Technische Universität München

Advance in pNRQCD studies for quarkonium production

Synergies between LHC and EIC for quarkonium physics

ECT*, Trento, 8 -12, July, 2024

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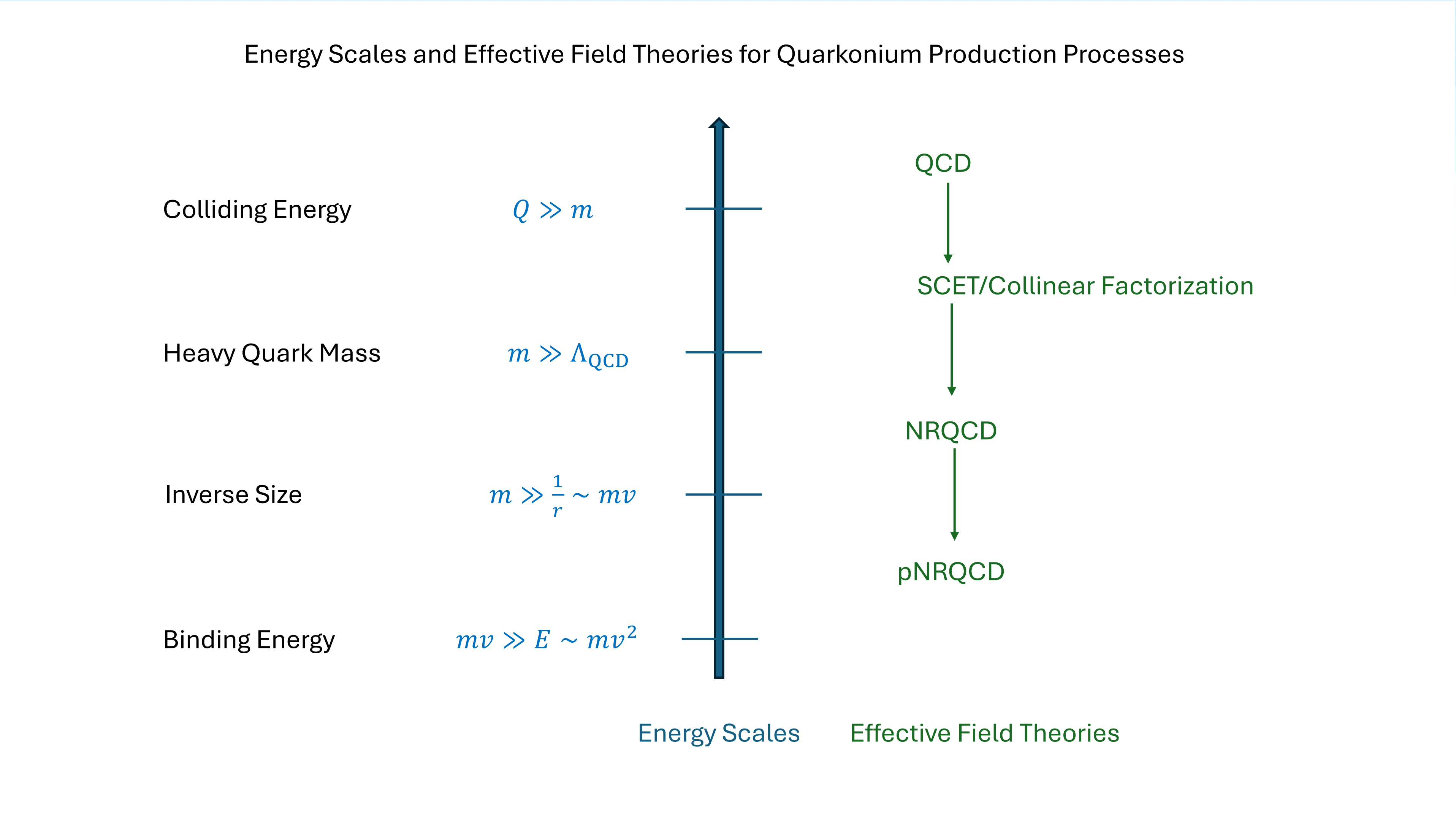
In collaboration with N. Brambilla, H. S. Chung, A. Vairo

Xiang-Peng Wang
08/07/2024

Contents

Introduction: Quarkonium Production and Energy Scales

- **Quarkonium production from colliders is typical complicated multi-scale problem:**



- **NRQCD factorization is the default framework to study quarkonium production**

Introduction: NRQCD Factorization

- **NRQCD factorization formula for inclusive quarkonium ($\mathcal{Q}$) production**

G. T. Bodwin, E. Bratten & G. P. Lapge, PRD 51 (1995) 1125, 2900+ citations

$$\sigma\left(A+B\to\mathcal{Q}+X\right)=\sum_{N}\hat{\sigma}\left(A+B\to Q\bar{Q}(N)+X\right)\,\langle\Omega|\,\mathcal{O}^{\mathcal{Q}}(N)\,|\Omega\rangle,$$

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  - $\hat{\sigma}$ **: Short-distance coefficients (SDCs), perturbatively calculable — α_s expansion**
  - $\langle\Omega|\mathcal{O}^{\mathcal{Q}}(N)|\Omega\rangle$ **: Universal non-perturbative long-distance-matrix-elements (LDMEs) defined using four fermion operators, has definite v scaling — v expansion**

$$\mathcal{O}^{\mathcal{Q}}(N_{\text{color singlet}})=\chi^\dagger\mathcal{K}_N\psi\mathcal{P}_{\mathcal{Q}(\mathbf{P}=\mathbf{0})}\psi^\dagger\mathcal{K}'_N\chi,$$

$$\mathcal{O}^{\mathcal{Q}}(N_{\text{color octet}})=\chi^\dagger\mathcal{K}_N T^a\psi\Phi_\ell^{\dagger ab}\mathcal{P}_{\mathcal{Q}(\mathbf{P}=\mathbf{0})}\Phi_\ell^{bc}\psi^\dagger\mathcal{K}'_N T^c\chi,$$

where Φ_ℓ **is a Wilson line along the path** ℓ **and** $\mathcal{P}_{\mathcal{Q}(\mathbf{P}=0)}=\sum_X \mathcal{Q}+X\rangle\langle\mathcal{Q}+X$

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  - **For J/ψ, N usually includes**: ${}^3S_1^{[1]}(v^0),\ {}^1S_0^{[8]}(v^3),\ {}^3S_1^{[8]}(v^4),\ {}^3P_J^{[8]}(v^4)\ldots$

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  - **Color-octet LDMEs are obtained through fitting NLO predictions to experimental data**

Introduction: Heavy Quark Spin Symmetry (HQSS)

- **Relates different P-wave LDMEs**

$$\langle \Omega \,|\, \mathcal{O}^{J/\psi}({}^3P_J^{[8]}) \,|\, \Omega \rangle = (2J+1) \langle \Omega \,|\, \mathcal{O}^{J/\psi}({}^3P_0^{[8]}) \,|\, \Omega \rangle + O(v^2),$$

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- **Relates different J/ψ LDMEs with η_c LDMEs (same for bottomonium)**

$$\langle \Omega | \mathcal{O}^{J/\psi}({}^3S_1^{[1/8]}) | \Omega \rangle = 3 \langle \Omega | \mathcal{O}^{\eta_c}({}^1S_0^{[1/8]}) | \Omega \rangle + O(v^2),$$

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Introduction: Successes and Challenges

- **Theory: NRQCD factorization is based on EFT, solved P-wave infrared divergence problem**

$$\langle \mathcal{O}^{Q}(^3S_1^{[8]})\rangle_{\rm NLO} = \langle \mathcal{O}^{Q}(^3S_1^{[8]})\rangle_{\rm Born} + \frac{2\alpha_s}{3\pi m_Q^2}\mu^{4-D}\left(\frac{1}{\epsilon_{\rm UV}} - \frac{1}{\epsilon_{\rm IR}}\right)\sum_{J=0}^{J=2}\left[\frac{C_F}{C_A}\langle \mathcal{O}^{Q}(^3P_J^{[1]})\rangle_{\rm Born} + (\frac{C_A}{2} - \frac{1}{C_A})\langle \mathcal{O}^{Q}(^3P_J^{[8]})\rangle_{\rm Born}\right].$$

