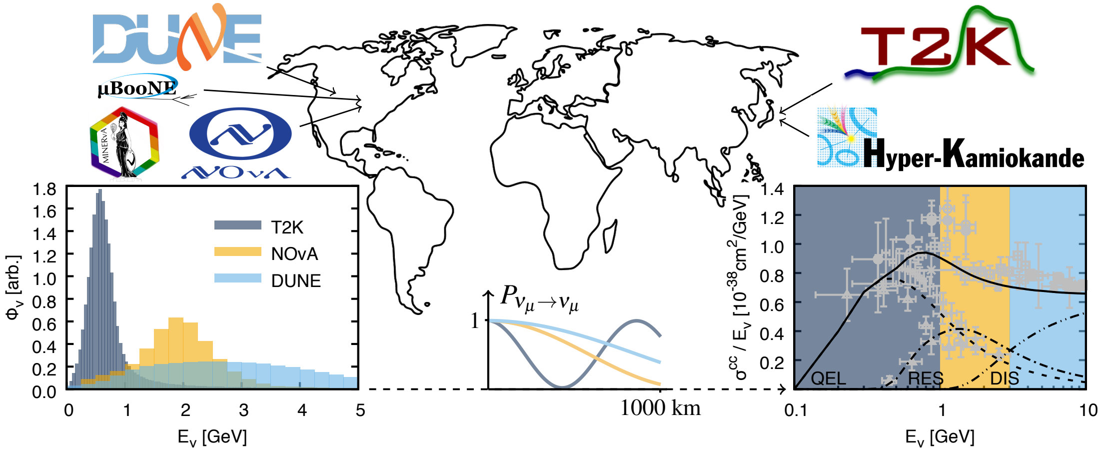
The Hadron Cascade

(and other related issues)

(a personal perspective)

Kajetan Niewczas



$$P(\nu_\mu \to \nu_e) \simeq \sin^2(2\theta)\sin^2\left(1.27\frac{\Delta m^2 L}{E_\nu}\right)$$

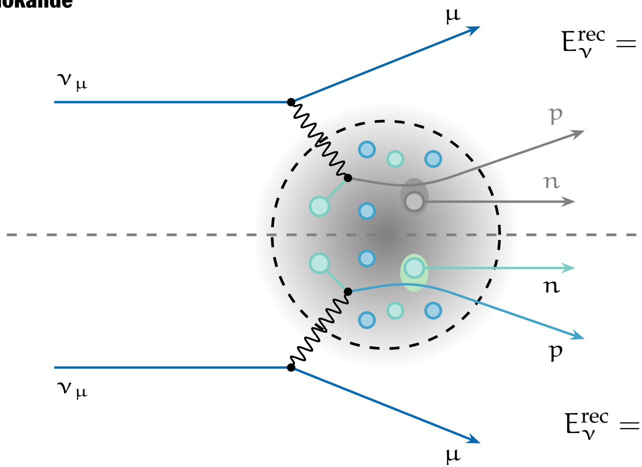
oscillation — amplitude — frequency

$$A_{CP} = \frac{P(\nu_\mu \to \nu_e) - P(\bar{\nu}_\mu \to \bar{\nu}_e)}{P(\nu_\mu \to \nu_e) + P(\bar{\nu}_\mu \to \bar{\nu}_e)}$$

asymmetry — oscillation ratio

Kinematical energy reconstruction

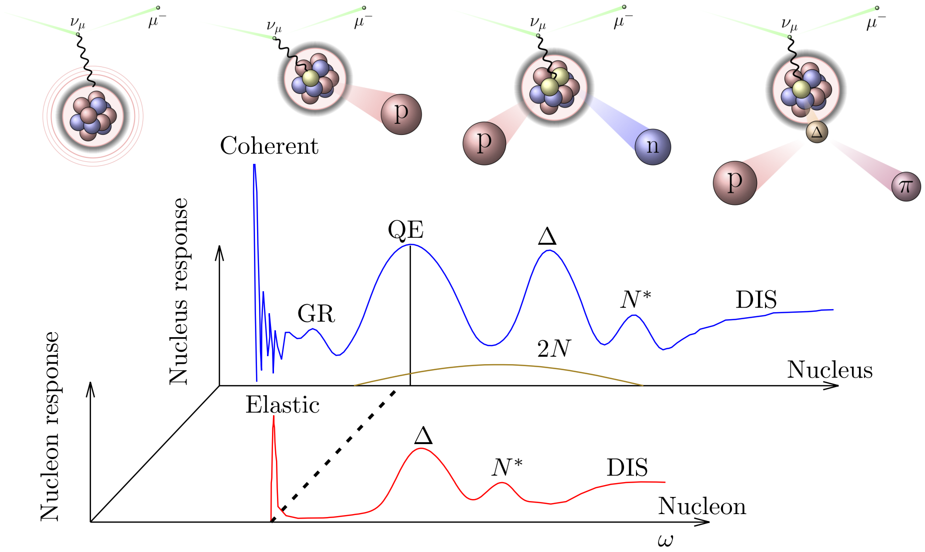
$$E_\nu^{rec} = \frac{2M_N E_\mu - m_\mu{}^2 + M_{N'}^2 - M_N^2}{2(M_N - E_\mu + p_\mu \cos\theta)}$$



$$E_\nu^{rec} = E_\mu - E_B + \sum_{nucl.} T_i + \sum_{mes.} E_j$$

Calorimetric energy reconstruction

Nuclear response



Modeling approximations

$$|\mathcal{M}|^2 \propto \underbrace{\left|\langle \Psi_A \,|\, i\hat{T} \,|\, X \rangle\right|^2}_{\textbf{Born approximation (IA)}}$$

CRPA

$$\approx \underbrace{\left|\sum_{\alpha} \langle \Psi_A \,|\, i\hat{T} \,|\, \Psi_\alpha \rangle \langle \Psi_\alpha \,|\, X \rangle\right|^2}_{\textbf{+ intermediate hadronic state}} \approx \underbrace{\sum_{\alpha} \left|\langle \Psi_A \,|\, i\hat{T} \,|\, \Psi_\alpha \rangle\right|^2 \, |\langle \Psi_\alpha \,|\, X \rangle|^2}_{\textbf{+ time separation of FSI (MC factorization)}}$$

DWIA

$$\approx \underbrace{\sum_{\alpha,h,p} \left|\langle \psi_h \,|\, \hat{O}_{1b} \,|\, \psi_p \rangle \langle \Psi_A \,|\, \hat{a}_h^\dagger \hat{a}_p \,|\, \Psi_\alpha \rangle\right|^2 \, |\langle \Psi_\alpha \,|\, X \rangle|^2}_{\textbf{+ one-body nuclear currents}}$$

