From BFKL to saturation physics in quarkonium production

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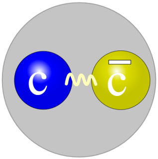
Motivations

- Heavy-flavored productions are recognized as excellent probe channels of Quantum Chromodynamics
- This resulted in remarkable interest over the last decades on both their formal and phenomenological aspects
- At modern colliders heavy-flavor production enters the two-scale regime: $S \gg M^2 \gg \Lambda^2_{QCD} \rightarrow$ large $\ln(S/M^2)$

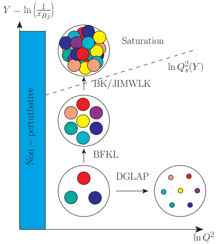
$$\downarrow$$

BFKL/saturation domain

- Quarkonium and small-x have an intriguing overlap

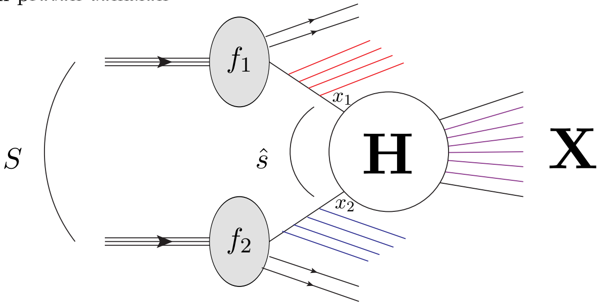


$$\longleftrightarrow$$



High-energy inclusive reactions

- Proton-proton collision



- High-energy logaritms

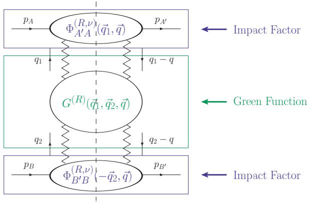
$$\ln\left(\frac{S}{Q^2}\right) = \ln\left(\frac{1}{x_1}\right) + \ln\left(\frac{1}{x_2}\right) + \ln\left(\frac{\hat{s}}{Q^2}\right)$$

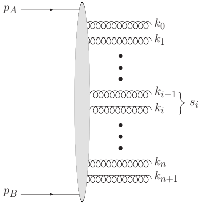
- $\ln(1/x_i) \to$ **small-x evolution** of the parton distribution $f_i(x_i)$
- $\ln\left(\frac{\hat{s}}{Q^2}\right) \to$ logarithmically enanched contributions in coefficients functions

BFKL resummation

- BFKL approach
  - *i.* ***Regge limit*** $\longrightarrow s \simeq -u \to \infty$, $t = q^2$ fixed (i.e not growing with s)
  - *ii.* Valid in **LLA** ($\alpha_s^n \ln^n s$ resummed) and **NLLA** ($\alpha_s^{n+1} \ln^n s$ resummed)

- BFKL factorization for $\Im \mathcal{A}_{AB}^{A'B'} \to$ convolution of a ***Green function*** (process independent) with the ***Impact factors*** (process dependent)





$$\Im \mathcal{A}_{AB}^{A'B'} = \frac{s}{(2\pi)^{D-2}} \int \frac{d^{D-2} q_1}{\vec{q}_1^{\,2} (\vec{q}_1 - \vec{q}\,)^{\,2}} \frac{d^{D-2} q_2}{\vec{q}_2^{\,2} (\vec{q}_2 - \vec{q}\,)^{\,2}}$$

$$\times \sum_{\nu} \Phi_{A'A}^{(R,\nu)}(\vec{q}_1, \vec{q}, s_0) \int \frac{d\omega}{2\pi i} \left[\left(\frac{s}{s_0} \right)^{\omega} G_{\omega}^{(R)}(\vec{q}_1, \vec{q}_2; \vec{q}\,) \right] \Phi_{B'B}^{(R,\nu)}(-\vec{q}_2, \vec{q}, s_0)$$

- $G_{\omega}^{(R)}(\vec{q}_1, \vec{q}_2; \vec{q}\,)$ satisfies the BFKL equation $\Longrightarrow \sigma_{AB \to X} = \frac{\Im \mathcal{A}_{AB}^{AB}}{s} \sim s^{\omega_0}$

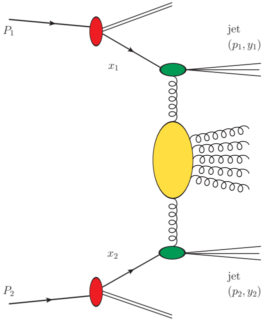
Forward/backward: Mueller-Navelet jets

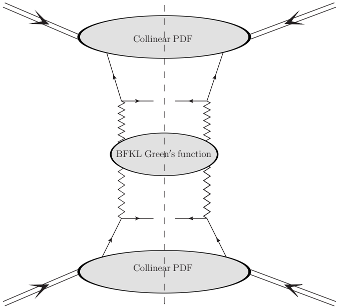
- Inclusive production of **rapidity-separated jets** in proton-proton collision
- Large energy logarithms → *BFKL resummed partonic cross section*
- Moderate values of parton x → *collinear PDFs*

[Mueller, Navelet (1987)]

- **Full next-to-leading analysis**

[Ducloué, Szymanowski, Wallon (2013,2014)]
[Caporale, Ivanov, Murdaca, Papa (2014,2015)]