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- **Phenomenology: NRQCD factorization can explain J/ψ large p_T distribution from hadron colliders, while color-singlet model predictions are two orders of magnitude smaller**

- **Dramatically different sets of J/ψ LDMEs are extracted by several groups through fitting**

	$p_{T,\rm min.}$ (Gev)	$\langle \mathcal{O}^{J/\psi}(^3S_1^{[1]})\rangle$ (Gev^3)	$\langle \mathcal{O}^{J/\psi}(^1S_0^{[8]})\rangle$ (10^{-2} Gev^3)	$\langle \mathcal{O}^{J/\psi}(^3S_1^{[8]})\rangle$ (10^{-2} Gev^3)	$\langle \mathcal{O}^{J/\psi}(^3P_0^{[8]})\rangle/m_c^2$ (10^{-2} Gev^3)
Chao *et al.* [1]	7	1.16	7.4	0.05	0
			0	1.11	1.89
Butenschön *et al.* [2]	3	1.32	3.04 ± 0.35	0.168 ± 0.046	-0.404 ± 0.072
Zhang *et al.* [3]	7	[0.24, 0.9]	[0.44, 1.13]	1.0 ± 0.3	1.7 ± 0.5
Bodwin *et al.* [4]	10	1.32	11.1 ± 1.4	-0.713 ± 0.364	-0.312 ± 0.151
Feng *et al.* [5]	7	1.16	5.66 ± 0.47	0.117 ± 0.058	0.054 ± 0.005

[1]. Y. Q. Ma, K. Wang & K. T. Chao, PRL 106 (2011) 042002
[2]. M. Butenschön & B. A. Kniehl, PRD 84 (2011) 051501
[3]. H. F. Zhang, Z. Sun, W. L. Sang & R. Li, PRL 114 (2015) 092006
[4]. G. T. Bodwin, K. T. Chao, H. S. Chung, U.~R.~Kim, J.~Lee & Y. Q. Ma, PRD 93 (2016) 034041
[5]. Y. Feng, B. Gong, C. H. Chang & J. X. Wang, PRD 99 (2019) 014044

Introduction: Successes and Challenges

- **Chao *et al.*** ($p_T > 7$ Gev)**, hadron production, fit two linear combinations**：

$$M_{0,\,r_0=3.9} = \langle \mathcal{O}^{J/\psi}(^1S_0^{[8]}) \rangle + \frac{r_0}{m_c^2} \langle \mathcal{O}^{J/\psi}(^3P_0^{[8]}) \rangle = (7.4 \pm 1.9) \times 10^{-2} \mathrm{Gev}^3,$$

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- **NRQCD factorization at NLO in small** p_T **region is not reliable? Or we need more accurate theoretical calculations, for instance, NNLO, relativistic corrections, resummation?**

Potential NRQCD

- **NRQCD still has several dynamic scales, $|p|$, E, $\Lambda_{\rm QCD}$, therefore, the NRQCD power counting is nonhomogeneous, it includes soft, potential, ultra-soft modes**

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pNRQCD: A. Pineda & J. Soto, Nucl. Phys. B Proc. Suppl. 64 (1998) 42

N. Brambilla, A. Pineda, J. Soto & A. Vairo, Nucl.Phys.B 566 (2000) 275; Rev.Mod.Phys. 77 (2005) 1423

$$\left.\begin{aligned}\mathcal{L} = \Phi(\mathbf{r})^\dagger\Big(& i\partial_0 - \frac{\mathbf{p}^2}{2m} - V^{(0)}(r) \\ & + \text{corrections to the potential} \\ & + \text{interactions with other low-} \\ & \text{energy degrees of freedom}\Big)\Phi(\mathbf{r})\end{aligned}\right\} \text{pNRQCD},$$

where $V^{(0)}(r)$ is the static potential ($-C_F\alpha_s/r$ in the perturbative case) and $\Phi(\mathbf{r})$ is the field associated with the $\bar{Q}$-Q state. This EFT turned out to meet all our expec-

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- **Weakly-coupled pNRQCD** ($|p| \gg E \gtrsim \Lambda_{\mathrm{QCD}}$)**: degree of freedom —** $S(r,R,t)$**,** $O(r,R,t)$**, including** $t\bar{t}$ **near threshold, and possibaly** $\Upsilon(1S)$**, perturbative matching**

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- **Strongly-coupled pNRQCD** ($\Lambda_{\rm QCD} \gg E$**, non-coulombic): degree of freedom —** $S(r,R,t)$**, including** $J/\psi,\ \psi(2S),\ \Upsilon(nS, n\geq 2)$ **and so on, nonperturbative matching**

LDMEs in pNRQCD

- **In strongly-coupled region, LDMEs can be factorized into products of wavefunction at the origin and gluonic correlators**

P-wave decay: N. Brambilla, D. Eiras, A. Pineda, J. Soto & A. Vairo, PRL 88 (2002) 012003

S-wave, P-wave decay & electromagnetic production: N. Brambilla, H. S. Chung, D. Müller & A. Vairo, JHEP 04 (2020) 095

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- **Generally, for a production LDMEs $\langle \Omega | \mathcal{O}^{\mathcal{Q}}(N) | \Omega \rangle$, applying pNRQCD, we have**

$$\langle\Omega|\mathcal{O}^{\mathcal{Q}}(N)|\Omega\rangle = \frac{1}{\langle \boldsymbol{P}=\boldsymbol{0}|\boldsymbol{P}=\boldsymbol{0}\rangle}\int d^3x_1 d^3x_2 d^3x_1' d^3x_2'\, \phi_{\mathcal{Q}}^{(0)}(\boldsymbol{x}_1-\boldsymbol{x}_2) \times \left[-V_{\mathcal{O}(N)}(\boldsymbol{x}_1,\boldsymbol{x}_2;\boldsymbol{\nabla}_1,\boldsymbol{\nabla}_2)\delta^{(3)}(\boldsymbol{x}_1-\boldsymbol{x}_1')\delta^{(3)}(\boldsymbol{x}_2-\boldsymbol{x}_2')\right]\phi_{\mathcal{Q}}^{(0)\dagger}(\boldsymbol{x}_1'-\boldsymbol{x}_2'), \tag{3.4}$$

where $V_{\mathcal{O}(N)}(\boldsymbol{x}_1,\boldsymbol{x}_2;\boldsymbol{\nabla}_1,\boldsymbol{\nabla}_2)$ is the contact term given by

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- $\phi_{\mathcal{Q}}^{(0)}$ **and** $V_{\mathcal{O}(N)}$ **have to be determined from matching the NRQCD LDMEs to pNRQCD**

Spin-1 S-wave LDMEs in pNRQCD

- **Based on perturbative quantum mechanics, up to $O(v^2, 1/N_c^2)$ corrections, we have**

$$\langle \mathcal{O}^V({}^3S_1^{[1]})\rangle = 2N_c \times \frac{3|R_V^{(0)}(0)|^2}{4\pi},$$

$$\langle \mathcal{O}^V({}^3S_1^{[8]})\rangle = \frac{1}{2N_c m^2}\frac{3|R_V^{(0)}(0)|^2}{4\pi}\mathcal{E}_{10;10},$$

$$\langle \mathcal{O}^V({}^1S_0^{[8]})\rangle = \frac{1}{6N_c m^2}\frac{3|R_V^{(0)}(0)|^2}{4\pi}c_F^2\mathcal{B}_{00},$$

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$$\mathcal{E}_{10;10} = \left| d^{dac}\int_0^\infty dt_1 t_1 \int_{t_1}^\infty dt_2 gE^{b,i}(t_2) \times \Phi_0^{bc}(t_1;t_2)gE^{a,i}(t_1)\Phi_0^{df}(0;t_1)\Phi_\ell^{ef}|\Omega\rangle\right|^2,$$

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- **Since the gluonic correlators have power divergence, and power divergences are dropped in the** $\overline{\mathrm{MS}}$ **scheme, the above gluonic correlators are not positive definite**

