Overview of theoretical modeling of pionless neutrino interactions and prospects for future development

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Workshop on “Measuring neutrino interactions for next-generation oscillation experiments”

ECT*, Trento, October 21, 2024

Pionless or Quasielastic-like: definition(s)

+ What’s a pionless or QE-like neutrino interaction?

It is **not** a reaction channel.

It is an experimental definition of a **selection of events with a given topology**.

+ Which topology?

It depends on the experiment (detector) and on the particular analysis.

Let’s see a few examples.

One muon and no pions

T2K (2016)
PRD93, 112012

Analysis I
"...events where a single muon (with a proton above or below detection threshold) is required and no other tracks."

T2K (2016)
PRD93, 112012

Analysis II
"...CCQE-like interactions are identified by vetoing the presence of pions in the final state..."

MINERvA (2016)
PRD99, 012004

"quasielastic-like signal, defined as those events with the following final state particles:
(i) One negatively charged muon of angle <20° with respect to the neutrino beam
(ii) Any number of protons or neutrons
(iii) No mesons
(iv) No heavy or excited baryons
(v) Any number of photons with energy ≤10 MeV"

One muon and at least one proton

TABLE II. Phase-space restrictions applied to the CC0π cross section measurements with one muon and at least one proton in the final state shown by T2K collaboration in [1].

T2K	k'	$\cos\theta_l$	p_N	$\cos\theta_N^L$	ϕ_N^L
TKI	> 0.25 GeV	> -0.6	0.45–1.0 GeV	> 0.4	...
IV	...	...	> 0.45 GeV	> 0.4	...

T2K

[1] https://doi.org/10.1103/PhysRevD.98.032003

TABLE III. Phase-space restrictions applied to the CC0π cross section measurements with one muon and at least one proton in the final state shown by MINERνA collaboration in [4,34].

MINERνA	k'	$\cos\theta_l$	p_N	$\cos\theta_N^L$	ϕ_N^L
All analyses	1.5–10 GeV	>0.939	0.45–1.2 GeV	>0.342	...

Tables from https://doi.org/10.1103/PhysRevD.106.113005.

MINERvA

[4] https://doi.org/10.1103/PhysRevLett.121.022504
[34] https://doi.org/10.1103/PhysRevD.101.092001

One muon and at least one proton, and only one proton within...

MicroBooNE

TABLE II. Phase-space restrictions applied to $\nu_\mu - {}^{40}$Ar CC0πNp [22] and CC0π1p [24] and $\nu_e - {}^{40}$Ar CC0πNp [23] cross section measurements performed by MicroBooNE collaboration. The opening angle $\theta_{\mu p}$ is defined as the angle between the muon and the ejected proton and $\delta p_T = |\mathbf{k}'_\mathbf{T} + \mathbf{p}_{\mathbf{N,T}}|$ is the transverse momentum imbalance [49] defined as the sum of the projections in the plane perpendicular to the neutrino direction of the muon and proton momenta. The index "L" over the proton angles means they are defined in the laboratory frame (neutrino direction fixed in the $\hat{z}$ axis).

	k'	$\cos\theta_l$	p_N	$\cos\theta_N^L$	ϕ_N^L	$\theta_{\mu p}$	δp_T
1 μCC0πNp							
	>0.1 GeV	...	0.3–1.2 GeV	...	...	...	...
1eCC0πNp							
	>30.5 MeV	...	>0.3105 GeV	...	...	...	...
1 μCC0π1p							
	0.1–1.5 GeV	$-0.65 < \cos\theta_l < 0.95$	0.3–1.0 GeV	>0.15	145–215°	35–145°	$\delta p_T < 0.35$ GeV

Table from https://doi.org/10.1103/PhysRevD.109.013004.

[22] https://doi.org/10.1103/PhysRevD.102.112013
[23] https://doi.org/10.1103/PhysRevD.106.L051102
[24] https://doi.org/10.1103/PhysRevLett.125.201803

Reaction channels contributing

+ Quasielastic scattering.

+ Processes leading to two-particle two-hole final state (more generally np-nh).

+ Pion production followed by absorption of the pion.

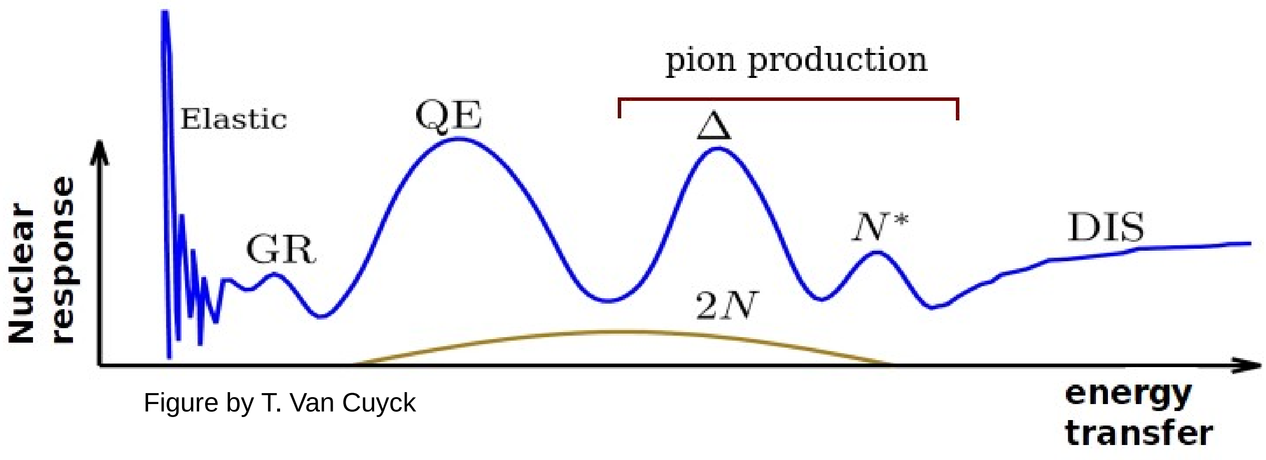
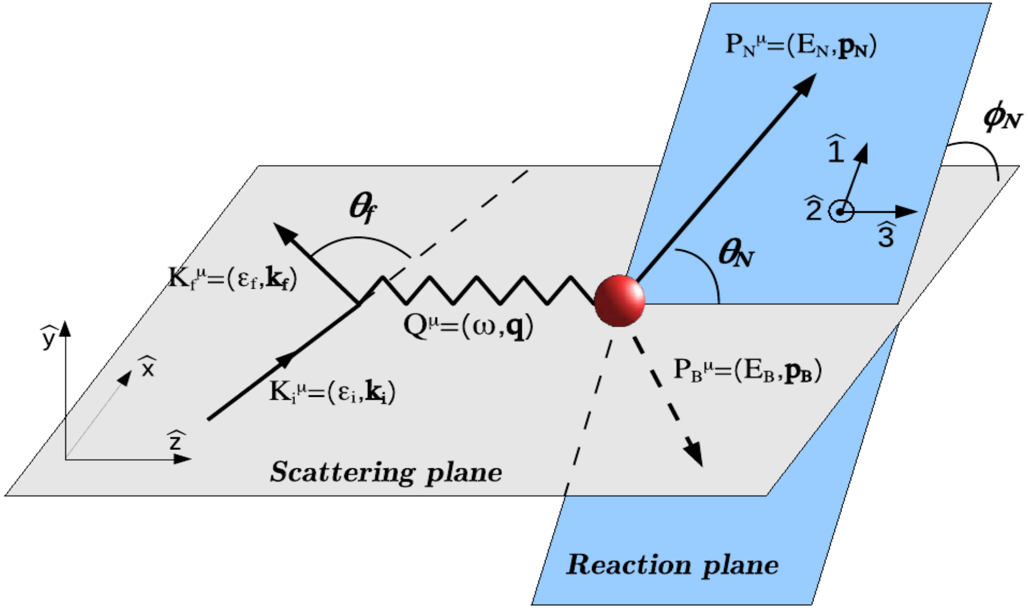
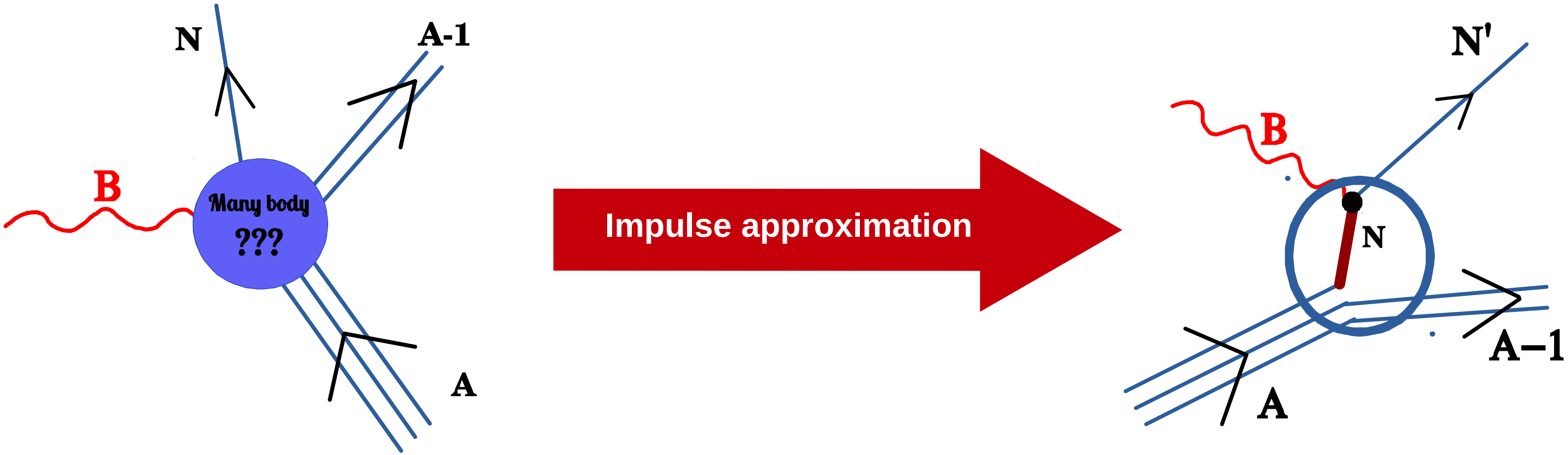


Figure by T. Van Cuyck

Modeling quasielastic scattering

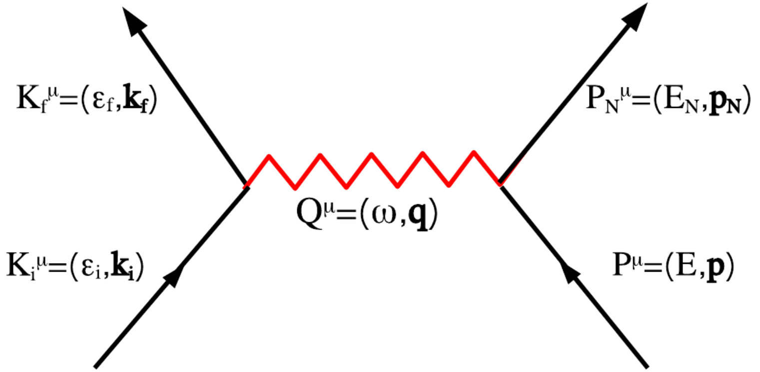


The impulse approximation



At first approximation, the quasielastic lepton-nucleus cross section is the incoherent sum of A (A=N+Z) lepton-nucleon elastic cross sections.

QE cross section $\propto \sum$ (number of nucleons) $\left| \text{diagram} \right|^2$



lepton-nucleon elastic cross sections:

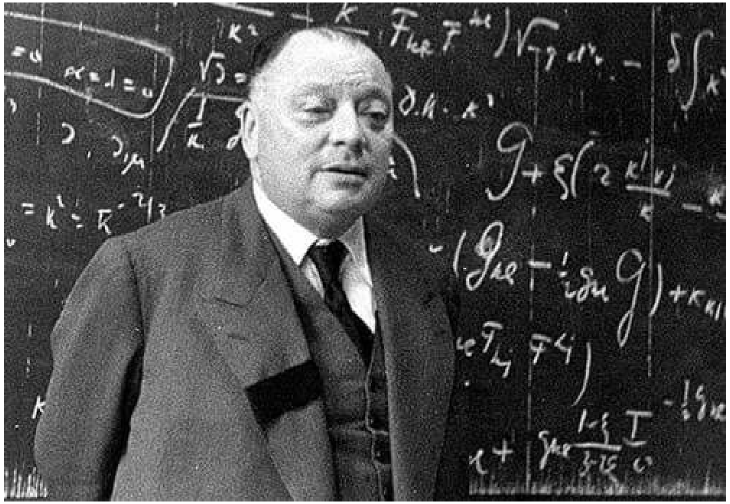
$$d^6\sigma = K(2\pi)^4\delta^4(K_f + P_N - K_i - P)L_{\mu\nu}H^{\mu\nu}\frac{d\mathbf{k}_f}{(2\pi)^3}\frac{d\mathbf{p}_N}{(2\pi)^3}$$

Important (or minimal) corrections to this picture:

1. Inside a nucleus, the nucleons are moving: **Fermi motion**.
2. **Pauli exclusion principle**.
3. Binding energy.

Such a model is the **relativistic Fermi gas (RFG)**: a relativistic gas of fermions at T=0K in an infinite volume.

Enrico Fermi



Wolfgang Pauli

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The QE cross section within the RFG model is obtained by taking the average of the elastic cross section off a free nucleon over the 4-momentum of the initial nucleon.

This is:

$$\frac{d^6\sigma}{d\mathbf{k}_f d\mathbf{p}_N} = \frac{\mathcal{N}}{4/3\pi p_F^3}\int dE\delta(E-\sqrt{p^2+M^2})\int d^3\mathbf{p}\ \Theta(p_F-p)\Theta(p_N-p_F)$$
$$\times \frac{K}{(2\pi)^2}\delta^4(K_f+P_N-K_i-P)L_{\mu\nu}H^{\mu\nu}\ .$$

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The QE cross sectio[...] the elastic cross section off a free nucleon ove[...]