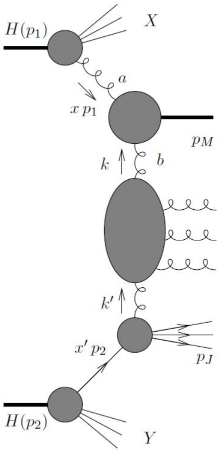




J/ψ plus jet production

- Process: proton(p_1) + proton(p_2) $\to J/\psi + X +$ jet

- **Hybrid collinear/BFKL approach**
- High-energy hadroproduction of a J/Ψ meson and a jet, with a remnant X
- Both the J/Ψ and the jet emitted with large transverse momenta and well separated in rapidity
- NLA BFKL + NLO jet + LO J/Ψ
  - LO J/Ψ IF calculated in **NRQCD** (Color-singlet and Color-octect)
  - LO J/Ψ IF calculated in **color evaporation model (CEM)**
- Realistic CMS and CASTOR rapidity ranges, fixed p_T final states

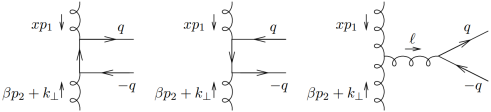


[Boussarie, Ducloué, Szymanowski, Wallon (2018)]

Impact factors in NRQCD

- Impact factor in **Color Evaporation Model**

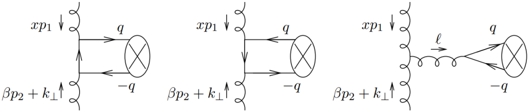
$$\mathcal{V}_{J/\psi} = F_{J/\psi} \int_{4m_c^2}^{4m_D^2} dM^2 \frac{d\mathcal{V}_{c\bar{c}}}{dM^2}$$



- **NRQCD expansion**

$$|J/\psi\rangle = O(1)\left|Q\bar{Q}\left[{}^3S_1^{(1)}\right]\right\rangle + O(v)\left|Q\bar{Q}\left[{}^3P_1^{(8)}\right]g\right\rangle + O\left(v^2\right)$$

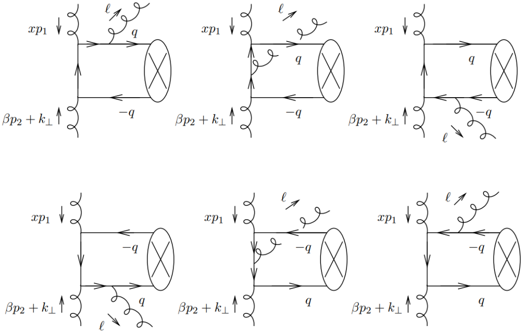
- Impact factor in the *Color Octect* case



$$\left[v\left(q\right)\bar{u}\left(q\right)\right]_{\alpha\beta}^{ji\to a} \to t_{ji}^a d_8 \left(\frac{\langle\mathcal{O}_8\rangle_{J/\psi}}{m}\right)^{\frac{1}{2}} \left[\hat{\varepsilon}_{J/\psi}^* \left(2\hat{q}+2m\right)\right]_{\alpha\beta}$$

Impact factors in NRQCD

- Impact factor in the *Color Singlet* case

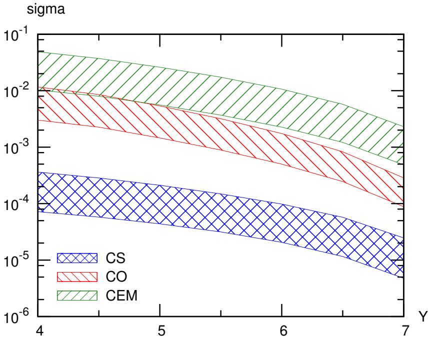


$$[v\,(q)\,\bar{u}\,(q)]^{ij}_{\alpha\beta} \to \frac{\delta^{ij}}{4N_c}\left(\frac{\langle \mathcal{O}_1\rangle_{J/\psi}}{m}\right)^{\frac{1}{2}}\left[\hat{\varepsilon}^*_{J/\psi}\,(2\hat{q}+2m)\right]_{\alpha\beta}$$

J/ψ plus jet production

- Realistic CMS and CASTOR rapidity ranges, fixed p_T final states

$$\frac{d\sigma}{d|k_{J/\psi}|d|k_{\rm jet}|dY}[\text{nb GeV}^{-2}] \qquad\qquad |\text{k}_{J/\psi}| = |\text{k}_{\text{jet}}| = 10 \text{ GeV}$$



[Boussarie, Ducloué, Szymanowski, Wallon (2018)]

J/ψ production from single parton fragmentation

- J/Ψ production from single parton fragmentation
  [Celiberto, M.F. (2022)] [Celiberto (2023)]

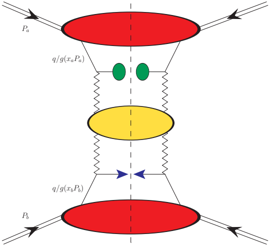
$$p(P_a) + p(P_b) \to \mathcal{Q}(p_{\mathcal{Q}}, y_{\mathcal{Q}}) + X + \text{jet}(p_J, y_J)$$

- **Hybrid cross section**

$$\frac{d\sigma}{dy_{\mathcal{Q}}\, dy_J\, d^2\vec{p}_{\mathcal{Q}}\, d^2\vec{p}_J} = \frac{1}{(2\pi)^2}$$

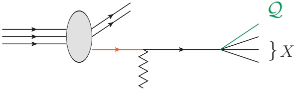
$$\times \int \frac{d^2\vec{q}_1}{\vec{q}_1^{\,2}} V_{\mathcal{Q}}(\vec{q}_1, x_g, \vec{p}_{\mathcal{Q}}) \int \frac{d^2\vec{q}_2}{\vec{q}_2^{\,2}} V_J(\vec{q}_2, x_J, \vec{p}_J)$$

$$\times \int_{\delta - i\infty}^{\delta + i\infty} \frac{d\omega}{2\pi i} \left(\frac{x_{\mathcal{Q}} x_J s}{s_0} \right)^{\omega} G_\omega(\vec{q}_1, \vec{q}_2)$$