Gluonic Correlator Evolutions

- **At one-loop level, the gluonic correlators satisfy the evolution equations ($\mathscr{E}_{00}$ does not evolve at one-loop level)**

$$\frac{d}{d\log\Lambda}\mathcal{B}_{00} = -\frac{\alpha_s C_A}{\pi}\mathcal{B}_{00} + O(\alpha_s^2),$$

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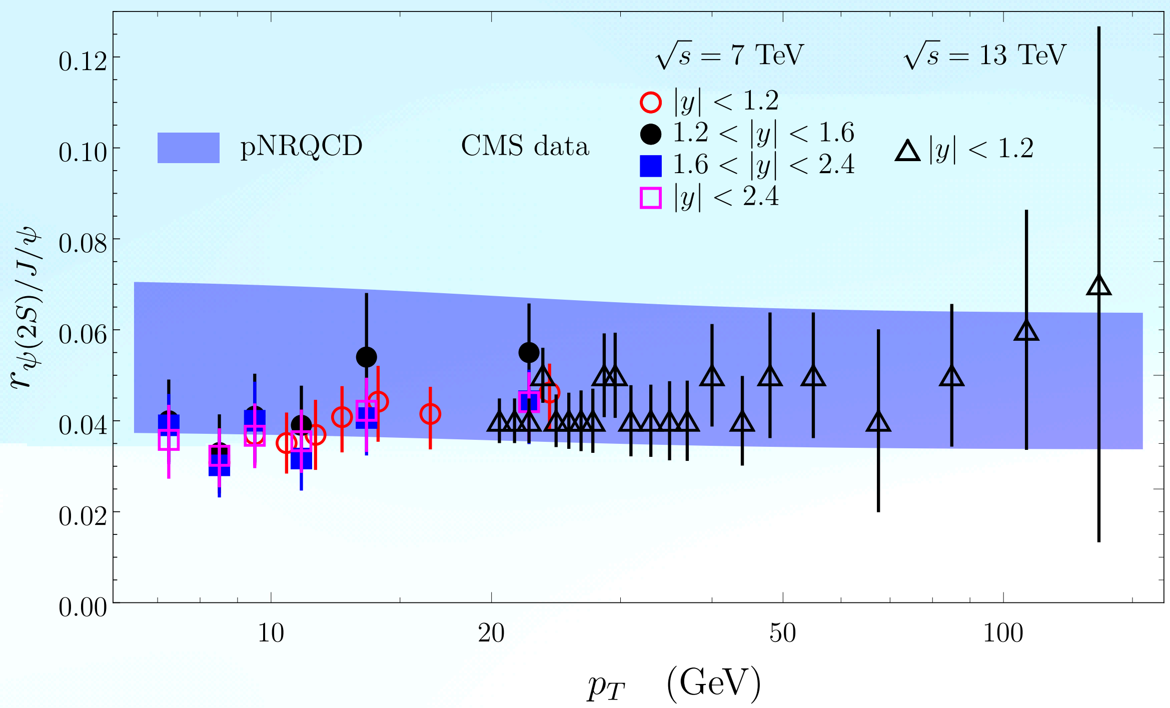
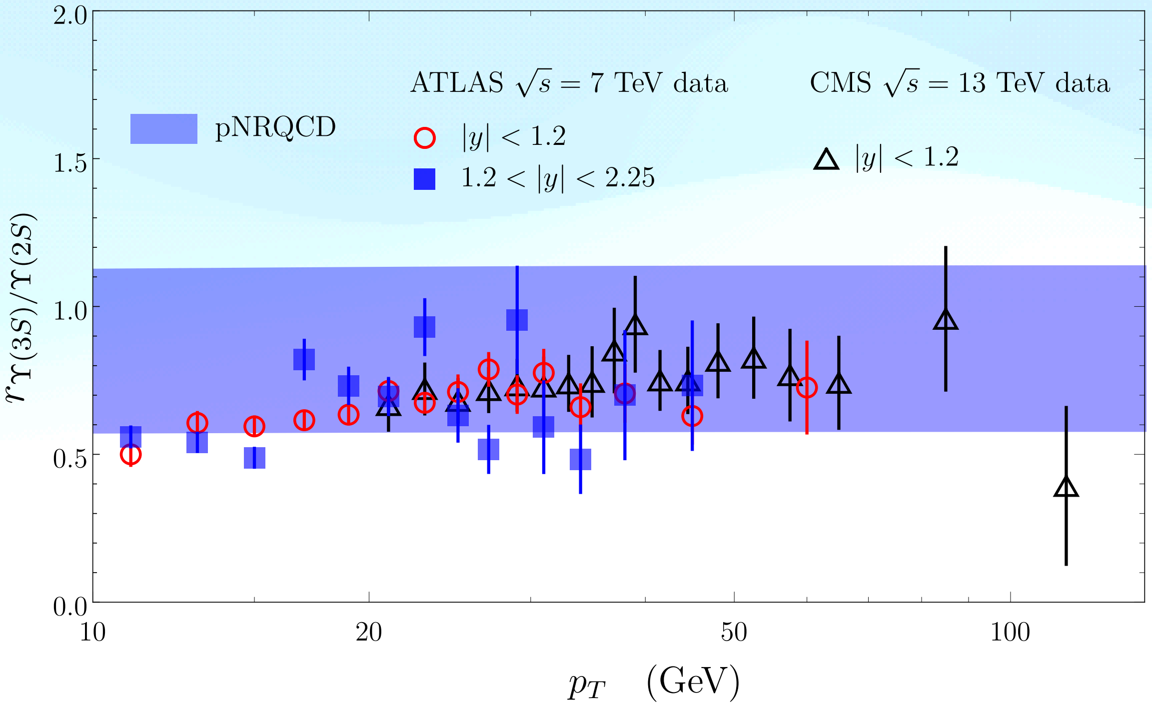
- **Theory and phenomennology impact： Relating different Spin- 1 S-wave quarkonium color-octet LDMEs through gluonic correlators, greatly reduced independent nonperturbative parameters for quarkonium production processes (from 12 and more to 3)**

Phenomenology 1： Cross Section Ratio Predictions

- **Without perturbative calculations, based on the NRQCD factroization and the LDMEs in pNRQCD, we give cross section ratio predictions for $\psi(2S)$ and J/ψ , $\Upsilon(3S)$ and $\Upsilon(2S)$**





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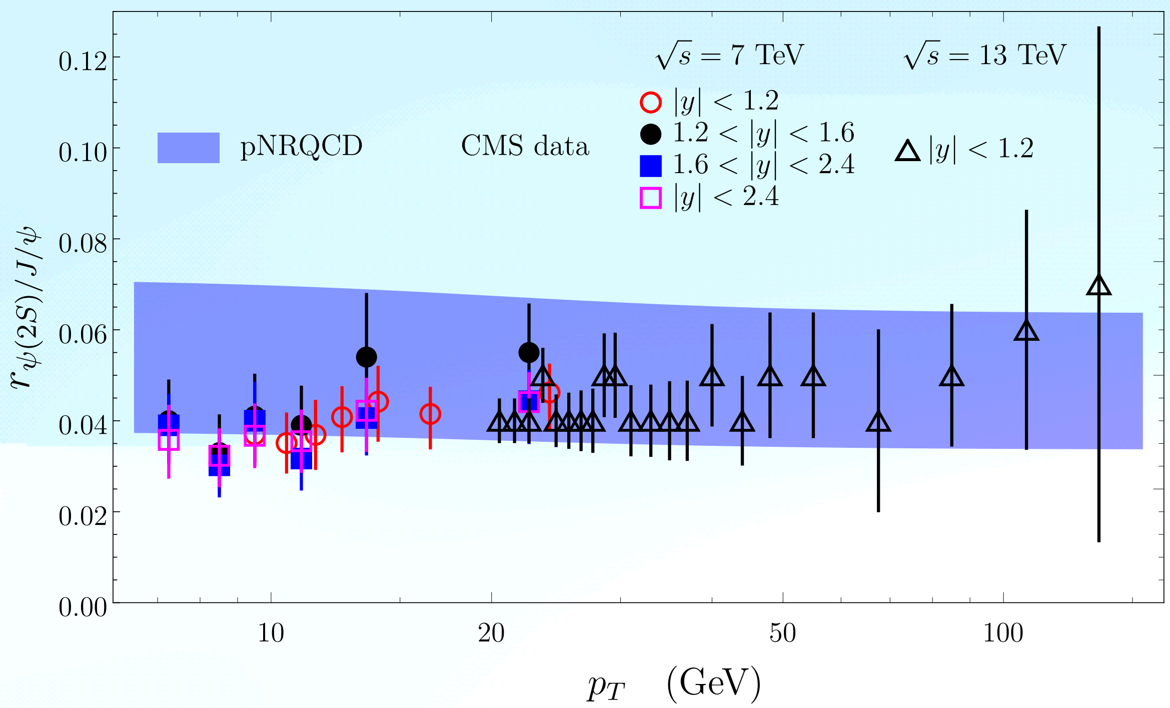

