Status and prospects of uncertainty treatment in nuclear theory models

Raúl González Jiménez

University of Seville, Spain

Grant **PID2021-127098NA-I00** and **RYC2022-035203-I** funded by MICIU/AEI/10.13039/501100011033, "ERDF a way of making Europe" and FSE+.

Workshop on "Measuring neutrino interactions for next-generation oscillation experiments"

ECT*, Trento, October 22, 2024

Nuclear model uncertainties in neutrino-nucleus interactions

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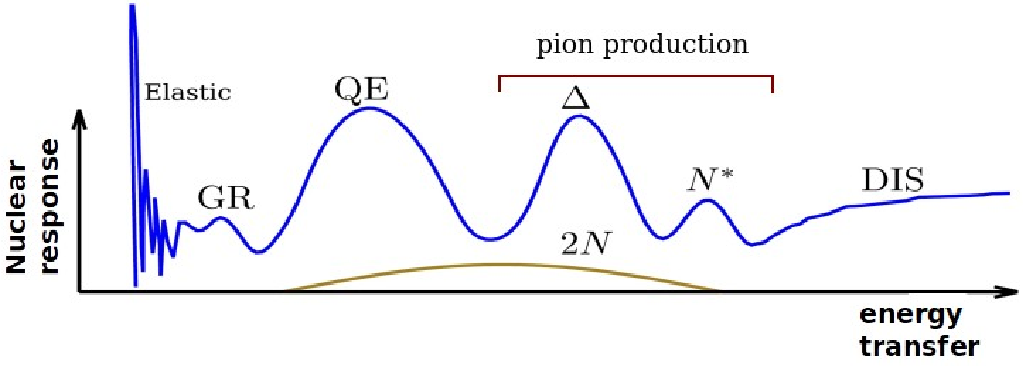
How do we model

Quasielastic scattering
and
Single-Pion production?



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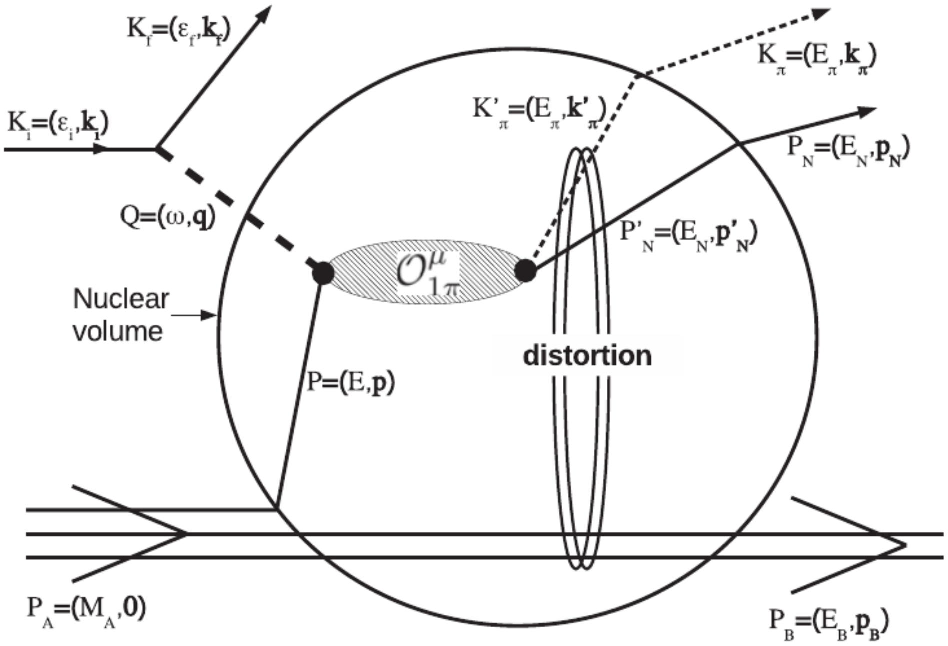
…

The IMPULSE APPROXIMATION (IA)

Single-Pion Production
(in the Impulse Approximation)



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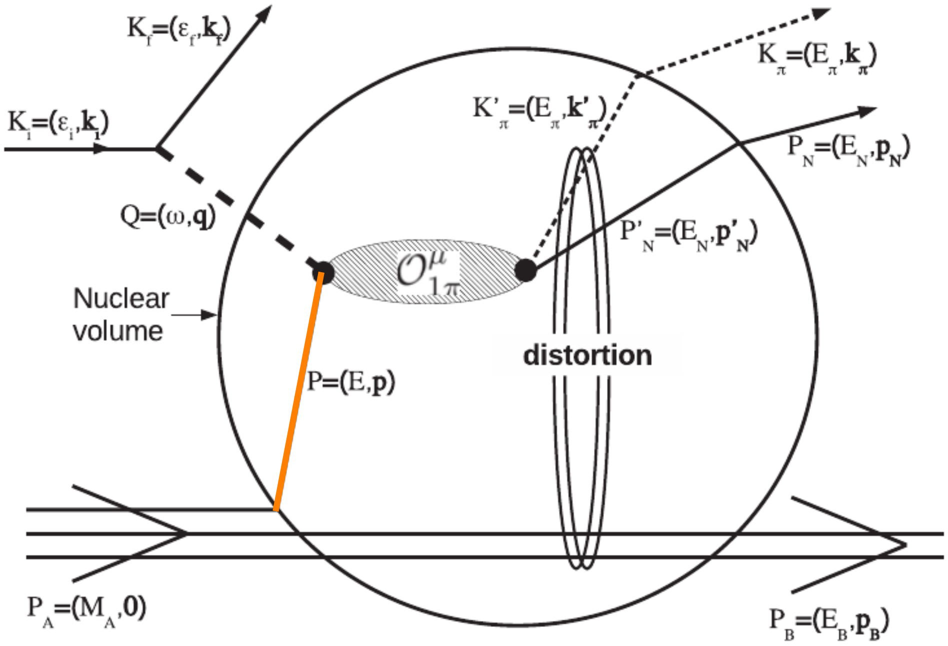

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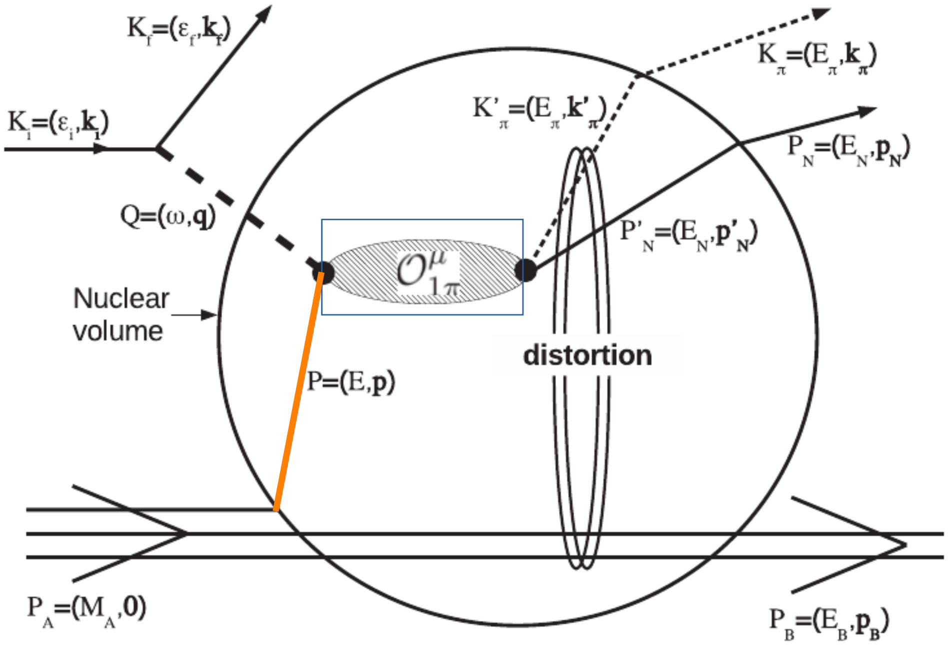

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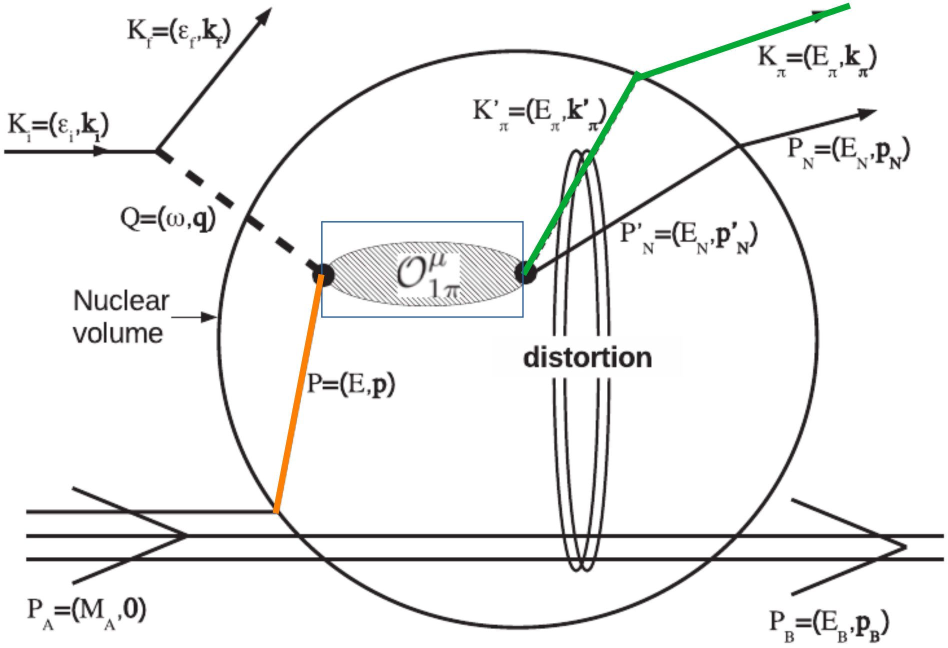

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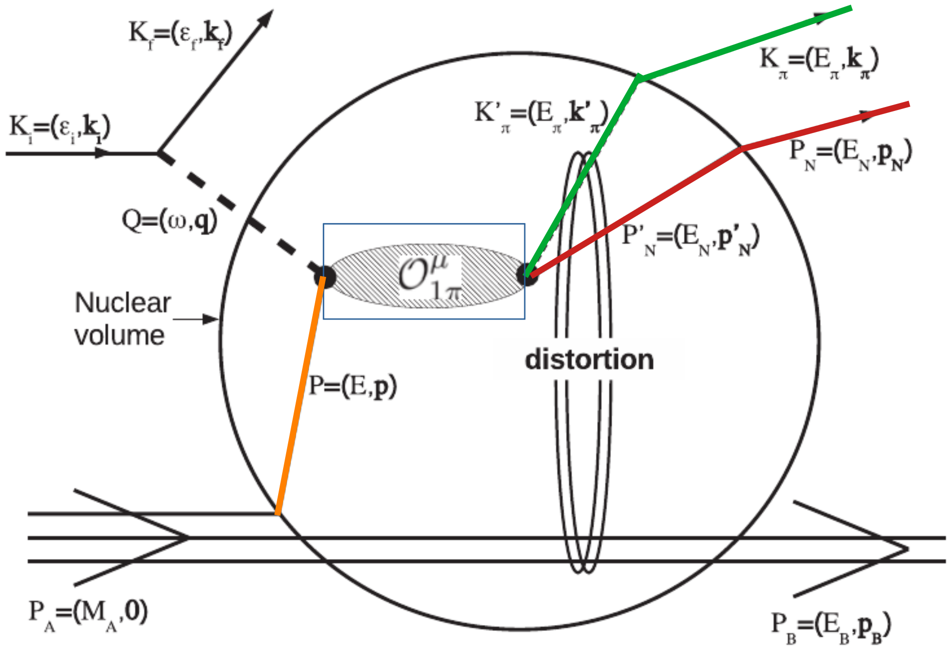