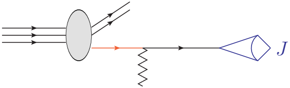


- Impact factors **[Ivanov, Papa (2012)]**

$$V_{\mathcal{Q}}(\vec{q}_1, x_{\mathcal{Q}}, \vec{p}_{\mathcal{Q}}) = f_{q/g} \otimes H \otimes D^{\mathcal{Q}}_{q/g}$$



$$V_J(\vec{q}_2, x_J, \vec{p}_J) = f_{q/g} \otimes H \cdot \mathcal{J}$$

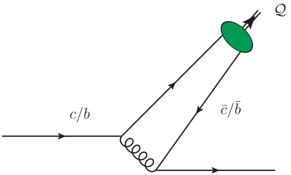


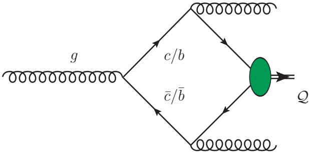
J/ψ production from single parton fragmentation

- J/Ψ fragmentation functions in the **NRQCD** framework

$$D_i^{\mathcal{Q}}(z,\mu_F) = \sum_{[n]} \mathcal{D}_i^{Q\bar{Q}}(z,\mu_F,[n])\langle \mathcal{O}^{\mathcal{Q}}([n])\rangle$$

- Spin-triplet (vector) and color-singlet quarkonium state, ${}^3S_1^{(1)}$
- Initial inputs for the DGLAP evolution





[Braaten, Cheung, Yuan (1993)] [Zheng, Chang, Wu (2019)] [Braaten, Yuan (1993)]

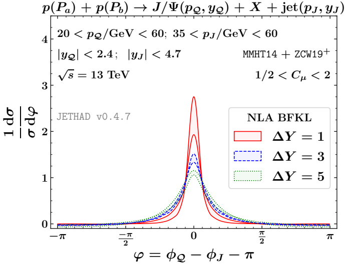
- Evolution performed by **APFEL++**

$$\left\{D_Q^{\mathcal{Q}}(z,3m_Q), D_g^{\mathcal{Q}}(z,2m_Q)\right\}_{\text{NRQCD}} \xrightarrow{\textbf{APFEL++}} \boxed{\textbf{ZCW19}^+ \textbf{ Onium FFs}}$$

J/ψ production from single parton fragmentation

- Azimuthal distribution

$$\frac{1}{\sigma}\frac{d\sigma}{d\varphi} = \frac{1}{2\pi}\left\{1 + 2\sum_{n=1}^{\infty}\cos(n\varphi)\langle\cos(n\varphi)\rangle\right\} = \frac{1}{2\pi}\left\{1 + 2\sum_{n=1}^{\infty}\cos(n\varphi)R_{n0}\right\} .$$



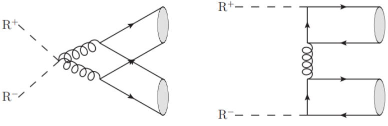
[Celiberto, M.F. (2022)]

- NLO analysis but only in the fragmentation region (high-p_Q^2 region)
- Higher sensitivity to high-energy effects at lower hard scales

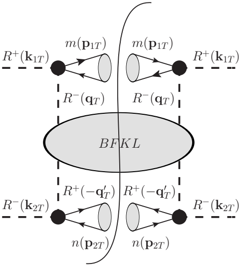
$$\alpha_s(Q^2)\ln\left(\frac{\hat{s}}{Q^2}\right) \sim 1$$

Double prompt J/Ψ hadroproduction

- Double prompt J/Ψ in Parton Reggeization Approach (PRA)
  [He, Kniehl, Nefedov, Saleev (2019)]



(a) (b)



- PRA cross-section

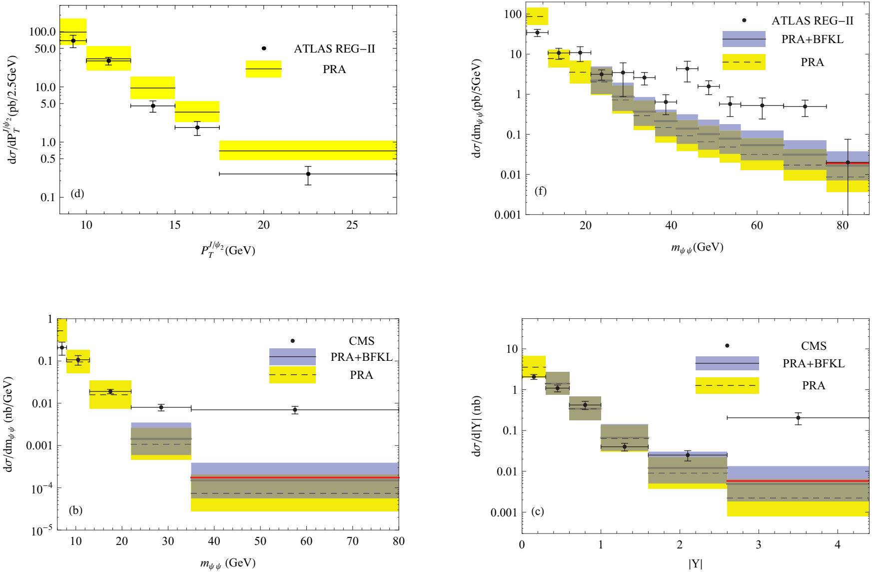
$$d\sigma^{\rm PRA}(AB \to 2J/\Psi + X) = \sum_{m,n,H_1,H_2} \int \frac{dx_1}{x_1} \int \frac{dx_2}{x_2} \int \frac{d^2\boldsymbol{k}_{1T}}{\pi} \int \frac{d^2\boldsymbol{k}_{2T}}{\pi}$$

$$\times \mathcal{U}^{+/A}(x_1, t_1, \mu^2)\, \mathcal{U}_{R^-/B}(x_2, t_2, \mu^2)\, d\sigma^{\rm PRA}_{mn}\, \langle \mathcal{O}^{H_1}(m) \rangle \, \langle \mathcal{O}^{H_2}(m) \rangle$$

