Superfluidity in neutron stars

Nicolas Chamel
Institute of Astronomy and Astrophysics
Université Libre de Bruxelles, Belgium

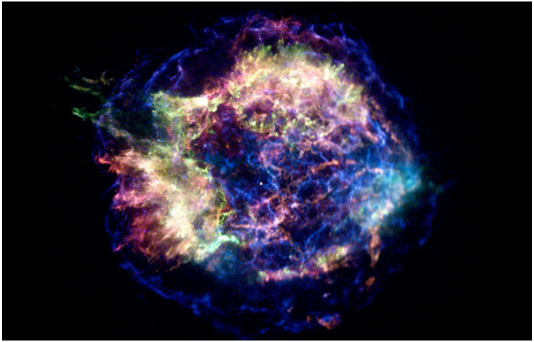
BLU-ULB
BRUSSELS LABORATORY OF THE UNIVERSE

fnrs
LA LIBERTÉ DE CHERCHER

UBEC 2024, ECT*, Trento, 6 November 2024

Neutron stars are cool

Neutron stars are dead “stars”: they are the extremely compact remnants of gravitational core-collapse supernova explosions.



Nuclear physics:

$M \sim 1 - 2M_\odot$
$R \sim 10$ km
$\Rightarrow \rho \sim 10^{15}$ g cm^{-3}

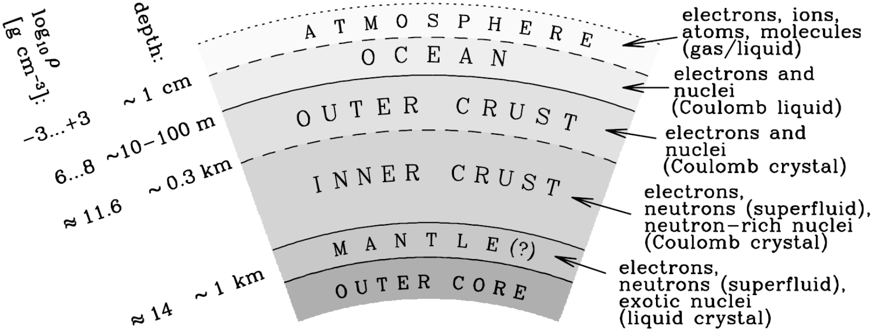
Energy scale: MeV

“cold” $\lesssim 10^{10}$ K $\lesssim$ “hot”

Neutron stars are initially very hot ($\sim 10^{12}$ K) but cool down to $\sim 10^9$ K within days by releasing neutrinos.

Their dense matter is thus expected to undergo various phase transitions, as observed in terrestrial materials at low-temperatures.

Neutron stars are “condensed” matter



picture taken from Haensel, Potekhin, Yakovlev, “Neutron Stars” (Springer, 2007)

Despite their name, neutron stars are not only made of neutrons!

Blaschke&Chamel, Astrophys. Space Sci. Lib. 457, eds L. Rezzolla, P. Pizzochero, D. I. Jones, N. Rea, I. Vidana p. 337-400 (Springer, 2018), arXiv:1803.01836

Outline

1. Overview of nuclear superfluidity in neutron stars
   - From theory
   - To observations
2. Neutron superfluidity in neutron-star crusts
   - Small superflow: superfluid density and sound modes
   - Large superflow: critical velocities, gapless superfluidity, vortices
   - Astrophysical implications
3. Conclusions & perspectives

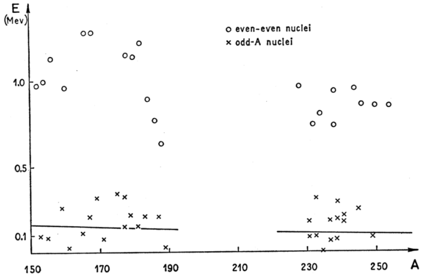
Nuclear superfluidity and superconductivity

The implications of the BCS theory (published in January 1957) for atomic nuclei were first discussed by A. Bohr, B. R. Mottelson, and D. Pines during the Summer of 1957.

D. Pines in "BCS: 50 Years" (World Scientific, 2011), pp.85-105

They speculated that pairing might explain the **energy gap** in the excitation spectra of nuclei.

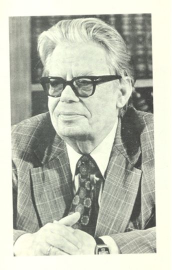
Phys. Rev. 110, 936 (1958)



They also anticipated that nuclear pairing could explain **odd-even mass staggering, and the reduced moments of inertia** of nuclei.

Superfluidity and superconductivity in neutron stars

In the 1960's, several superconductors had been found but ^{4}He was the only superfluid known.



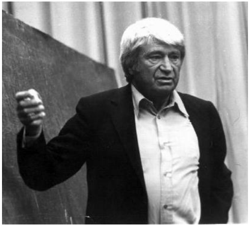
Bogoliubov developed a microscopic theory of superfluidity and superconductivity, and was the first to explore its application to nuclear matter.

Dokl. Ak. nauk SSSR 119, 52 (1958)

Neutron-star superfluidity was predicted by Arkady Migdal in 1959, and first studied by Ginzburg & Kirzhnits in 1964 **before the discovery of pulsars** in 1967.

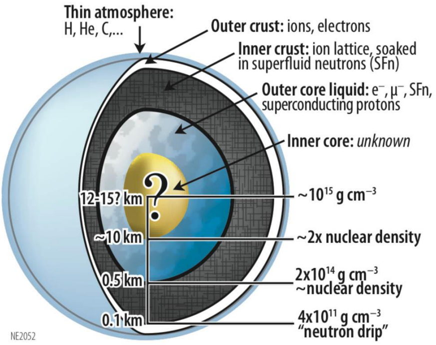
Migdal, Nucl. Phys. 13, 655 (1959)

Ginzburg & Kirzhnits, Zh. Eksp. Teor. Fiz. 47, 2006 (1964)



Superstars

The huge gravity of neutron stars produces the highest-T_c and largest superfluids and superconductors known in the Universe!

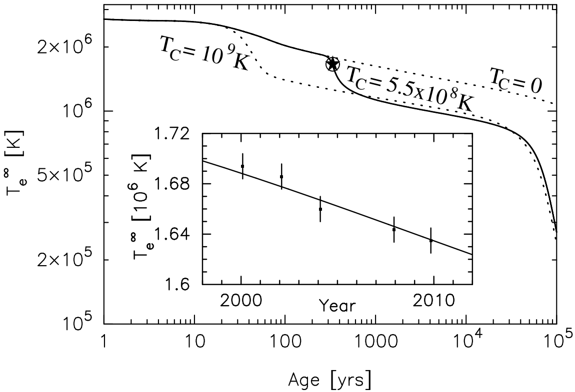


Neutron stars	**$\sim 10^{10}$ K**
⋮	⋮
$LaH_{10\pm x}$	260 K
Cuprates	$1 - 130$ K
Electrons in metals	$1 - 25$ K
Helium-4	2.17 K
Helium-3	2.491×10^{-3} K
Bosonic condensates	$\sim 10^{-6}$ K
Fermionic condensates	$\sim 10^{-8}$ K

Predicted long ago, these quantum condensates can be probed through astrophysical observations.
Chamel, J. Astrophys. Astron. 38, 43 (2017)

Rapid cooling of neutron stars and superfluidity

Observations of the youngest known neutron star in Cassiopeia A provide evidence for **neutron superfluidity in its core**:

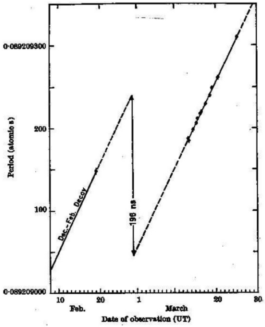


The neutrino emission is enhanced by Cooper pair breaking and formation at T_c leading to fast cooling.

