Lattice QCD outlook and prospects for neutrino physics

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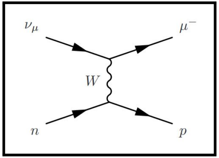
ECT* workshop “Measuring neutrino interactions for next-generation oscillation experiments”, 24th October, Trento

Overview of lattice input

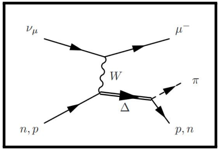
Neutrino-nucleon scattering

Elastic



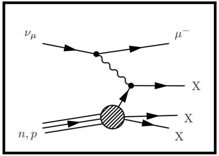
$0.1 < E_\nu < 10$ GeV

Resonance



$0.3 < E_\nu < 50$ GeV

Deep inelastic



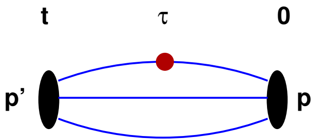
$E_\nu > 2$ GeV

- ⋆ **Elastic**: $\langle N|J|N\rangle$, $J = A_\mu$, axial form factors. Already results with all systematics under control for $0 < Q^2 < 1$ GeV^2.
- ⋆ **Resonance region above pion production threshold**: $N \xrightarrow{J} N\pi$. $\langle N|J|N\pi\rangle$, $J = V_\mu, A_\mu$, with enhancement due to resonances. Possible, first steps taken.
- ⋆ **Shallow and deep-inelastic**: $\langle N|JJ|N\rangle$, hadronic tensor, (quasi-, pseudo-) parton distribution functions (PDFs). Systematics still to be explored. $\langle N|J|N\rangle$, moments of PDFs, results with all systematics under control.

Neutrino-nucleus scattering

- ⋆ Structure of nuclei. So far investigations of $\langle A|J|A\rangle$ with $A = 2, 3$ for $Q^2 = 0$.

Lattice details

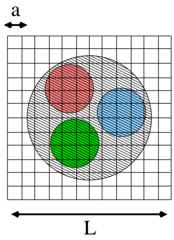


Extract matrix elements from correlation functions, e.g. for $\langle N|J|N\rangle$.

Challenge: signal to noise decays exponentially fast with the source-sink separation (t).

$$C_{3pt,\Gamma_i}^{\vec{p}',\vec{p},J}(t,\tau) = \sum_{n,m} Z_n Z_m^* e^{-E^n_{\vec{p}'}(t-\tau)} e^{-E^m_{\vec{p}}\tau} \langle \mathbf{n(\vec{p}')|J|m(\vec{p})} \rangle \overset{t,\tau\to\infty}{\longrightarrow} \sim e^{-E_{\vec{p}'}(t-\tau)} e^{-E_{\vec{p}}\tau} \langle \mathbf{N(\vec{p}')|J|N(\vec{p})} \rangle .$$

Challenge: ground state + excited state contributions (resonances, $N\pi$, $N\pi\pi$, ...).



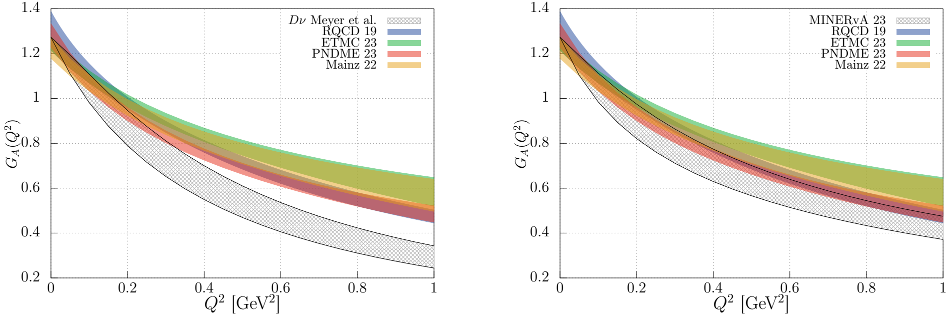
Repeat analysis on several ensembles to explore

- **Finite volume effects**: exponentially suppressed $\sim e^{-LM_\pi}$, $LM_\pi > 4$.
- **Cut-off effects**: $\mathcal{O}(a)$ or $\mathcal{O}(a^2)$, larger for larger $|\vec{p}|$, $|\vec{p}'|$.
- **Quark mass dependence**: chiral pert. theory (ChPT) $M_\pi \to M_\pi^{phys}$.