- BFKL/PRA matching

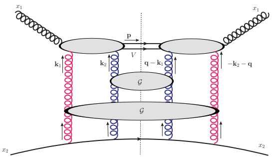
$$d\sigma = d\sigma^{\rm PRA} + d\sigma^{\rm BFKL} - d\sigma^{{\rm BFKL},\Delta Y = 0}$$

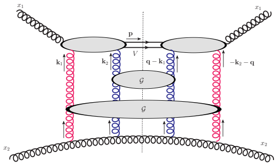
Double prompt J/Ψ hadroproduction



J/Ψ hadroproduction and Pomeron loops

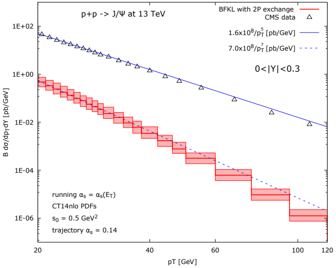
- Color Singlet Mechanism (CSM)
- Pomeron loop contributions to the inclusive J/Ψ hadroproduction





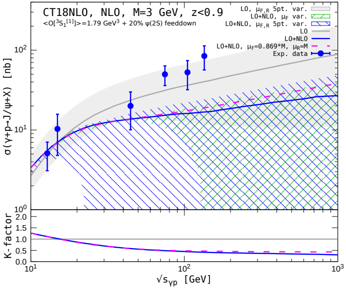
- A careful analysis shows the suppression of these contributions

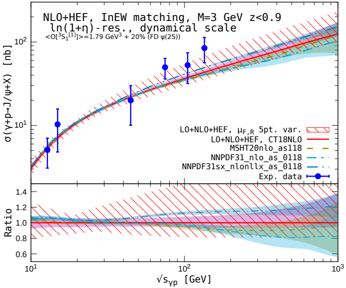
[Kotko, Motyka, Sadzikowski, Stasto (2019)]



Curing high-energy instability of quakonium production

- Instabilities in the inclusive J/Ψ photoproduction for $\sqrt{s} > 20$ GeV





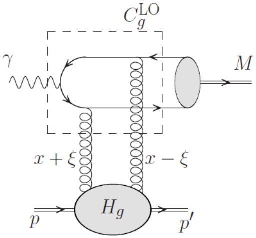
- Similar path observed in the inclusive η_c hadroproduction for $\sqrt{s} > 1$ TeV
- The instabilities are due to a lack of resummation of terms $\alpha_s^n \ln^{n-1}(M^2/\hat{s})$
- High-energy factorization treatment in the DLA

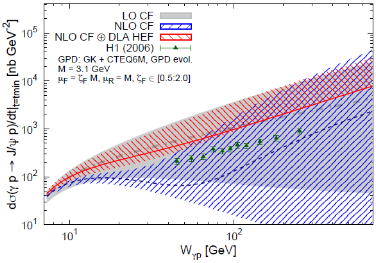
$$\frac{d\hat{\sigma}_{i\gamma}}{dz}\left(\frac{M^2}{\hat{s}}, \mu_F, \mu_R\right) = \frac{1}{M^2}\int_0^\infty dk_T^2 C_{gi}(X, k_T^2, \mu_F, \mu_R)\int_{1/z}^\infty \frac{dy}{y}\frac{d\mathcal{H}}{dz}(k_T^2, y, z)$$

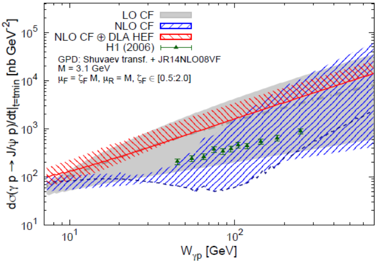
[Lansberg, Nefedov, Ozelick (2023)]

Curing high-energy instability of quakonium production

- Exclusive J/Ψ photoproduction
- Instabilities for $\sqrt{s} > 30$ GeV
- High-energy resummation allows to get more stable result
- Data are systematically below the HEF prediction



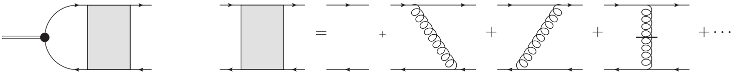




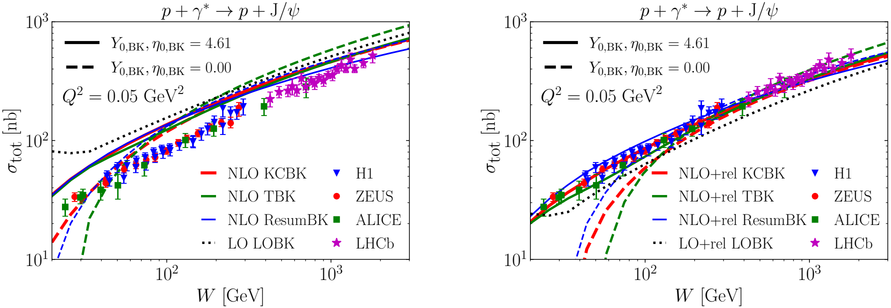
[Flett, Lansberg, Nabeebaccus, Nefedov, Sznajder, Wagner (to appear)]

