadowing)
- ✓ unique opportunity to access the real part of the scattering amplitudes in a wide range of t
- ✓ The position of the minimum is extremely sensitive to dynamics and the structure!

#### Results for J/Ψ exclusive production @EIC: xB ≈ 10-3





Error bars account:

-10% of variation for Bo

-15 of variation in  $< \sigma^2 >$ 

=Multi parton interactions effects

- ✓ 1 PRECISE MEASUREMENTS ontrol: very good chances to dise

ring)

ARE NECESSARY!

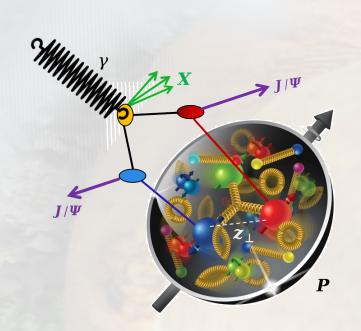
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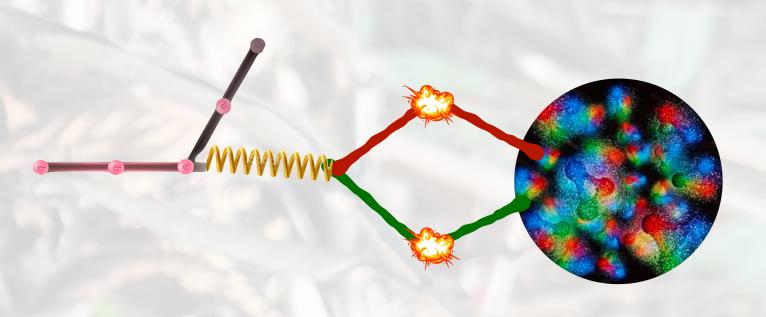
A NEW LINK BETWEEN NUCLEAR DPS PHYSICS AND DIFFRACTIVE PROCESSES

#### CONCLUSIONS

- 1) We demonstrated DPS represents a new way to access new information of hadrons
- 2) Several experimental analyses and theoretical developments are on going
- 3) We proposed to consider DPS initiated via photon-proton interactions:
  - a) DPS@EIC



b) Nuclear DPS@EIC



- a) DPS contributes, in particular in the 4-jets photoproduction
- b) We have estimated SPS and DPS cross sections for quarkonium-pair photoproduction at the EIC using the NRQCD framework
- c) The dependence of  $\sigma_{\rm eff}^{\gamma p}(Q^2)$  on  $Q^2$  can unveil the mean distance of partons in the proton
- d) Quarkonium-pair photoproduction is a promising channel to probe the gluonic content of the photon structure

M.R. in progress

$$\text{For example in DPS1:} \quad \tilde{F}^1_{a_1a_2}(x_1,x_2,k_\perp) = \sum_{N=p,n} \int \frac{1}{\xi} \tilde{F}^N_{a_1a_2}\left(\frac{x_1}{\xi},\frac{x_2}{\xi},k_\perp\right) \boxed{\rho_A^N(\xi,p_{t,N})} \frac{d\xi}{\xi} d^2p_{t,N}$$

Let us check sum rules:

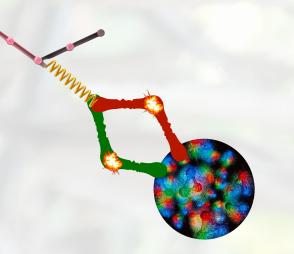
$$\int_{0}^{1} dx_{1} \int_{0}^{1-x_{1}} dx_{2} F_{i_{1}i_{2}}(x_{1}, x_{2}, k_{\perp} = 0) = \begin{cases} N_{i_{1}} N_{i_{2}} & \text{for } i_{1} \neq i_{2} \\ \left(N_{i_{1}} - 1\right) N_{i_{2}} & \text{for } i_{1} = i_{2} \end{cases}$$

Gaunt's sum rules

J. R. Gaunt and W. J. Stirling, JHEP 03, 005 (2010)

M.R. in progress

For example in DPS1: 
$$\tilde{F}_{a_1a_2}^1(x_1,x_2,k_\perp) = \sum_{N=p,n} \int \frac{1}{\xi} \tilde{F}_{a_1a_2}^N \left(\frac{x_1}{\xi},\frac{x_2}{\xi},k_\perp\right) \boxed{\rho_A^N(\xi,p_{t,N})} \frac{d\xi}{\xi} d^2p_{t,N}$$



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Gaunt's sum rules

J. R. Gaunt and W. J. Stirling, JHEP 03, 005 (2010)

However for the nuclear case one needs also the DPS2



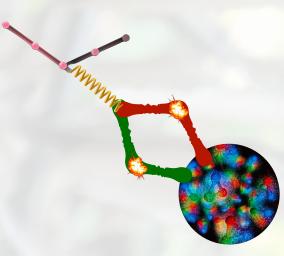
Thus we can introduce approximated partial sum rules (APSR)

M.R. in progress

For example in DPS1: 
$$\tilde{\mathsf{F}}_{\mathsf{a}_1\mathsf{a}_2}^1(\mathsf{x}_1,\mathsf{x}_2,\mathsf{k}_\perp) = \sum_{\mathsf{N}=\mathsf{p},\mathsf{n}} \int \frac{1}{\xi} \tilde{\mathsf{F}}_{\mathsf{a}_1\mathsf{a}_2}^\mathsf{N} \left(\frac{\mathsf{x}_1}{\xi},\frac{\mathsf{x}_2}{\xi},\mathsf{k}_\perp\right) \boxed{\rho_\mathsf{A}^\mathsf{N}(\xi,\mathsf{p}_\mathsf{t},\mathsf{N})} \frac{\mathsf{d}\xi}{\xi} \mathsf{d}^2 \mathsf{p}_\mathsf{t},\mathsf{N}$$

$$\left(\frac{\mathsf{x}_1}{\xi}, \frac{\mathsf{x}_2}{\xi}, \mathsf{k}_\perp\right)$$

$$\rho_{\mathsf{A}}^{\mathsf{N}}(\xi,\mathsf{p}_{\mathsf{t},\mathsf{N}})\frac{\mathsf{d}\xi}{\xi}\mathsf{d}^{2}\mathsf{p}$$



$$\int_{0}^{1} dx_{1} \int_{0}^{1-x_{1}} dx_{2} F_{i_{1}i_{2}}(x_{1}, x_{2}, k_{\perp} = 0) = \begin{cases} N_{i_{1}} N_{i_{2}} & \text{for } i_{1} \neq i_{2} \\ \left(N_{i_{1}} - 1\right) N_{i_{2}} & \text{for } i_{1} = i_{2} \end{cases}$$

Gaunt's sum rules

J. R. Gaunt and W. J. Stirling, JHEP 03, 005 (2010)

**APSR**: Since 
$$f_n^A(\xi) = \int d^2p_{t,N} \, \rho_A^N(\xi, p_{t,N})$$
 is peaked around 1/A

$$\int_{0}^{A} dx_{1} \int_{0}^{A-x_{1}} dx_{2} \tilde{F}_{i_{1}i_{2}}^{A,1} (x_{1}, x_{2}, 0) \sim \sum_{n=N,P} \int d\xi f_{n}^{A}(\xi) \left\{ \begin{pmatrix} N_{i_{1}}^{n} - 1 \end{pmatrix} N_{i_{2}}^{n} & i_{1} = i_{2} \\ N_{i_{1}}^{n} N_{i_{2}}^{n} & i_{1} \neq i_{2} \end{pmatrix} \right\}$$

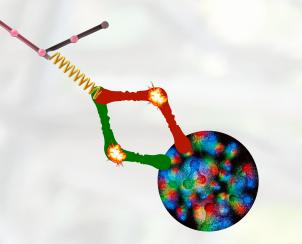
$$\begin{cases} \left(N_{i_1}^n - 1\right) N_{i_2}^n & i_1 = i_2 \\ N_{i_1}^n N_{i_2}^n & i_1 \neq i_2 \end{cases}$$

Gaunt's sum rules for the nucleon DPD: numbers of quarks with given flavor i in the nucleon n

Normalized to 1

M.R. in progress

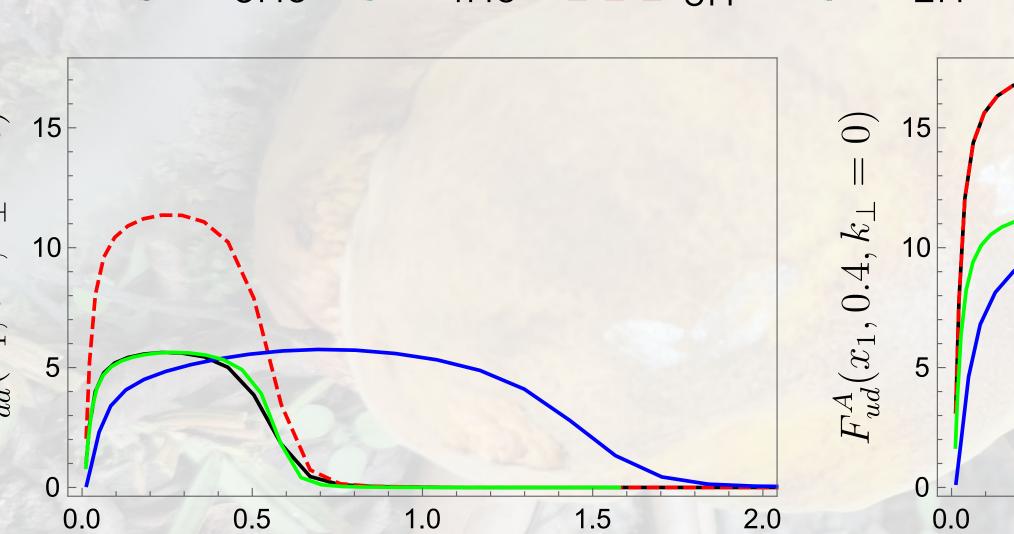
$$\text{For example in DPS1:} \quad \tilde{F}^1_{a_1a_2}(x_1,x_2,k_\perp) = \sum_{N=p,n} \int \frac{1}{\xi} \tilde{F}^N_{a_1a_2}\left(\frac{x_1}{\xi},\frac{x_2}{\xi},k_\perp\right) \boxed{\rho^N_A(\xi,p_{t,N})} \frac{d\xi}{\xi} d^2p_{t,N}$$