PWIA

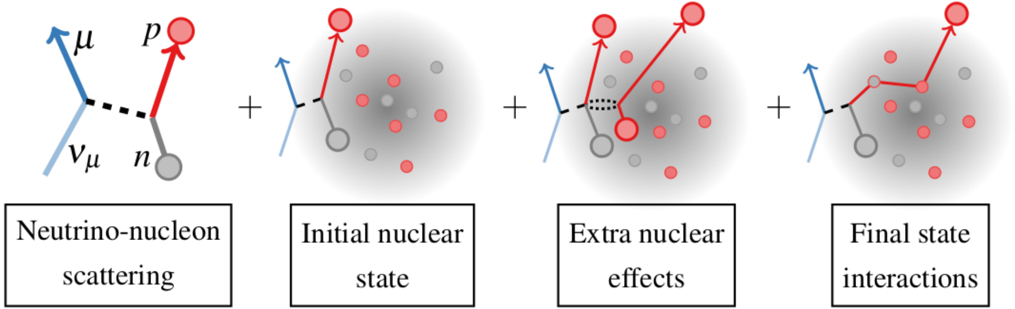
$$\approx \underbrace{\sum_{h,p} \left|\langle \psi_h \,|\, \hat{O}_{1b} \,|\, \psi_p \rangle\right|^2 \, \left|\langle \Psi_A \,|\, \hat{a}_h^\dagger \,|\, \Psi_{A-1} \rangle\right|^2 \, |\langle \Psi_{A-1}, \psi_p \,|\, X \rangle|^2}_{\textbf{+ cross section factorization (PWIA)}}$$

Generators

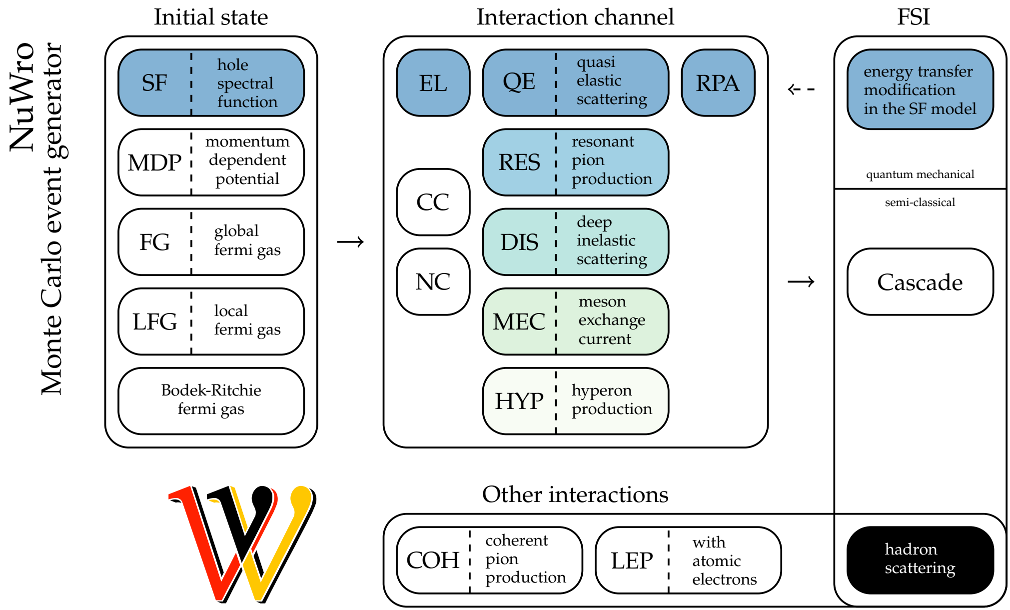
$$\approx \underbrace{\sum_{h,p} \sigma_{hp} \, S_h(E,p) \, P(p|X)}_{\textbf{+ intranuclear cascade}}$$

KN, MSc
A. Nikolakopoulos et al., arXiv:2406.09244

Monte Carlo event generators



→ Cross sections are evaluated in a **factorized scheme**


NuWro
Monte Carlo event generator
Initial state
SF
hole spectral function
MDP
momentum dependent potential
FG
global fermi gas
LFG
local fermi gas
Bodek-Ritchie fermi gas
Interaction channel
EL
QE
quasi elastic scattering
RPA
CC
NC
RES
resonant pion production
DIS
deep inelastic scattering
MEC
meson exchange current
HYP
hyperon production
FSI
energy transfer modification in the SF model
quantum mechanical
semi-classical
Cascade
Other interactions
COH
coherent pion production
LEP
with atomic electrons
hadron scattering

Intranuclear cascade

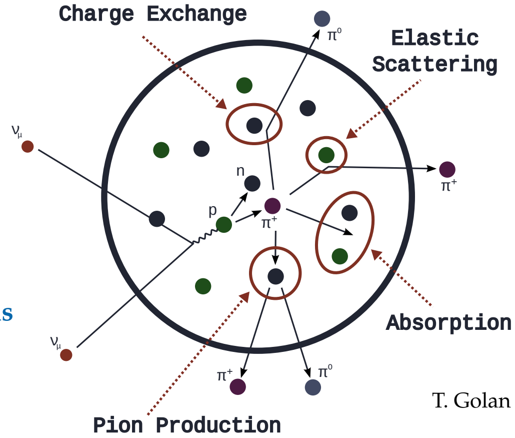
- **Propagates particles** through the nuclear medium
- **Probability** of passing a distance λ:

$$P(\lambda) = e^{-\lambda/\tilde{\lambda}}$$

where $\tilde{\lambda} \equiv (\rho\sigma)^{-1}$ and
ρ - local density
σ - cross section

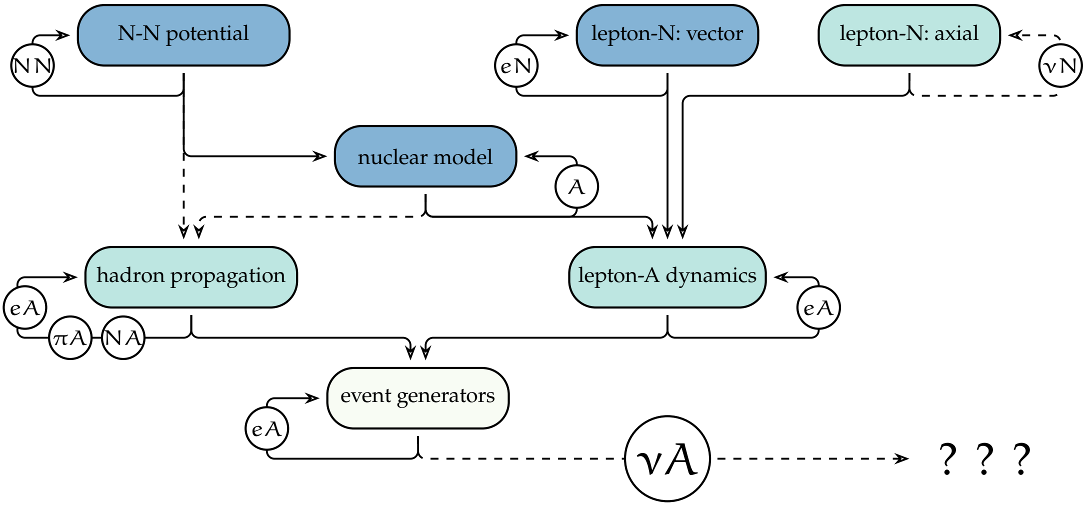
→ Implemented for **nucleons**, **pions** and **kaons**

T. Golan, C. Juszczak, J.T. Sobczyk, Phys.Rev. C 86 (2012) 015505



T. Golan

What can we rely on?