Cost of HMC $\propto 1/(a^{\gtrsim 6}\, m_\pi^{\approx 7.5})$

Elastic scattering: $\langle N(p')|A_\mu|N(p)\rangle = \bar{u}_N(p')\left[G_A(Q^2)\gamma_\mu - i\frac{\tilde{G}_P(Q^2)}{2m_N}Q_\mu\right]\gamma_5 u_N(p)$

Axial form factor $G_A(Q^2)$



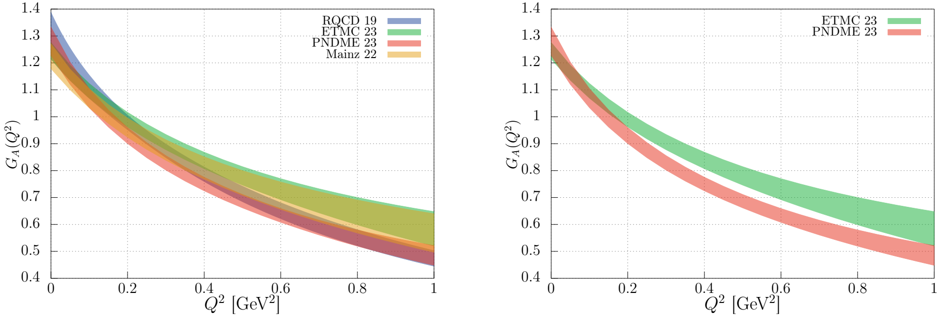
[RQCD,1911.13150], [Mainz,2207.03440], [PNDME,2305.11330], [ETMC,2309.05774].

All studies perform continuum, physical point $M_\pi \to M_\pi^{phys}$, finite volume extrapolations.

Left: νD fit from [Meyer et al.,1603.03048]. Right: $\bar{\nu}H$ fit from [MINERva,Nature 614, 48 (2023)].

Fits to expt., $Q^2 = 0$ fixed using $G_A(0) = g_A$. Not the case for the lattice results.

Axial form factor



Fit data for Q^2 up to ~ 1 GeV^2. Individual total uncertainties $\sim$ 4%–12% (0–1 GeV^2).

Overall spread at 1 GeV^2 $\sim$ 18%. Slight tension in the lattice results around 0.5 GeV^2 of $\sim 2\sigma$.

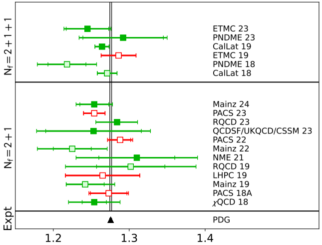
Next 2-5 years: more results expected including continuum limit and m_q, V extrapolations, see, e.g., [NME,2103.05599], [CalLat,2111.06333], [PACS,2311.10345].

FLAG average of form factor likely.

Improved precision and extend range to 2 GeV^2? Possible with available methods.

Lattice benchmarks

$Q^2 = 0$: $G_A(0) \equiv g_A$



$0 \leq Q^2 \leq 1$ **GeV^2**

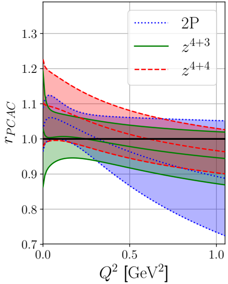
Axial Ward identity:

$$r_{\text{PCAC}} = \frac{m_q G_P(Q^2) + \frac{Q^2}{4m_N}\tilde{G}_P(Q^2)}{m_N G_A(Q^2)} = 1$$

Pseudoscalar form factor:

$$\langle p(p')|P|n(p)\rangle = \overline{u}_p i\gamma_5 G_P(Q^2) u_n.$$

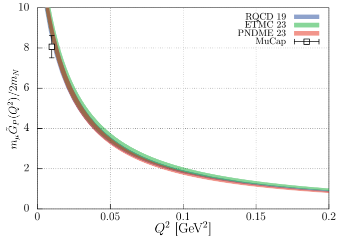
[RQCD,1911.13150]



Induced pseudoscalar form factor $\tilde{G}_P$ at the muon capture point $(\mu^- p \to \nu_\mu n)$ is reproduced:

[MuCap,1210.6545]