This is:

$$\frac{d^6\sigma}{d\mathbf{k}_f d\mathbf{p}_N} = \ldots)\Theta(p_N - p_F)$$

$$\times \ldots$$

Reminder, the average of a function f(x,y,z) over the variable z, which is given by a probability density function $\rho(z)$, is given by:

$$\langle f(x,y) \rangle = \frac{\int dz\ \rho(z)\ f(x,y,z)}{\int dz\ \rho(z)}$$

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elastic lepton-nucleon cross section

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This is:

average over energy and momentum

$$\frac{d^6\sigma}{d\mathbf{k}_f d\mathbf{p}_N} = \frac{\mathcal{N}}{4/3\pi p_F^3}\int dE\,\delta(E-\sqrt{p^2+M^2})\int d^3\mathbf{p}\;\Theta(p_F-p)\Theta(p_N-p_F)$$
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momentum distribution

$$\frac{d^6\sigma}{d\mathbf{k}_f d\mathbf{p}_N} = \frac{\mathcal{N}}{4/3\pi p_F^3} \int dE\delta(E - \sqrt{p^2 + M^2}) \int d^3\mathbf{p}\, \Theta(p_F - p)\Theta(p_N - p_F)$$
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step function

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The QE cross section within the RF erage of the elastic cross section off a free nucleon over the 4-mome

normalization of the momentum distribution

$$\frac{\mathcal{N}}{4/3\pi p_F^3}\int d\mathbf{p}\,\Theta(p_F-p)=\mathcal{N}\,.$$

This is:

momentum distribution

$$\frac{d^6\sigma}{d\mathbf{k}_f d\mathbf{p}_N} = \frac{\mathcal{N}}{4/3\pi p_F^3}\int dE\,\delta(E-\sqrt{p^2+M^2})\int d^3\mathbf{p}\,\Theta(p_F-p)\Theta(p_N-p_F)$$
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energy distribution

$$\frac{d^6\sigma}{d\mathbf{k}_f d\mathbf{p}_N} = \frac{\mathcal{N}}{4/3\pi p_F^3}\int dE\,\delta(E-\sqrt{p^2+M^2})\int d^3\mathbf{p}\;\Theta(p_F-p)\Theta(p_N-p_F)$$
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On shell

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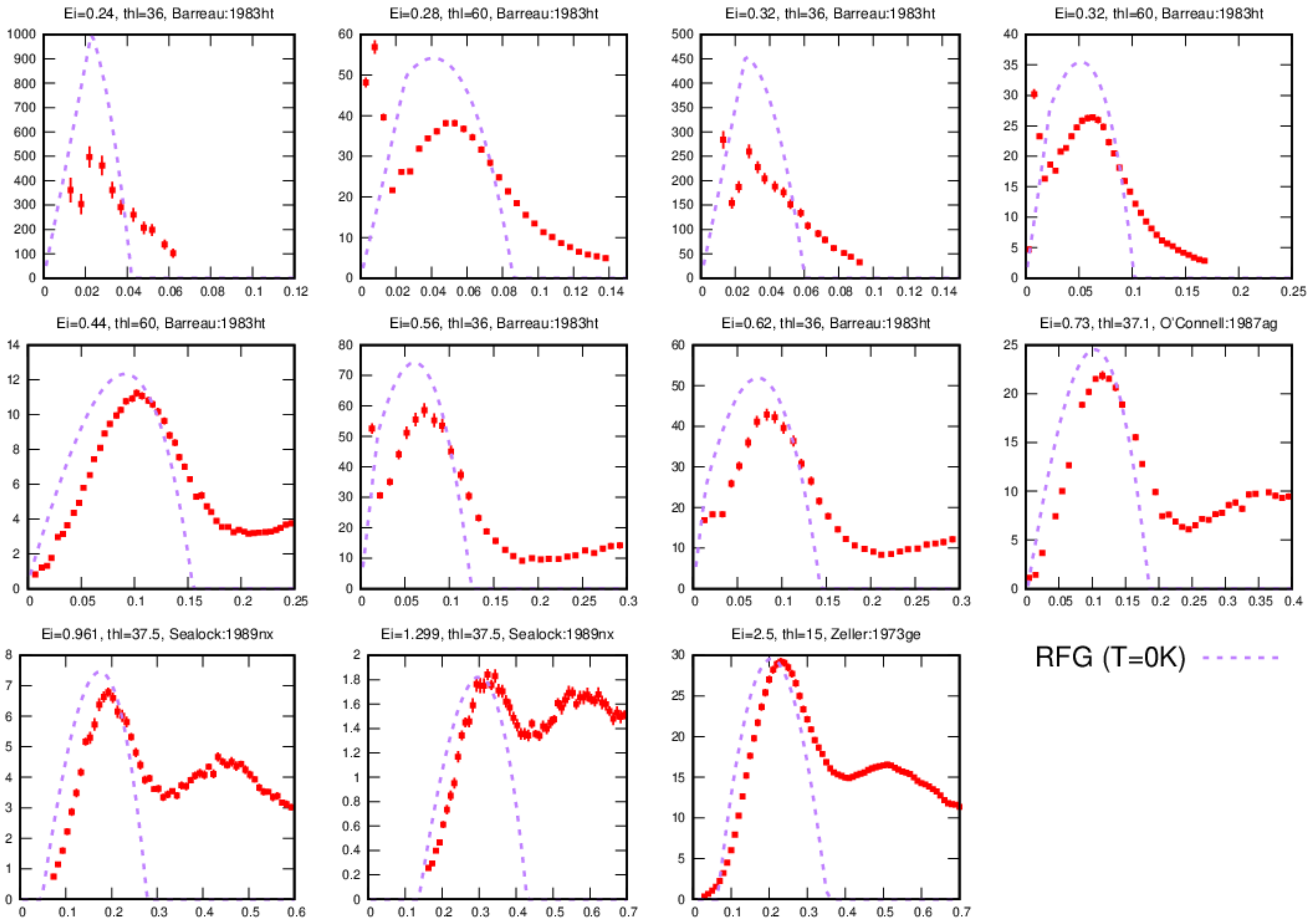
Pauli blocking

$^{12}C(e,e')$
cross sections with the RFG at T=0K


Ei=0.24, thl=36, Barreau:1983ht
Ei=0.28, thl=60, Barreau:1983ht
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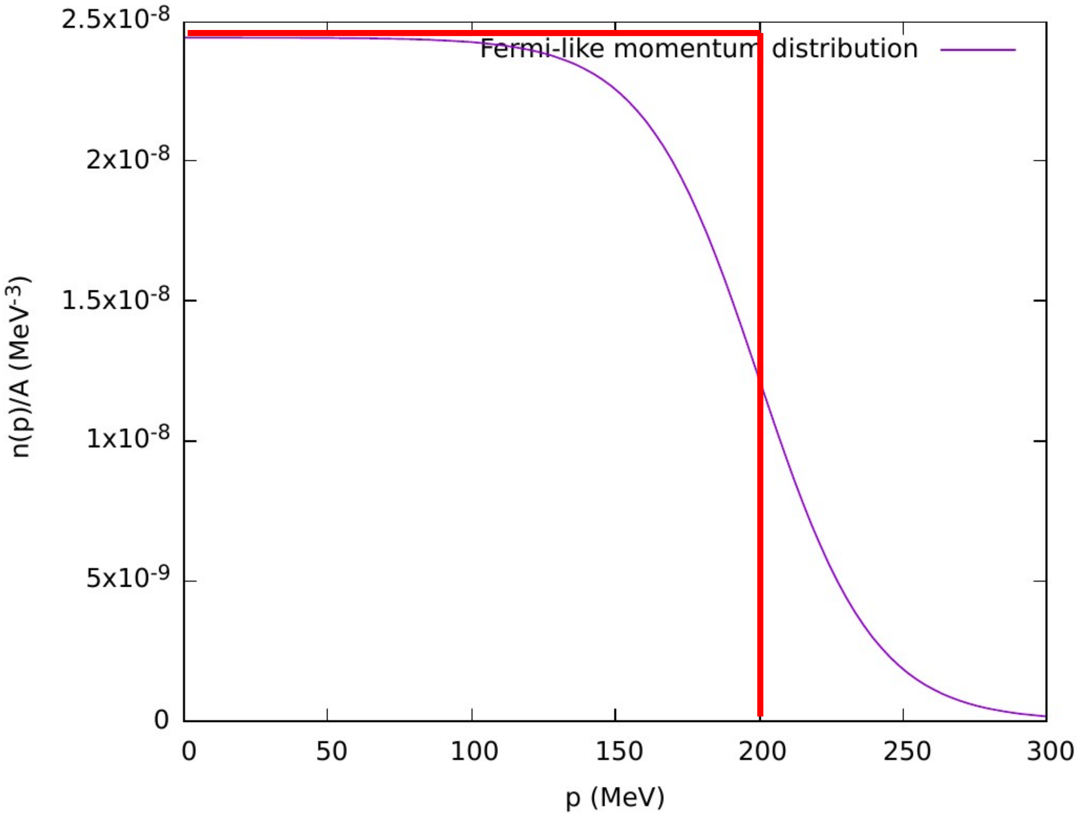
Ei=0.961, thl=37.5, Sealock:1989nx
Ei=1.299, thl=37.5, Sealock:1989nx
Ei=2.5, thl=15, Zeller:1973ge
RFG (T=0K)

Let’s replace the step function (zero temperature, T=0K) by a Fermi-like distribution (T>0K).



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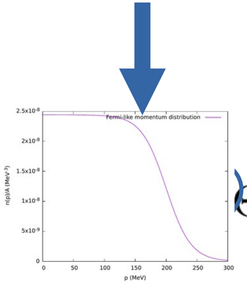
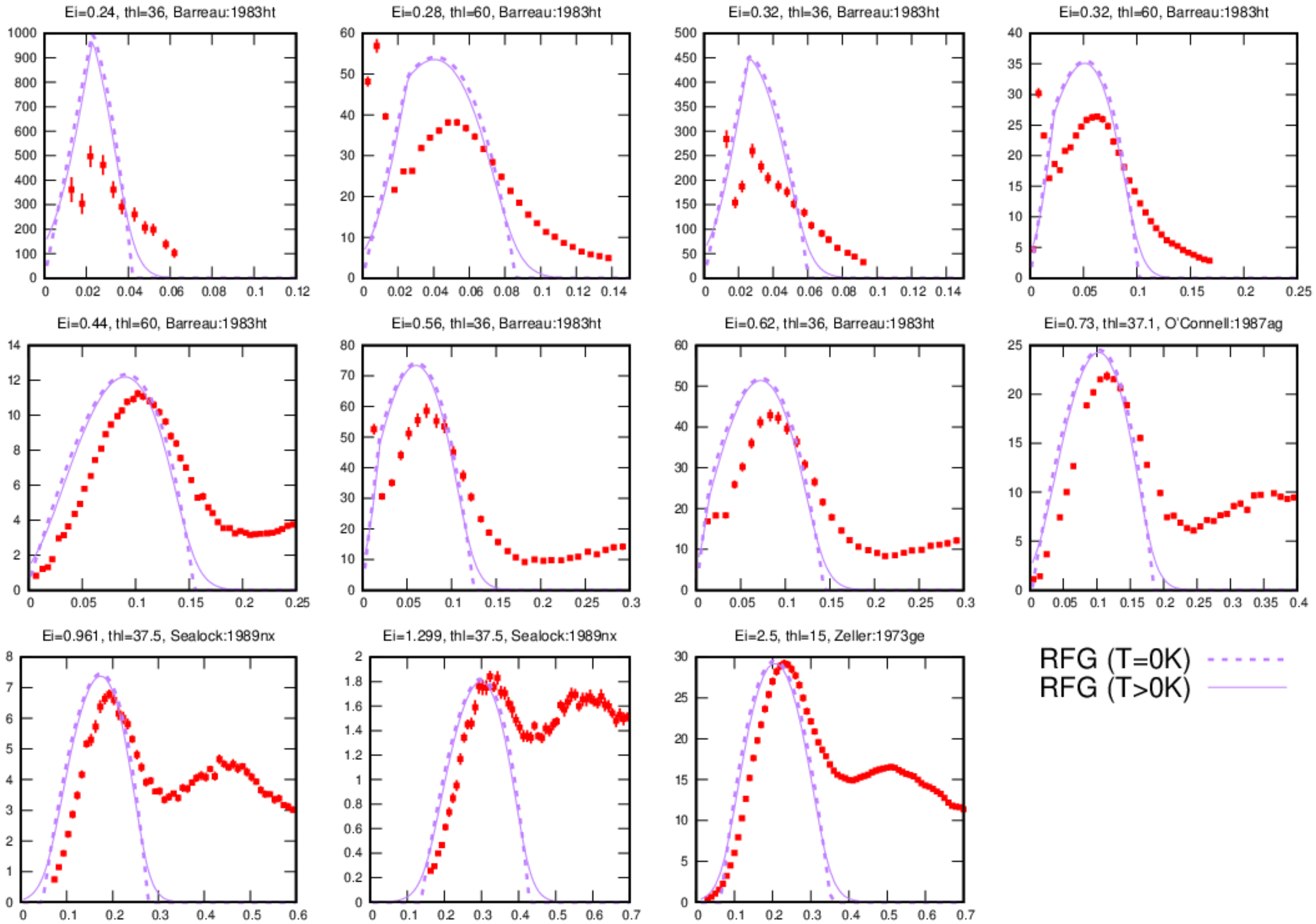


$$\Theta(p_N - p_F)$$

$$\times \frac{K}{(2\pi)^2} \delta^4(K_f + P_N - K_i - P) L_{\mu\nu} H^{\mu\nu} \, .$$

$^{12}C(e,e')$
cross sections with the RFG
at T>0K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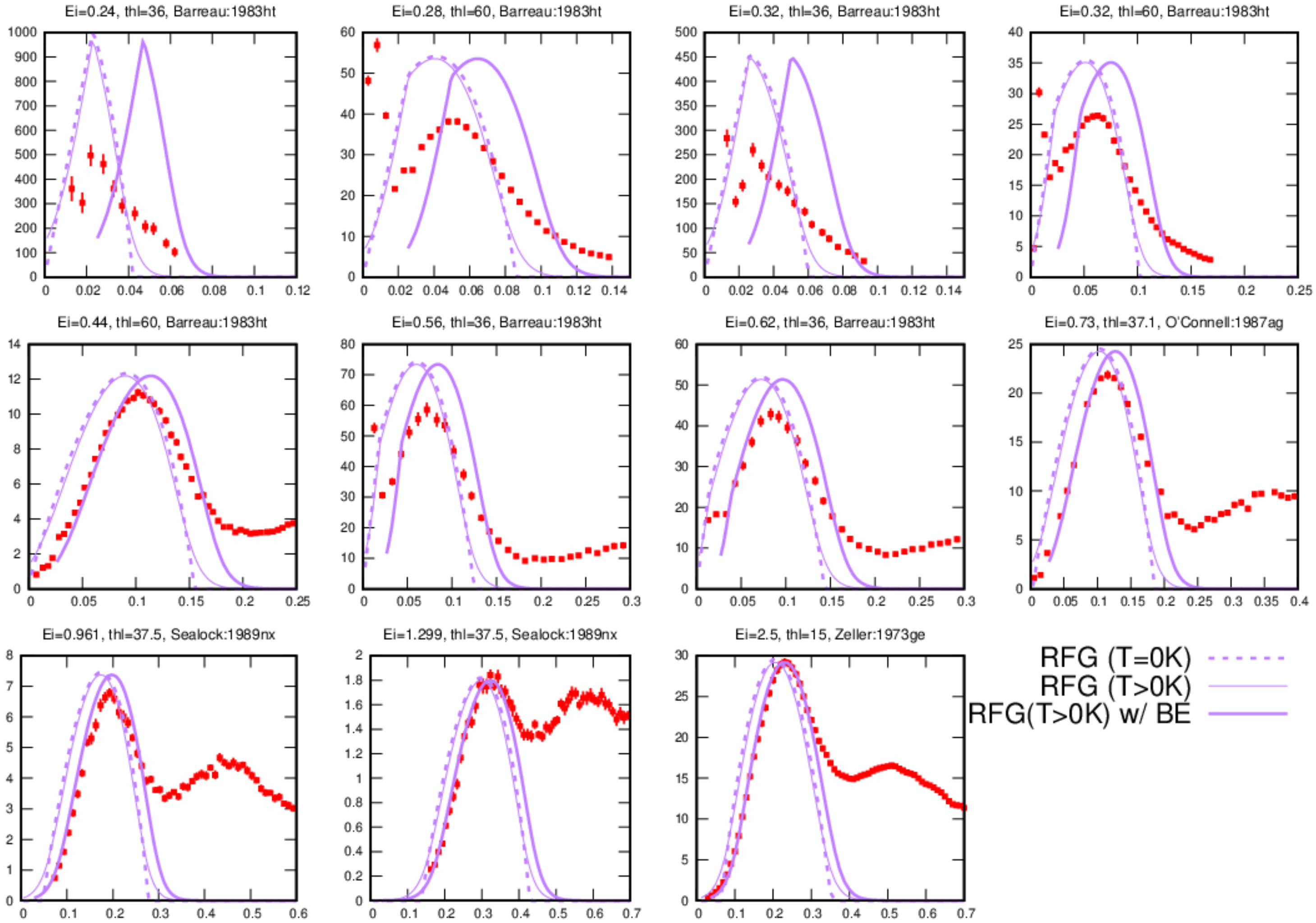
RFG (T=0K)
RFG (T>0K)

