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Meson Productions in Lepton- Deuteron scattering

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*ECT - Trento

Oct. 24, 2024

Why is Deuterium Important?

- **Essential Substitute for Hydrogen:** While hydrogen is ideal for neutrino cross-section models, deuterium, with its simple two-nucleon structure, is the best alternative.
- There are no differential cross-section measurements for neutrino and anti-neutrino interactions on hydrogen at energies relevant to modern neutrino experiments.

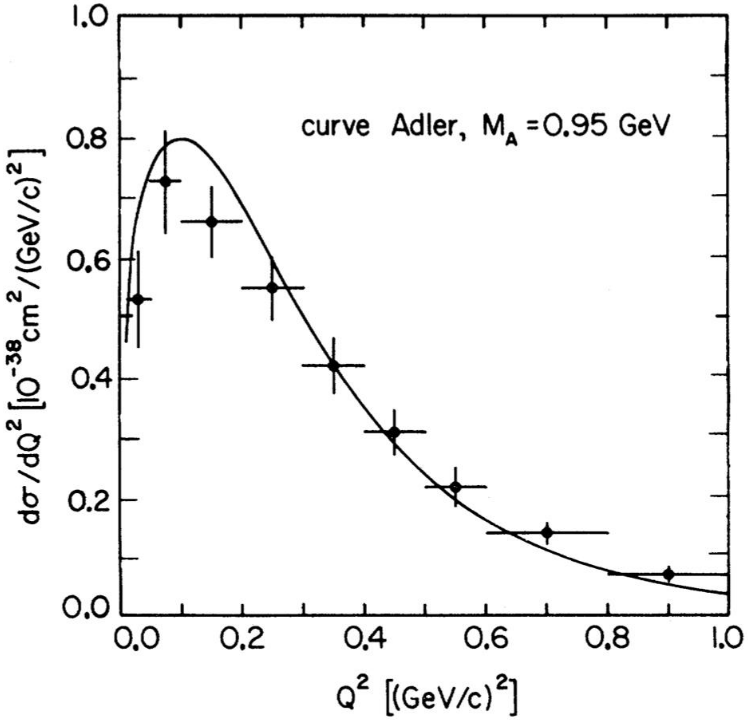
Neutrino data: ANL & BNL

- In the ANL experiment, the bubble chamber was first filled with hydrogen, then switched to deuterium for the rest of the experiment. Event rates combine data from both hydrogen (30%) and deuterium fills.
- In the BNL experiment, results are separated for hydrogen and deuterium, but no differential cross-section measurement for single pion production on a hydrogen target exists.

Available cross-section data: ANL

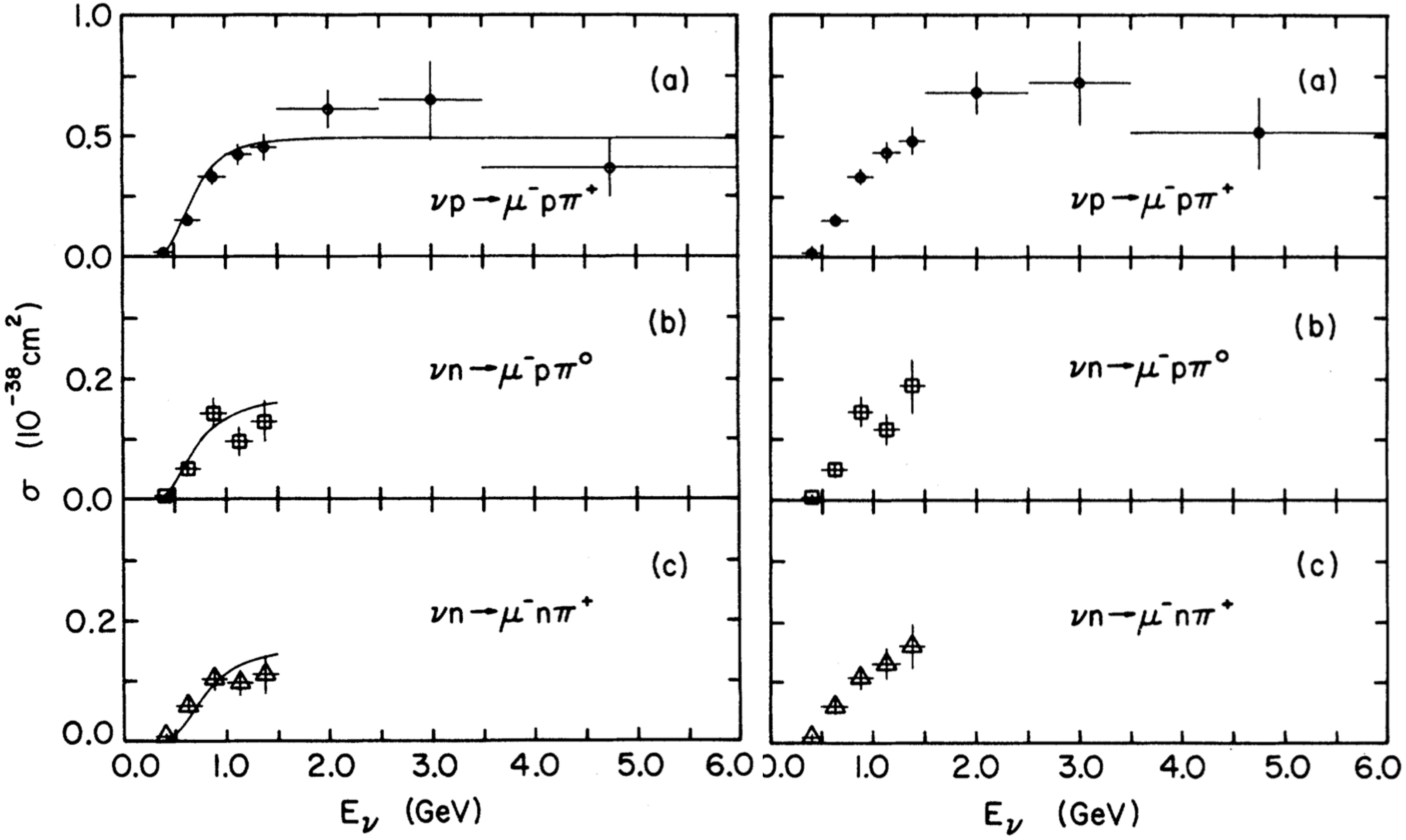
- Deuterium target

W<1.4 GeV



W<1.4 GeV

W<1.6 GeV



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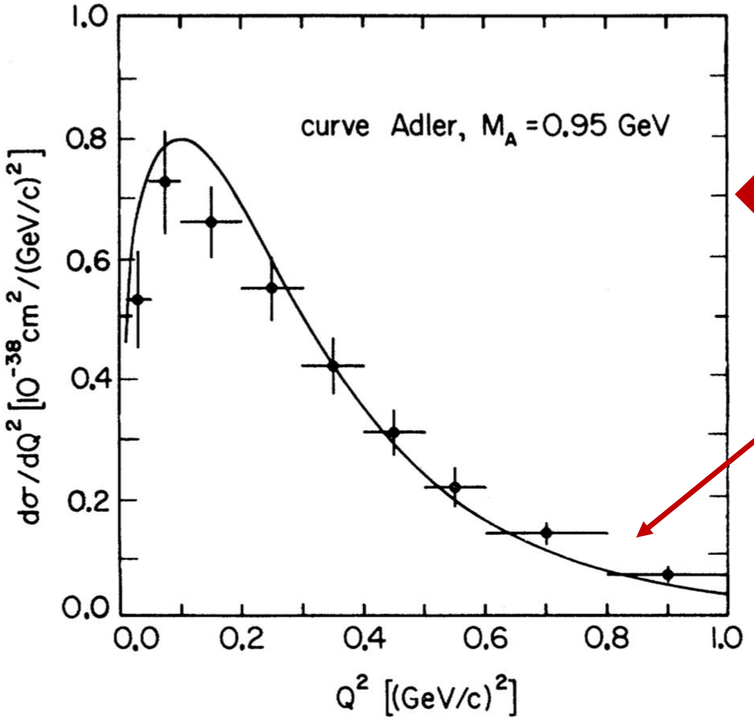

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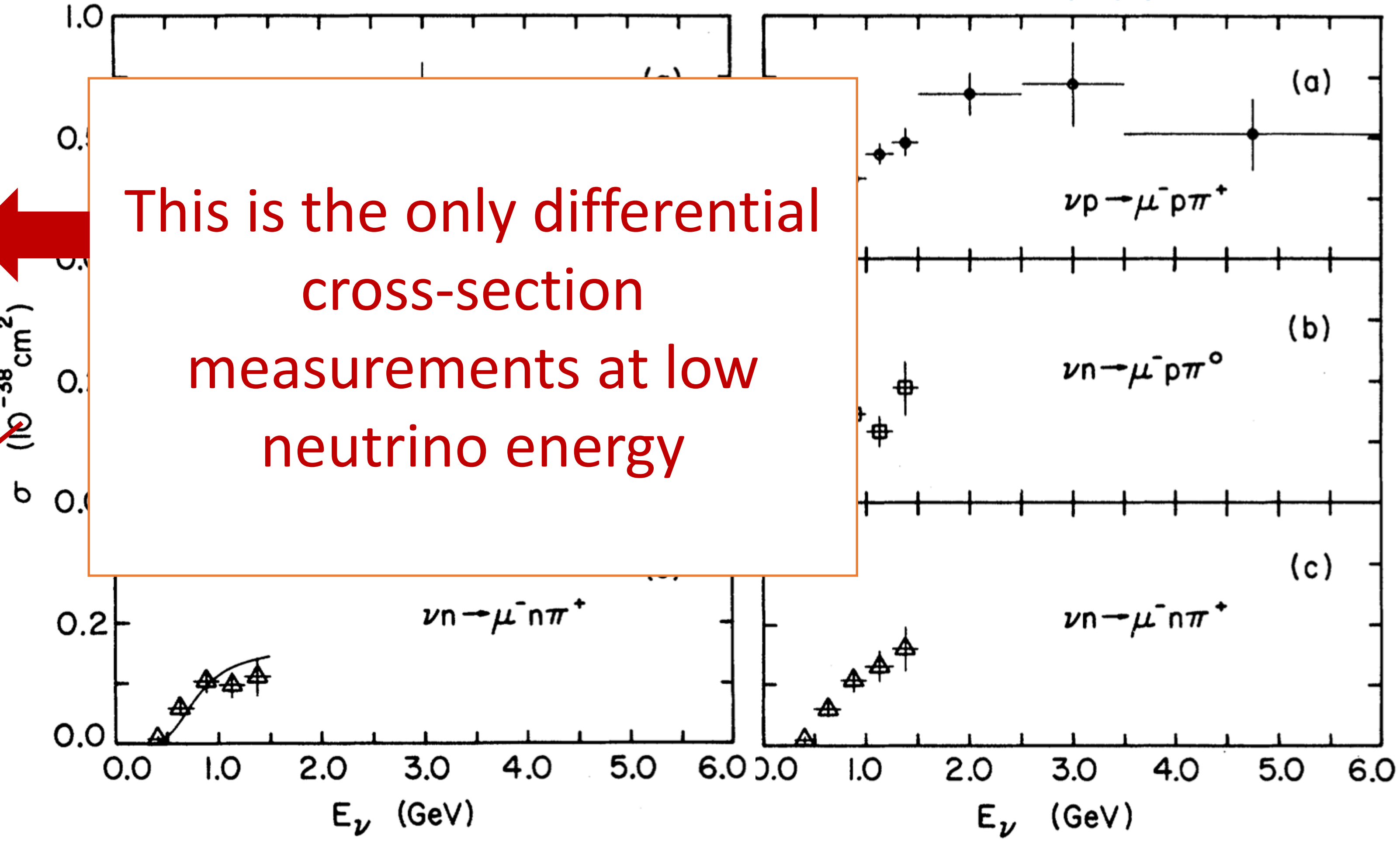

This is the only differential cross-section measurements at low neutrino energy

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Why is Deuterium Important?

- **Essential Substitute for Hydrogen**
- **Broader Measurement Opportunities:** Unlike hydrogen, which has only protons, deuterium targets enable interactions on both protons and neutrons, allowing for the study of all reaction channels.

ν beam CC-channels	$\bar{\nu}$ beam CC-channels	Contributed resonances
$\nu\, p \;\rightarrow l^- p + \pi^+$	$\bar{\nu}\, n \;\rightarrow l^+ n + \pi^-$	Iso-spin 3/2
$\nu\, n \;\rightarrow l^- p + \pi^0$	$\bar{\nu}\, p \;\rightarrow l^+ n + \pi^0$	Iso-spin 3/2 & 1/2
$\nu\, n \;\rightarrow l^- n + \pi^+$	$\bar{\nu}\, p \;\rightarrow l^+ p + \pi^-$	Iso-spin 3/2 & 1/2

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$\nu\, n \rightarrow l^{-} n + \pi^{+}$	$\bar{\nu}\, p \rightarrow l^{+} p + \pi^{-}$	Iso-spin 3/2 & 1/2

Resonance	M_R	Γ_0	χ_E
$P_{33}(1232)$	1232	117	1
$P_{11}(1440)$	1430	350	0.65
$D_{13}(1520)$	1515	115	0.60
$S_{11}(1535)$	1535	150	0.45
$P_{33}(1600)$	1600	320	0.18
$S_{31}(1620)$	1630	140	0.25
$S_{11}(1650)$	1655	140	0.70
$D_{15}(1675)$	1675	150	0.40
$F_{15}(1680)$	1685	130	0.67
$D_{13}(1700)$	1700	150	0.12
$D_{33}(1700)$	1700	300	0.15
$P_{11}(1710)$	1710	100	0.12
$P_{13}(1720)$	1720	250	0.11
$F_{35}(1905)$	1880	330	0.12
$P_{31}(1910)$	1890	280	0.22
$P_{33}(1920)$	1920	260	0.12
$F_{37}(1950)$	1930	285	0.40

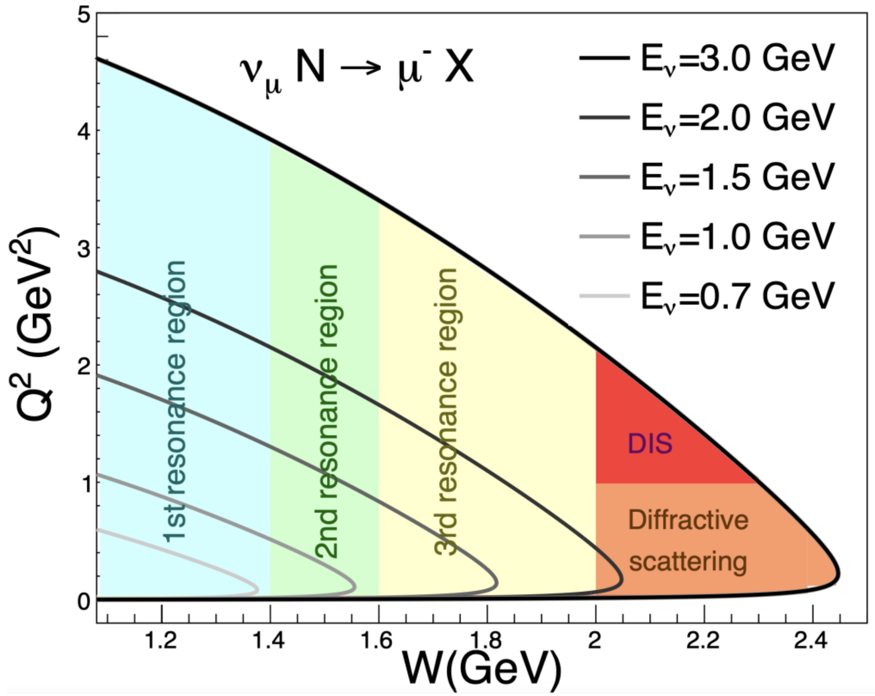
Why is Deuterium Important?

- **Essential Substitute for Hydrogen**
- **Broader Measurement Opportunities**
- **Extensive Cross-Section Data with Electron and Photon Beams:** A vast collection of exclusive cross-section measurements on deuterium with electron or photon beams provides a crucial resource for studying nuclear medium effects.

Why Do I Care?

- I have been developing a single-pion production model for neutrino experiments for over a decade without having enough neutrino data.

- Unlike (quasi-) elastic scattering, there are two independent scalar variables in inelastic scattering.



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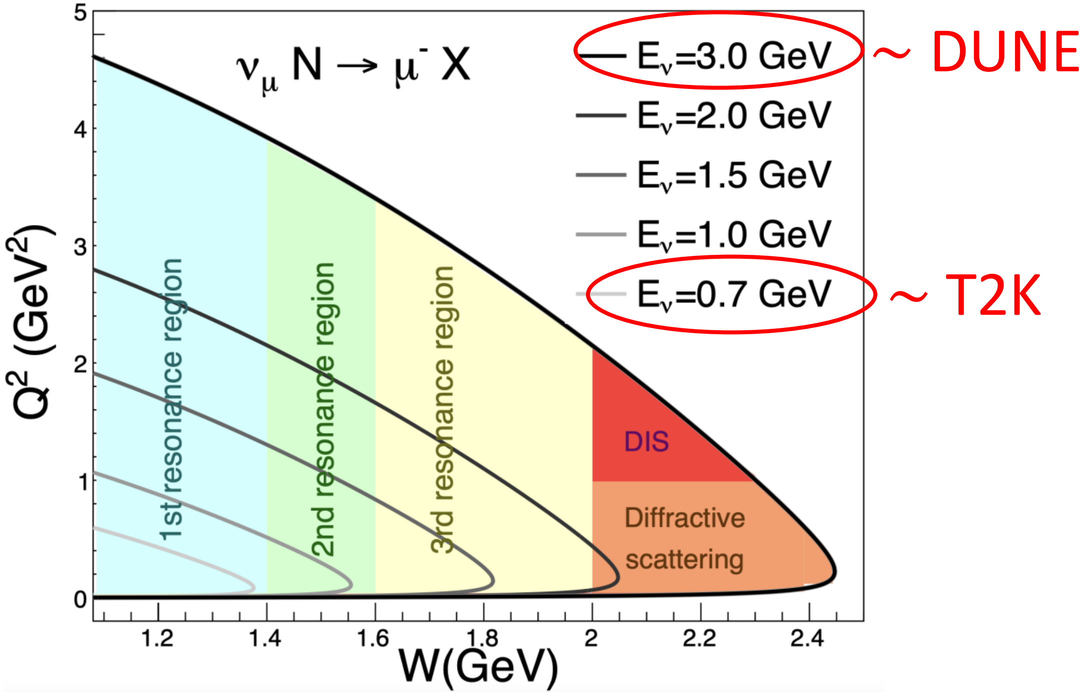
Why Do I Care?

- **Comprehensive Model**: To accurately describe both low- and high-energy neutrino experiments, the model must incorporate multiple processes, including 18 resonances and non-resonant interactions.

- **Optimising with Limited Data**: Strategies to maximise insights and make reliable predictions when data is scarce.