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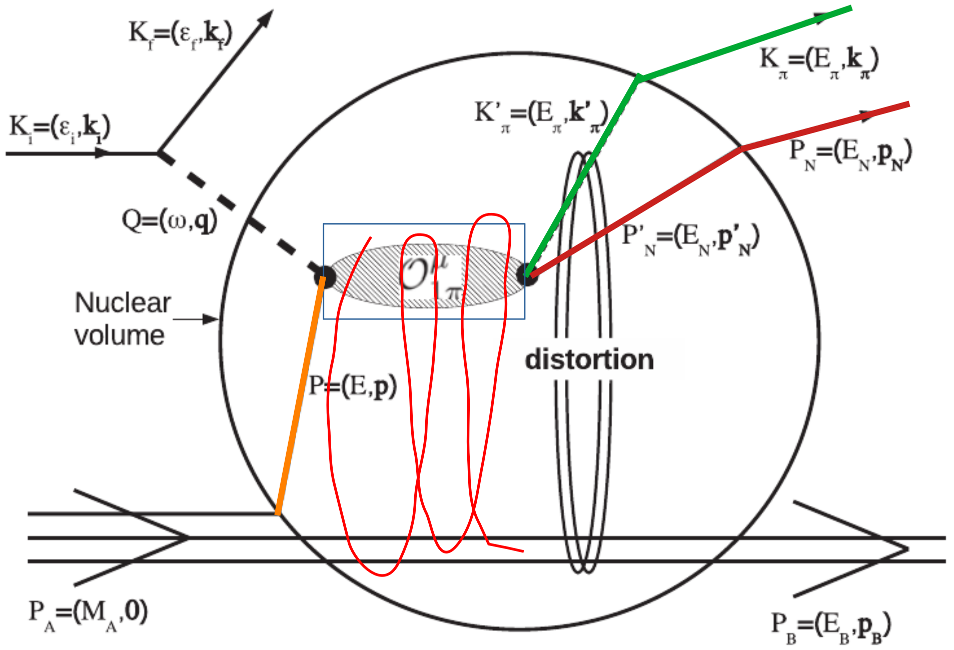

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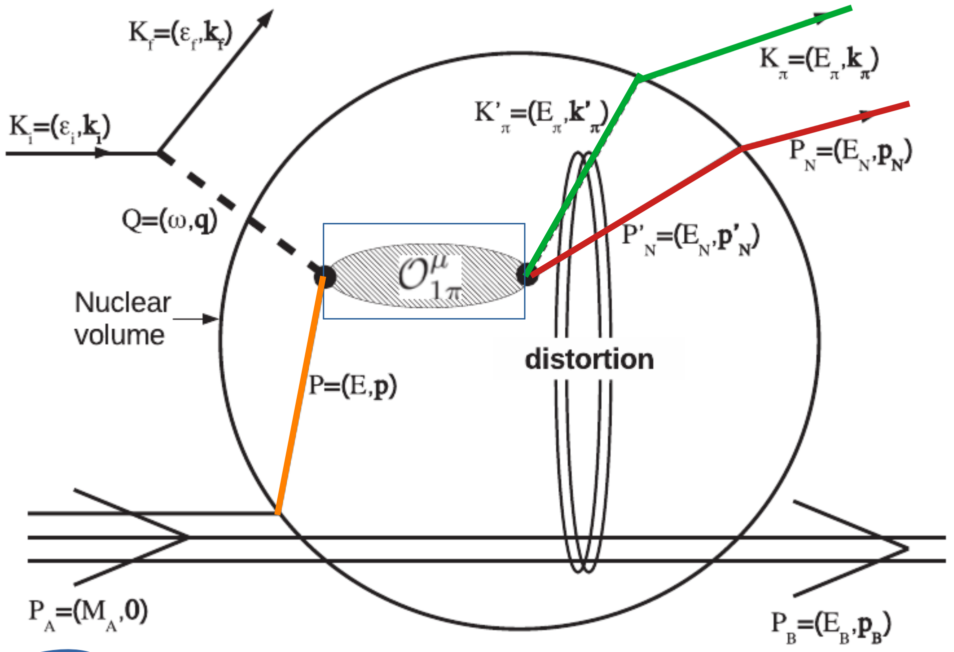

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From complex to simple

Amaro et al. https://doi.org/10.1088/1361-6471/abb128
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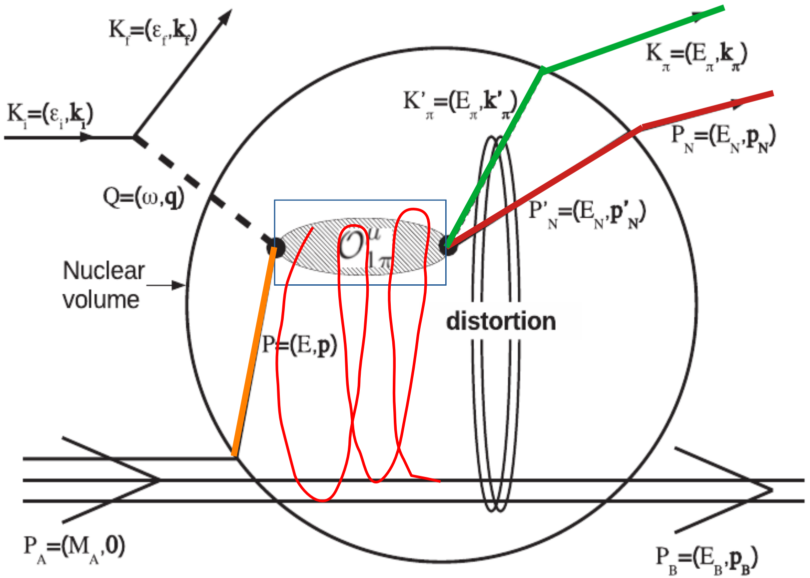

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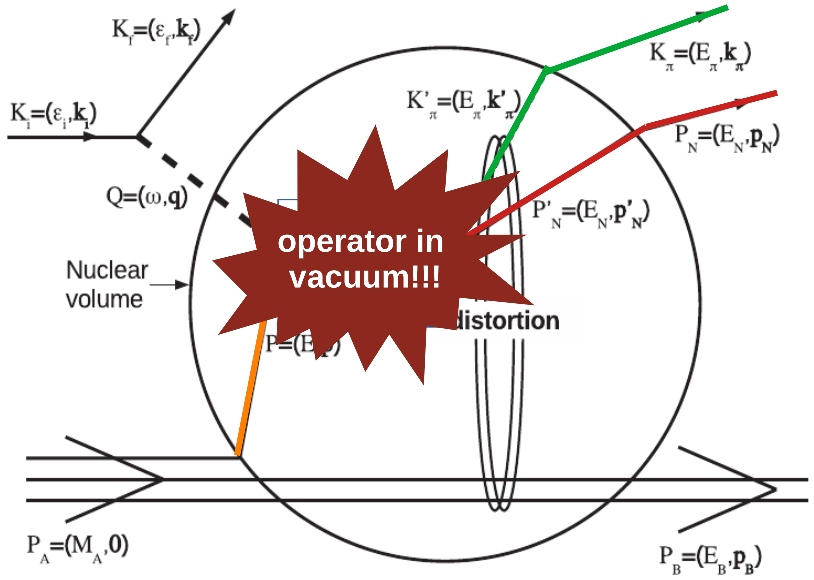

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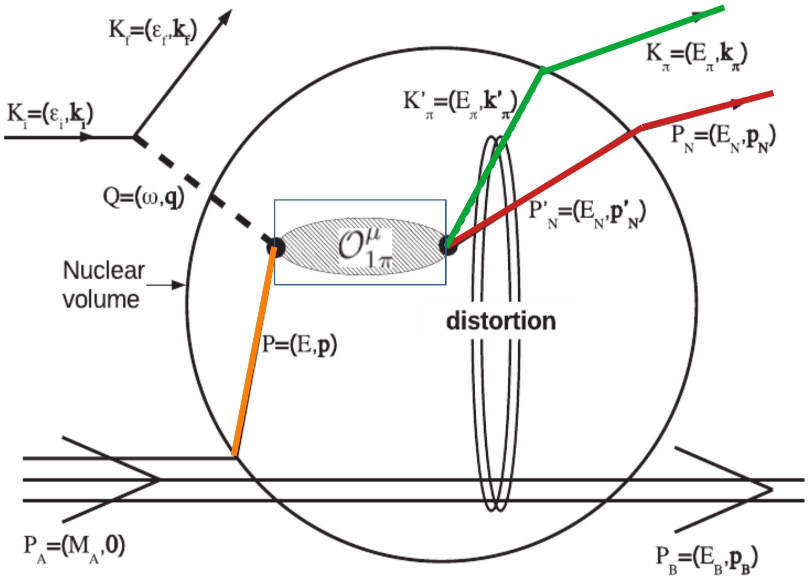

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Asymptotic (or local) operator approximation

$$J^\mu_{\rm had} = \int d\mathbf{p} \int d\mathbf{p}'_N \overline{\Psi}_F(\mathbf{p}'_N,\mathbf{p}_N)\phi^*_\pi(\mathbf{p}+\mathbf{q}-\mathbf{p}'_N,\mathbf{k}_\pi)\mathcal{O}^\mu_{\rm asymp.}(Q,K_\pi,P_N)\Psi_B(\mathbf{p})$$

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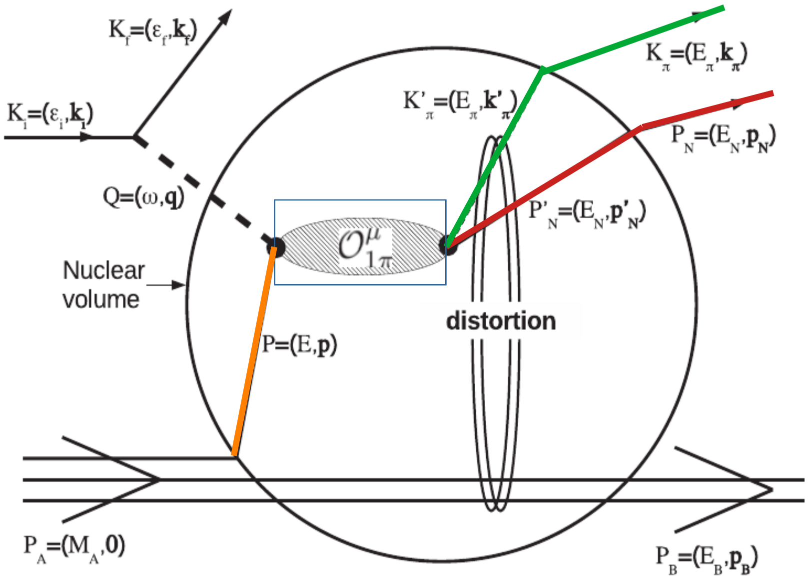

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Asymptotic (or local) operator approximation
In coordinate space

$$J^\mu_{\rm had} = \int d\mathbf{r} \int d\mathbf{r}' \overline{\Psi}_F(\mathbf{r}', \mathbf{p}_N) \phi^*_\pi(\mathbf{r}', \mathbf{k}_\pi) \mathcal{O}^\mu_{\rm local}(Q, K_\pi, P_N) \Psi_B(\mathbf{r}) e^{i\mathbf{q}\cdot\mathbf{r}}$$

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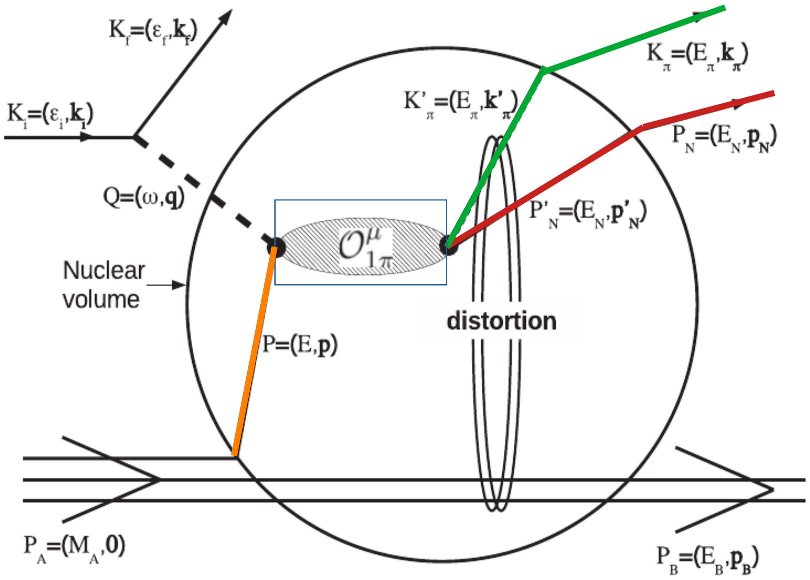

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Pion as a plane wave

$$J^\mu_{\rm had} = \frac{1}{\sqrt{2E_\pi}} \int d\mathbf{p} \overline{\Psi}_F(\mathbf{q}+\mathbf{p}-\mathbf{k}_\pi, \mathbf{p}_N) \mathcal{O}^\mu_{1\pi}(Q, P, K_\pi) \Psi_B(\mathbf{p})$$