$^{12}C(e,e')$
cross sections with the RFG
at T>0K
+
binding energy

(simply a shift of the distributions by 24 MeV)



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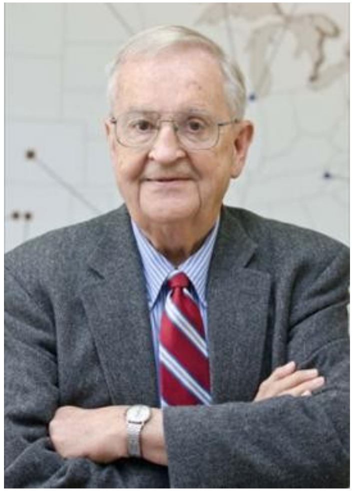
RFG (T=0K)
RFG (T>0K)
RFG(T>0K) w/ BE

The Independent-Particle Shell-Model (IPSM)

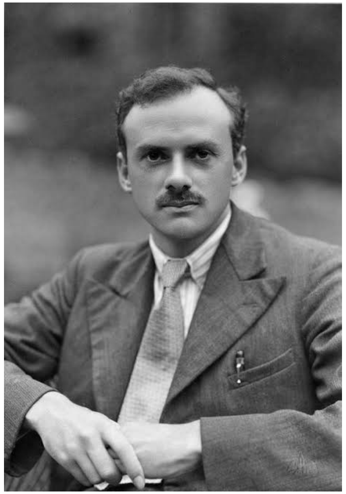
Dirac equation for nucleons (within an extension of **Walecka** model):

$$[-i\boldsymbol{\alpha}\cdot\boldsymbol{\nabla} + V(r) + \beta(M + S(r)]\Psi_{\boldsymbol{i}}(\boldsymbol{r}) = E_i\Psi_{\boldsymbol{i}}(\boldsymbol{r})$$

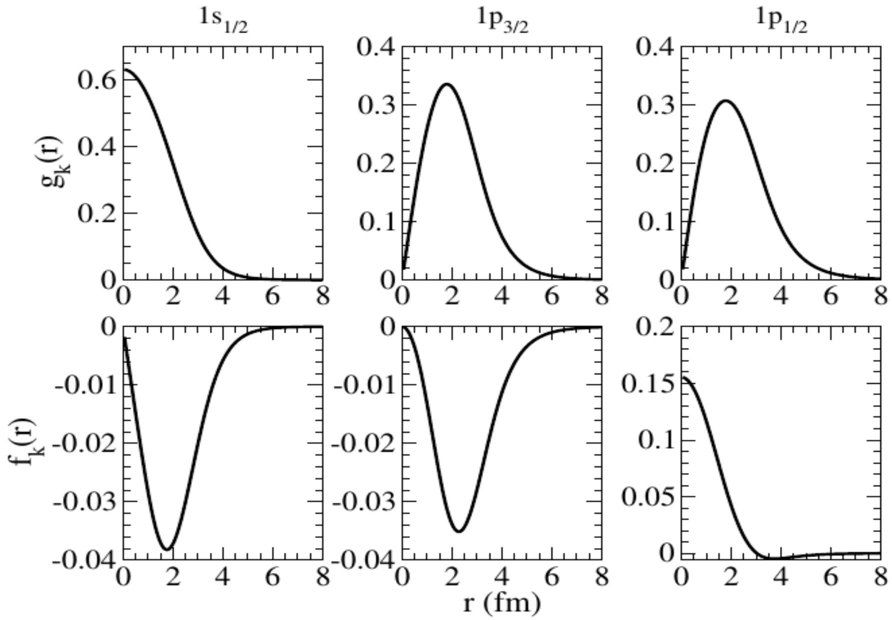
$$\Psi_k^{m_j}(\boldsymbol{r}) = \begin{pmatrix} g_k(r)\varphi_k^{m_j}(\Omega_r) \\ if_k(r)\varphi_{-k}^{m_j}(\Omega_r) \end{pmatrix},$$



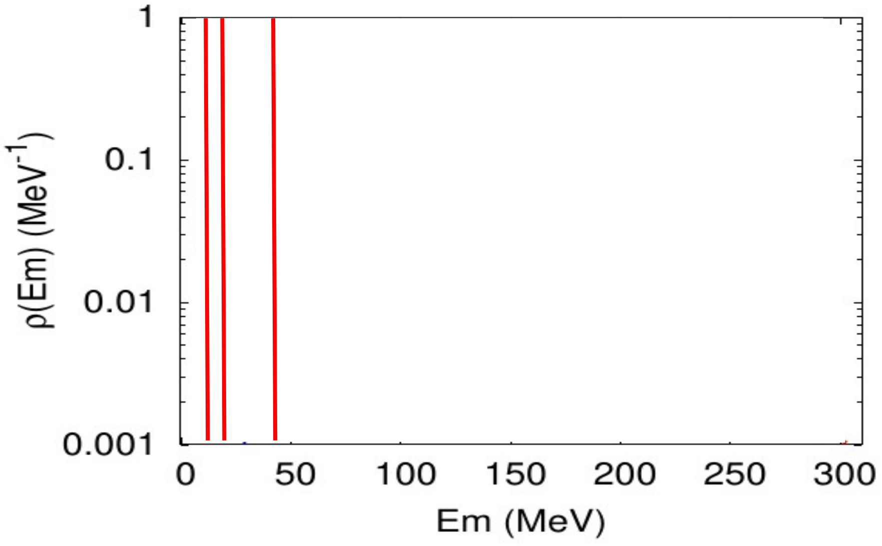
J. Dirk Walecka



Paul Dirac



Energy distribution in an IPSM: a set of Dirac deltas



$$\rho_\kappa(E_m) = \delta(E_m - E_m^\kappa)$$

The position of the shells is given by the eigenvalues of the wave functions.

Momentum distributions in an IPSM

Nuclear density:

$$\rho_\kappa(r) = \sum_{m_j} \int d\Omega_{\mathbf{r}} |\Psi_\kappa^{m_j}(\mathbf{r})|^2$$

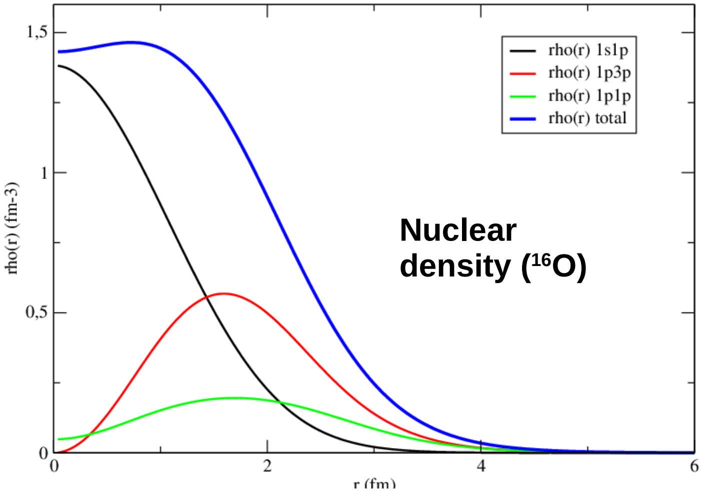
Fourier transform of the wave function:

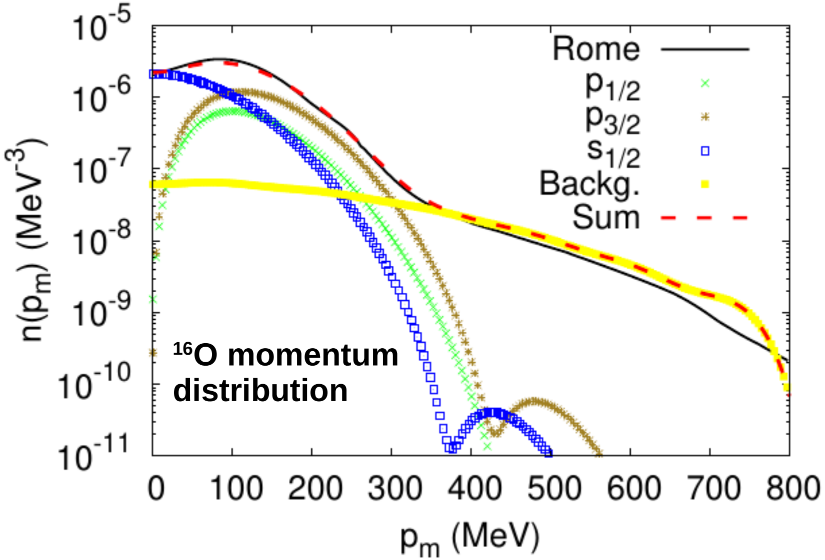
$$\Psi_k^{m_j}(\boldsymbol{p}) = \frac{1}{(2\pi)^{3/2}} \int d\boldsymbol{r} e^{-i\boldsymbol{p}\boldsymbol{r}} \Psi_k^{m_j}(\boldsymbol{r})$$

Joseph Fourier

The **momentum distribution** of each shell is computed from the wave functions in momentum space:

$$\boxed{n_\kappa(p) = \sum_{m_j} \int d\Omega_{\mathbf{p}} |\Psi_\kappa^{m_j}(\mathbf{p})|^2}$$





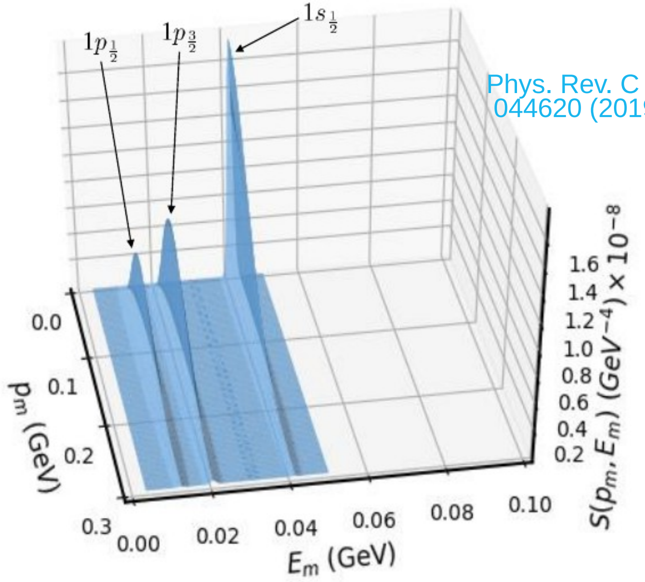
The **spectral function** is constructed from the momentum and energy distribution of each shell:

$$S(E_m, p_m) = \sum_{\kappa} n_\kappa(p_m)\delta(E_m - E_m^\kappa)$$

It is normalized to the total number of nucleons:

$$n(p_m) = \int dE_m S(E_m, p_m)$$

$$\int d^3\mathbf{p}_m n(p_m) = 8$$



Phys. Rev. C 100, 044620 (2019)

Realistic?

PRC49, 955 (1994)
Experiment at Nikhef

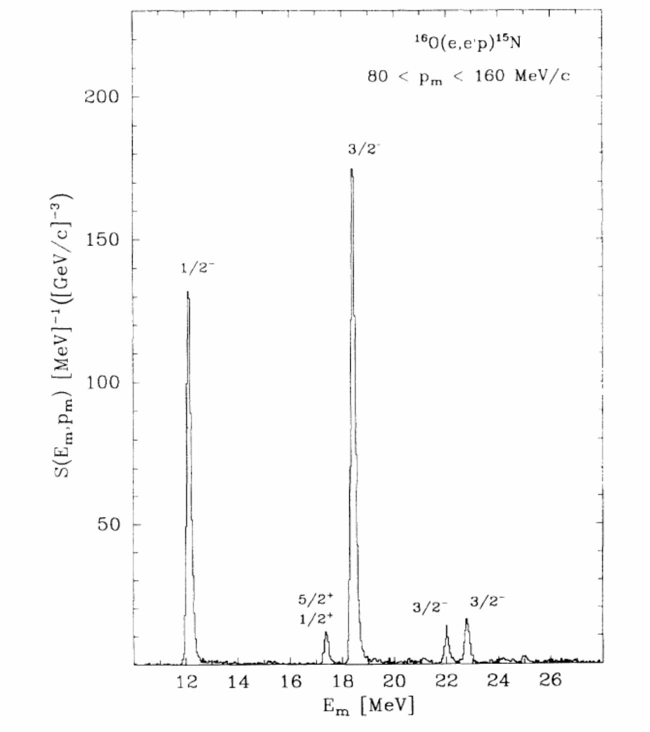
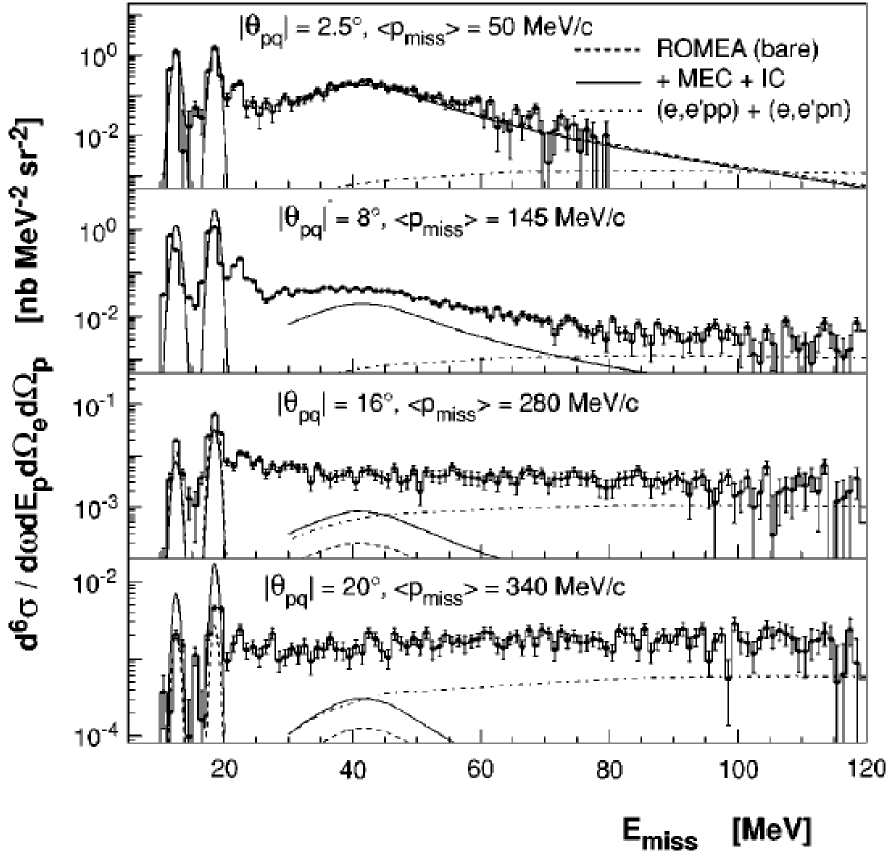


FIG. 1. $^{16}O(e,e'p)^{15}N$ missing energy spectrum for the kinematics centered about $p_m = 120$ MeV/c.