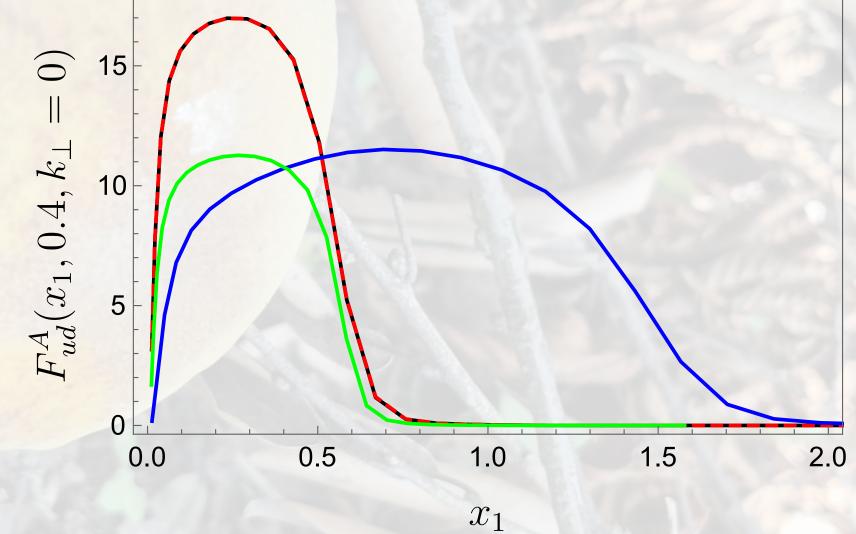
 $0 < x_i < A$ 

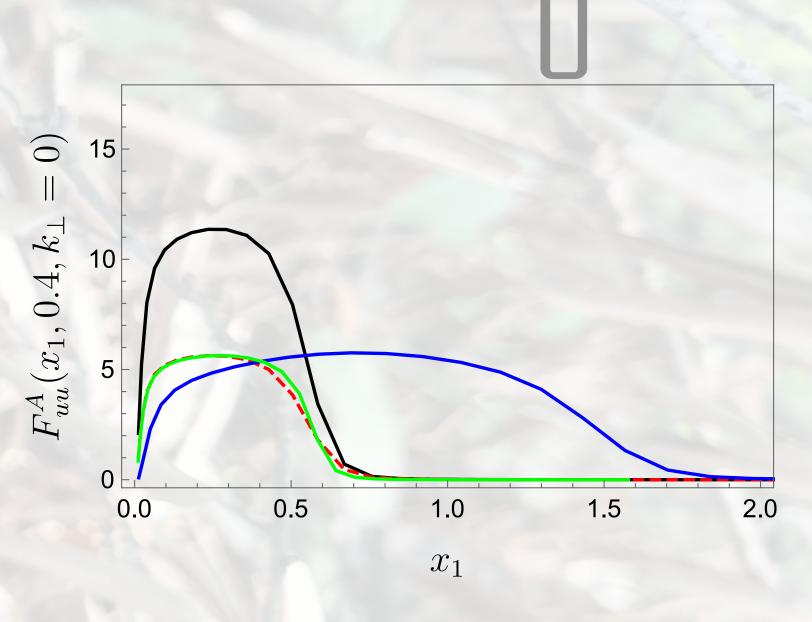
- Only for valence quarks. We used a relativistic constituent quark model for the nucleon DPD
- verified approximated partial sum rules (numerically)

- 3He - 4He - 3H 2H



 $x_1$ 

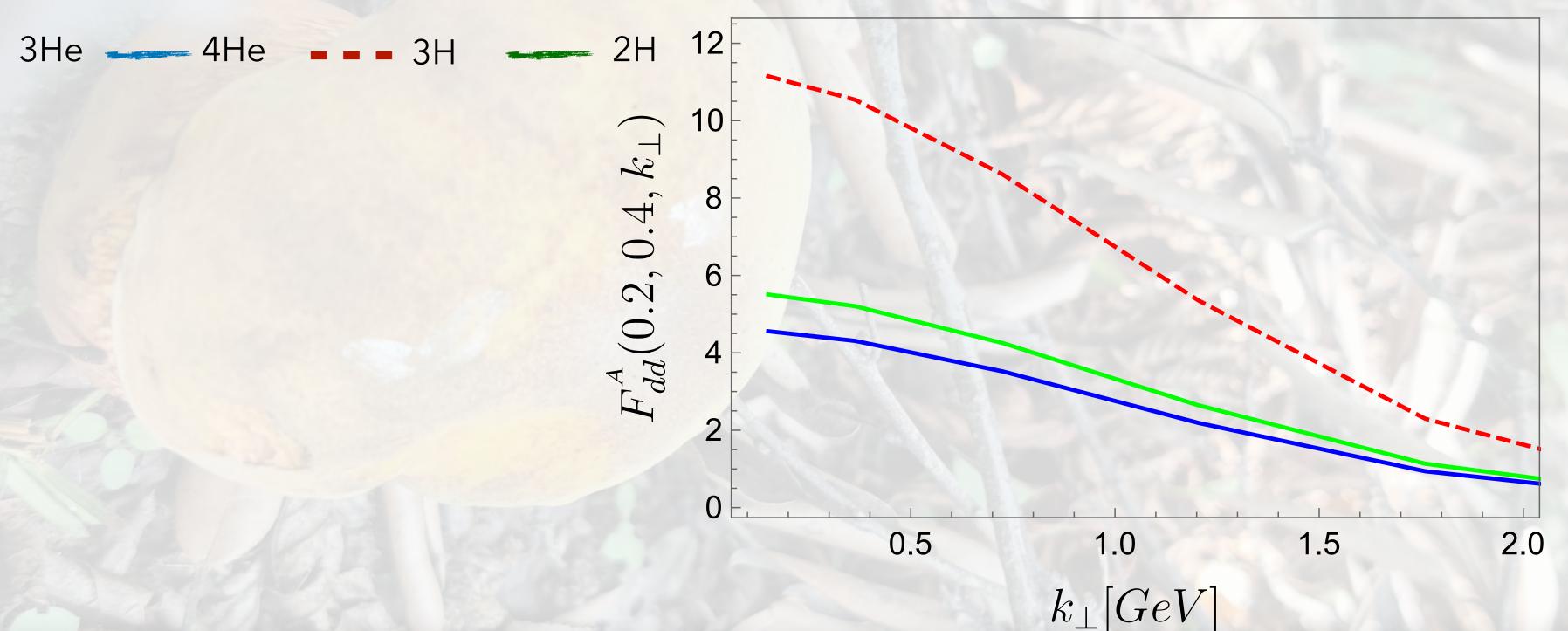




M.R. in progress

$$\text{For example in DPS1:} \quad \tilde{F}^1_{a_1a_2}(x_1,x_2,k_\perp) = \sum_{N=p,n} \int \frac{1}{\xi} \tilde{F}^N_{a_1a_2}\left(\frac{x_1}{\xi},\frac{x_2}{\xi},k_\perp\right) \boxed{\rho_A^N(\xi,p_{t,N})} \frac{d\xi}{\xi} d^2p_{t,N}$$

- Only for valence quarks. We used a relativistic constituent quark model for the nucleon DPD
- verified approximated partial sum rules (numerically)



### CONCLUSIONS



Matteo Rinaldi

# Backup - $\sigma_{\rm eff}^{\gamma p}(Q^2 \to \infty)$

1) we show that high virtual behavior of the effective cross sections correctly follows the result in **J.R. Gaunt JHEP 01, 042 (2013)**, i.e.:

$$\sigma_{eff}^{\gamma p}(Q^2 \to \infty) = \sigma_{1v2}^{pp} = \left[ \int \frac{d^2 k_{\perp}}{(2\pi)^2} T_p(k_{\perp}) \right]^{-1}$$

2) In Ref. M.Rinaldi and F.A: Ceccopieri JHEP 09, 097 (2019), we prove, in a general framework:

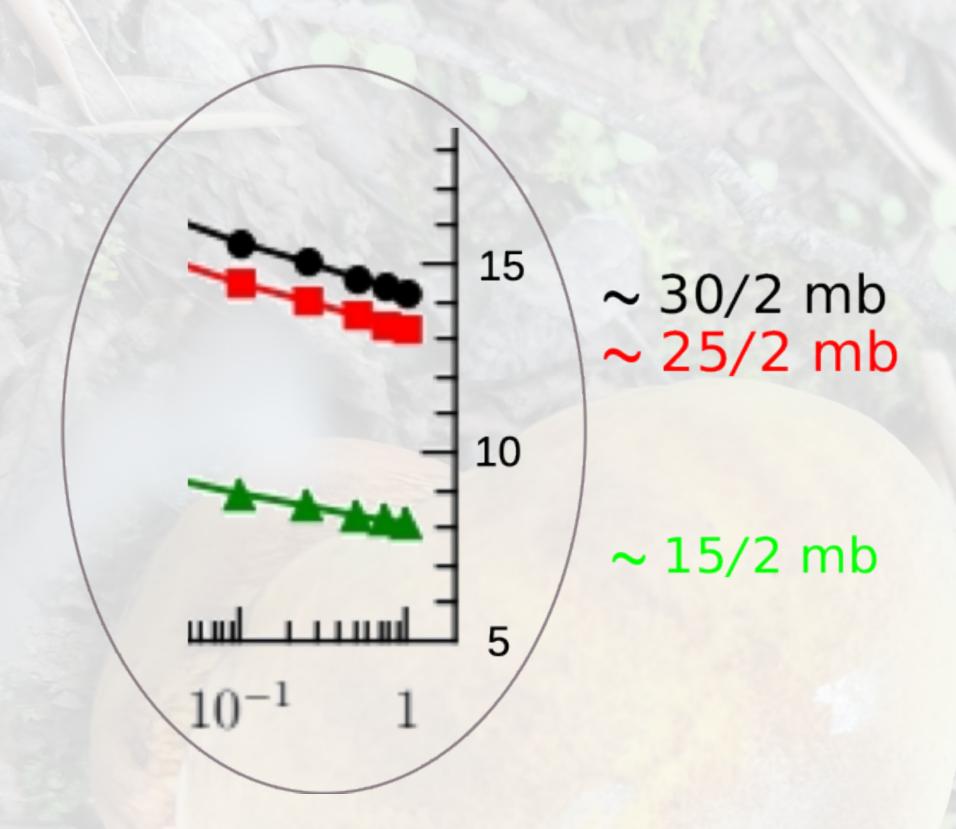
$$\frac{\pi}{2} \langle b^2 \rangle \le \sigma_{eff}^{\gamma p} (Q^2 \to \infty) \le 2\pi \langle b^2 \rangle$$

Being: 
$$\sigma_{\rm eff}^{\gamma p}(Q^2 \to \infty) = \sigma_{\rm eff}^{2v1}$$

$$\frac{\sigma_{eff}^{pp}}{6} \le \sigma_{eff}^{\gamma p}(Q^2 \to \infty) \le 2\sigma_{eff}^{pp}$$

Extracted from data

# Backup - $\sigma_{\rm eff}^{\gamma p}(Q^2 \to \infty)$



$$[\sigma_{eff}^{\gamma p}(Q^2)]^{-1} = \int \frac{d^2k_{\perp}}{(2\pi)^2} T_p(k_{\perp}) T_{\gamma}(k_{\perp}; Q^2)$$

$$[\sigma_{eff}^{\gamma p}(Q^2)]^{-1} \sim \int_{Q^2 >>1} \frac{d^2 k_{\perp}}{(2\pi)^2} T_p(k_{\perp}) \times 1$$

For the proton models we have used:

$$\int \frac{d^2k_{\perp}}{(2\pi)^2} T_p(k_{\perp}) \sim 2 \int \frac{d^2k_{\perp}}{(2\pi)^2} T_p(k_{\perp})^2$$

$$\sigma_{eff}^{\gamma p}(Q^2 >> 1~{\rm GeV}^2) \sim \sigma_{eff}^{pp}/2$$

Thus for QED:  $Q^2 > 1 \text{ GeV}^2$  almost approximates the asymptotic

### DPS in pA collisions

The DPS cross-section

$$d\sigma_{DPS}^{ML} = \frac{m}{2} \sum_{i,j,k,l} d\hat{\sigma}_{ik}^{M} d\hat{\sigma}_{jl}^{L} \int d^{2}b_{\perp} \; F_{p}^{ij}(x_{1},x_{2},\vec{b}_{\perp}) \int d^{2}B \bigg\{$$

the thickness function as a function of the impact parameter B

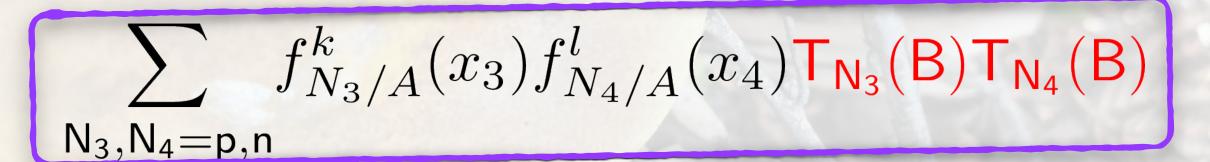
$$\bar{T}(\vec{b}_{\perp} + \vec{B}) \sim \bar{T}(\vec{B})$$

$$\bar{T}_N(B) = \int dz \, \rho_N(\sqrt{B^2 + z^2})$$

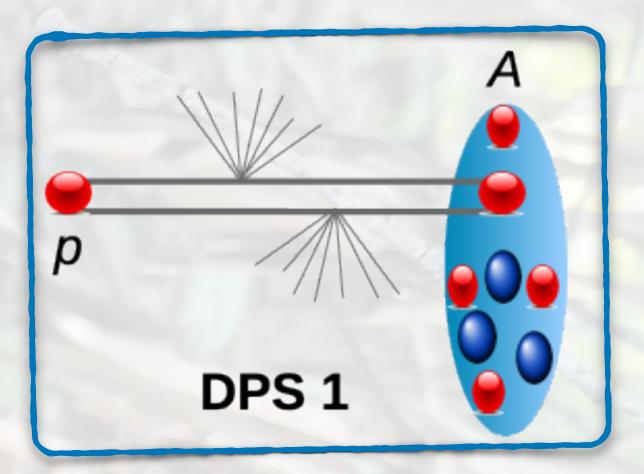
Wood-Saxon distribution for pb normalized to A

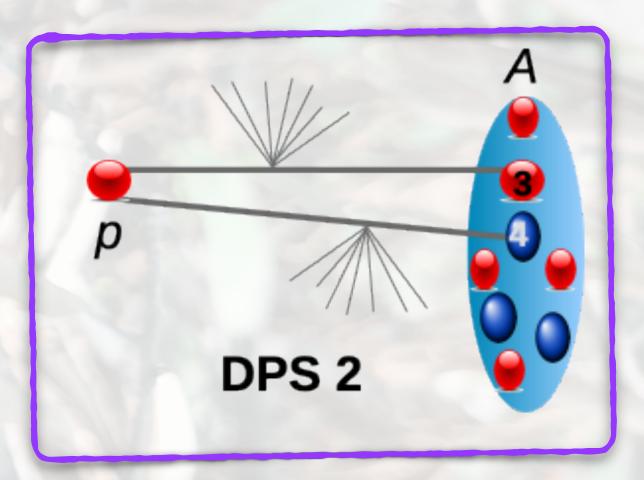
$$\sum_{N=p,n} F_N^{kl}(x_3,x_4,\vec{b}_\perp) \overline{T}_N(B)$$



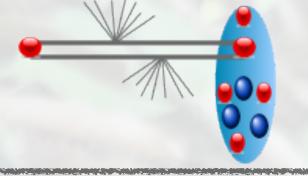


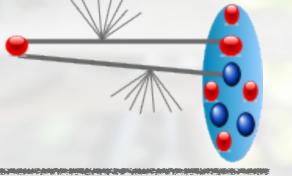






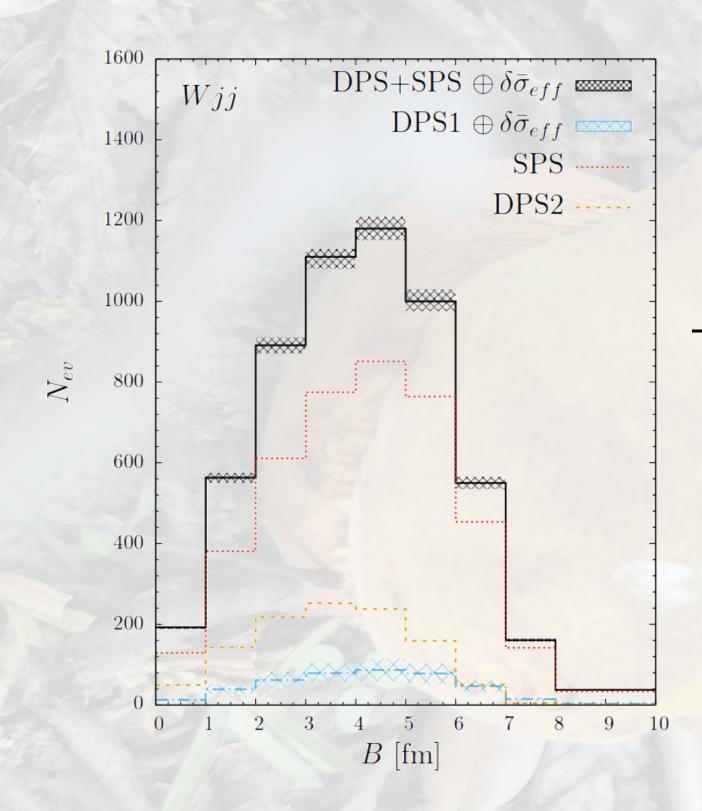
### DPS in pA collisions





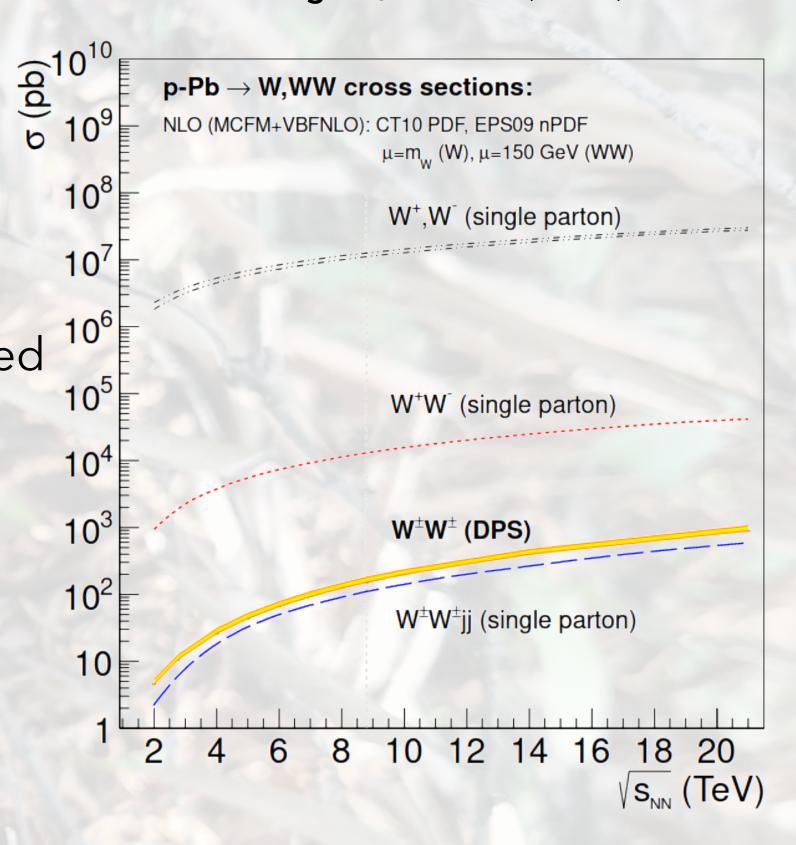
Some examples of predictions:

W+di-jets
B. Blok and F. A. Ceccopieri EPJC (2020) 80, 278



- SPS dominant
- DPS2 bigger then DPS1 has expected

Same sign WW D. D'Enterria and Snigirev, PLB 718 (2013) 1395-1400



For example in DPS2:

$$\begin{split} \tilde{\textbf{F}}_{a_{1}a_{2}}^{2}(\textbf{x}_{1},\textbf{x}_{2},\vec{\textbf{k}}_{\perp}) &\propto \int \frac{1}{\xi_{1}\xi_{2}} \prod_{i=1}^{i=A} \frac{\text{d}\xi_{i} \text{d}^{2}\textbf{p}_{ti}}{\xi_{i}} \delta \Biggl( \sum_{i} \xi_{i} - A \Biggr) \delta^{(2)} \Biggl( \sum_{i} \textbf{p}_{ti} \Biggr) \psi_{A}^{*}(\xi_{1},\xi_{2},\textbf{p}_{t1},\textbf{p}_{t2}) \psi_{A} \Biggl( \xi_{1},\xi_{2},\textbf{p}_{t1} + \vec{\textbf{k}}_{\perp},\textbf{p}_{t2} - \vec{\textbf{k}}_{\perp} \Biggr) \\ &\times \textbf{G}_{a_{1}}^{\textbf{N}_{1}} \Biggl( \frac{\textbf{x}_{1}}{\xi_{1}},|\vec{\textbf{k}}_{\perp}| \Biggr) \textbf{G}_{a_{2}}^{\textbf{N}_{2}} \Biggl( \frac{\textbf{x}_{2}}{\xi_{2}},|\vec{\textbf{k}}_{\perp}| \Biggr); \end{split}$$

if we approximate:  $\xi_i \sim 1$  we get:

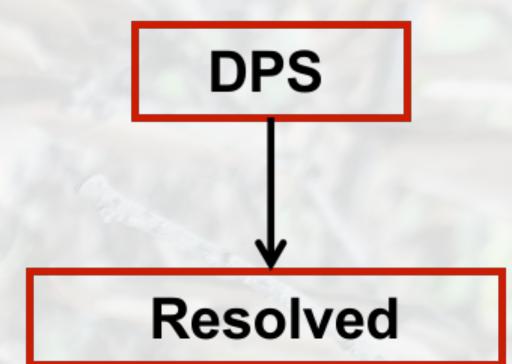
### Di J/w photo-production@EIC

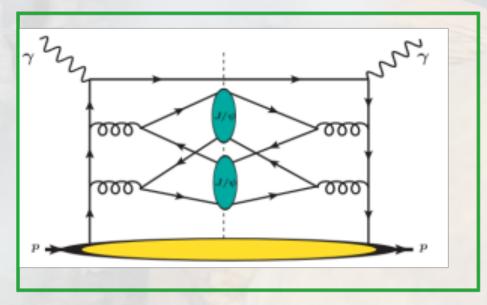
F. A. Ceccopieri, H. S. Shao, J. P. Lansberg, M. R. and R. Sangem in prep.

\*Slide from R. Sangem

SPS

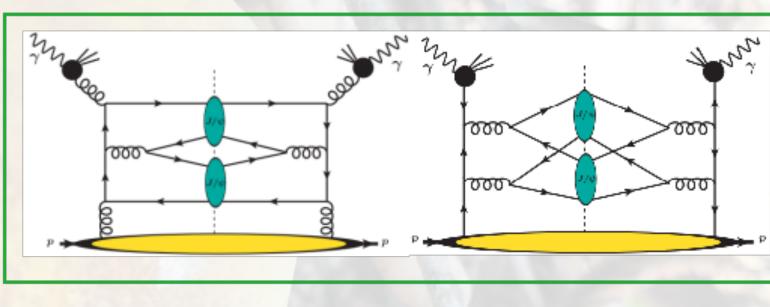
Resolved





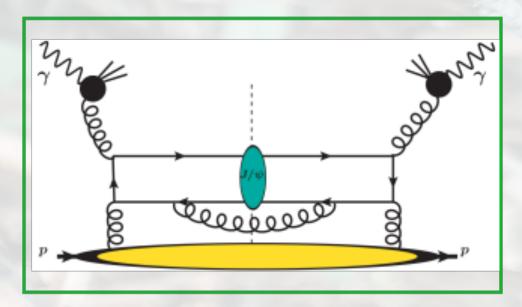
Unresolved





$$i + j \rightarrow J/\psi + J/\psi$$

$$i, j = g, q$$



$$|g+g\to J/\psi+g|^2$$

#### Range of cross sections in CSM =100 GeV



$$\sigma_{SPS}^{(J/\psi,J/\psi)} \times Br^2 = 4 - 30 \text{ fb}$$

$$\sigma_{DPS}^{(J/\psi,J/\psi)} \times Br^2 = 0.2 - 5 \text{ fb}$$

$$\sigma_{SPS}^{(J/\psi,J/\psi)} \times Br^2 = 2 - 12 \text{ fb}$$

(Resolved)  $\sigma_{eff}^{\gamma p} = 10 \text{ mb for DPS}$ 

(Unresolved)

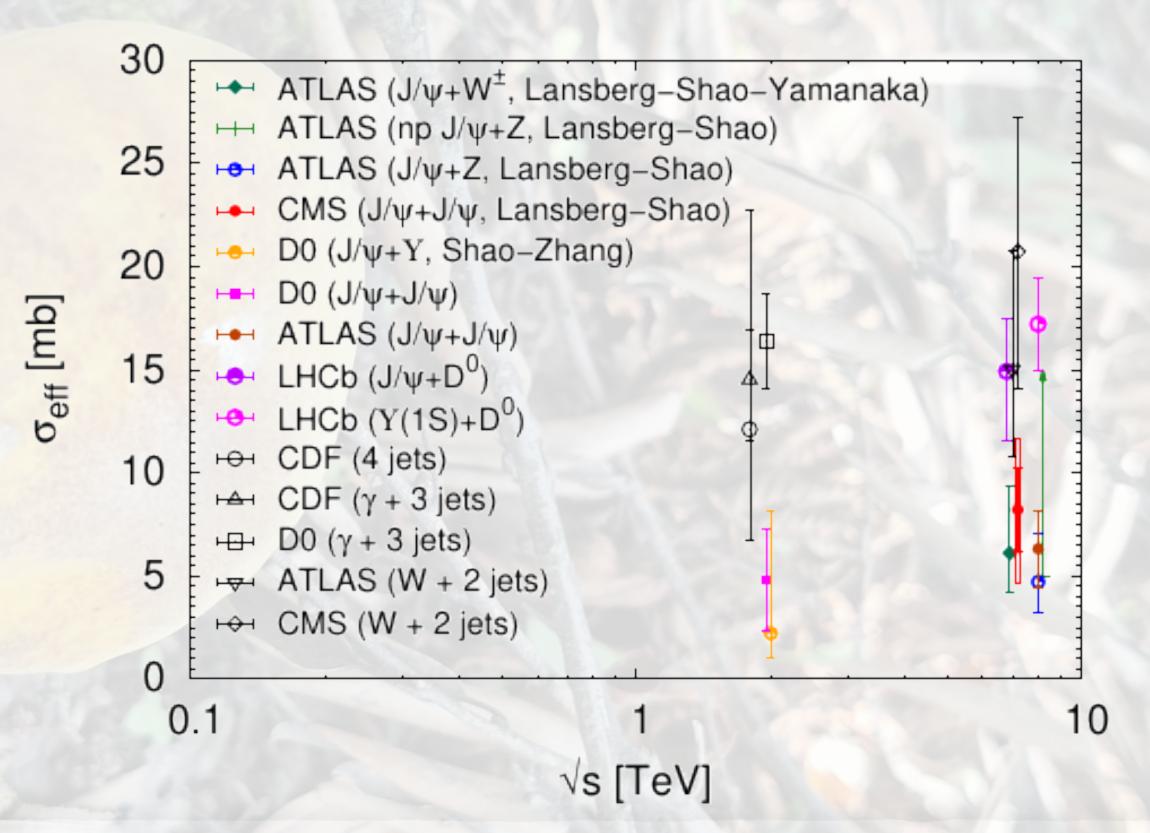
#### Some Data and Effective Cross Section

$$\sigma_{\text{eff}}^{\text{pp}} = \frac{m}{2} \frac{\sigma_{\text{A}}^{\text{pp}} \sigma_{\text{B}}^{\text{pp}}}{\sigma_{\text{DPS}}^{\text{pp}}}$$