Page et al., PRL 106, 081101; Shternin et al.,MNRAS 412, L108 (2011)

Pulsar frequency glitches and superfluidity

Pulsars are spinning very rapidly with **extremely stable periods** $\dot{P} \gtrsim 10^{-21}$, outperforming the best atomic clocks!



Still, some pulsars have been found to **suddenly spin up** (in less than a minute!).

682 glitches have been detected in 225 pulsars.

`http://www.jb.man.ac.uk/pulsar/glitches.html`

Recent review: *Antonopoulou, Haskell, Espinoza, Rep. Prog. Phys. 85, 126901 (2022)*

Experimental glitches with ultracold atoms

Poli et al., PRL 131, 223401 (2023)

Pulsar glitches provide strong evidence for the existence of a **neutron superflow in neutron-star crusts** driven by the pinning of quantized vortices. But the superfluid dynamics remains poorly understood.

Time-dependent Hartree-Fock-Bogoliubov theory

The dynamics of nuclear superfluids ($q = n, p$) is here described by the **time-dependent Hartree-Fock-Bogoliubov equations**:

$$\begin{pmatrix} h_q(\boldsymbol{r},t) - \lambda_q & \Delta_q(\boldsymbol{r},t) \\ \Delta_q(\boldsymbol{r},t)^* & -h_q(\boldsymbol{r},t)^* + \lambda_q \end{pmatrix} \begin{pmatrix} \psi_1^{(q)}(\boldsymbol{r},t) \\ \psi_2^{(q)}(\boldsymbol{r},t) \end{pmatrix} = \mathrm{i}\hbar \frac{\partial}{\partial t} \begin{pmatrix} \psi_1^{(q)}(\boldsymbol{r},t) \\ \psi_2^{(q)}(\boldsymbol{r},t) \end{pmatrix}$$

Time-dependent Hartree-Fock-Bogoliubov theory

The dynamics of nuclear superfluids ($q = n, p$) is here described by the **time-dependent Hartree-Fock-Bogoliubov equations**:

$$\begin{pmatrix} h_q(\boldsymbol{r},t) - \lambda_q & \Delta_q(\boldsymbol{r},t) \\ \Delta_q(\boldsymbol{r},t)^* & -h_q(\boldsymbol{r},t)^* + \lambda_q \end{pmatrix} \begin{pmatrix} \psi_1^{(q)}(\boldsymbol{r},t) \\ \psi_2^{(q)}(\boldsymbol{r},t) \end{pmatrix} = \mathrm{i}\hbar \frac{\partial}{\partial t} \begin{pmatrix} \psi_1^{(q)}(\boldsymbol{r},t) \\ \psi_2^{(q)}(\boldsymbol{r},t) \end{pmatrix}$$

They resemble the **Bogoliubov-de Gennes equations** BUT

- the Hamiltonian $h_q(\boldsymbol{r},t)$ takes a much **more complicated form**,
- both $h_q(\boldsymbol{r},t)$ and $\Delta_q(\boldsymbol{r},t)$ are **internal fields**,
- the BdG equations for **neutrons and protons** are coupled.

Time-dependent Hartree-Fock-Bogoliubov theory

The dynamics of nuclear superfluids ($q = n, p$) is here described by the **time-dependent Hartree-Fock-Bogoliubov equations**:

$$\begin{pmatrix} h_q(\boldsymbol{r},t) - \lambda_q & \Delta_q(\boldsymbol{r},t) \\ \Delta_q(\boldsymbol{r},t)^* & -h_q(\boldsymbol{r},t)^* + \lambda_q \end{pmatrix} \begin{pmatrix} \psi_1^{(q)}(\boldsymbol{r},t) \\ \psi_2^{(q)}(\boldsymbol{r},t) \end{pmatrix} = \mathrm{i}\hbar \frac{\partial}{\partial t} \begin{pmatrix} \psi_1^{(q)}(\boldsymbol{r},t) \\ \psi_2^{(q)}(\boldsymbol{r},t) \end{pmatrix}$$

$$h_q(\boldsymbol{r},t) \equiv -\boldsymbol{\nabla} \cdot \frac{\hbar^2}{2m_q^{\oplus}(\boldsymbol{r},t)} \boldsymbol{\nabla} + U_q(\boldsymbol{r},t) - \frac{\mathrm{i}}{2} \left\{ \boldsymbol{I_q}(\boldsymbol{r},t), \boldsymbol{\nabla} \right\} + \dots$$

$$\frac{\hbar^2}{2m_q^{\oplus}(\boldsymbol{r},t)} = \frac{\delta E}{\delta \tau_q(\boldsymbol{r},t)}, \quad U_q(\boldsymbol{r},t) = \frac{\delta E}{\delta n_q(\boldsymbol{r},t)}, \quad \boldsymbol{I_q}(\boldsymbol{r},t) = \frac{\delta E}{\delta \boldsymbol{j_q}(\boldsymbol{r},t)}$$

$$\Delta_q(\boldsymbol{r},t) = 2 \frac{\delta E}{\delta \widetilde{n_q}(\boldsymbol{r},t)^*} = |\Delta_q(\boldsymbol{r},t)| e^{\mathrm{i}\phi_q(\boldsymbol{r},t)}$$

Time-dependent Hartree-Fock-Bogoliubov theory

The dynamics of nuclear superfluids ($q = n, p$) is here described by the **time-dependent Hartree-Fock-Bogoliubov equations**:

$$\begin{pmatrix} h_q(\boldsymbol{r},t)-\lambda_q & \Delta_q(\boldsymbol{r},t) \\ \Delta_q(\boldsymbol{r},t)^* & -h_q(\boldsymbol{r},t)^*+\lambda_q \end{pmatrix} \begin{pmatrix} \psi_1^{(q)}(\boldsymbol{r},t) \\ \psi_2^{(q)}(\boldsymbol{r},t) \end{pmatrix} = \mathrm{i}\hbar\frac{\partial}{\partial t} \begin{pmatrix} \psi_1^{(q)}(\boldsymbol{r},t) \\ \psi_2^{(q)}(\boldsymbol{r},t) \end{pmatrix}$$

$$h_q(\boldsymbol{r},t) \equiv -\boldsymbol{\nabla}\cdot\frac{\hbar^2}{2m_q^{\oplus}(\boldsymbol{r},t)}\boldsymbol{\nabla} + U_q(\boldsymbol{r},t) - \frac{\mathrm{i}}{2}\left\{\boldsymbol{I_q}(\boldsymbol{r},t),\boldsymbol{\nabla}\right\} + \dots$$

$$\frac{\hbar^2}{2m_q^{\oplus}(\boldsymbol{r},t)} = \frac{\delta E}{\delta\tau_q(\boldsymbol{r},t)}, \quad U_q(\boldsymbol{r},t) = \frac{\delta E}{\delta n_q(\boldsymbol{r},t)}, \quad \boldsymbol{I_q}(\boldsymbol{r},t) = \frac{\delta E}{\delta \boldsymbol{j_q}(\boldsymbol{r},t)}$$

$$\Delta_q(\boldsymbol{r},t) = 2\frac{\delta E}{\delta\widetilde{n_q}(\boldsymbol{r},t)^*} = |\Delta_q(\boldsymbol{r},t)|e^{\mathrm{i}\phi_q(\boldsymbol{r},t)}$$

all these fields are not external but **self-induced** via thermal averaged matrix densities expressible in terms of $\psi_1^{(q)}(\boldsymbol{r},t)$ and $\psi_2^{(q)}(\boldsymbol{r},t)$

$$n_q(\boldsymbol{r},\sigma;\boldsymbol{r'},\sigma';t) = < c_q(\boldsymbol{r'},\sigma';t)^\dagger c_q(\boldsymbol{r},\sigma;t) >$$

$$\widetilde{n}_q(\boldsymbol{r},\sigma;\boldsymbol{r'},\sigma';t) = -\sigma' < c_q(\boldsymbol{r'},-\sigma';t) c_q(\boldsymbol{r},\sigma;t) >$$