PRC70, 034606 (2004)
Experiment at JLab



There are effects beyond IPSM, due to short- and long-range correlations.

Experimentally one observes:
+ fragmentation of the strength
+ new bound states
+ some width of the peaks, specially the deeper shells
+ spectroscopic factors (< 1)

+ SRC: ~20% of the nucleons appear in the high E_m and p_m region.

The spectral function approach (SFA)

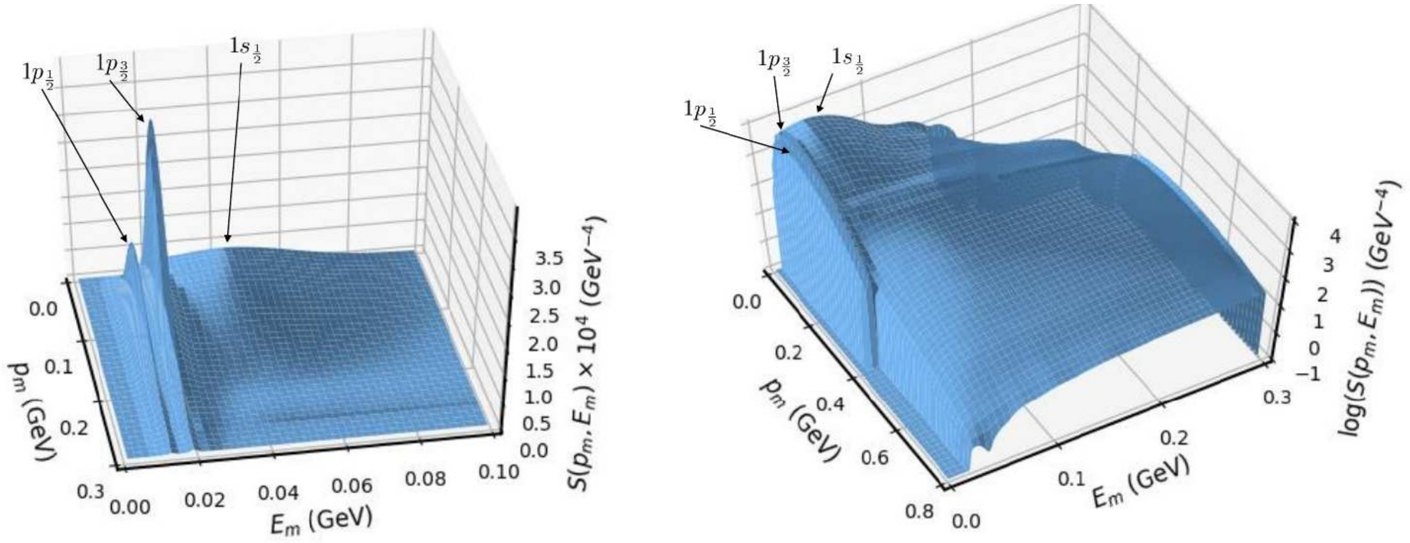


Image from Phys. Rev. C 100, 044620 (2019); spectral function from Nuclear Physics A 579, 493 (1994); Phys. Rev. D 72, 053005 (2005)

"The initial-state spectral function (SF) gives the probability density of knocking out a nucleon from the nucleus, leaving it with an excitation energy E_m and a recoil momentum $\mathbf{p}_m$."

Average (of the elastic lepton-nucleon cross section) over initial energy and momentum of the bound nucleon, where the SF is the PDF.
Notice that contrary to the RFG model, the bound nucleon is not in its mass shell: one additional degree of freedom.

$$d^6\sigma = \int d\mathbf{p} \int dE \; S(E,\mathbf{p})\, K(2\pi)^4\delta^4(K_f + P_N - K_i - P) L_{\mu\nu} H^{\mu\nu} \frac{d\mathbf{k}_f}{(2\pi)^3}\frac{d\mathbf{p}_N}{(2\pi)^3}$$

Normalization:

$$\int d\mathbf{p} \int dE \; S(E,p) = \mathcal{N}$$

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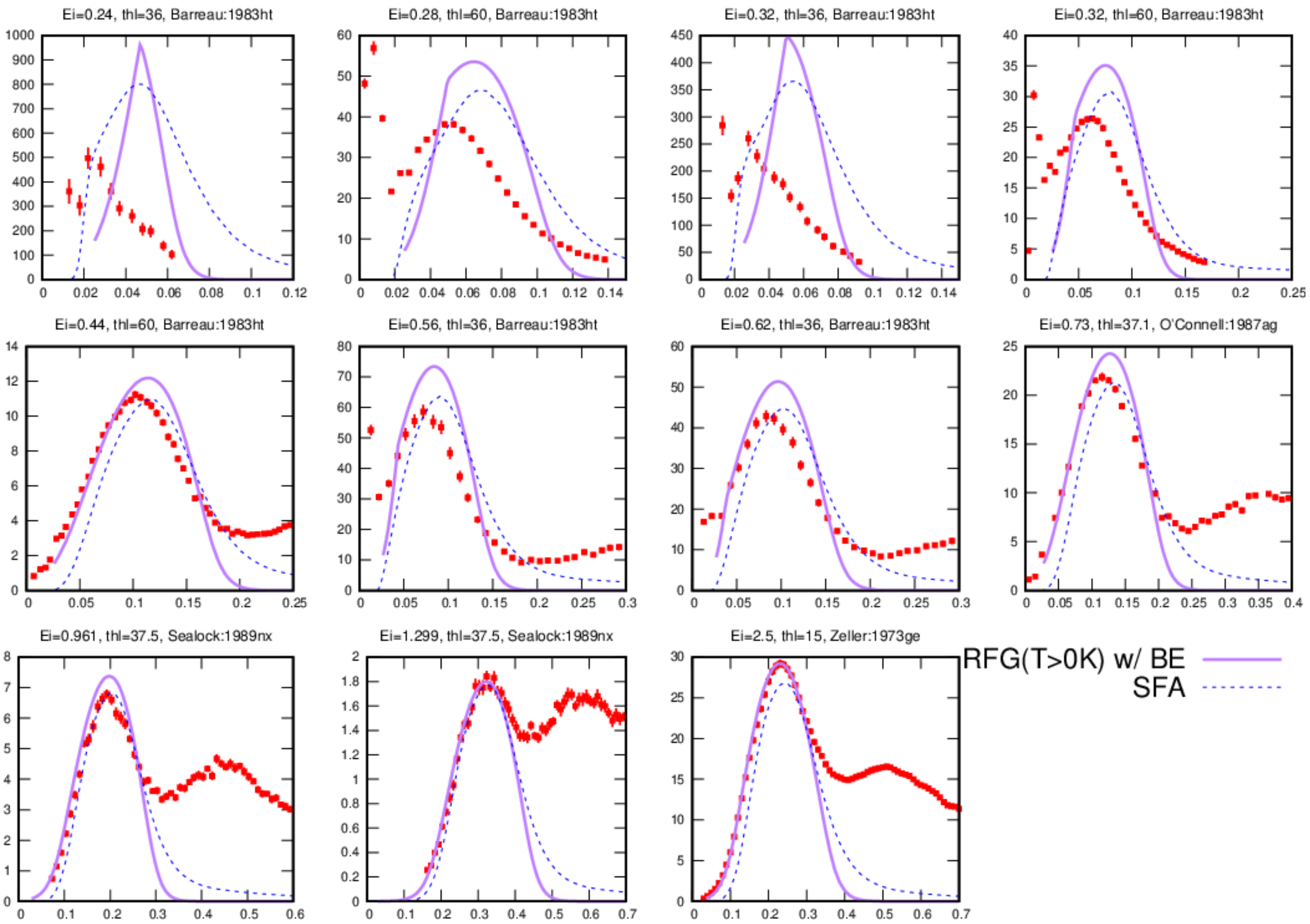
$^{12}C(e,e')$
cross sections with the RFG
at T>0K+ binding energy
+
SFA



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RFG(T>0K) w/ BE
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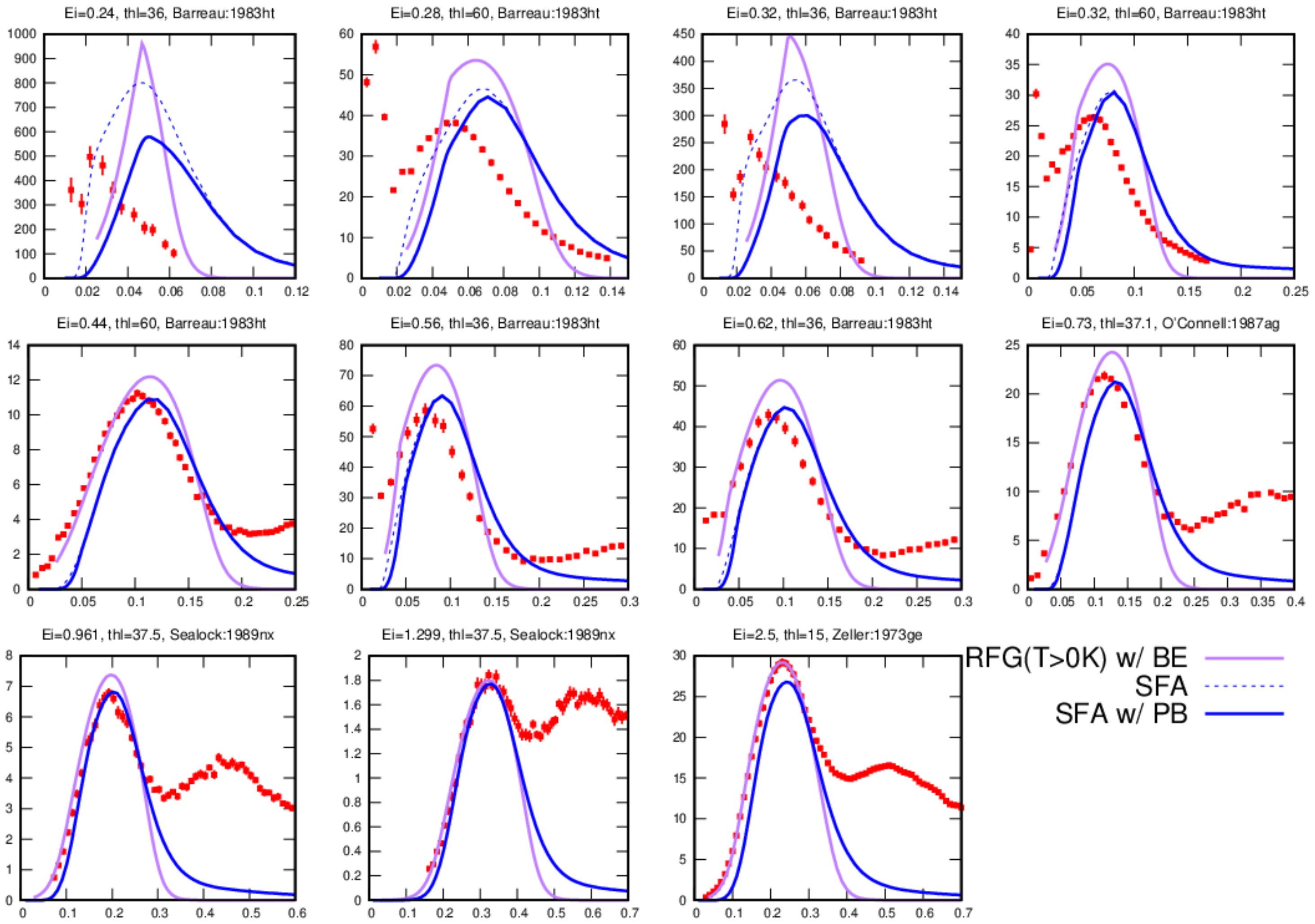
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RFG(T>0K) w/ BE
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What is missing?

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What is missing?

QUANTUM MECHANICHS!!!

What is missing? A consistent treatment of the initial and final states

$$J^{\mu}_{had} = \int d\mathbf{p}\,\overline{\Psi}_F(\mathbf{p}+\mathbf{q},\mathbf{p}_N)\;\mathcal{O}^{\mu}_{\text{one body}}\;\Psi_B(\mathbf{p})$$

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Wave functions of the final and bound nucleons must be solution of the **same wave equation,** it ensures:

+ **orthogonality** of the states (Pauli blocking),

+**current conservation**

+ and the **distortion** of the final nucleon (whatever it means...)

Emmy Noether

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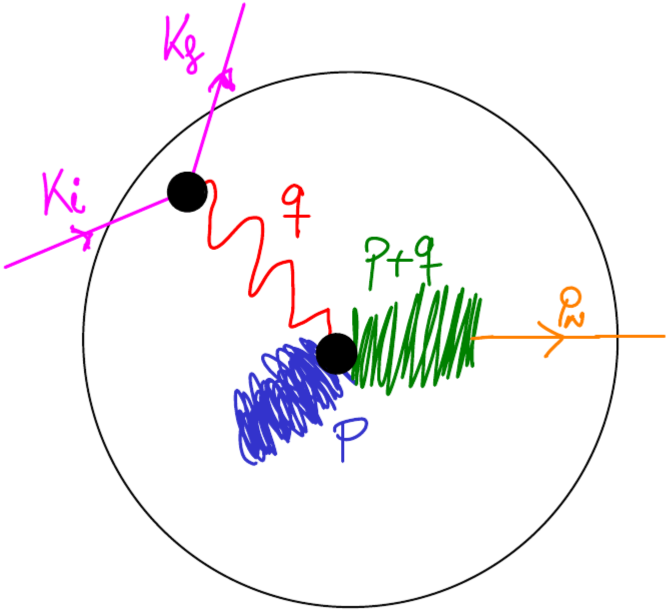

The **momentum of the nucleons** inside the nucleus is given by the wave functions (PDFs).

In other words: the nucleons do not have a momentum, but many. The nucleons do not have a wave length, but many. (That’s why we average over them.)