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Meson production (W evolution)

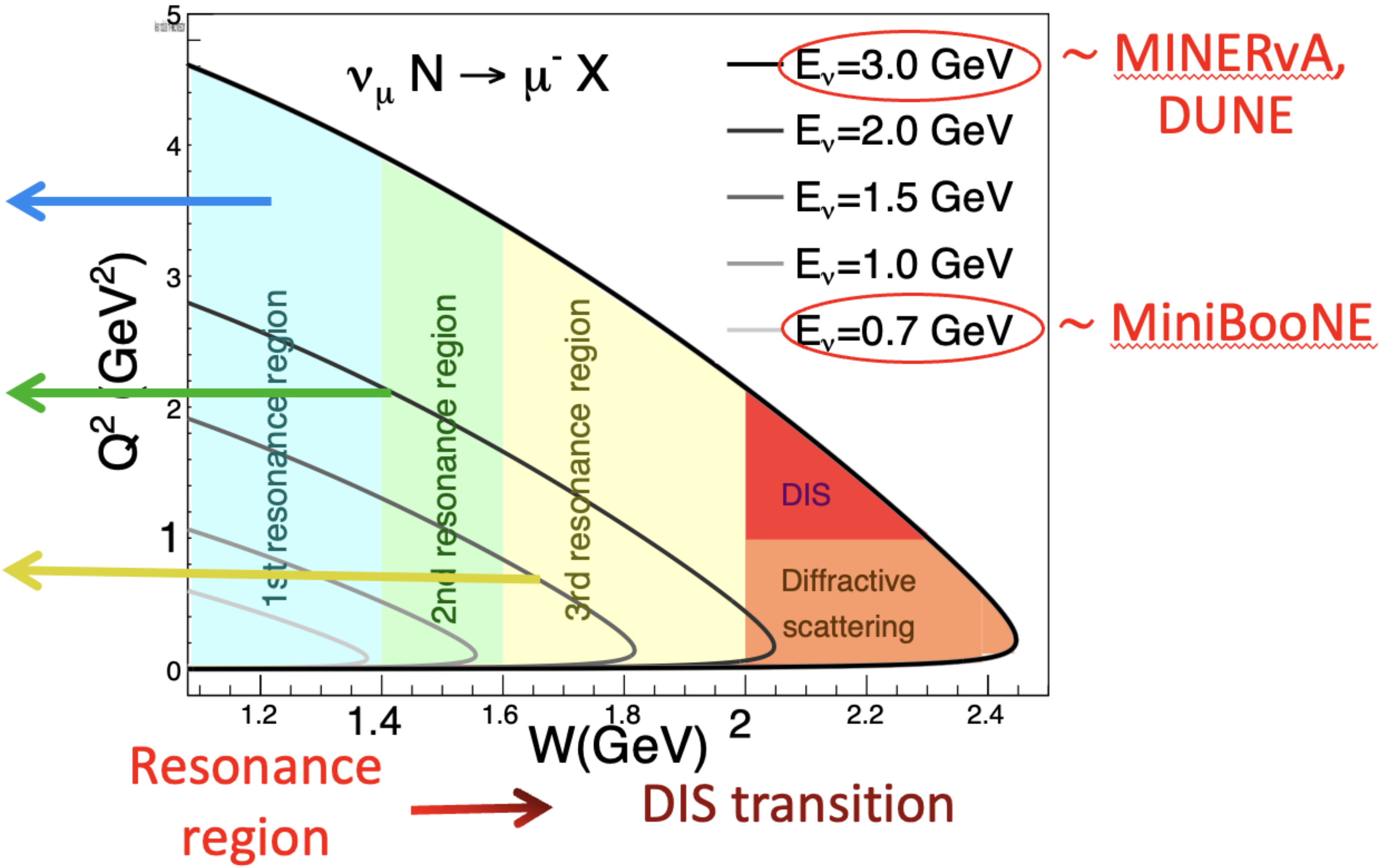
Resonant interaction

- $\Delta(1232)$ resonance
- 1-π production

- $P_{11}(1440), D_{13}(1520)$ and $S_{11}(1535)$

- 2-π, η, etc. production

- About 13 resonances overlap



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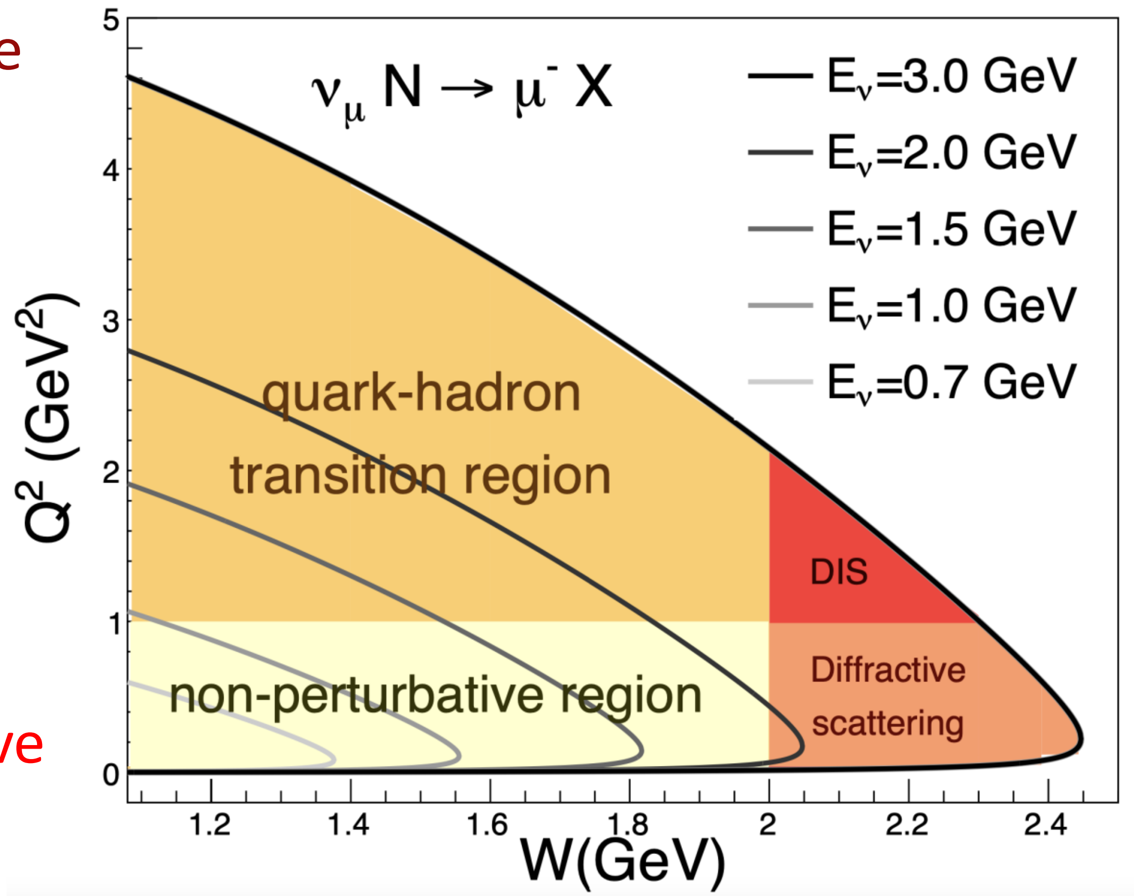
Meson production (Q^2 evolution)

Unlike quasi-elastic interactions, the onset of the quark-hadron transition occurs at relatively low momentum transfers, starting as early as Q^2 =1.0 GeV^2.



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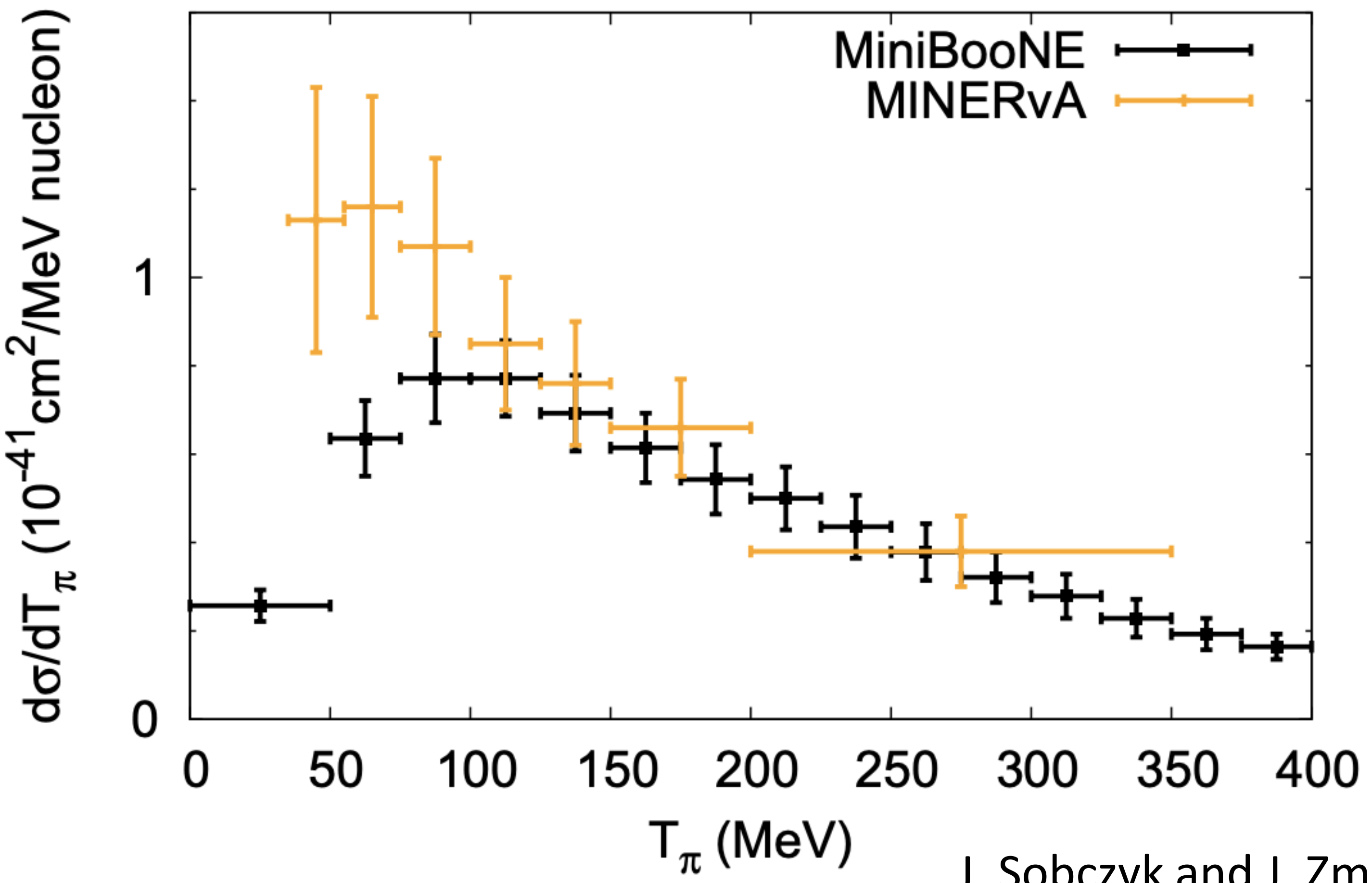
Tensions between **MiniBooNE** and **MINERvA**

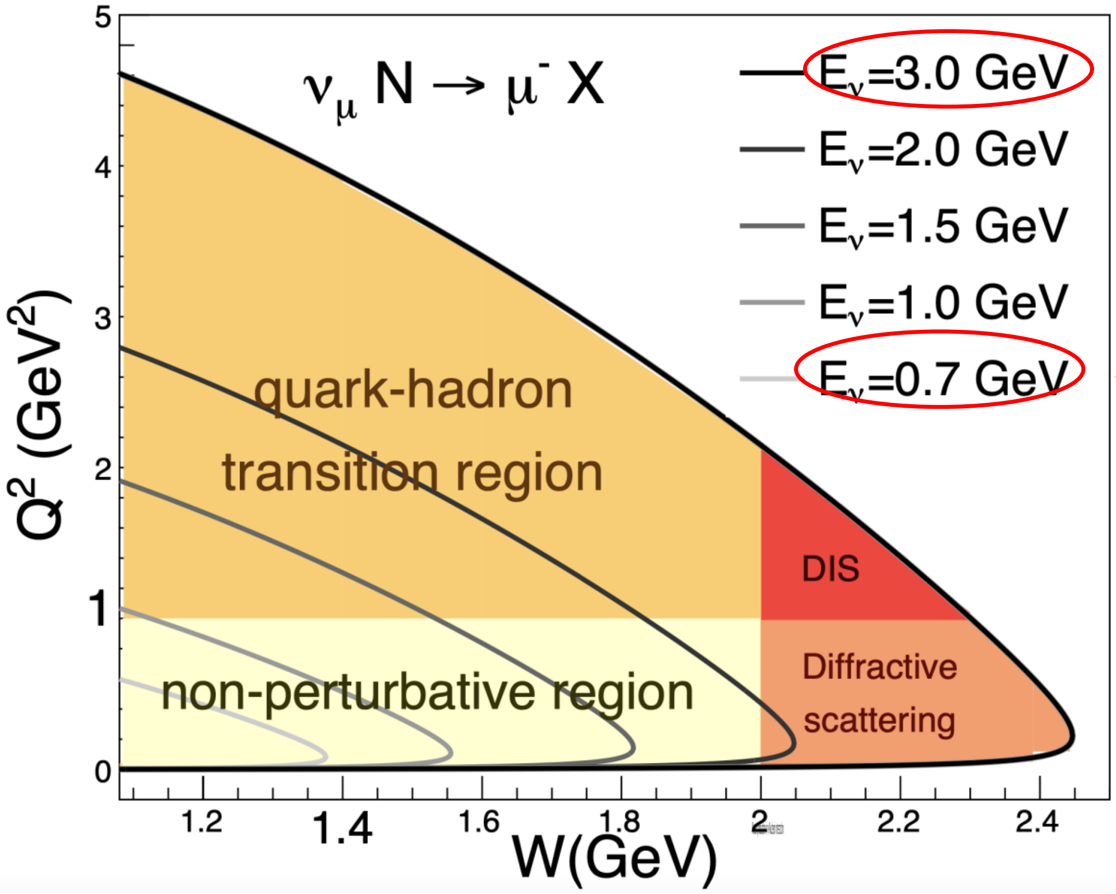
- Tensions between MiniBooNE and MINERvA for single pion production measurements on CH_2 and CH targets in the **first resonance region**.



J. Sobczyk and J. Zmuda
Phys. Rev. C **91** (2015)

Meson production in ν-Nucleon interaction

- Degree of freedom at E < 1 GeV (MiniBooNE) is **hadrons**
- Degree of freedom at E > 1 GeV (MINERvA) is a mixture of **hadrons** and **partons**



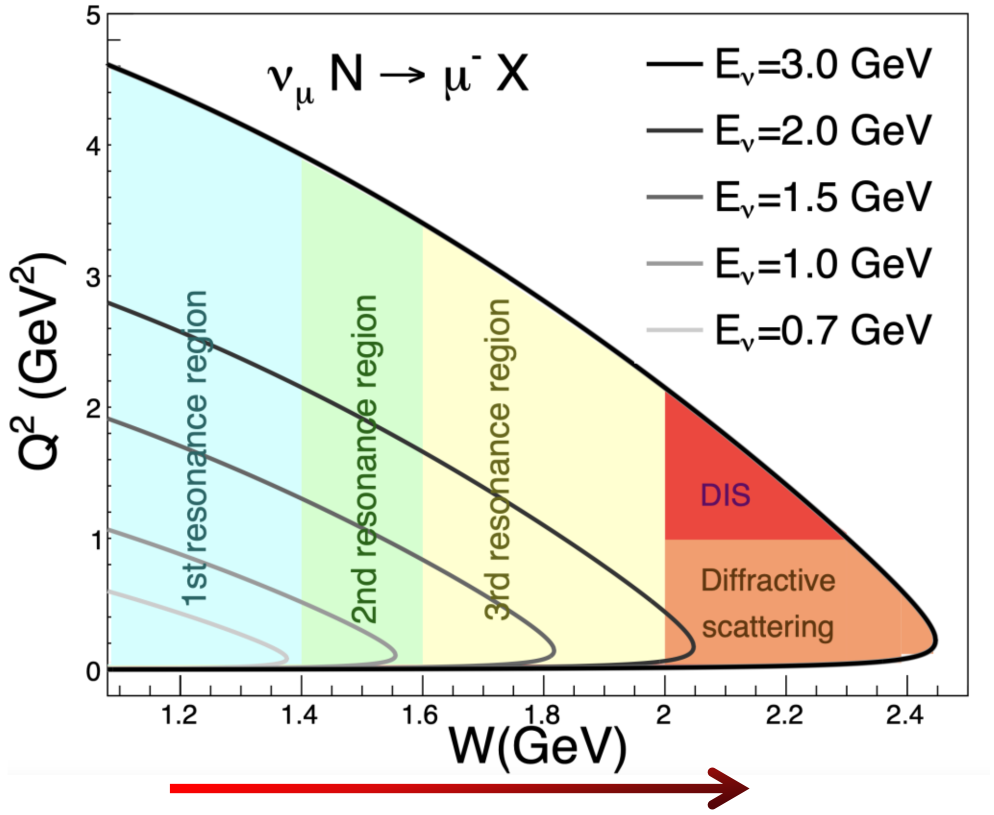
~ MINERvA, DUNE

~ MiniBooNE

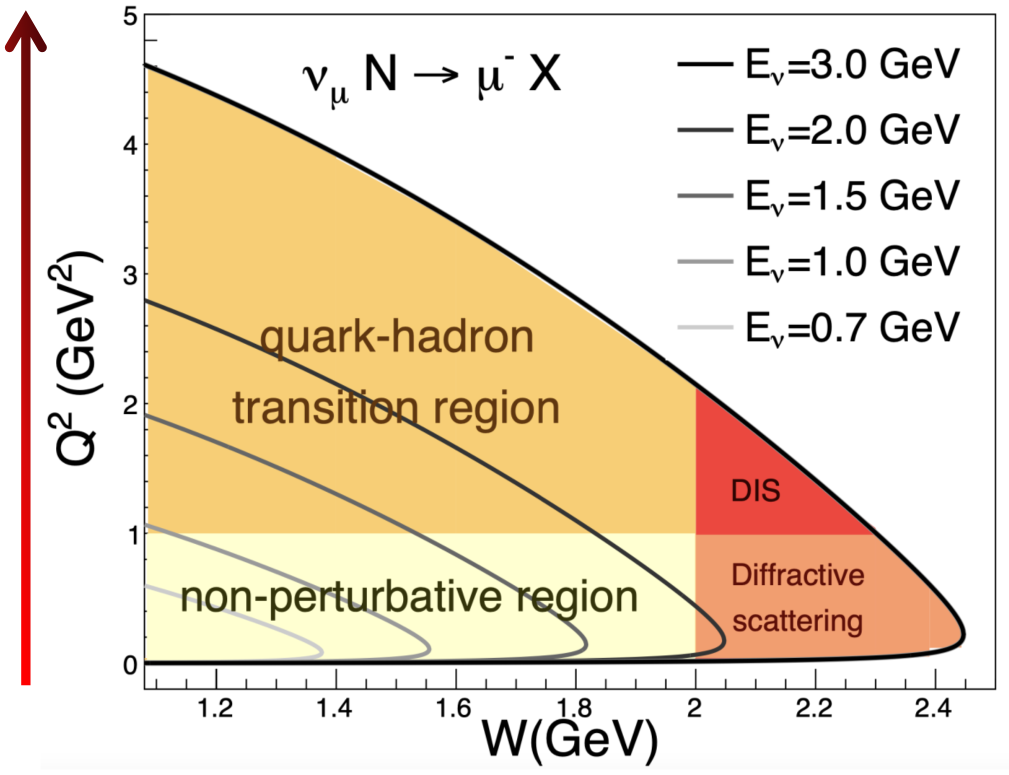
perturbative region

Transition region

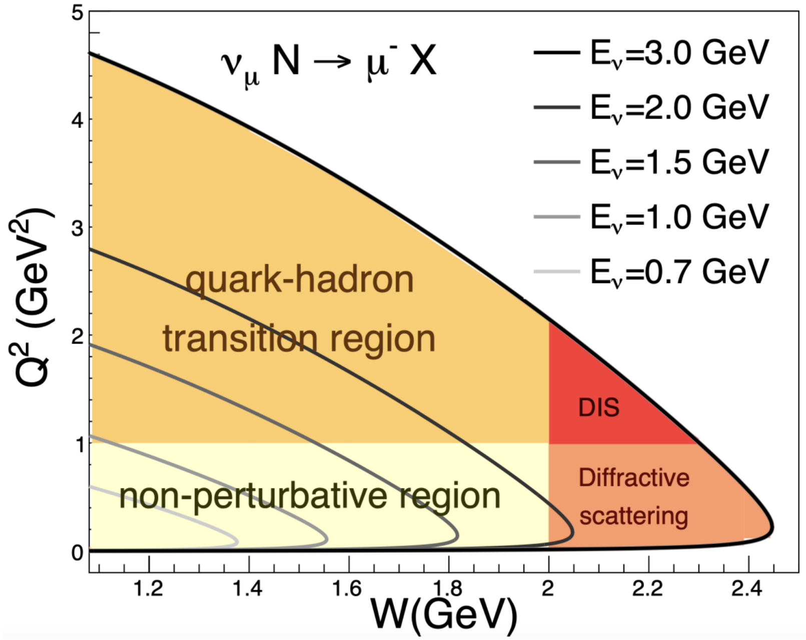
W evolution



Q^2 evolution



Shallow Inelastic Scattering (SIS)



- Refers to the nonresonant meson production and non-perturbative **multi-quark** meson production.
- SIS is not a well-defined region. It refers to two different regions:
  1. Nonresonant meson production region.
  2. Transition region ($Q^2 > 1$ GeV); interactions occur through multi-quark processes until Q^2 increases sufficiently to enter the meson production regime via single quark perturbative QCD DIS scattering.