Superfluid velocity, momentum and mass transport

The **superfluid velocity** defined through the phase of the pairing field

$$\Delta_q(\boldsymbol{r}, t) = |\Delta_q(\boldsymbol{r}, t)| e^{\mathrm{i}\phi_q(\boldsymbol{r}, t)} \qquad \Rightarrow \boldsymbol{V_q}(\boldsymbol{r}, t) = \frac{\hbar}{2m_q} \nabla \phi_q(\boldsymbol{r}, t)$$

Superfluid velocity, momentum and mass transport

The **superfluid velocity** defined through the phase of the pairing field

$$\Delta_q(\boldsymbol{r},t) = |\Delta_q(\boldsymbol{r},t)| e^{\mathrm{i}\phi_q(\boldsymbol{r},t)} \qquad \Rightarrow \boldsymbol{V_q}(\boldsymbol{r},t) = \frac{\hbar}{2m_q}\nabla\phi_q(\boldsymbol{r},t)$$

is neither equal to $\hbar\boldsymbol{j_q}/\rho_q$ where $\boldsymbol{j_q}$ is the **momentum density**

$$\boldsymbol{j_q}(\boldsymbol{r},t) = -\frac{i}{2}\sum_{\sigma=\pm1}\int \mathrm{d}^3\boldsymbol{r'}\,\delta(\boldsymbol{r}-\boldsymbol{r'})(\nabla-\nabla')n_q(\boldsymbol{r},\sigma;\boldsymbol{r'},\sigma;t)$$

Superfluid velocity, momentum and mass transport

The **superfluid velocity** defined through the phase of the pairing field

$$\Delta_q(\boldsymbol{r},t) = |\Delta_q(\boldsymbol{r},t)| e^{\mathrm{i}\phi_q(\boldsymbol{r},t)} \qquad \Rightarrow \boldsymbol{V_q}(\boldsymbol{r},t) = \frac{\hbar}{2m_q}\nabla\phi_q(\boldsymbol{r},t)$$

is neither equal to $\hbar\boldsymbol{j_q}/\rho_q$ where $\boldsymbol{j_q}$ is the **momentum density**

$$\boldsymbol{j_q}(\boldsymbol{r},t) = -\frac{i}{2}\sum_{\sigma=\pm 1}\int \mathrm{d}^3\boldsymbol{r'}\,\delta(\boldsymbol{r}-\boldsymbol{r'})(\nabla-\nabla')n_q(\boldsymbol{r},\sigma;\boldsymbol{r'},\sigma;t)$$

nor to the velocity associated with **mass transport**

$$\boldsymbol{v_q}(\boldsymbol{r},t) = \frac{m_q}{m_q^{\oplus}(\boldsymbol{r},t)}\frac{\hbar\boldsymbol{j_q}(\boldsymbol{r},t)}{\rho_q(\boldsymbol{r},t)} + \frac{\boldsymbol{I_q}(\boldsymbol{r},t)}{\hbar}, \qquad \frac{\partial\rho_q}{\partial t} + \nabla\cdot(\rho_q\boldsymbol{v_q}) = 0$$

Superfluid velocity, momentum and mass transport

The **superfluid velocity** defined through the phase of the pairing field

$$\Delta_q(\boldsymbol{r},t) = |\Delta_q(\boldsymbol{r},t)| e^{\mathrm{i}\phi_q(\boldsymbol{r},t)} \qquad \Rightarrow \boldsymbol{V_q}(\boldsymbol{r},t) = \frac{\hbar}{2m_q}\nabla\phi_q(\boldsymbol{r},t)$$

is neither equal to $\hbar \boldsymbol{j_q}/\rho_q$ where $\boldsymbol{j_q}$ is the **momentum density**

$$\boldsymbol{j_q}(\boldsymbol{r},t) = -\frac{i}{2}\sum_{\sigma=\pm1}\int \mathrm{d}^3\boldsymbol{r'}\,\delta(\boldsymbol{r}-\boldsymbol{r'})(\nabla-\nabla')n_q(\boldsymbol{r},\sigma;\boldsymbol{r'},\sigma;t)$$

nor to the velocity associated with **mass transport**

$$\boldsymbol{v_q}(\boldsymbol{r},t) = \frac{m_q}{m_q^{\oplus}(\boldsymbol{r},t)}\frac{\hbar\boldsymbol{j_q}(\boldsymbol{r},t)}{\rho_q(\boldsymbol{r},t)} + \frac{\boldsymbol{I_q}(\boldsymbol{r},t)}{\hbar}, \qquad \frac{\partial\rho_q}{\partial t} + \nabla\cdot(\rho_q\boldsymbol{v_q}) = 0$$

All these velocities only coincide for a one-component superfluid:

$$\boldsymbol{v} = \boldsymbol{V} = \frac{\hbar\boldsymbol{j}}{\rho}$$

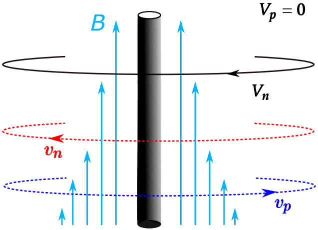
Chamel & Allard, PRC100, 065801 (2019); *Allard & Chamel, PRC103, 025804 (2021)*

Entrainment and dissipation in neutron-star cores

Neutrons and protons are **mutually entrained**: $\rho_q \boldsymbol{v_q} = \sum_{q'} \rho_{qq'} \boldsymbol{V_{q'}}$

Entrainment and dissipation in neutron-star cores

Neutrons and protons are **mutually entrained**: $\rho_q \boldsymbol{v_q} = \sum_{q'} \rho_{qq'} \boldsymbol{V_{q'}}$



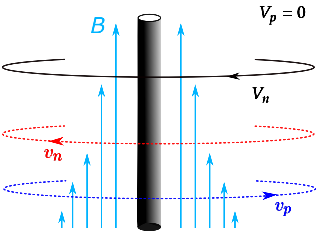
Neutron vortices thus carry a **fractional magnetic quantum flux**

$$\Phi_\star = \oint \boldsymbol{A} \cdot \mathrm{d}\boldsymbol{\ell} = k\Phi_0 \,, \quad k = \frac{\rho_{pn}}{\rho_{pp}} , \Phi_0 \equiv \frac{hc}{2e}$$

Sedrakyan&Shakhabasyan, Astrofizika 8, 557 (1972)

Entrainment and dissipation in neutron-star cores

Neutrons and protons are **mutually entrained**: $\rho_q \boldsymbol{v_q} = \sum_{q'} \rho_{qq'} \boldsymbol{V_{q'}}$



Neutron vortices thus carry a **fractional magnetic quantum flux**

$$\Phi_\star = \oint \boldsymbol{A} \cdot \mathrm{d}\boldsymbol{\ell} = k\Phi_0 \,, \quad k = \frac{\rho_{pn}}{\rho_{pp}}, \Phi_0 \equiv \frac{hc}{2e}$$

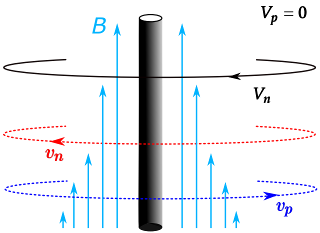
Sedrakyan&Shakhabasyan, Astrofizika 8, 557 (1972)

Due to electrons scattering off the magnetic field of the vortex lines, **the core superfluid is strongly coupled to the crust**.