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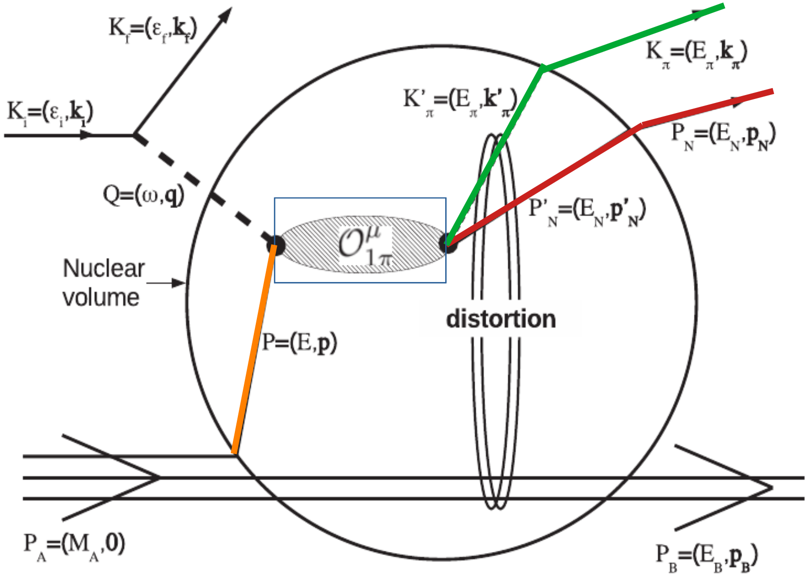

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Pion and final nucleon as plane waves

$$J^{\mu}_{\rm had} = (2\pi)^{3/2} \sqrt{\frac{M_N}{2E_\pi E_N}}\, \overline{u}(\mathbf{p}_N, s_N) \mathcal{O}^{\mu}_{1\pi}(Q, P_N, K_\pi) \Psi_B(\mathbf{p}_N + \mathbf{k}_\pi - \mathbf{q})$$

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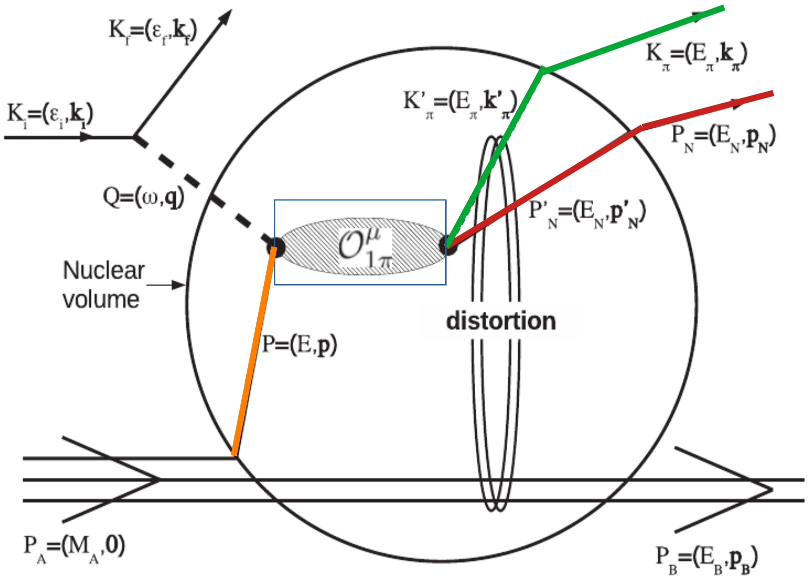

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Pion and final nucleon as plane waves

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All particles as **plane waves**

$$J^\mu_{\rm had} = (2\pi)^3 \delta^3(\mathbf{p}_N + \mathbf{k}_\pi - \mathbf{q} - \mathbf{p}) \sqrt{\frac{M_N^2}{2E_\pi E_N E}}\, \overline{u}(\mathbf{p}_N, s_N) \mathcal{O}^\mu_{1\pi}(Q, P_N, K_\pi) u(\mathbf{p}, s)$$

After each approximation, we left some physics behind…

As a consequence, we should somehow try to evaluate the error that we have introduced.

But that’s difficult, **because** after each approximation the approach (or framework) is different, new parameters *or new features* are introduced to effectively account for the physics that was left out.
(Examples of this: the role of correlations in Fermi gas based models versus mean-field approaches, non-relativistic approaches versus relativistic ones.)

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IMPORTANT, we should work with models that have passed the **electron-scattering test**.
If they don't, not worth it.

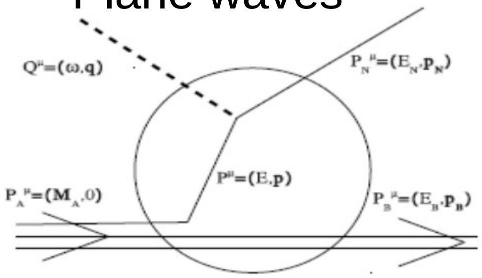
We cannot validate **nuclear models** using neutrino data. (New high-statistics measurements may change this picture.)
An example in what follows.

Pauli blocking and elastic FSI

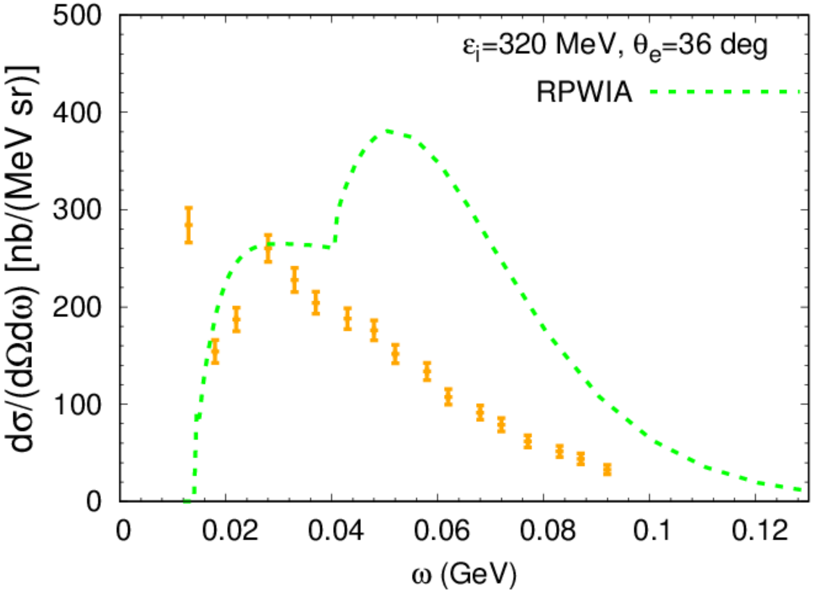
Pauli blocking and elastic FSI

Inclusive electron scattering **at low q:**

Plane waves



Phys. Rev. C 100 045501 (2019)



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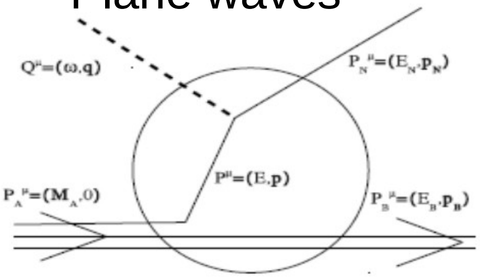

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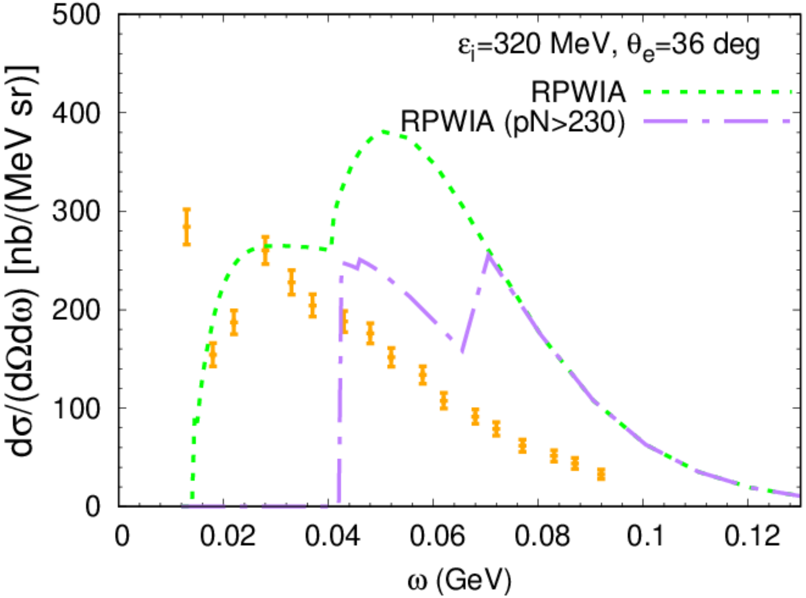

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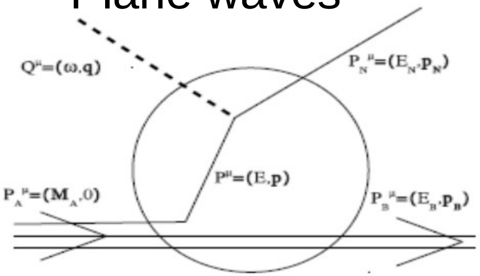

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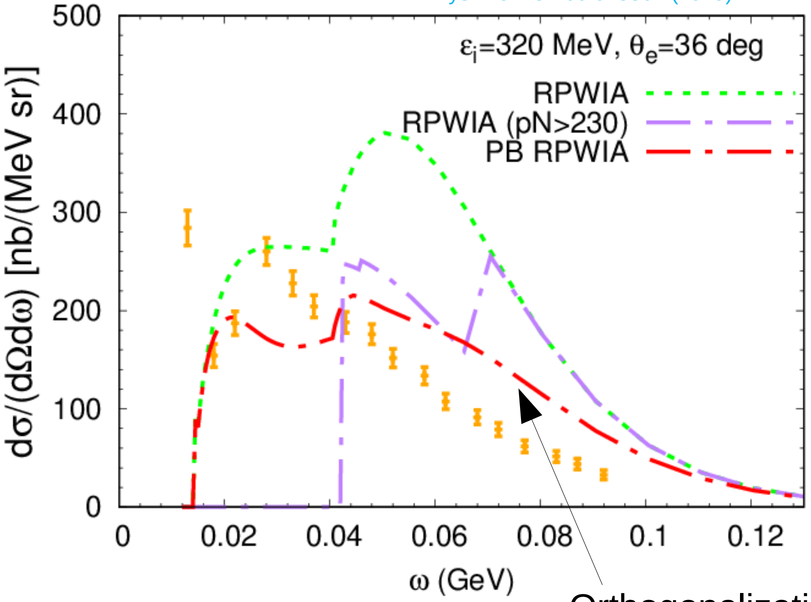

Orthogonalization

$$|\Psi^{s_N}(\mathbf{p}_N)\rangle = \left|\psi^{s_N}_{pw}(\mathbf{p}_N)\right\rangle - \sum_{\kappa,m_j}\left[C^{m_j,s_N}_{\kappa}(\mathbf{p}_N)\right]^{\dagger}\left|\psi^{m_j}_{\kappa}\right\rangle$$

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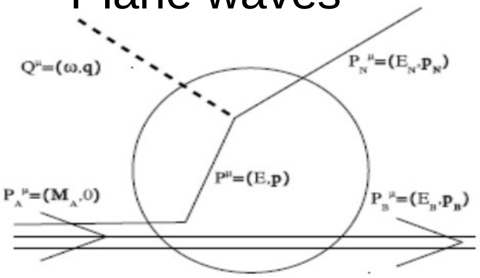

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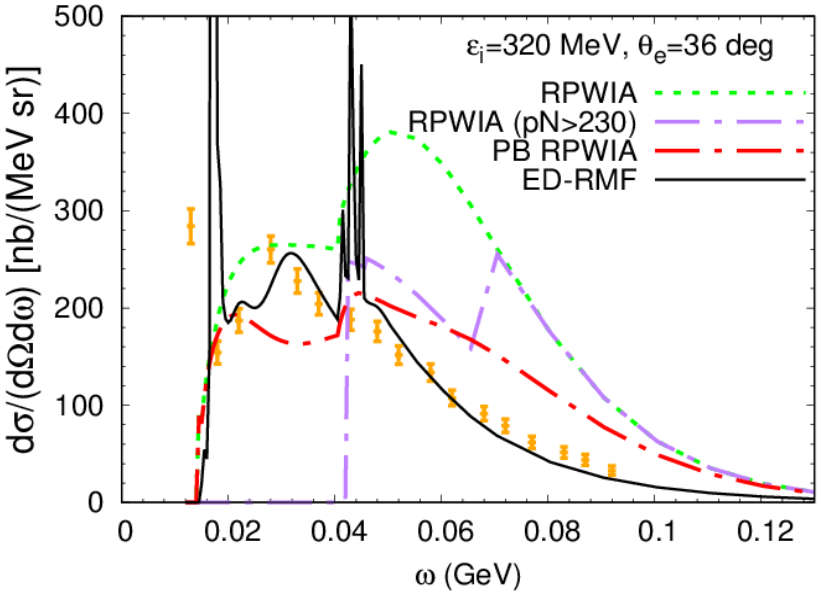
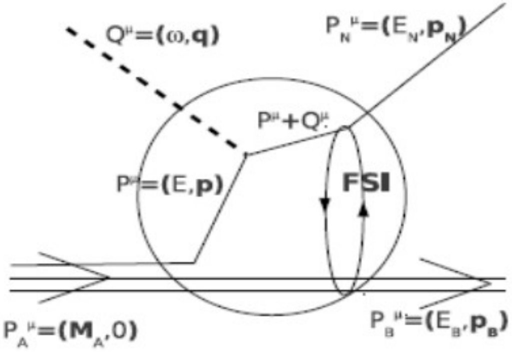


