The Axial Form Factor Extracted from Elementary Targets

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October 24, 2024

Measuring Neutrino Interactions for Next-Generation Oscillation Experiments – ECT*

This work is supported in part by:
Lawrence Livermore National Security, LLC #DE-AC52-07NA27344,
Neutrino Theory Network Program Grant #DE-AC02-07CHI11359,
U.S. Department of Energy Award #DE-SC0020250.

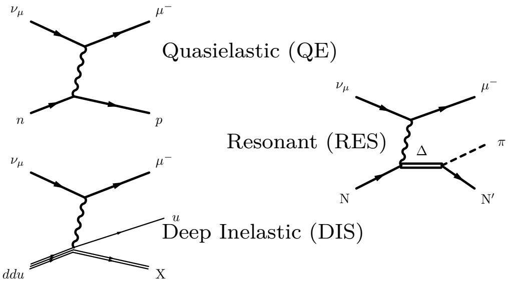
Outline

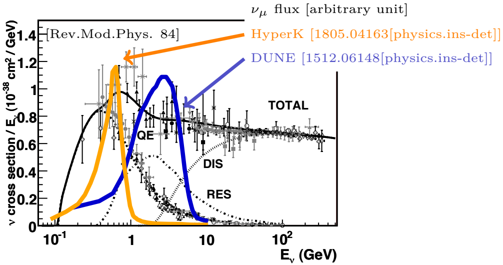
- Introduction
- Combined Hydrogen+Deuterium Fitting
- LQCD Intro
- LQCD Averaging
- LQCD Implications
- Conclusions

Note: all references in online slides are hyperlinked

Introduction

Neutrino Cross Sections





Energy range spans several *nucleon* interaction topologies

Nucleon amplitudes used to build *nuclear* cross sections
$\implies$ inputs to Monte Carlo simulations, E_ν reconstruction

Goal: isolate, quantify, improve *nucleon* amplitudes
Precise, theoretically robust *nucleon* inputs $\rightarrow$ definitive statements about *nuclear* uncertainties

Neutrino Event Topologies

Larger nucleus

$\implies$ more nucleons to interact with

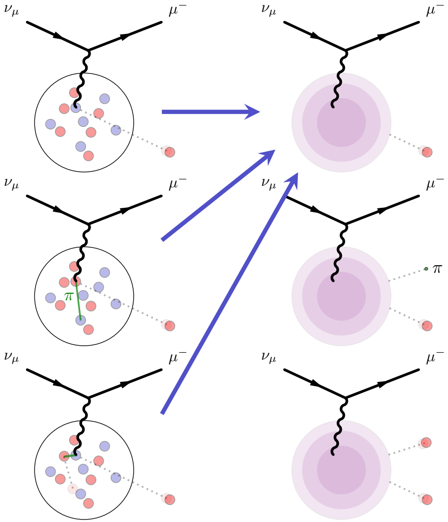
$\implies$ larger cross sections

Nuclear environment complicates measurements:

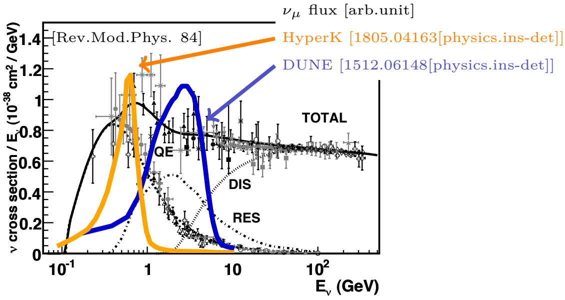
- Many allowed kinematic channels
- Reinteractions within nucleus
- Only final state particles are observable

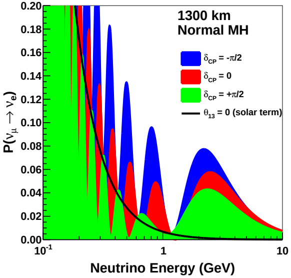
Precise cross sections need precise nucleon amplitudes

Nucleon amplitudes assumed to be precisely known



Neutrino Cross Sections from Elementary Targets





Quasielastic is lowest E_ν, simplest $\implies$ most important

Question:

How well do we know free nucleon quasielastic cross section from elementary target sources?

Three main sources:

- Hydrogen scattering (new!)
- Deuterium scattering
- Lattice QCD

Combined Hydrogen–Deuterium Fits

Form Factor Parameterizations

Dipole ansatz — $$F_A(Q^2) = g_A \left(1 + \frac{Q^2}{m_A^2}\right)^{-2}$$

- Overconstrained by both experimental and LQCD data
- Inconsistent with QCD, requirements from unitarity bounds
- Motivated by $Q^2 \to \infty$ limit, data restricted to low Q^2