Alpar, Langer, Sauls, ApJ 282, 533 (1984)

Entrainment and dissipation in neutron-star cores

Neutrons and protons are **mutually entrained**: $\rho_q \boldsymbol{v_q} = \sum_{q'} \rho_{qq'} \boldsymbol{V_{q'}}$



Neutron vortices thus carry a **fractional magnetic quantum flux**

$$\Phi_\star = \oint \boldsymbol{A} \cdot \mathrm{d}\boldsymbol{\ell} = k\Phi_0 \,, \quad k = \frac{\rho_{pn}}{\rho_{pp}}, \Phi_0 \equiv \frac{hc}{2e}$$

Sedrakyan&Shakhabasyan, Astrofizika 8, 557 (1972)

Due to electrons scattering off the magnetic field of the vortex lines, **the core superfluid is strongly coupled to the crust**.

Alpar, Langer, Sauls, ApJ 282, 533 (1984)

At the scale of the star, **general relativity leads to additional fluid couplings** due to frame-dragging effects!

B. Carter, Ann. Phys. 95, 53 (1975);Sourie et al., MNRAS 464, 4641(2017)

Neutron superfluidity in neutron-star crusts

The **breaking of translational symmetry** leads to the depletion of the superfluid reservoir.

Leggett, PRL 25, 1543 (1970)

Neutron superfluidity in neutron-star crusts

The **breaking of translational symmetry** leads to the depletion of the superfluid reservoir.

Leggett, PRL 25, 1543 (1970)

In the presence of a **superflow** with velocity $\boldsymbol{V_n}$, the average neutron mass current in the rest frame of the neutron-star crust is

$$\bar{\rho}_n^i \equiv \frac{1}{V}\int \mathrm{d}^3r\, \rho_n(\boldsymbol{r},t) v_n^i(\boldsymbol{r},t) = \sum_j \rho_{n,\mathrm{S}}^{ij}\, \bar{V}_{n\,j}$$

Treating the crust as a polycrystal $\bar{\boldsymbol{\rho}}_{\boldsymbol{n}} = \rho_{n,\mathrm{S}} \bar{\boldsymbol{V}}_{\boldsymbol{n}} = \rho_n \dfrac{m_n}{m_n^\star} \bar{\boldsymbol{V}}_{\boldsymbol{n}}$.

The **superfluid density** $\rho_{n,\mathrm{S}} < \rho_n$ ($m_n^\star > m_n$) is a current-current response function.

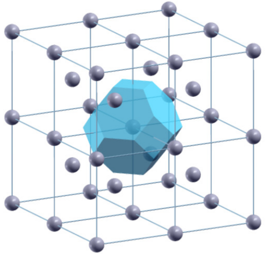
This "*is a derived concept and is not the density of anything*".

Feynman, Statistical Mechanics: A Set of Lectures.

Review: *Chamel, J. Low Temp. Phys. 189, 328 (2017)*

Neutron superfluidity in neutron-star crusts

In the presence of neutron superflow $\bar{\boldsymbol{V}}_{\boldsymbol{n}} \equiv \dfrac{\hbar \boldsymbol{Q}}{m_n}$:



$$\psi_{1\alpha\boldsymbol{k}}(\boldsymbol{r}) = e^{\mathrm{i}\,(\boldsymbol{k}+\boldsymbol{Q})\cdot\boldsymbol{r}}\,\widetilde{\psi}_{1\alpha\boldsymbol{k}}(\boldsymbol{r})$$
$$\psi_{2\alpha\boldsymbol{k}}(\boldsymbol{r}) = e^{\mathrm{i}\,(\boldsymbol{k}-\boldsymbol{Q})\cdot\boldsymbol{r}}\,\widetilde{\psi}_{2\alpha\boldsymbol{k}}(\boldsymbol{r})$$

where $\widetilde{\psi}_{1,2\alpha\boldsymbol{k}}(\boldsymbol{r})$ are periodic

α is the band index
$\boldsymbol{k}$ is the Bloch wave vector

3D HFB computations remain very challenging:

- lattice spacing can be large $\sim$ 100 fm vs cluster size $\sim$ 10 fm,
- huge number of neutrons in the Wigner-Seitz cell ($\sim 10^2 - 10^3$).

Neutron superfluid density

In most regions of the crust, superfluid neutrons are in the weak coupling BCS regime.

For **small currents**, the neutron superfluid density is given by

$$\rho_{n,\mathrm{S}} = \frac{m_n^2}{24\pi^3\hbar^2}\sum_\alpha \int_{\mathrm{BZ}} \mathrm{d}^3k\, |\nabla_{\boldsymbol{k}}\varepsilon_{\alpha\boldsymbol{k}}|^2 \frac{\Delta_{\alpha\boldsymbol{k}}^2}{\sqrt{(\varepsilon_{\alpha\boldsymbol{k}}-\mu)^2+\Delta_{\alpha\boldsymbol{k}}^2}^3}$$

Carter,Chamel,Haensel,Nucl.Phys.A748,675 (2005); Nucl.Phys.A759,441(2005)

Neutron superfluid density

In most regions of the crust, superfluid neutrons are in the weak coupling BCS regime.

For **small currents**, the neutron superfluid density is given by

$$\rho_{n,\mathrm{S}} = \frac{m_n^2}{24\pi^3\hbar^2}\sum_\alpha \int_{\mathrm{BZ}} \mathrm{d}^3k \, |\nabla_{\boldsymbol{k}}\varepsilon_{\alpha\boldsymbol{k}}|^2 \frac{\Delta_{\alpha\boldsymbol{k}}^2}{\sqrt{(\varepsilon_{\alpha\boldsymbol{k}}-\mu)^2+\Delta_{\alpha\boldsymbol{k}}^2}^3}$$

Carter,Chamel,Haensel,Nucl.Phys.A748,675 (2005); Nucl.Phys.A759,441(2005)

In the limit $\Delta_{\alpha\boldsymbol{k}}/\varepsilon_{\mathrm{F}} \to 0$

$$\rho_{n,\mathrm{S}} \approx \frac{m_n^2}{12\pi^3\hbar^2}\sum_\alpha \int_{\mathrm{BZ}} \mathrm{d}^3\boldsymbol{k} \, \delta(\varepsilon_{\alpha\boldsymbol{k}}-\mu)|\nabla_{\boldsymbol{k}}\varepsilon_{\alpha\boldsymbol{k}}|^2$$

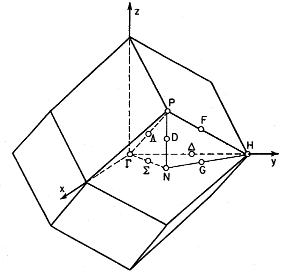
reduces to the expression obtained for a dilute Fermi superfluid in a 1D external optical potential

Pitaevskii, Stringari, Orso, Phys. Rev. A 71, 053602 (2005)