Final-state interactions in a many-body nuclear problem

Hadron propagation

Nucleon-Nucleus (NA) dynamics problem

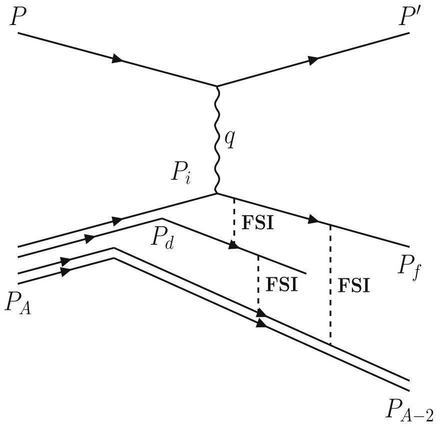
- can be constrained by **hadron scattering**
- the nucleon should only propagate outwards

→ e.g., optical potential, etc.

Built from **Nucleon-Nucleon** (NN) dynamics

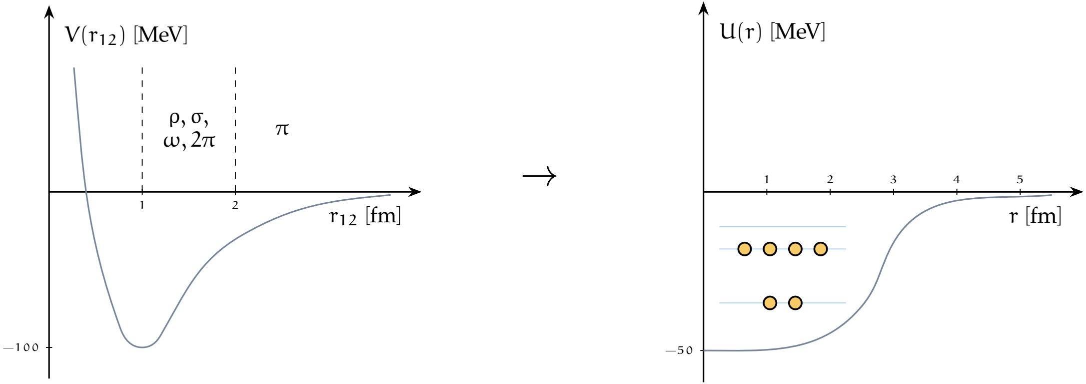
- well-constrained **nucleon-nucleon potential**
- common practice across many-body nuclear problems

→ e.g., *ab initio*, HF, etc.



A. Bodek, T. Cai, ERJ C (2019) 79

Mean-field nuclear picture



→ let's try to use a realistic **nucleon-nucleon potential** to derive the **central nuclear potential**

The iterative Hartree-Fock method

→ start with an **initial guess** for the **average field** or the **wave functions**

→ using the nucleon-nucleon potential $V(\mathbf{r},\mathbf{r}')$ **solve the equation**

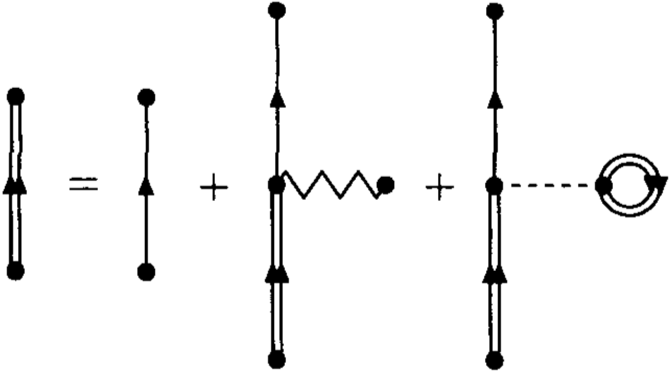
$$-\frac{\hbar^2}{2m}\nabla^2\phi_i(\mathbf{r}) + U_H(\mathbf{r})\phi_i(\mathbf{r}) - \int U_F(\mathbf{r},\mathbf{r}')\phi_i(\mathbf{r}')d\mathbf{r}' = \epsilon_i\phi_i(\mathbf{r})$$

→ determine new values of $U_H(\mathbf{r})$, $U_F(\mathbf{r},\mathbf{r}')$, $\phi_i(\mathbf{r})$, ϵ_i

$$\begin{array}{ccccccc}
U_{H(F)}^{(0)}(\boldsymbol{r}) & & U_{H(F)}^{(1)}(\boldsymbol{r}) & & U_{H(F)}^{(2)}(\boldsymbol{r}) & \ldots \\
\downarrow & \nearrow & \downarrow & \nearrow & \downarrow & \nearrow \\
\varphi_i^{(0)}(\boldsymbol{r}) & & \varphi_i^{(1)}(\boldsymbol{r}) & & \varphi_i^{(2)}(\boldsymbol{r}) & \\
\varepsilon_i^{(0)} & & \varepsilon_i^{(1)} & & \varepsilon_i^{(2)} & .
\end{array}$$

→ at convergence: the **final field** $U_H(\mathbf{r})$, **wave function** $\phi_i(\mathbf{r})$, and **single-particle energy** ϵ_i

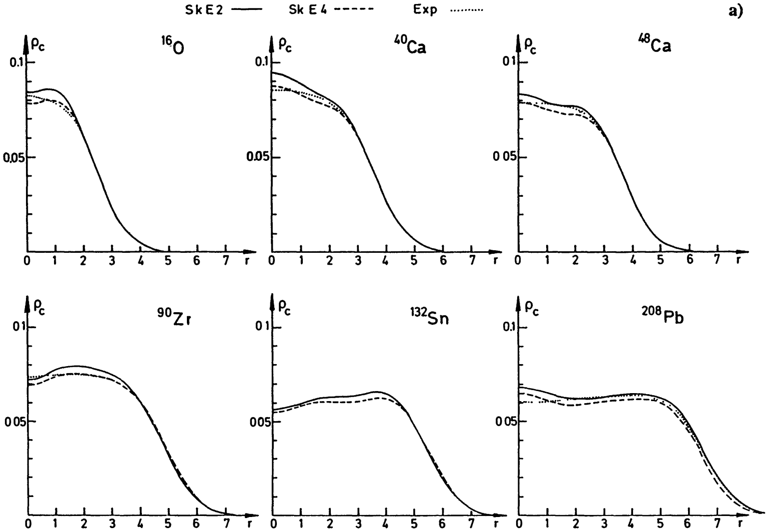
Nucleons in the mean-field potential



W. H. Dickhoff, D. Van Neck, Many-body Theory Exposed! (2005)

→ **nucleon lines are dressed** according to the Hartree-Fock procedure

Charge densities from the mean-field framework



K. Heyde, The Nuclear Shell Model (1990)

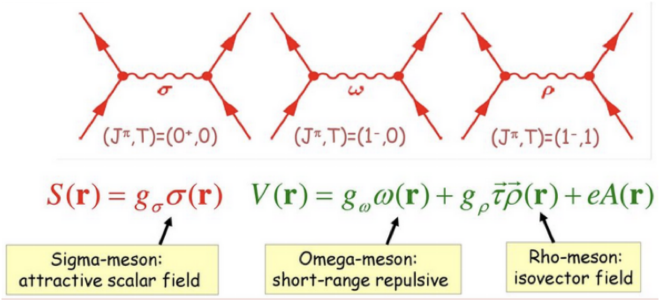
Relativistic mean-field

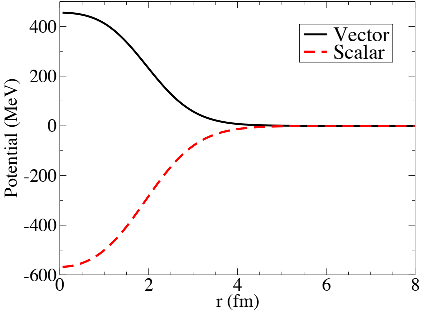
All of this can be also done in a **relativistic** framework:

- Schrödinger equation → **Dirac equation**,
- Wave functions → **Dirac spinors**,
- Spin-orbit term comes for free!

$$\left(\tilde{E}\gamma_0 - \vec{p}\cdot\vec{\gamma} - \tilde{M}\right)\psi = 0$$

$$\tilde{E} = E - V(r)$$
$$\tilde{M} = M - S(r)$$





Final-state interactions

- RPWIA:

→ **plane-wave outgoing nucleon**

- RDWIA:

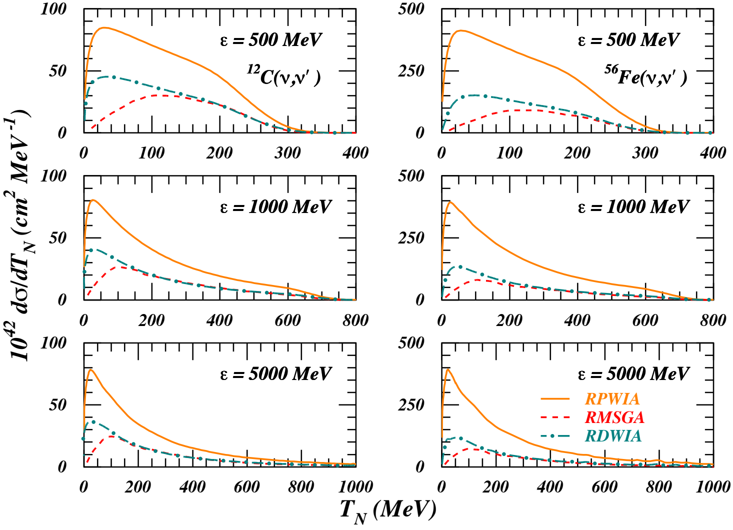
→ **optical potential from NA**

- RMSGA:

→ **Glauber approximation from NN**

(notice the low energy behavior)

→ **for more results see the talks by Raúl or Juanma!**



M.C. Martinez et al., Phys.Rev. C 73 (2006) 024607

Modeling the fate of hadrons in final-state interactions

Classical particle propagating through a medium

Probability for a particle to propagate over a distance x with **no interactions** is

$$P(x) = \frac{1}{\lambda}\exp(-x/\lambda)$$

where $\lambda = (\rho\sigma)^{-1}$ is the **mean free path**, while ρ is **target density** and σ is **interaction cross section**

We can try to apply it to nucleons in nuclei because:

$$\tilde{\lambda} \ll d < \lambda < R$$

where $\tilde{\lambda}$ is the de Broglie **wavelength**, d is the **distance** between targets, and R is the **nuclear radius**

N. Metropolis et al., Phys.Rev. 188 (1958) 185, Phys.Rev. 188 (1958) 204

Application to nuclei (space-like approach)

- Pick a random starting point in the nucleus
- Propagate the nucleon in discrete steps, e.g., $\Delta x = 0.2$ fm
- At every step, we sample x from $P(x) = \lambda^{-1} \exp(-x/\lambda)$

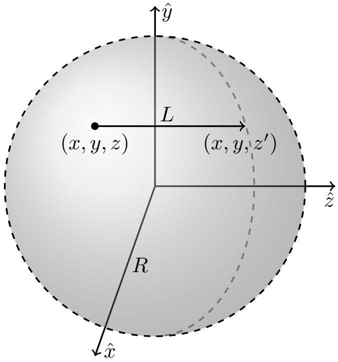
→ If $x < \Delta x$, then the nucleon-nucleon interaction happens

- The probability that the nucleon leaves the nucleus with no re-interactions is called **transparency**

→ Our procedure solves an integral

$$T = \int_0^{2R} f_R(z) e^{-z/\lambda} \, dz$$

where $f_R(z)$ is the distribution of the starting points



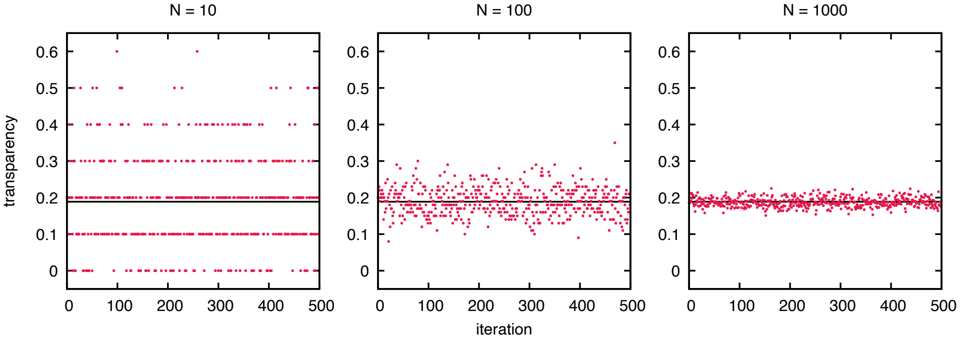
Nuclear transparency

- For fixed density (uniform ball), the **solution is given analytically**

$$T = 3e^{-A}\left(\frac{1}{A^2} + \frac{1}{A^3}\right) + 3\left(\frac{1}{2A} - \frac{1}{A^3}\right)$$

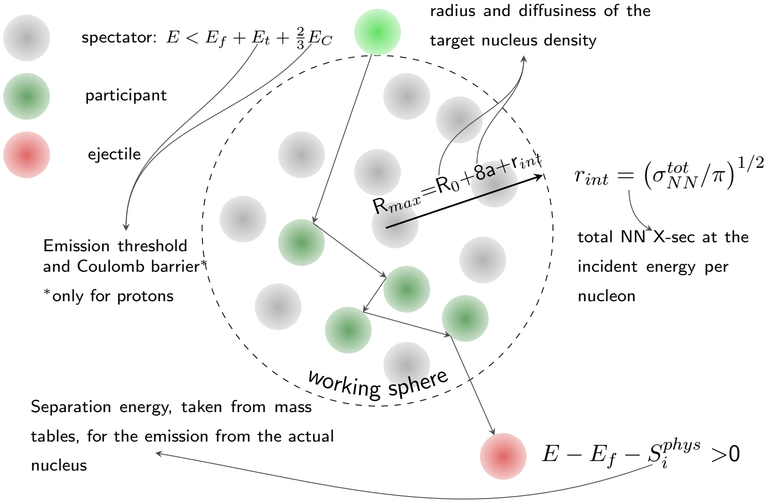
where $A = 2R/\lambda = 2R\rho\sigma$

→ e.g., for $\rho = 0.16\ \text{fm}^{-3}$, $\sigma = 40$ mb, and $R = 6$ fm, we get $T \simeq 0.189$