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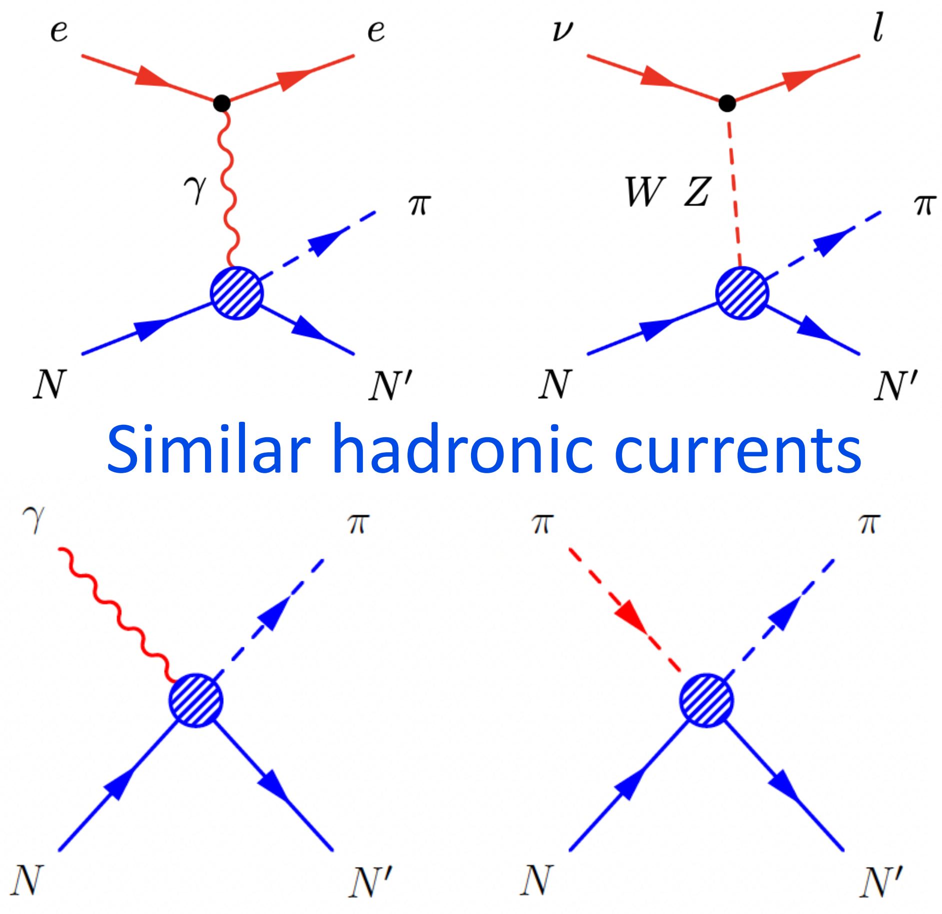
Phys. Rev. D **97** (2018)
Phys. Rev. D **102** (2020)
Phys.Rev.C **107** (2023)

MK model

- The MK model comprehensively describes single-pion production in interactions involving **photons, electrons, and neutrinos** with nucleons.
- Phenomenological models in this region must account for numerous processes and parameters.
- A unified model is essential for interpreting all interactions and **maximising data utilisation**.



Data used in the Joint analysis

# data point	Photon, electron, pion, Neutrino Channels	Q^2 Range $(GeV/C)^2$	W Range GeV	Form Factors	
≈ 9800	$\gamma\, p \rightarrow n + \pi^+ ,\ \ \gamma p \rightarrow p + \pi^0$	0	1.08 – 2.0	Proton	Vector
≈ 31000	$ep \rightarrow en + \pi^+,\ \ ep \rightarrow ep + \pi^0$	0.16 – 6.0	1.08 – 2.0		
≈ 2500	$\gamma n \rightarrow p + \pi^-$	0	1.08 – 2.0	Neutron	
≈ 700	NEW $en \rightarrow ep + \pi^-$	0.4 – 1.0	1.08 – 1.8		
≈ 400	$\pi^+ p \rightarrow p + \pi^+,\ \ \pi^- p \rightarrow p + \pi^-$	0	1.08 – 2.0	Axial-Vector	
<100	$\nu N \rightarrow l^- N + \pi\ ,\bar{\nu} N \rightarrow l^+ N + \pi$	$Q^2>0$ Integrated	1.08 – 2.0 Integrated		

Benefits from the Joint Analysis

- **Model Parameterisation**: Parametrise the model to approach perturbative QCD at large Q^2 while respecting CVC and PCAC constraints at low Q^2.
- **Uncertainty Evaluation**: Crucially assess systematic uncertainties, especially where theoretical insights are limited.
- **Interference Effects**: Improve predictions by better understanding interference between resonances and non-resonant interactions.
- **Rigorous Control Over NC Channels**: Achieves consistent descriptions of both electromagnetic and CC weak interactions, ensuring precise control over the NC vector current.

Motivation for the joint analysis

- **Complexity of Form Factors**: Despite CVC and PCAC constraints, numerous form factors still need to be defined. Some resonances (spin 3/2, isospin 1/2) require 3 (proton) + 3 (neutron) + 3 (axial) form factors.
- **Large Parameter Space**: Many unknowns lead to a broad range of parameters across the energy spectrum.
- **Reducing Axial Current Parameters**: Use insights from vector form factors to minimise unknowns in axial current, particularly at large and low Q^2.

Existing form-factors determination

- Some of neutrino models, utilised the helicity amplitudes determined in the MAID analysis to extract form factors.

$$D_V = \left(1 + \frac{Q^2}{M_V^2}\right)^2,$$

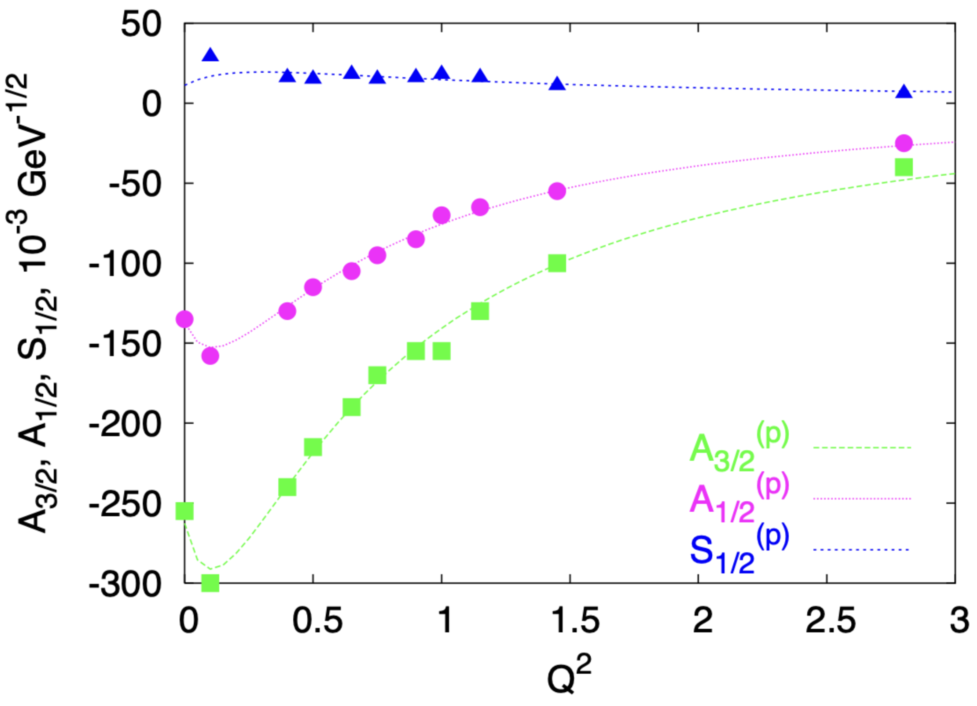
$$M_V = 0.84\ GeV$$

$$C_3^{(p)} = \frac{2.13/D_V}{1 + Q^2/4M_V^2},$$

$$C_4^{(p)} = \frac{-1.51/D_V}{1 + Q^2/4M_V^2},$$

$$C_5^{(p)} = \frac{0.48/D_V}{1 + Q^2/0.776M_V^2}.$$

- Using pre-defined form-factors I couldn't predict high-Q^2 region.



Fitted model to MAID analysis for $P_{33}(1232)$ resonance.
From Lalakulich et. al. (2006)

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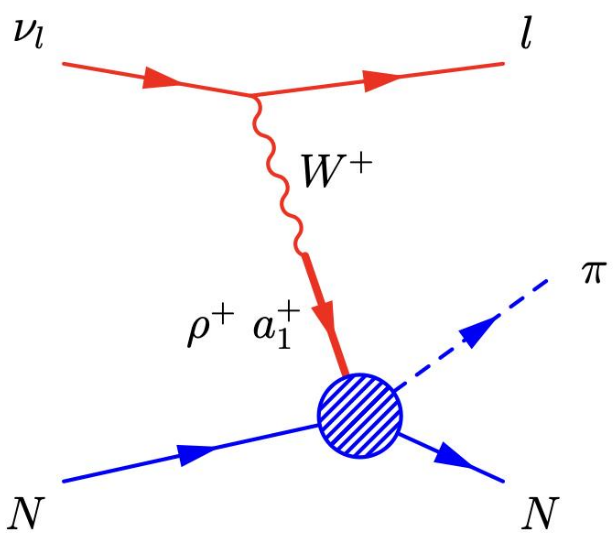
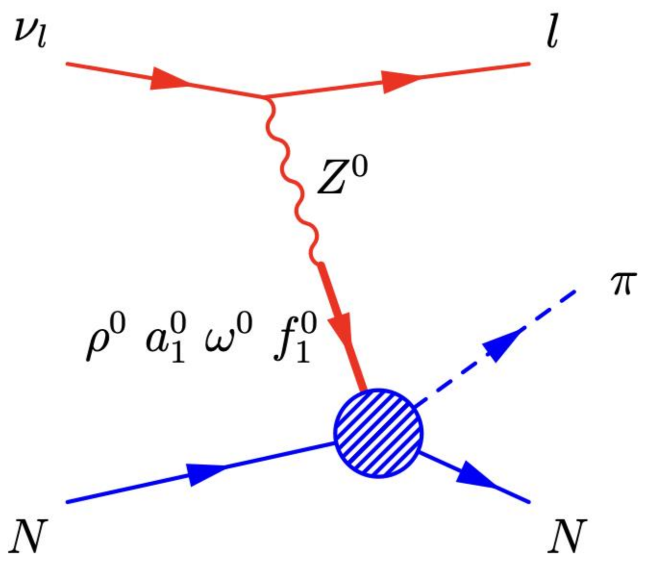
Meson Dominance (MD) model

- The MD model is rooted in the effective Lagrangian of quantum field theory.

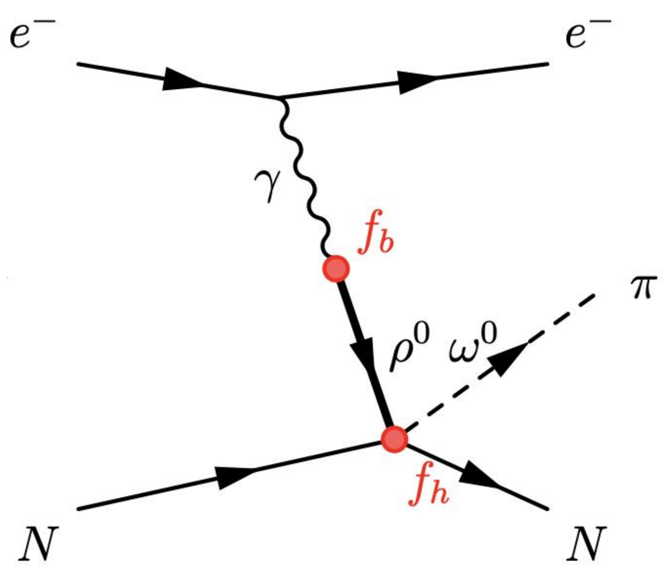
1. J. J. Sakurai, Annals Phys.11, 1 (1960)
2. M. Gell-Mann and F. Zachariasen, Phys. Rev. 124, 953 (1961)

- It establishes connections between vector and axial currents and corresponding meson fields with analogous quantum properties.
- This framework explains the interaction between neutrinos and nucleons through meson exchange.





Meson Dominance (MD) model



- MD form factors can be expressed in terms of the meson masses and the coupling strengths, summing over all possible mesons:

$$\mathrm{F_N}(Q^2) = \sum_{j=1}^{n} \frac{m_j^2}{m_j^2 - Q^2} \left(\frac{f_h}{f_b}\right)$$

- Although they do not inherently comply to the unitarity condition (analytic model) or accurately predict behaviour at high Q^2, they can be **imposed**!

C. Adamuscin *et al*. Eur. Phys. J. C 28, 115 (2003)

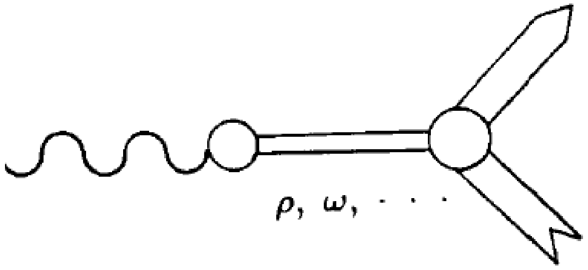
k	ρ -group	$m_{(\rho)k}$[GeV]	ω -group	$m_{(\omega)k}$[GeV]
1	$\rho(770)$	0.77526	$\omega(782)$	0.78265
2	$\rho(1450)$	1.465	$\omega(1420)$	1.410
3	$\rho(1700)$	1.720	$\omega(1650)$	1.670
4	$\rho(1900)$	1.885	$\omega(1960)$	1.960
5	$\rho(2150)$	2.150	$\omega(2205)$	2.205
k	a_1 -group	$m_{(a_1)k}$[GeV]	f_1 -group	$m_{(f_1)k}$[GeV]
1	$a_1(1260)$	1.230	$f_1(1285)$	1.2819
2	$a_1(1420)$	1.411	$f_1(1420)$	1.4263
3	$a_1(1640)$	1.655	$f_1(1510)$	1.518
4	$a_1(2095)$	2.096	$f_1(1970)$	1.1971

Meson Dominance (MD) model

Representation of electron scattering:

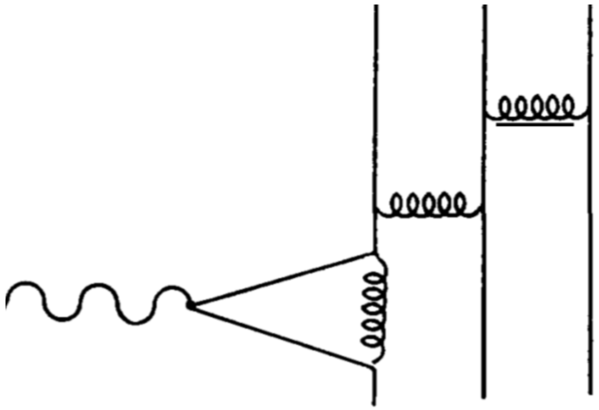
P. Stoler (1991): Phys. Rev. D **44**, 73

Non-perturbative (low Q^2)



- mesons propagate between the virtual gauge boson and the nucleon

Perturbative (high Q^2)



- schematic quark model of MD model

Asymptotic behaviour of form factor

- At large Q2, resonance form factors must align with the perturbative QCD constraints.
- For spin 3/2 resonance:

G. Vereshkov and N. Volchanskiy
(PRD 2007)

$$F_\alpha(Q^2) \cong \left(\frac{4M_N^2}{Q^2}\right)^{p_\alpha} \frac{f_\alpha}{\ln^{n_\alpha}\left({Q^2}/{\Lambda_{QCD}^2}\right)}, \qquad (\alpha = 1-3)$$

$$p_1 = 3, p_2 = p_3 = 4,$$
$$n_3 > n_1 > n_2, \qquad n_1 \cong 3$$

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MD form factors used in the model

- For spin 3/2 resonance:

$$F_\alpha(Q^2) = \frac{f_\alpha}{L_\alpha(Q^2)} \sum_{k=1}^{K} \frac{a_{\alpha k}\, m_k^2}{m_k^2 + Q^2}, \qquad (\alpha = 1 - 3)$$

$$L_\alpha(Q^2) = 1 + g_\alpha \ln^{n_\alpha}\left(1 + \frac{Q^2}{\Lambda_{QCD}^2}\right)$$

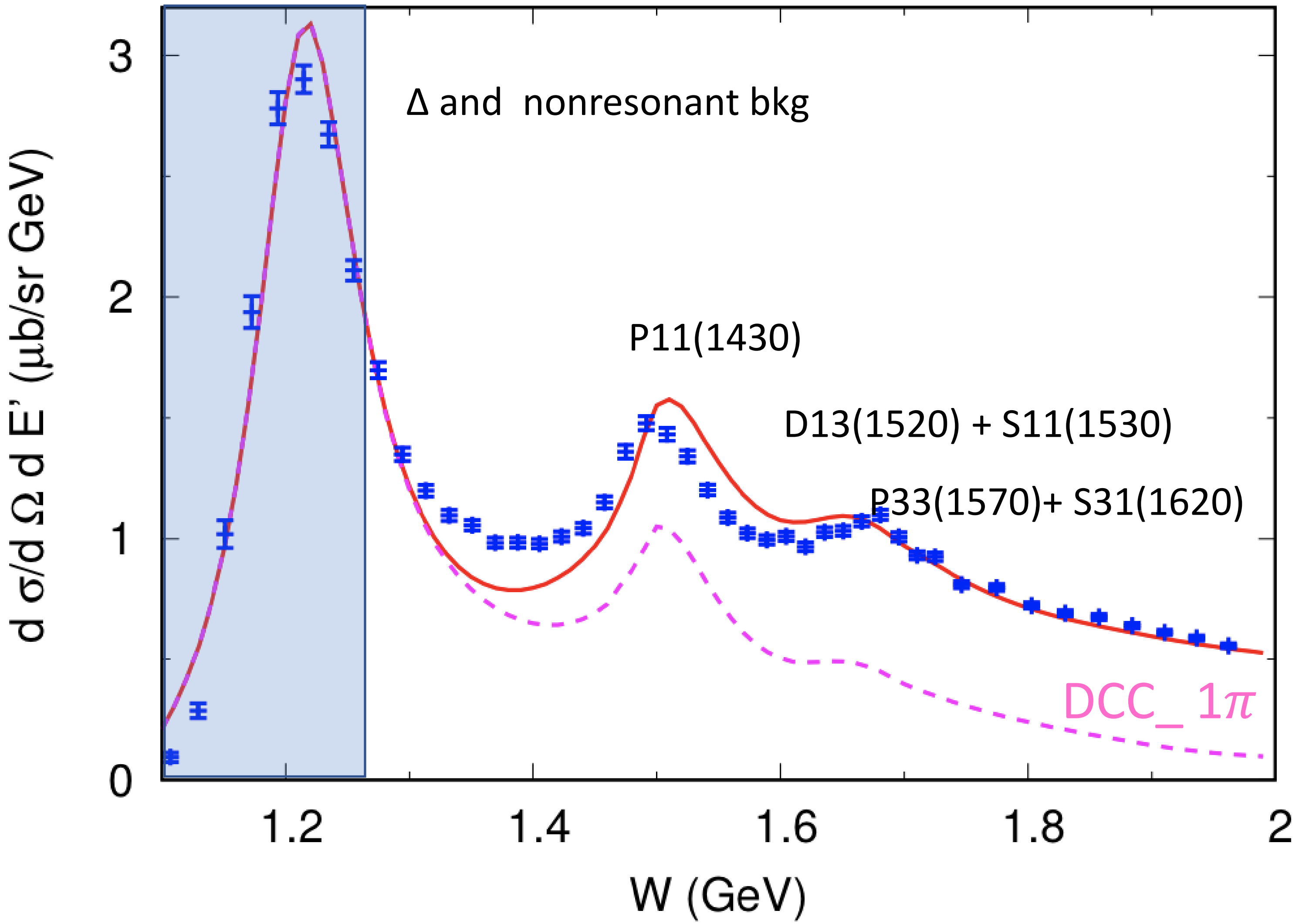
$n_3 > n_1 > n_2, \quad n_1 \simeq 3$

$\Lambda_{QCD} \in [0.19 - 0.24]$ GeV

- $a_{\alpha k}$ are constrained by **unitarity conditions** that also satisfy asymptotic QCD requirements.