- Exclusive diffractive J/Ψ production

  [Mäntysaari, Penttala (2022)]
- Tool: Light-cone perturbation theory + NRQCD
- NLO corrections to the photon-J/Ψ wave functions overlap



- Comparison with H1, ZEUS, HERA and LHCb data



(a) No relativistic corrections

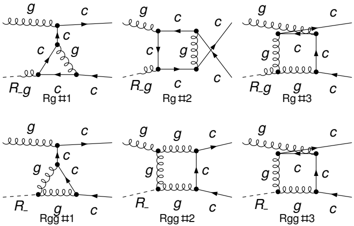
(b) NLO relativistic corrections

Towards NLL BFKL resummation for Quarkonium

- NLO impact factor for the $\eta_{c,b}$ hadroproduction **[Nefedov (2023)]**

$$R_+(k) + g(q) \to Q\bar{Q}\left[{}^1S_0^{[1]}\right](p)$$

- Some diagrams of the off-shell coefficient functions



- Gauge invariant **Lipatov effective action** $\to$ eikonal approximation and factorization in rapidity space
- Regularization of rapidity divergences through **tilted Wilson lines** allows automation of computational steps (IBP reductions, ecc...)

- DIS total cross-section

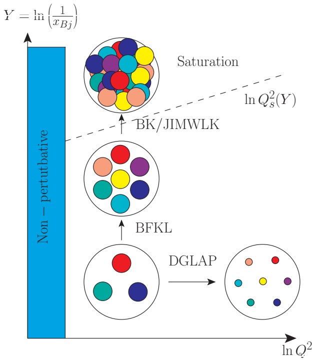
$$\sigma_{\gamma^* P}(x) = \Phi_{\gamma^*\gamma^*}(\vec{k}\,) \otimes_{\vec{k}} \mathcal{F}(x,\vec{k})$$

$$\downarrow$$

$$\sigma_{\gamma^* P}(x) \sim \left(\frac{s}{Q^2}\right)^{\omega_0} = \left(\frac{1}{x}\right)^{\omega_0}$$

- ***Martin-Froissart bound***

$$\sigma_{tot} \lesssim c \ln^2 s$$



- The violation of Martin-Froissart bound means a breakdown of the unitarity
- The violation is physically interpretable as an *infinite growth* of the unintegrated gluon density at small value of the Bjorken-x

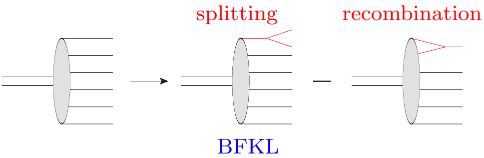
$$\boxed{\Delta x_\perp = \frac{1}{Q}}$$

Saturation physics

- ***Saturation effects***

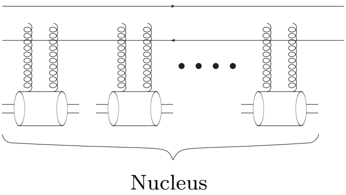
  i. Very dense system $\Longrightarrow$ *Recombination effects*

  [Gribov, Levin, Ryskin, Mueller and Qui (1980-1983)]



  ii. In large nuclei $\Longrightarrow$ *Multiple re-scattering* ($\alpha_s^2 A^{1/3}$ resummation)

  [Glauber (1959)—Gribov (1969)], [Kovchegov (1999)]



Color glass condensate

- Characteristic ***Saturation scale***

$$Q_s^2 \sim \left(\frac{A}{x}\right)^{1/3} \Lambda^2_{\rm QCD} \qquad \alpha_s(Q_s^2) \ll 1 \implies \textit{Weakly coupled QCD}$$

Saturation window: $Q^2 < Q_s^2$

- The small-x gluon field can be obtained by solving the classical Yang-Mills equation (MV-model)

[McLerran, Venugopalan (1994)]

- The solution that is obtained has three main properties:

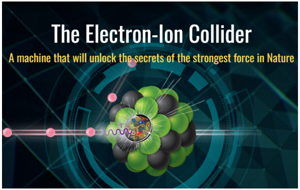
  i. **Color** $\rightarrow$ dominated by colored particle (gluons)
  ii. **Condensate** $\rightarrow$ very high-density of gluons
  iii. **Glass** $\rightarrow$ well-separated time scales between small-x and large-x, with this latter appearing as "frozen"

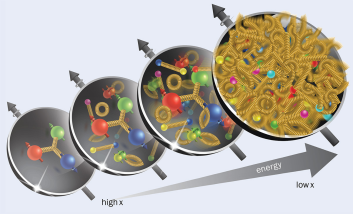
- *Quantum corrections* to MV model $\rightarrow$ non-linear small-x evolution

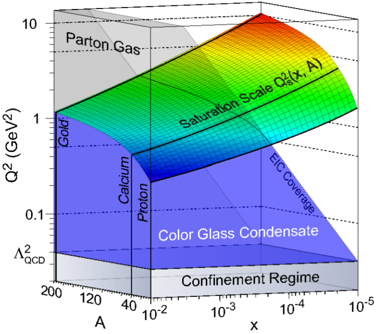
[Balitsky (1995)]

[Jalilian-Marian, Iancu, McLerran, Weigert, Kovner, Leonidov]

Saturation at the Electron-Ion collider (EIC)