Time-like approach

- tracking **all particles**
- evolution over dt
- calculating **collision probability** in a $(\mathrm{dt}, \mathrm{d}^3\mathrm{x})$ box



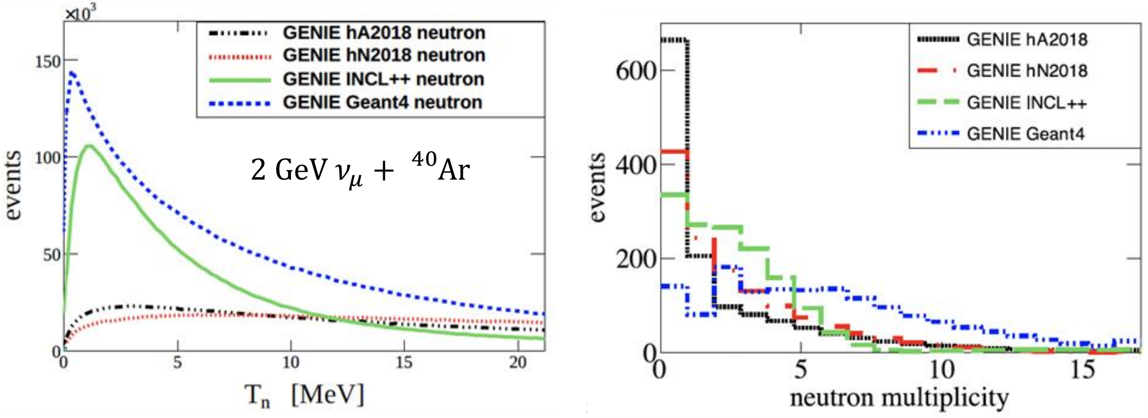
Anna Ershova • NuInt 2024 • April 15, 2024

Cascade models

	algorithm	nuclear model	cross sections	potential
GENIE hA	space-like	LFG	SAID	-
GENIE hN	space-like	LFG	data + ?	-
NEUT	space-like	LFG	data + ?	-
NuWro	space-like	LFG	data + in-medium	-
Achilles	time-like	*ab initio*	data	-
INCL	time-like	HF	data + ?	constant
GiBUU	time-like	RMF and HF (LFG)	off-shell	RMF and HF

→ **see the talk by Patrick for an overview of MC generators' efforts!**

Highlights: GENIE



J. Plows, NuFACT 2024

→ **variety of models available**: homegrown **hA** and **hN**, **INCL++**, **Bertini** (Geant4)

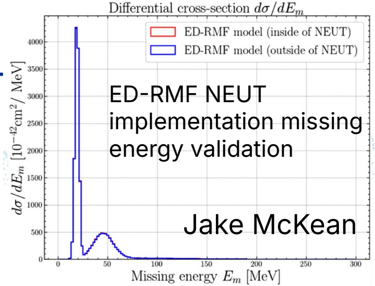
Relativistic Mean Field

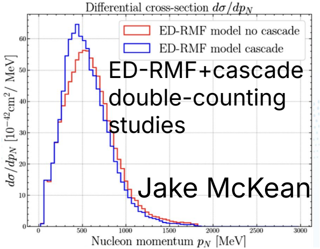
- First implementation of a macroscopic model based on a Relativistic Mean Field optical models into NEUT:
  - Jake McKean, Raul González-Jiménez, Minoo Kabirnezhad
- Potential for new theory-motivated systematic uncertainty studies in NEUT.
- Possible consideration of alternative operators for different processes.
  - *E.g.* Kabirnezhad inelastic pion production model operator.

Neutrino-nucleon Operator

$$J^\mu \propto \int d\mathbf{p}\ \bar{\Psi}_{Scattered}\ \mathcal{O}^\mu\ \Psi_{Bound}$$

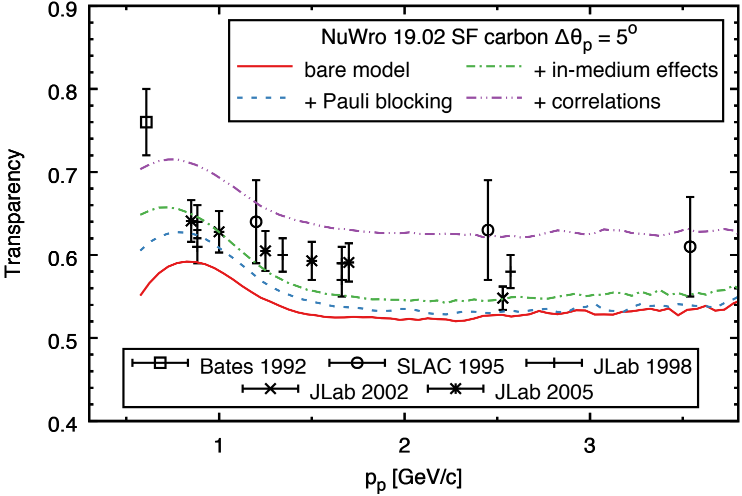
Hadronic Current — Scattering Potential — Bound State Wavefunction