C. Adamuscin *et al*. Eur. Phys. J. C 28, 115 (2003)

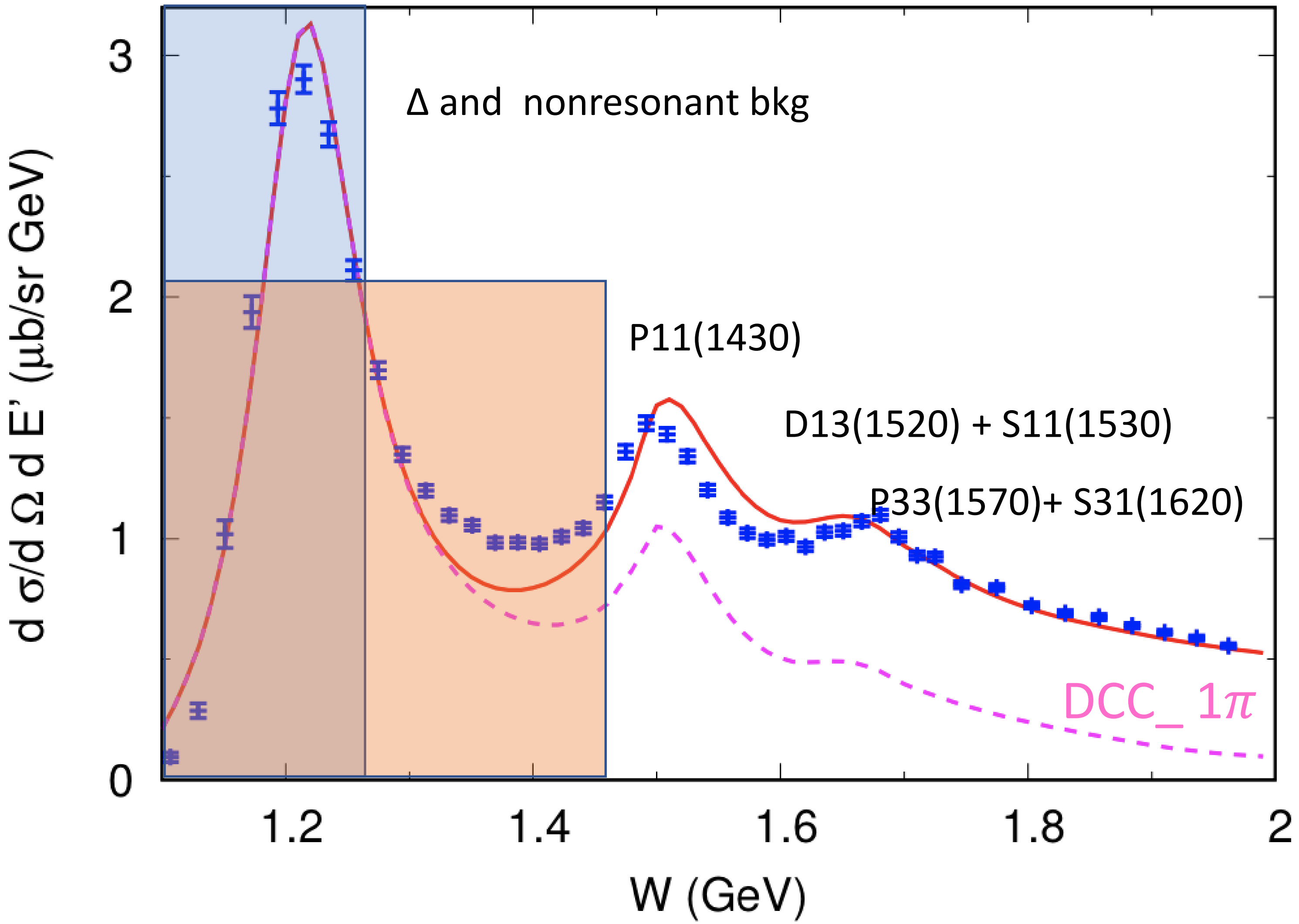
Leveraging Electron-Proton Data for the Analysis



- Select Data in 1.08<W<1.28 GeV region to choose the best Δ and bkg form factors.

- Throw random starting point for minuit.

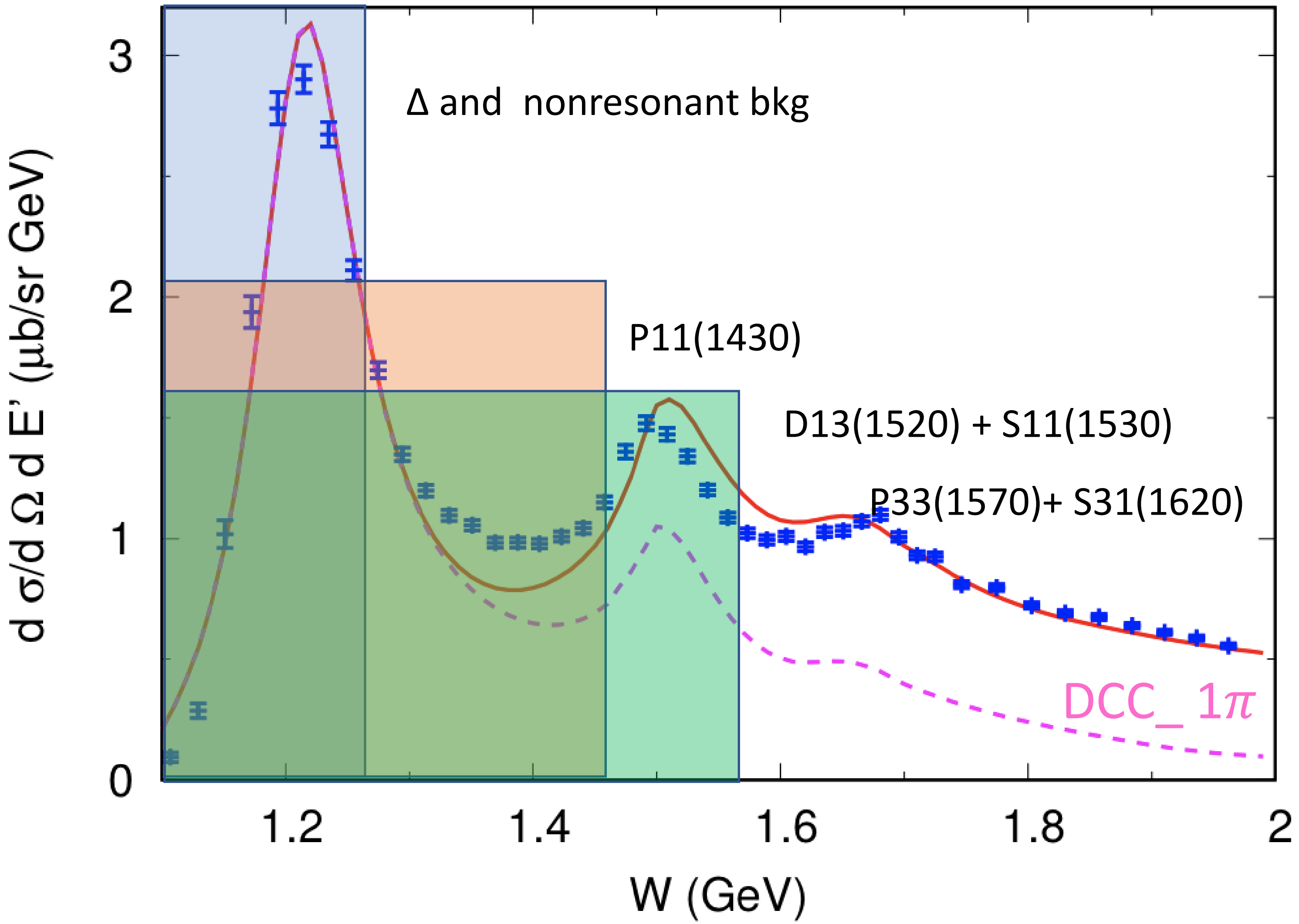
Leveraging Electron-Proton Data for the Analysis



- Add data in 1.28<W<1440 MeV to choose the best P11(1430) resonance's form factor and best bkg_cut.

- Throw random starting point for minuit.

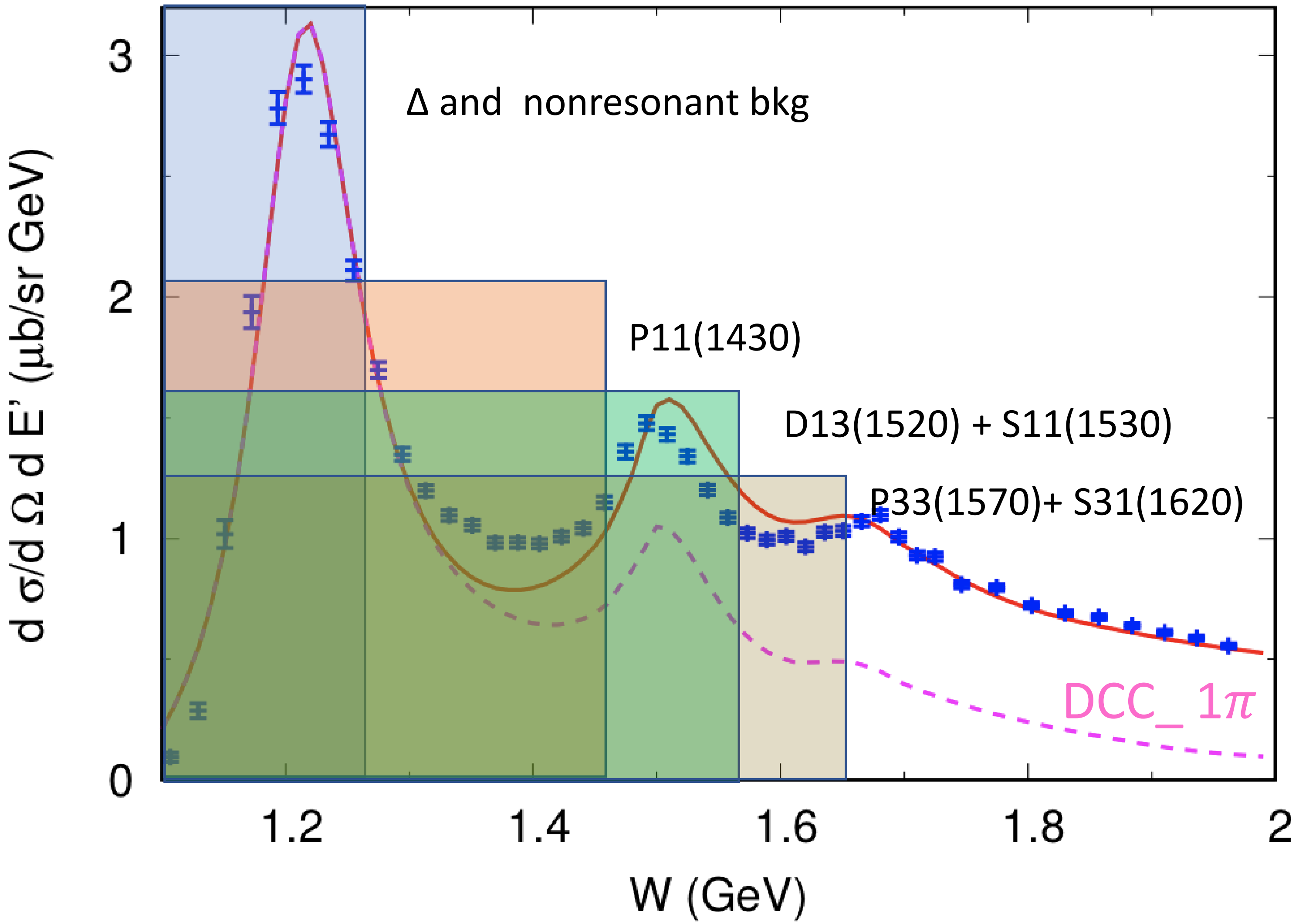
Leveraging Electron-Proton Data for the Analysis



- Add data in 1440<W<1540 MeV region to choose D13(1520) + S11(1530) form factorsand the best bkg cut.

- Throw random starting point for minuit.

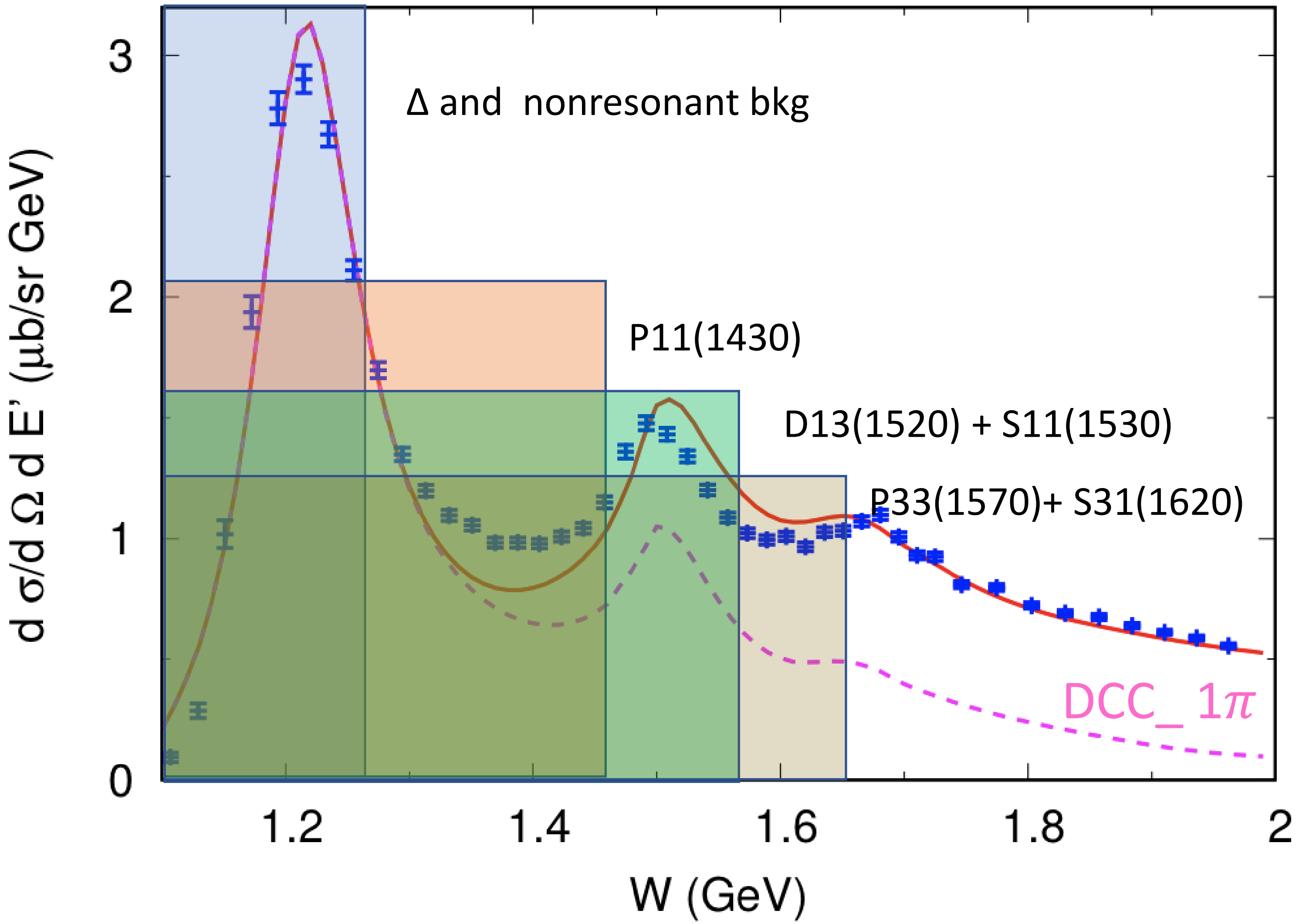
Leveraging Electron-Proton Data for the Analysis



- Add data in 1540<W<1640 MeV region to P33(1570)+ S31(1620) form factor.

- Throw random starting point for minuit.

Leveraging Electron-Proton Data for the Analysis



- In the final step all the parameters in the form-factors and the phases between these resonances and the nonresonant helicity amplitudes were fit.