Model independent alternative: z expansion [Phys.Rev.D 84 (2011)] —

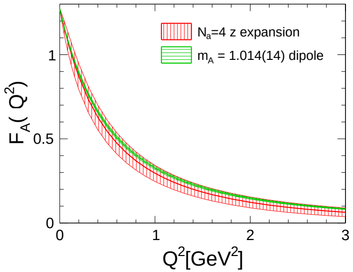
$$F_A(z) = \sum_{k=0}^{\infty} a_k z^k \qquad z(Q^2; t_0, t_{\mathrm{cut}}) = \frac{\sqrt{t_{\mathrm{cut}} + Q^2} - \sqrt{t_{\mathrm{cut}} - t_0}}{\sqrt{t_{\mathrm{cut}} + Q^2} + \sqrt{t_{\mathrm{cut}} - t_0}} \qquad t_{\mathrm{cut}} \leq (3M_\pi)^2$$

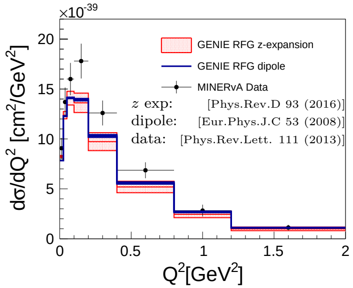
- Rapidly converging expansion
- Controlled procedure for introducing new parameters
- Sum rule constraints to regulate large-Q^2 behavior

Previous Deuterium Constraints

Fits: [Phys.Rev.D 93 (2016)]

- Outdated bubble chamber experiments
- Dipole overconstrained by data
  underestimated uncertainty $\times 10$
- Discrepancies in nuclear cross sections
  could be nucleon and/or nuclear origins





New Data

Previous datasets:

- ANL, BNL, FNAL deuterium bubble chamber event distributions

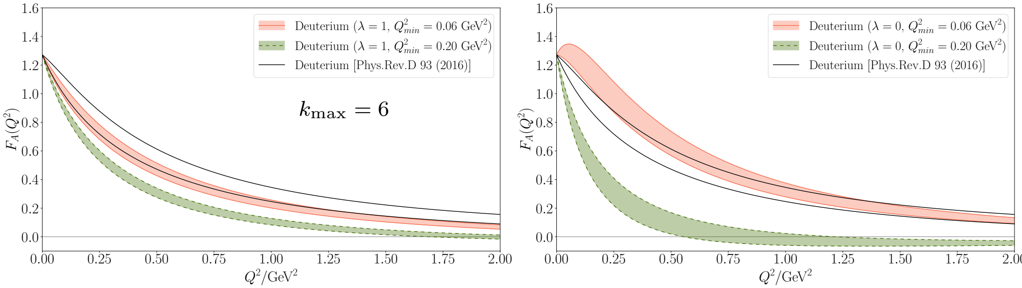
Two additional datasets:

- MINERνA 2023 $\bar{\nu}_\mu p \to \mu^+ n$ [Nature 614 (2023)]
  Special thanks: Tejin Cai, Kevin McFarland, Miriam Moore
- BEBC 1990 $\nu_\mu D \to \mu^- pp$ [Nucl.Phys.B 343]
  Special thanks: Clarence Wret, NUISANCE

Updated vector form factors:

- Use Borah *et al.* z expansion [Phys.Rev.D 102 (2020)]

Normalization Degeneracy



Change in philosophy: use χ^2_{data} vs $\chi^2_{\mathrm{penalty}}$ to inform prior width (not assumptions from unitarity)

$$\chi^2_{\mathrm{penalty}} = \lambda \sum_{k=1}^{k_{\max}} \left(a_k / a_0 \sigma_k \right)^2 \qquad (\lambda = 1 \text{ in [Phys.Rev.D 93]})$$

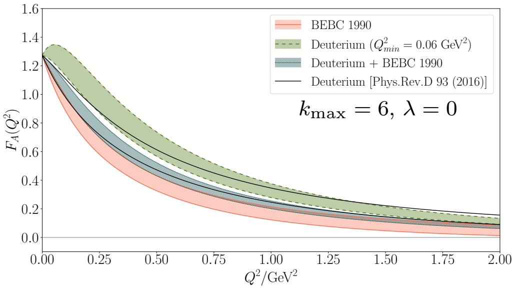
Strong dependence on prior width, manifests as dependence on Q^2 cuts

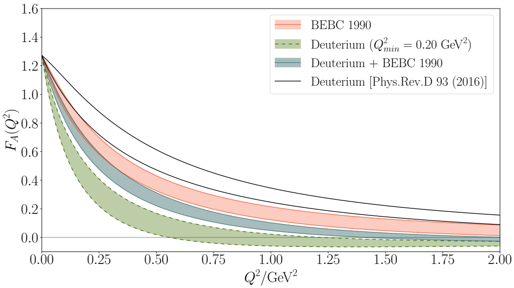
Degeneracy between free normalization, axial form factor

BEBC & MINERνA provide differential cross section, not event distributions

$\implies$ might resolve degeneracy?