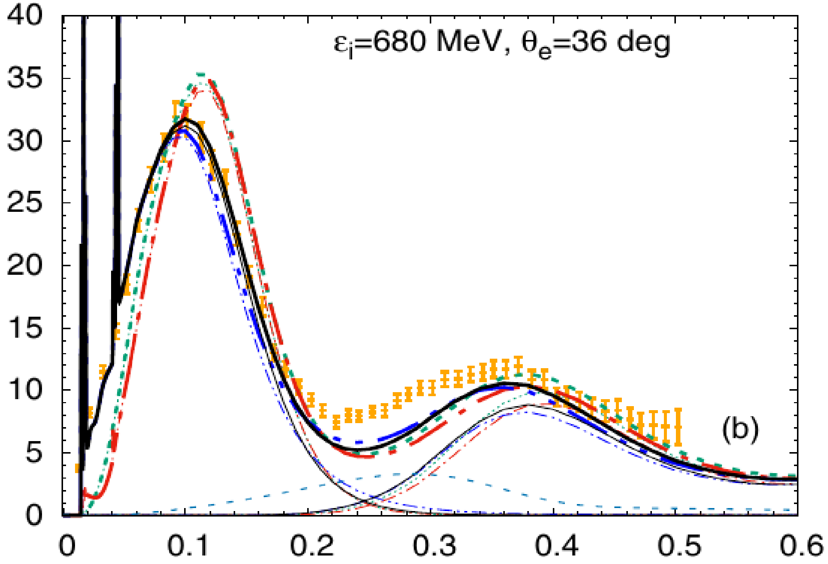
Distorted waves

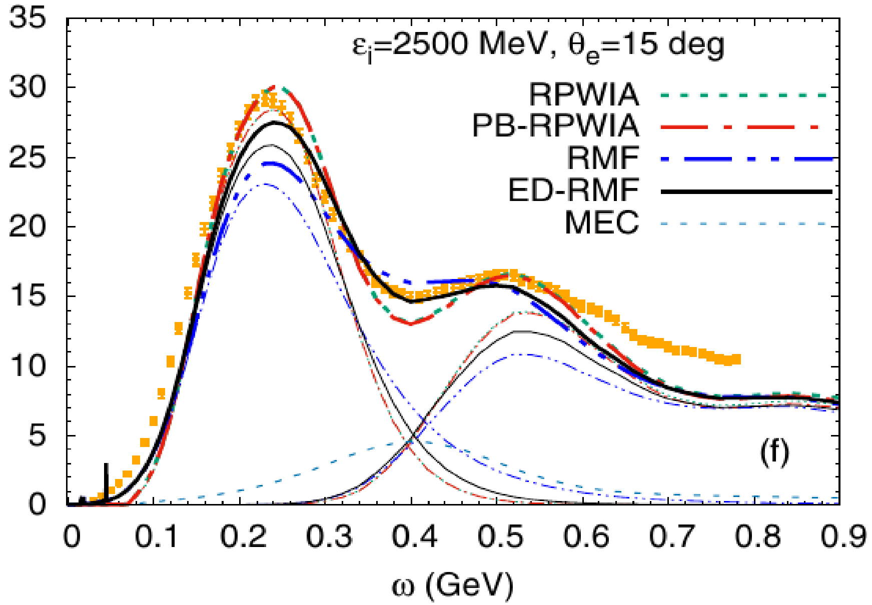


Pauli blocking and elastic FSI

Inclusive electron scattering **at intermediate q:**

Phys. Rev. C 100 045501 (2019)

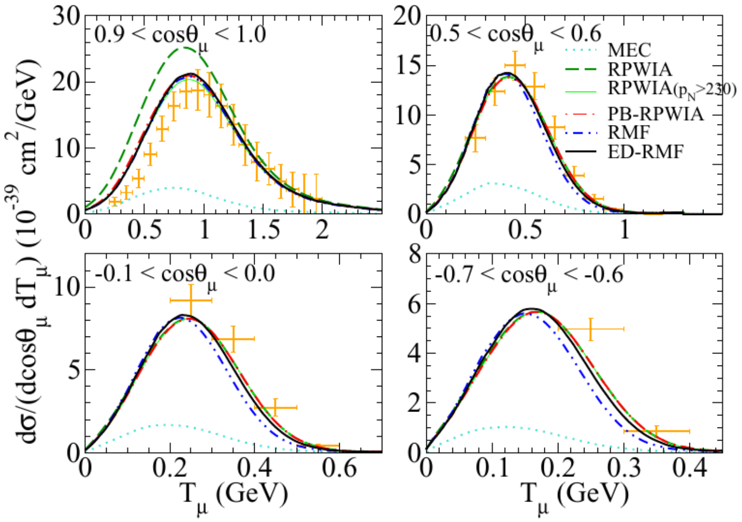


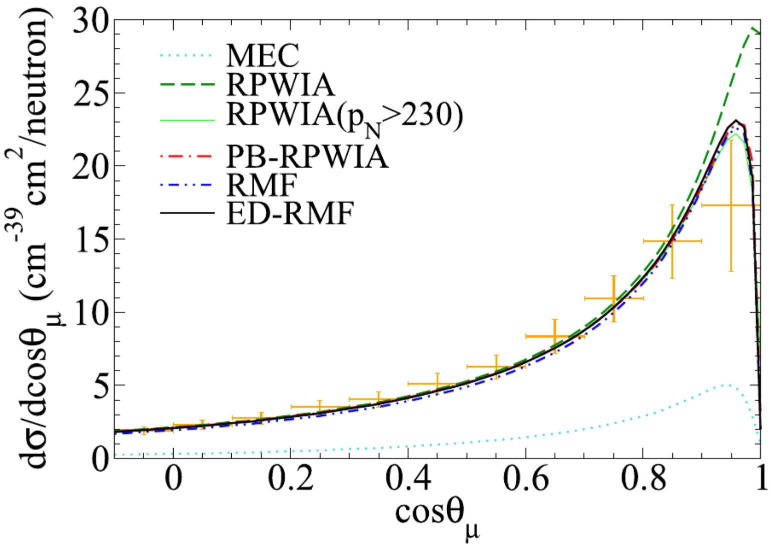


Distortion of the outgoing nucleon (elastic FSI in a Quantum Mechanical way) is important at intermediate energies too !!!

Pauli blocking and elastic FSI

MicroBooNE data, neutrino-nucleus CCQE-like scattering:





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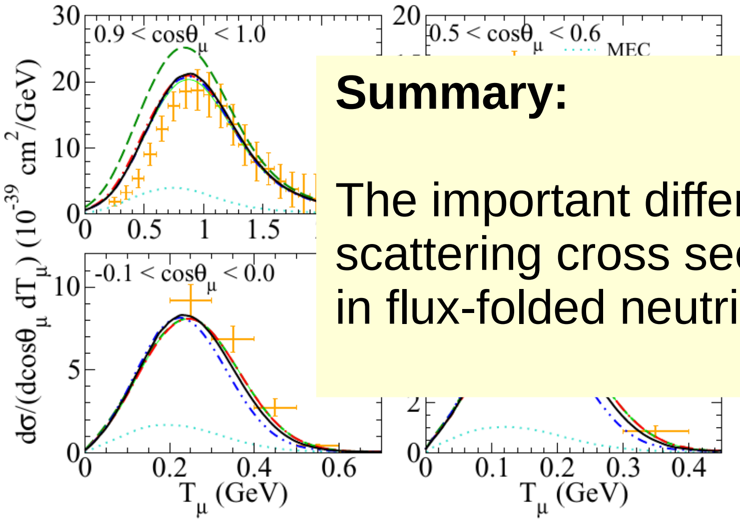
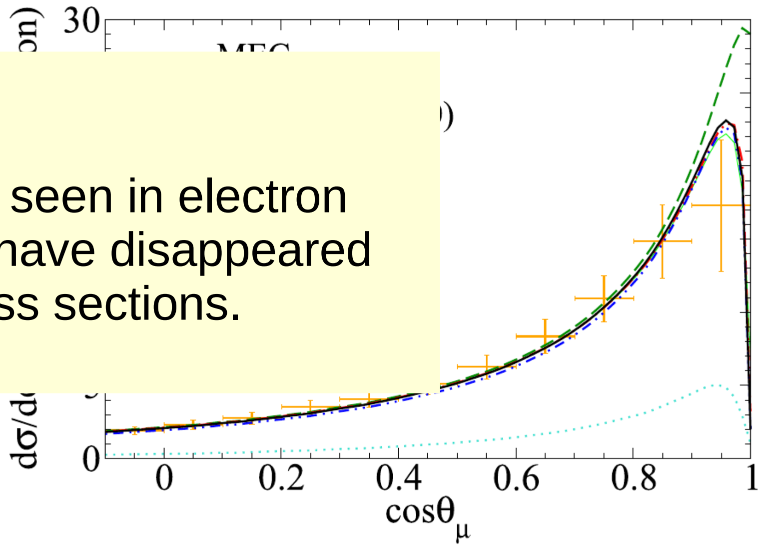


Summary:

The important differences seen in electron scattering cross sections have disappeared in flux-folded neutrino cross sections.



https://doi.org/10.1103/PhysRevC.100.045501

Sources of uncertainties, three categories:

+ Uncertainties in the modeling of the **neutrino-nucleon** interaction: photon- and electron-induced reactions are useful to constrain vector current, but one needs neutrino data for the axial part. Parity violating electron scattering could be useful too.

+ Uncertainties due to the **nuclear model**: no need for neutrino data, one could use photon- and electron-induced reactions.

+ Uncertainties in the **cascade model**. Assuming we want to use a cascade approach, we need to somehow find ways to constrain it.

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+ Uncertainties due to the **nuclear model**: no need for neutrino data, one could use photon- and electron-induced reactions.

+ Uncertainties in the **cascade model**. Assuming we want to use a cascade approach, we need to somehow find ways to constrain it.

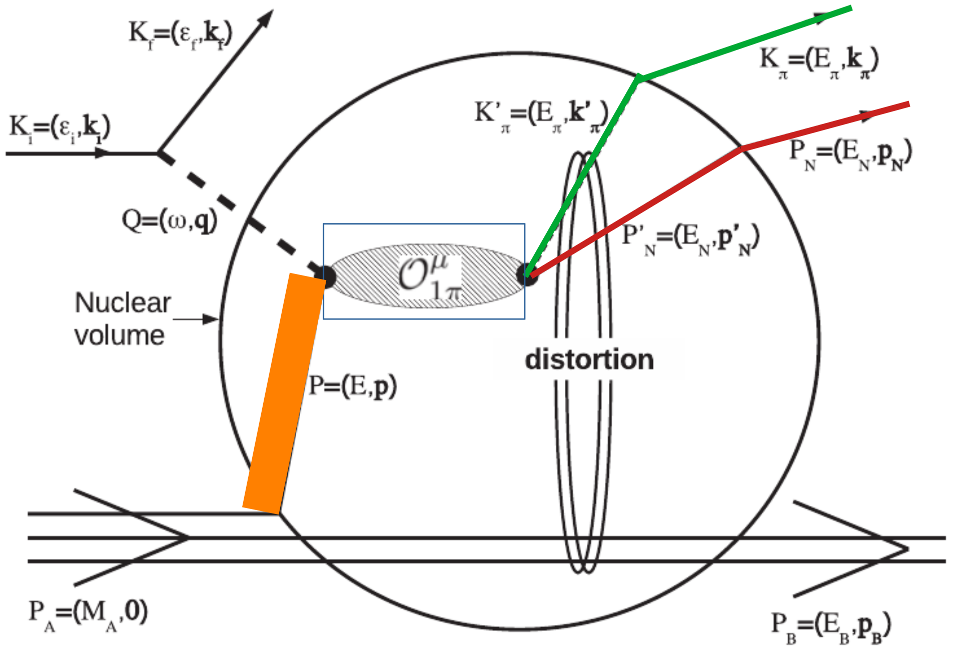
Small list of **nuclear effects** in the cross sections:

+ **Initial state**: binding energy, Fermi motion, Pauli blocking, short- and long-range correlations

+ **Interaction:** everything beyond one-body current is very difficult, modification of the interaction operator due to in medium effects