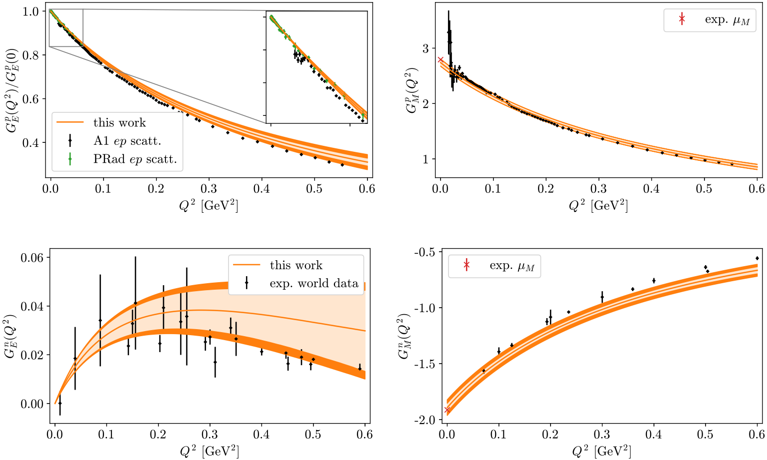
$$g_P^* = \frac{m_\mu}{2m_N}\tilde{G}_P(Q^2 = 0.88\, m_\mu^2) = 8.06(55)$$



Lattice benchmarks

Isospin limit ($m_u = m_d$): (weak) Pauli and Dirac vector form factors can be related to the electromagnetic form factors $G^{n,p}_{E,M}$ extracted from $e - p$ scattering data.

[Mainz,2309.06590,2309.07491] $N_f = 2 + 1$, $a = 0.09 - 0.05$ fm, $M_\pi = 130 - 290$ MeV, $LM_\pi \gtrsim 4$.



Above the pion production threshold: resonance region

Transition matrix elements: $N \xrightarrow{J} N\pi$, with enhancement from resonances $R = \Delta(1232), N^*, N(1440), \ldots$ for vector and axial currents.

Straightforward to determine $\langle N|J|R\rangle$ if choose unphysically heavy m_π such that the resonance is stable, i.e. $m_R < m_N + m_\pi$.

See e.g. [ETMC,0710.4621,0706.3011] ($N \xrightarrow{J} \Delta$), [Lin, Cohen,1108.2528] ($N \xrightarrow{J}$ Roper) and earlier $N_f = 0$ works.

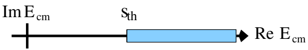
At m_π^{phys}, $m_R > m_N + m_\pi$ and the resonance is unstable.

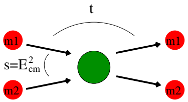
Euclidean space-time $\to$ transition matrix elements cannot be directly extracted from the correlation functions.

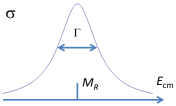
Properties of resonances from the lattice

Properties of resonances cannot be extracted directly on a (Euclidean) lattice.

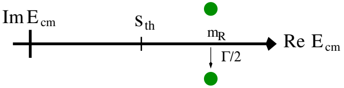
Infinite volume: continuous spectrum.



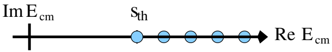


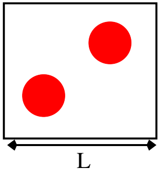


From discrete E_n constrain t_ℓ^{-1} then analytically continue to the complex plane.



Finite volume: discrete spectrum.



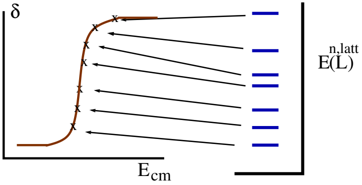


[Lüscher,Commun.Math.Phys. 105 (1986)] and others.

$$\det\left[\tilde{K}^{-1}_{\ell,J}(E_{cm})\delta_{\ell\ell'}\delta_{JJ'} - B^{\vec{P},\Lambda}_{\ell' J',\ell J}(E_{cm})\right] = 0,$$