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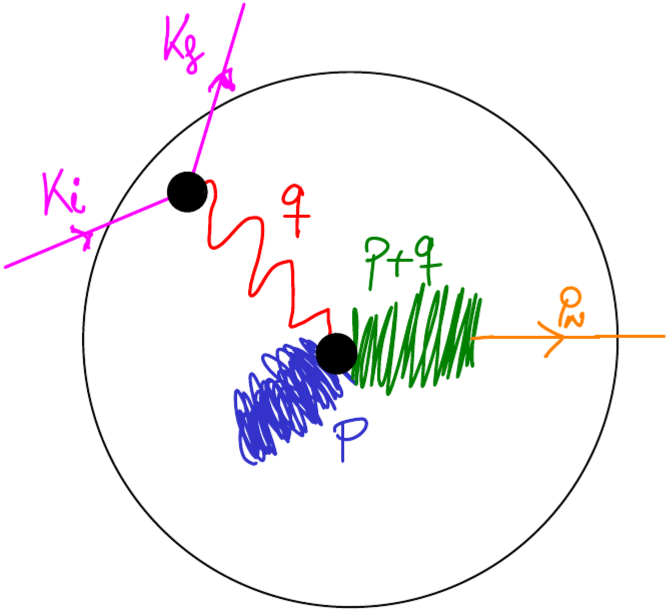

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Analogously **in coordinate space**: the nucleons are not in a particular point but in many at the same time, actually, in the whole nucleus. (That's why we average over the whole nuclear volume.)

For the final nucleon, we know that its asymptotic momentum is $\mathbf{p}_N$. This is the momentum that one can measure in a detector **if and only if nothing else happens after the primary interaction**.

$$J^{\mu}_{\text{had}} = \int d\mathbf{r} \overline{\Psi}_F(\mathbf{r}, \mathbf{p}_N) \mathcal{O}^{\mu}_{one\ body} \Psi_B(\mathbf{r}) e^{i\mathbf{q}\cdot\mathbf{r}}$$

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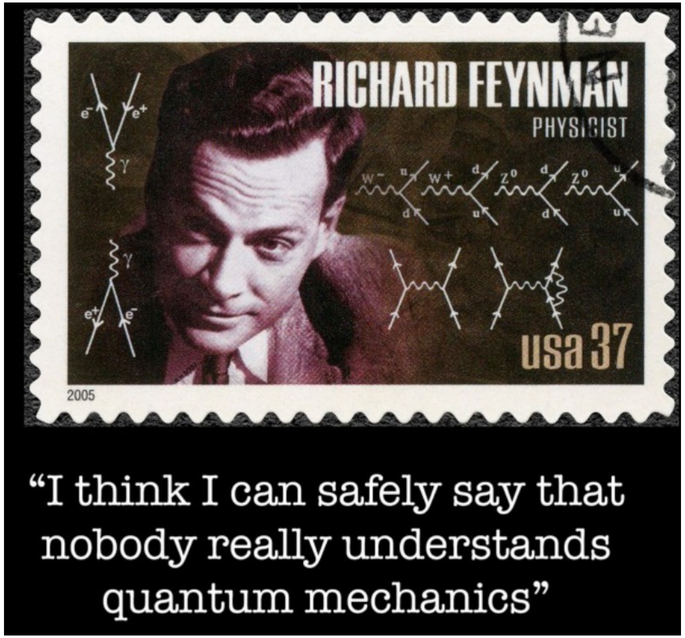

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Summarizing...

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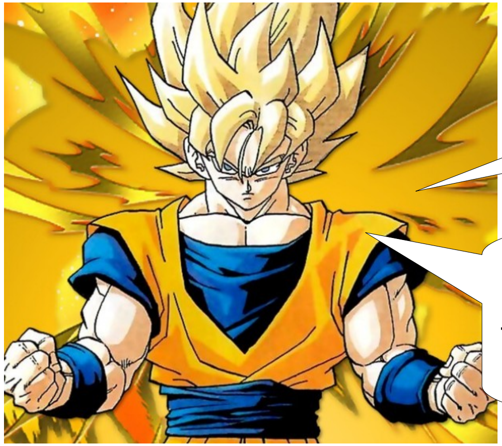

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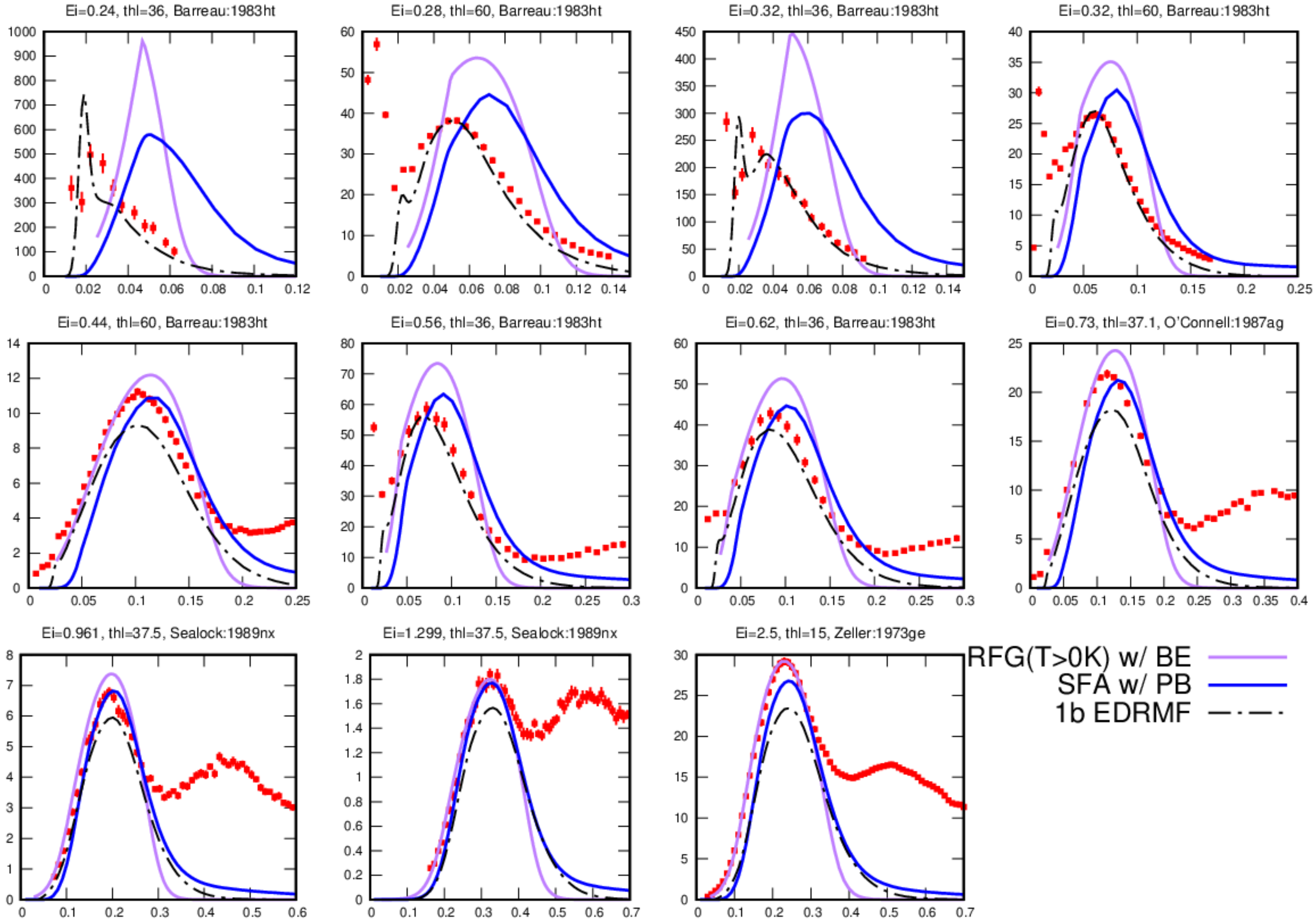
So… like in the Fermi gas model.

NO!!
The average here is at the amplitude level!
The wave functions are the probability density distributions!
This is QM!!!

Cross sections with the RFG
at T>0K+ binding energy
+
SFA w/ PB
+
EDRMF 1b



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RFG(T>0K) w/ BE
SFA w/ PB
1b EDRMF

https://arxiv.org/abs/2203.09996v3
https://arxiv.org/abs/2306.10823

Beyond Impulse Approximation: two-body currents in the 1p-1h sector

$$J^{\mu}_{had} = \int d\mathbf{p}\, \overline{\Psi}_F(\mathbf{p}+\mathbf{q}, \mathbf{p}_N) \left(\mathcal{O}^{\mu}_{\text{one body}} + \mathcal{O}^{\mu}_{\text{two body}}\right) \Psi_B(\mathbf{p})$$

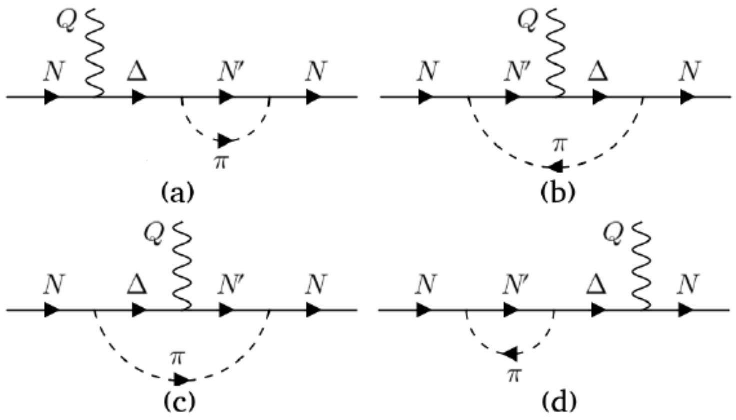


FIG. 1. Delta contributions.

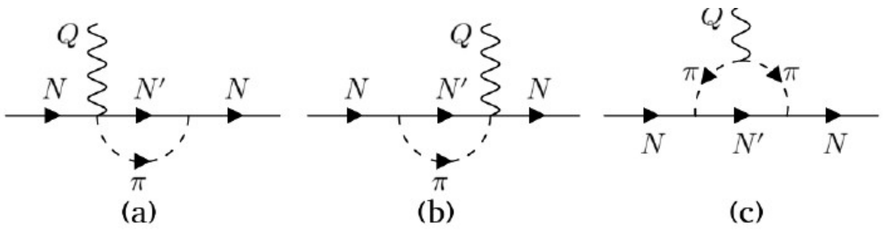
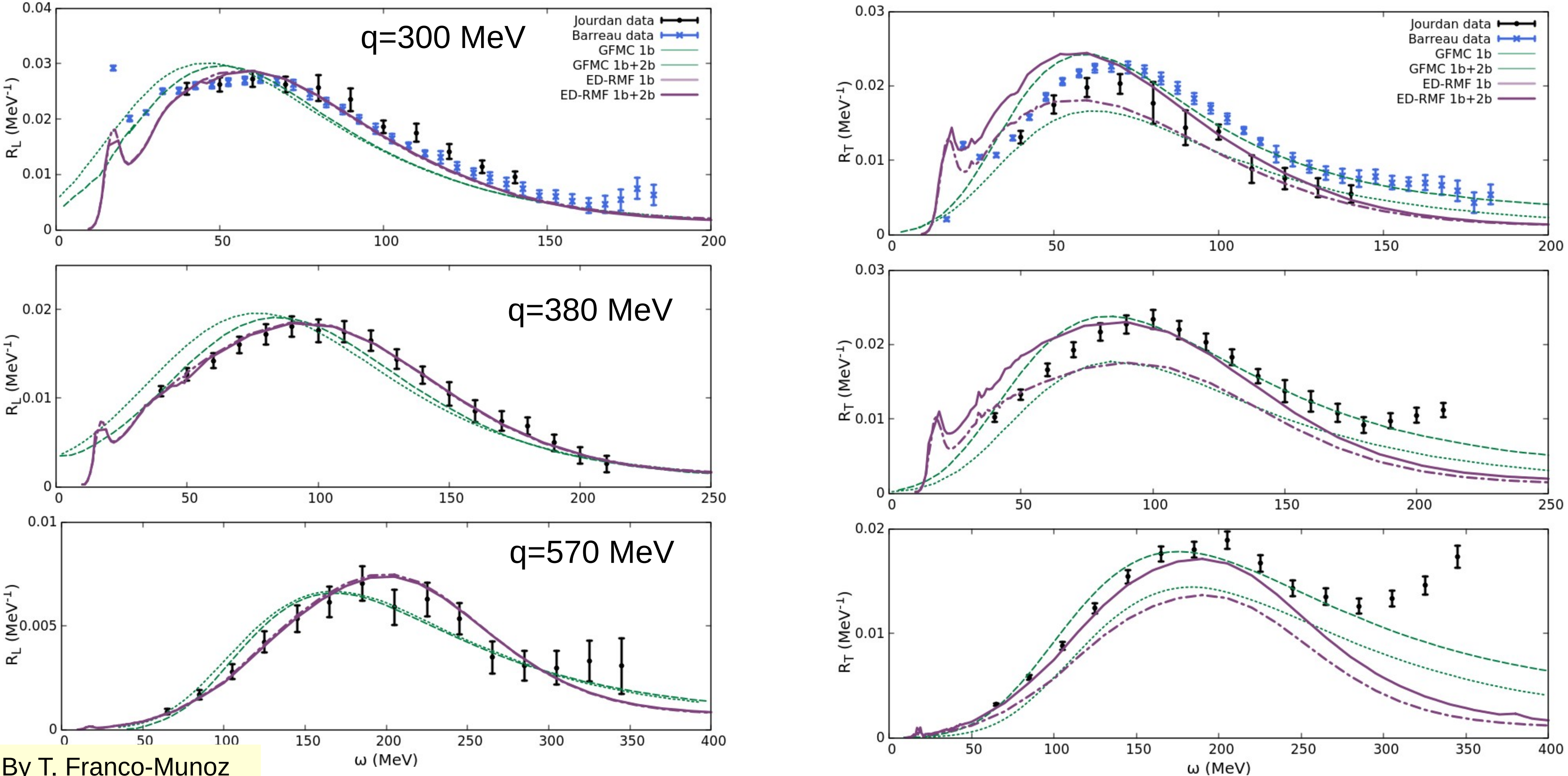


FIG. 2. Background contributions: seagull or contact [CT, (a) and (b)] and pion-in-flight [PF, (c)].

