Mathematical methods for neutrino cross-section extraction

Based on arXiv:2401.04065

$$U = \bigoplus_{\mathfrak{b}=0} U_{\mathfrak{b}} = U_0 \oplus U_1 \oplus \cdots = \begin{pmatrix} U_0 & 0 & 0 & \cdots \\ 0 & U_1 & 0 & \cdots \\ 0 & 0 & \ddots & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \qquad \langle \sigma \rangle_\mu = \left\langle \frac{d^n\sigma}{d\mathbf{x}} \right\rangle_\mu \cdot \Delta\mathbf{x}_\mu$$

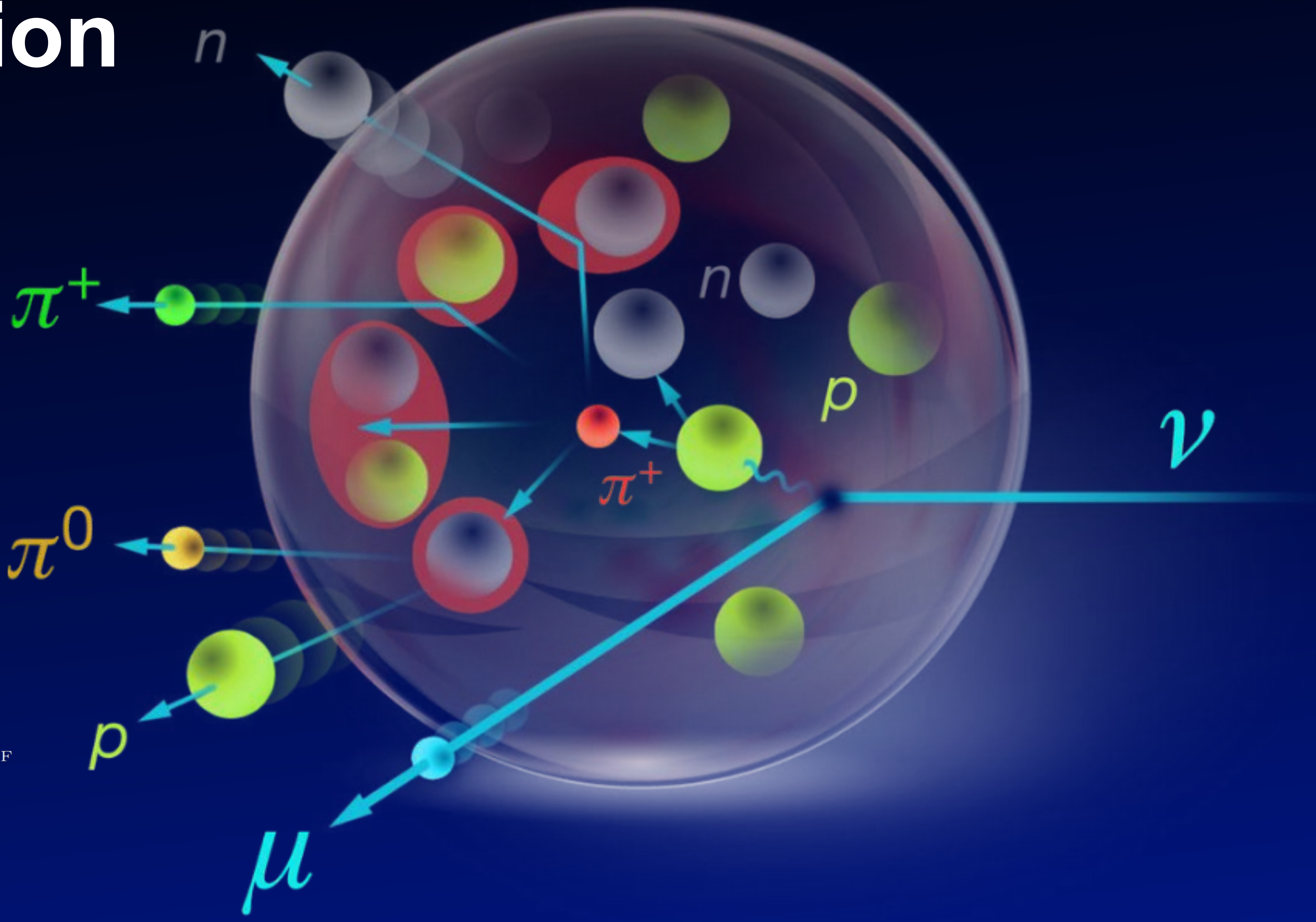
$$\mathcal{P}_a = \beta_a n_a - D_a + D_a \log\left(\frac{D_a}{\beta_a n_a}\right) + \frac{\beta_a^2 - 1}{2\sigma_a^2}$$

$$\mathfrak{E}^{i+1}_{\mu a} = \frac{\partial \hat{\phi}^{i+1}_\mu}{\partial d_a} = U^i_{\mu a} + \frac{\hat{\phi}^{i+1}_\mu}{\hat{\phi}^i_\mu} \mathfrak{E}^i_{\mu a} - \sum_{\lambda, b} \epsilon_\lambda \frac{d_b}{\hat{\phi}^i_\lambda} U^i_{\mu b} U^i_{\lambda b} \mathfrak{E}^i_{\lambda a} \qquad B_a = \sum_{\mathfrak{p}} \alpha_{\mathfrak{p}} B_{a\mathfrak{p}}$$

$$\mathbf{B}^{\mathrm{constr,CV}}_S = \mathbf{B}^{\mathrm{CV}}_S + V_{\mathbf{B}_S \mathbf{n}_C} \cdot V^{-1}_{\tilde{\mathbf{n}}_C \tilde{\mathbf{n}}_C} \cdot \left(\mathbf{D}_C - \mathbf{n}^{\mathrm{CV}}_C\right)$$

$$V^{\mathrm{constr}}_{\mathbf{B}_S \mathbf{B}_S} = V_{\mathbf{B}_S \mathbf{B}_S} - V_{\mathbf{B}_S \mathbf{n}_C} \cdot V^{-1}_{\tilde{\mathbf{n}}_C \tilde{\mathbf{n}}_C} \cdot V^T_{\mathbf{B}_S \mathbf{n}_C} \qquad H_{\text{чъ}} = -\left.\frac{\partial^2 \log(\mathcal{L})}{\partial K_{\text{ч}}\, \partial K_{\text{ъ}}}\right|_{\mathbf{K}=\mathbf{K}^{\mathrm{PF}}}$$

$$V_{\mathbf{m}_S \mathbf{m}_S} = \mathrm{Cov}(\mathbf{m}_S, \mathbf{m}_S) = V_{\boldsymbol{\phi}_S \boldsymbol{\phi}_S} + V^{\mathrm{constr}}_{\boldsymbol{\phi}_S \mathbf{B}_S} + V^{\mathrm{constr}}_{\mathbf{B}_S \boldsymbol{\phi}_S} + V^{\mathrm{constr}}_{\mathbf{B}_S \mathbf{B}_S}$$



Steven Gardiner (gardiner@fnal.gov)

Measuring neutrino interactions for next-generation oscillation experiments

ECT*, 25 October 2024

Fermilab

Scope of the talk

- **Trailer for v2** of my recent methods paper
  - Main ideas unchanged since v1
  - Some details refined upon further thought and feedback

- Summary of approaches used in current analyses
  - Quick overview here, Lukas covered many elements already

- "Cookbook" for applying proposed innovations across experiments

- MicroBooNE "**data driven model validation**"
  - Distinct from these techniques = not a change to extraction procedure itself
  - Also likely interesting to this community

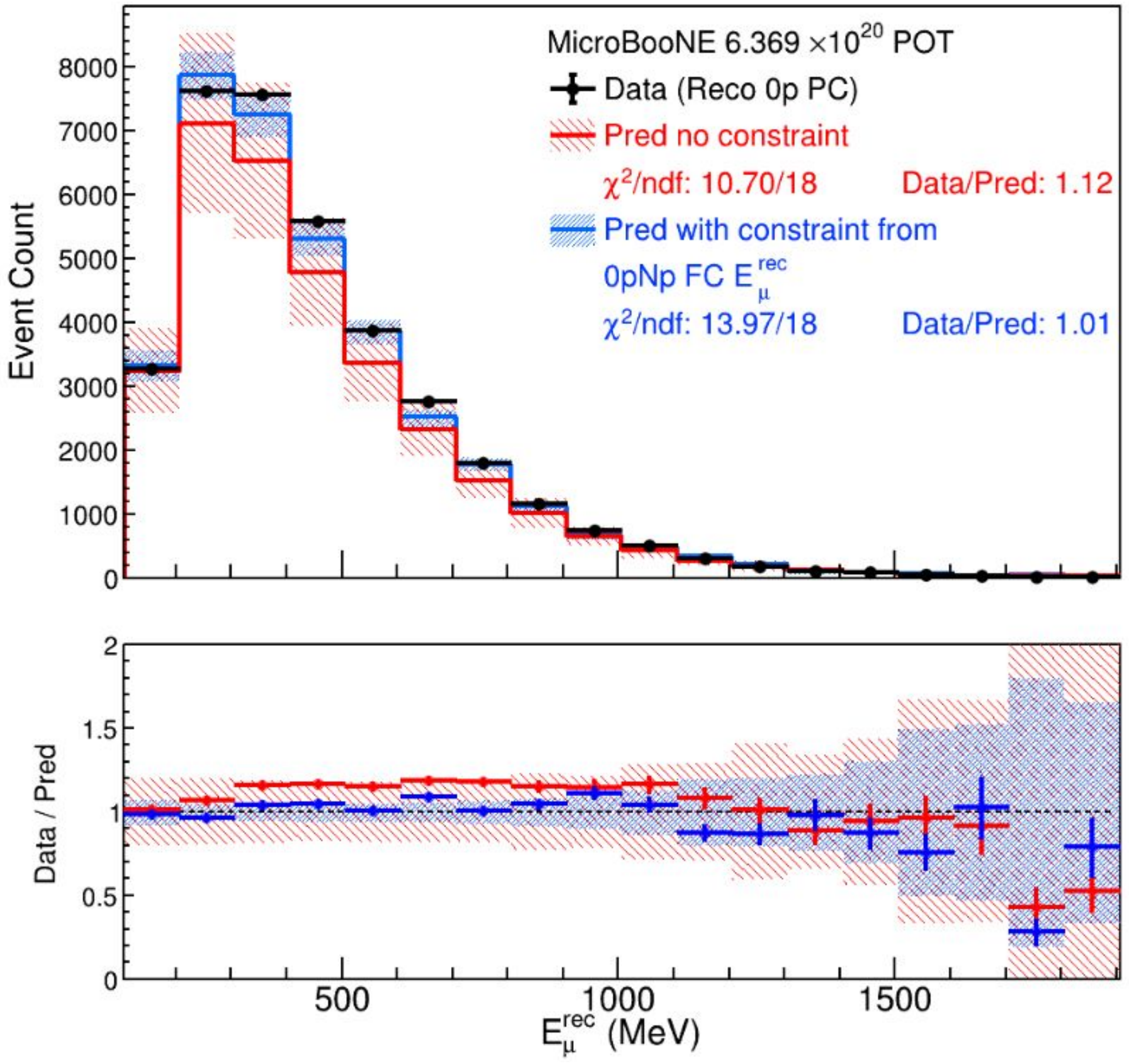
arXiv:2401.04065

FERMILAB-PUB-23-692-CSAID

Mathematical methods for neutrino cross-section extraction

Steven Gardiner*
Fermi National Accelerator Laboratory, Batavia, Illinois 60510 USA
(Dated: October 25, 2024)