**POCKET FORMULA** 

▶ Differential X-section single parton scattering for the process:  $pp \longrightarrow A(B) + X$ 

 $\rightarrow$  Differential X-section double parton scattering for the process:  $pp \longrightarrow A + B + X$ 



First observation of same sign WW via DPS:

$$\sigma_{
m eff}=12.2^{+2.9}_{-2.2}~{
m mb}$$
 [CMS coll.], PRL 131 (2023) 091803

$$\sigma^{\rm DPS} \sim 6.28~{\rm fb}$$

#### Some Data and Effective Cross Section

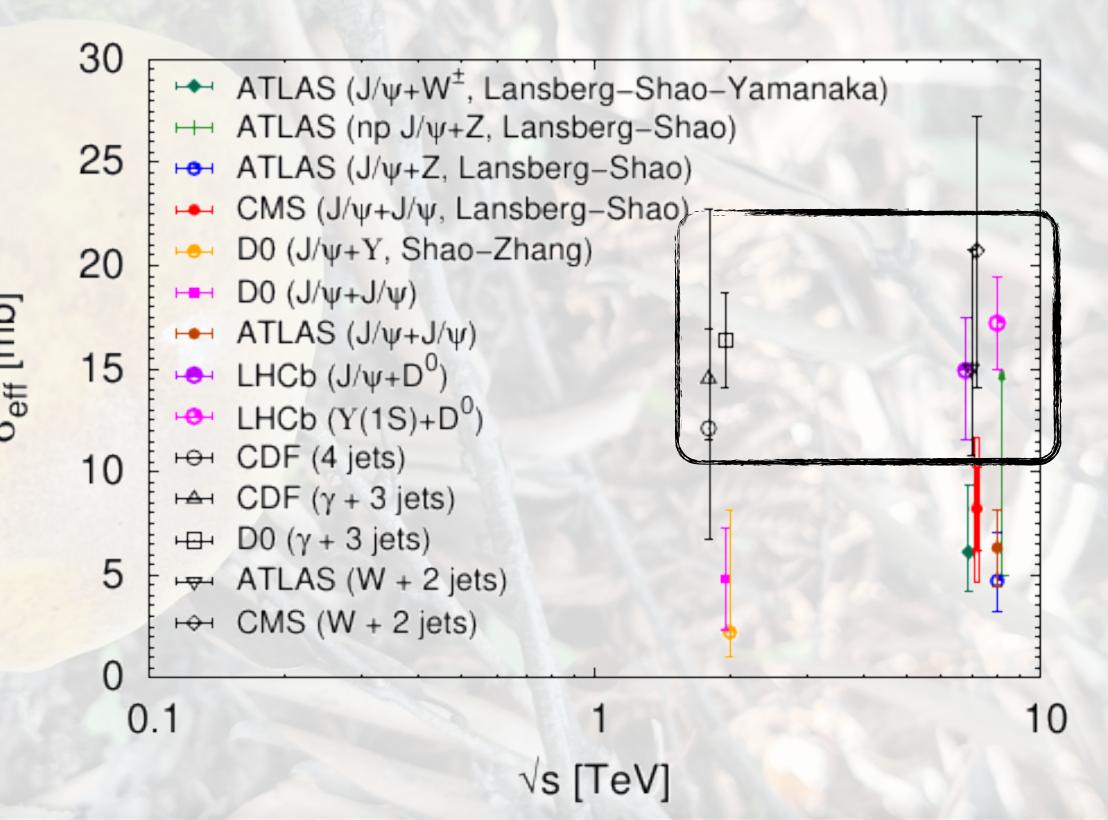
$$\sigma_{\text{eff}}^{\text{pp}} = \frac{m}{2} \frac{\sigma_{\text{A}}^{\text{pp}} \sigma_{\text{B}}^{\text{pp}}}{\sigma_{\text{DPS}}^{\text{pp}}}$$

▶ Differential X-section single parton scattering for the process:  $pp \longrightarrow A(B) + X$ 

 $\rightarrow$  Differential X-section double parton scattering for the process:  $pp \longrightarrow A + B + X$ 

#### **POCKET FORMULA**

Results for W, Jet productions...



First observation of same sign WW via DPS:

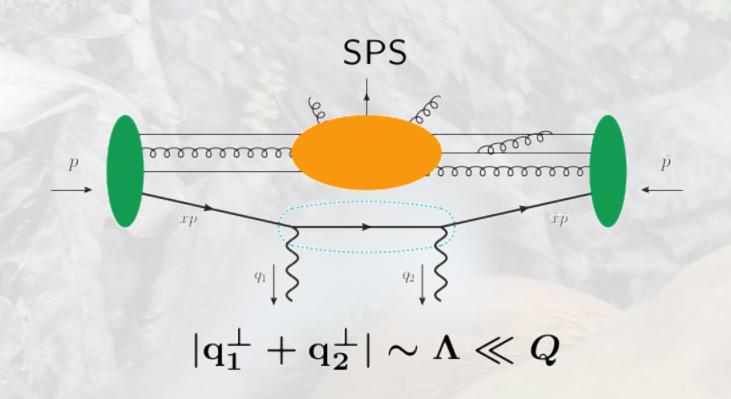
$$\sigma_{
m eff}=12.2^{+2.9}_{-2.2}~{
m mb}$$
 [CMS coll.], PRL 131 (2023) 091803

$$\sigma^{\rm DPS} \sim 6.28~{\rm fb}$$

Matteo Rinaldi

### Double Parton Scattering scales

Scale analysis of SPS and DPS processes



First appearance in theory studies:

Politzer

Paver, Treleani

Mekhfi

Other ground-setting works:

Gaunt, Stirling

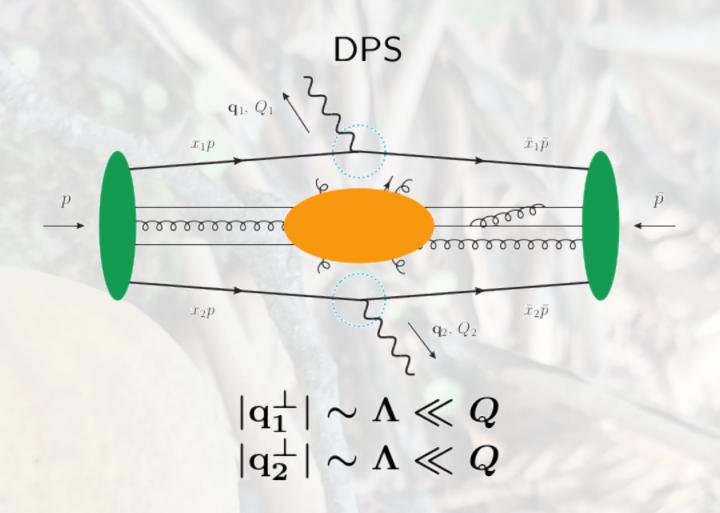
Blok et al.

Diehl et al.

Manohar, Waalewijn

Ryskin, Snigierev

. . .



#### where:

- $-Q = min(Q_1, Q_2)$
- A transverse momentum scale
- 1000 << 1 << Q

#### Usually:

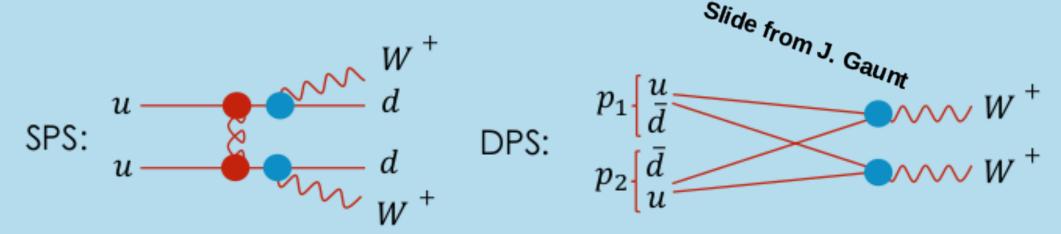
$$\frac{\text{d}^2\sigma_{SPS}}{\text{d}^2q_1~\text{d}^2q_2} \sim \frac{\text{d}^2\sigma_{DPS}}{\text{d}^2q_1~\text{d}^2q_2}$$

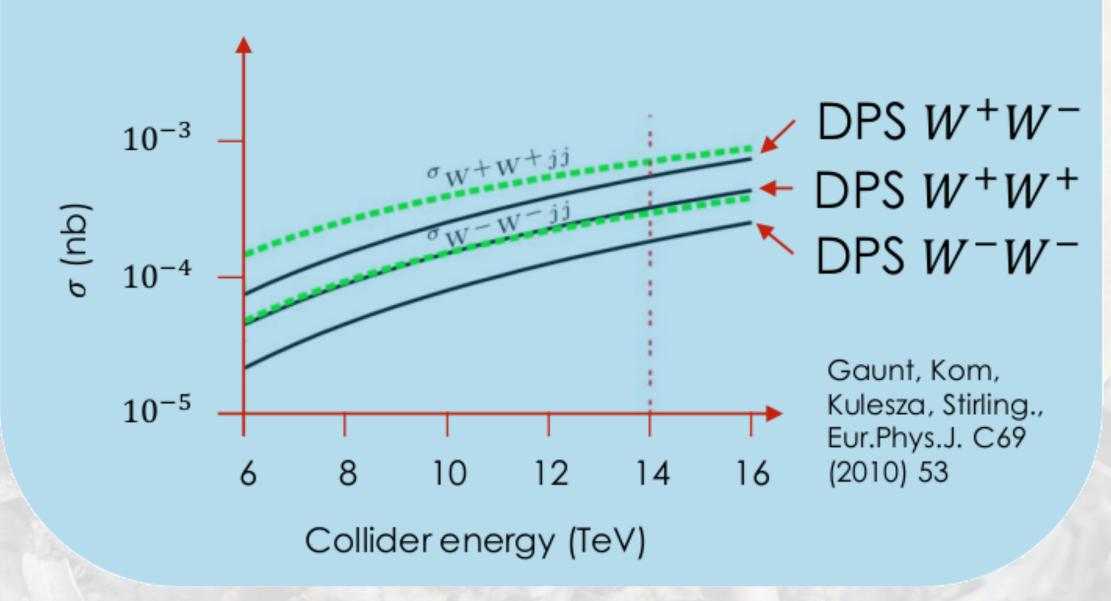
$$rac{\sigma_{\mathsf{DPS}}}{\sigma_{\mathsf{SPS}}} \sim \mathcal{O}\left(rac{\Lambda^2}{\mathsf{Q}^2}
ight)$$

Nagar's slides MPI 2021

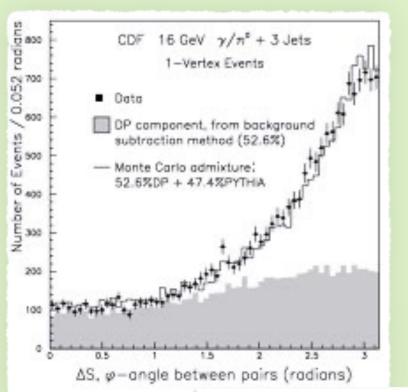
## Where and Why DPS?