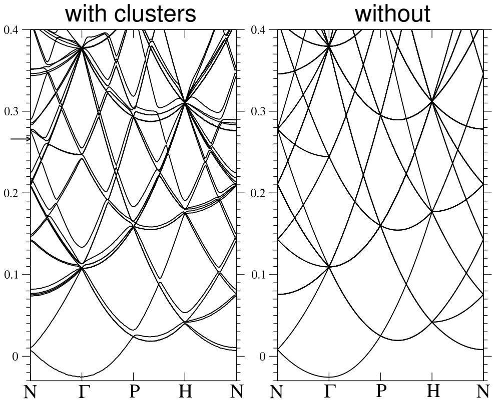
Neutron superfluid fraction in shallow region

Neutron band structure (s.p. energy in MeV vs $\boldsymbol{k}$) in a body-centered cubic (bcc) lattice at $\bar{n} = 0.0003$ fm^{-3} ($Z = 50, A = 200$):

First Brillouin zone:



Chamel,Phys.Rev.C85,035801(2012)

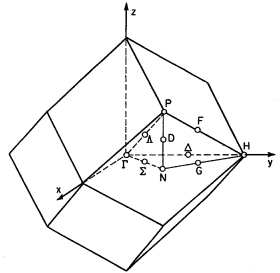


The spectrum is similar that of free neutrons: $\rho_{n,\mathrm{S}}/\rho_n = 83\%$.

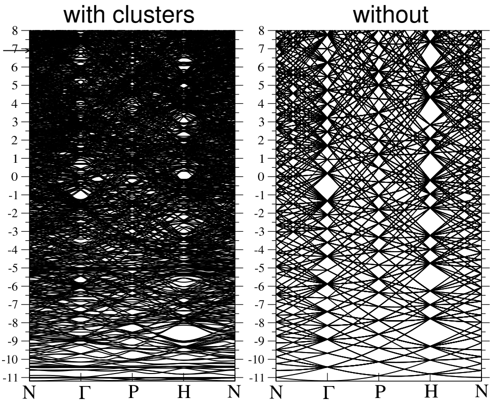
Neutron superfluid fraction in deep region

Neutron band structure (s.p. energy in MeV vs $\boldsymbol{k}$) in a body-centered cubic (bcc) lattice at $\bar{n} = 0.03$ fm^{-3} ($Z = 40, A = 1590$):

First Brillouin zone:



Chamel,Phys.Rev.C85,035801(2012)



The spectrum is very different: $\rho_{n,S}/\rho_n = 7\%$. Neutron superfluidity is almost entirely suppressed!

Band structure and Fermi surface

cluster size $\sim \lambda_F \ll$ lattice spacing
potential depth $\sim 2\varepsilon_F \gg \Delta$

Bragg scattering leads to strong distortions of the Fermi surface.

Avoided band crossings where

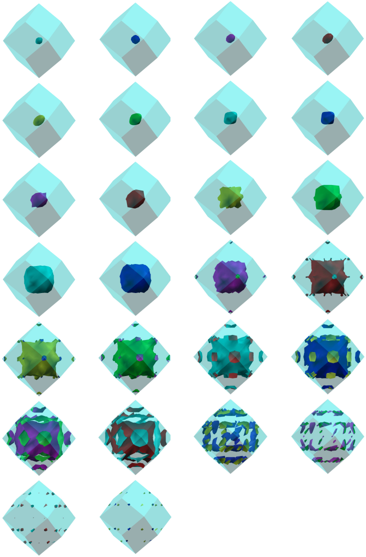
$$|\nabla_{\boldsymbol{k}} \varepsilon_{\alpha \boldsymbol{k}}| \approx 0$$

translate into necks and holes **reducing the Fermi surface** $\mathcal{S}$.

Both effects suppress the superfluid density:

$$\rho_{n,\mathrm{S}} = \frac{m_n^2}{12\pi^3\hbar^2} \sum_{\alpha} \int_{\mathrm{F}} |\nabla_{\boldsymbol{k}} \varepsilon_{\alpha \boldsymbol{k}}| \mathrm{d}\mathcal{S}^{(\alpha)}$$

Chamel,Phys.Rev.C85,035801(2012)

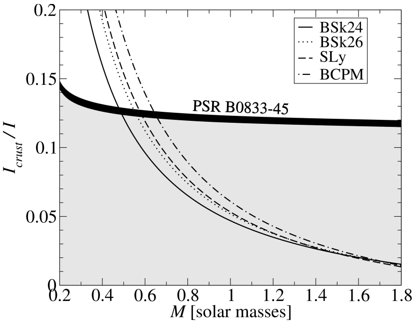


Picture made with XCrySDen

Superfluid reservoir and giant pulsar glitches

The fractional moment of inertia of the superfluid in the crust can be inferred from pulsar glitches.

Chamel&Carter,MNRAS368,796(2006)



From **pulsar timing**: spin frequency $\Omega(t)$ including glitches $\Delta\Omega_i$

$$\frac{I_s}{I} \geq \frac{1}{t|\dot{\Omega}|}\sum_i \Delta\Omega_i$$

The allowed mass is much lower than expected from measured masses and supernova simulations.

Delsate et al., PRD 94, 023008 (2016)

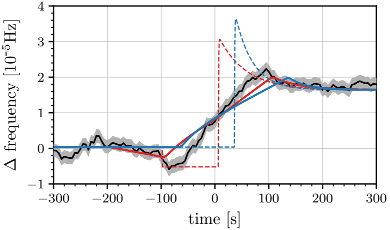
The superfluid in the crust does not carry enough angular momentum.

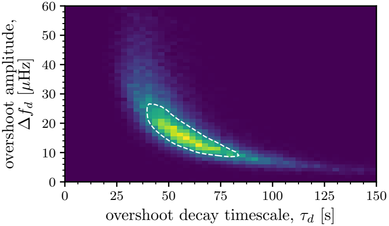
Chamel, PRL 110, 011101 (2013)

Core induced glitches?

Timing of the Crab and Vela pulsars have recently revealed very peculiar evolutions of their spin frequency during the rise of a glitch.

- Analyses of a Vela glitch in 2016 suggest a rotational-frequency **overshoot and a fast relaxation ($\sim$ min)** following the glitch.





Ashton, Lasky, Graber, Palfreyman, Nature Astronomy 3, 1143 (2019)

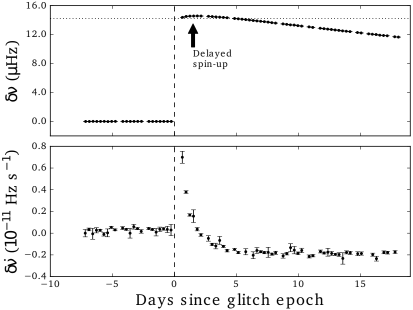
Core induced glitches?

Timing of the Crab and Vela pulsars have recently revealed very peculiar evolutions of their spin frequency during the rise of a glitch.

- Analyses of a Vela glitch in 2016 suggest a rotational-frequency **overshoot and a fast relaxation ($\sim$ min)** following the glitch.

  Ashton, Lasky, Graber, Palfreyman, Nature Astronomy 3, 1143 (2019)

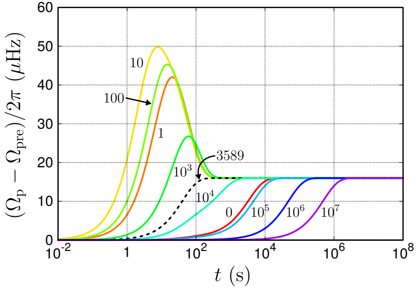
- A **delayed spin-up** has been detected in the 1989, 1996 and 2017 Crab glitches.