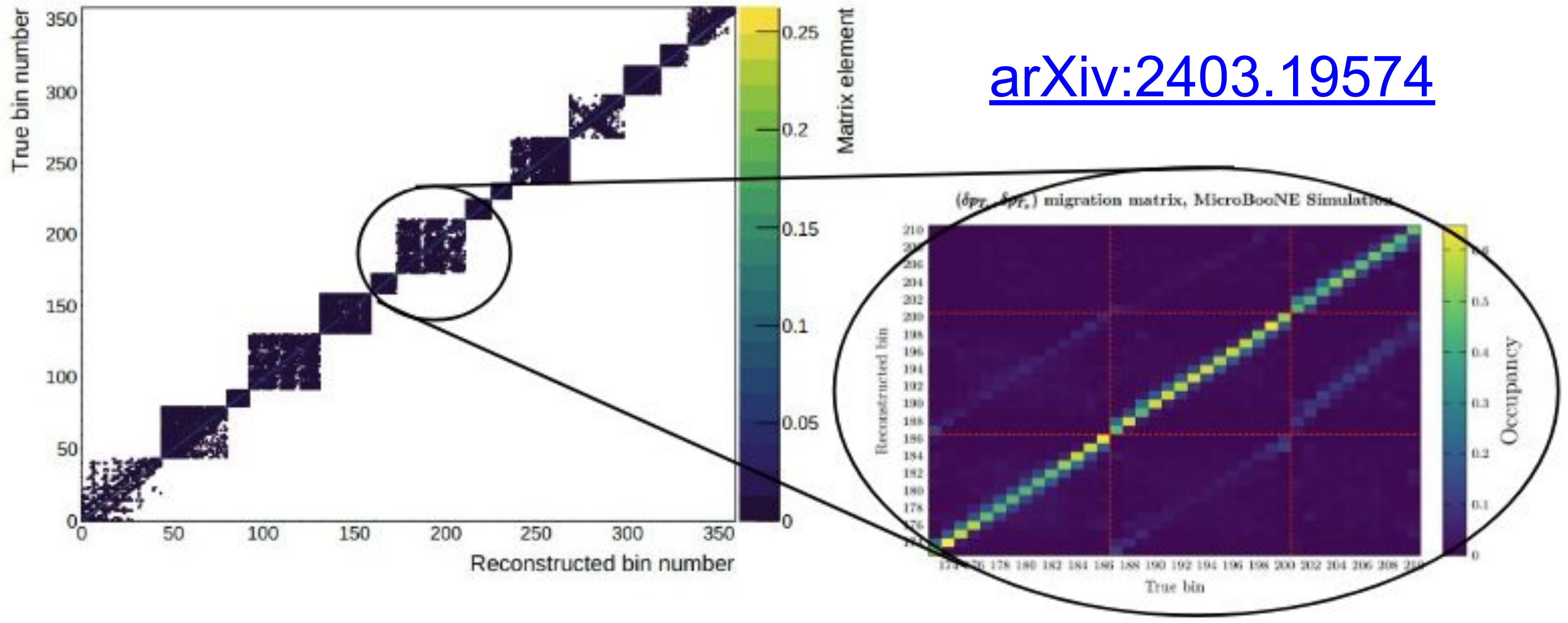
Precise modeling of neutrino-nucleus scattering is becoming increasingly important as accelerator-based oscillation experiments seek definitive answers to open questions about neutrino properties. To guide the needed model refinements, a growing number of experimental collaborations are pursuing a wide-ranging program of neutrino interaction measurements at GeV energies. A key step in most such analyses is cross-section extraction, in which measured event counts are corrected for background contamination and imperfect detector performance to yield cross-section results that are directly comparable to theoretical predictions. In this paper, I review the major approaches to cross-section extraction in the literature using representative examples from the MINERvA, MicroBooNE, and T2K experiments. I then present two mathematical techniques, blockwise unfolding and the conditional covariance background constraint, which overcome some limitations of typical cross-section extraction procedures.



Phys. Rev. D 110, 013006 (2024), Sec. VI

Overview of new methods

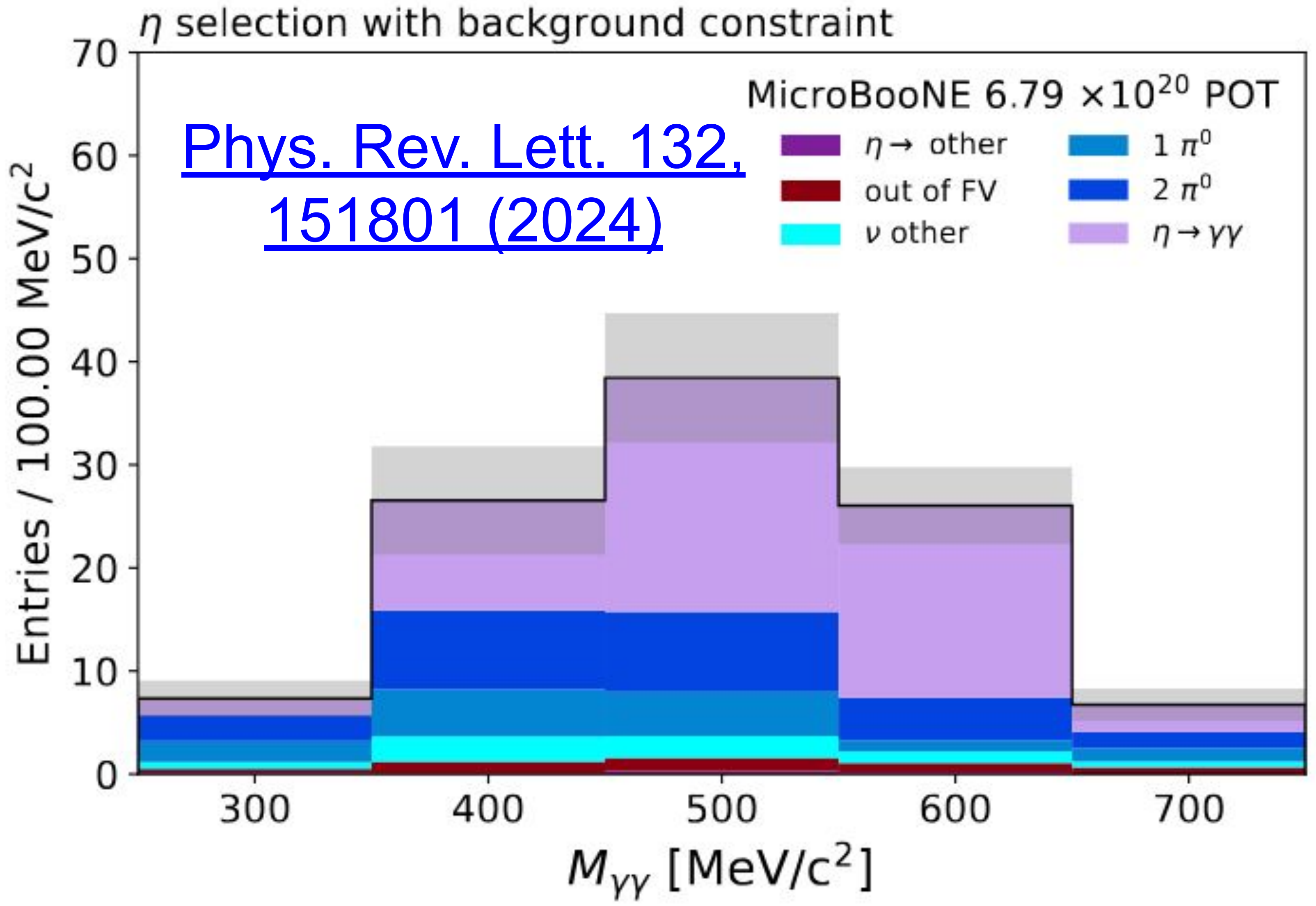
Blockwise unfolding



[arXiv:2403.19574](https://arxiv.org/abs/2403.19574)

- An argument that we should report correlations much more thoroughly
- Practical advice for how to do that
- Recipes for avoiding some potential pitfalls
- Used in several recent MicroBooNE papers

Conditional Covariance Background Constraint (CCBC)

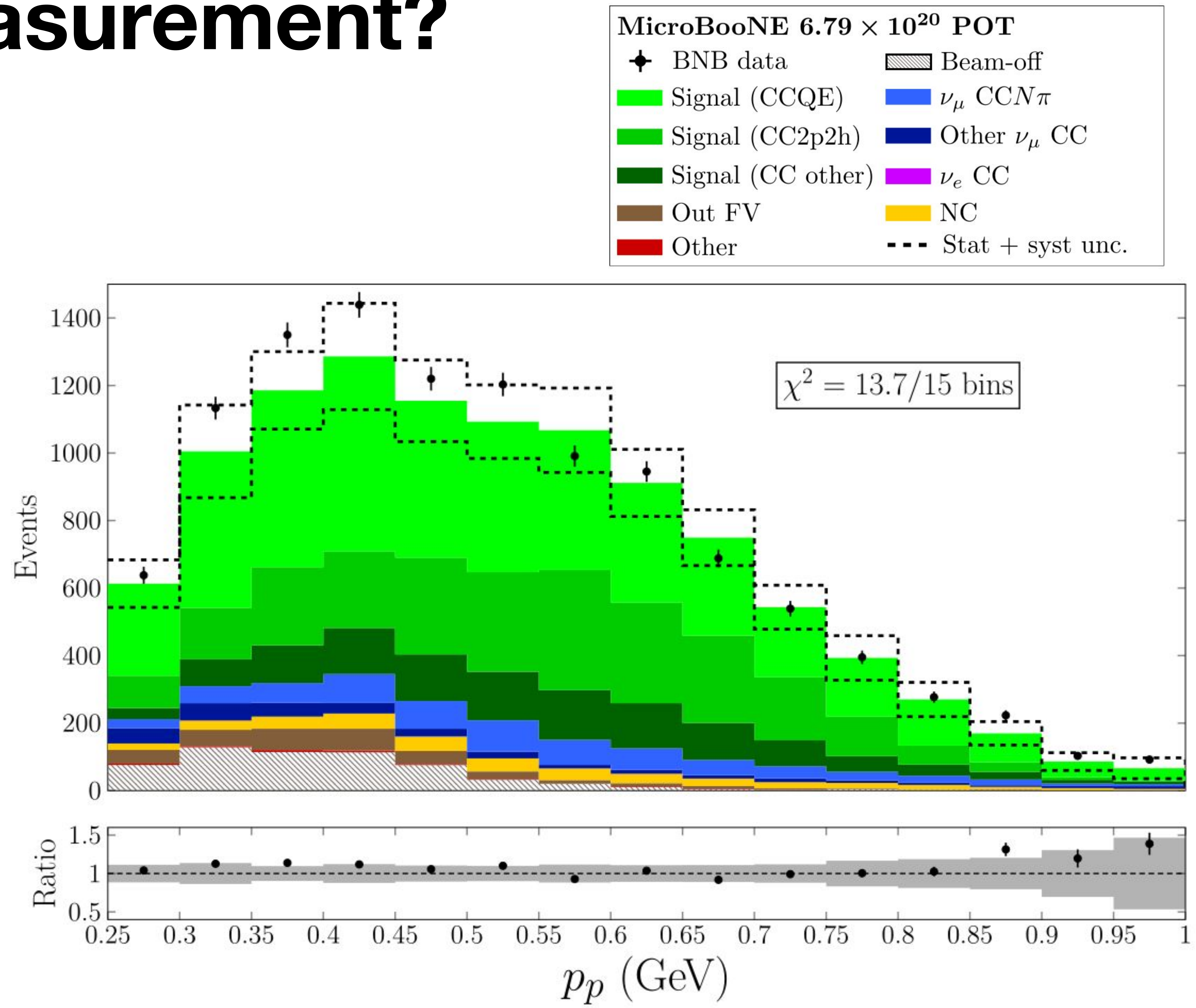


Phys. Rev. Lett. 132, 151801 (2024)

- Use Gaussian statistics to refine a background prediction
- Achieves compatibility with preferred MicroBooNE extraction procedure
  - No background constraint in *any* differential μBooNE result so far
- Inspired by μBooNE η production analysis

How do we perform the measurement?

- Counting experiment: bin for variable(s) of interest
- Raw event counts comparable to simulation
  - Only feasible by the experimental collaboration
- **Cross-section extraction**
  - Converts this measurement to a result anyone can use
  - Details vary across experiments
- Many subtleties, care must be taken to avoid bias



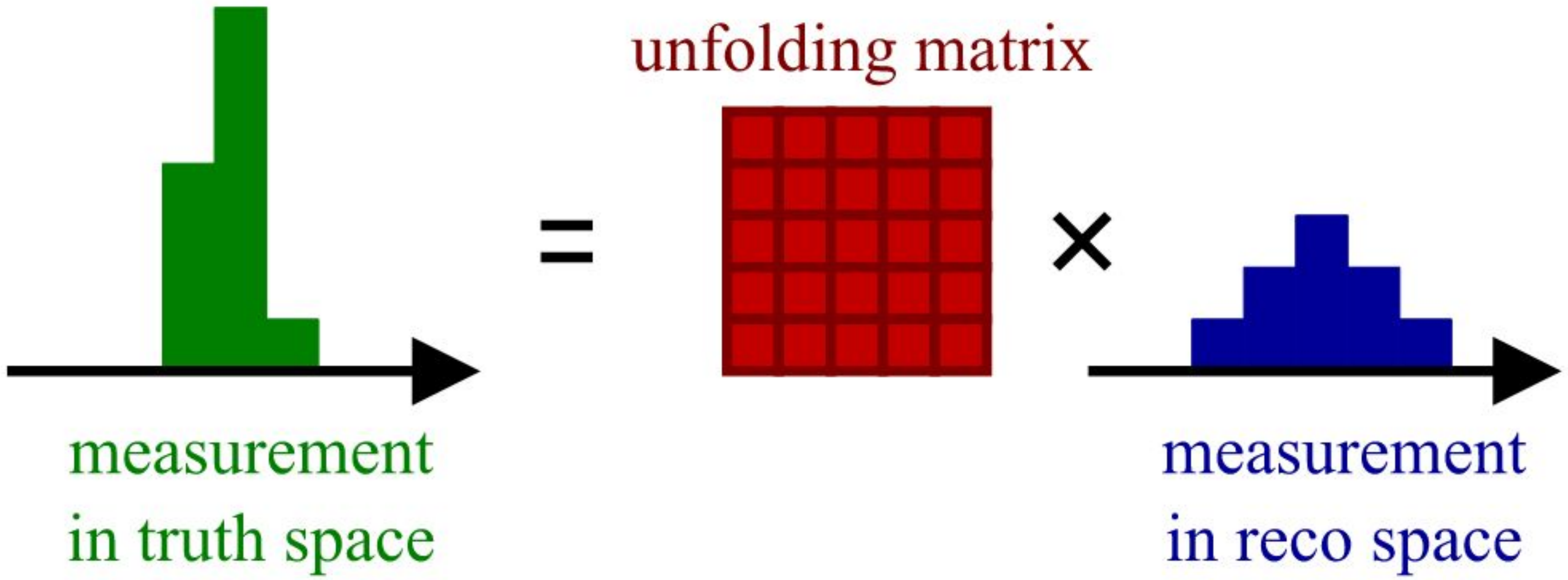
How do we perform the measurement?