Previous Deuterium + BEBC

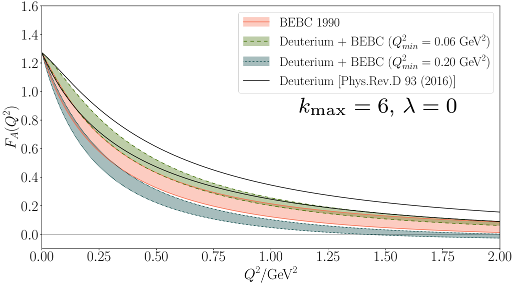




Normalization of BEBC restricted to 1 ± 0.1 (flux uncertainty)

Other deuterium pulls BEBC to limit of 1σ uncertainty on normalization

Previous Deuterium + BEBC



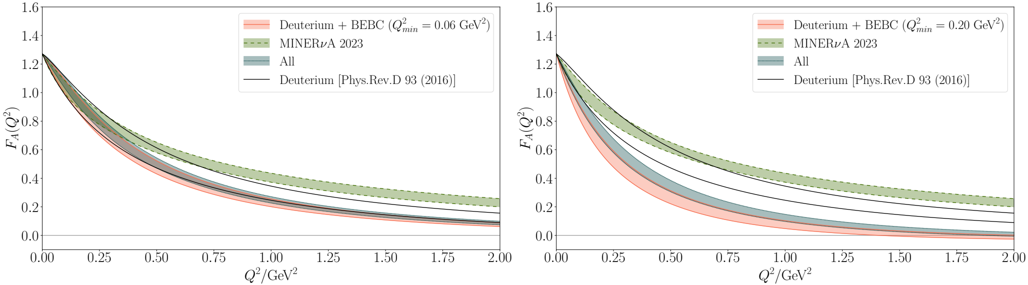
	$Q^2_{\min} = 0.06$ GeV^2		$Q^2_{\min} = 0.20$ GeV^2	
fit	χ^2_{data}/DoF	$p_{\Delta\chi^2}$	χ^2_{data}/DoF	$p_{\Delta\chi^2}$
BEBC 1990	6.3/ 9		6.3/ 9	
Deuterium	107.8/104		91.8/ 98	
Deut+BEBC	116.1/111		100.4/105	
$\Delta\chi^2$	1.9/ 1	0.16	2.4/ 1	0.12

$\Delta\chi^2 = \chi^2_{A+B} - \chi^2_A - \chi^2_B \implies$ 1 degree of freedom χ^2 test of compatibility

p values are sensible – Q^2 cuts have different pulls

BEBC is consistent with deuterium, partial resolution of degeneracy?

Deuterium + MINERvA



MINERvA incompatible with deuterium

Remnant deuterium effects?

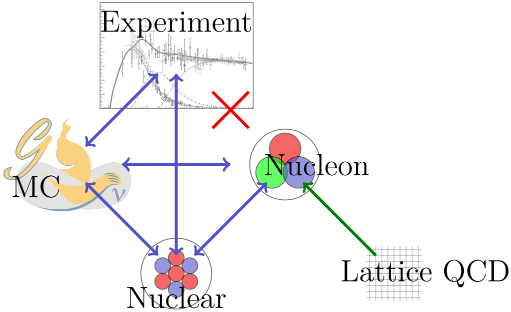
	$Q^2_{\text{min}} = 0.06$ GeV^2		$Q^2_{\text{min}} = 0.20$ GeV^2	
fit	χ^2_{data}/DoF	$p_{\Delta\chi^2}$	χ^2_{data}/DoF	$p_{\Delta\chi^2}$
All Deuterium	116.1/111		100.4/105	
MINERvA	9.1/ 16		9.1/ 16	
All	129.5/125		120.9/119	
$\Delta\chi^2$	4.3/ 1	4×10^{-2}	11.3/ 1	8×10^{-4}

LQCD Introduction

LQCD as Disruptive Technology

LQCD is a complement to experiment

- ✓ No nuclear effects
- ✓ Realistic uncertainty estimates
- ✓ Systematically improvable
- ✓ Computers are (relatively) inexpensive



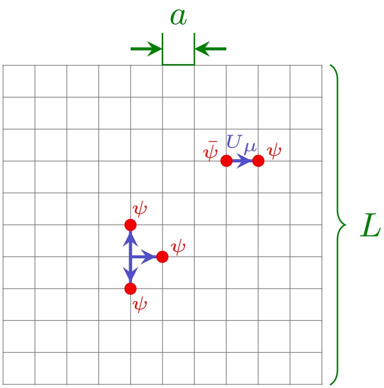
Build from the ground up:

Nucleon amplitudes from first principles

Robust uncertainty quantification

Well motivated theory inputs to nuclear models/EFTs

Lattice QCD Formalism



Numerical evaluation of path integral
Quark, gluon DOFs —

$$\langle\mathcal{O}\rangle = \frac{1}{Z}\int \mathcal{D}\psi\,\mathcal{D}\overline{\psi}\,\mathcal{D}U\ \exp(-S)\,\mathcal{O}_\psi\,[U]$$

Parameters: $am_{(u,d),\mathrm{bare}}$
$am_{s,\mathrm{bare}}$
$\beta = 6/g^2_{\mathrm{bare}}$