Data/models comparison at low Q^2

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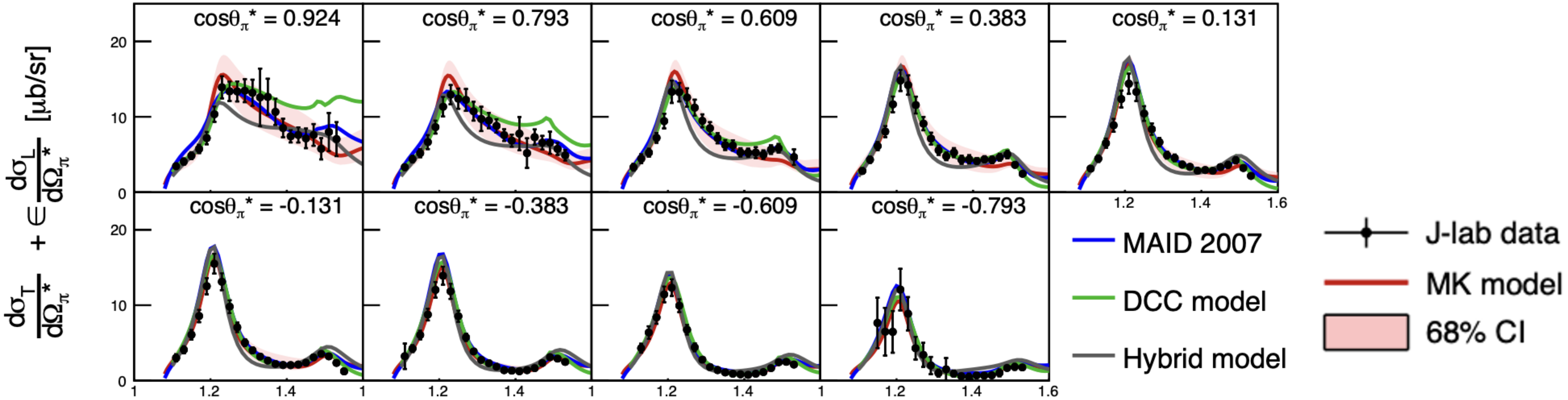
Phys.Rev.C 107 (2023)

$ep \rightarrow en + \pi^+$



E= 1.515 GeV

Q^2=0.4 GeV^2

1.1<W<1.41 GeV

Data/models comparison at high Q^2

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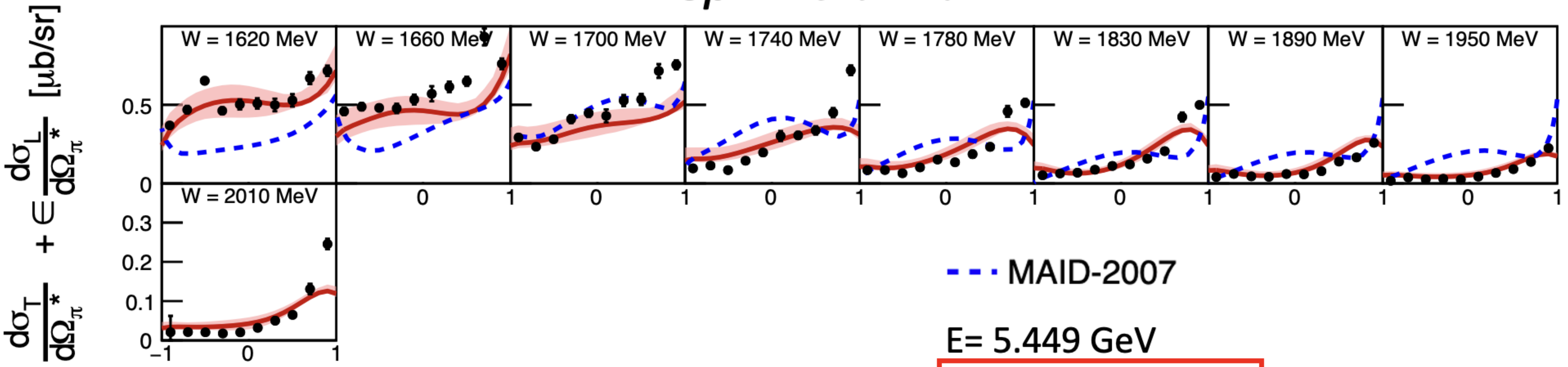
Phys.Rev.C 107 (2023)

- Only MAID model provide prediction for high Q^2

$$ep \rightarrow en + \pi^+$$



E= 5.449 GeV

SIS region

Q^2=2.6 GeV^2

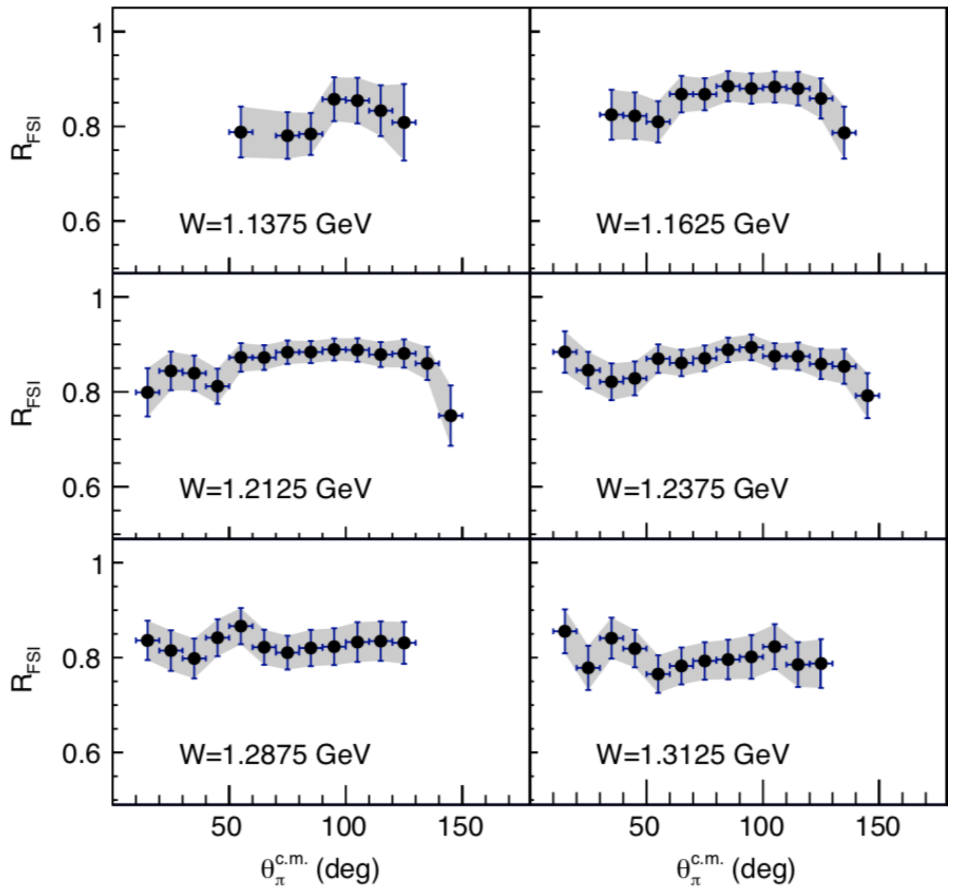
1.62 <W< 2.01 GeV

Electron-neutron scattering data

- The measurements indicate that the deuterium nuclear effect cannot be ignored.
- This ratio represents the comparison between the full exclusive and quasi-free cross sections within the 0.6<Q2<0.8 (GeV/c)^2 range.

$$R = \frac{d\sigma(\gamma^* n \rightarrow p\pi^-) / d\Omega^{c.m.}}{d\sigma(\gamma^* n(p) \rightarrow p\pi^-(p)) / d\Omega^{c.m.}}$$

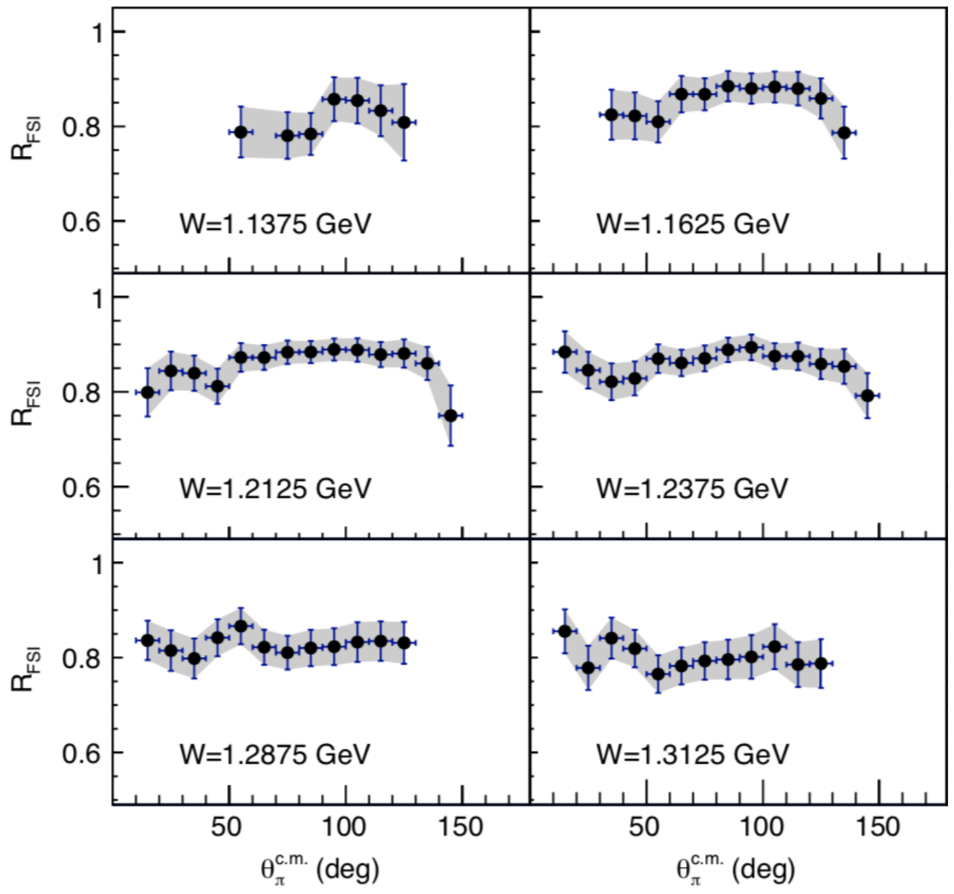
0.8 GeV2 < Q2 < 1.0 GeV2



Do we understand Deuterium?

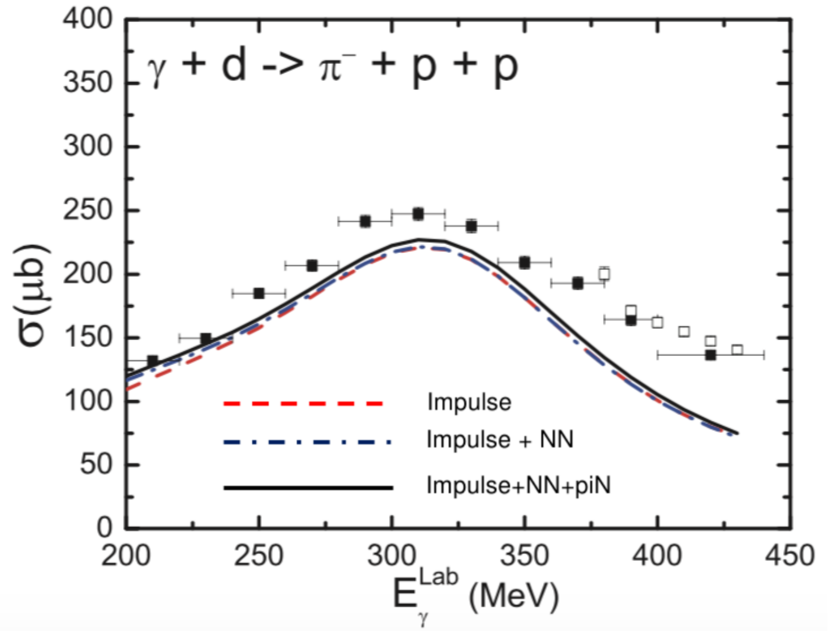
- What if nuclear effects in deuterium are significant for ANL & BNL data?
- Comparison of model predictions with exclusive neutrino data on deuterium is extremely limited. However, comparisons with exclusive photon scattering data are more common and can provide insights.

0.8 GeV2 < Q2 < 1.0 GeV2



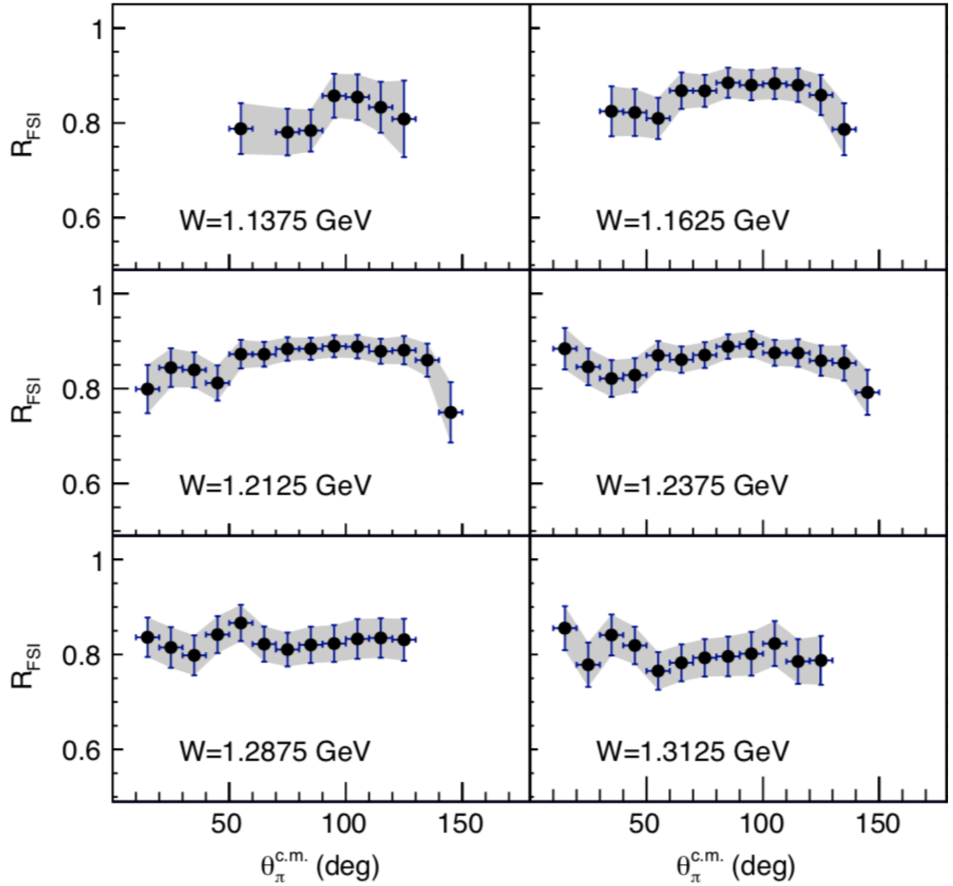
Do we understand Deuterium?

T. Sato, CETUP*2014

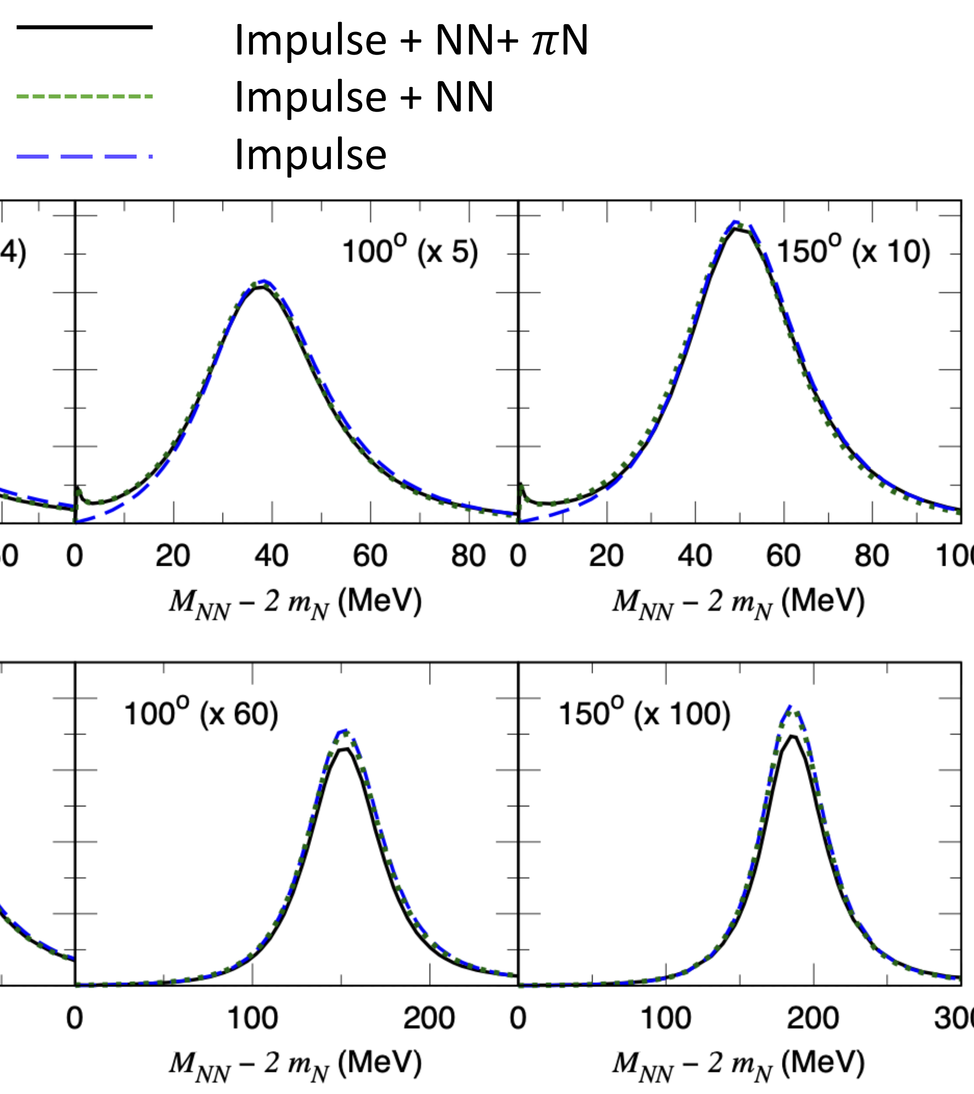


$$R = \frac{d\sigma(\gamma^* n \rightarrow p\pi^-)/_{d\Omega^{c.m.}}}{d\sigma(\gamma^* n(p) \rightarrow p\pi^-(p))/_{d\Omega^{c.m.}}}$$