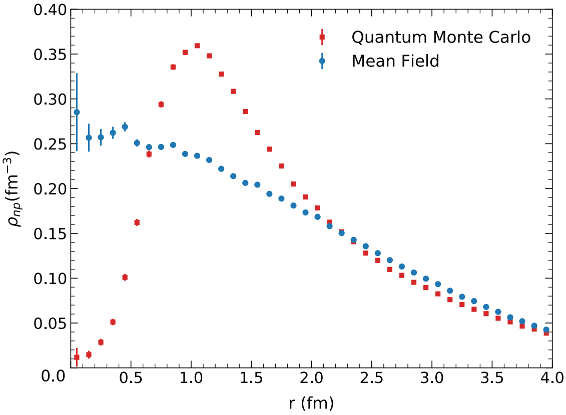
L. Pickering, NuFACT 2024

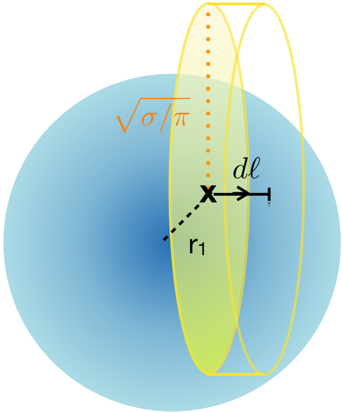
Highlights: NuWro



KN, J. Sobczyk, Phys.Rev. C 100 (2019) 015505

Highlights: Achilles

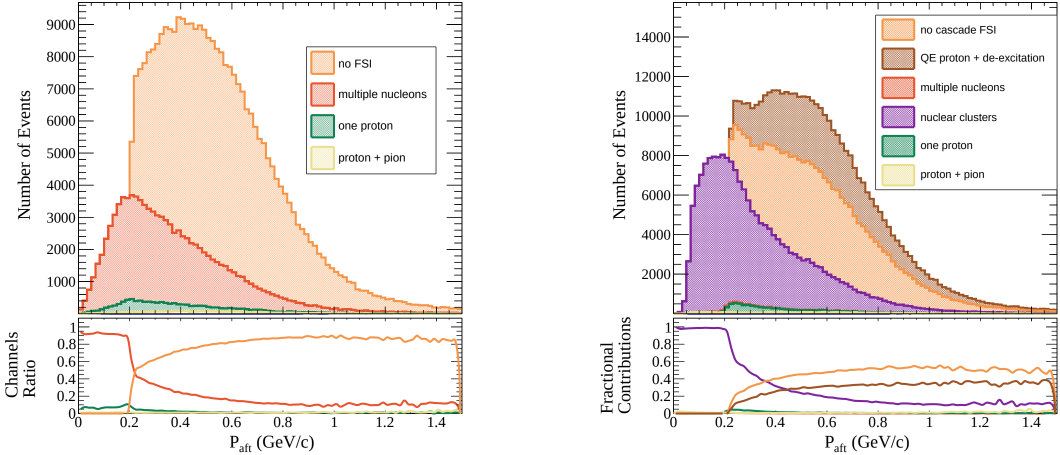




J. Isaacson et al., Phys.Rev. C 103 (2021) 015502

→ **time-like cascade** based on ***ab initio* nucleon distributions**

Highlights: INCL



A. Ershova et al., Phys.Rev. D 106 (2022) 032009

→ **softer interactions** with higher ingredient complexity, e.g. cluster emission

Highlights: GiBUU

Nuclear ground state:

- **density distribution**: Woods-Saxon
- **particle momenta**:

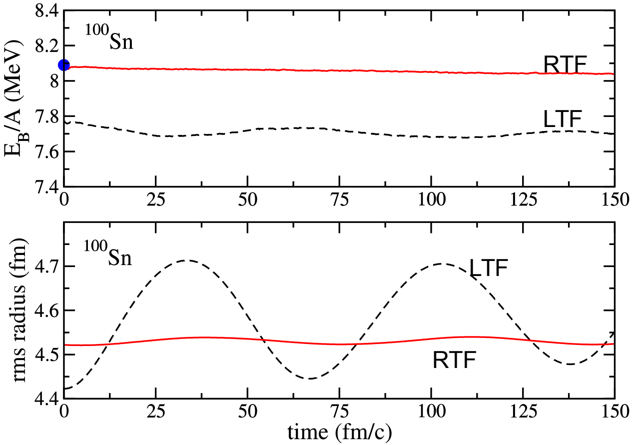
$$f(\vec{r},\vec{p}) = \Theta\left[p_F(\vec{r}) - |\vec{p}|\right]$$

- **Fermi-momentum**:

$$p_F(\vec{r}) = \left(3\pi^2\rho(\vec{r})\right)^{1/3}$$

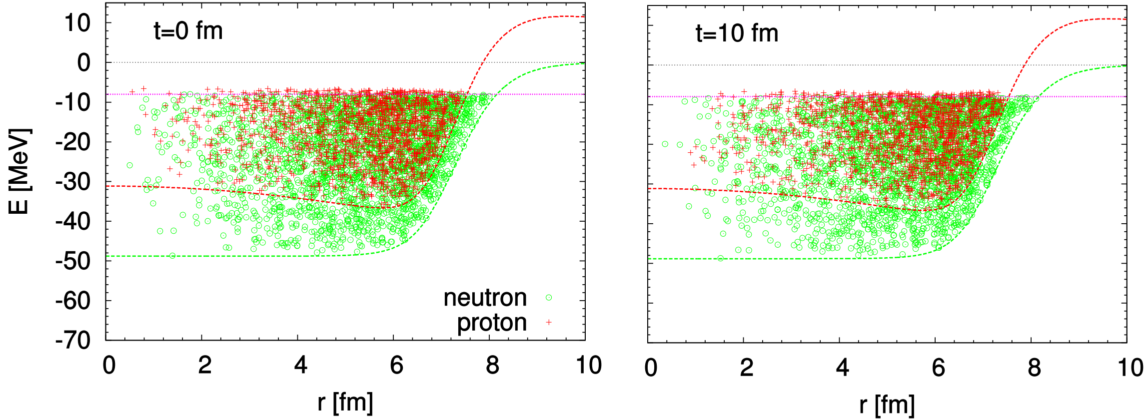
- **Fermi-energy**:

$$E_F = \sqrt{p_F^2 + M_N^2} + U_{MF}(\vec{r}, p_F)$$



K. Gallmeister, NuSTEC School 2017

Highlights: GiBUU



K. Gallmeister, NuSTEC School 2017

Constraining through electron scattering

(and other probes)

Exclusive electron scattering

Exclusive QE proton knockout
at **fixed kinematics**:

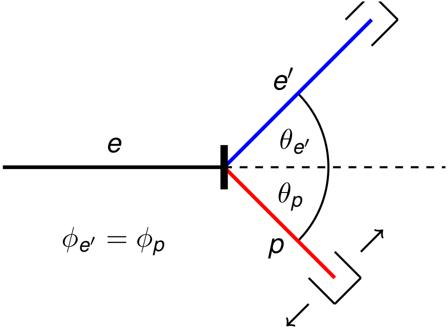
- beam: E_e
- electron: $E_{e'}, \theta_{e'}, \phi_{e'}$
- proton: E_p, θ_p, ϕ_p

With provided: $\frac{\Delta p}{p}, \Delta\theta, \Delta\phi$

Cuts on "missing" variables:

- energy: $E_m = \omega - T_p - T_{A-1}$
- momentum: $\vec{p}_m = \vec{p}_p - \vec{q}$

$$E_m < 80 \text{ MeV}, \quad |\vec{p}_m| < 300 \text{ MeV/c}$$



Transparency:

$$\langle T \rangle_{\theta_p} = \frac{\sigma_{\text{exp}}}{\sigma_{\text{PWIA}}} = \frac{[\text{after FSI}]}{[\text{total}]}$$

σ_{PWIA} - expected value without FSI (model dependent)

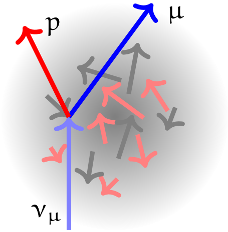
Nuclear transparency

Definition

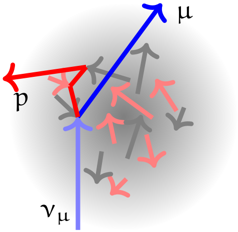
Nuclear transparency is the average **probability** for a knocked-out **proton** to **escape** the nucleus **without significant reinteraction**.

e.g. measured for Carbon: $T \simeq 0.60$ [D. Abbott *et al.*, PRL 80 (1998), 5072]