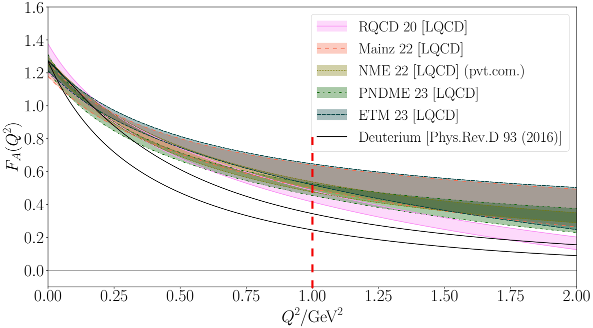
Matching: e.g. $\frac{M_\pi}{M_\Omega}$, $\frac{M_K}{M_\Omega}$, M_Ω
1 per parameter

Results — first principles predictions from QCD, gluons+$q\bar{q}$ loops to all orders

"Complete" error budget $\implies$ extrapolation in a, L, M_π guided by EFT, FVχPT

- $a \to 0$ (continuum limit)
- $L \to \infty$ (infinite volume limit)
- $M_\pi \to M_\pi^{\mathrm{phys}}$ (chiral limit)

LQCD Axial Form Factor Summary



LQCD results maturing:

- Many results, complete error budgets
- Small systematic effects observed (expectation: largest at $Q^2 \to 0$)
- Nontrivial consistency checks from PCAC

LQCD fits only up to $Q^2 = 1~\mathrm{GeV}^2$, extrapolated to higher Q^2

LQCD prediction of slow Q^2 falloff

LQCD averaging

LQCD references

Reference	N_{ens}	$N_{\text{ens}}^{\text{phys}}$	quark action	fit method	parameterization
[RQCD 2020]	37	2	clover	χPT-inspired	z^2, ~~sum rules~~
[NME 2022]	7	0	clover	Bayesian prior exponential	Padé[0/2]
[Mainz 2022]	14	1	clover	summation + z expansion	z^2, ~~SR~~
[PNDME 2023]	13	2	clover (on HISQ)	Bayesian prior exponential	z^2, ~~SR~~
[ETM 2023]	3	3	twisted mass	Bayesian prior exponential	z^3, ~~SR~~

LQCD collaborations provide nontrivial consistency checks of each other

Larger N_{ens} essential for removing systematics (> 5 maybe okay, > 10 good)

"Chiral extrapolation" hard, more physical mass ensembles $N_{\text{ens}}^{\text{phys}}$ better

Notes:

- $g_A \sim a_0$ is an output for LQCD; fixed by constraint in ν fit (not in input LQCD parameterizations)
- these fits will use sum rules (input LQCD parameterizations do not)

Averaging Strategy

Want same z expansion parameterization used in experiment fitting:

- g_A fixed to exact value
- 4 sum rules regulate large-Q^2 behavior
- no priors imposed

Procedure for matching LQCD results with different F_A parameterizations:

1. fit to derivatives $(d/dQ^2)^n F_A(Q^2)$ for $n \in \{0, ..., n_{\max}\}$ with $n_{\max} \leq k_{\max}$
2. derivatives evaluated at $Q^2 = \max[\,|-t_0|, 0.1\ \mathrm{GeV}^2\,]$;

Without offset from $Q^2 = 0$, intercept parameter a_0 would be ignored in $t_0 = 0$ fits

NME 2022 3-parameter Padé (ratio Q^2 polynomials)

$\implies$ treated same as a 3-parameter z expansion with $t_0 = 0$

LQCD Averaging Fits

Q^2_{compare}	$k_{\max}$	$n_{\max}$	ETM?	χ^2/DoF	p
	6	2	yes	56.40/13	2×10^{-7}
	6	2	no	16.63/10	0.08
	7	1	yes	6.60/ 7	0.47
	7	1	no	4.01/ 5	0.55
	6	1	yes	10.34/ 8	0.24
	6	**1**	**no**	**5.05/ 6**	**0.54**
min@$Q^2 = 0.05$ GeV^2	6	1	no	3.55/ 6	0.74
min@$Q^2 = 0.15$ GeV^2	6	1	no	7.59/ 6	0.26
all@$Q^2 = 0.15$ GeV^2	6	1	no	8.39/ 6	0.21
all@$Q^2 = 0.50$ GeV^2	6	1	no	9.37/ 6	0.15

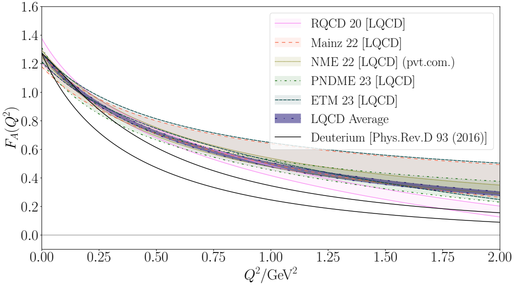
Good consistency between all except ETM result