0.8 GeV2 < Q2 < 1.0 GeV2



ANL-Osaka

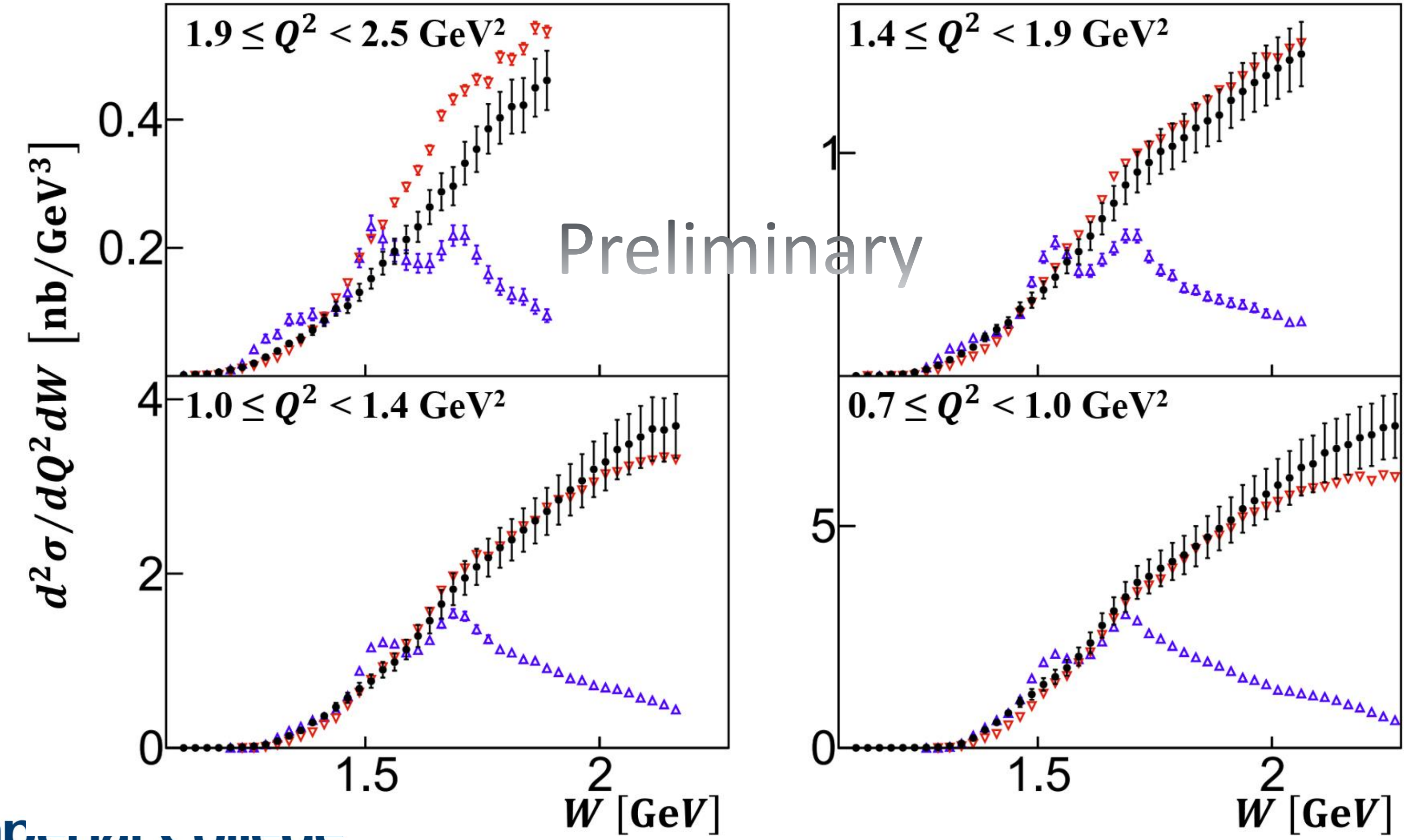


Eγ = 305 MeV

Eγ = 705 MeV

S. X. Nakamura et al. Arxiv: 1804.04757"

4.2 GeV d(e,e'pi+) CLAS12 PRELIMINARY cross sections



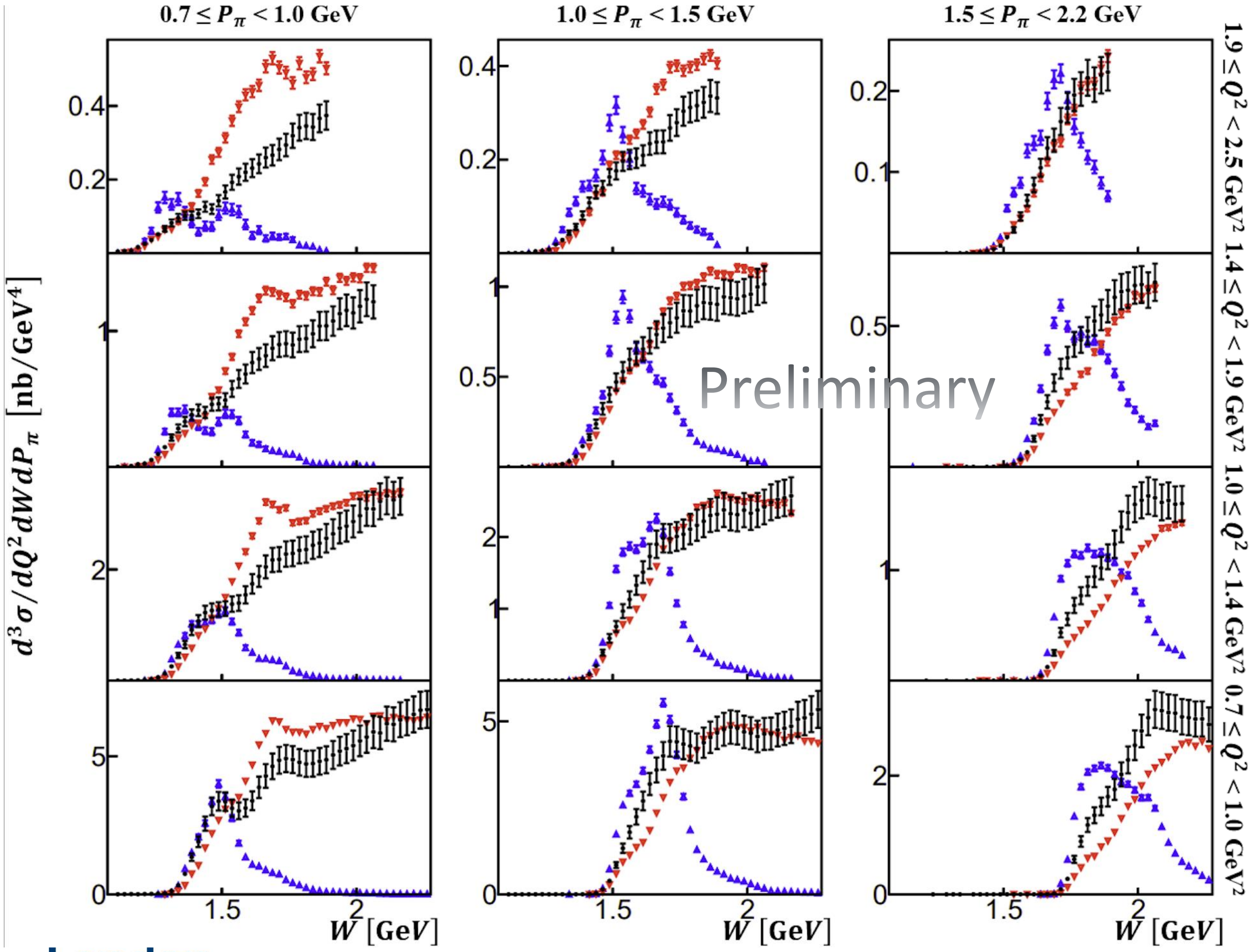
$ed \rightarrow e\pi^+ + X$

- Data

Onepigen (MAID model)

eGENIE-18_02a Rein & Sehgal model + nuclear effects

Analyzer: Caleb Fogler, Old Dominion University CLAS and e4nu collaborations



4.2 GeV d(e,e'pi+) PRELIMINARY 3D cross sections

- Data
- Onepigen (MAID model)
- eGENIE-18_02a Rein & Sehgal model + nuclear effects

Analyzer: Caleb Fogler,
Old Dominion University
CLAS and e4nu collaborations

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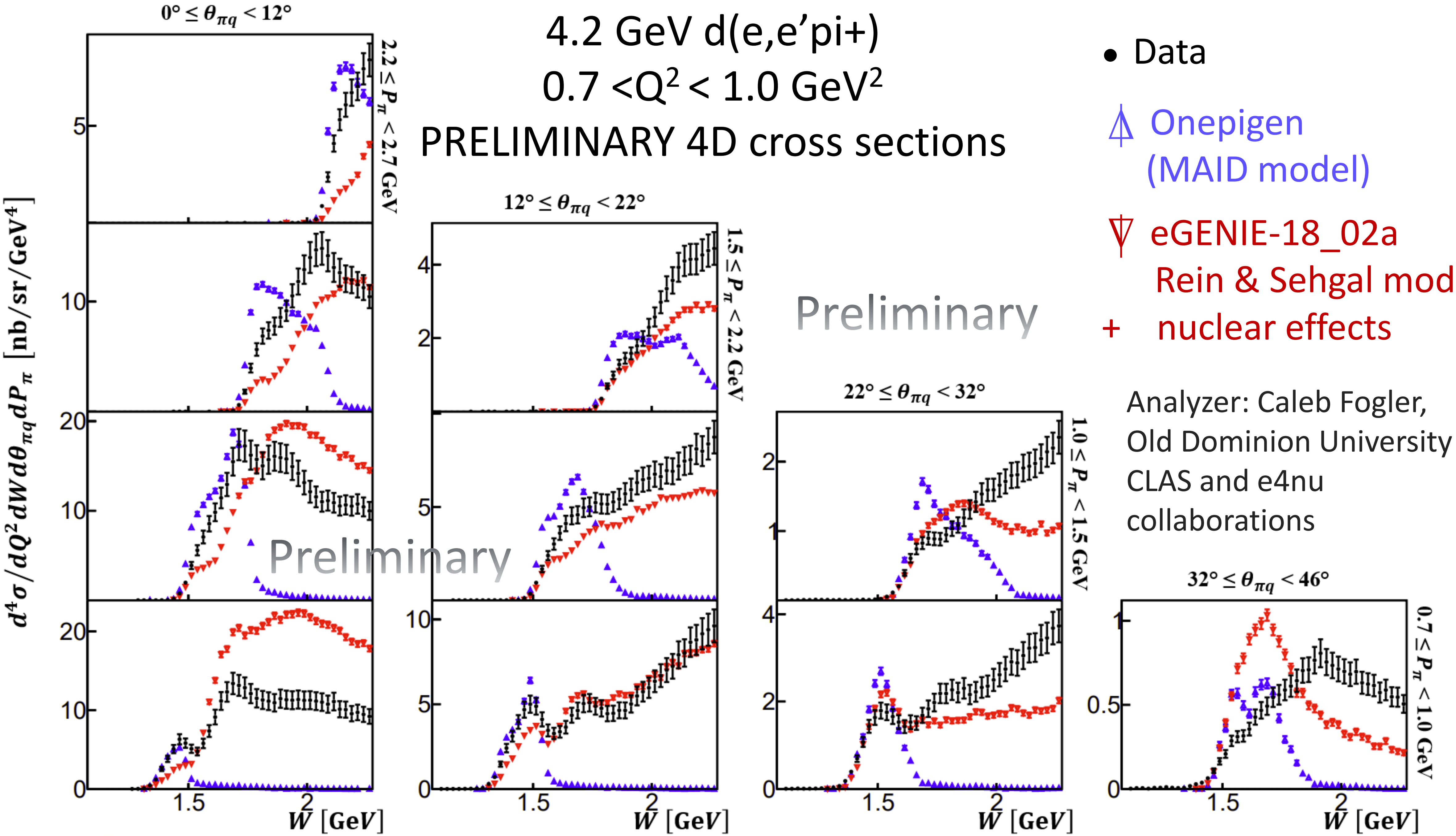
4.2 GeV d(e,e’pi+)
0.7 <Q^2 < 1.0 GeV^2
PRELIMINARY 4D cross sections



- Data
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Reducing independent parameters

- Consistency with unitarity, isospin symmetry, and perturbative QCD constraints.
- Leverage insights from vector form factors to reduce parameters in axial current, particularly at large Q^2
- Apply CVC principles, supported by photon scattering data, to constrain vector form factors at low Q^2.
- Incorporate PCAC principles and pion scattering data to refine axial current parameters at low Q^2.

$$\frac{\mathrm{d}\sigma}{dQdW}\bigg|_{Q^2=0} \propto \sigma(\pi N \rightarrow \pi N)$$

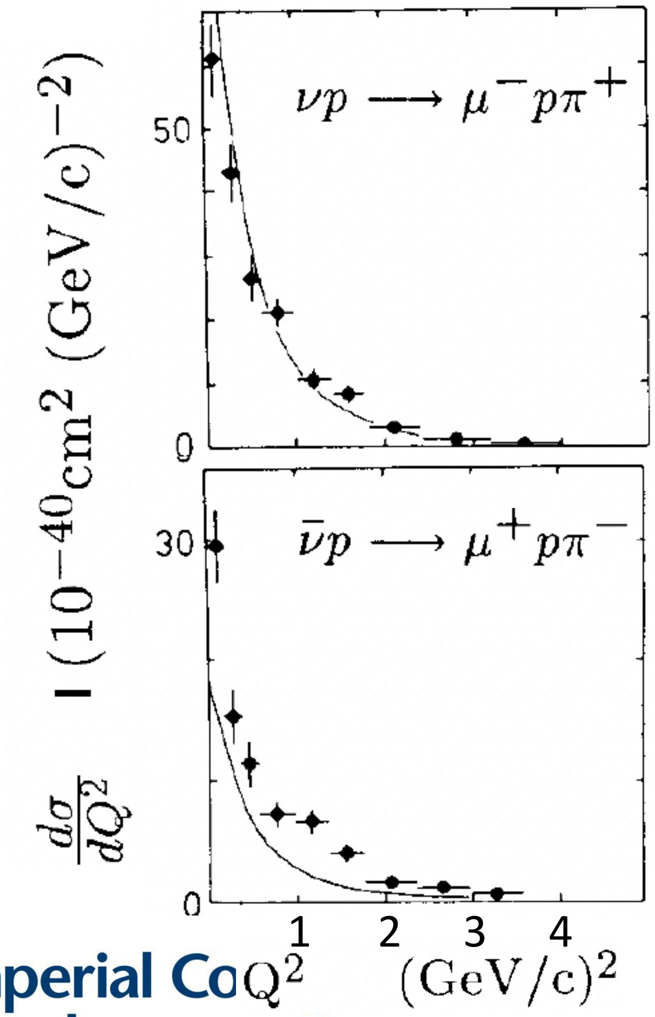
Neutrino data Used in the joint analysis

1. ANL & BNL measurements on Deuterium or mixed with Hydrogen targets ($E_\nu \approx 1$ GeV)
   - Employing the ratio of channels in the joint fit

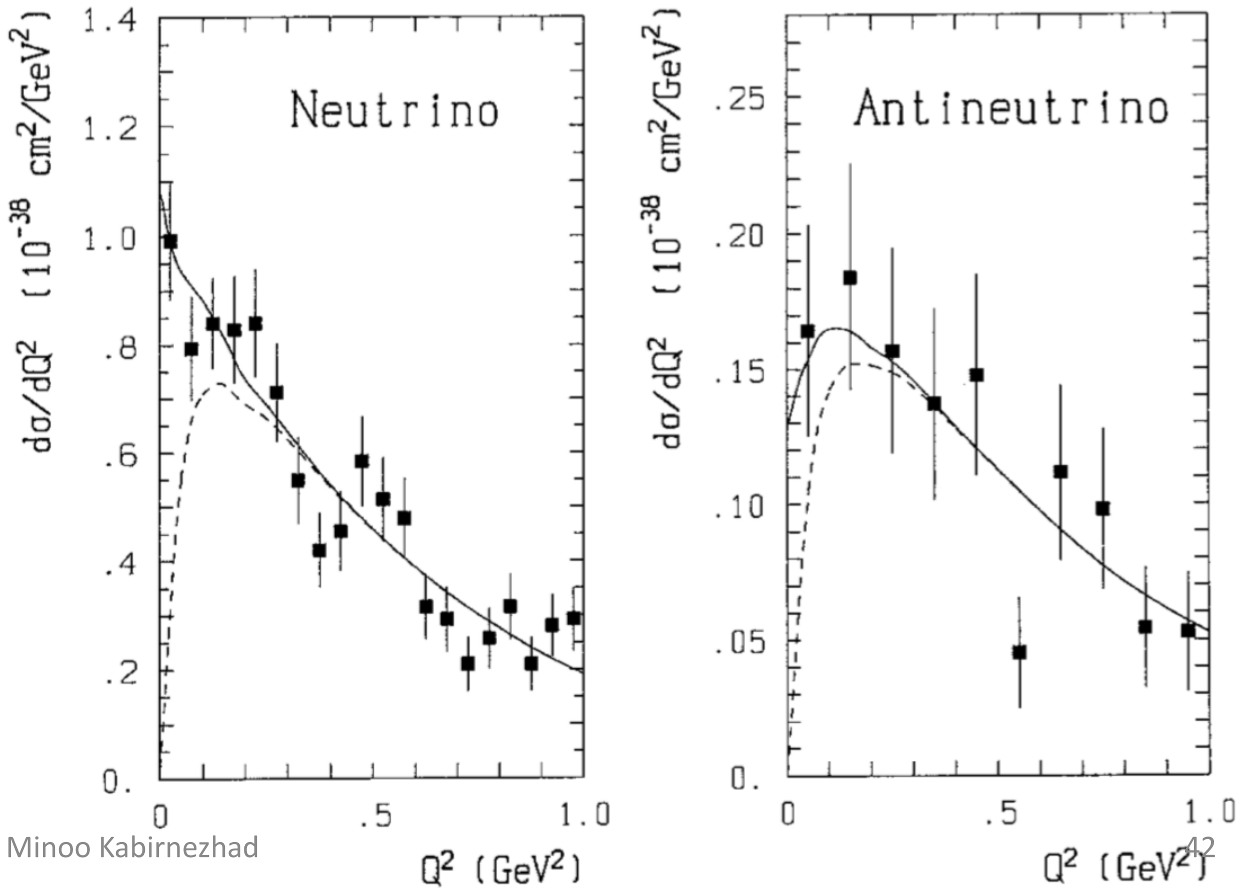
$$R = \frac{\sigma[\nu p(n_s) \to \mu p \pi^+(n_s)]}{\sigma[\nu n(p_s) \to \mu n \pi^+(p_s)]}$$

2. BEBC measurements on Hydrogen & Deuterium ($E_\nu \approx 20$ GeV)
   - Employing data on hydrogen and utilising the ratio of channels on deuterium in the fit
   - Despite high energy beam, cross-section is measured at low Q^2 and W

BEBC measurements (D_2 vs LH_2), W<1.4 GeV



Liquid hydrogen



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Final words about my analysis

- **Minimisation Technique**: The Minuit minimiser is employed in the joint analysis for parameter optimization.
- **Systematic Uncertainty Evaluation**: Systematic uncertainties are quantified using the covariance (correlation) matrix.
- **Parameter Sensitivity**: Parameters with sensitivity from the analysis can be applied to improve precision in neutrino measurements.

Summary on meson production model

- **Transition (SIS) Region**: Both the evolution of Q^2 and W must be thoroughly studied to understand the transition region.
- **Parametrisation:** After thorough calculations, it is crucial to ensure that the model remains valid in the relevant kinematic region, particularly in areas where theoretical insights are constrained.
- **Systematic Uncertainties**: In meson production, the main source of systematic uncertainty arises from nucleon-level interactions.

Summary on Deuterium

- **Deuterium Effects in SPP**: JLab data suggests a significant deuterium effect in single pion production (SPP), but current models struggle to predict this, raising the question of additional nuclear effects such as short-range correlations (SRC).
- **Modeling**: Ab-initio models that include pion production (like Neomi's model) may provide insights into this phenomenon.
- **Implications for Neutrino Interactions**: If these deuterium effects apply to neutrino interactions, using ANL and BNL deuterium data to fit neutrino-nucleon interaction models may not be reliable.

Backup

Resonance productions (spin 1/2 & 3/2)

$$J^{\mu}_{\frac{1}{2}(\frac{3}{2})} = \bar{u}(p')\Gamma^{\mu}_{\frac{1}{2}(\frac{3}{2})}u(p), \qquad \Gamma^{\mu}_{\frac{1}{2}(\frac{3}{2})} \propto \left(\mathcal{V}^{\mu}_{\frac{1}{2}(\frac{3}{2})} - \mathcal{A}^{\mu}_{\frac{1}{2}(\frac{3}{2})}\right)$$

$$\mathcal{V}^{\mu}_{1/2} = \frac{\mathcal{F}_1}{(2M)^2}(Q^2\gamma^{\mu} + \not{q}q^{\mu}) + \frac{\mathcal{F}_2}{2M}i\sigma^{\mu\nu}q_{\nu}$$

$$-\mathcal{A}^{\mu}_{1/2} = \mathcal{F}_A\gamma^{\mu}\gamma^5 + \frac{\mathcal{F}_p}{M}q^{\mu}\gamma^5$$

$$\mathcal{V}^{\nu\mu}_{3/2} = \frac{\mathcal{C}^V_3}{M}(g^{\nu\mu}\not{q} - q^{\nu}\gamma^{\mu}) + \frac{\mathcal{C}^V_4}{M^2}(g^{\nu\mu}q.p' - q^{\nu}p'^{\mu}) + \frac{\mathcal{C}^V_5}{M^2}(g^{\nu\mu}q.p - q^{\nu}p^{\mu}) + g^{\nu\mu}\mathcal{C}^V_6$$

$$-\mathcal{A}^{\mu}_{3/2} = [\frac{\mathcal{C}^A_3}{M}(g^{\nu\mu}\not{q} - q^{\nu}\gamma^{\mu}) + \frac{\mathcal{C}^A_4}{M^2}(g^{\nu\mu}q.p' - q^{\nu}p'^{\mu}) + \mathcal{C}^A_5 g^{\nu\mu} + \frac{\mathcal{C}^V_6}{M^2}q^{\nu}q^{\mu}]\gamma^5$$

Resonance production (spin 1/2 & 3/2)

$$J^{\mu}_{\frac{1}{2}\left(\frac{3}{2}\right)} = \bar{u}(p')\Gamma^{\mu}_{\frac{1}{2}\left(\frac{3}{2}\right)}u(p), \qquad \Gamma^{\mu}_{\frac{1}{2}\left(\frac{3}{2}\right)} \propto \left(\mathcal{V}^{\mu}_{\frac{1}{2}\left(\frac{3}{2}\right)} - \mathcal{A}^{\mu}_{\frac{1}{2}\left(\frac{3}{2}\right)}\right)$$

$$\mathcal{V}^{\mu}_{1/2} = \frac{\mathcal{F}_1}{(2M)^2}(Q^2\gamma^{\mu} + \not{q}q^{\mu}) + \frac{\mathcal{F}_2}{2M} i\sigma^{\mu\nu}q_{\nu}$$

$$-\mathcal{A}^{\mu}_{1/2} = \mathcal{F}_A\gamma^{\mu}\gamma^5 + \frac{\mathcal{F}_p}{M}q^{\mu}\gamma^5$$

$$\mathcal{V}^{\nu\mu}_{3/2} = \frac{\mathcal{C}^V_3}{M}(g^{\nu\mu}\not{q} - q^{\nu}\gamma^{\mu}) + \frac{\mathcal{C}^V_4}{M^2}(g^{\nu\mu}q.p' - q^{\nu}p'^{\mu}) + \frac{\mathcal{C}^V_5}{M^2}(g^{\nu\mu}q.p - q^{\nu}p^{\mu}) + g^{\nu\mu}\mathcal{C}^V_6$$

$$-\mathcal{A}^{\mu}_{3/2} = [\frac{\mathcal{C}^A_3}{M}(g^{\nu\mu}\not{q} - q^{\nu}\gamma^{\mu}) + \frac{\mathcal{C}^A_4}{M^2}(g^{\nu\mu}q.p' - q^{\nu}p'^{\mu}) + \mathcal{C}^A_5 g^{\nu\mu} + \frac{\mathcal{C}^A_6}{M^2}q^{\nu}q^{\mu}]\gamma^5$$

Resonances with spin > 3/2

- Any formalism describing resonances with spin greater than 3/2 is highly complicated
- In most of the models, resonance with spin>3/2 either ignore or treated with spin 3/2 formalism.