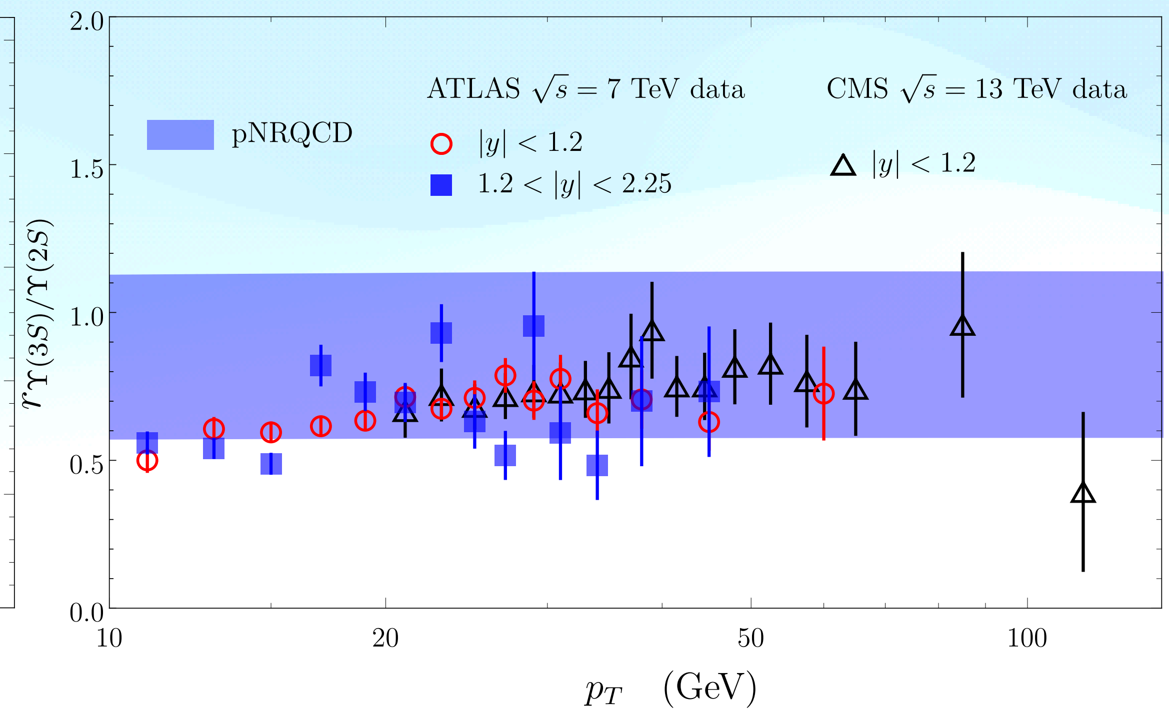

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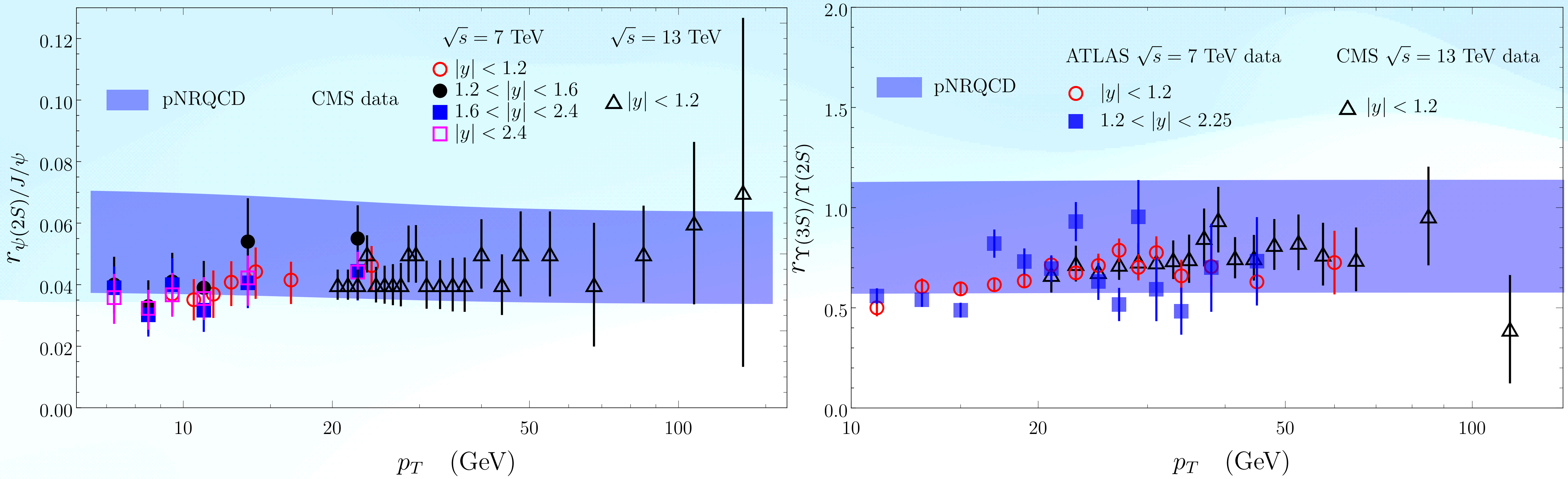

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- **The deviations from data indicate that NRQCD factorization maynot be reliable in such p_T regions because of significant nonperturbative (or/and) relativistic correction effect?**

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p_T cut	$\mathcal{E}_{10;10}$	$c_F^2\mathcal{B}_{00}$	$\mathcal{E}_{00}$
$p_T/(2m) > 3$	1.14 ± 0.12	-7.13 ± 2.89	18.9 ± 2.16
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- **In terms of LDMEs, we have (units of $10^{-2}\ \mathrm{Gev}^3$)**