- **Flux-averaged differential cross section**
  - true bins *μ*, reco bins *a*
  - Average value in true bin *μ*

- **Unfolding matrix** U accounts for inefficiency and bin migrations

- **Unfolded space ≈ true space**
  - Systematics must be considered carefully

$$\left\langle \frac{d^n\sigma}{d\mathbf{x}} \right\rangle_\mu = \frac{\sum_a U_{\mu a}\left(D_a - B_a\right)}{\Phi\, T\, \Delta \mathbf{x}_\mu}$$



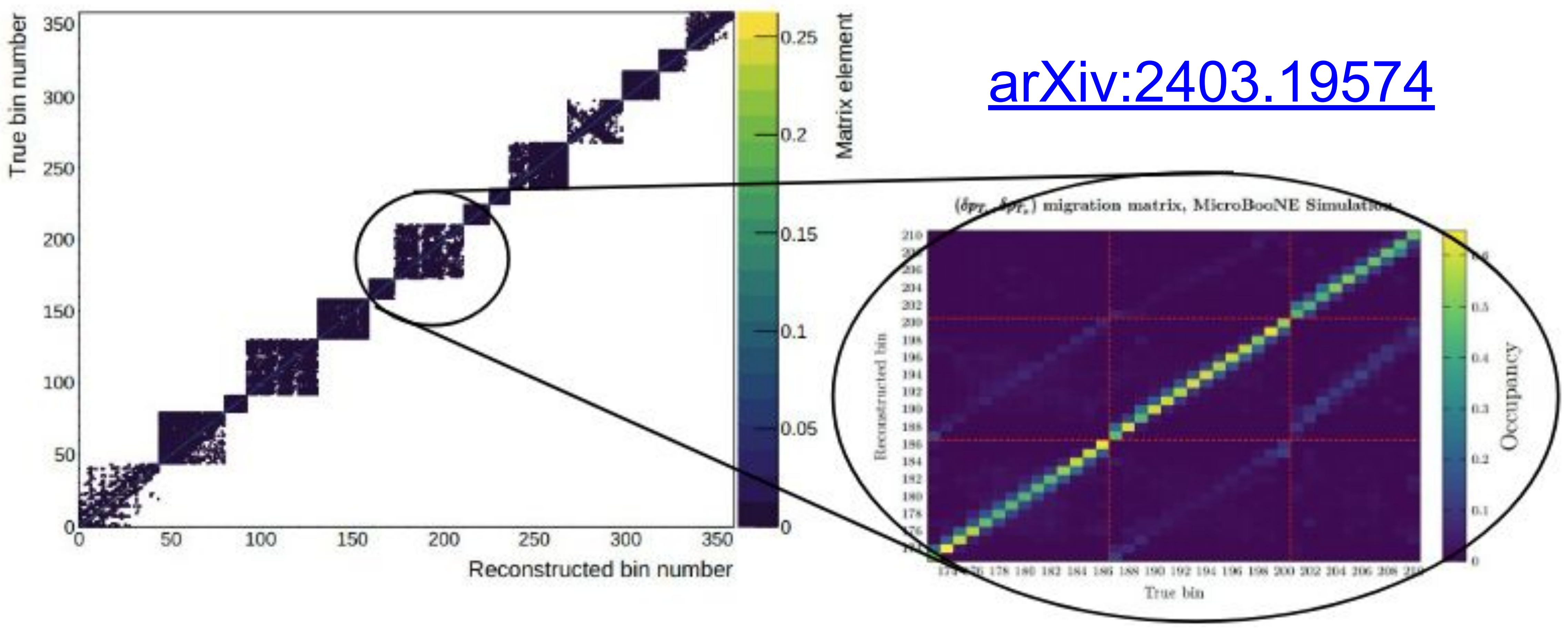
$$U_{\mu a} = \frac{P_{\mu a}}{\epsilon_\mu}$$

"Styles" of cross-section extraction

- Superficially, everyone plays the same game, but differently
  - **3 major approaches** at GeV scale, the rest are perturbations
  - Details are often not spelled out, especially for *Phys. Rev. Lett*.

- **MINERvA**
  - D'Agostini iterative recipe for building unfolding matrix U
  - Uncertainties: repeat extraction, take spread between "universes"
- **MicroBooNE**
  - Wiener-SVD unfolding
  - Compute total covariance on event counts, propagate through unfolding
- **T2K**
  - Perform likelihood fit to event counts (huge number of parameters)
  - Uncertainties can be treated two ways
    - Repeat the fit across many universes (MINERvA-esque)
    - Vary parameters according to post-fit covariance matrix

Blockwise unfolding



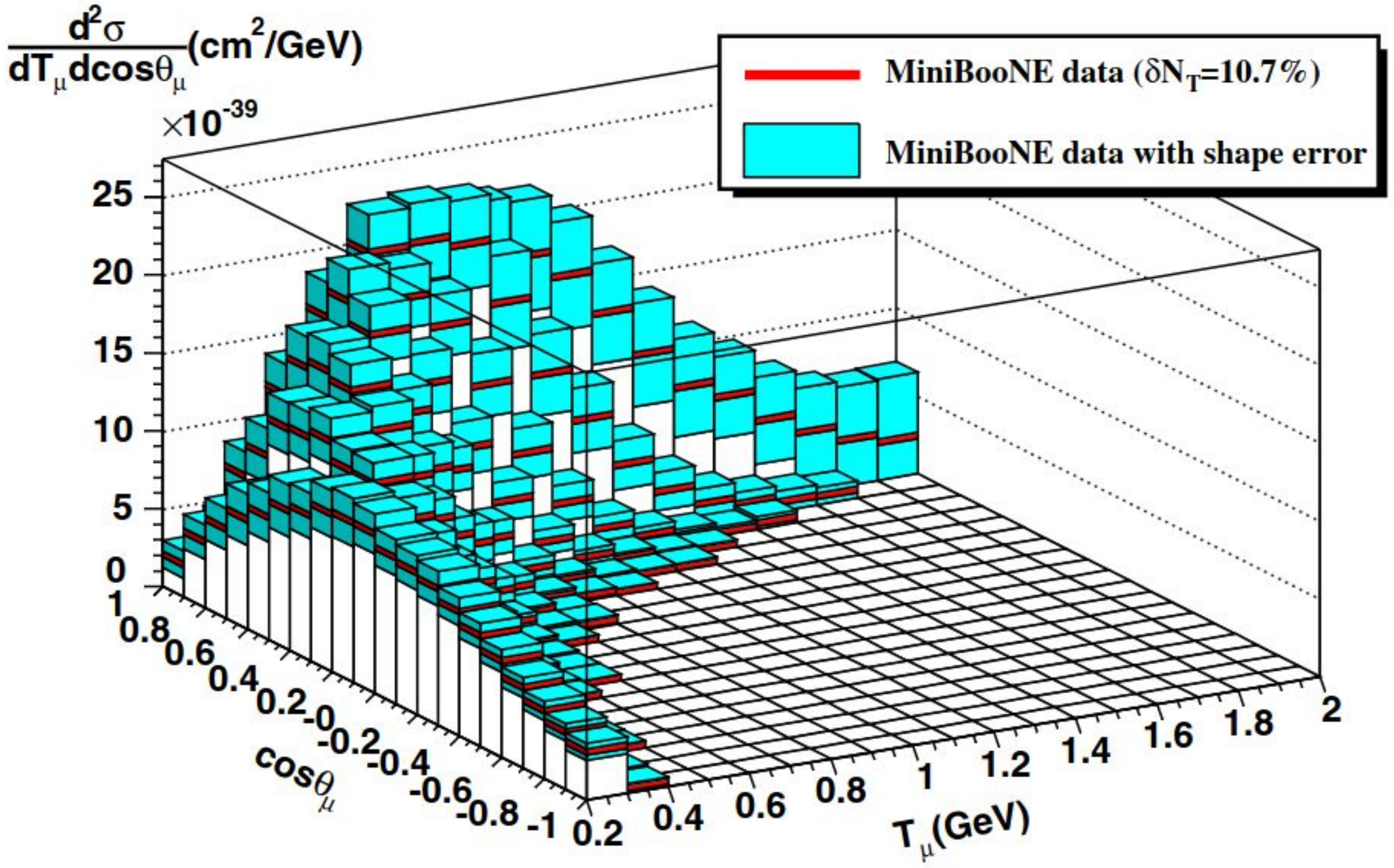
arXiv:2403.19574

"Blockwise unfolding": motivation

Phys. Rev. D 81, 092005 (2010)

- **MiniBooNE**: pioneering neutrino experiment at Fermilab
  - Many cross-section analysis practices established
  - Key early measurements

- Several data releases report binwise uncertainties but ***not correlations***
  - Large & important
  - Both systematic (e.g., flux) and statistical (unfolding)



2D result for CH target

Problematic for quantitative comparisons (χ^2, etc.)

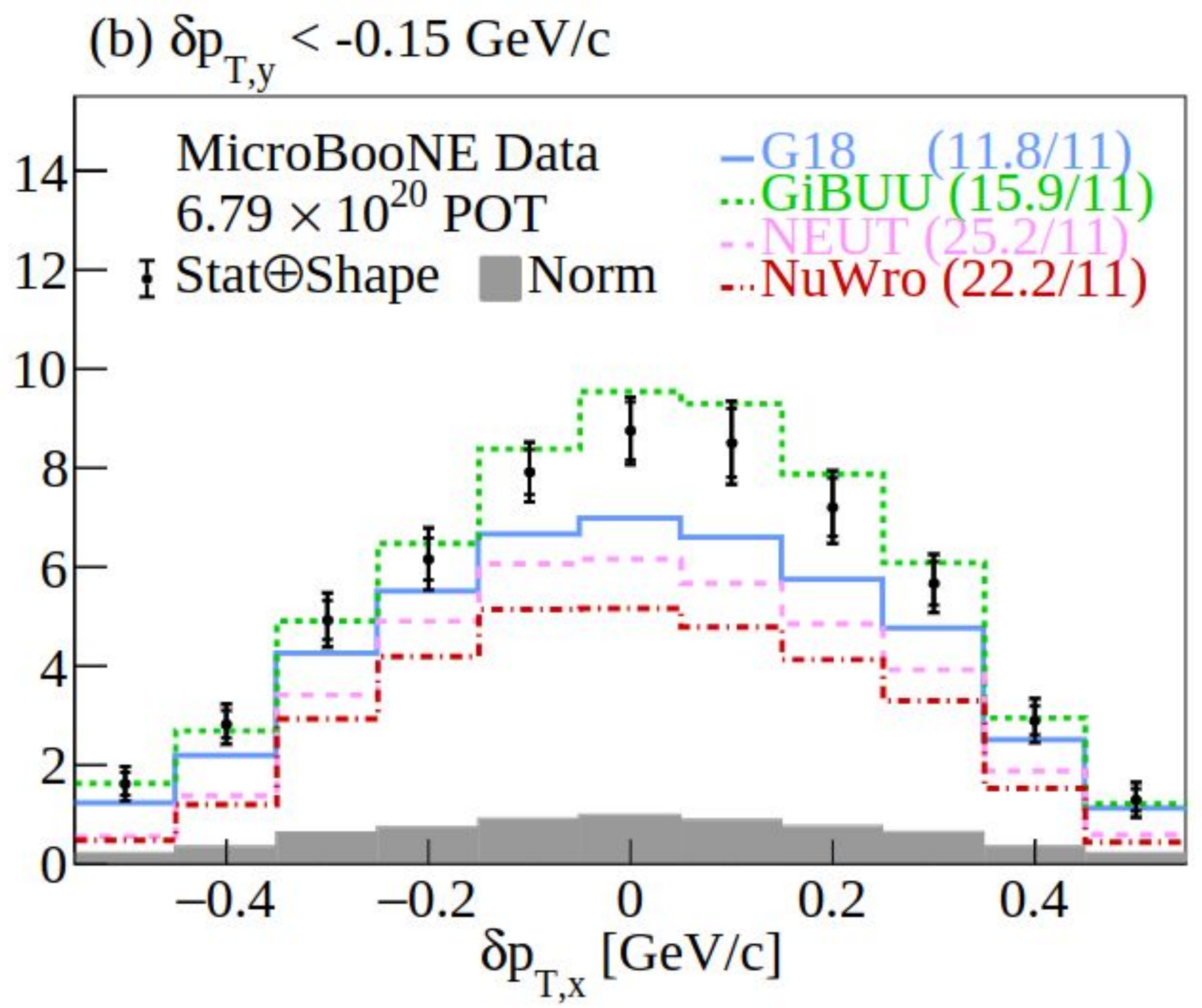
Standard practice is now to provide a full covariance matrix