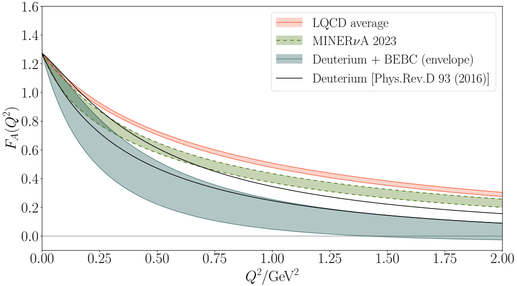
$\Longrightarrow$ ETM requires $k_{\max} = 7$ to get good fit, others sufficient at $k_{\max} = 6$

$\left(d/dQ^2\right)^2$ terms increase χ^2 significantly, not well constrained anyway $\Longrightarrow$ dropped

No large sensitivity to Q^2 where form factor matching occurs

LQCD Average vs Experiment





MINERvA consistent with LQCD central value

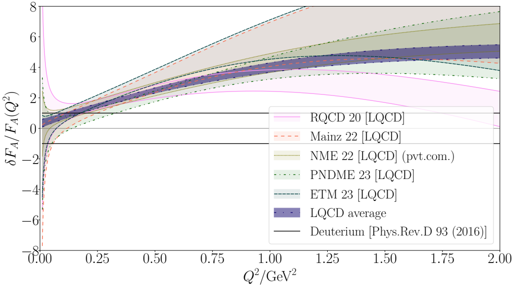
LQCD too precise to compare MINERvA minimum

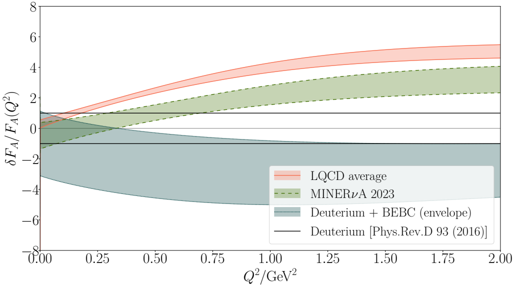
$\Longrightarrow$ consistency between LQCD/MINERvA

	LQCD $\{a_k\}$		MINERvA $\{a_k\}$	
	χ^2/DoF	p	χ^2/DoF	p
LQCD $\partial^n_{Q^2} F_A$	5.0/ 6	0.54	83.5/ 8	1×10^{-14}
MINERvA data	12.6/ 14	0.56	9.1/ 12	0.69

uncertainties	$F_A(0.50\ \mathrm{GeV}^2)$	r_A^2
LQCD average	2%	15%
MINERvA	6%	34%

LQCD Average vs Experiment





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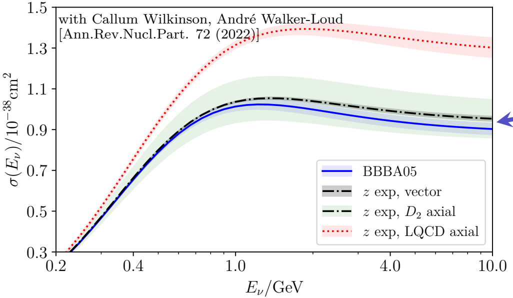
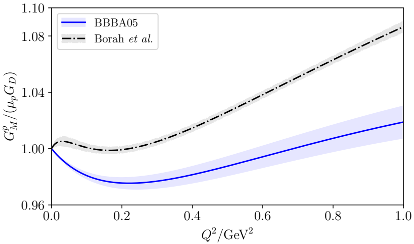
LQCD Implications

Free Nucleon Cross Section





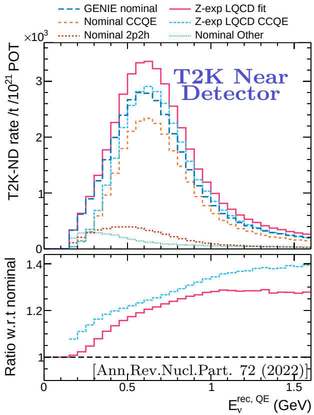
LQCD prefers 30-40% enhancement of ν_μ CCQE cross section

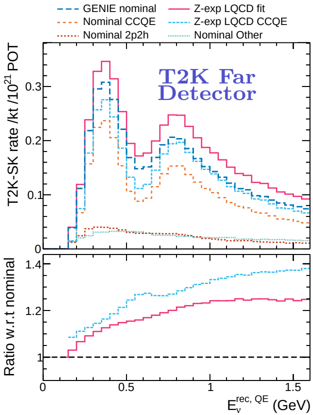
Recent Monte Carlo tunes require 20% enhancement of QE [Phys.Rev.D 105 (2022)] [Phys.Rev.D 106 (2022)]

With improved precision, sensitive to

- BBA vs z expansion vector FF difference [Phys.Rev.D 102 (2020)] [Nucl.Phys.B Proc.Suppl. 159 (2006)]
- isospin-breaking corrections? [Phys.Rev.Lett. 129 (2022)]