https://arxiv.org/abs/2306.10823

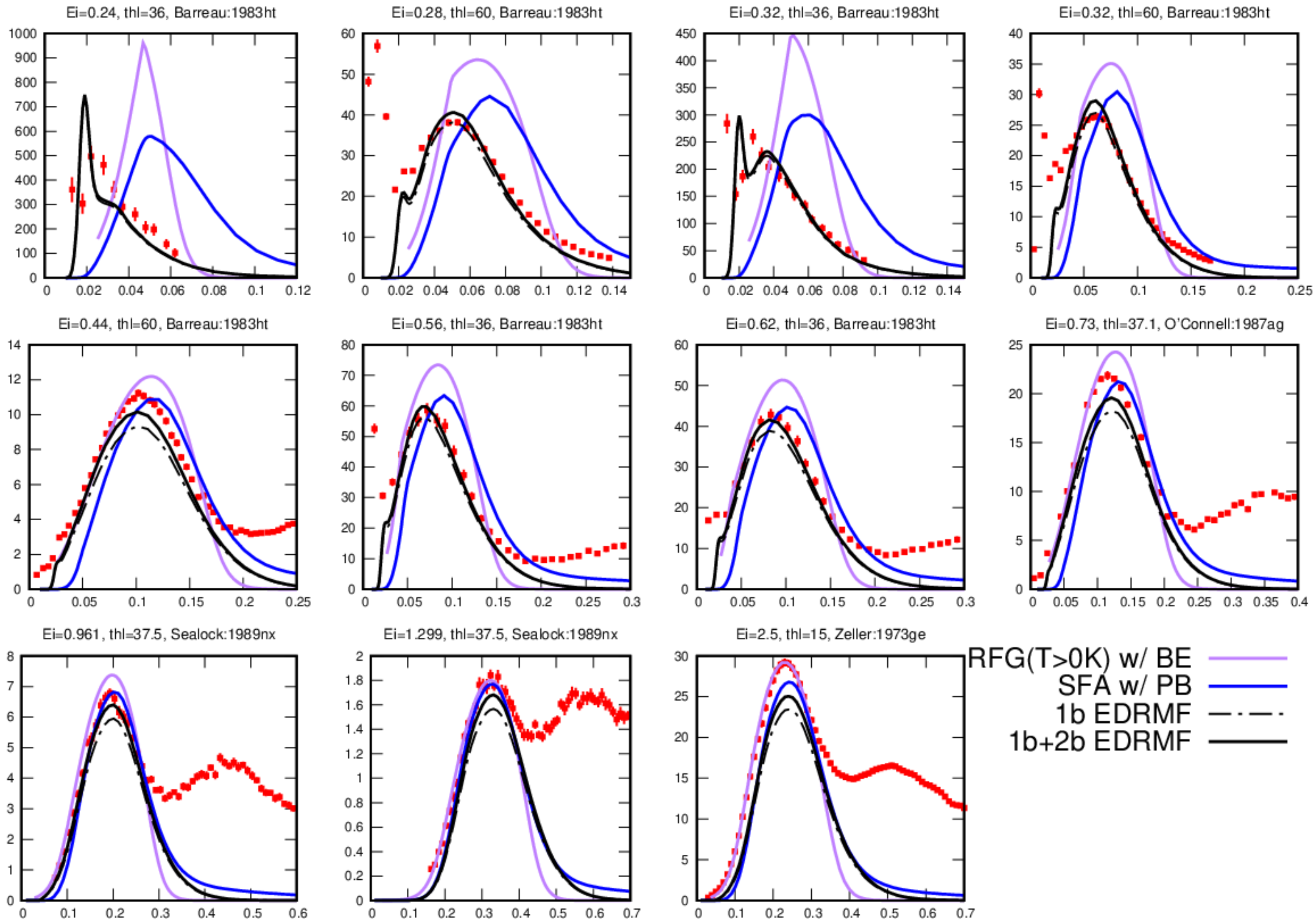
Carbon 12 responses

green lines from Lovato et al.
PRL 117, 082501 (2016)



By T. Franco-Munoz
as part of her PhD.

EDRMF 1b
+
EDRMF 1b+2b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RFG(T>0K) w/ BE
SFA w/ PB
1b EDRMF
1b+2b EDRMF

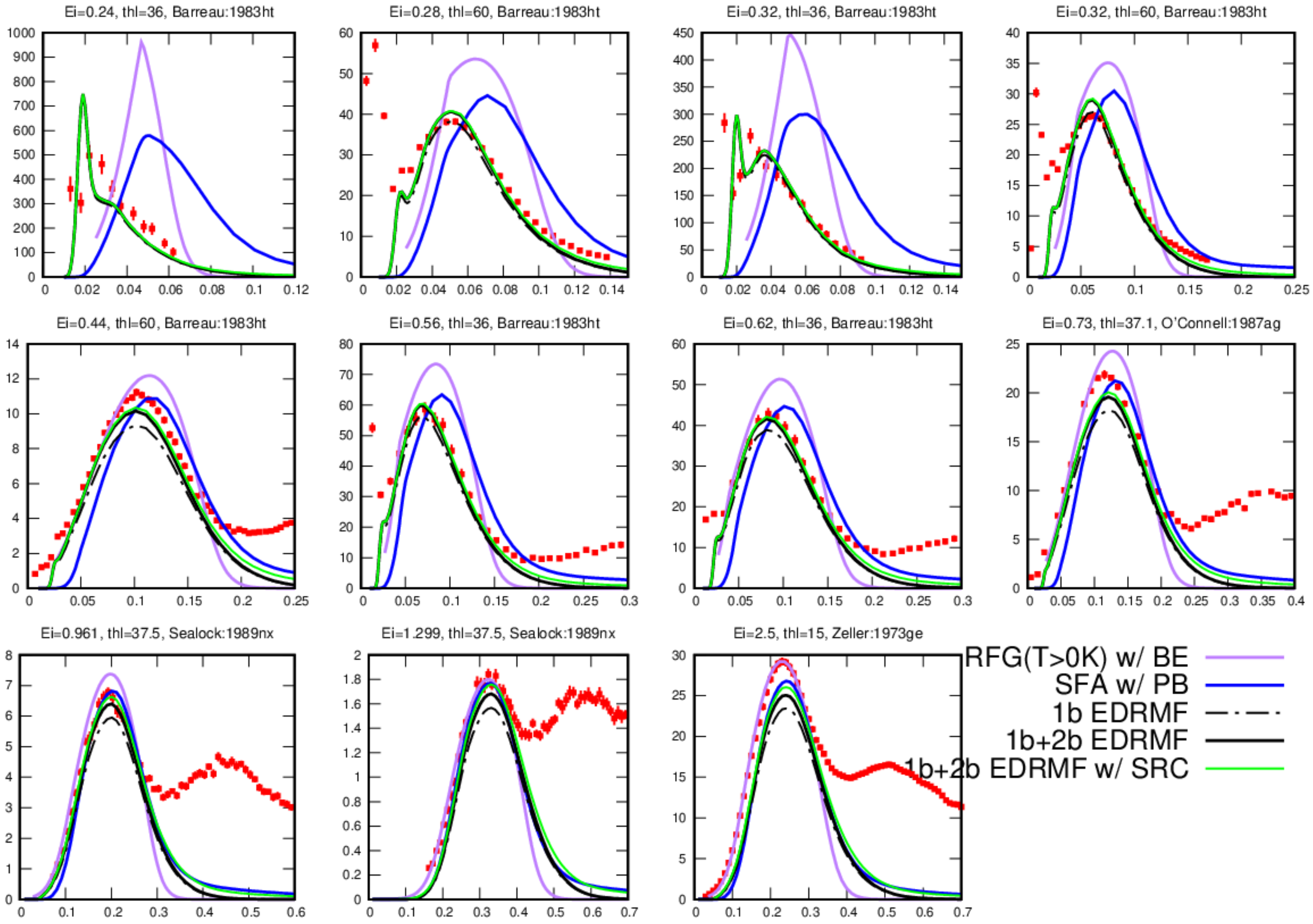
EDRMF 1b

+

EDRMF 1b+2b

+

SRC contribution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RFG(T>0K) w/ BE
SFA w/ PB
1b EDRMF
1b+2b EDRMF
1b+2b EDRMF w/ SRC

What is the best seed for a cascade?

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We believe a model like EDRMF approach is the best seed for a cascade model.

Or in other words, it is the best approach to be implemented in a MC event generators:
+ it gives a fair description of the inclusive cross section
+ it provides information about the final nucleon

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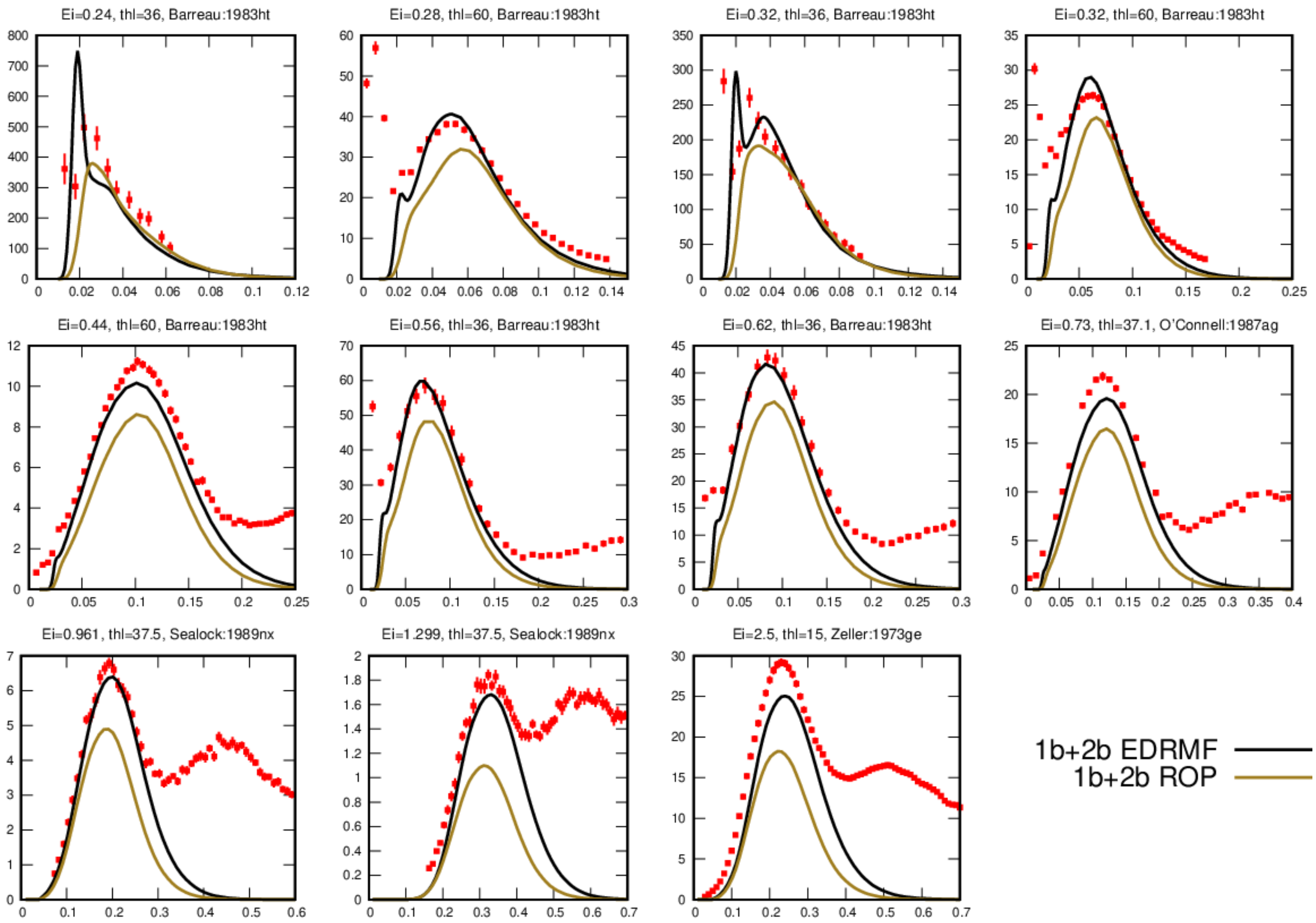
What about, **Double counting the FSI?**

I don’t think so… (keep watching).

EDRMF 1b+2b
+
ROP 1b+2b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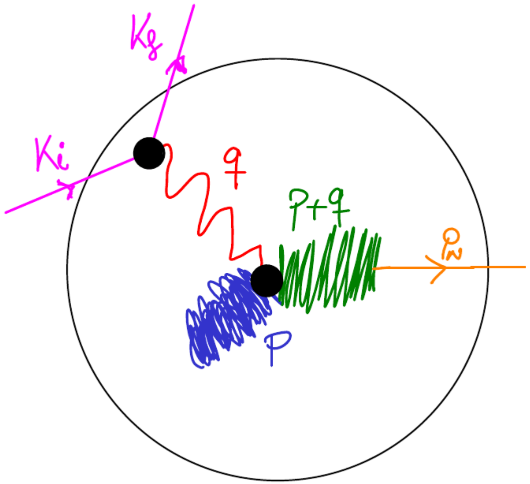
1b+2b EDRMF
1b+2b ROP

The model **ROP** uses a complex Relativistic Optical Potential (ROP). It predicts the cross section for the case in which the struck nucleon suffers only elastic final-state interactions *: so the **final state consists in “the lepton + only one nucleon”**, this is the Golden Channel.

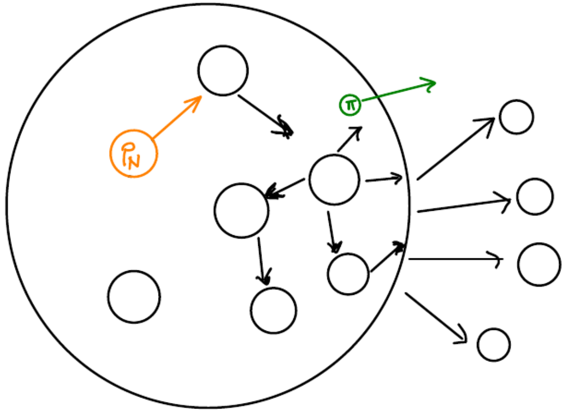
Only a fraction of the strength corresponds to the “only one nucleon” case.

So… why the EDRMF approach works well for the inclusive?

(*)In a MC generator, it corresponds to the case in which the nucleon propagates through the nucleus (using the intranuclear cascade model) without interacting at all. Useful to benchmark cascade models.
https://arxiv.org/abs/2406.09244,https://doi.org/10.1103/PhysRevC.105.054603



Some time goes by...



This happens first.

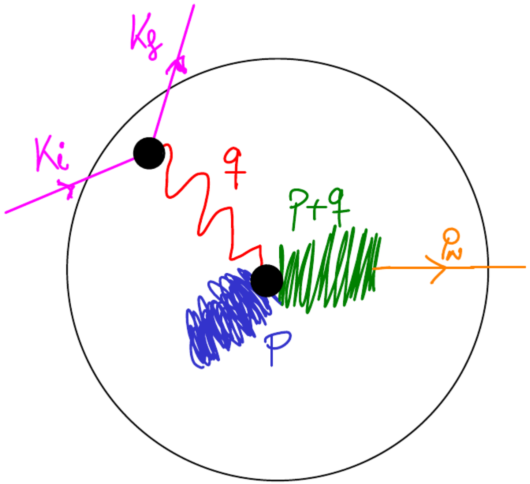
To describe this we need ALL INGREDIENTS discussed earlier.

We use EDRMF or analogous approach.

Then, rescattering(s) can happen.

Hopefully, a cascade model is able to handle this.
Whatever happens here, the inclusive cross section remains the same.