- At the **EIC**, the saturation scale Q_s will be in the perturbative range

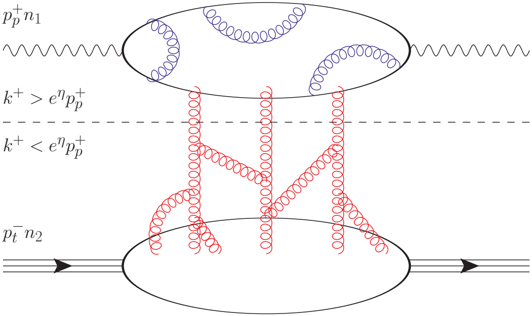
$$Q_s^2 \sim \left(\frac{A}{x}\right)^{1/3} \Lambda^2_{\mathrm{QCD}}$$

- **Perturbative control on gluonic saturation**

$$\Lambda^2_{QCD} \ll Q^2 < Q_s^2$$

Shockwave approach

- High-energy approximation $s = (p_p + p_t)^2 \gg \{Q^2\}$



- Separation of the gluonic field into "fast" (quantum) part and "slow" (classical) part through a rapidity parameter $\eta < 0$

[Balitsky (1996-2001)]

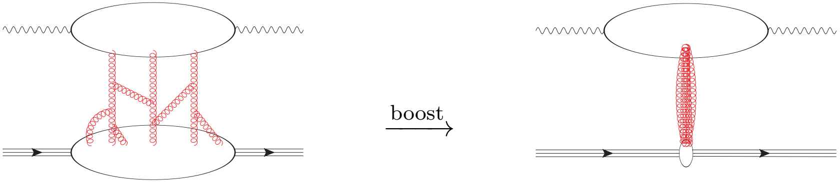
$$\mathcal{A}^\mu(k^+, k^-, \vec{k}) = A^\mu(k^+ > e^\eta p_p^+, k^-, \vec{k}) + b^\mu(k^+ < e^\eta p_p^+, k^-, \vec{k})$$

$e^\eta \ll 1$

Shockwave approach

- Large longitudinal Boost: $\Lambda = \sqrt{\frac{1+\beta}{1-\beta}} \sim \frac{\sqrt{s}}{m_t}$

$$\begin{cases} b^+(x^+, x^-, \vec{x}) & = \Lambda^{-1} b_0^+(\Lambda x^+, \Lambda^{-1} x^-, \vec{x}) \\ b^-(x^+, x^-, \vec{x}) & = \Lambda b_0^-(\Lambda x^+, \Lambda^{-1} x^-, \vec{x}) \\ b^i(x^+, x^-, \vec{x}) & = b_0^i(\Lambda x^+, \Lambda^{-1} x^-, \vec{x}) \end{cases}$$



$b_0^\mu(x)$

$b^\mu(x^+, x^-, \vec{x}) = \delta(x^+) \mathbf{B}(\vec{x}) n_2^\mu + \mathcal{O}\left(\Lambda^{-1}\right)$

Shockwave approximation

- Light-cone gauge $A \cdot n_2 = 0 \implies A \cdot b = 0 \implies$ *Simple effective Lagrangian*

$$\mathcal{L}_{\text{int}}^S = ig T^c_{ab} b^{-c} g_{\perp\perp}^{\alpha\beta} \left(\frac{\partial A^a_\beta}{\partial x^-} \right) A^b_\alpha + \bar{\psi} g b^- \gamma^+ \psi \ ,$$

CGC description of J/Ψ production

- J/Ψ productions at low-p_T provides a semi-hard scale comparable to the expected saturation scales in high-energy proton-nucleus and electron-nucleus collisions

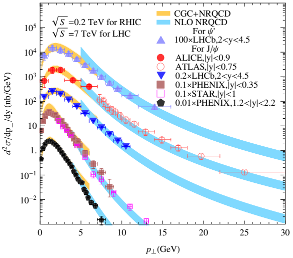
$\downarrow$

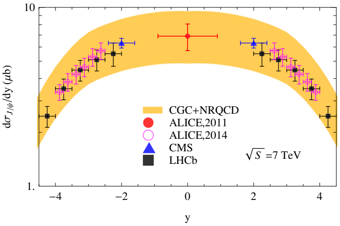
CGC/Shockwave description at low-p_T

[Kang, Ma, Venugopalan (2014)]
[Qiu, Sun, Xiao, Yuan (2014)]
[Ma, Venugopalan, Zhang (2015)]
[Ma, Stebel, Venugopalan (2018)]

- Inclusive J/Ψ production in p-p collisions

[Ma, Venugopalan (2014)]





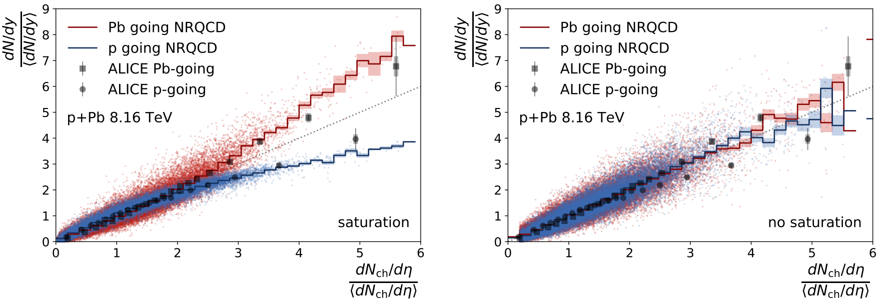
Probing saturation through the J/Ψ production

- J/Ψ mass act as a good scale to compare to the **saturation scale** $Q_s(x)$
- However, heavier mesons are considerably less sensitive to saturation effects with respect to light mesons

↓

- Rapidity dependent J/Ψ production in hadronic collisions, in combination with charged hadron production

[F. Salazar, B. Schenke, Soto-Ontoso (2022)]



- Inclusive J/Ψ plus jet in DIS at small-x

[Kang, Li, Salazar (2023)]