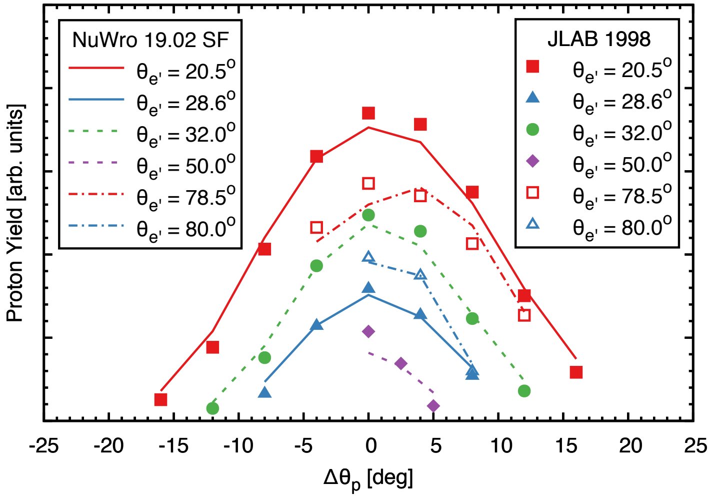
$\sim 60\%$ without FSI



$\sim 40\%$ with FSI

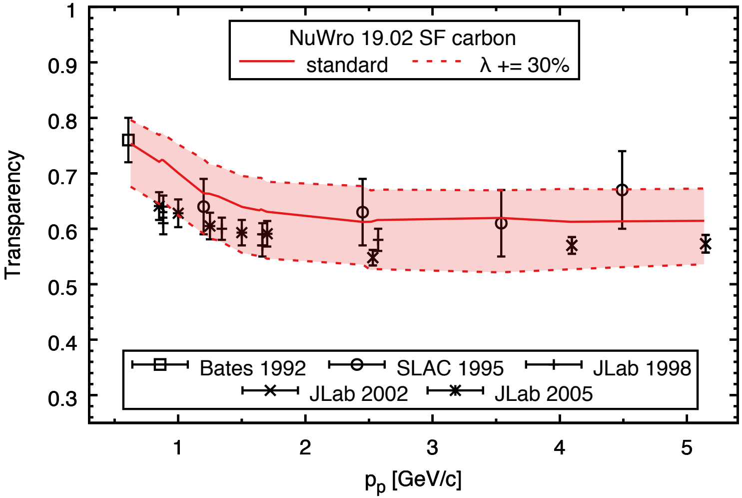


Nuclear transparency

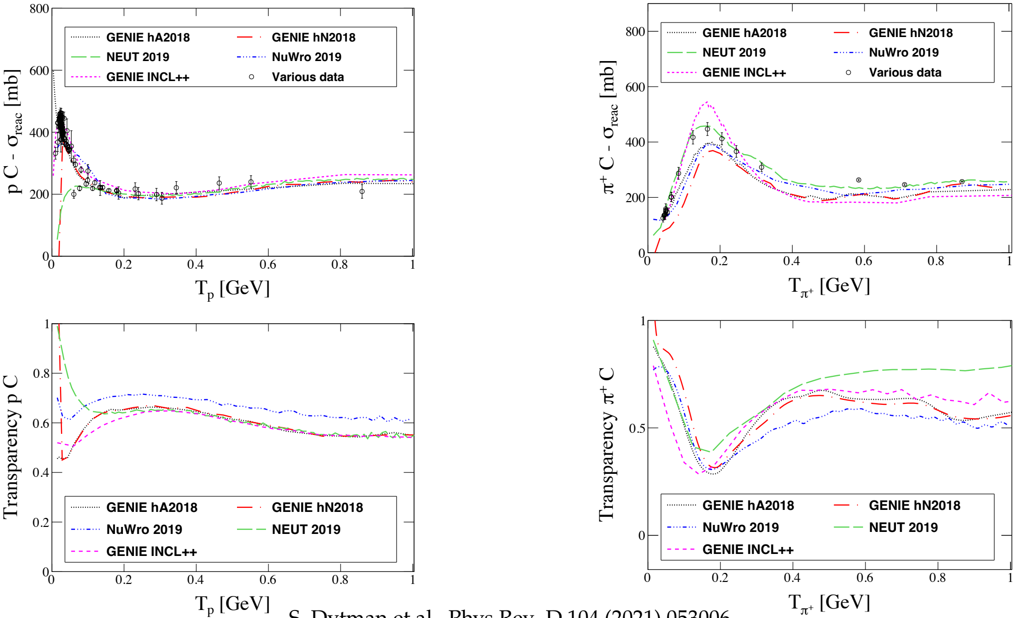


KN, J. Sobczyk, Phys.Rev. C 100 (2019) 015505

Nuclear transparency

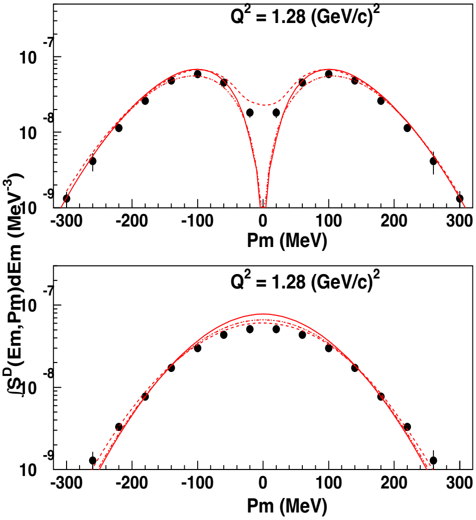


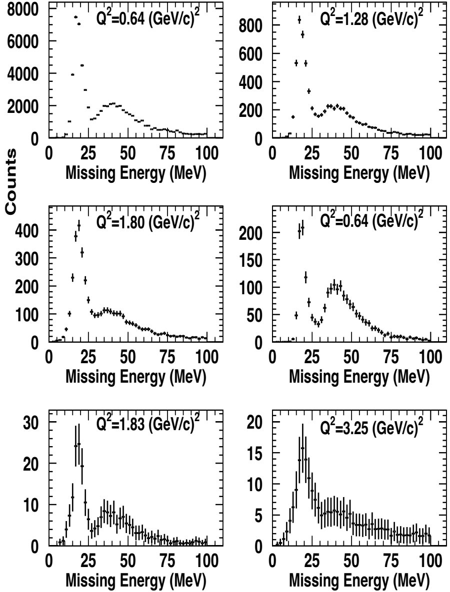
KN, J. Sobczyk, Phys.Rev. C 100 (2019) 015505