(I think elastic interactions should be avoided in the cascade because they were already included in the modeling of the primary interaction, but I don't really know...)



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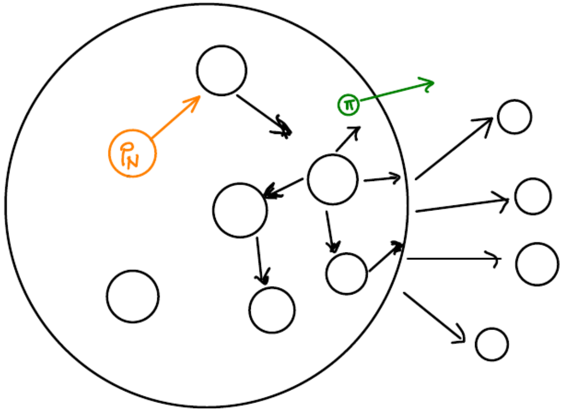

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This is my opinion in October 2024. It may change with time.

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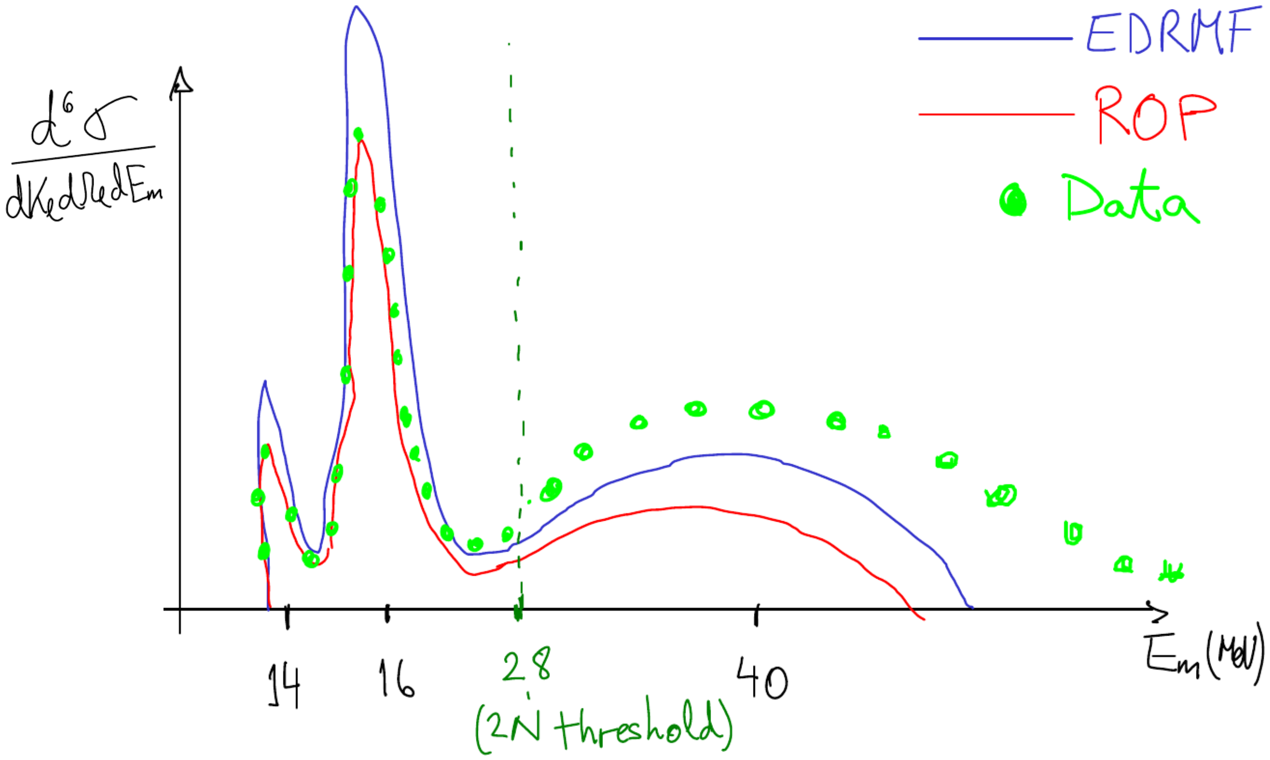
Fictitious experiment

$^{16}O(e,e'p)^{15}N$, integrated over the whole solid angle of the nucleon.

(And let's imagine that the only process that exists is QE scattering, so there is no MEC 2p-2h or SRC inducing 2p2h.)

What do we expect to see?



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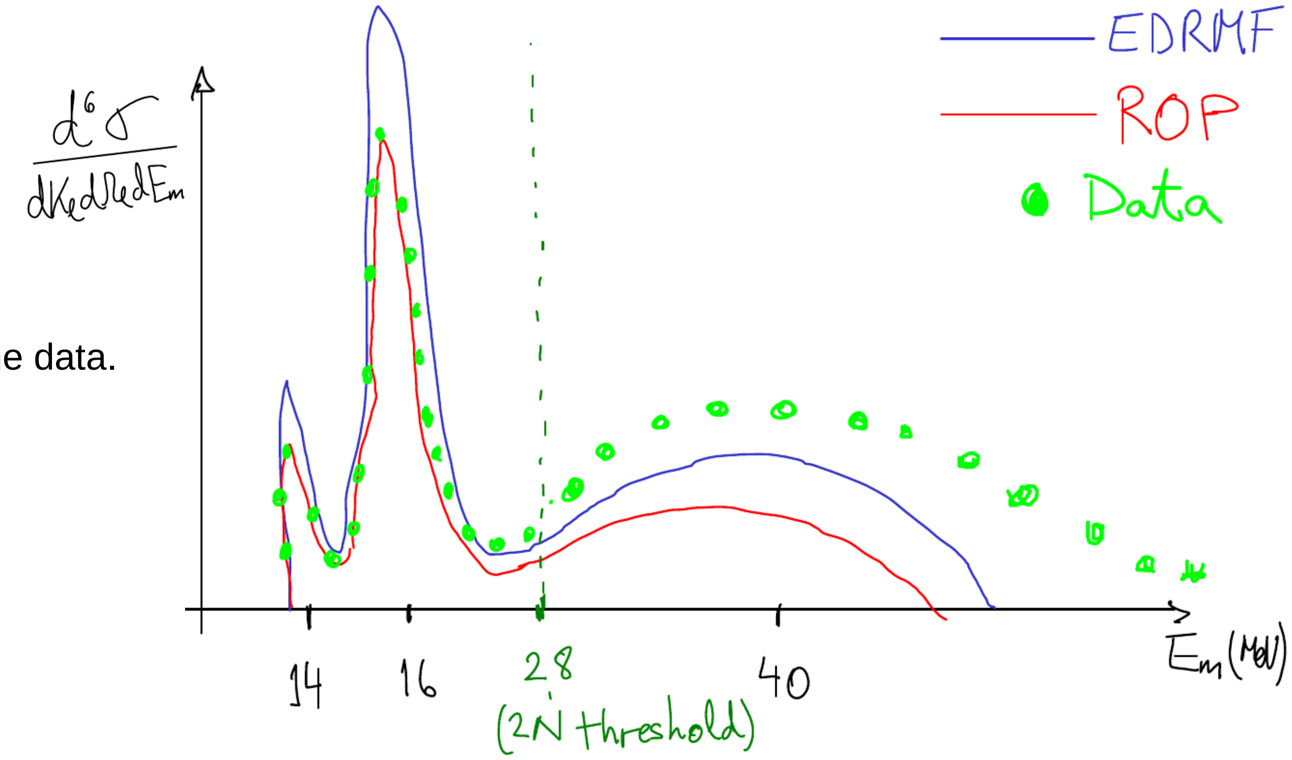

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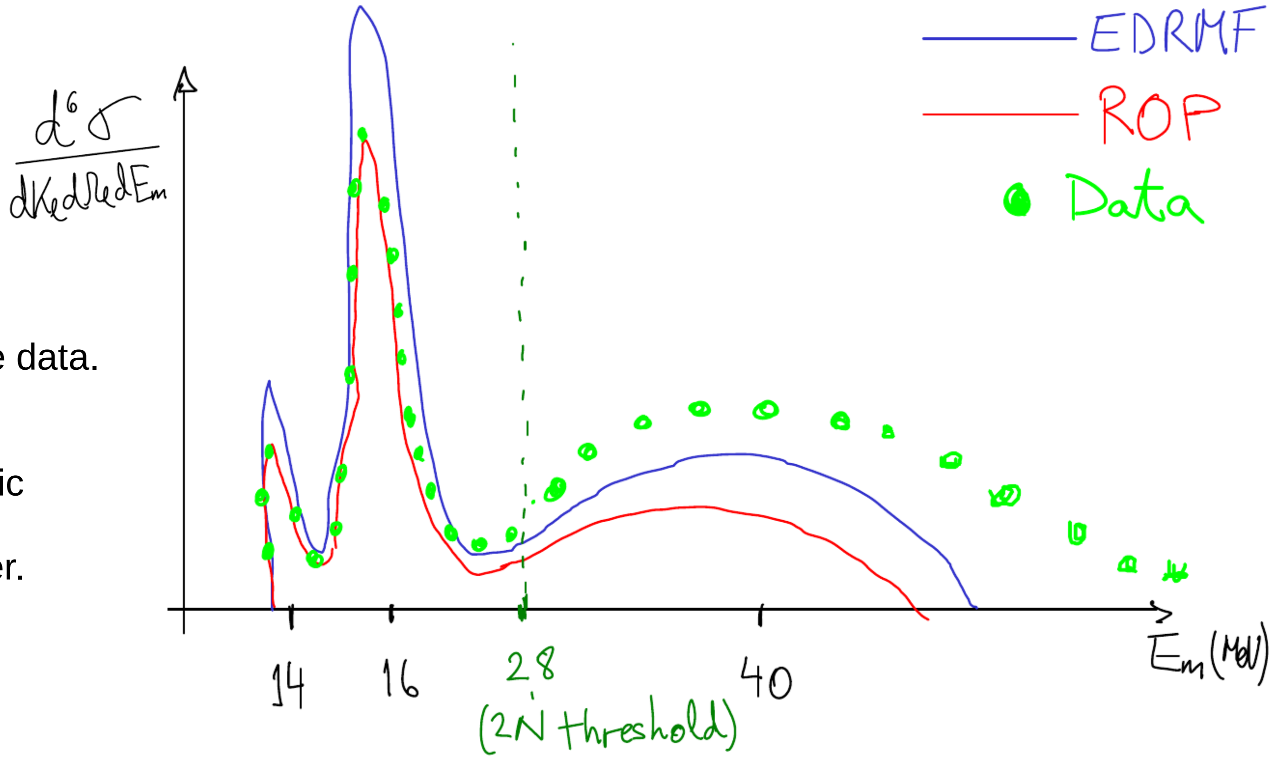

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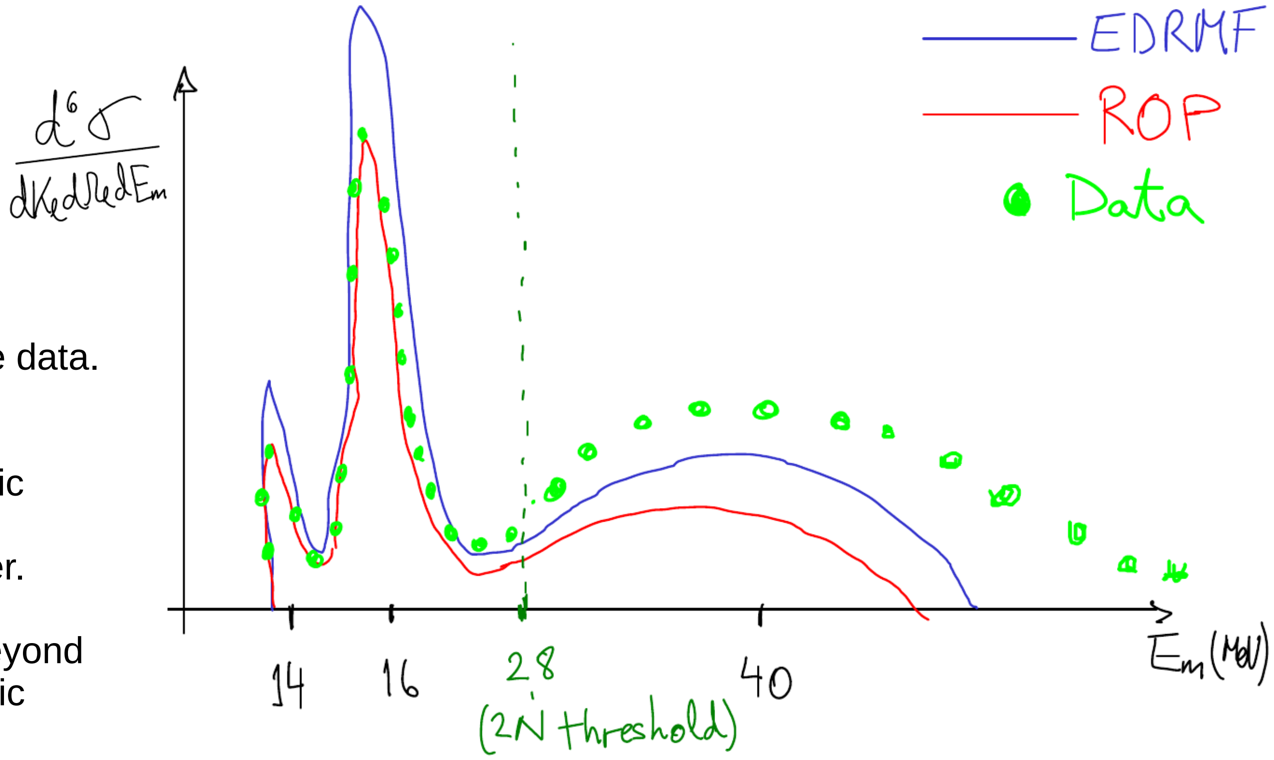