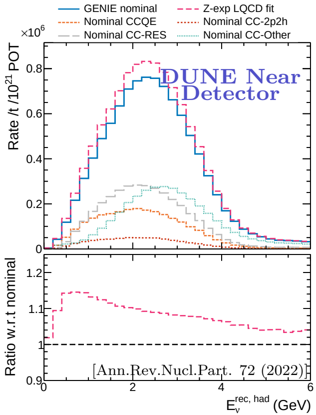
T2K Implications

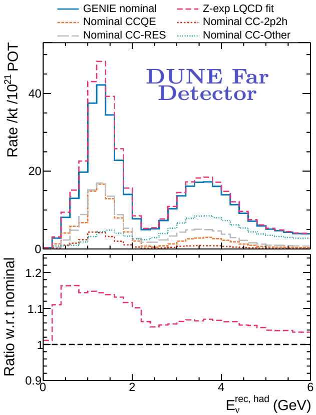




- Dashed dark blue (GENIE nominal) vs solid magenta (z exp LQCD fit)
- QE enhancements produce 10-20% event rate enhancement, E_ν-dependent
- Monte Carlo tuning invalidates more sophisticated comparisons

DUNE Implications





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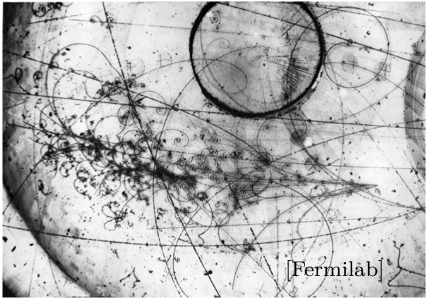
Concluding Remarks

Outlook

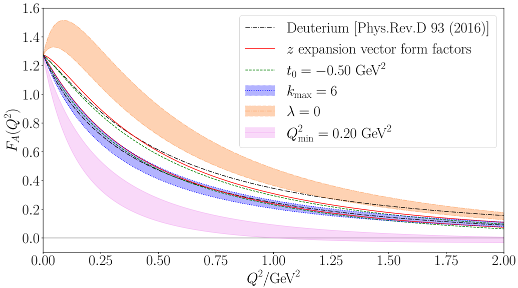


- Nucleon axial form factor uncertainty historically significantly underestimated
- Evidence that QE cross section underestimated, beyond published deuterium 1σ uncertainty band
- LQCD as proxy for (or complementary to) experimental data
- Indications of consistency between hydrogen/LQCD $\Longrightarrow$ experiment/theory parameterizations
- Fit deuterium shape inconsistent with hydrogen/LQCD shape
- Nontrivial implications of LQCD/hydrogen for oscillation experiments

Thank you for your attention!

Backup

Cumulative Updates to Deuterium



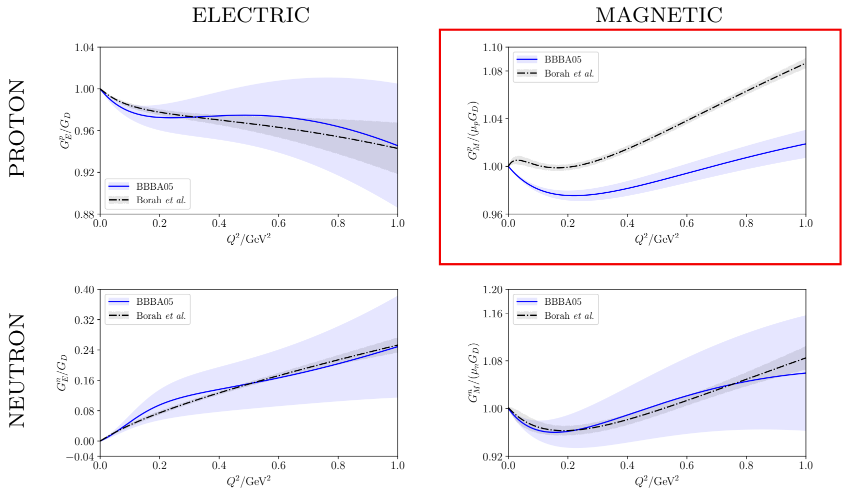
Cumulative changes between fits

$\implies$ moving down legend labels, fits include same modifications as fits above them