DPS can give a significant contribution to processes where SPS is suppressed by small/multiple coupling constants:

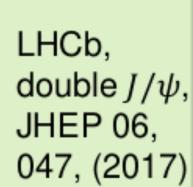


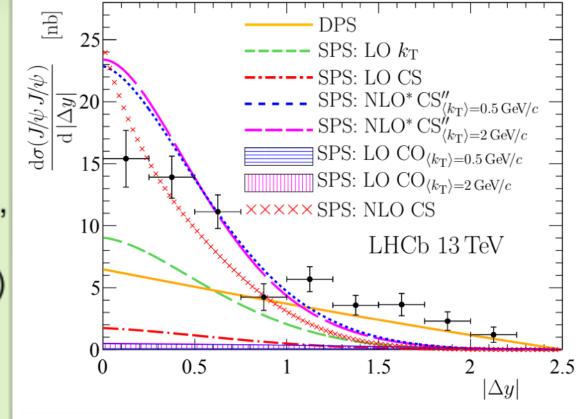


## ...or in certain phase space regions



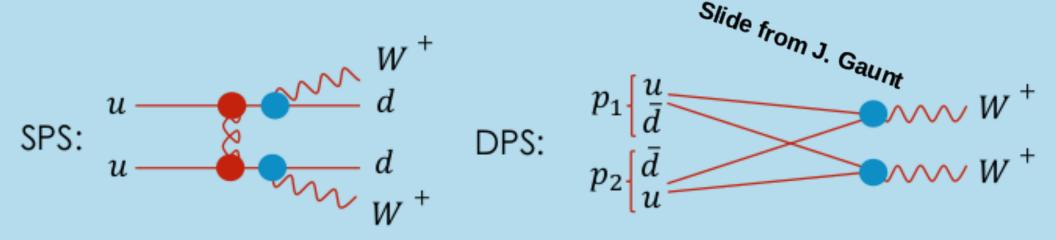
CDF, γ + 3j, Phys.Rev. D56 (1997) 3811-3832

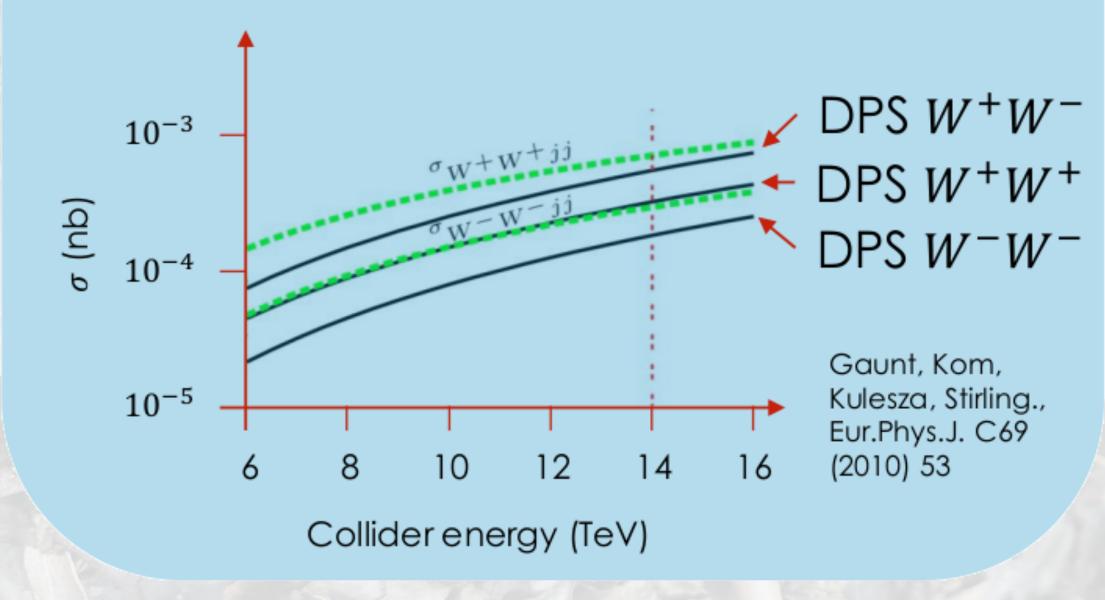




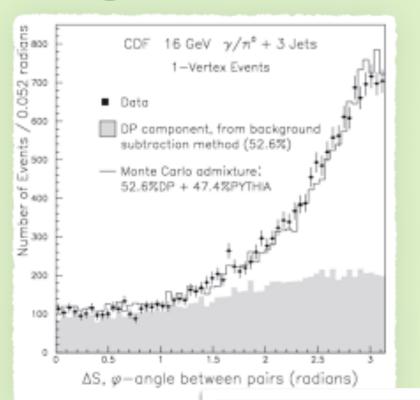
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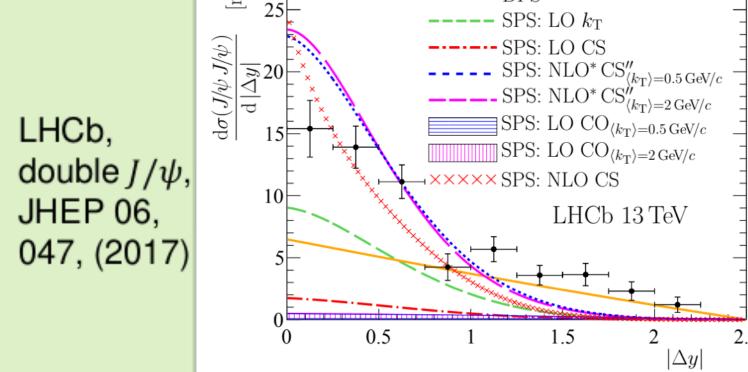




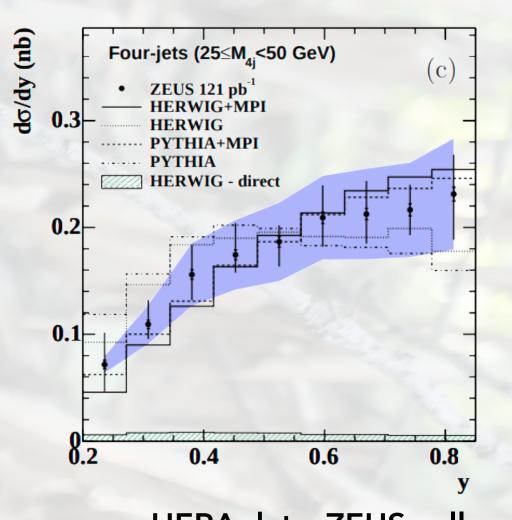
### ...or in certain phase space regions



CDF, γ + 3j, Phys.Rev. D56 (1997) 3811-3832



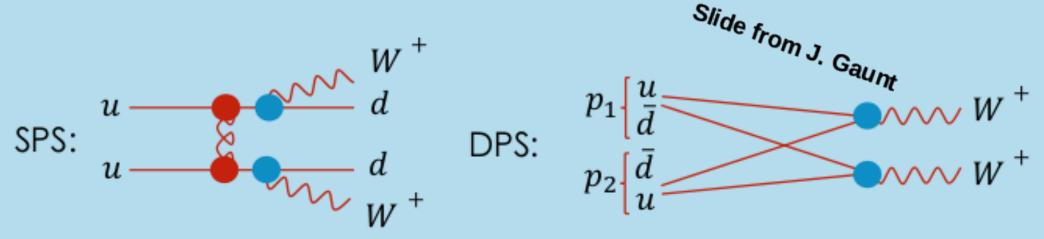
#### in ep Colliders?