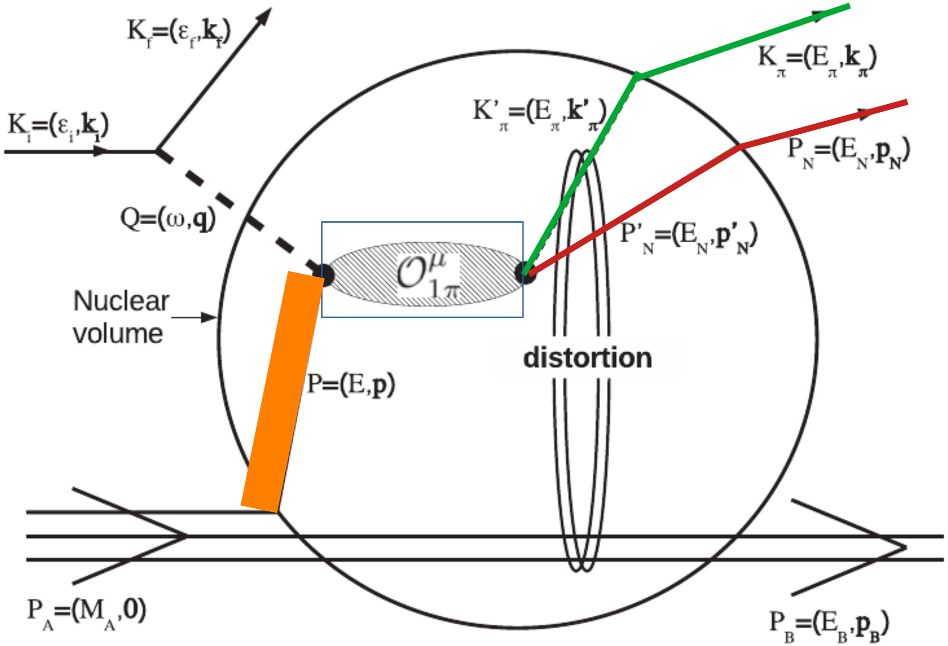
+ **Final state interactions**: distortion effects or elastic FSI, Pauli blocking, inelastic FSI (cascade)

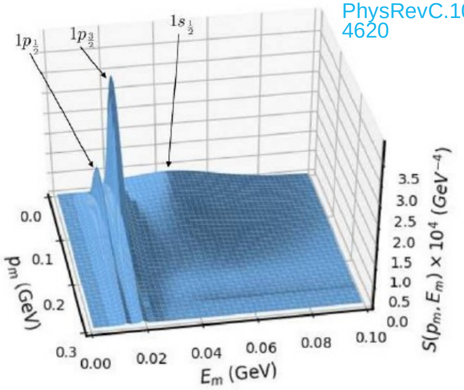
Single-Pion Production
(in the Impulse Approximation)



$$J^{\mu}_{\rm had} = \int d\mathbf{p} \int d\mathbf{p}'_N \overline{\Psi}_F(\mathbf{p}'_N, \mathbf{p}_N) \phi^*_\pi(\mathbf{p} + \mathbf{q} - \mathbf{p}'_N, \mathbf{k}_\pi) \mathcal{O}_{1\pi}(Q, P'_N, P) \Psi_B(\mathbf{p})$$

Single-Pion Production
(in the Impulse Approximation)





PhysRevC.100.044620

Initial state

$$J^{\mu}_{\mathrm{had}} = \int d\mathbf{p} \int d\mathbf{p}'_N \overline{\Psi}_F(\mathbf{p}'_N, \mathbf{p}_N)\phi^*_{\pi}(\mathbf{p}+\mathbf{q}-\mathbf{p}'_N, \mathbf{k}_{\pi})\mathcal{O}_{1\pi}(Q, P'_N, P)\Psi_B(\mathbf{p})$$

https://arxiv.org/abs/2203.09996v3

^{40}Ca independent particle shell model

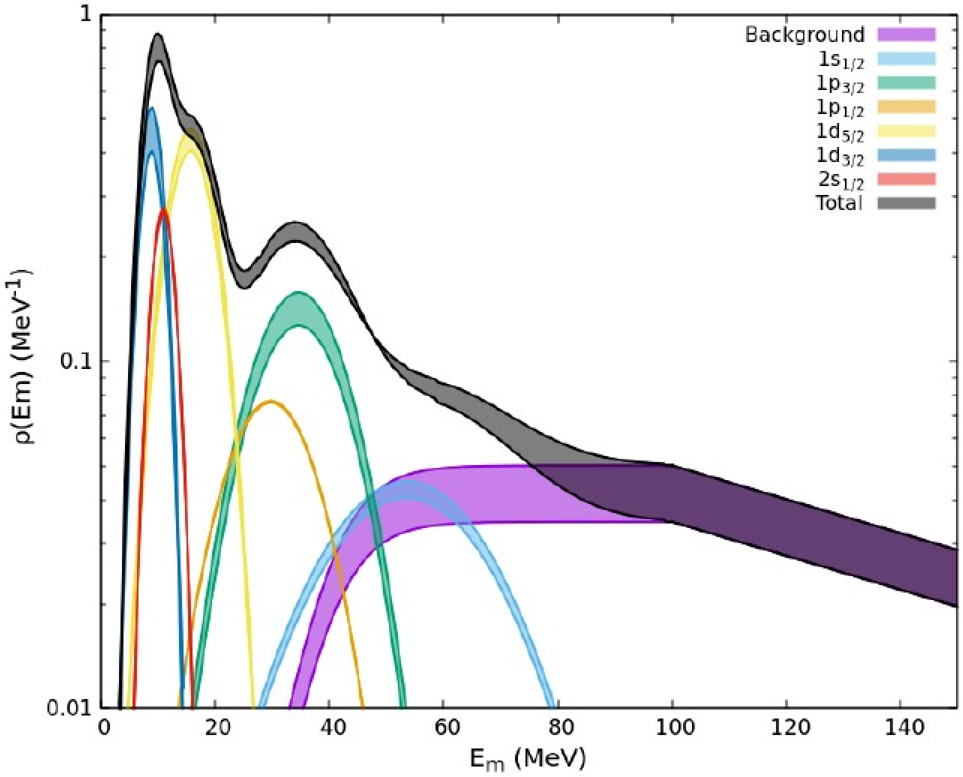
UNIVERSIDAD COMPLUTENSE MADRID
gfn UCM Grupo de Física Nuclear

- *Realistic* treatment of nuclear structure:
  - Reduced shell model occupations

(The total number of nucleons is preserved = 20p+20n)

Shell model state	Occupation probability
$1d_{3/2}$	0.5 – 0.7
$2s_{1/2}$	0.5 – 0.7
$1d_{5/2}$	0.6 – 0.8
$1p_{3/2}+1p_{1/2}$	0.6 – 0.8
$1s_{1/2}$	0.7 – 0.85

  - Background due to short range correlations
  - Continuous missing energy profile



https://arxiv.org/abs/2203.09996v3

^{40}Ca electromagnetic inclusive cross section

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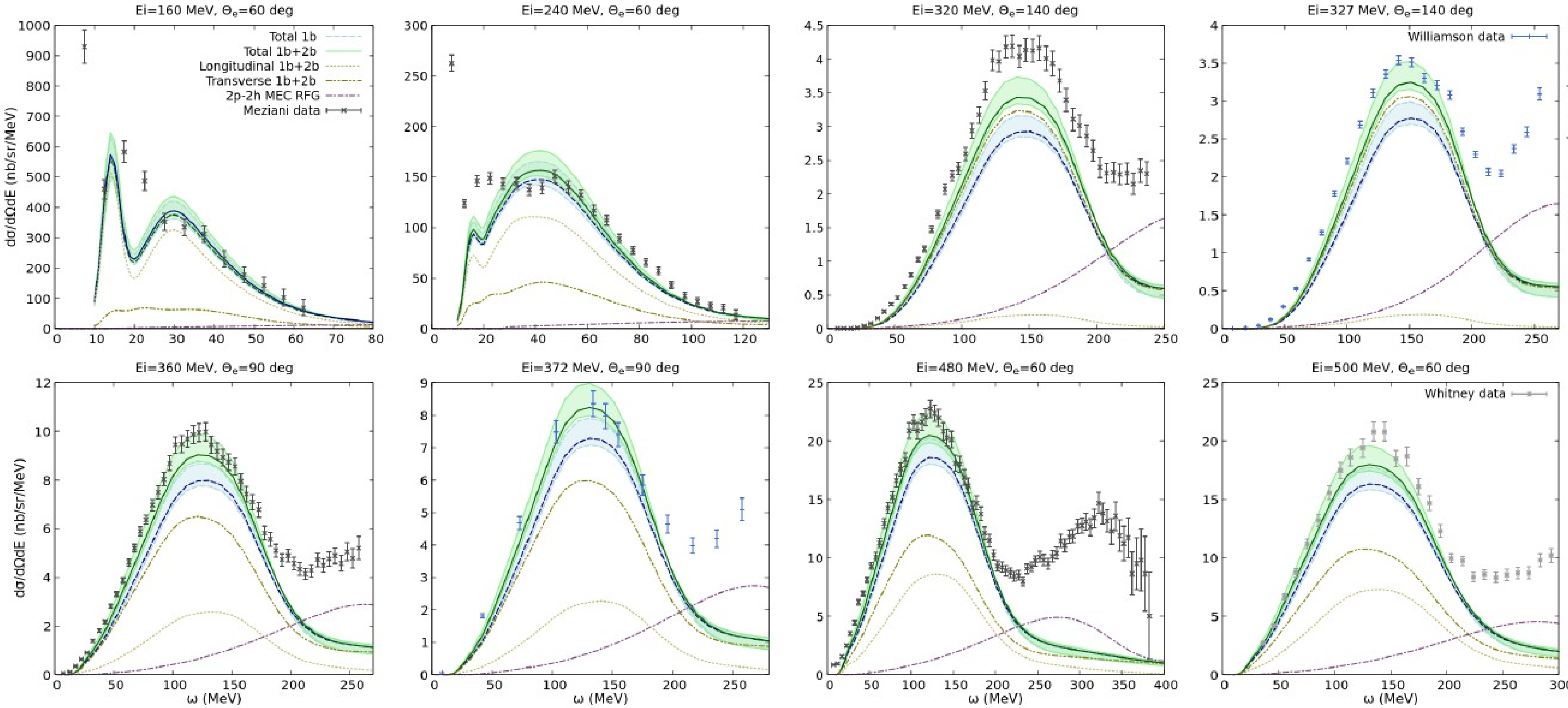


Data: discovery.phys.virginia.edu/research/groups/qes-archive/data/40Ca.html

https://arxiv.org/abs/2203.09996v3
https://arxiv.org/abs/2306.10823

Beyond Impulse Approximation: two-body currents in the 1p-1h sector

$$J^{\mu}_{had} = \int d\mathbf{p}\, \overline{\Psi}_F(\mathbf{p}+\mathbf{q}, \mathbf{p}_N) \left(\mathcal{O}^{\mu}_{\text{one body}} + \mathcal{O}^{\mu}_{\text{two body}}\right) \Psi_B(\mathbf{p})$$

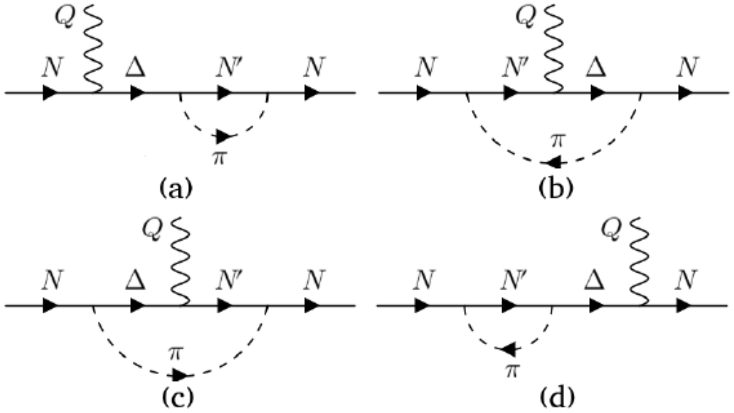


FIG. 1. Delta contributions.