Fits all 1σ consistent until regularization removed

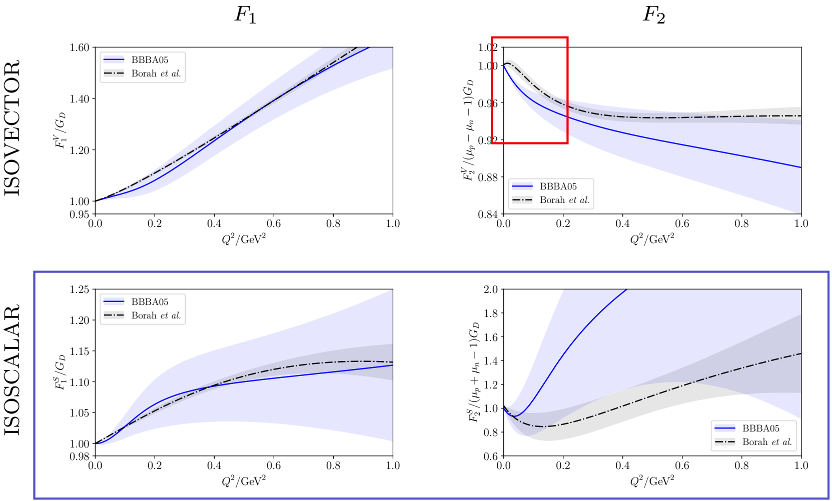
Q^2 cut emphasizes axial form factor + normalization degeneracy

Vector Form Factors - Proton/Neutron



Large tension in proton magnetic form factor

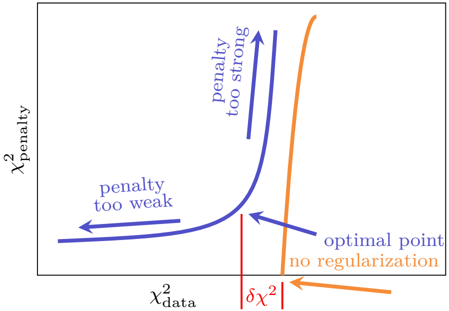
Vector Form Factors - Isospin Symmetric

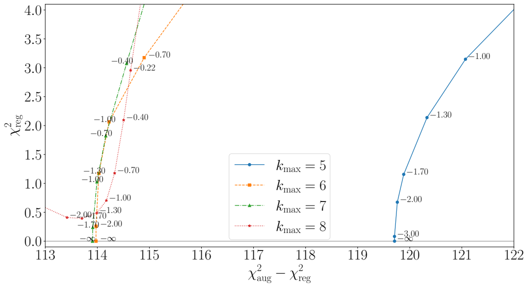


Uncertain slope of F_2^V

Large uncertainty on isoscalar form factors

L-curve Basics





$F_A(z) = \sum_{k=0}^{k_{\max}} a_k z^k$ L-curve heuristic to choose $k_{\max}$, λ

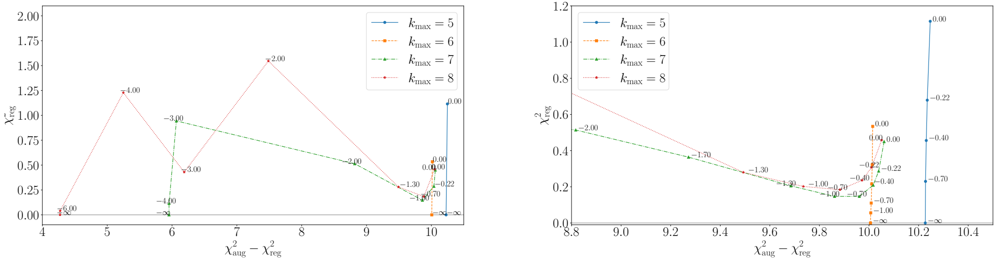
Optimal λ from minimum curvature on L-curve (or $\lambda = 0$), optimal $k_{\max}$ where $\delta\chi^2 < 1$

Regularization term: $\chi^2_{\text{reg}}(\lambda) = \lambda \sum_k \left|\frac{a_k}{\sigma_k}\right|^2$, $\sigma_k = |a_0| \cdot \min[5, 25/k]$; $\log_{10}\lambda$ printed on curves

L-curve Studies

Hydrogen

(zoomed)



Hydrogen preference for $k_{\max} = 5$, $\lambda = 0$

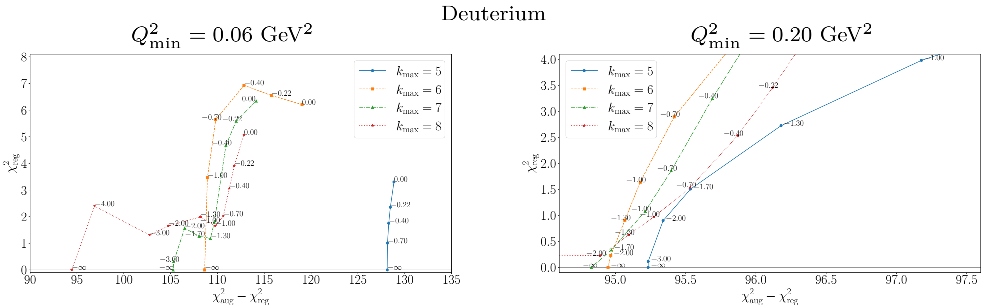
Deuterium preference depends on Q^2_{cut}; compromise $k_{\max} = 6$, $\lambda = 0$

$t_0 = -0.50\ \mathrm{GeV}^2$, $k_{\max} = 6$, $\lambda = 0$ for nominal studies here