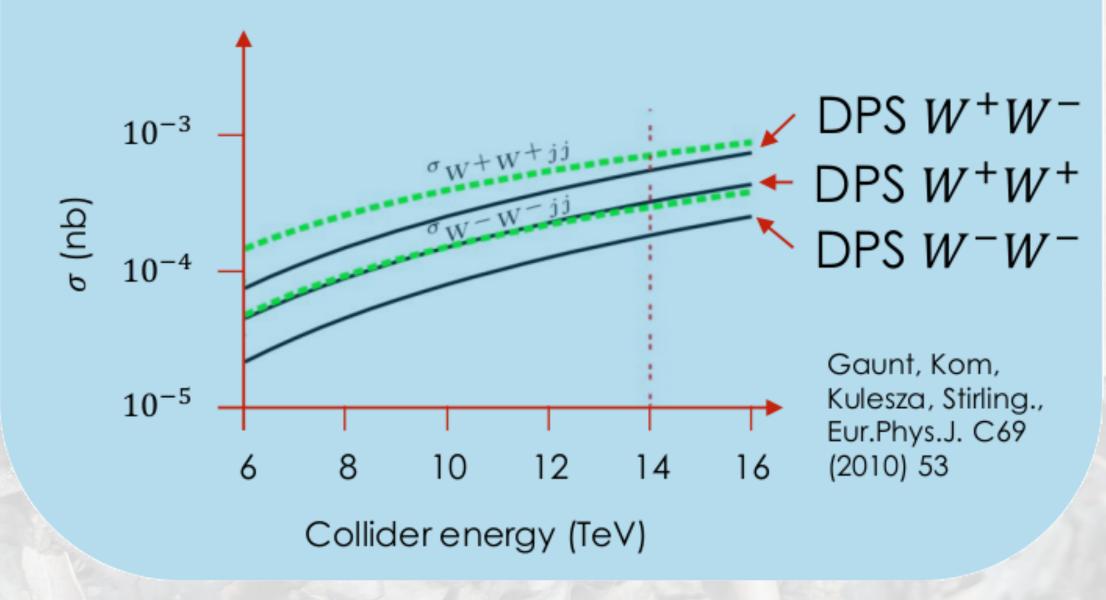


HERA data, ZEUS coll, Nucl. Phys. B 729, 1 (2008)

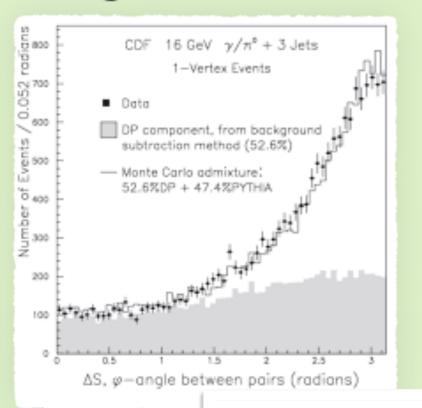
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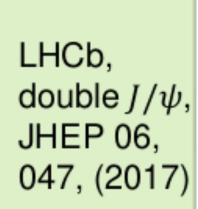


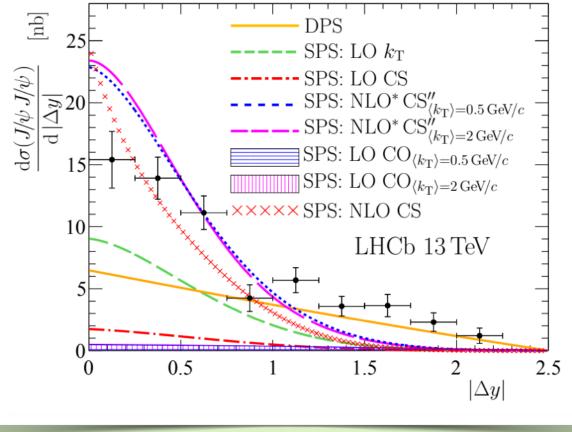


### ...or in certain phase space regions

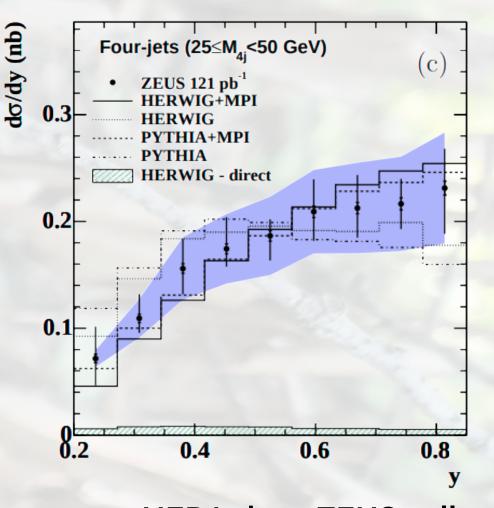


CDF, γ + 3j, Phys.Rev. D56 (1997) 3811-3832





#### in ep Colliders?



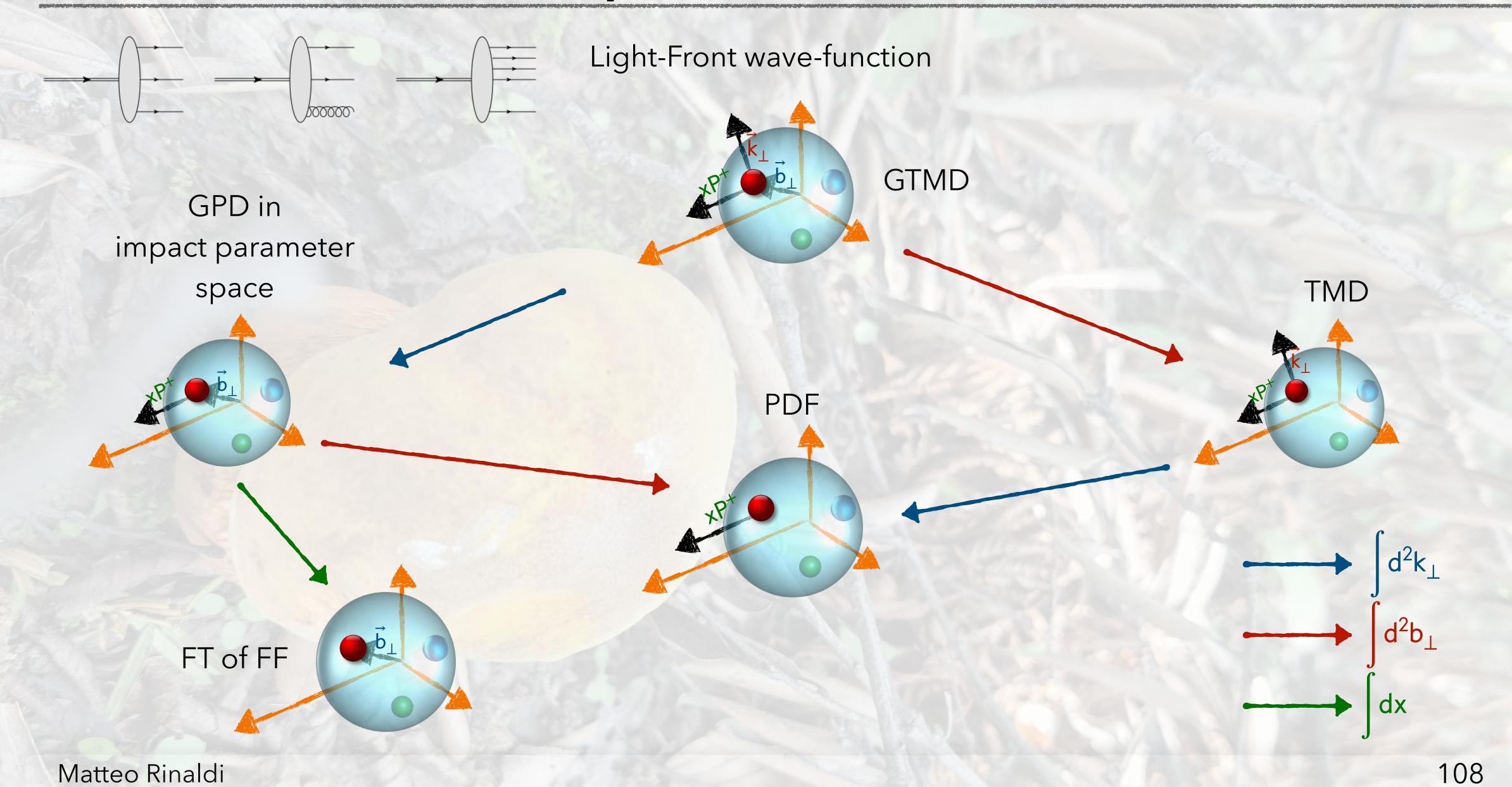
HERA data, ZEUS coll, Nucl. Phys. B 729, 1 (2008)