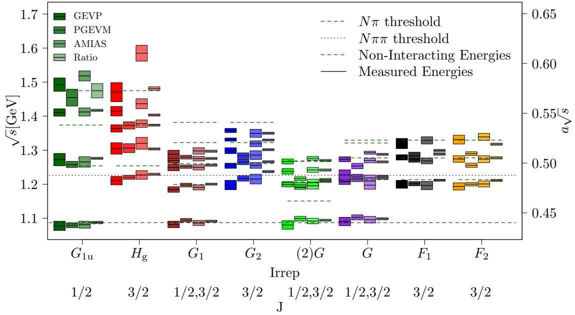
$$t_\ell^{-1} = \frac{2}{E_{cm}p^{2\ell}}\tilde{K}^{-1}_{\ell J} - i\rho, \quad \tilde{K}^{-1}_{\ell J} = p^{2\ell+1}\cot\delta_{\ell J}.$$

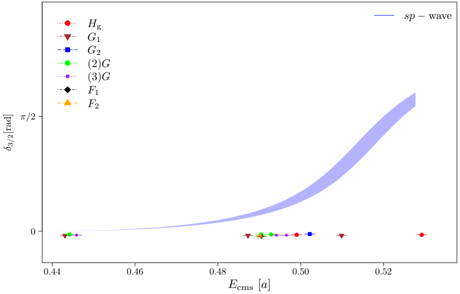
Elastic scattering:
$E_n \to \delta$
(one partial wave)



Elastic $N\pi \to N\pi$ scattering in the Δ channel

[ETMC,2307.12846] $I = \frac{3}{2}$, $N_f = 2+1+1$, $a = 0.08$ fm, $m_\pi \sim m_\pi^{phys}$,
$|\vec{P}/(2\pi/L)| = 0\ (G_{1u}, H_g), 1\ (G_{1,2}), \sqrt{2}\ (G), \sqrt{3}\ (F_{1,2})$.

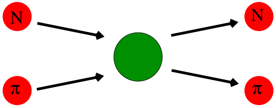




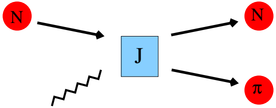
$m_R = 1269(45)$ MeV, $\Gamma = 144(181)$ MeV.

See also [Lang et al.,1610.01422], [Anderson et al.,1710.01557], [Bulava et al.,2208.03867] for $I = 1/2$ and $3/2$.

$N \to N\pi$ (resonance) transition form factors from the lattice

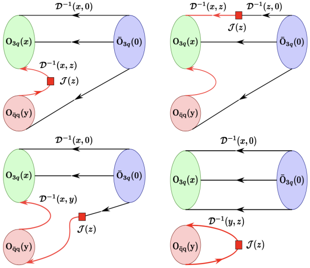


$$C^{2pt}_{3q,3q} = \langle O_{3q}(t,\vec{p})\overline{O}_{3q}(0,\vec{p})\rangle$$
$$C^{2pt}_{3q,5q} = \langle O_{3q}(t,\vec{p})\overline{O}_{5q}(0,\vec{p})\rangle$$
$$C^{2pt}_{5q,5q} = \langle O_{5q}(t,\vec{p})\overline{O}_{5q}(0,\vec{p})\rangle$$



$$C^{3pt}_{3q,3q} = \langle O_{3q}(t,\vec{p}\,')J(\tau,\vec{q})\overline{O}_{3q}(0,\vec{p})\rangle$$
$$C^{3pt}_{3q,5q} = \langle O_{3q}(t,\vec{p}\,')J(\tau,\vec{q})\overline{O}_{5q}(0,\vec{p})\rangle$$
$$C^{3pt}_{5q,5q} = \langle O_{5q}(t,\vec{p}\,')J(\tau,\vec{q})\overline{O}_{5q}(0,\vec{p})\rangle$$

Quark-line diagrams for $C^{3pt}_{3q,5q}$.



$C^{2pt} \to$ eigenvalues of GEVP $\to$ FV spectrum, E_n. Eigenvectors of GEVP + $C_{3pt} \to$ finite volume $\langle N|J|N\pi\rangle_L$.

$1 \to J \to 2$ formalism: $\langle N|J|N\pi\rangle_L \longrightarrow \langle N|J|N\pi\rangle_\infty \to$ form factors.