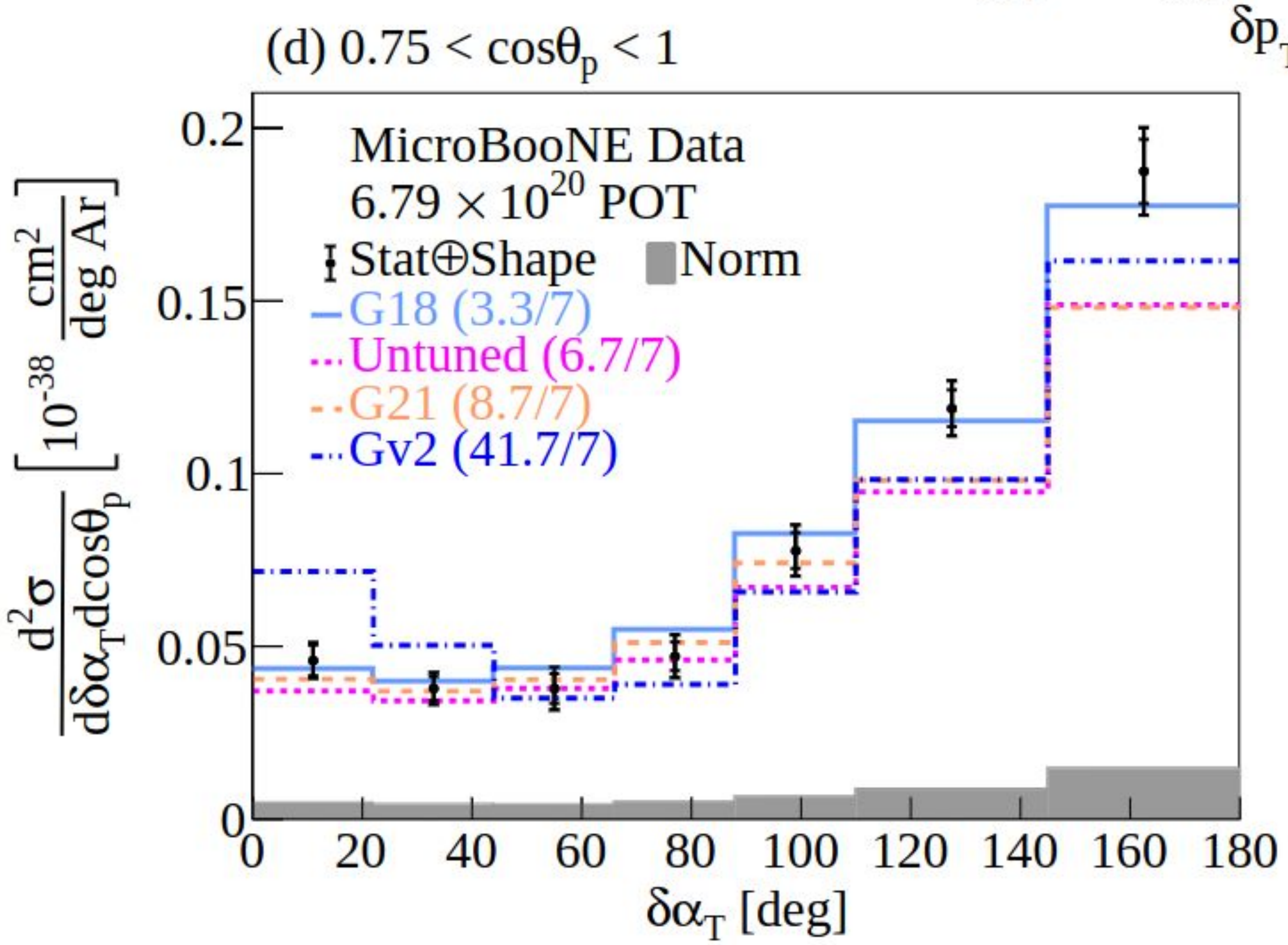
"Blockwise unfolding": motivation

Phys. Rev. D 108, 053002 (2023)

- Experiments often report **multiple kinematic distributions**
  - Same analysis or complementary ones
- **Correlated uncertainties** between distributions are still not typically reported
  - All the same drawbacks as before

ν_μ CC 1p0π data from MicroBooNE



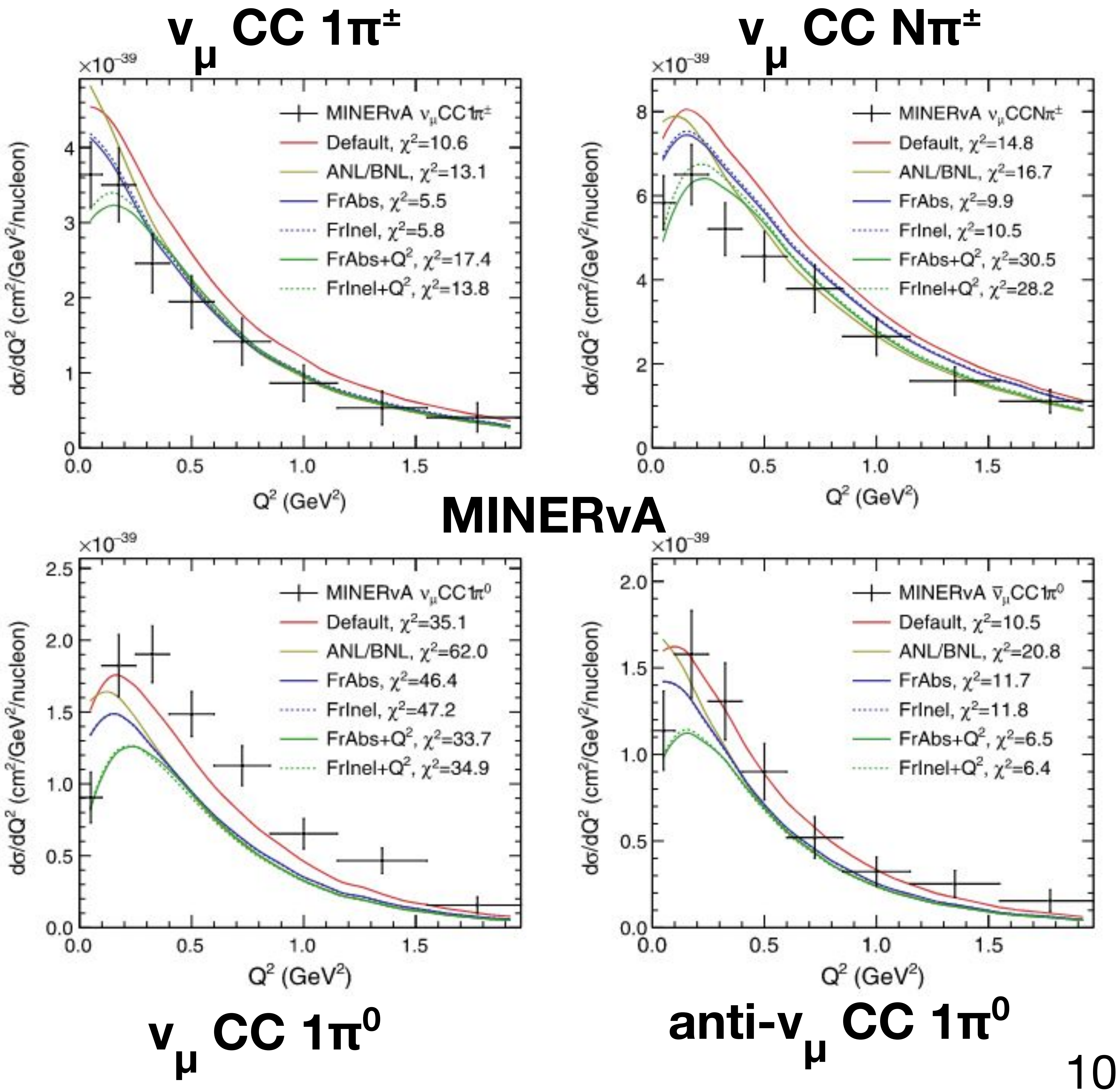


Measurements use same set of ~9000 events

"Blockwise unfolding": motivation

[Phys. Rev D. 100, 072005 (2019)]

- Experiments often report **multiple kinematic distributions**
  - Same analysis or complementary ones
- **Correlated uncertainties** between distributions are still not typically reported
  - All the same drawbacks as before
- **Limitations** discussed in MINERvA paper tuning GENIE to π production data



"Blockwise unfolding": motivation

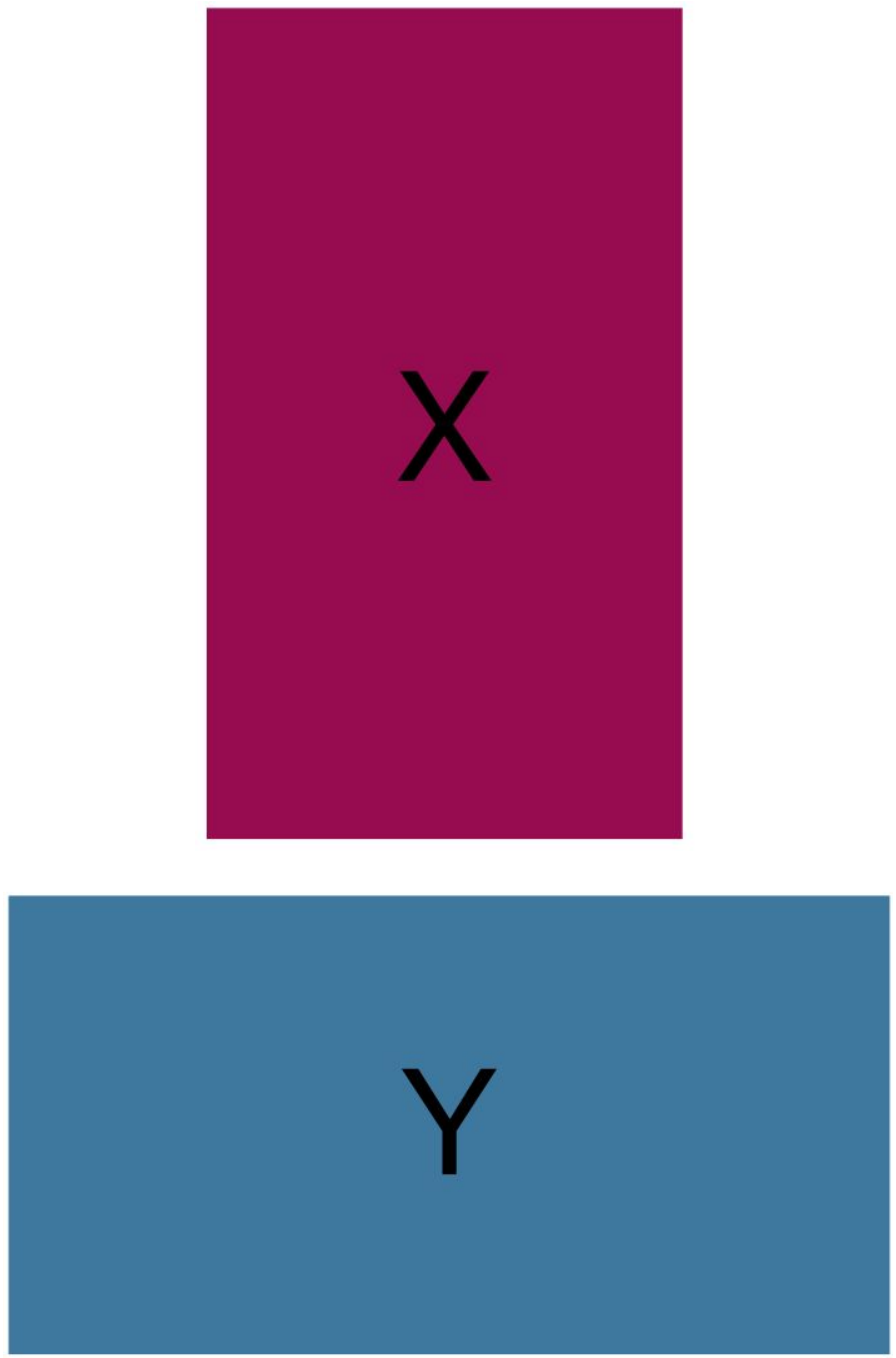
> The published cross sections are one dimensional with correlations provided between the bins within each distribution. No correlations are provided between measurements of different final states, or between different one-dimensional projections of the same measurement. These correlations are expected to be large, coming predominantly from flux and detector uncertainties. Additionally, the ν_μCC1$\pi^\pm$ event sample is a subset (~64%) of the ν_μCC$N\pi^\pm$ event sample, and including both channels introduces a statistical correlation. Not assessing correlations between the distributions, while a common practice in this field, is a limitation when tuning models to multiple datasets. It introduces a bias in the χ^2 statistic that is difficult to quantify, and requires imposing *ad hoc* uncertainties [4] as the test statistic is not expected to follow a χ^2 distribution for the given degrees of freedom.