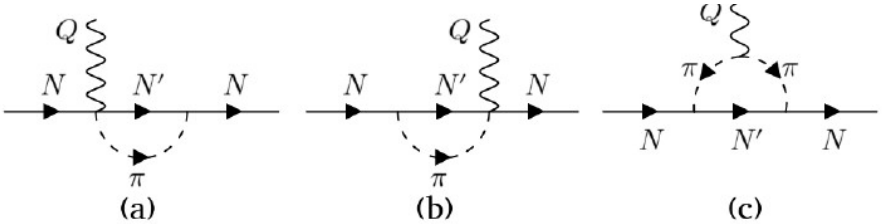
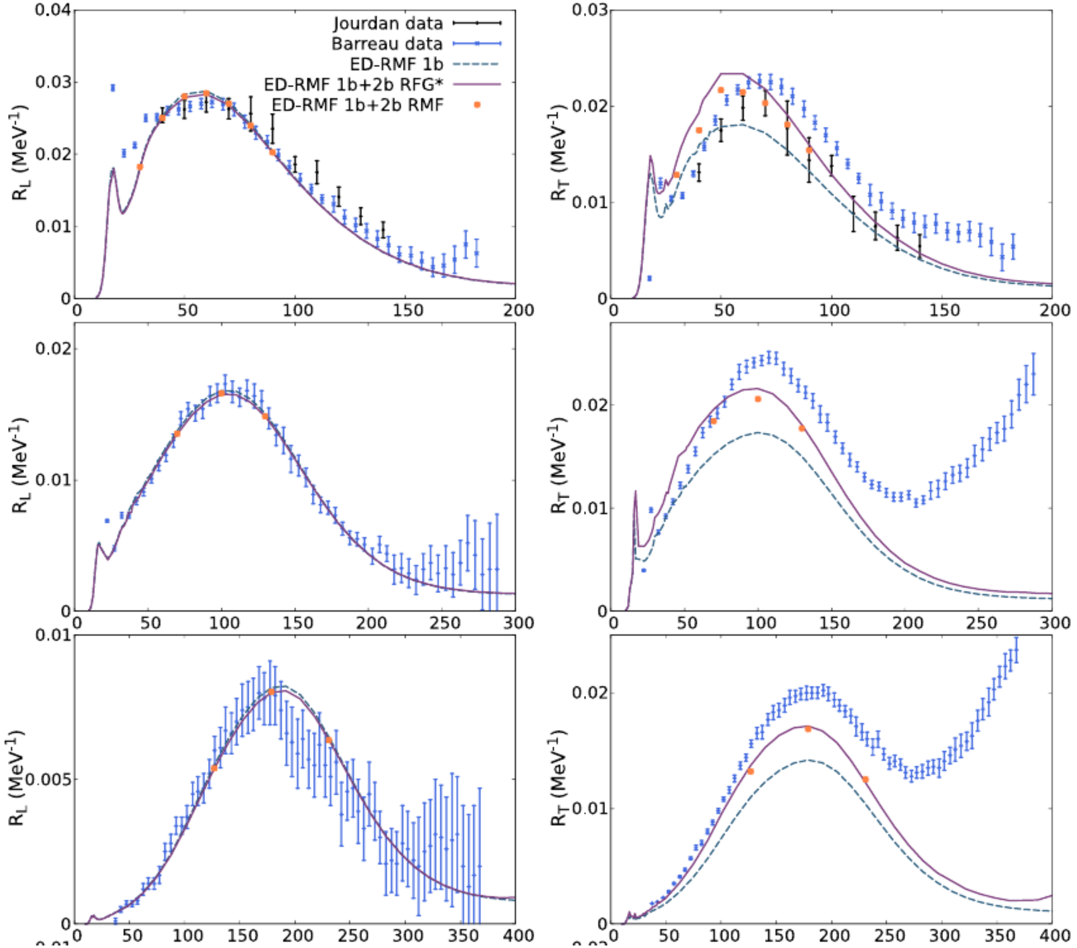


FIG. 2. Background contributions: seagull or contact [CT, (a) and (b)] and pion-in-flight [PF, (c)].

https://arxiv.org/abs/2306.10823

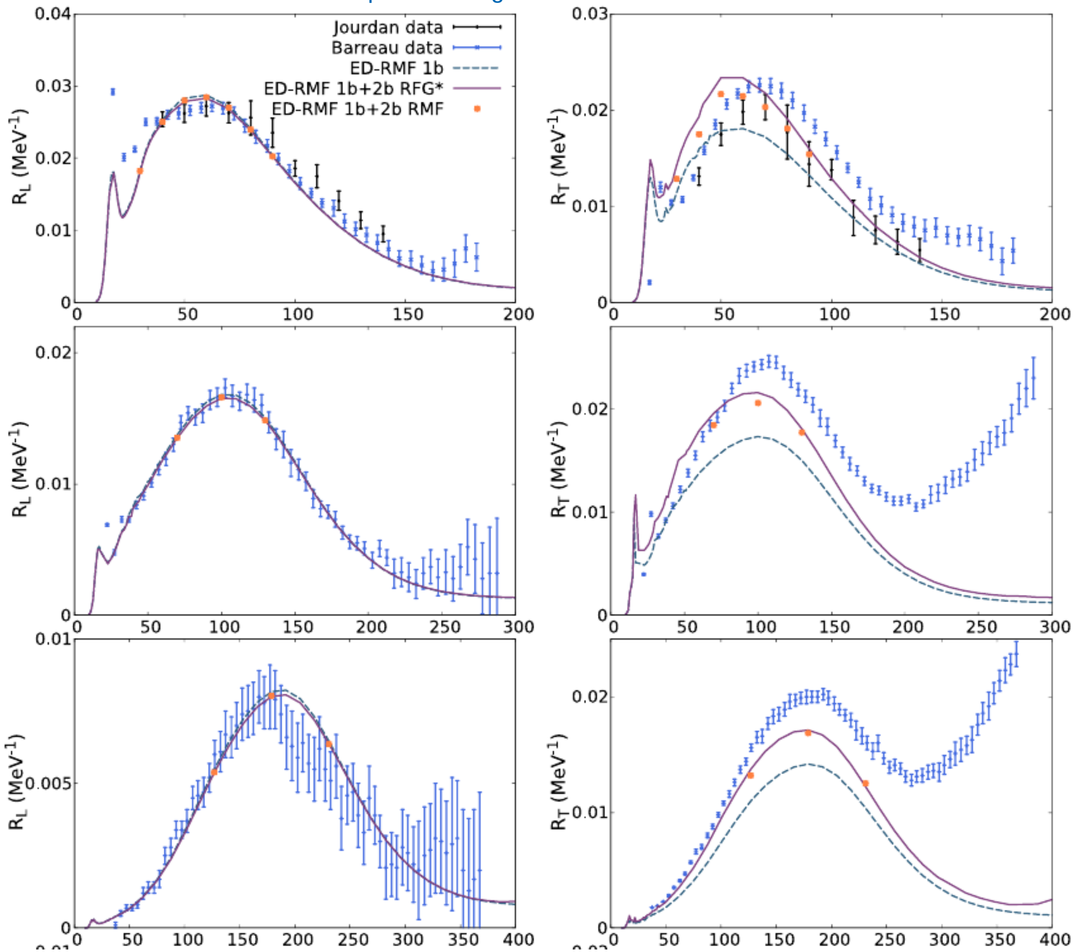


Caption: ^{12}C longitudinal (left) and transverse (right) responses. The transferred momentum q is (from up to bottom) 300, 400 and 550 MeV/c. Results: the **intermediate bound-nucleon state** is described in terms of free particles in an RFG with a modified mass and energy (**RFG***), and **RMF** nucleons.

The full model is too expensive computationally, so we have to make approximations…

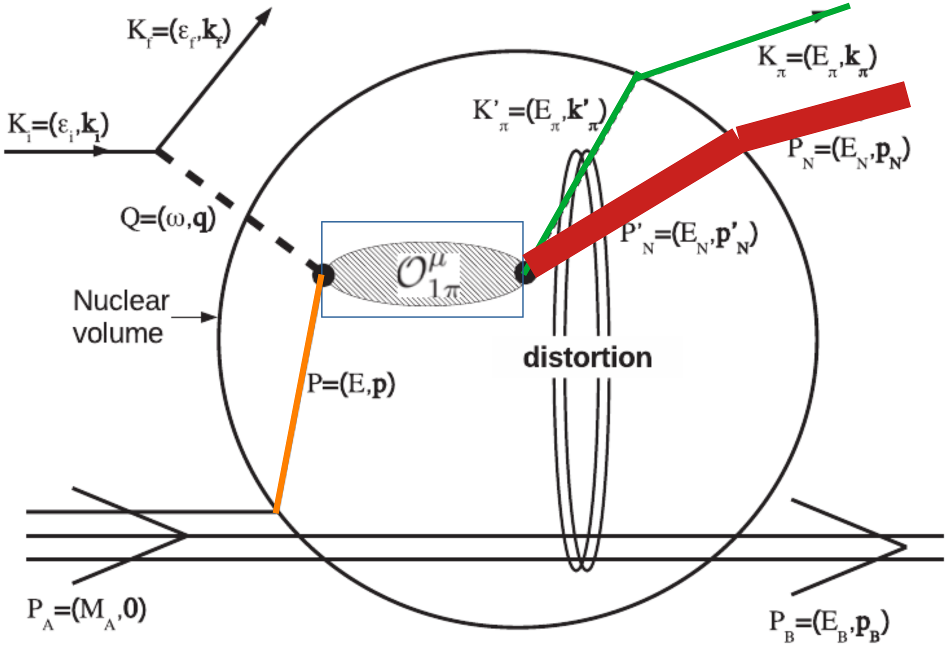
An uncertainty could be attached to the model prediction.

https://arxiv.org/abs/2306.10823



Caption: ^{12}C longitudinal (left) and transverse (right) responses. The transferred momentum q is (from up to bottom) 300, 400 and 550 MeV/c. Results: the **intermediate bound-nucleon state** is described in terms of free particles in an RFG with a modified mass and energy (**RFG***), and **RMF** nucleons.

Single-Pion Production
(in the Impulse Approximation)



$$J^{\mu}_{\rm had} = \int d\mathbf{p} \int d\mathbf{p}'_N \overline{\Psi}_F(\mathbf{p}'_N, \mathbf{p}_N) \phi^*_\pi(\mathbf{p}+\mathbf{q}-\mathbf{p}'_N, \mathbf{k}_\pi) \mathcal{O}_{1\pi}(Q, P'_N, P) \Psi_B(\mathbf{p})$$

J. García-Marcos et al., Towards a more complete description of nucleon distortion in lepton-induced single-pion production at low-Q2
https://doi.org/10.48550/arXiv.2310.18056

Distortion of the nucleon wave function
(or Elastic FSI of the nucleon)

and

Asymptotic approximation for the SPP operator
(or local versus non-local operator)

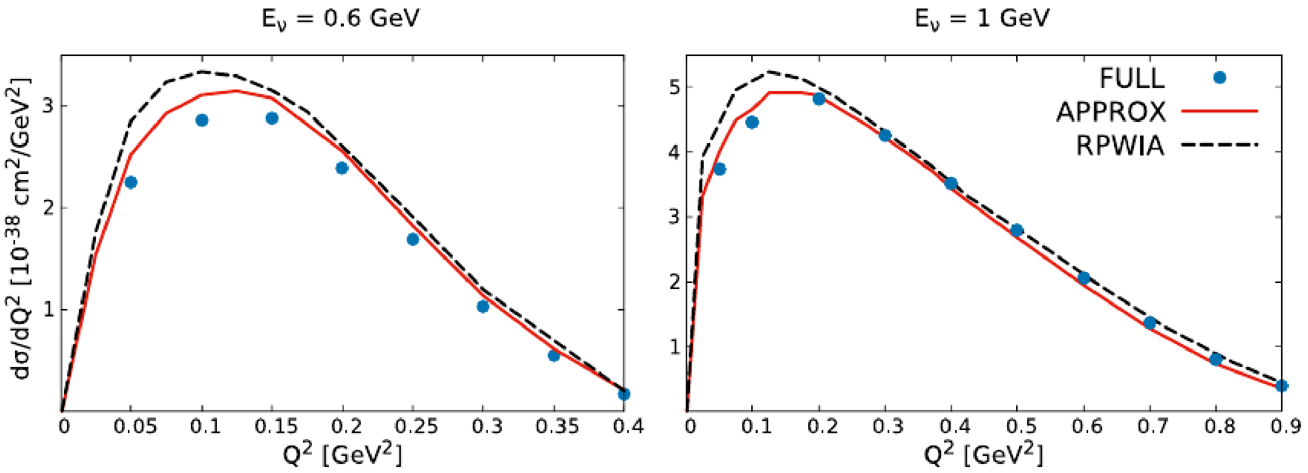


FIG. 8. Single π^+ production off ^{12}C in terms of Q^2 for two neutrino energies: $E_\nu = 0.6$ GeV and $E_\nu = 1$ GeV. Blue points represent ED-RMF without asymptotic approximation and red lines represent ED-RMF with asymptotic approximation. Dashed black lines stand for RPWIA.

The full model is too expensive computationally, so we have to make approximations…
An uncertainty could be attached to the model prediction.