V	p_T cut	$\langle\mathcal{O}^V(^3S_1^{[8]})\rangle$	$\langle\mathcal{O}^V(^1S_0^{[8]})\rangle$	$\langle\mathcal{O}^V(^3P_0^{[8]})\rangle/m^2$
J/ψ	$p_T/(2m) > 3$	1.66 ± 0.18	-3.47 ± 1.41	3.07 ± 0.35
	$p_T/(2m) > 5$	1.40 ± 0.42	-0.63 ± 3.22	2.59 ± 0.83
$\psi(2S)$	$p_T/(2m) > 3$	0.99 ± 0.11	-2.07 ± 0.84	1.83 ± 0.21
	$p_T/(2m) > 5$	0.84 ± 0.25	-0.37 ± 1.92	1.55 ± 0.49
$\Upsilon(2S)$	$p_T/(2m) > 3$	1.79 ± 0.20	-1.12 ± 0.46	1.28 ± 0.15
	$p_T/(2m) > 5$	1.52 ± 0.47	-0.20 ± 1.04	1.08 ± 0.35
$\Upsilon(3S)$	$p_T/(2m) > 3$	1.39 ± 0.16	-0.87 ± 0.35	0.99 ± 0.11
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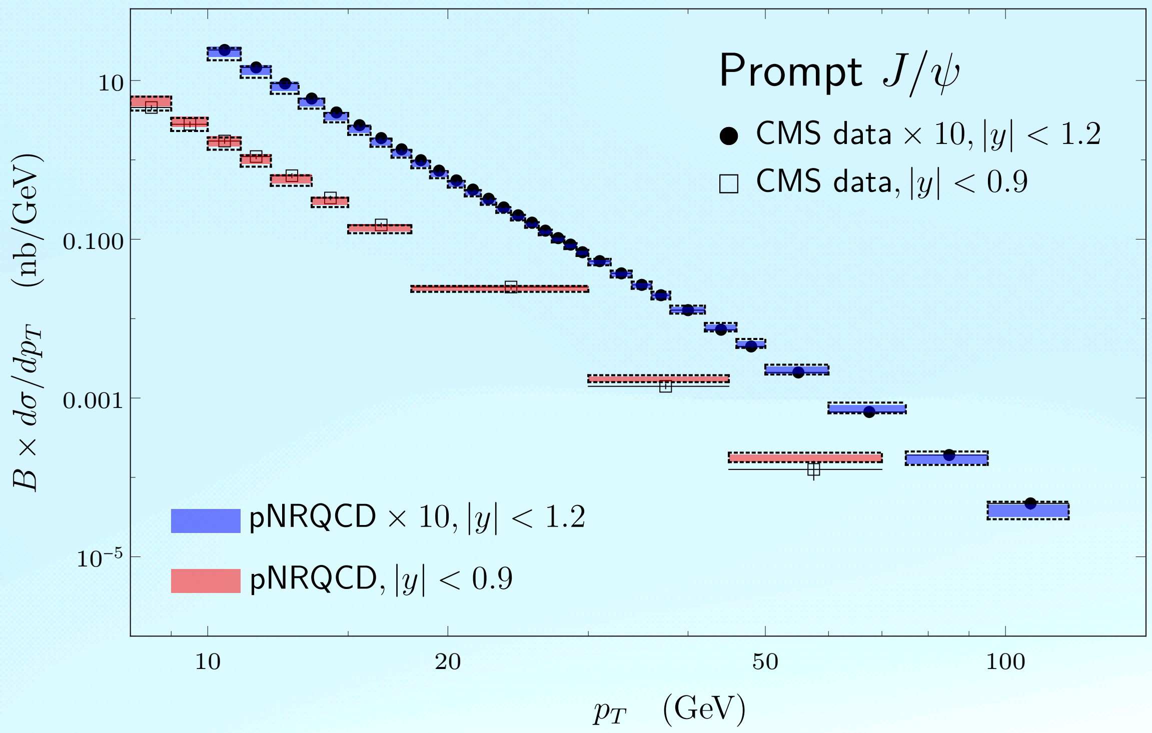
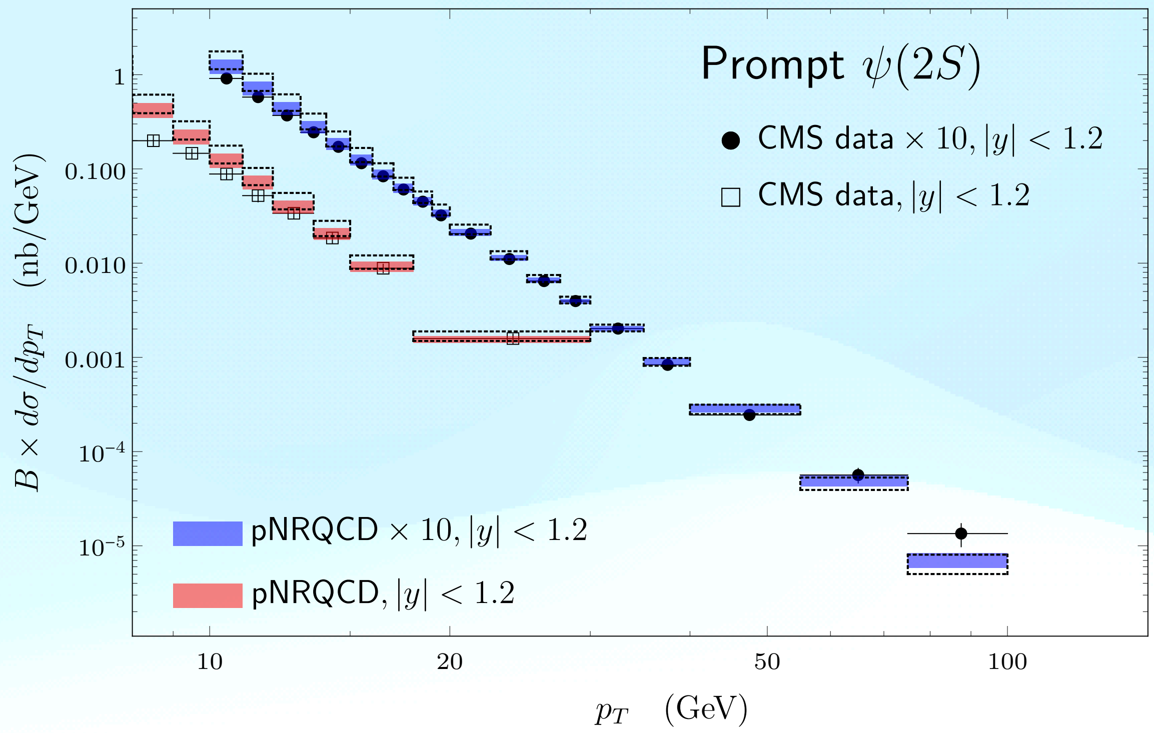
Fittings