[Bernard et al.,1205.4642], [Agadjanov et al.,1405.3476], [Briceno, Hansen,1502.04314]

So far $C^{3pt}_{3q,3q}$, $C^{3pt}_{3q,5q}$ computed for $I = 1/2$ and used to remove $N \xrightarrow{J} N\pi$ contributions to $C^{3pt}_{3q,3q}$.
[Barca et al.,2211.12278] $N_f = 3$ $(m_\ell = m_s)$ $m_\pi = 420$ MeV, $a = 0.098$ fm
[ETMC,2312.15737] $N_f = 2+1+1$ $m_\pi = 346$ MeV, $a = 0.095$ fm, $N_f = 2$ $m_\pi = 131$ MeV, $a = 0.094$ fm.

Resonance transition form factors from the lattice

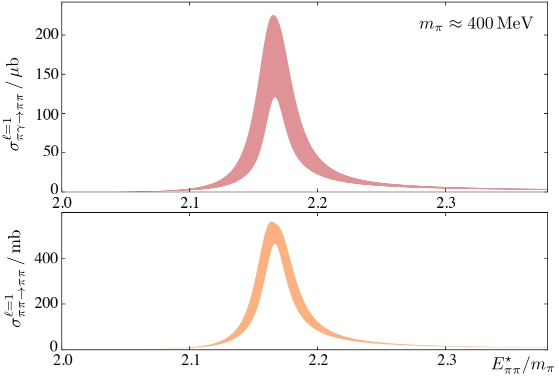
[HadSpec,1604.03530]

$N_f = 2+1$, $a_s = 0.12$ fm, $a_t = a_s/3.5$.

$\rho \xrightarrow{V_\mu} \pi$ form factor.

Shown: P-wave $\pi\gamma \to \pi\pi$ and $\pi\pi \to \pi\pi$ cross-sections.

$E^\star_{\pi\pi}$: $\pi\pi$ center of momentum energy.



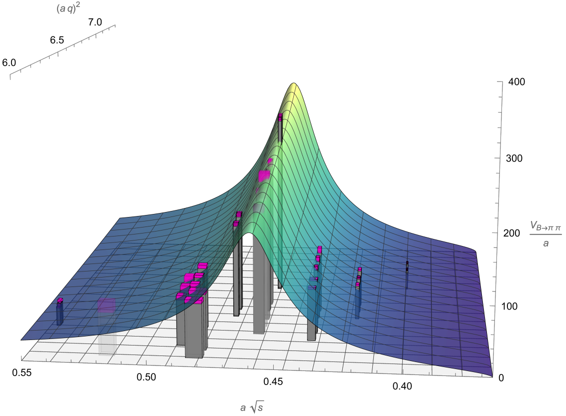
[Leskovec et al.,PoS (EuroPLEx 2023) 018]

$N_f = 2+1$, $a = 0.11$ fm, $m_\pi = 320$ MeV.

$B \to \rho\ell\nu$ form factor.

Shown: transition amp. $V_{B\to\pi\pi}(s, q^2) \propto \langle\pi\pi; P|V^\mu|B; P_B\rangle_\infty$.

$q^2 = (P_B - P)^2$, $s = (E^2_{\pi\pi} - \vec{P}^2) = E^{\star 2}_{\pi\pi}$.



$N \to$ resonance transition form factors from the lattice

Resonances from $N\pi$ scattering: $\Delta(1232)$, N^*, $N(1440)$.

- ⋆ Formalism for relating $N \to \xrightarrow{J} N\pi$ in a finite volume to infinite volume is available.
- ⋆ Requires information from $N\pi \to N\pi$ scattering. Several studies already for $I = 1/2$ and $3/2$.
- ⋆ Requires evaluation of $N \xrightarrow{J} N\pi$ correlation functions. First calculations performed.

Technically far more challenging than extracting $N \xrightarrow{J} N$ form factors and computationally more expensive (factor of order 10).

- ⋆ Larger basis of operators needed O_{3q}, O_{5q}.
- ⋆ Larger number of quark-line diagrams to calculate (each diagram more expensive to compute).
- ⋆ Worse statistical signal for correlation functions with O_{5q}.

Next 1-5 years, first determinations of the transition form factors on single ensembles, possibly at m_π^{phys}.

Transition form factor for $N \to \xrightarrow{J} N\pi\pi$, resonances, some years away.

Light nuclei

Challenges: large choice of interpolating operator,
number of quark-line diagrams grows factorially with number of quarks

→ various techniques have been developed, see, e.g., [NPLQCD,1908.07050], [Günther et al.,1301.4895], [Doi and Endres,1205.0585], [Detmold and Orginos,1207.1452].