Possible (naive?) strategy to estimate uncertainties:

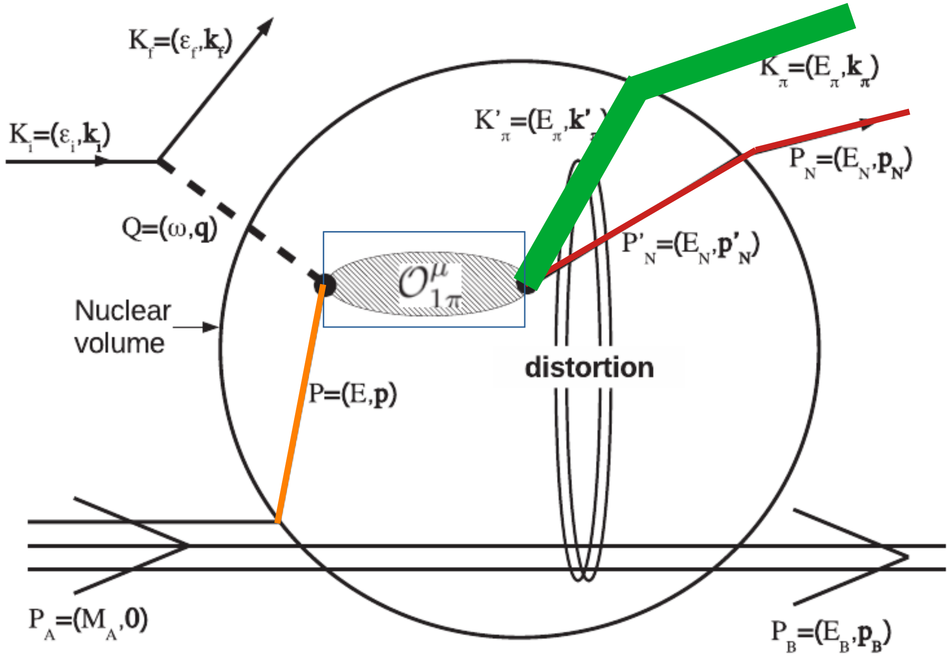
1. Find the relevant variable (e.g. energy of the nucleon in previous slide)

2. Compare cross section(s) for the approximated model(s) with the full model, as a function of the relevant variable

3. The difference between the full model and the approximated ones gives an estimate of the error introduced by the approximations, as a function of the chosen variable

Problems:
+ We're projecting on one dimension a multi-dimension problem.
+ Sometimes (often) we do not have the "full" model to compare with.
+ No correlations between uncertainties.

Single-Pion Production
(in the Impulse Approximation)



$$J^{\mu}_{\rm had} = \int d\mathbf{p} \int d\mathbf{p}'_N \overline{\Psi}_F(\mathbf{p}'_N, \mathbf{p}_N) \phi^*_\pi(\mathbf{p}+\mathbf{q}-\mathbf{p}'_N, \mathbf{k}_\pi) \mathcal{O}_{1\pi}(Q, P'_N, P) \Psi_B(\mathbf{p})$$

Pion wave function in the nuclear medium

There are some works on **photoproduction** and a few on **electroproduction** but they usually include the distortion of the nucleon and pion all together and compare it to the case of plane-wave approach. That makes it difficult to isolate the effect of distortion of the pion.

Coherent pion production is a cleaner way to study this effect:

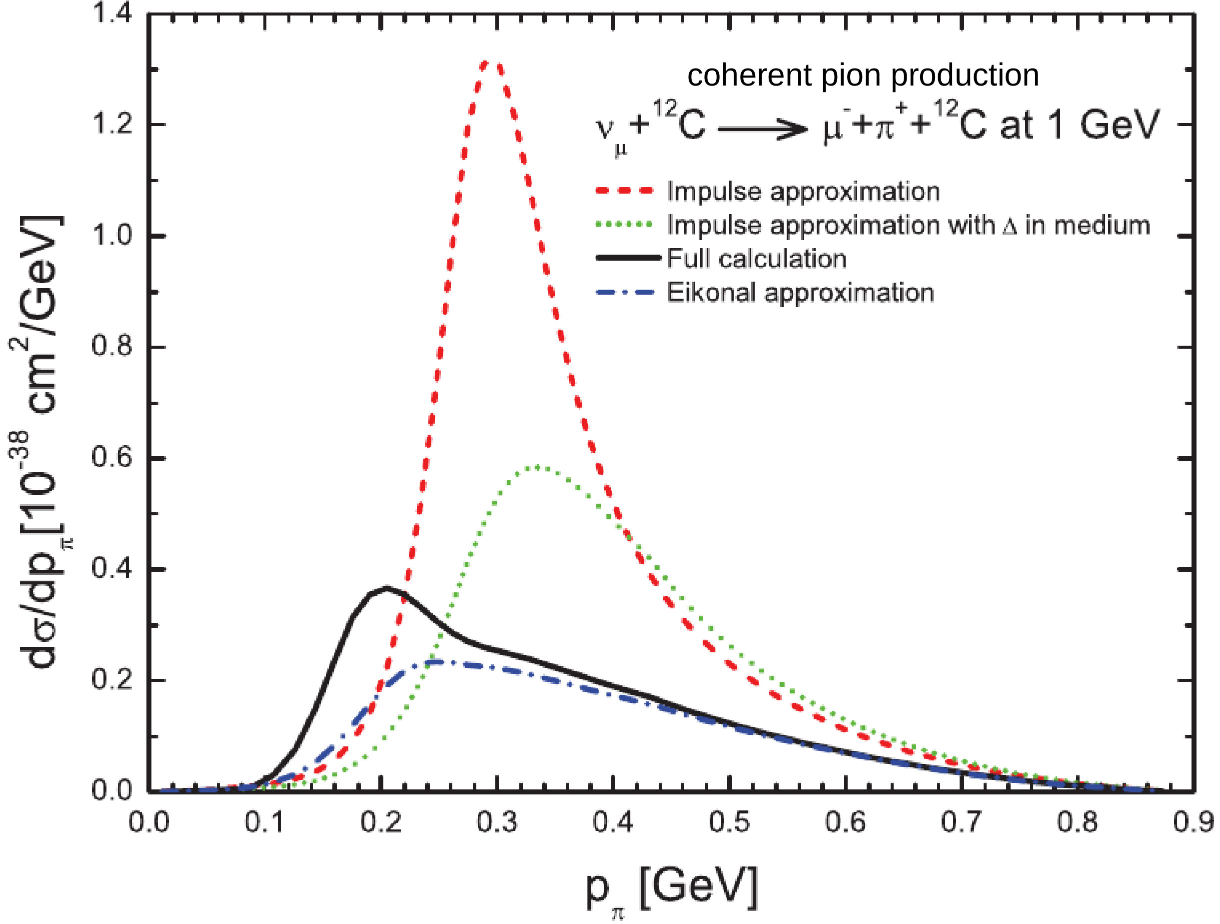
$$\gamma + A \text{ -----> } \pi^0 + A$$

or

$$\text{lepton} + A \text{ -----> } \text{lepton}' + \pi + A$$

Pion wave function in the nuclear medium

Alvarez-Ruso et al. (2007), http://dx.doi.org/10.1103/PhysRevC.75.055501

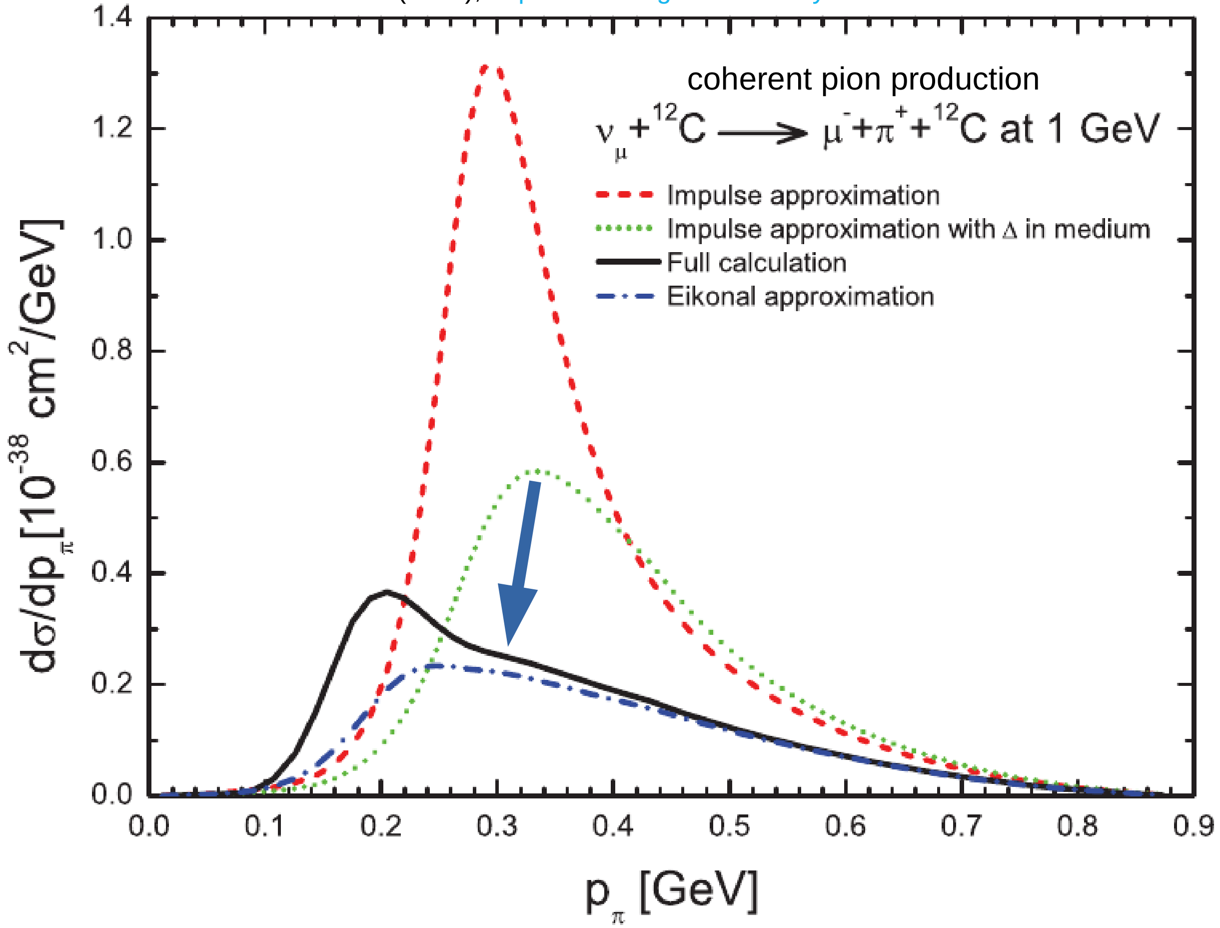


Pion wave function in the nuclear medium

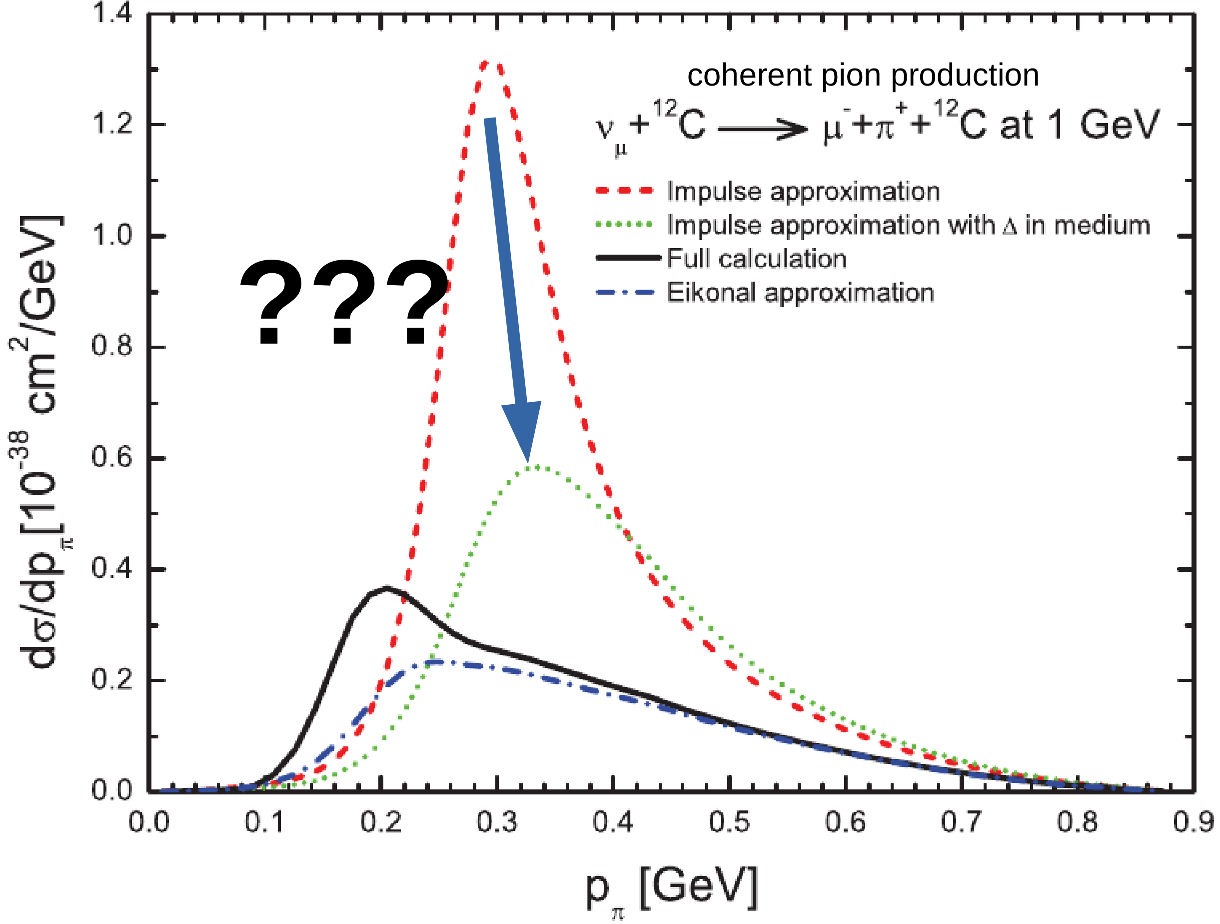
FSI of the pion:
+ elastic FSI (real part of the nuclear potential) mostly changes the shape. Cascade cannot do this.