S. Dytman et al., Phys.Rev. D 104 (2021) 053006

Exclusive electron scattering



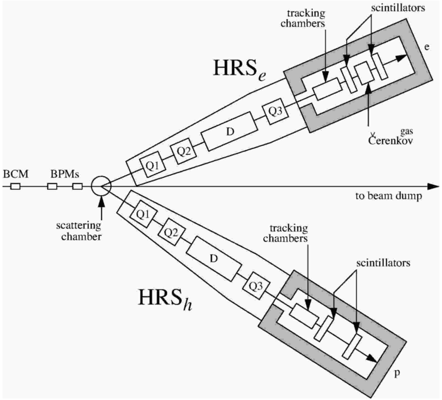


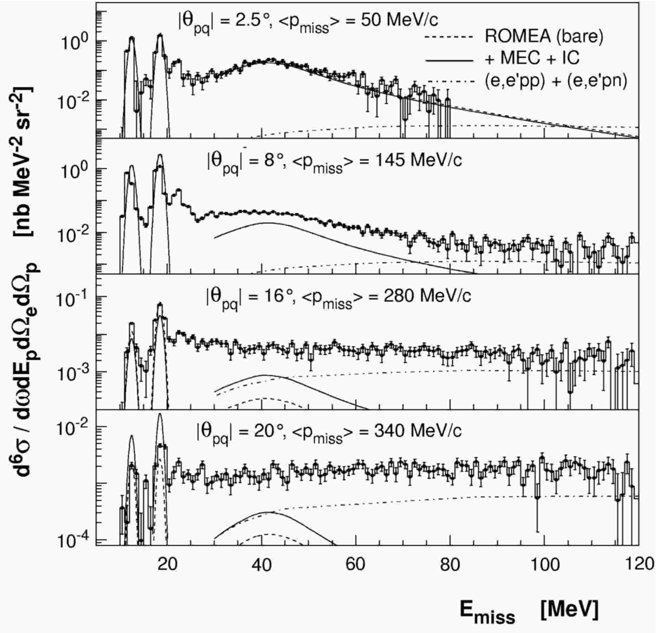
JLab E91013 (D. Dutta et al.), Phys.Rev. C 68 (2003) 064603

Issues with transparency measurements

- Performed at **high energies** (color transparency)
- Valid up to the **first inelastic interaction** (imaginary part of the optical potential)
- Results are **model-dependent** (spectroscopic factors)
- **Issues with consistency** across experimental analyses

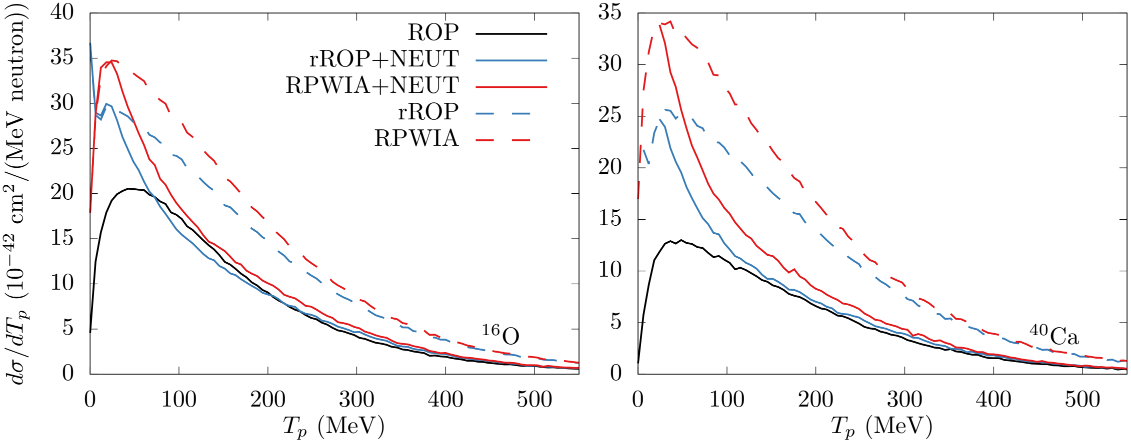
Other exclusive electron data





K. G. Fissum et al., Phys.Rev. C 70 (2004) 034606

Merging together the optical potential and intranuclear cascade



A. Nikolakopoulos et al., Phys.Rev. C 105 (2022) 054603

→ see the talk by Anna for more results and comparisons!

Summary

- We have **many independent models** capable of **propagating hadrons** through nuclear matter
- Although sometimes simplistic, these solutions **work remarkably well**

  ◦ Comparisons to **hadron and electron scattering** provide vital information

  ◦ We need more dedicated, **neutrino-like experimental analyses** to test our models:

→ Data should focus on **clear experimental signals** to test various dynamics

→ This requires the best possible knowledge of the **primary interaction**

Supplementary material

Mean-field potential

Single-particle **radial Schrödinger equation**:

$$(T + U(r))\,\phi_a(r) = \epsilon_a \phi_a(r) \tag{1}$$

Nuclear **Hamiltonian**:

$$H_0 = \sum_{i=1}^{A} (T_i + U(r_i)) = \sum_{i=1}^{A} h_0(i), \qquad E_0 = \sum_{i=1}^{A} \epsilon_{a_i}(r_i) \tag{2}$$

Nuclear wave function is a **Slater determinant**:

$$\Phi_{a_1,\dots,a_A}(r_1,\dots,r_A) = \frac{1}{\sqrt{A!}} \begin{vmatrix} \phi_{a_1}(r_1) & \dots & \phi_{a_1}(r_A) \\ \vdots & \ddots & \vdots \\ \phi_{a_A}(r_1) & \dots & \phi_{a_A}(r_A) \end{vmatrix} \tag{3}$$

Let's restrict ourselves to two-body interactions only and evaluate the **mean-field potential**:

$$H = \sum_{i=1}^{A} T_i + \frac{1}{2} \sum_{i,j=1}^{A} V_{ij} \tag{4}$$

$$H = \sum_{i=1}^{A} (T_i + U(r_i)) + \left(\frac{1}{2} \sum_{i,j=1}^{A} V_{ij} - \sum_{i=1}^{A} U(r_i) \right) = H_0 + H_{res} = \sum_{i=1}^{A} h_0(i) + H_{res}, \tag{5}$$

Hartree-Fock methods

Let's consider a density in terms of the **occupied single-particle states**:

$$\rho(\mathbf{r}) = \sum_b \phi_b^*(\mathbf{r})\phi_b(\mathbf{r}) \tag{6}$$

The **Hartree potential** at a given point generated by the **two-body interaction**:

$$U_H(\mathbf{r}) = \sum_b \int \phi_b^*(\mathbf{r}')V(\mathbf{r},\mathbf{r}')\phi_b(\mathbf{r}')d\mathbf{r}' \tag{7}$$

The **Schrödinger equation** becomes:

$$\begin{aligned} -\frac{\hbar^2}{2m}\nabla^2\phi_i(\mathbf{r}) &+ \sum_b \int \phi_b^*(\mathbf{r}')V(\mathbf{r},\mathbf{r}')\phi_b(\mathbf{r}')d\mathbf{r}' \cdot \phi_i(\mathbf{r}) \\ &- \sum_b \int \phi_b^*(\mathbf{r}')V(\mathbf{r},\mathbf{r}')\phi_b(\mathbf{r})\phi_i(\mathbf{r}')d\mathbf{r}' = \epsilon_i\phi_i(\mathbf{r}) \end{aligned} \tag{8}$$

$$-\frac{\hbar^2}{2m}\nabla^2\phi_i(\mathbf{r}) + U_H(\mathbf{r})\phi_i(\mathbf{r}) - \int U_F(\mathbf{r},\mathbf{r}')\phi_i(\mathbf{r}')d\mathbf{r}' = \epsilon_i\phi_i(\mathbf{r}) \tag{9}$$

where the exchange term is driven by the **Fock potential**:

$$U_F(\mathbf{r}) = \sum_b \phi_b^*(\mathbf{r}')V(\mathbf{r},\mathbf{r}')\phi_b(\mathbf{r}) \tag{10}$$