Shaw et al., MNRAS, 478, 3832 (2018)

Role of vortex pinning to fluxoids

These differences can be interpreted from the interactions between superfluid vortices and proton fluxoids in neutron-star cores.



The number N_p of fluxoids attached to vortices turns out to be a key parameter governing the global dynamics of the star:

- $N_p < N_p^{crit}$: overshoot $\Delta\Omega_{over} < \Delta\Omega/(1 - I_n^{free}/I)$,
- $N_p < N_p^{crit}$: smooth spin-up on a longer timescale.

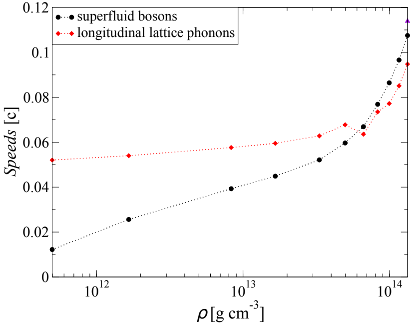
Sourie&Chamel, MNRAS 493, L98 (2020)

Superfluid density and sound propagation

The suppression of the superfluid fraction impacts sound modes:

- transverse lattice phonons (clusters are effectively heavier),
- longitudinal lattice and superfluid phonons are mixed:

Chamel,Page,Reddy,PRC87,035803(2013); J.Phys.Conf.Ser.665, 012065(2016)



no suppression of $\rho_{n,S}$ and no mixing:

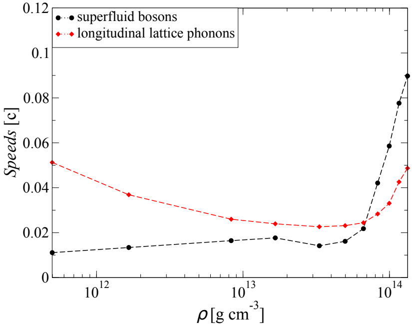
- lattice phonon velocity v_ℓ^0
- superfluid phonon velocity v_ϕ^0

Superfluid density and sound propagation

The suppression of the superfluid fraction impacts sound modes:

- transverse lattice phonons (clusters are effectively heavier),
- longitudinal lattice and superfluid phonons are mixed:

Chamel,Page,Reddy,PRC87,035803(2013); J.Phys.Conf.Ser.665, 012065(2016)



suppression of $\rho_{n,S}$ and no mixing:

$$v_\ell = \sqrt{\frac{\rho_{cluster}}{\rho_{cluster} + \rho_n - \rho_{n,S}}}\, v_\ell^0 < v_\ell^0$$

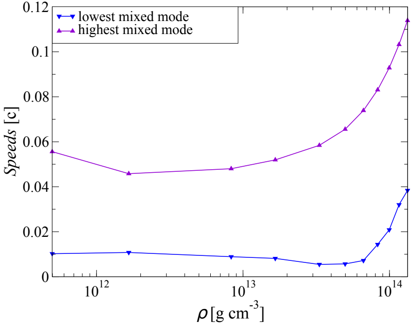
$$v_\phi = \frac{\rho_{n,S}}{\rho_n} v_\phi^0 < v_\phi^0$$

Superfluid density and sound propagation

The suppression of the superfluid fraction impacts sound modes:

- transverse lattice phonons (clusters are effectively heavier),
- longitudinal lattice and superfluid phonons are mixed:

Chamel,Page,Reddy,PRC87,035803(2013); J.Phys.Conf.Ser.665, 012065(2016)



suppression of $\rho_{n,\rm S}$ and mixing:

$$v_\pm = \frac{V}{\sqrt{2}}\sqrt{1 \pm \sqrt{1 - \frac{4 v_\ell^2 v_\phi^2}{V^4}}}$$

$$V = \sqrt{v_\ell^2 + v_\phi^2 + g_{\rm mix}^2}$$

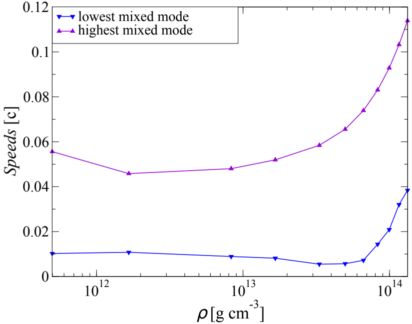
$$g_{\rm mix} = v_\phi \sqrt{\frac{(\rho_{n,cluster} + \rho_n - \rho_{n,\rm S})^2}{(\rho_{cluster} + \rho_n - \rho_{n,\rm S})\rho_{n,\rm S}}}$$

Superfluid density and sound propagation

The suppression of the superfluid fraction impacts sound modes:

- transverse lattice phonons (clusters are effectively heavier),
- longitudinal lattice and superfluid phonons are mixed:

Chamel,Page,Reddy,PRC87,035803(2013); J.Phys.Conf.Ser.665, 012065(2016)



suppression of $\rho_{n,\mathrm{S}}$ and mixing:

$$v_{\pm} = \frac{V}{\sqrt{2}}\sqrt{1 \pm \sqrt{1 - \frac{4v_\ell^2 v_\phi^2}{V^4}}}$$

$$V = \sqrt{v_\ell^2 + v_\phi^2 + g_{\mathrm{mix}}^2}$$

$$g_{\mathrm{mix}} = v_\phi \sqrt{\frac{(\rho_{n,cluster} + \rho_n - \rho_{n,\mathrm{S}})^2}{(\rho_{cluster} + \rho_n - \rho_{n,\mathrm{S}})\rho_{n,\mathrm{S}}}}$$

This has implications for the oscillations and cooling of neutron stars.

Finite superflow and Landau's critical velocity

What happens for large superflows?

Finite superflow and Landau's critical velocity

What happens for large superflows?

Ignoring spatial inhomogeneities, the HFB equations can be solved analytically. An **effective superfluid velocity** naturally emerges:

$$\mathbb{V}_q \equiv \frac{m_q}{m_q^{\oplus}} \boldsymbol{V}_q + \frac{\boldsymbol{I}_q}{\hbar}$$

Finite superflow and Landau's critical velocity

What happens for large superflows?

Ignoring spatial inhomogeneities, the HFB equations can be solved analytically. An **effective superfluid velocity** naturally emerges:

$$\mathbb{V}_{\boldsymbol{q}} \equiv \frac{m_q}{m_q^{\oplus}} \boldsymbol{V_q} + \frac{\boldsymbol{I_q}}{\hbar}$$

The order parameter Δ_q of the superfluid phase remains unchanged provided $\mathbb{V}_q < \mathbb{V}_{Lq}$, the critical velocity from **Landau's criterion**

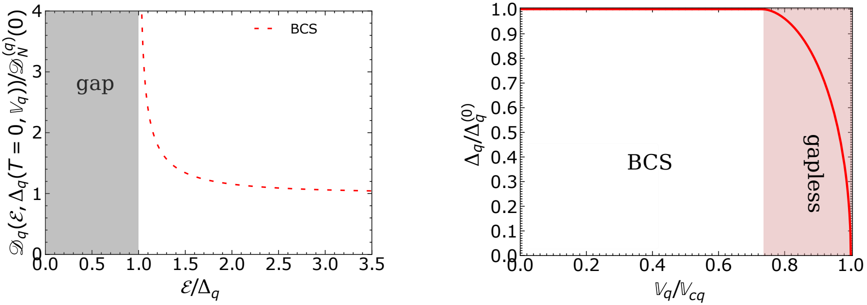
$$\mathbb{V}_{Lq} \equiv V_{Fq} \sqrt{\frac{\mu_q}{2\varepsilon_{Fq}} \left[\sqrt{1 + \left(\frac{\Delta_q}{\mu_q} \right)^2} - 1 \right]} \approx \frac{\Delta_q}{\hbar k_{Fq}}$$