Resonance	M_R	Γ_0	χ_E
$P_{33}(1232)$	1232	117	1
$P_{11}(1440)$	1430	350	0.65
$D_{13}(1520)$	1515	115	0.60
$S_{11}(1535)$	1535	150	0.45
$P_{33}(1600)$	1600	320	0.18
$S_{31}(1620)$	1630	140	0.25
$S_{11}(1650)$	1655	140	0.70
$D_{15}(1675)$	1675	150	0.40
$F_{15}(1680)$	1685	130	0.67
$D_{13}(1700)$	1700	150	0.12
$D_{33}(1700)$	1700	300	0.15
$P_{11}(1710)$	1710	100	0.12
$P_{13}(1720)$	1720	250	0.11
$F_{35}(1905)$	1880	330	0.12
$P_{31}(1910)$	1890	280	0.22
$P_{33}(1920)$	1920	260	0.12
$F_{37}(1950)$	1930	285	0.40

Highlight 2: NC channels

- Data from NC channels are exceptionally rare.
- The model successfully describes both electromagnetic and weak interactions simultaneously, and ensures rigorous control over the NC vector current.

- $V^{\mu}_{1,2,3}$ form a vector in isospin space (isovector current).
- V^{μ}_{Y} is hypercharge current

$$J^{\mu}{}_{EM} = \frac{1}{2} V^{\mu}_{Y} + V^{\mu}_{3}$$

$$J^{\mu}{}_{CC} = V^{\mu}_{1} + iV^{\mu}_{2}$$

$$J^{\mu}{}_{NC} = (1 - 2\sin^2\theta_W)V^{\mu}_{3} - \sin^2\theta_W V^{\mu}_{Y}$$

Highlight 3: Low Q^2 region

- The model is designed to address the low Q^2 region, where existing models struggle to predict empirical data:

  1. For the vector current, it incorporates Conserved Vector Current (CVC) principles and photon scattering data.
  2. For the axial current, it utilises Partially Conserved Axial Current (PCAC) principles and pion scattering data.

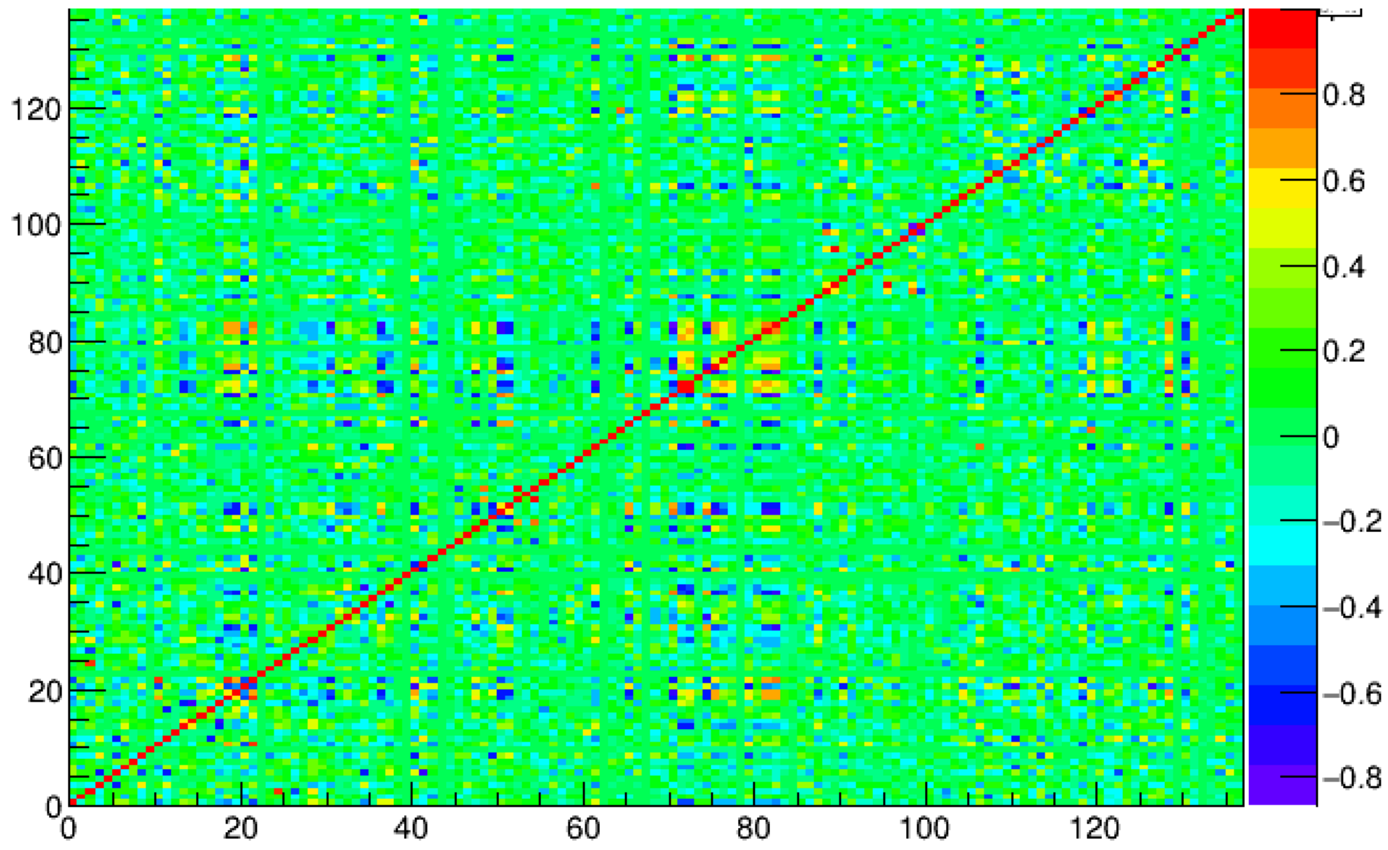
$$\frac{\mathrm{d}\sigma}{dQdW}\bigg|_{Q^2=0} \propto \sigma(\pi N \rightarrow \pi N)$$

Highlight 4: high Q^2 region

- The model remains consistent with perturbative QCD at high Q^2, thereby addressing the transition region between perturbative and non-perturbative regimes.
- This capability is crucial as it allows for a single model to predict data across various regions, ensuring versatility in its application.
- Neutrino experiments with wide energy beams operate within both the non-perturbative and transition regions, underscoring the significance of a model that can accurately describe phenomena in these diverse regimes.

Systematic Uncertainties

- **Minuit** minimiser is used in the joint analysis.
- Systematic uncertainties are assessed by employing the covariance (correlation) matrix.
- The parameters with sensitivity can be used in neutrino measurements.



Preliminary

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Conclusion

- The MK model is applicable in the resonance region (W < 2.0 GeV), covering both resonance and nonresonant interactions.
- It utilises a form-factor model (Meson Dominance) that complies with the **unitary condition**, respects **CVC and PCAC**, and is consistent with QCD principles. Consequently, the model provides accurate predictions across both low and high Q^2 regions.
- All form factors (neutron, proton, CC, and NC) are determined through a **joint fit** incorporating approximately 50,000 data points on **electron, photon, pion, and neutrino scattering data,** providing **covariance matrix**.

MK model

M. Kabirnezhad
Phys. Rev. D **97** (2018)
Phys. Rev. D **102** (2020)
Phys.Rev.C **107** (2023)

The MK model comprehensively describes single-pion production in interactions involving **photons, electrons, and neutrinos** with nucleons.

- Meson Dominance (MD) form factor: Maintains **unitarity** and integrates **QCD principles** for both resonant and non-resonant interactions.
- **CVC and PCAC** fulfilment: Ensures model consistency at low Q^2.
- Q^2 evolution: Utilises QCD calculations and **quark-hadron duality**.
- W evolution: Applies **Regge trajectory** and the Hybrid model.

R. González-Jiménez, *et al*
Phys. Rev. D **95** (2017)

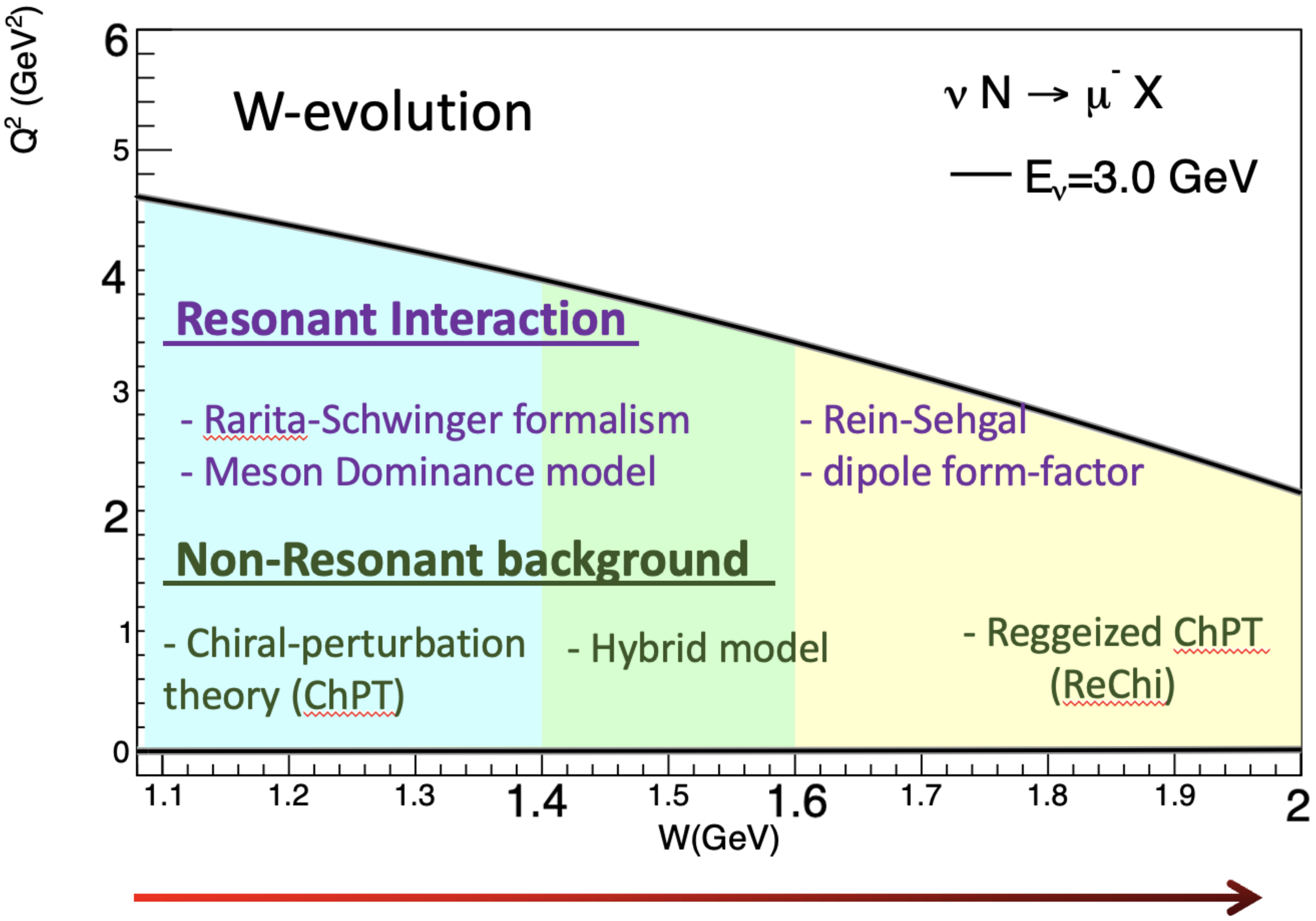
MK model

Resonant interaction

- Several resonances contribute at different invariant mass (W)

Non-resonant bkg

- Chiral perturbation at low W < 1.4 GeV
- Regge trajectory at high W
- Hybrid model



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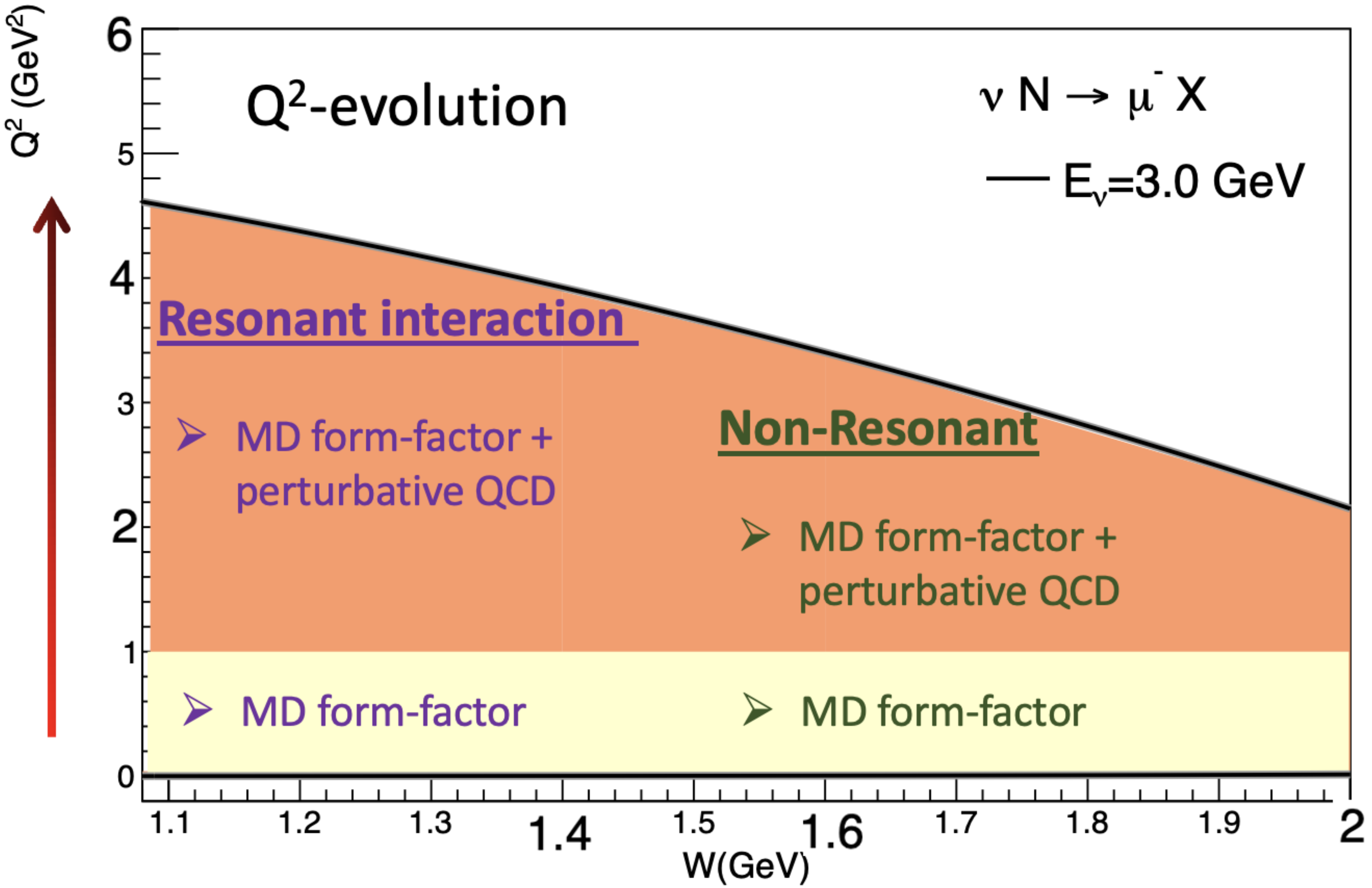
MK model

- Meson Dominance (MD) model describes form-factors in non-perturbative domain

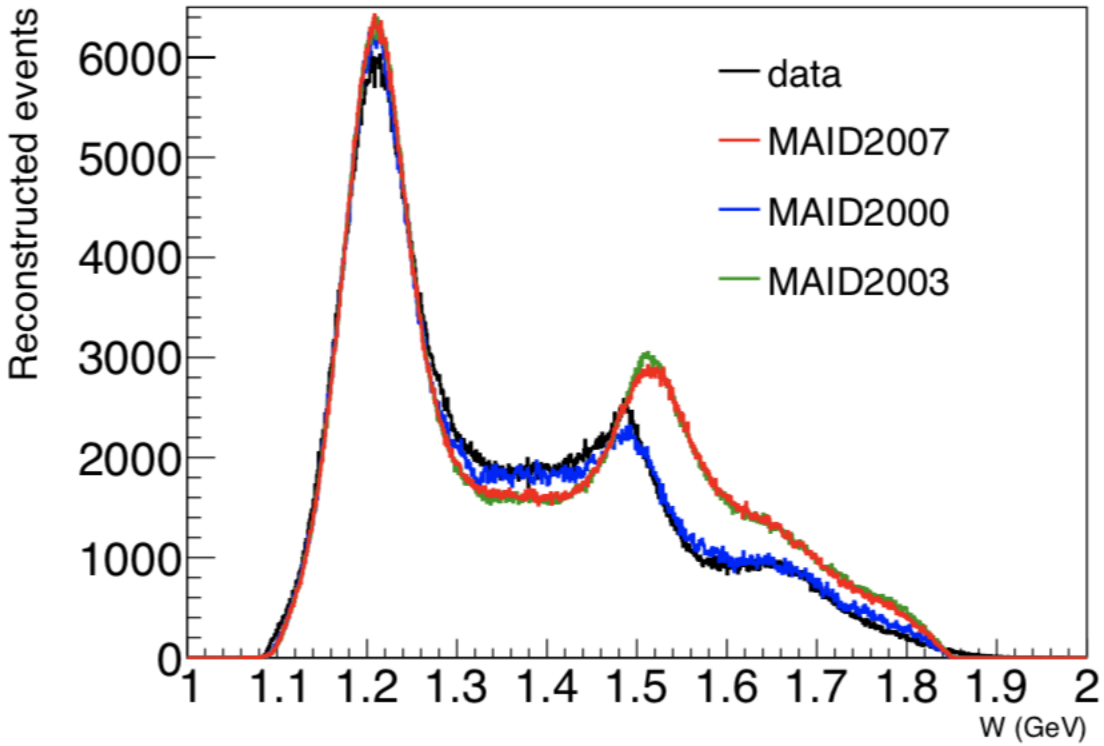
- It can reproduce Q^2-evolution of form-factors to asymptotically join QCD expectations



Valid kinematic region region for MK model

Electron-neutron scattering data

- A First-Time Endeavour!
- Utilisation of Data for Fitting Isospin ½ Resonances (Second Region)
- MAID2007, the latest version, is used by theorists to fit neutron form-factors.