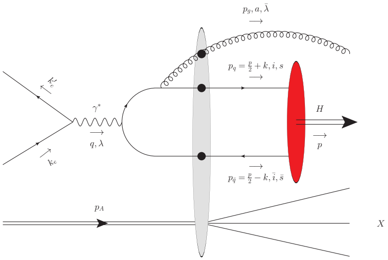
- Color Glass Condensate EFT

+

- NRQCD framework

$$d\sigma^H = \sum_k d\hat{\sigma}_k \langle \mathcal{O}_k^H \rangle \qquad k = {}^{2S+1}L_J^{[c]}$$

- S-wave (color octect and singlet)



Outlook

- Integrating over the kinematical variables of the additional gluon

↓

(real) NLO corrections to the Quarkonium production in DIS

- Limit of small transverse momentum imbalance between $\mathcal{Q}$ and the jet

↓

Overlap with the TMD formalism domain **[Maxia, Yuan (2024)]** **[Luca's talk]**

Summary and conclusions

- There is a fascinating link between quarkonium and small-x physics
- Several production channels of the J/Ψ are strongly influenced by the **BFKL dynamics**
  i. Inclusive J/Ψ plus jet hadroproduction
  ii. Inclusive J/Ψ photoproduction
  iii. Exclusive J/Ψ photoproduction
- From a different perspective, the quarkonium represent an opportunity for the small-x community from many points of view
  i. Developing efficient computational tools
  ii. Testing various approaches to deal with small-x
  iii. Probing the **gluonic saturation** inside nuclei
- The challenge in the coming years will be to continue pushing studies to a precision level

Summary and conclusions

- There is a fascinating link between quarkonium and small-x physics
- Several production channels of the J/Ψ are strongly influenced by the **BFKL dynamics**
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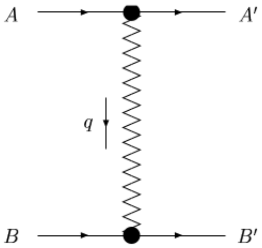
Thank you for the attention!

Backup

The Reggeized gluon

Scattering process $A+B \longrightarrow A'+B'$

- Gluon quantum numbers in the t-channel
- Regge limit: $s \simeq -u \to \infty$, t fixed (i.e not growing with s)
- All-order resummation:
  leading logarithmic approximation (LLA): $(\alpha_s \ln s)^n$
  next-to-leading logarithmic approximation (NLA): $\alpha_s(\alpha_s \ln s)^n$



$$(\mathcal{A})_{AB}^{A'B'} = \Gamma^c_{A'A}\left[\left(\frac{-s}{-t}\right)^{j(t)} - \left(\frac{s}{-t}\right)^{j(t)}\right]\Gamma^c_{B'B}$$

$$j(t) = 1+\omega(t), \qquad j(0)=1$$

$j(t)$-**Reggeized gluon trajectory**

$$\Gamma^c_{A'A} = g\,\langle A'|T^c|A\rangle\,\Gamma_{A'A}$$

T^c- fundamental(quarks) or adjoint(gluons)

- LLA

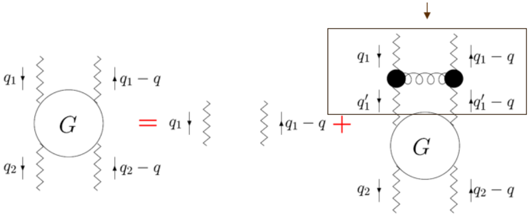
[Balitsky, Fadin, Lipatov (1979)]

$$\Gamma^{(0)}_{A'A} = \delta_{\lambda_{A'}\lambda_A}, \quad \omega^{(1)}(t) = \frac{g^2 t}{(2\pi)^{(D-1)}}\frac{N}{2}\int\frac{d^{D-2}k_\perp}{k_\perp^2(q-k)_\perp^2} = -g^2\frac{N\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}}\frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)}(\vec{q}^{\,2})^\epsilon$$

BFKL resummation

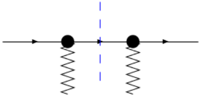
- $G_\omega^{(R)}(\vec{q}_1, \vec{q}_2; \vec{q}\,)$-Mellin transform of the Green function for the Reggeon-Reggeon scattering

$$\omega G_\omega^{(R)}(\vec{q}_1, \vec{q}_2; \vec{q}\,) = \vec{q}_1^{\,2}(\vec{q}_1 - \vec{q}\,)^2 \delta^{(D-2)}(\vec{q}_1 - \vec{q}_2) + \int \frac{d^{D-2}q_1'}{\vec{q}_1^{\,\prime 2}(\vec{q}_1^{\,\prime} - \vec{q}\,)^2} \mathcal{K}^{(R)}(\vec{q}_1, \vec{q}_1^{\,\prime}; \vec{q}\,) G_\omega^{(R)}(\vec{q}_1^{\,\prime}, \vec{q}_2; \vec{q}\,)$$



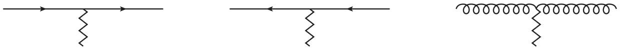
- $\Phi_{P'P}^{(R,\nu)}$- LO impact factor in the t-channel color state (R, ν)

$$\Phi_{PP'}^{(R,\nu)} = \langle cc'|\hat{\mathcal{P}}|\nu\rangle \sum_{\{f\}} \int \frac{ds_{PR}}{2\pi} d\rho_f \Gamma^c_{\{f\}P} (\Gamma^{c'}_{\{f\}P'})^*$$



LO Impact factors

- Parton-Parton-Reggeon (PPR) vertices



- LO jet impact factor

$$c_J(n,\nu,|\vec{p}\,|,x) = 2\sqrt{\frac{C_F}{C_A}}(|\vec{p}\,|^2)^{i\nu-1/2}\left(\frac{C_A}{C_F}f_g(x) + \sum_{\beta=q,\bar{q}} f_\beta(x)\right) \;,$$