Phys. Rev D. 100, 072005 (2019)

- Not trivial to add this information after the fact
- Correlations calculable with **suitable planning ahead**
  - Maximize impact from cross-section analyses
- **Two issues**
  - Event overlaps (statistical covariances)
  - Unfolding treatment
- **Methods paper** (arXiv:2401.04065) gives recipes for solving these problems

Statistical covariances

- Events belong to multiple bins
  ⇒ correlated stat uncertainties
- **Easily calculable** if the problem is framed properly
- Arbitrary bins X and Y
  - Event count n_X in bin X follows a Poisson distribution
- Estimator for the mean: n_X
- Estimator for the variance: n_X
- Bin Y is similar. How to get the covariance?

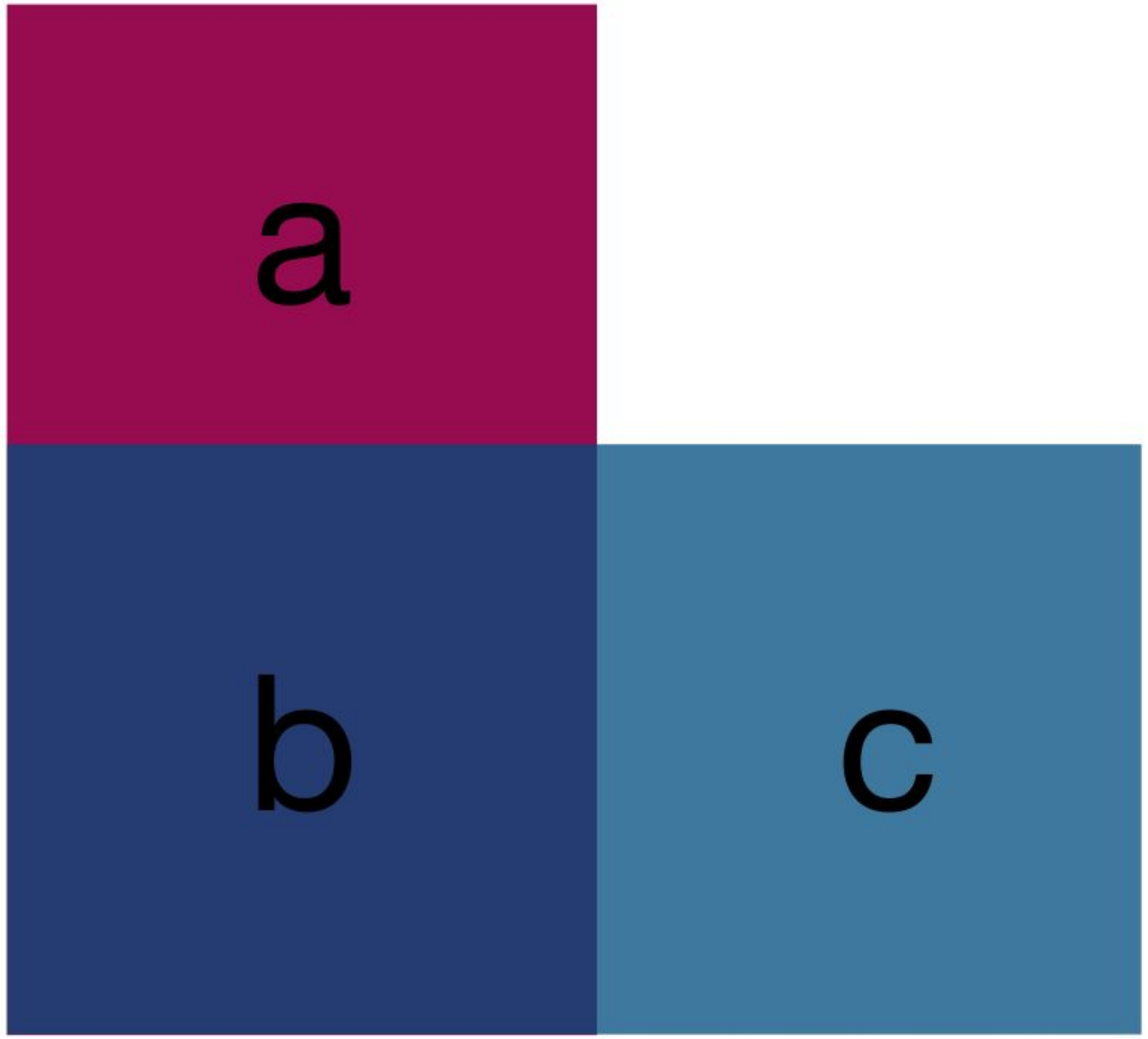


Statistical covariances

- The trick: one may always rebin 2 $\longrightarrow$ 3
- Bins a, b, and c are **non-overlapping**
- Independent Poisson distributions

$$
\begin{aligned}
\mathrm{cov}(X, Y) &= \mathrm{cov}(a+b, b+c) \\
&= \mathrm{cov}(a,b) + \mathrm{cov}(a,c) + \mathrm{cov}(b,b) + \mathrm{cov}(b,c) \\
&= 0 + 0 + \mathrm{var}(b) + 0 \\
&\approx n_b
\end{aligned}
$$

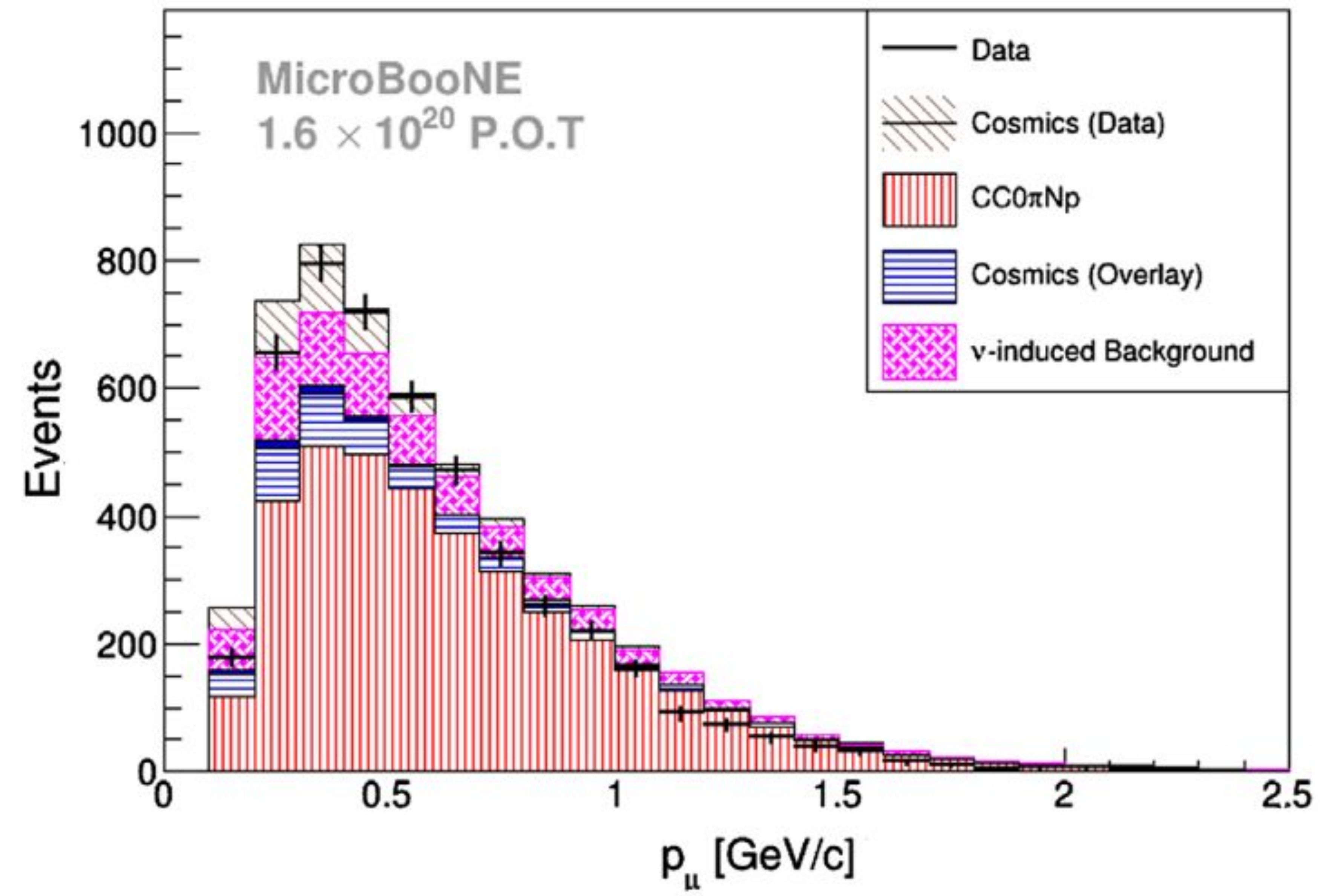
- Estimator for statistical covariance is just the **number of events that bins X and Y have in common**
- Details can change for MINERvA/T2K, but solution is conceptually similar



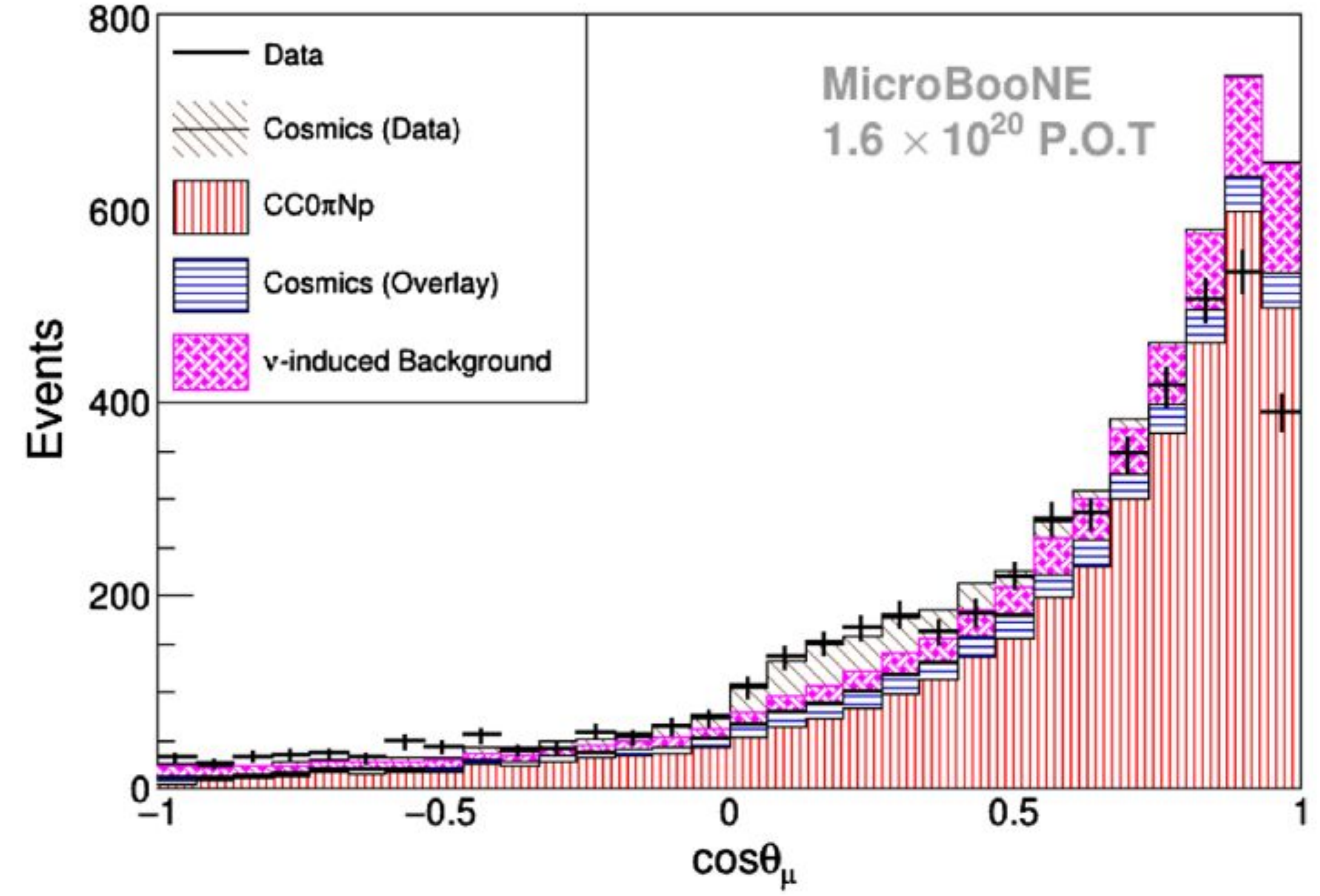
Note that this behaves as expected for X = Y as well as disjoint bins