Y. Tian *et al.* [CLAS]
Phys. Rev. C **107** (2023)

Vector Form Factor

- The **Proton** and **neutron** form-factors, for 17 resonances and nonresonant interactions define the vector form-factor in the weak CC and NC interactions.

	for $I = 1/2$	for $I = 3/2$
$e^- p \to e^- R^+$	F_i^p	F_i^N
$e^- n \to e^- R^0$	F_i^n	F_i^N
$\nu p \to \ell^- R^{++}$	-	$\sqrt{3} F_i^V = -\sqrt{3} F_i^N$
$\nu n \to \ell^- R^+$	$F_i^V = F_i^p - F_i^n$	$F_i^V = -F_i^N$
$\nu p \to \nu R^+$	$\tilde{F}_i^p = (\frac{1}{2} - 2\sin^2\theta_W) F_i^p - \frac{1}{2} F_i^n - \frac{1}{2} F_i^s$	$\tilde{F}_i^N = (1 - 2\sin^2\theta_W) F_i^N$
$\nu n \to \nu R^0$	$\tilde{F}_i^n = (\frac{1}{2} - 2\sin^2\theta_W) F_i^n - \frac{1}{2} F_i^p - \frac{1}{2} F_i^s$	$\tilde{F}_i^N = (1 - 2\sin^2\theta_W) F_i^N$

Isospin relations for the vector form factors

From T. Leitner thesis

Unitarity and QCD conditions

- Unitarization and analytic model: the inclusion of the continua contributions and the instability of MD model, leads to a simultaneous description of the space-like and time-like data.
- F_N is saturated by n different meson pole terms

$\mathrm{F_N}(Q^2) = \sum_{j=1}^{n} \frac{m_j^2}{m_j^2 - Q^2} a_j$ (a_j is the ratio of the coupling strength)

Unitarity and QCD conditions

- F_N is saturated by n different meson pole terms

$$F_N(Q^2) = \sum_{j=1}^{n} \frac{m_j^2}{m_j^2 - Q^2} a_j$$ (a_j is the ratio of the coupling strength)

- assume that the asymptotic behavior is: $F_N(Q^2)_{Q^2 \to \infty} \sim (Q^2)^m$

- Transforming $F_N(Q^2)$ into a common denominator, we obtain a rational function with a polynomial of degree (n−1) in the numerator, and putting in the latter the first (m−1) coefficients from the highest powers of t to zero, one obtains the linear homogeneous algebraic equations for a_j

$$\sum_{j=1}^{n} m_j^2 a_j = 0,$$

$$\sum_{\substack{i=1 \\ i \neq j}}^{n} m_i^2 \sum_{j=1}^{n} m_j^2 a_j = 0,$$

$$\sum_{\substack{i_1, i_2 = 1 \\ i_1 < i_2, i_r \neq j}}^{n} m_{i_1}^2 m_{i_2}^2 \sum_{j=1}^{n} m_j^2 a_j = 0,$$

$$\sum_{\substack{i_1, i_2, i_3 = 1 \\ i_1 < i_2 < i_3, i_r \neq j}}^{n} m_{i_1}^2 m_{i_2}^2 m_{i_3}^2 \sum_{j=1}^{n} m_j^2 a_j = 0,$$

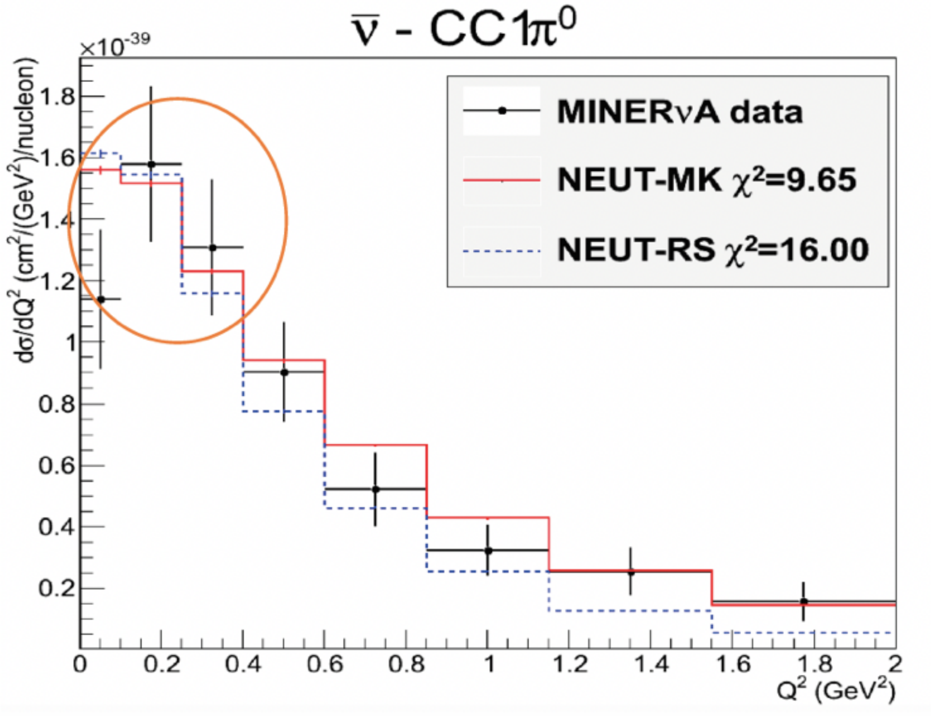
$$\cdots\cdots\cdots$$

$$\sum_{\substack{i_1, i_2, \cdots i_{m-2} = 1 \\ i_1 < i_2 \cdots < i_{m-2}, i_r \neq j}}^{n} m_{i_1}^2 m_{i_2}^2 \cdots m_{i_{m-2}}^2 \sum_{j=1}^{n} m_j^2 a_j = 0.$$

Improving the axial current

- **PCAC** relation allow us to use pion scattering data at Q^2=0. At low Q^2(<0.2 GeV), the axial current has the main contribution (due to the conservation of vector current).

Mainly axial contributions



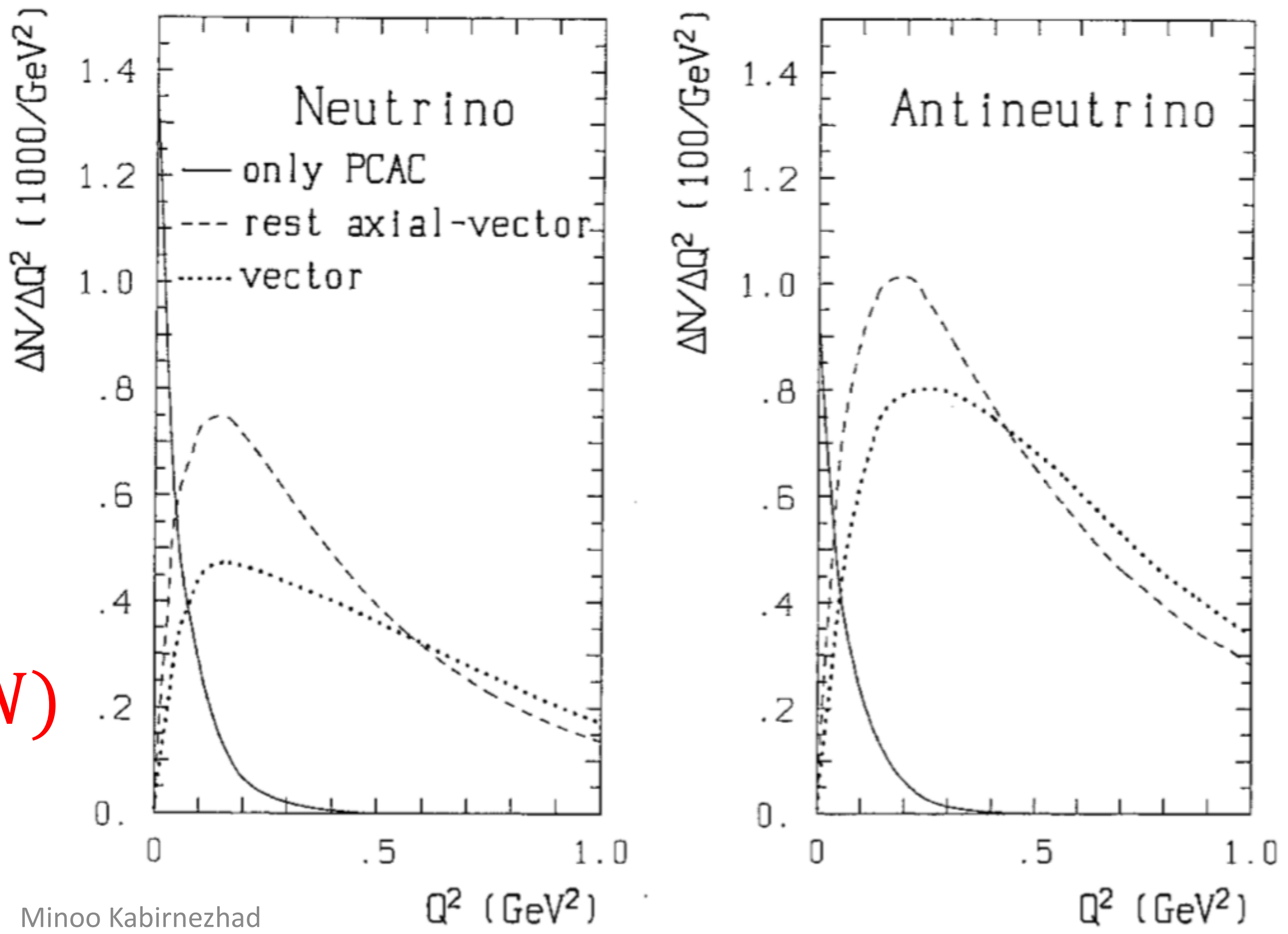
Improving the axial part

$$\frac{d\sigma}{dQ^2} = \left(\frac{d\sigma}{dQ^2}\right)^V + \left(\frac{d\sigma}{dQ^2}\right)^A$$

$$\left(\frac{d\sigma}{dQ^2}\right)^{VT} + \left(\frac{d\sigma}{dQ^2}\right)^{VL}$$

$$\left(\frac{d\sigma}{dQ^2}\right)^{AT} + \left(\frac{d\sigma}{dQ^2}\right)^{AL}$$

$$\left(\frac{d\sigma}{dQ^2\, dW}\right)^{AL} \Big|_{Q^2=0} \propto \sigma(\pi N \rightarrow \pi N)$$



$$\frac{d\sigma}{d\Omega dE'} = \frac{G^2}{16\pi^2}\cos^2\theta_C \frac{E'}{E} L_{\mu\nu}\mathcal{W}^{\mu\nu}$$

$$L_{\mu\nu} = Tr[\gamma_\mu(1-\gamma_5)\not{k}\gamma_\nu\not{k}']$$

$$\mathcal{W}^{\mu\nu} = \frac{1}{2m_N}\sum\langle p|J^\mu(0)|\Delta\rangle\langle\Delta|J^\nu(0)|p\rangle\delta(W^2-M_R^2)$$

$$\delta(W^2-M_R^2) = \frac{M_R\Gamma_R}{\pi}\frac{1}{(W^2-M_R^2)^2+M_R^2\Gamma_R^2}$$

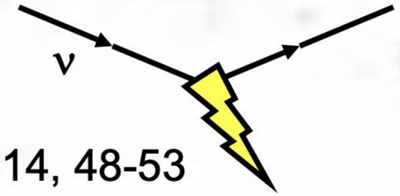
Resonance has an observable width,

$$\langle\Delta^{++}|J^\nu|p\rangle = \sqrt{3}\bar{\psi}_\lambda(p')d^{\lambda\nu}u(p)$$

$$d^{\lambda\nu} = g^{\lambda\nu}\left[\frac{C_3^V}{m_N}\not{q} + \frac{C_4^V}{m_N^2}(p'q) + \frac{C_5^V}{m_N^2}(pq) + C_6^V\right]\gamma_5 - q^\lambda\left[\frac{C_3^V}{m_N}\gamma^\nu + \frac{C_4^V}{m_N^2}p'^\nu + \frac{C_5^V}{m_N^2}p^\nu\right]\gamma_5$$
$$+g^{\lambda\nu}\left[\frac{C_3^A}{m_N}\not{q} + \frac{C_4^A}{m_N^2}(p'q)\right] - q^\lambda\left[\frac{C_3^A}{m_N}\gamma^\nu + \frac{C_4^A}{m_N^2}p'^\nu\right] + g^{\lambda\nu}C_5^A + q^\lambda q^\nu\frac{C_6^A}{m_N^2}.$$

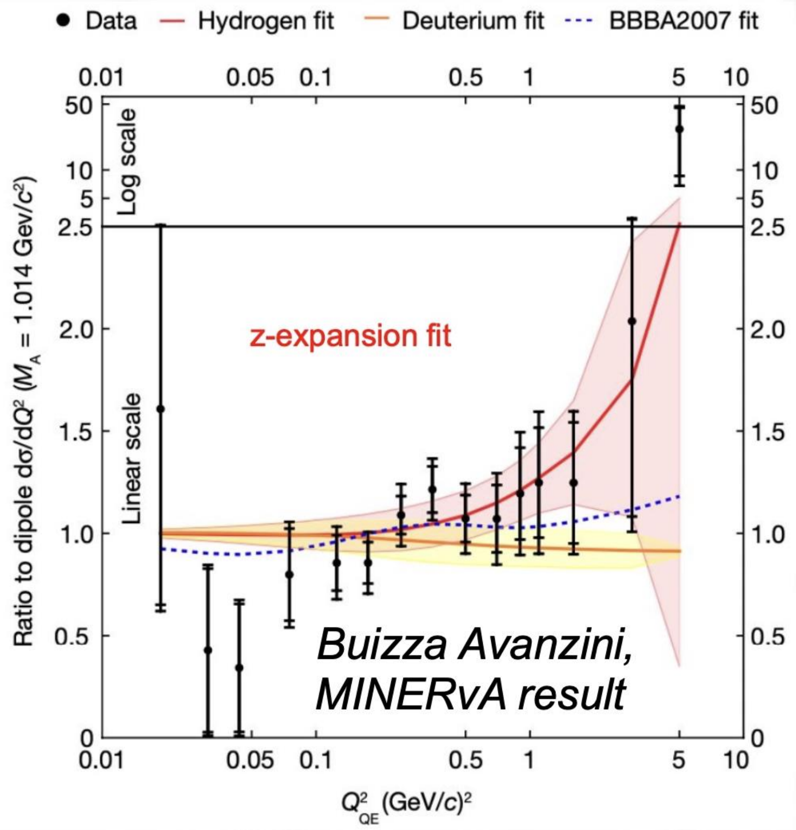
Kevin's talk at Neutrino24

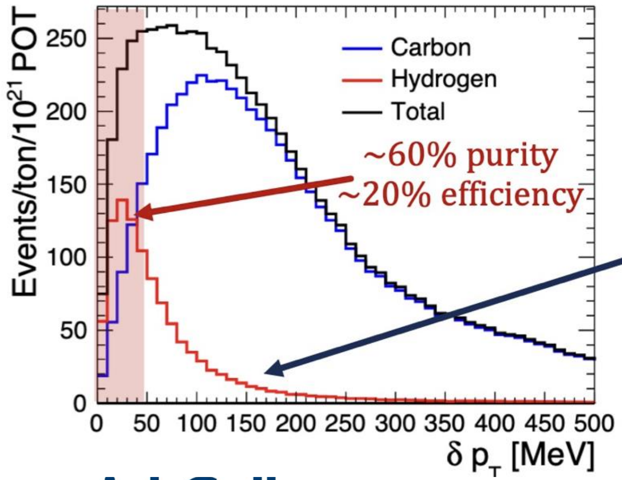
Neutron and Axial Form Factor



Nature, 614, 48-53

- MINERvA used neutron reconstruction and ability to isolate events on hydrogen (only with direction!) to measure $F_A(Q^2)$ with useful precision $0.06 < Q^2 < 2$ GeV^2.
- SuperFGD will have two handles, direction and energy, to isolate hydrogen scattering.





Antineutrinos:
Peak from interactions on hydrogen

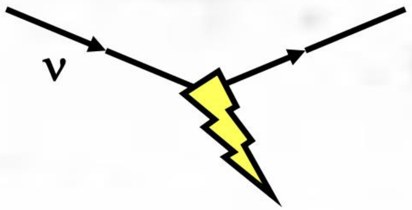
No nuclear effects

Possible thanks to **neutron detection**!

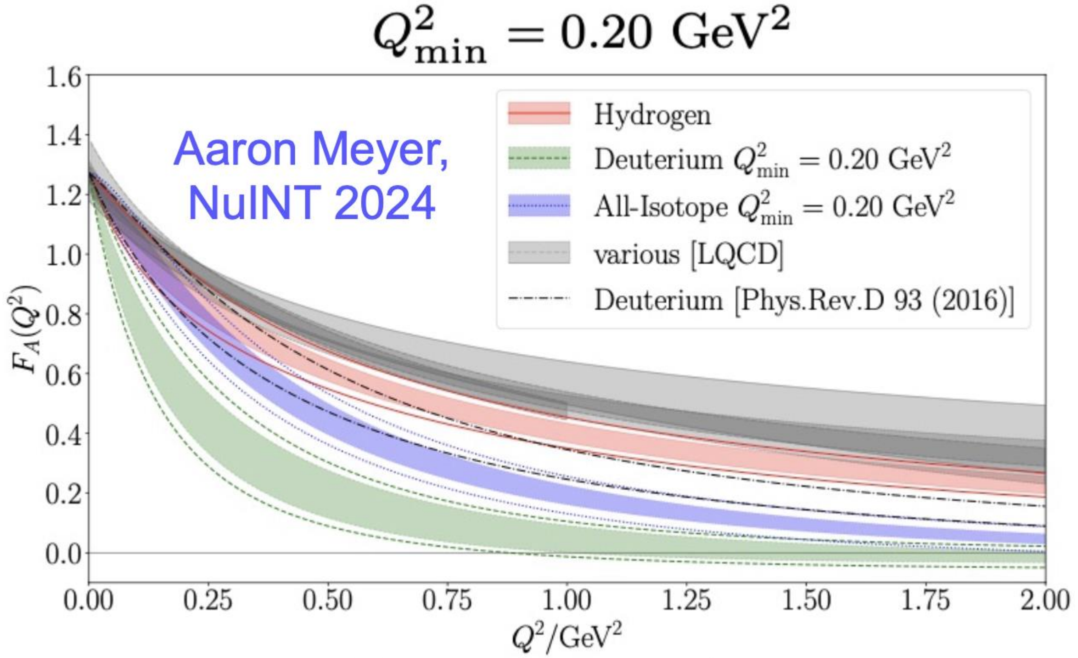
Phys. Rev. D 101, 092003 (2020),
assembled figure from L. Munteanu

Kevin’s talk at Neutrino24

Nucleons vs Nuclei



- By contrast, we are struggling to understand cross-sections on free nucleons as a base for calculating cross-sections on nucleons.
- In $F_A(Q^2)$, there are significant tensions between the deuterium bubble chamber legacy data, and either the MINERvA hydrogen or lattice QCD calculations.
- Why? It’s possible that nuclear model assumptions in the analysis of the deuterium data played a role.



	$\{a_k\}_D$	p_D	$\{a_k\}_H$	p_H
χ^2_D/DoF_D	94.9/94	0.45	167.7/96	8.3×10^{-6}
χ^2_H/DoF_H	23.3/15	0.08	10.0/13	0.69

Deuterium is incompatible with hydrogen, LQCD