Allard & Chamel, Phys. Rev. C 108, 015801 (2023)

This is the generalization to nuclear superfluids of the expression obtained for a one-component Fermi gas
Combescot, Yu Kagan, Stringari, Phys. Rev. A 74, 042717 (2006)

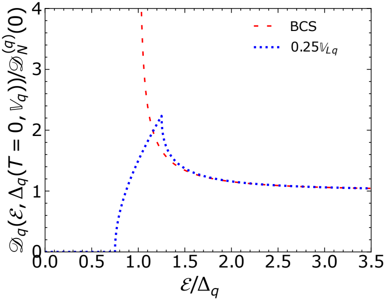
Gapless superfluidity

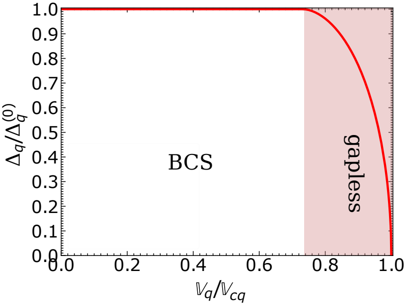
Superfluidity is not destroyed for super Landau superflow $\mathbb{V}_q \geq \mathbb{V}_{Lq}$, but Δ_q decreases and eventually vanishes for $\mathbb{V}_q = \mathbb{V}_{cq} \approx 1.36\mathbb{V}_{Lq}$. In this intermediate regime, superfluidity becomes gapless:



Gapless superfluidity

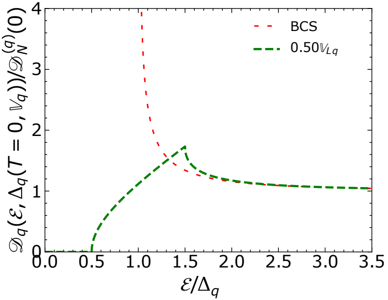
Superfluidity is not destroyed for super Landau superflow $\mathbb{V}_q \geq \mathbb{V}_{Lq}$, but Δ_q decreases and eventually vanishes for $\mathbb{V}_q = \mathbb{V}_{cq} \approx 1.36\mathbb{V}_{Lq}$. In this intermediate regime, superfluidity becomes gapless:

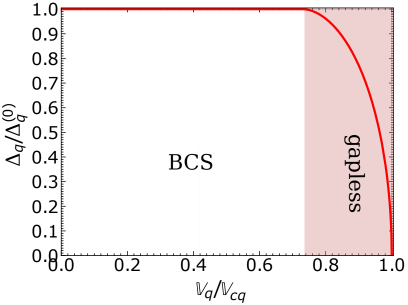




Gapless superfluidity

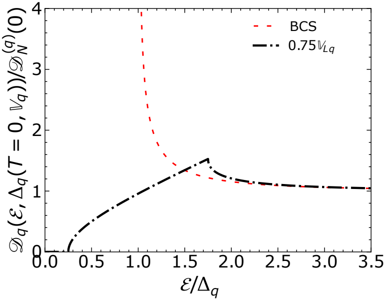
Superfluidity is not destroyed for super Landau superflow $\mathbb{V}_q \geq \mathbb{V}_{Lq}$, but Δ_q decreases and eventually vanishes for $\mathbb{V}_q = \mathbb{V}_{cq} \approx 1.36\mathbb{V}_{Lq}$. In this intermediate regime, superfluidity becomes gapless:

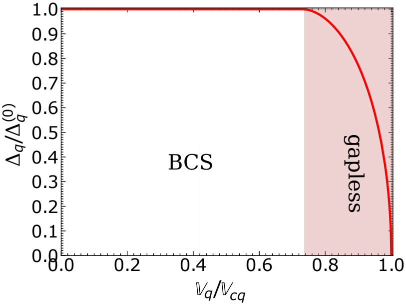




Gapless superfluidity

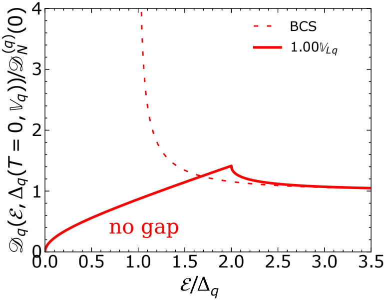
Superfluidity is not destroyed for super Landau superflow $\mathbb{V}_q \geq \mathbb{V}_{Lq}$, but Δ_q decreases and eventually vanishes for $\mathbb{V}_q = \mathbb{V}_{cq} \approx 1.36\mathbb{V}_{Lq}$. In this intermediate regime, superfluidity becomes gapless:

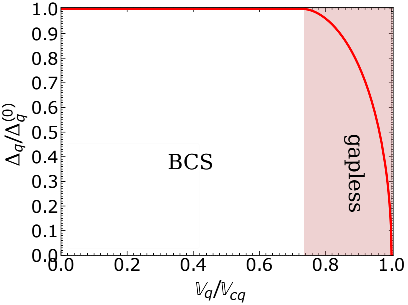




Gapless superfluidity

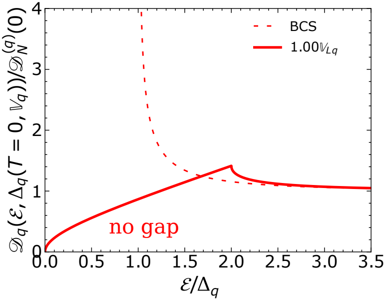
Superfluidity is not destroyed for super Landau superflow $\mathbb{V}_q \geq \mathbb{V}_{Lq}$, but Δ_q decreases and eventually vanishes for $\mathbb{V}_q = \mathbb{V}_{cq} \approx 1.36\mathbb{V}_{Lq}$. In this intermediate regime, superfluidity becomes gapless:

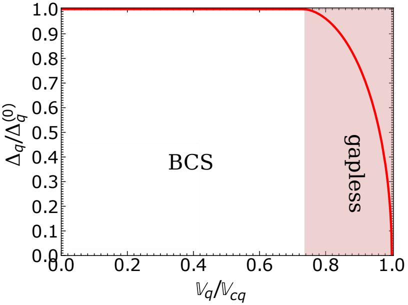




Gapless superfluidity

Superfluidity is not destroyed for super Landau superflow $\mathbb{V}_q \geq \mathbb{V}_{Lq}$, but Δ_q decreases and eventually vanishes for $\mathbb{V}_q = \mathbb{V}_{cq} \approx 1.36\mathbb{V}_{Lq}$. In this intermediate regime, superfluidity becomes gapless:



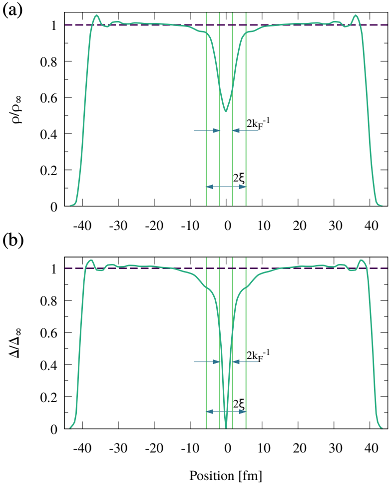


A normal fluid of quasiparticles excitations is present even at $T = 0$.