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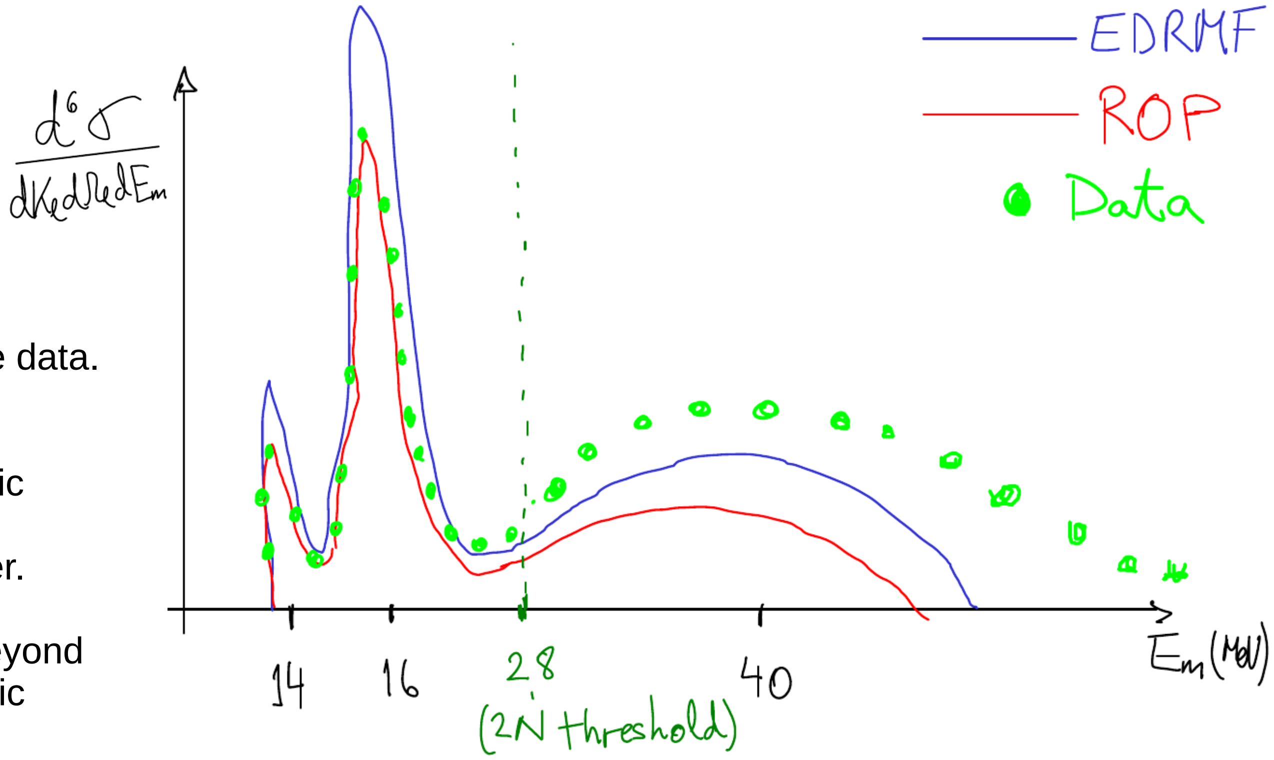

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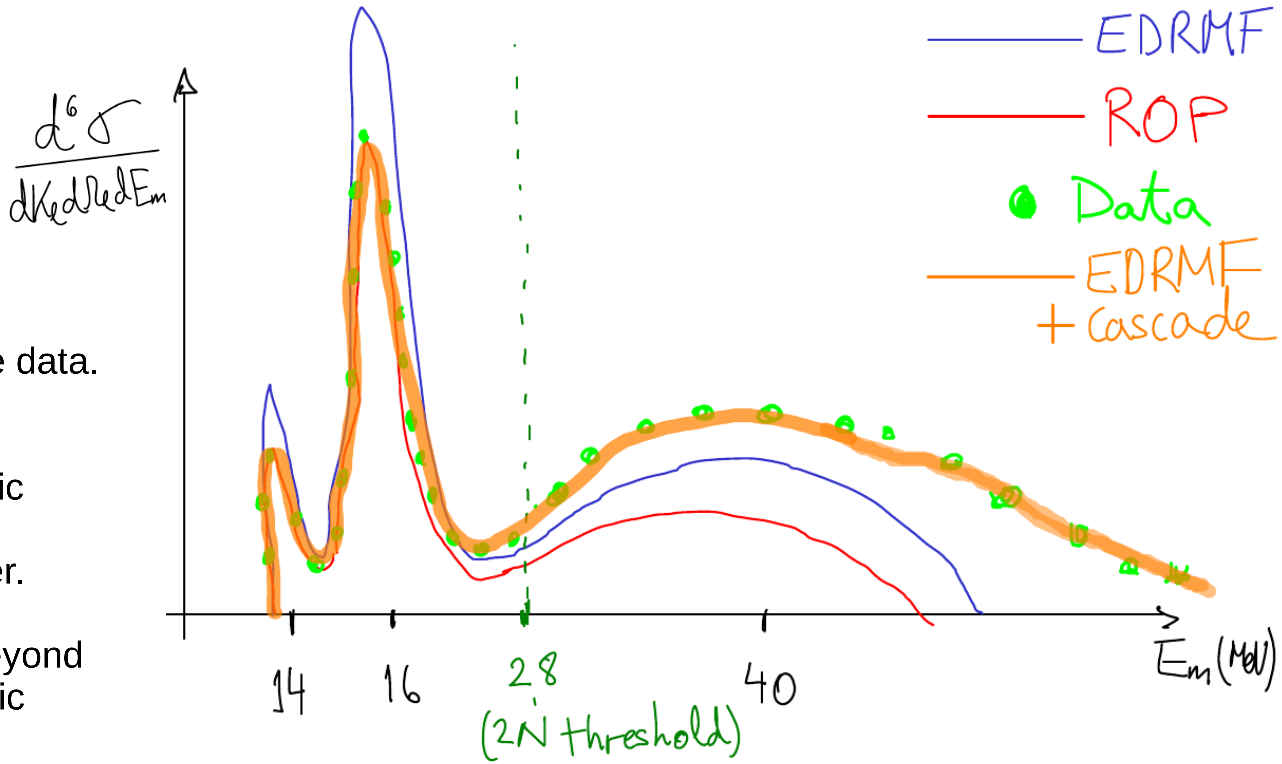

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5. EDRMF+cascade (hopefully) matches the data nicely.

Final remarks

1. Nuclear models must pass the **electron scattering test**. If they don't, take another one.
(It's up to you to set the 'goodness threshold'...)

2. Possibilities to improve the reliability of MC event generators' predictions:
+ Use of realistic models that provide good <u>inclusive results</u> and <u>information on the hadron(s)</u>.
+ Benchmarking the cascade model by systematically comparing the elastic ("only-1-proton-in-the-final-state") signal with the predictions from well-and-widely-tested models (like ROP).

3. Models must contain as much physics as possible, e.g.: spectroscopic factors, experimental binding energies, charge densities, two body currents, coulomb effects, etc.
(In my opinion, *tuning* is not bad if you know what you're doing.)

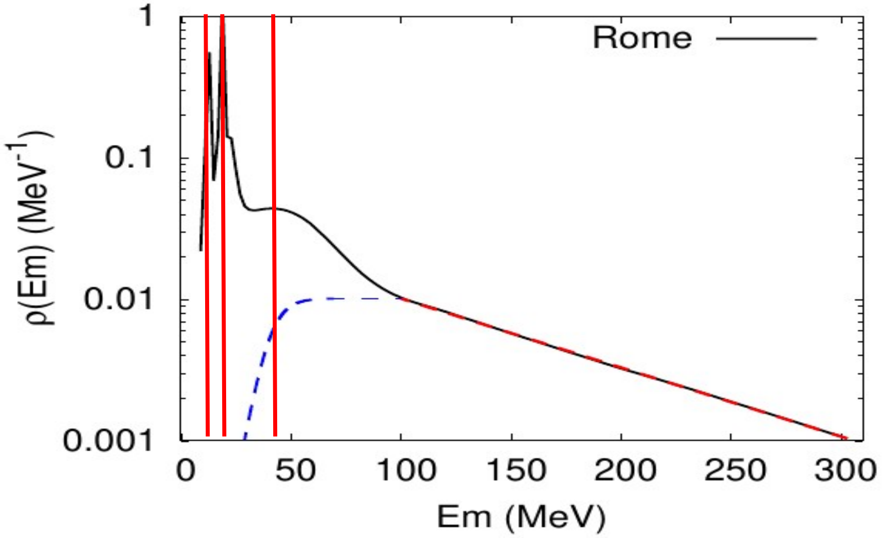
4. A quantum mechanical description seems to be essential to reproduce inclusive and exclusive cross sections. It ensures:
+ correct implementation of Pauli exclusion principle
+ distortion effects (or elastic FSI, or consistency, or whatever name we choose for it…)

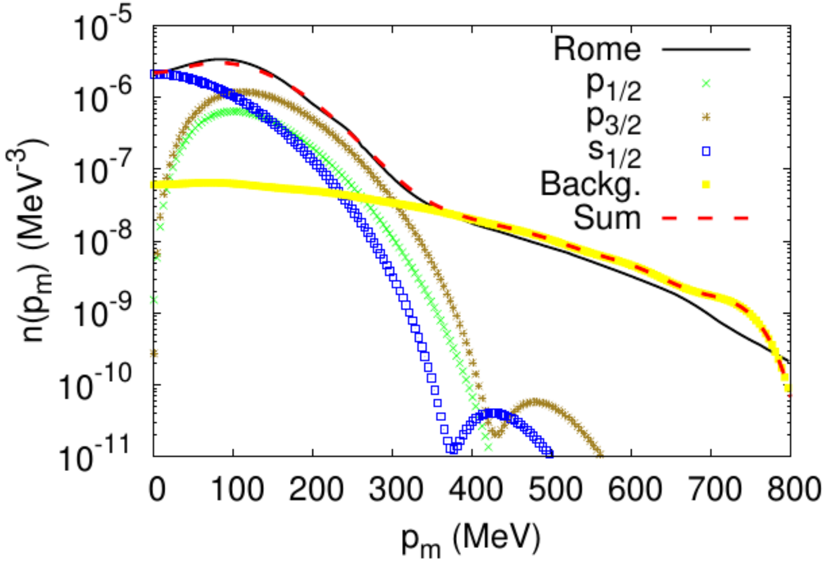
5. For the health of the community, the weaknesses of the models must be exposed (not hidden).

Grazie per la tua attenzione

Just in case material

Missing energy and momentum distributions from the Rome spectral function (O. Benhar et al. NPA 579, 493 (1994); PRD 72, 053005 (2005)) and the shell model used in this work:





E_m (MeV)	Shells	^{16}O
0 – 16.5	$p_{1/2}$	1.51
16.5 – 25	$p_{3/2}$	3.47
25 – 100	$s_{1/2}$ + backg.	2.22
	$s_{1/2}$	1.62
	backg.	0.60
100 – 300	backg.	0.80

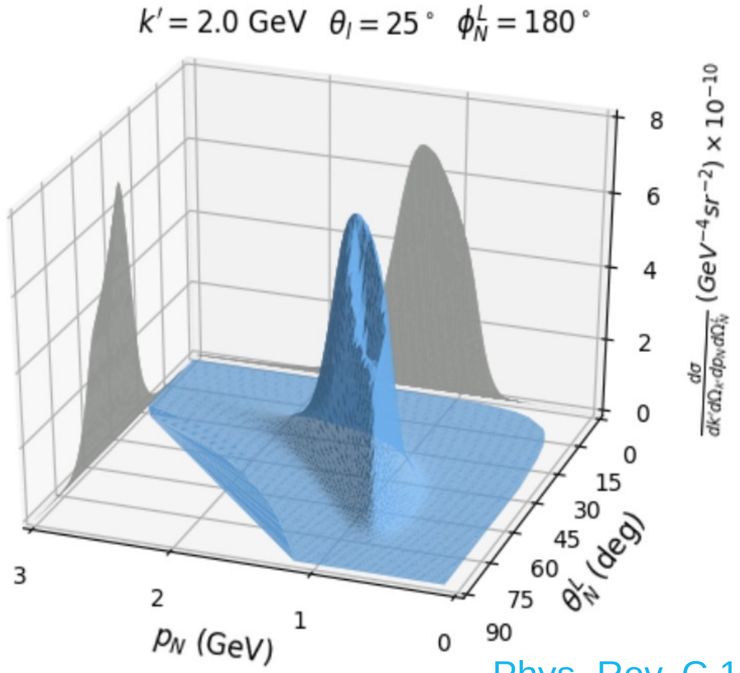
$$n(p_m) = \int dE_m S(E_m, p_m)$$

$$\int d^3\mathbf{p}_m n(p_m) = 8$$

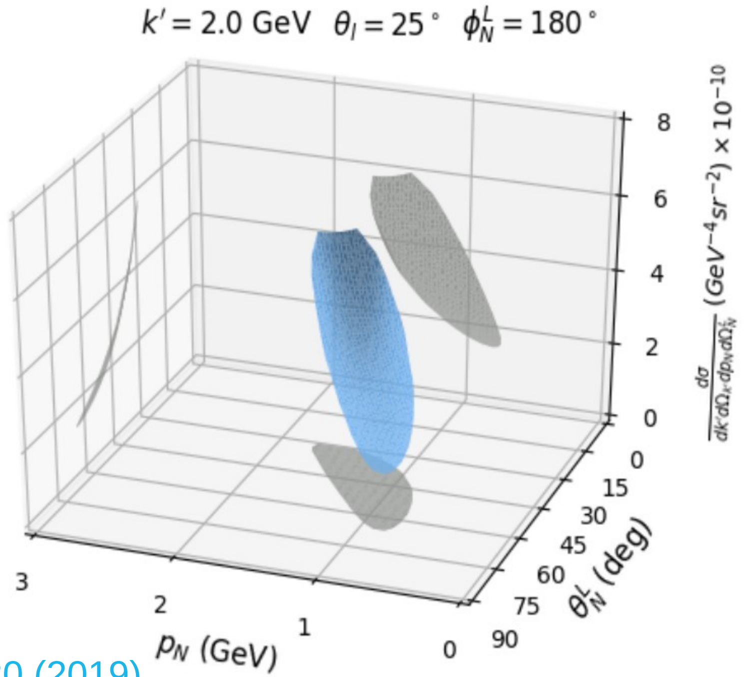
TABLE: Correspondence between missing energy regions and shells in oxygen. The last column are the occupation numbers.

Though for inclusive results both the SFA and RFG model may give similar results, the RFG model fails dramatically when applied to exclusive or semi-inclusive scenarios.

Spectral function approach



RFG model



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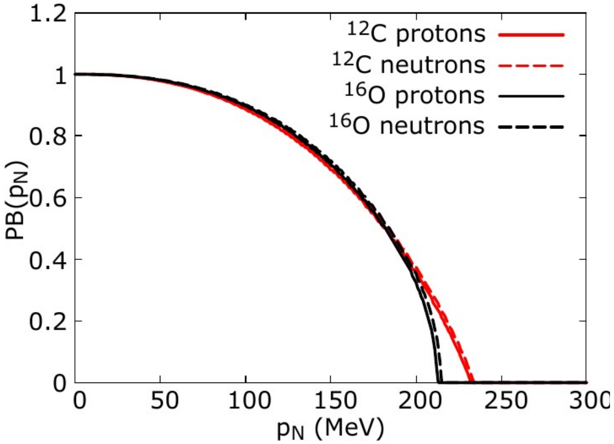
Pauli blocking

Inspired by what is done in a local Fermi gas (PRD 91, 033005 (2015)).

$$\left.\frac{d^6\sigma}{d\mathbf{k}_\mu d\mathbf{p}_N}\right|_{\text{Pauli blocked}} = \frac{d^6\sigma}{d\mathbf{k}_\mu d\mathbf{p}_N}\left(1 - \int d\mathbf{r}\ \rho_\mathcal{N}\ \theta[k_F(r) - p_N]\right)$$

$\rho_\mathcal{N}$ is the proton or neutron density

$$k_F(r) = \left(\frac{3\pi^2}{2}\mathcal{N}\rho_\mathcal{N}(r)\right)^{1/3}$$



Pauli blocking function
(it depends on mom. of final nucleon).