Prompt J/ψ
CMS data × 10, $|y| < 1.2$
CMS data, $|y| < 0.9$
pNRQCD × 10, $|y| < 1.2$
pNRQCD, $|y| < 0.9$
$B \times d\sigma/dp_T$ (nb/GeV)
p_T (GeV)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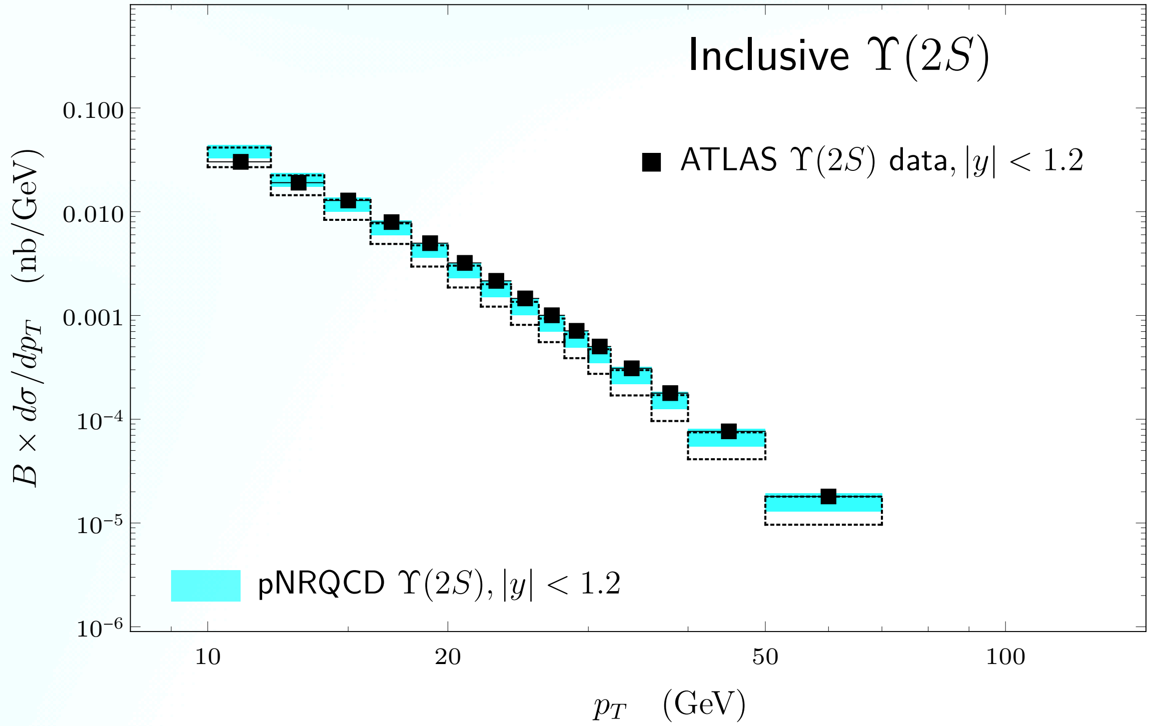
Inclusive $\Upsilon(2S)$
ATLAS $\Upsilon(2S)$ data, $|y| < 1.2$
pNRQCD $\Upsilon(2S)$, $|y| < 1.2$
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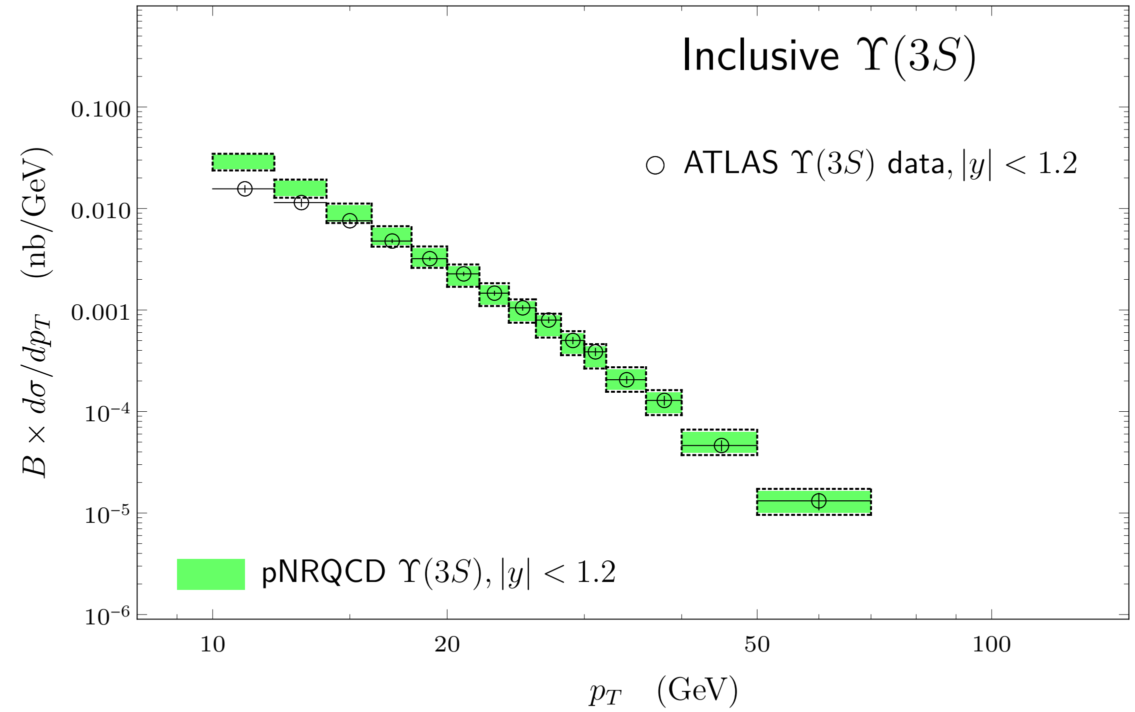
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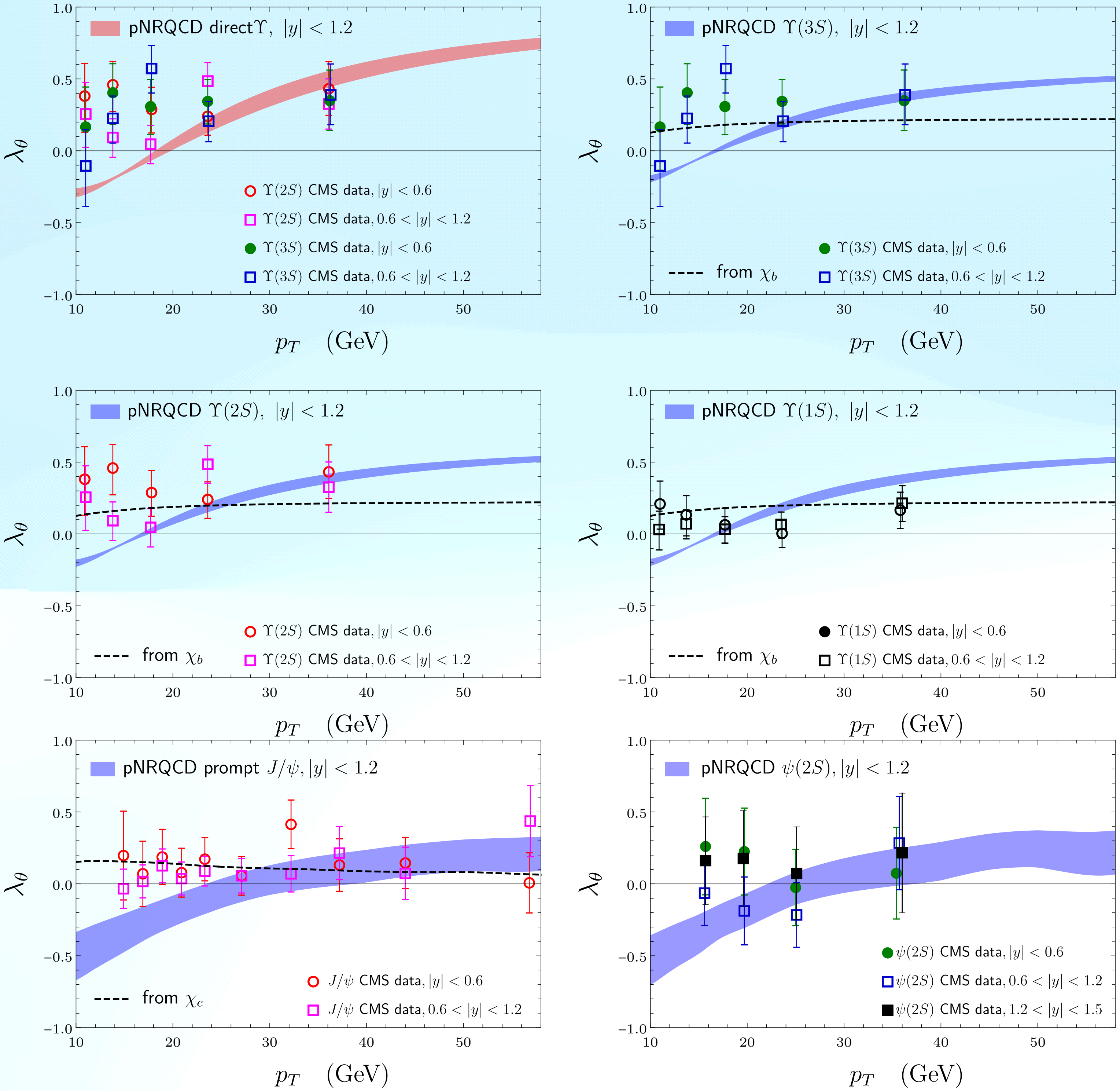
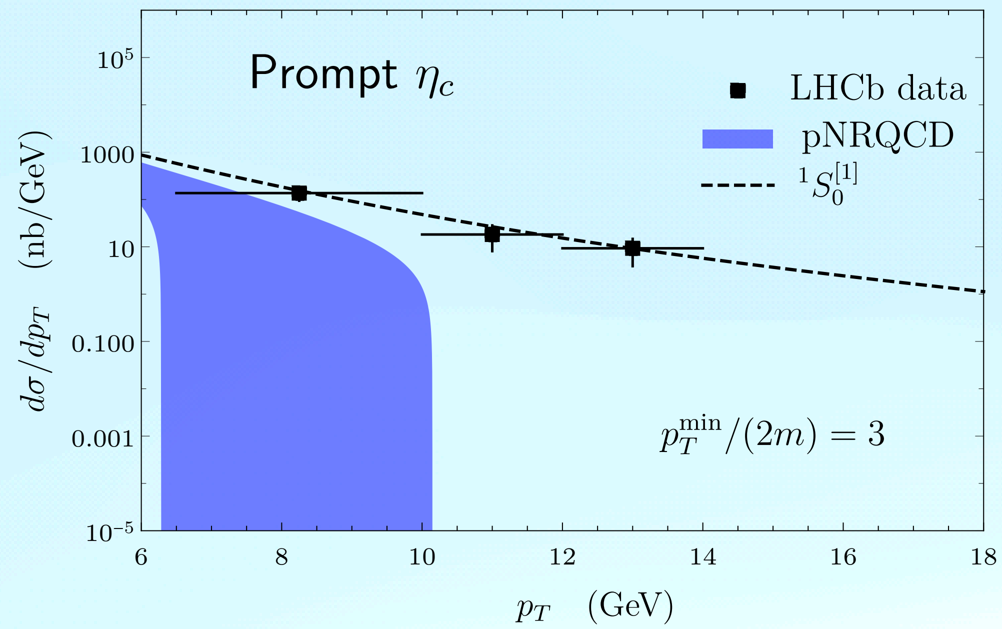
Other Applications — Polarizations