Unfolding with correlated uncertainties

- Group bins belonging to the same kinematic distribution in a "**block**"
- An event should belong to a maximum of one reco bin and one true bin in each block → avoids double-counting
- Observables can be abstracted away by working in "bin number space"
  - Trivially generalizes to 2D, 3D, etc.
- Example:
  - Bins 0-19 represent p_μ → block #0
  - Bins 20-49 represent $\cos\theta_\mu$ → block #1



Phys. Rev. D 102, 112013 (2020)



A "blockwise" unfolding matrix

- Build an unfolding matrix U_b for the b-th block according to one's preferred approach
- Overall unfolding matrix U is block-diagonal
- Results for individual blocks are the **same** as for stand-alone measurements of each
- This organization allows reporting of correlated uncertainties between all bins in all blocks
  - Details depend on extraction style, but fully documented in paper

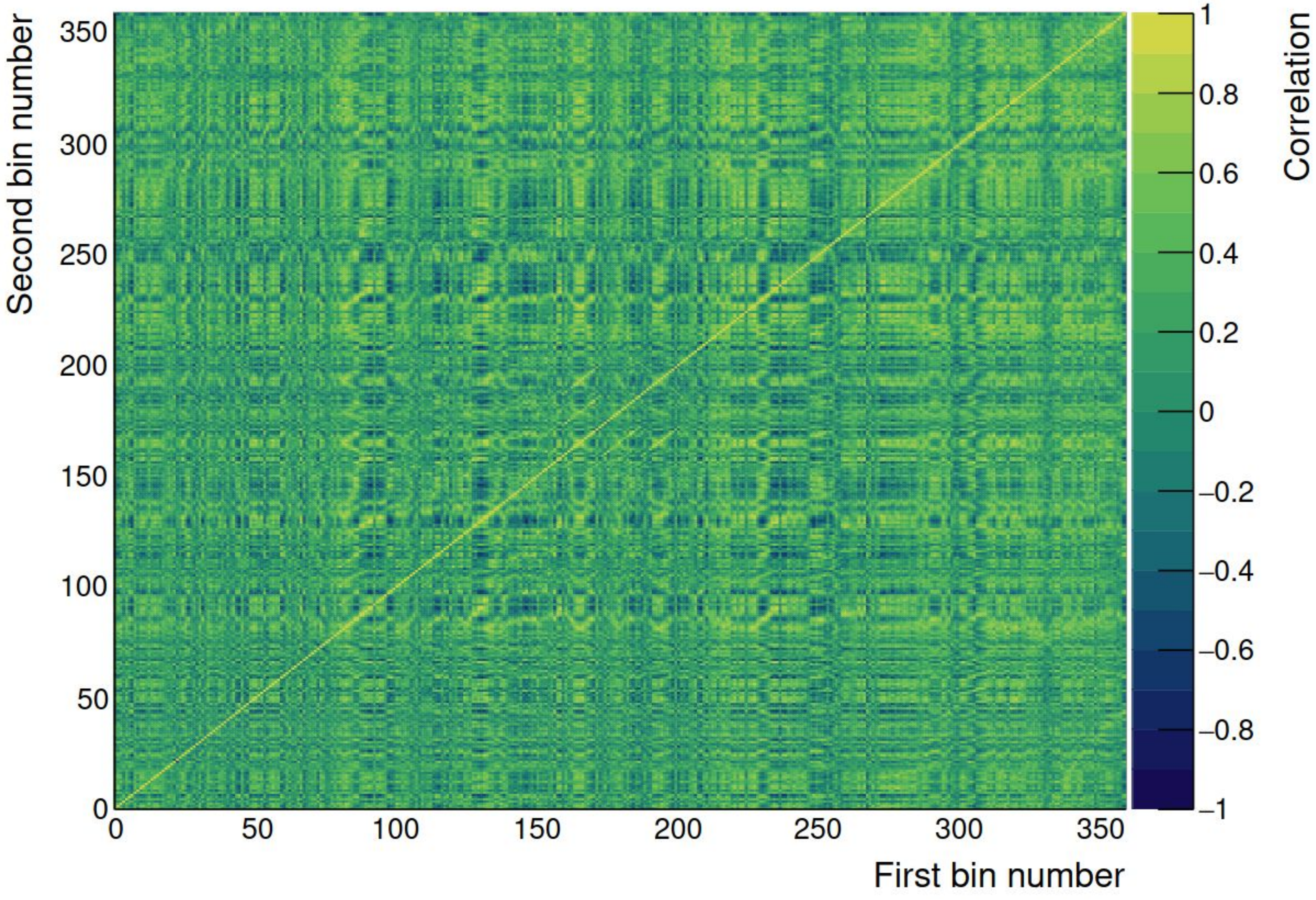
$$U = \bigoplus_{b=0} U_b = U_0 \oplus U_1 \oplus \cdots = \begin{pmatrix} U_0 & 0 & 0 & \dots \\ 0 & U_1 & 0 & \dots \\ 0 & 0 & \ddots & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Inter-distribution correlations

- Enable χ^2 comparisons to entire data set!
- **Annoying detail:** differential cross sections vary in their units
  - Can lead to confusion when reporting covariances
- **My recommendation**:
  - Re-express as *total* cross sections per bin
  - Makes inclusion of under/overflow bins easy

MicroBooNE, [arXiv:2403.19574](https://arxiv.org/abs/2403.19574)

Total correlation matrix for measured CC0πNp cross sections



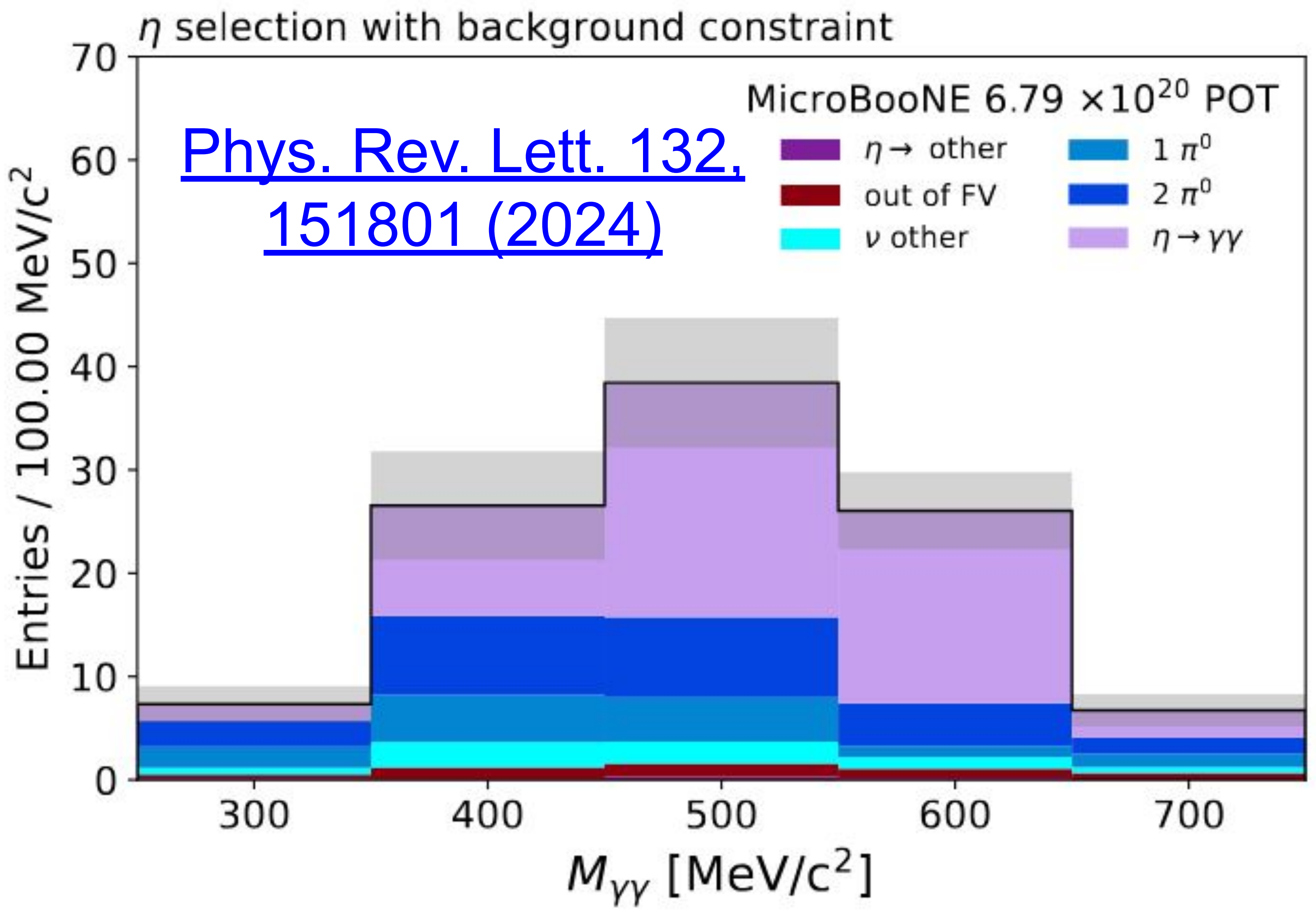
$$\mathrm{Cov}\Big(\langle\sigma\rangle_\mu, \langle\sigma\rangle_\lambda\Big) = \frac{\sum_{a,b} \mathfrak{E}_{\mu a}\, \mathrm{Cov}(D_a, D_b)\, \mathfrak{E}_{\lambda b}}{\Phi_\mu\, \Phi_\lambda\, T_\mu\, T_\lambda}$$

$$\langle\sigma\rangle_\mu = \left\langle \frac{d^n\sigma}{d\mathbf{x}} \right\rangle_\mu \cdot \Delta\mathbf{x}_\mu \qquad \langle\sigma\rangle_\mu = \frac{\hat{\phi}_\mu}{\Phi_\mu\, T_\mu} = \frac{\sum_a U_{\mu a}\,(D_a - B_a)}{\Phi_\mu\, T_\mu}$$

Outlook for the blockwise unfolding technique

- Theorists and generator developers can fit to all measured distributions simultaneously
  - Increases discrimination power of the data: can the model describe the correlations as well as each individual block?
- No need for ad hoc estimates of flux-related covariances, etc.
  - All uncertainties come from the experiment itself
- Potential for **inter-analysis covariances** with two ingredients:
  - Bookkeeping for event overlaps (statistical uncertainties)
  - Consistent systematic variations
- **Latest MicroBooNE analyses** report model goodness-of-fit χ^2 over hundreds of bins in this way
  - Other experiments can do this too!