- LO light hadron impact factor

$$c_Q(n,\nu,|\vec{p}\,|,x) = 2\sqrt{\frac{C_F}{C_A}}(|\vec{p}\,|^2)^{i\nu-1/2}\int_x^1 \frac{d\zeta}{\zeta}\left(\frac{\zeta}{x}\right)^{2i\nu-1}$$
$$\times\left[\frac{C_A}{C_F}f_g(\zeta)D_g^Q\left(\frac{x}{\zeta}\right) + \sum_{\alpha=q,\bar{q}} f_\alpha(\zeta)D_\alpha^Q\left(\frac{x}{\zeta}\right)\right]$$

- Both known at NLO

[Bartels, Colferai, Vacca (2003)]
[Caporale, Ivanov, Murdaca, Papa (2014)]
[Ivanov, Papa (2012)]

Treatment of heavy-quarks masses

- The mass of light quarks ($q = u, d, s$) is always set to zero. They are always present in the initial state
- The presence in the initial state and the way one must treat the mass of an heavy-quark ($Q = c, b$) depends on kinematical conditions
- **Fixed flavor number scheme (FFNS)**
  - $m_Q \neq 0$
  - Heavy quark is present only in the final state
  - Logarithms of $p_{T,Q}^2/m_Q^2$ missed by the scheme
  - It is appropriate in regions of moderate $p_{T,Q}^2$
- **Zero-mass variable flavor number scheme (0M-VFNS)**
  - $m_Q = 0$
  - Heavy quark is present in the initial state above a fixed threshold.
  - Powers of $m_Q^2/p_{T,HQ}^2$ missed by the scheme
  - It is appropriate in region of high $p_{T,HQ}^2 \gg m_Q^2$
- **General-mass variable flavor number schemes (GM-VFNS)**
  - It is a matching between the previous schemes

J/Ψ fragmentation: heavy-quark channel

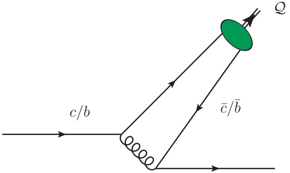
- NRQCD FFs

$$D_i^{\mathcal{Q}}(z,\mu_F) = \sum_{[n]} \mathcal{D}_i^{Q\bar{Q}}(z,\mu_F,[n])\langle \mathcal{O}^{\mathcal{Q}}([n])\rangle$$

- Spin-triplet (vector) and color-singlet quarkonium state, ${}^3S_1^{(1)}$
- Heavy-quark fragmentation function computed at $\mu_0 = 3m_Q$ in NRQCD

$$D_Q^{\mathcal{Q}}(z,\mu_F \equiv \mu_0) = D_Q^{\mathcal{Q},\mathrm{LO}}(z)$$

$$+\frac{\alpha_s^3(\mu_R)}{m_Q^3}\,|\mathcal{R}_{\mathcal{Q}}(0)|^2\,\Gamma^{\mathcal{Q},\mathrm{NLO}}(z)$$



- LO fragmentation function

[Braaten, Cheung, Yuan (1993)]

$$D_Q^{\mathcal{Q},\mathrm{LO}}(z) = \frac{\alpha_s^2(\mu_R)}{m_Q^3}\,\frac{8z(1-z)^2}{27\pi(2-z)^6}\,|\mathcal{R}_{\mathcal{Q}}(0)|^2\,(5z^4-32z^3+72z^2-32z+16)$$

- The NLO correction is given by a polynomial function

[X.C. Zheng, C.H. Chang, and X.G. Wu (2019)]

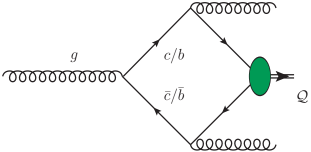
J/Ψ fragmentation: gluon and light-quark channels

- Gluon fragmentation function computed at $\mu_0 = 2m_Q$ in NRQCD

[Braaten, Yuan (1993)]

$$D_g^{\mathcal{Q}}(z, 2m_Q) = \frac{5}{36(2\pi)^2}\alpha_s^3(2m_Q)\frac{|\mathcal{R}_{\mathcal{Q}}(0)|^2}{m_{\mathcal{Q}}^3}\int_0^z d\xi \int_{(\xi+z^2)/2z}^{(1+\xi)/2} d\tau \frac{1}{(1-\tau)^2(\tau-\xi)^2(\tau^2-\xi)^2}$$

$$\sum_{i=1}^{2} z^i \left[f_i^{(g)}(\xi,\tau) + g_i^{(g)}(\xi,\tau)\frac{1+\xi-2\tau}{2(\tau-\xi)\sqrt{\tau^2-\xi}} \ln\left(\frac{\tau-\xi+\sqrt{\tau^2-\xi}}{\tau-\xi-\sqrt{\tau^2-\xi}}\right)\right]$$



- FFs evolved from the initial scale through the DGLAP evolution equations
- Light-quarks FFs (LQFFs) are zero at the initial scale: $D_q^{\mathcal{Q}}(z, 2m_Q) = 0$
- At higher scales LQFFs dynamically generated by evolution

$$D_q^{\mathcal{Q}}(z, \mu > 2m_Q) = \underbrace{D_q^{\mathcal{Q}}(z, 2m_Q)}_{0} + \frac{\alpha_s}{2\pi}\ln\left(\frac{\mu^2}{(2m_Q)^2}\right)\int_z^1 \frac{dx}{x} P_{qg}(x) D_g^{\mathcal{Q}}\left(\frac{z}{x}, 2m_Q\right)$$