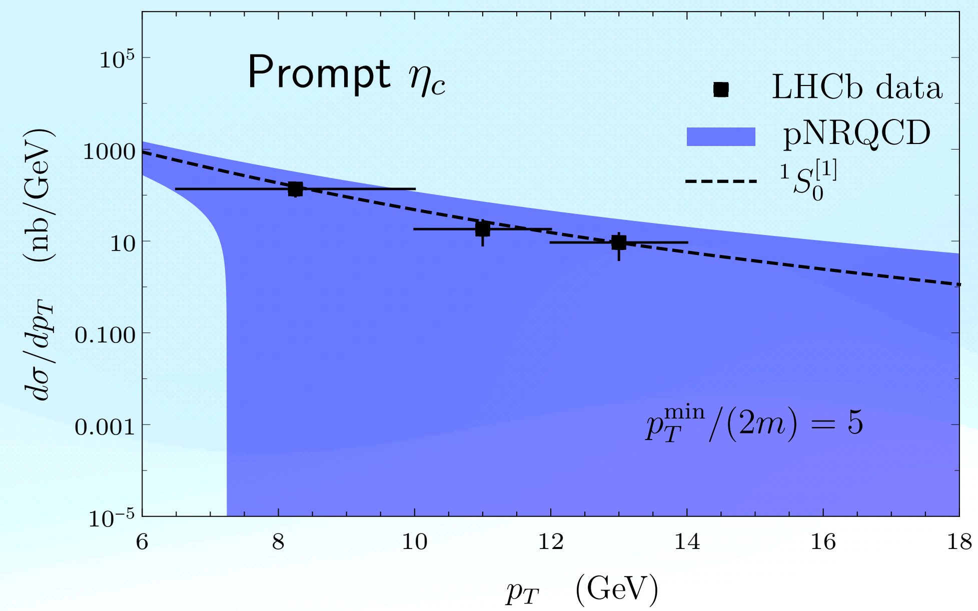


- $\Upsilon(nS)$ are more transversely polarized compared with $J/\psi, \psi(2S)$
- This is consistent with the fitted positive value of $\mathscr{E}_{00}$ ($^3P_0^{[8]}$ LDMEs) in the cancellation scenario because it leads to larger value of $\mathscr{E}_{10;10}$ ($^3S_1^{[8]}$ LDMEs) with larger scale Λ

Other Applications

- η_c **hadroproduction**





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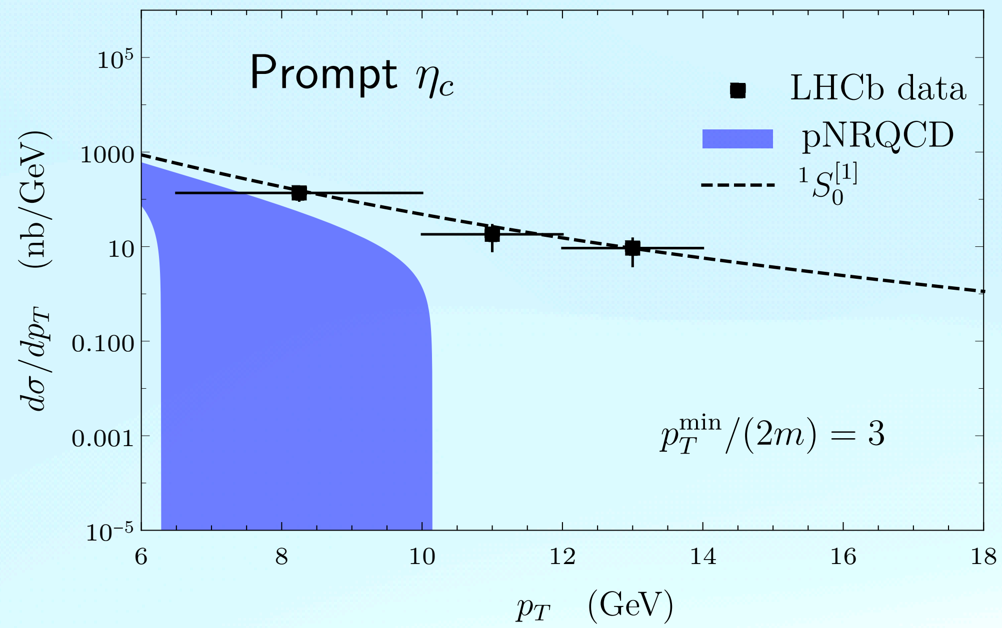
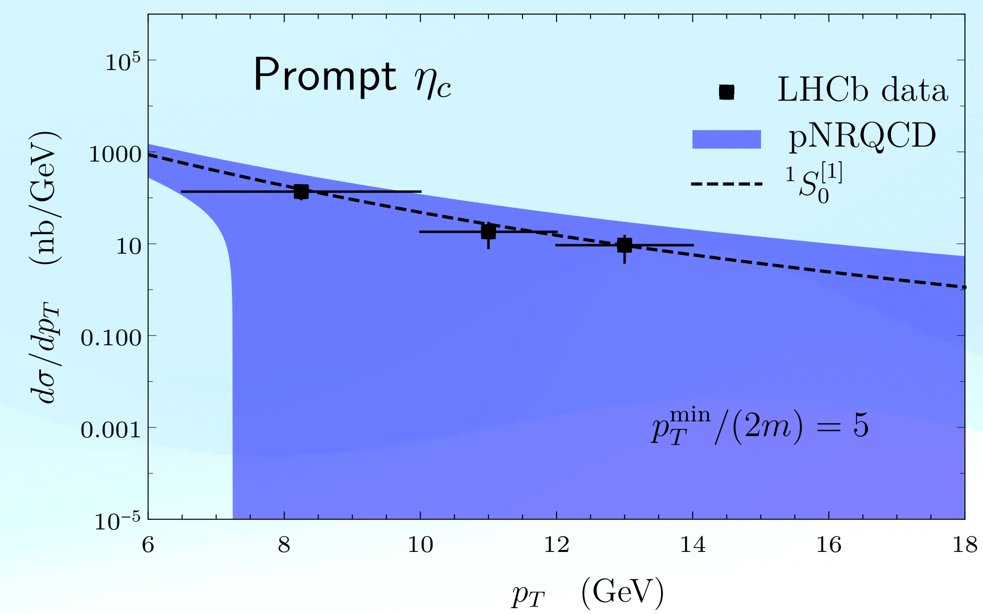
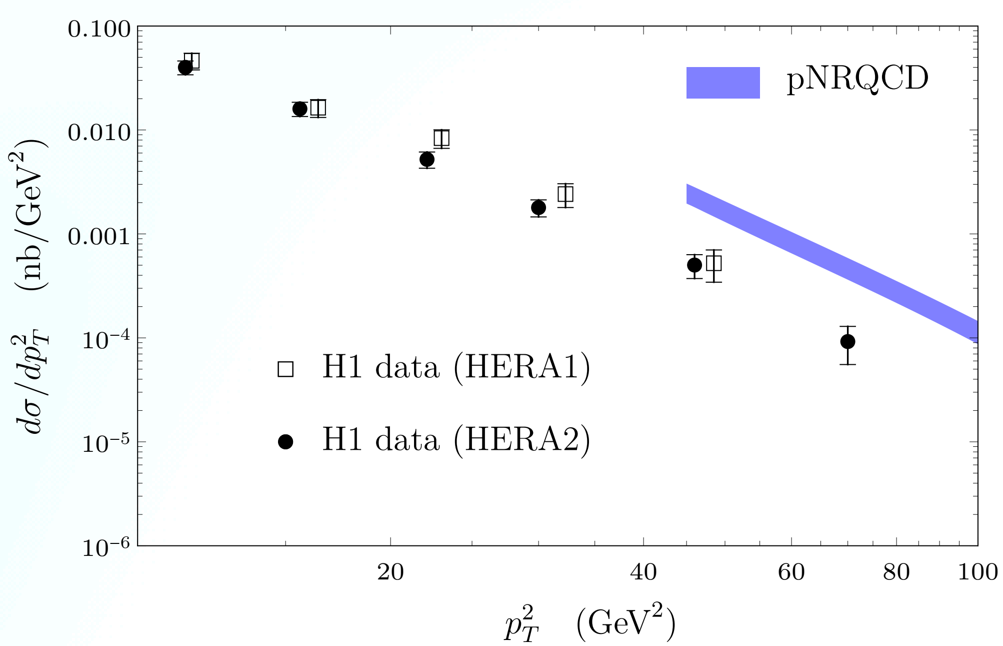
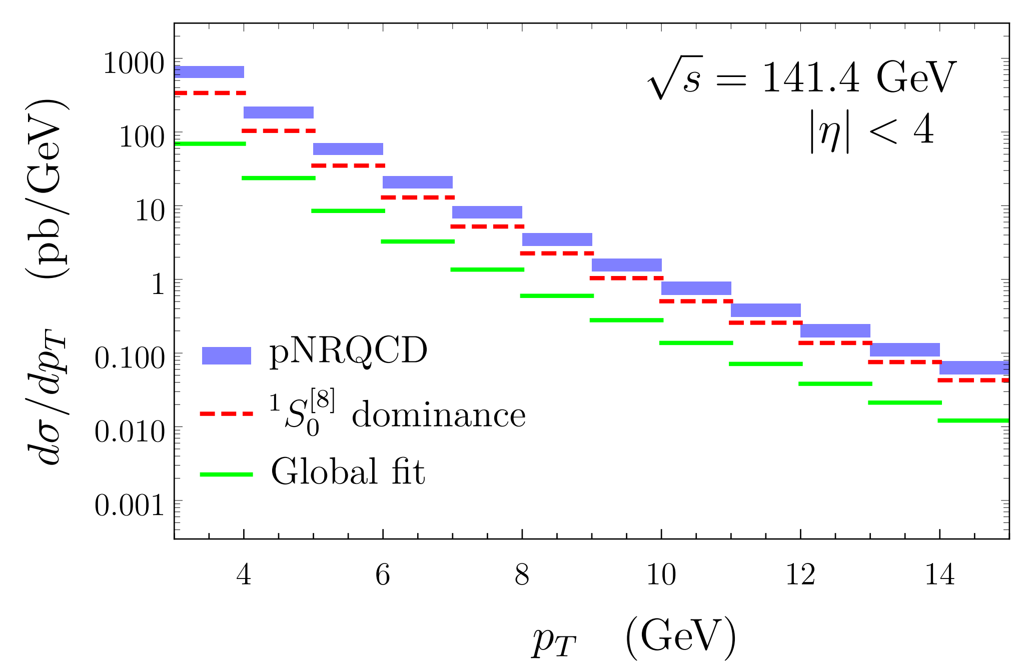