$\implies$ similar quality to $k_{\max} \geq 7$, but no regularization

$t_0 = -0.28\ \mathrm{GeV}^2$, $k_{\max} = 8$, $\lambda = 1$ in published deuterium result [Phys.Rev.D 93 (2016)]

L-curve Studies



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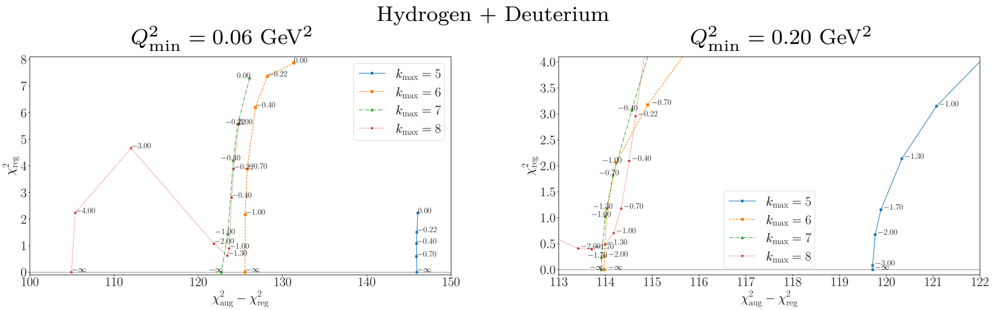

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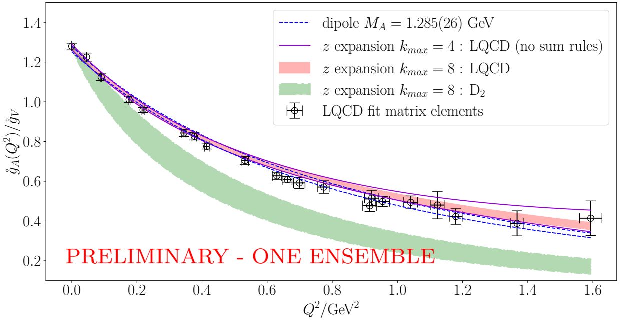
Axial Form Factor Fit

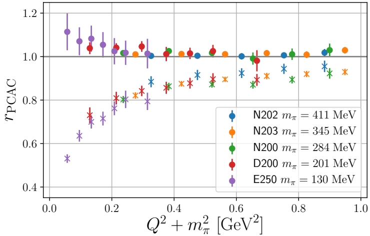


Trend of high-Q^2 enhancement seen in other LQCD results

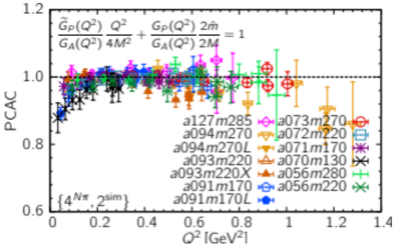
2–4% LQCD uncertainty vs 10% uncertainty on D_2 result

PCAC Checks

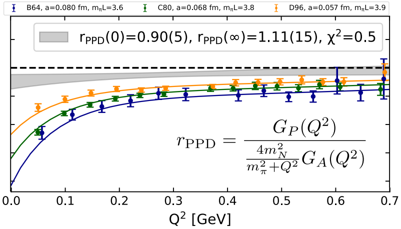
RQCD [JHEP 05 (2020)]



NME [prelim]

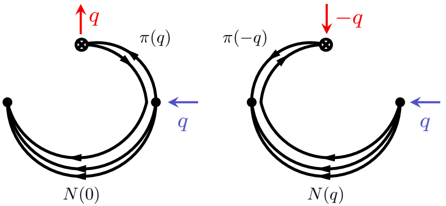


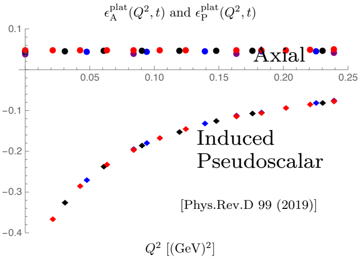
ETMC [prelim]



- Relation btw F_A, F_P, $\tilde{F}_P$ via PCAC
- Contamination in F_A and $\tilde{F}_P$, F_P very different
  $\implies$ nontrivial consistency check
  [Phys.Rev.D 99 (2019)]

LQCD Excited States — χPT and $N\pi$





Contamination in $g_A(Q^2)$ primarily from enhanced $N\pi$, mostly from induced pseudoscalar

Correlator fits without axial current not sensitive to $N\pi$ [Phys.Rev.C 105 (2022)] [Phys.Rev.D 105 (2022)]

Alternate fit strategies:

- explicit $N\pi$ operators
- include $\mathcal{A}_4$ (strong $N\pi$ coupling)
- Kinematic constraints ($F_P = 0$)

Prediction from χPT: [Phys.Rev.D 99 (2019)]

First demonstration of $N\pi$: [Phys.Rev.Lett. 124 (2020)]

χPT-inspired fit methods for fitting form factor data
[Phys.Rev.D 105 (2022)] [JHEP 05 (2020) 126]