Conditional Covariance Background Constraint (CCBC)



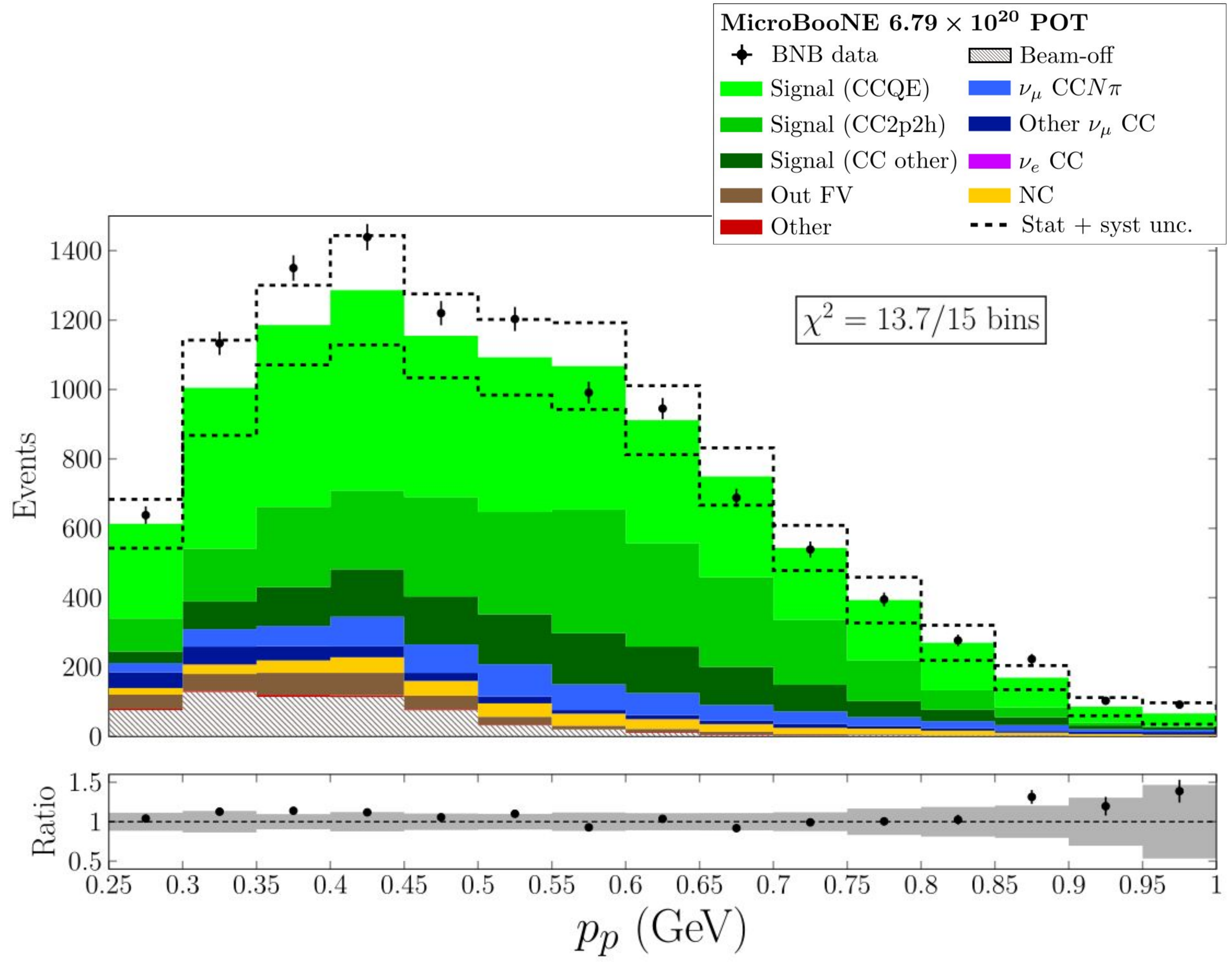
[Phys. Rev. Lett. 132, 151801 (2024)]

Background control samples

- Minimizing model dependence is critical
  - We want to learn about Nature, not our simulation!

$$\left\langle \frac{d^n\sigma}{d\mathbf{x}} \right\rangle_\mu = \frac{\sum_a U_{\mu a}\left(D_a - B_a\right)}{\Phi\, T\, \Delta\mathbf{x}_\mu}$$

- Risk of biasing the measurement in both the unfolding (U) and **background subtraction** (B)
  - Sometimes we have to rely on the prediction
  - **Is it good enough to do this?** If not, how do we fix it?

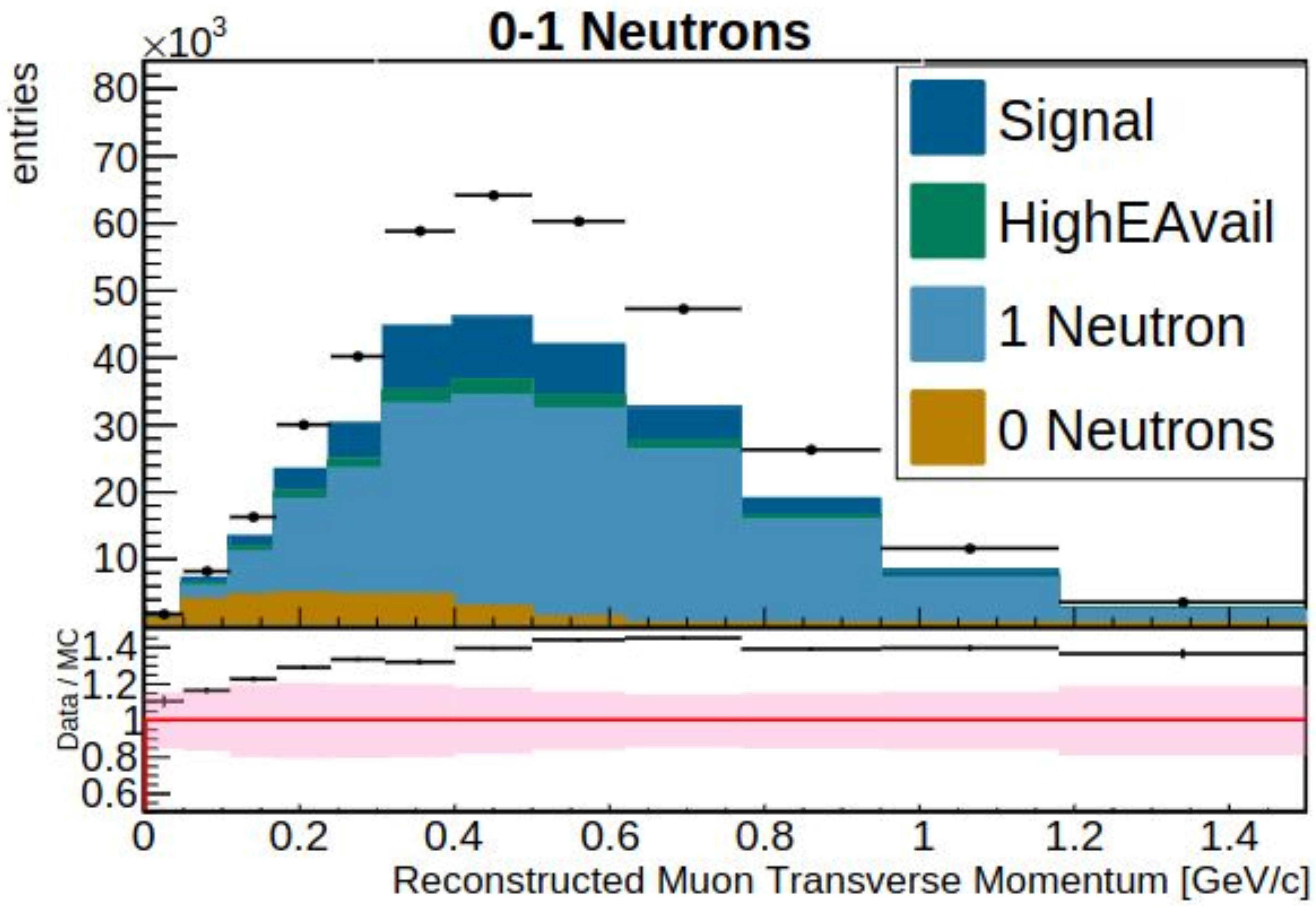


Background control samples

Phys. Rev. D 108, 112010 (2023)

- **Control samples**: **check/correct background model** based on parallel measurement
  - Background-enhanced selection
- Also often referred to as "**sidebands**"
  - I use the terms interchangeably in the paper
- I propose a semi-new way of using these for cross-section analyses

anti-ν_μ CC 2+ neutrons (MINERvA)



Few-neutron sideband (**pre-fit**)

Background control samples

Phys. Rev. D 108, 112010 (2023)

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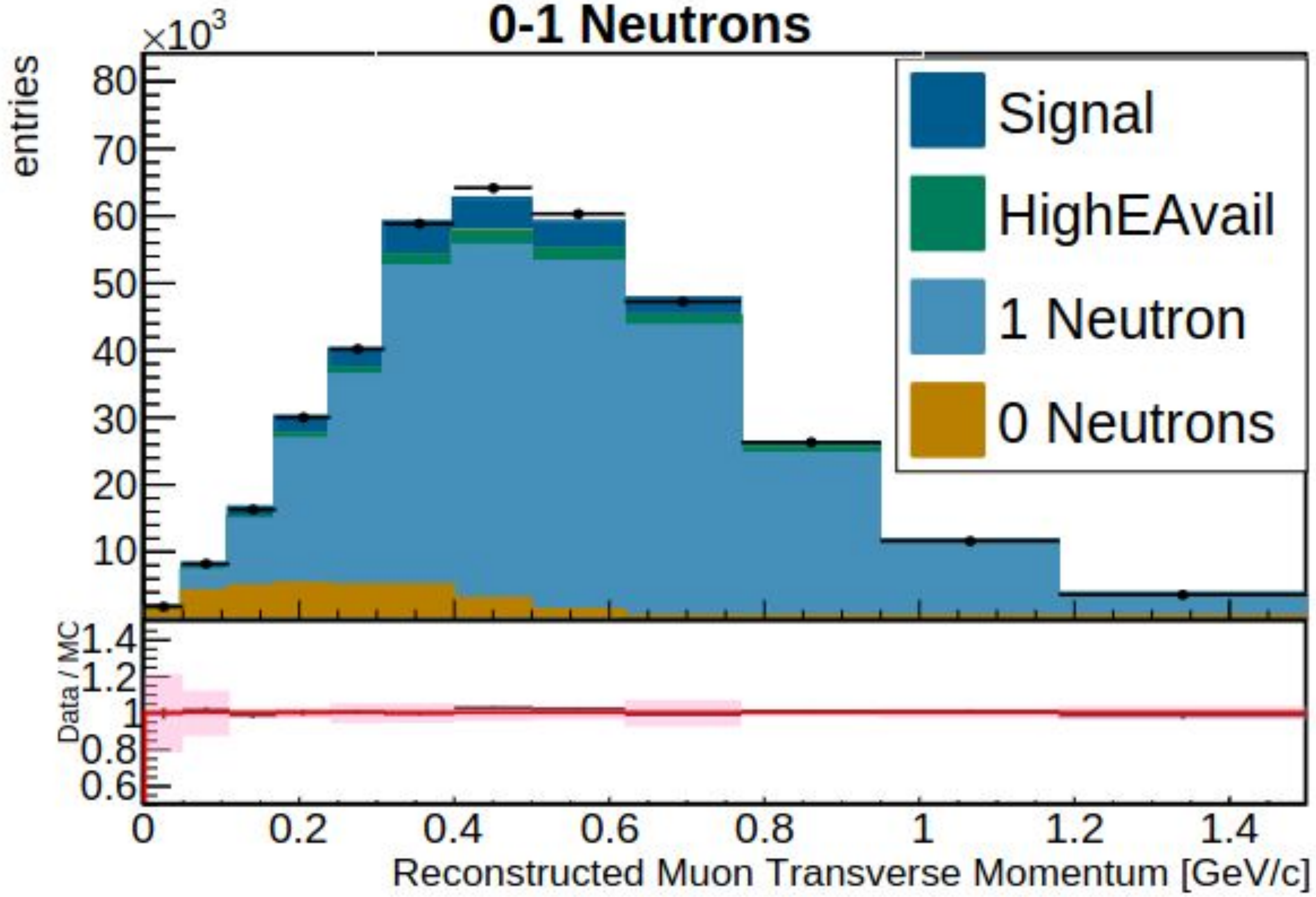


Few-neutron sideband (**post-fit**)