Allard & Chamel, Phys. Rev. C 108, 015801 (2023); Phys. Rev. C 108, 045801 (2023)

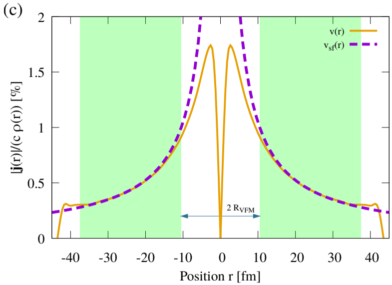
Microscopic dynamics of a vortex

Full HFB calculations of a neutron superfluid vortex:



In the vortex core, $V_n > V_{Ln}$:

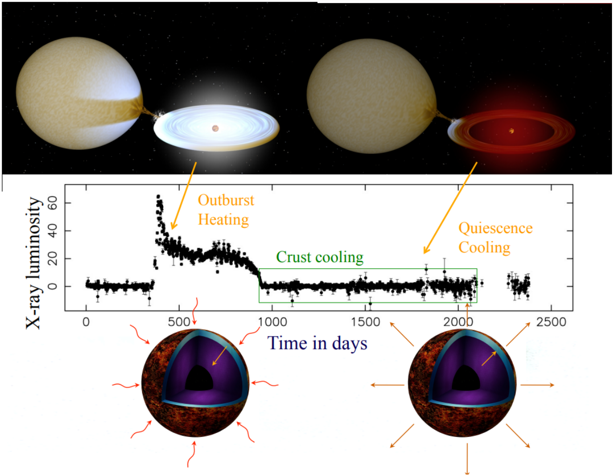
- Cooper pairs are broken ($\Delta \to 0$ but ρ_n finite)
- $\boldsymbol{V_n} \neq \boldsymbol{v_n} \neq \dfrac{\hbar \boldsymbol{j_n}}{\rho_n}$



Peçak, Chamel, Magierski, Wlazlowski, Phys. Rev. C 104, 055801 (2021)

Transiently accreting neutron stars

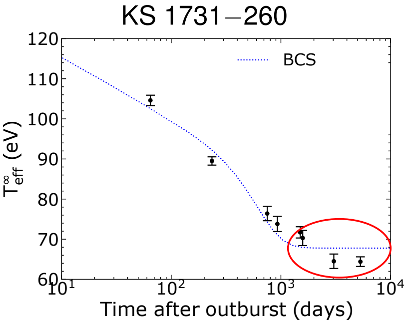
Superfluidity can be probed from the cooling of neutron-star crusts after the end of an accretion episode

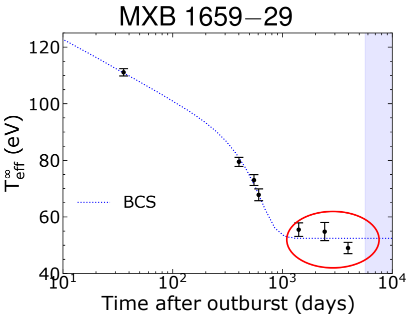


Wijnands, Degenaar, Page, J. Astrophys. Astron. 38, 49, (2017)

Observational evidence of gapless superfluidity

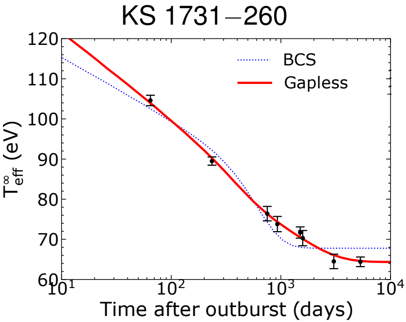
Some sources appeared **colder than expected**:

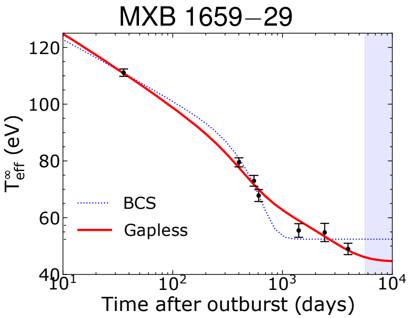




Observational evidence of gapless superfluidity

Some sources appeared **colder than expected**:



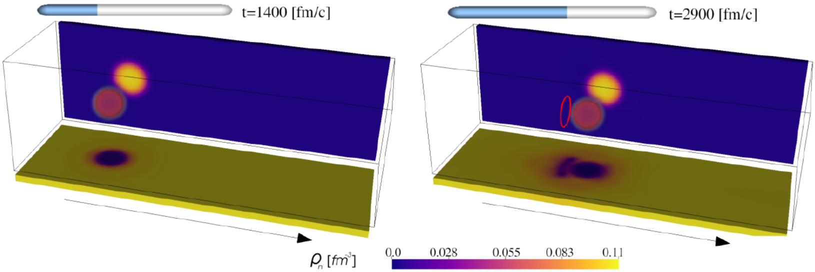


Gapless superfluidity can naturally explain the observed late-time cooling due to the huge enhancement of the neutron specific heat.

Allard & Chamel, PRL 132, 181001 (2024); Allard & Chamel, EPJA 60, 116 (2024)

Stability of the super Landau superflow?

Fully self-consistent time-dependent HFB simulations of the **motion of a single cluster through the neutron superfluid**:



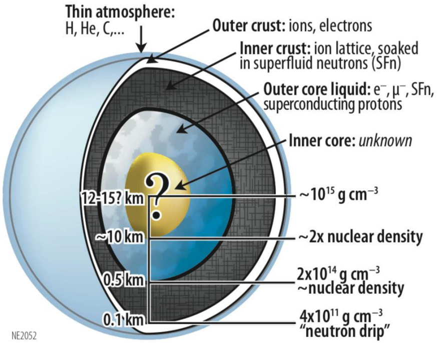
The gapless superfluid is stable in deep crust but in shallow layers Cooper pair breaking leads to the **formation of vortex rings**: onset of quantum turbulence? Glitch triggering mechanism?

Peçak, Chamel, Magierski, Wlazlowski, Phys. Rev. C 104, 055801 (2021)

Peçak, Chamel, Zdanowicz et al., Phys. Rev. X in press

Summary

Neutron stars are the **most extreme superfluids and superconductors** (supersolid?) known in the Universe.



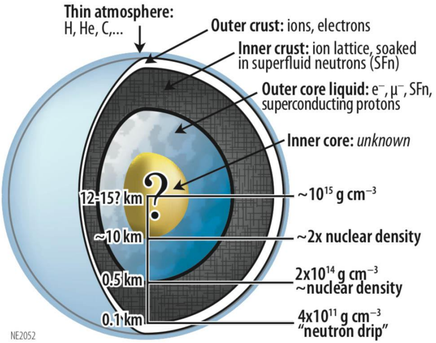
Nucleonic condensates are supported by **independent observations** (glitches, cooling).

However, many aspects still remain poorly understood.

Additional challenge to relate the local dynamics of vortices to the **global dynamics** of the star.

Summary

Neutron stars are the **most extreme superfluids and superconductors** (supersolid?) known in the Universe.



Nucleonic condensates are supported by **independent observations** (glitches, cooling).

However, many aspects still remain poorly understood.

Additional challenge to relate the local dynamics of vortices to the **global dynamics** of the star.

The neutron-star physics is very rich. Insights from terrestrial experiments are crucially needed!