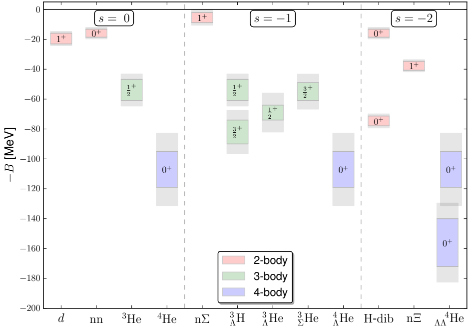
In addition: statistical errors grow exponentially with A. Dense spectrum of states. Large volumes are needed.

[NPLQCD,1206.5219v2] $A < 5$

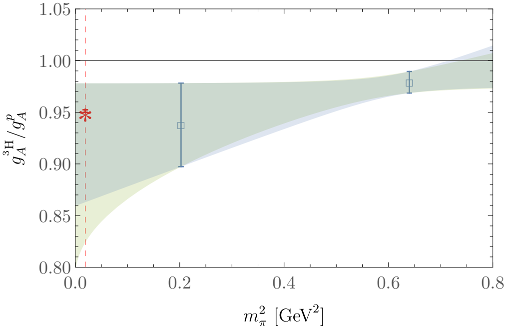
$N_f = 3$ $(m_u = m_d = m_s)$,

$m_\pi = 807$ MeV, $a = 0.145$ fm,

$L = 3.4, 4.5, 6.7$ fm.



Light nuclei



[NPLQCD,1610.04545,2102.03805]

Axial charge of tritium. Shown $g_A^{^3H}/g_A^p$.

$m_\pi = 450, 807$ MeV, $a = 0.12, 0.145$ fm

Also: $A \leq 3$

[NPLQCD,1709.00395] gluon momentum fraction and transversity.

[NPLQCD,2009.05522] isovector quark momentum fraction.

Lattice predictions of spectra and matrix elements of light nuclei can be used to

constrain the parameters of low-energy effective field theories,

test and constrain phenomenological models of nuclei based on nucleon degrees of freedom.

Next five years, move to lighter m_π and away from the forward limit.

Summary and outlook

Lattice can provide ab-initio, systematically improveable results relevant for neutrino-nucleon and neutrino-nucleus scattering.

- ⋆ **Axial form factor** $G_A(Q^2)$ for $0 < Q^2 < 1$ GeV^2.

  Results with all systematics under control are available.

  Next 5 years, more results. Improved precision and extending the range of Q^2 possible.

  Also results for isoscalar (neutral) currents, $G_A^s(Q^2)$, see, e.g., [ETMC,2106.13468].

- ⋆ $N \xrightarrow{J} N\pi$ **transition form factors** $J = V_\mu, A_\mu$**, resonances** $\Delta(1232), N^*, N(1440)$**.**

  Possible, first steps taken.

  First results in the next 1-5 years, likely not all systematics will be explored.

- ⋆ **Hadronic tensor**: can provide information on (elastic), shallow and deep-inelastic region.

  Compute $\langle N|JJ|N\rangle$, see, e.g., [Liu and Dong,hep-ph/9306299].

  Nucleon momentum does not need to be large.

  Exploratory: first calculations have been perfomed, see, e.g., [χQCD,1906.05312].

  Laplace transform connects Euclidean and Minkowski hadronic tensor $\rightarrow$ inverse problem.

Summary and outlook

⋆ **Quasi-, pseudo-PDFs**: deep-inelastic region.

Compute $\langle N|JJ|N\rangle$.

Isovector unpolarised, helicity and transversity PDFs, strange quark and gluon PDFs have been computed.

Some results at the physical point after continuum extrapolation.

Rapid progress will continue. Challenge to achieve large enough momentum (small lattice spacings, signal to noise problems, dense spectrum of states), ...

First and second moments of PDFs: results with all systematics under control.
Over ext five years calculations of higher moments can be expected, see, e.g., [Shindler,2311.18704].

Flavour separation is straightfoward on the lattice. Present and future: lattice results used as additional input to phenomenological global fits of experimental data.

⋆ **Spectra and matrix elements of light nuclei** $A = 2, 3, 4$ have been determined. Next five years, move to lighter m_π and away from the forward limit.