Use by experiments

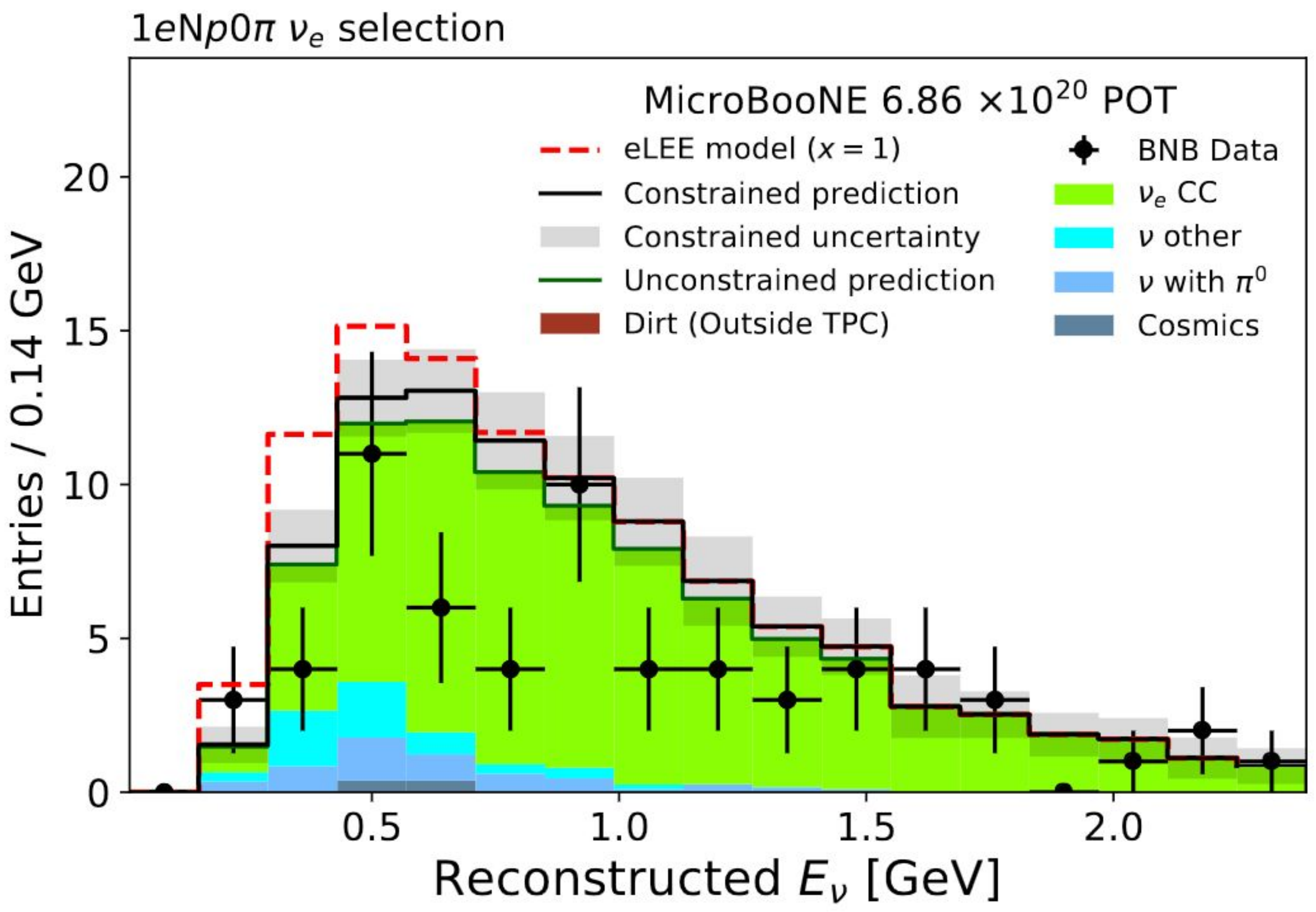
- **T2K** gets background model constraints "for free"
  - Just include bins from the sideband(s) in the fit!
- **MINERvA**: normalization scale factor approach
  - **Pre-fit**: $\alpha_þ = 1$ for all background classes þ
  - **Post-fit** values obtained from sidebands
    - Details vary widely
  - Shape from simulation unaltered*
  - Implicit 100% correlation between $\alpha_þ$ in sidebands and **signal region**
- **MicroBooNE**: no sidebands used as a constraint for any multi-bin cross-section result so far
  - I **generalize and improve** a method used for single-bin η analysis

$$B_a = \sum_{þ} \alpha_{þ} \, B_{aþ}$$

Data-driven constraint in MicroBooNE LEE analyses

- MicroBooNE built to investigate anomalous excess of ν_e-like events seen by MiniBooNE at low energies ("**LEE**")
- First results October 2021
  - Data prefer **no excess**
- Judged relative to prediction of "MicroBooNE GENIE tune" with **data-driven, analysis-specific adjustments**
- All based on a **conditional covariance** treatment

Phys. Rev. D 105, 112004 (2022)



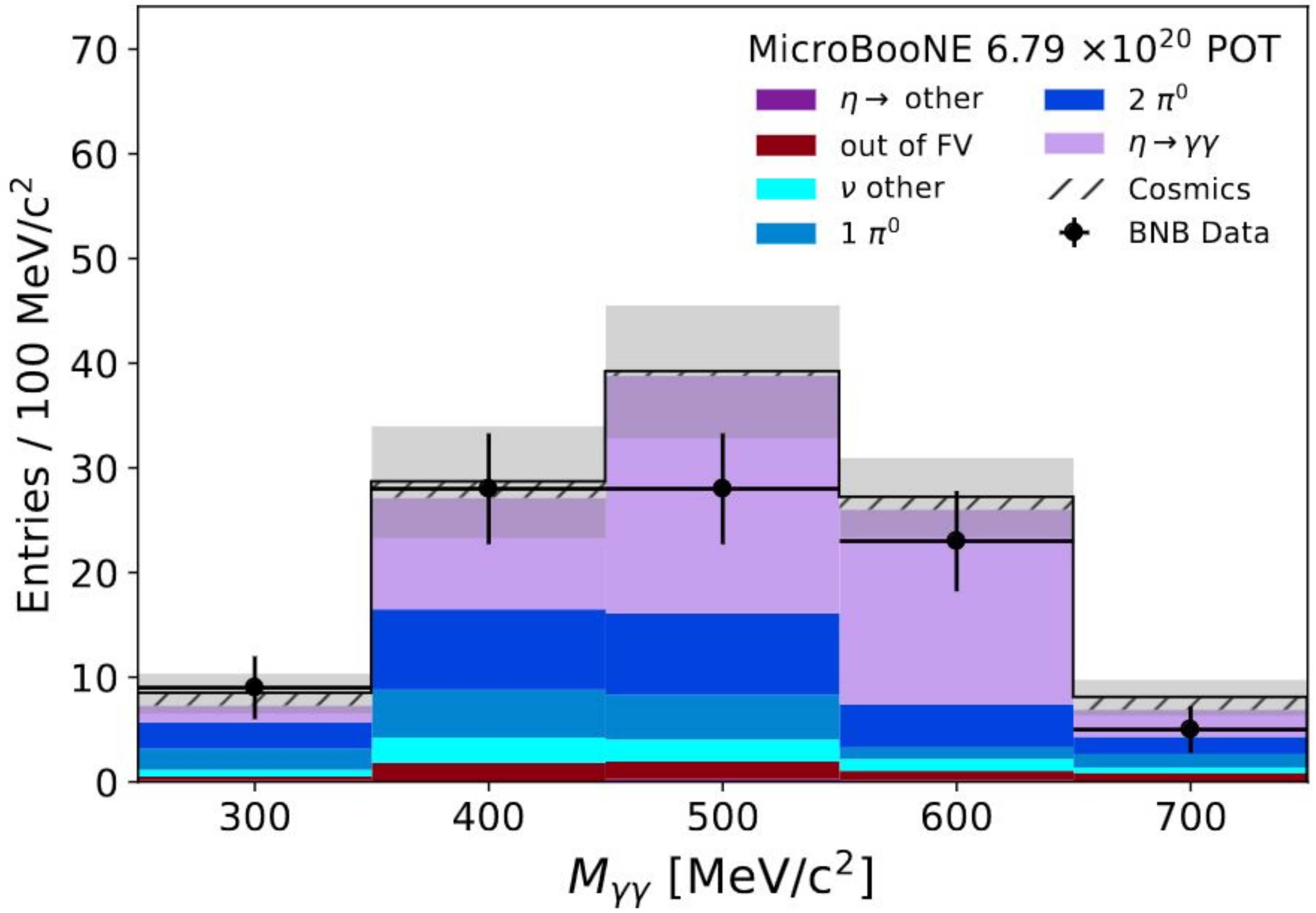
$$m^{e\,\text{constrained}} = m^e + C^{e\mu}(C^{\mu\mu})^{-1}(n^\mu - m^\mu)$$

$$C^{ee\,\text{constrained}} = C^{ee} - C^{e\mu}(C^{\mu\mu})^{-1}C^{\mu e}$$

Use for a background model constraint

- MicroBooNE η production study
  - Signal is two photons with the η invariant mass
- Dominant backgrounds are single- and multi-π^0 production
  - Each constrained separately with a single sideband bin
- I **generalize this procedure** for multiple bins and simultaneous fits to multiple backgrounds
  - Treatment suitable for MicroBooNE-style extraction

Phys. Rev. Lett. 132, 151801 (2024)



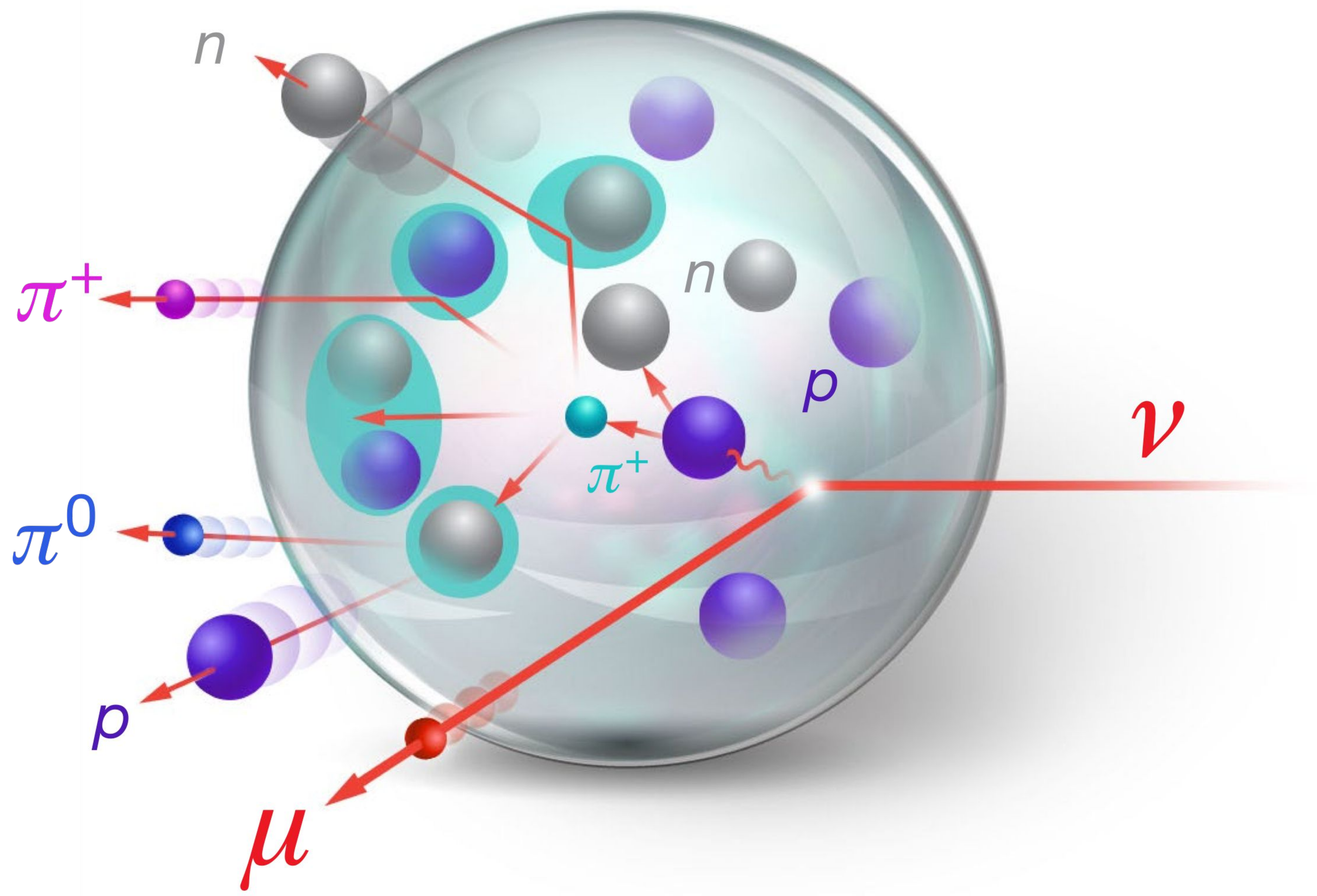
$$N_{MC}^{S,constrained} = N_{MC}^{S} + \frac{\sigma^{corr}}{(\sigma^{B})^2} \times \left(N_{data}^{B} - N_{MC}^{B}\right)$$

$$\left(\sigma^{S,\ constrained}\right)^2 = \left(\sigma^{S}\right)^2 - \frac{\left(\sigma^{corr}\right)^2}{\left(\sigma^{B}\right)^2}$$

Can also be adapted to MINERvA's style (no 100% correlation assumption)

Conclusion

- Recent paper (arXiv:2401.04065) proposes some adjustments to how we extract neutrino cross section data
- "**Blockwise unfolding**" enables full reporting of correlated uncertainties
  - Make our hard work even more informative!
- **CCBC** provides somewhat new way of refining background predictions with data
  - Basic ideas have existed for some time, now applied to cross-section extraction



Backup