#### Access to:

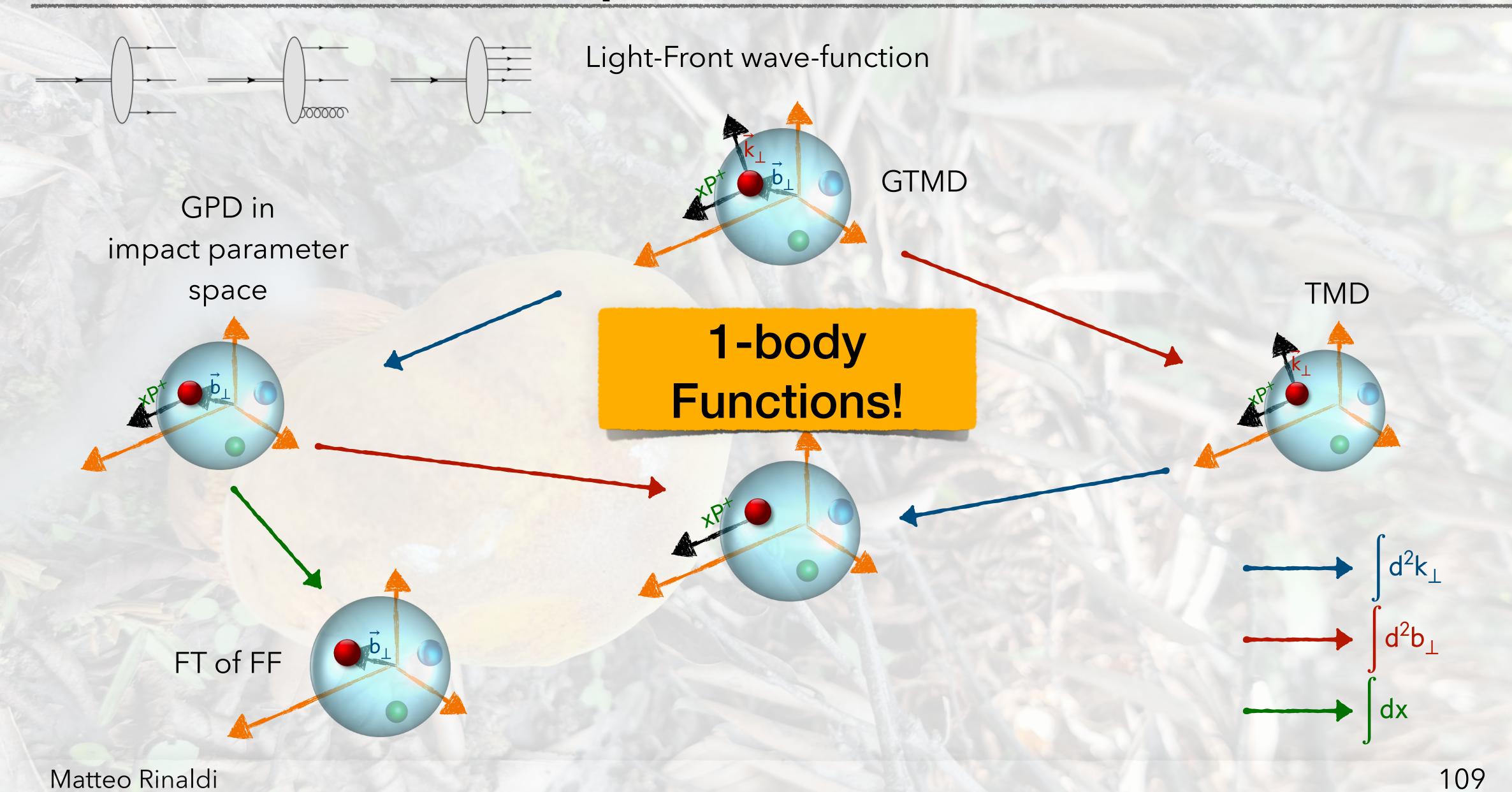
- double parton correlations
- the transverse distance distribution of partons!!

all UNKNOWN

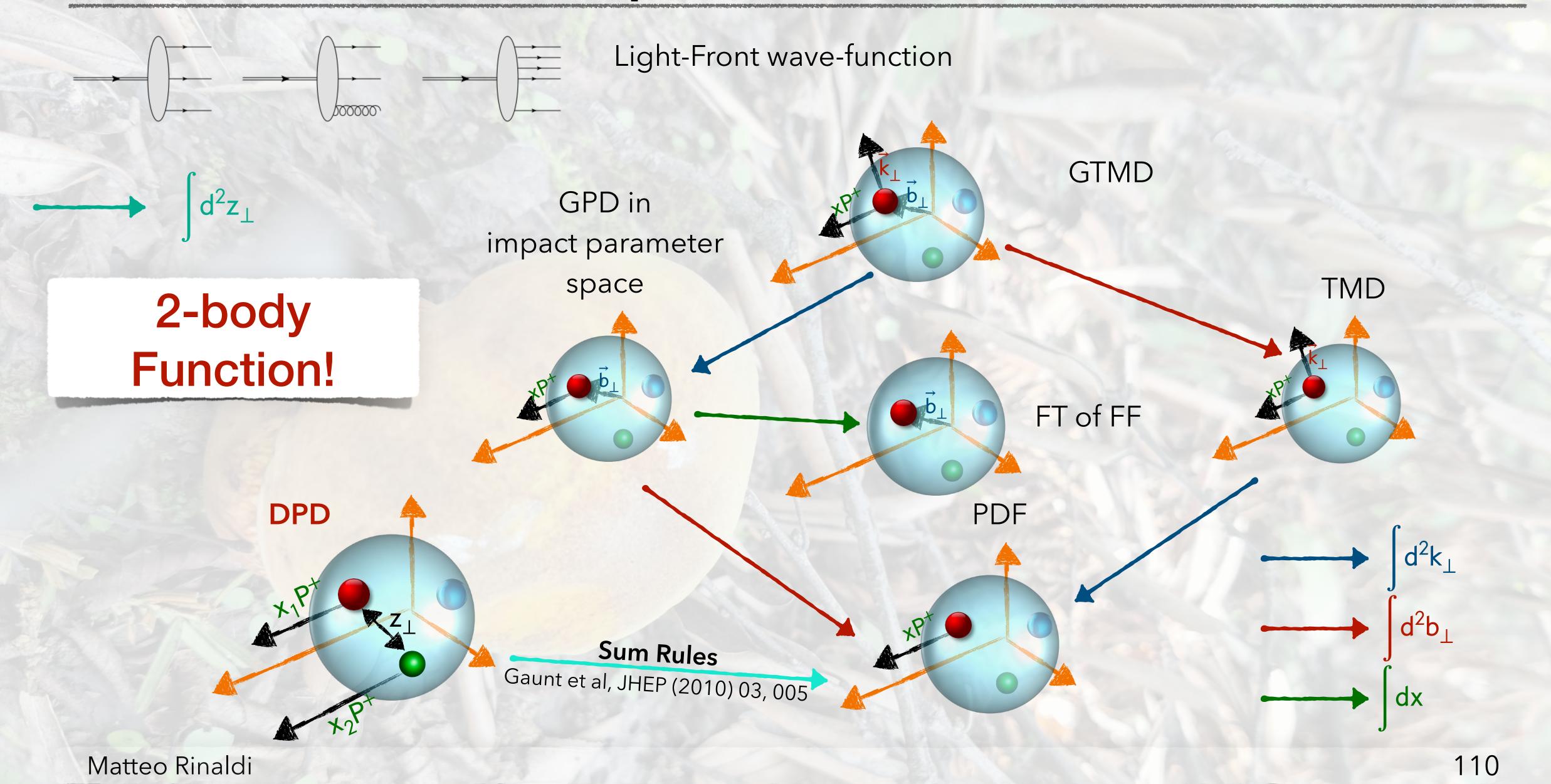
## Multidimensional picture of hadrons



## Multidimensional picture of hadrons

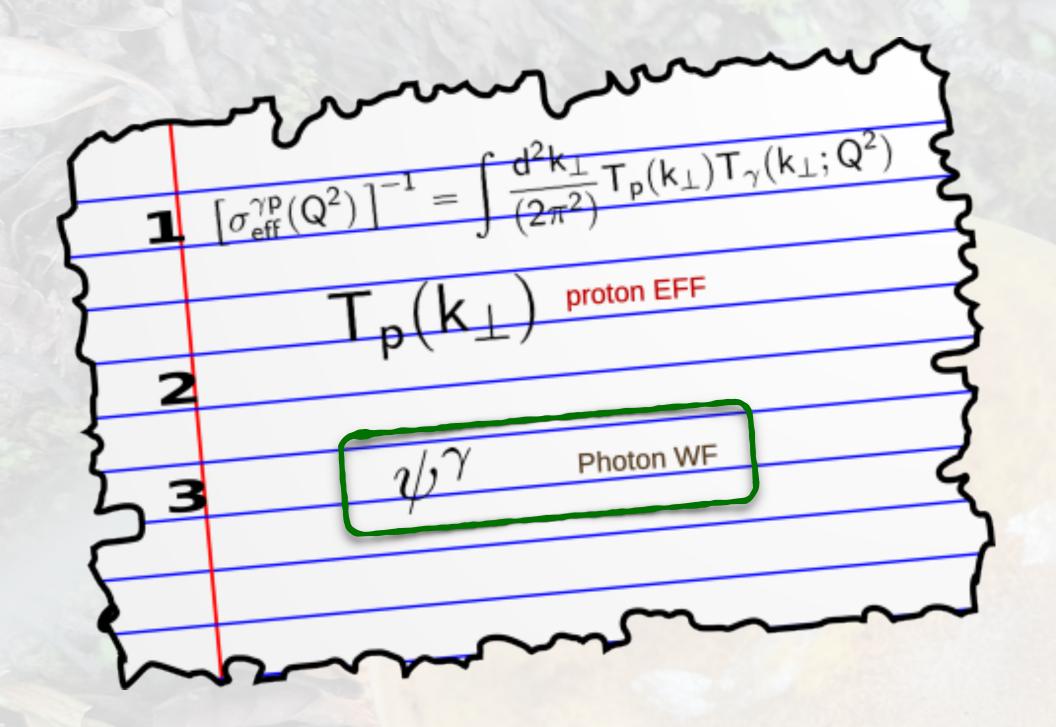


## Multidimensional picture of hadrons



### The $\gamma$ – p effective cross-section

The main ingredients of the calculations:



For the photon W.F. use has been made of two choices representing two extreme cases:

1) QED at LO (S.J. Brodsky et al. PRD50, 3134 (1994)):

$$\psi_{q,\bar{q}}^{\lambda=\pm}(x,k_{1\perp};Q^2) = -e_f \frac{\bar{u}_q(k) \; \gamma \cdot \varepsilon^{\lambda} \; v_{\bar{q}}(q-k)}{\sqrt{x(1-x)} \left[Q^2 + \frac{k_{1\perp}^2 + m^2}{x(1-x)}\right]}$$

2) Non-Pertubative (NP) effects (E.R.Arriola et al, PRD74,054023 (2006))

$$\begin{split} \psi_A^{\gamma}(x,k_{\perp 1};Q^2) &= \frac{6(1+Q^2/m_{\rho}^2)}{m_{\rho}^2 \left(1+4\frac{k_{\perp 1}^2+Q^2x(1-x)}{m_{\rho}^2}\right)^{5/2}} \end{split}$$