- J/ψ **photoproduction (HERA data) and** J/ψ **production from EIC (ep frame)**





Other Applications — $J/\psi + W/Z$ from LHC

- **4th column: theory prediction based on our fittings**

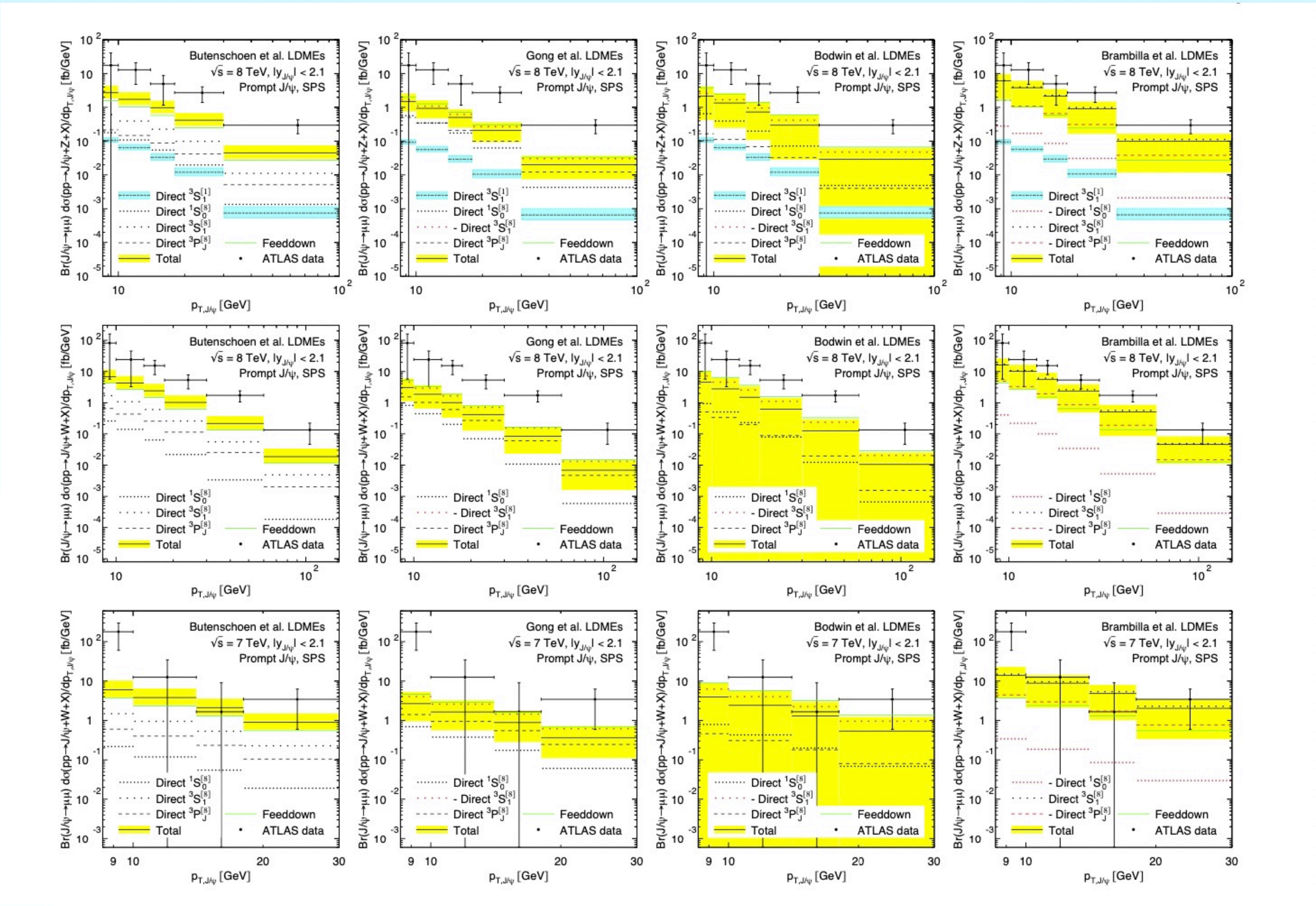


Figure taken from M. Butenschoen, B. Kniehl, PRL 130 (2023) 4, 041901

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