+ inelastic FSI (imaginary part of the nuclear potential) removes strength from this channel. Cascade could do this.

Alvarez-Ruso et al. (2007), http://dx.doi.org/10.1103/PhysRevC.75.055501



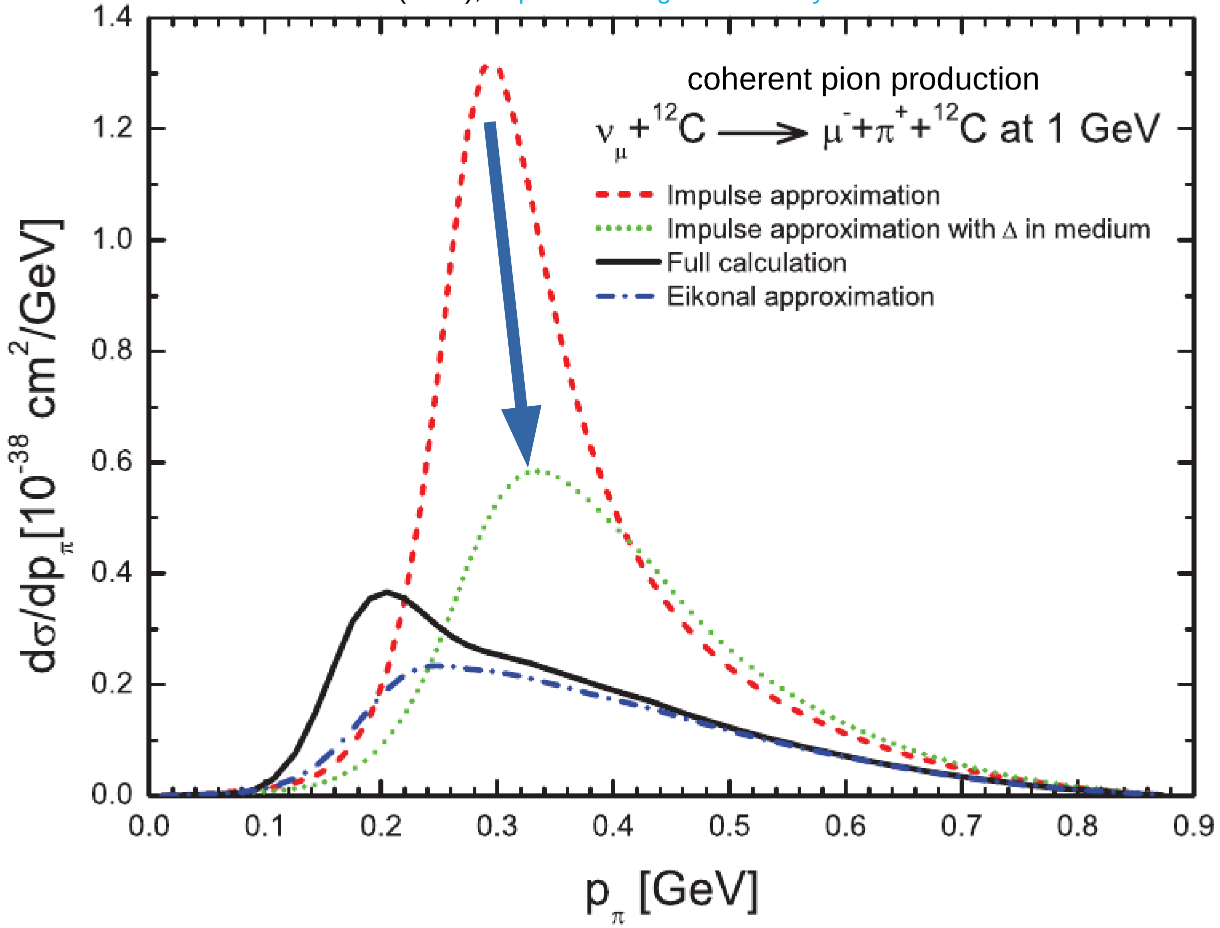
Alvarez-Ruso et al. (2007), http://dx.doi.org/10.1103/PhysRevC.75.055501



Alvarez-Ruso et al. (2007), http://dx.doi.org/10.1103/PhysRevC.75.055501

In-medium modification of the resonance properties

In this approach, the width and mass of the delta are modified due to interactions with the nucleons in the nucleus.
Globally, the delta width gets wider: lower cross section.



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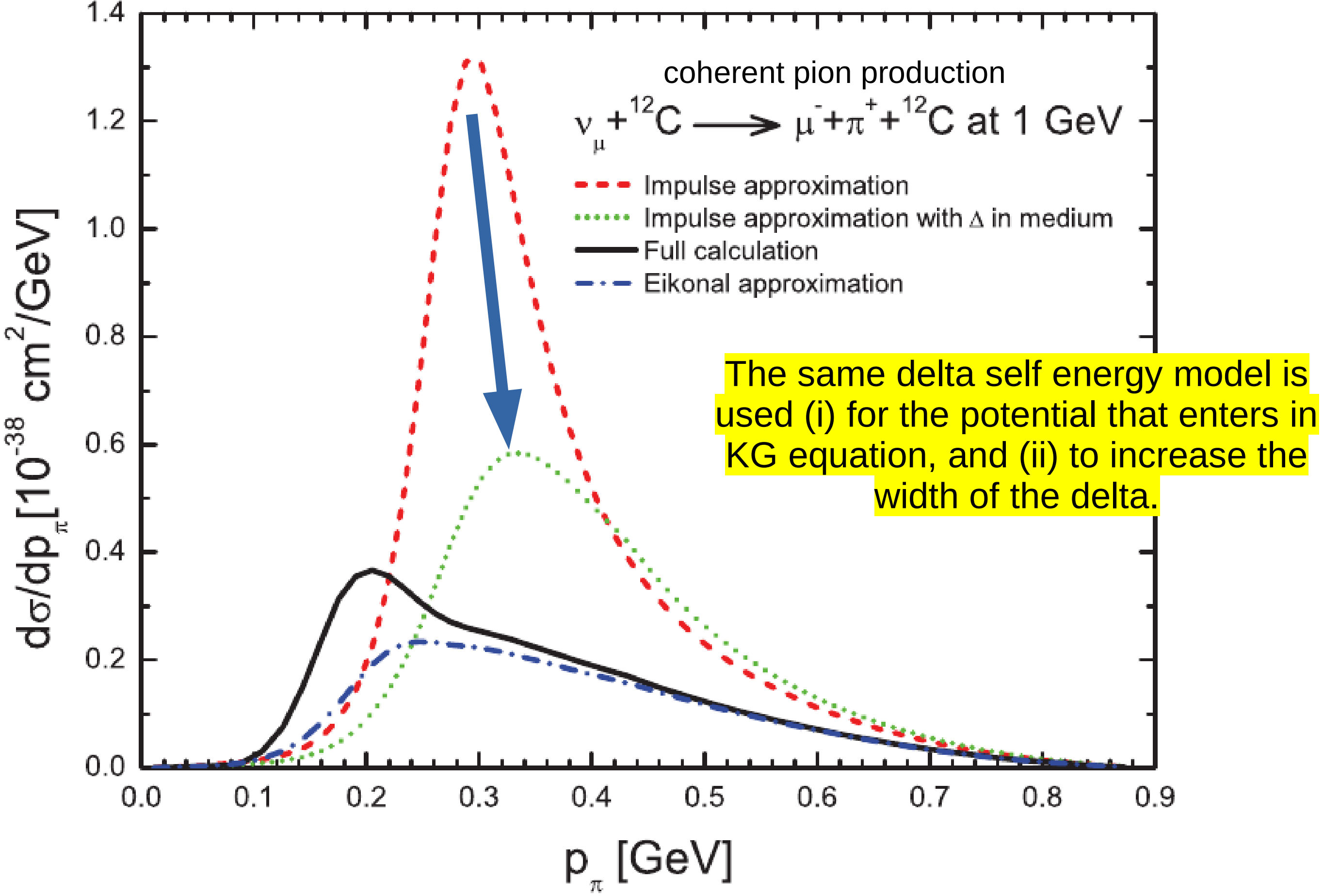
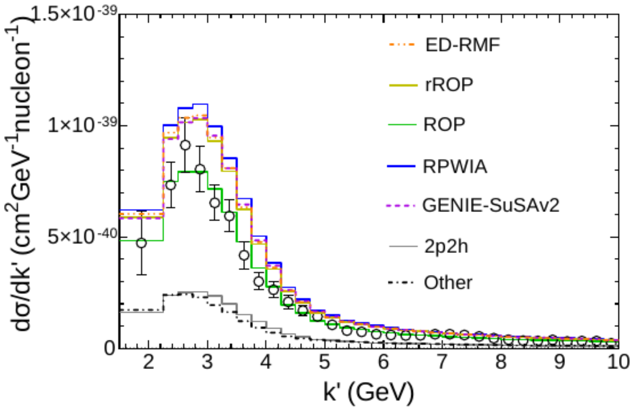
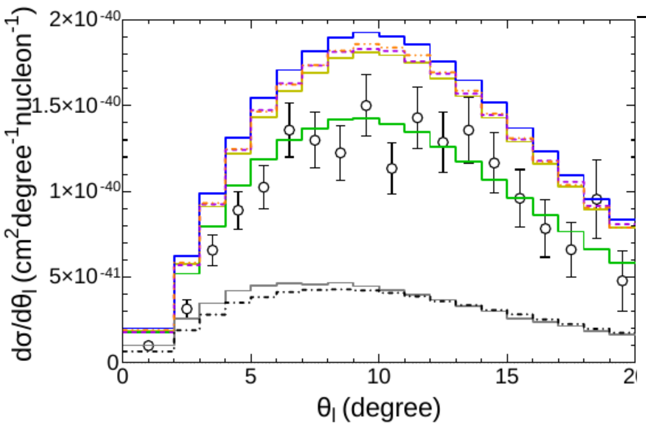


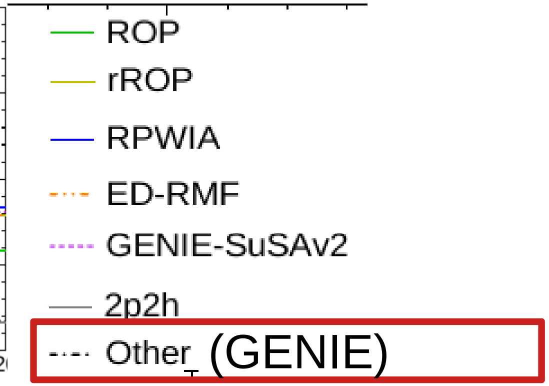
IMPORTANT FOR NEUTRINO INTERACTIONS?

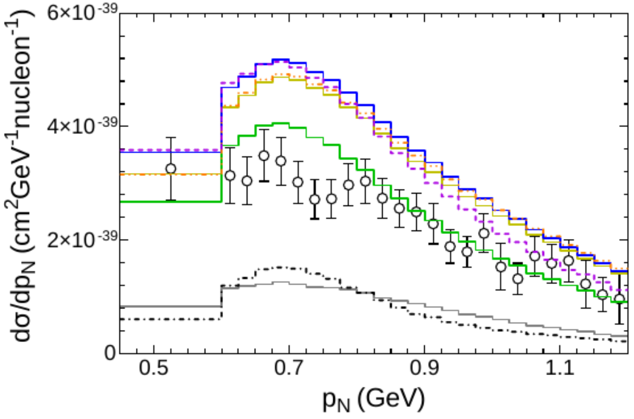
MINERνA no-pion ν_μ-^{12}C cross section

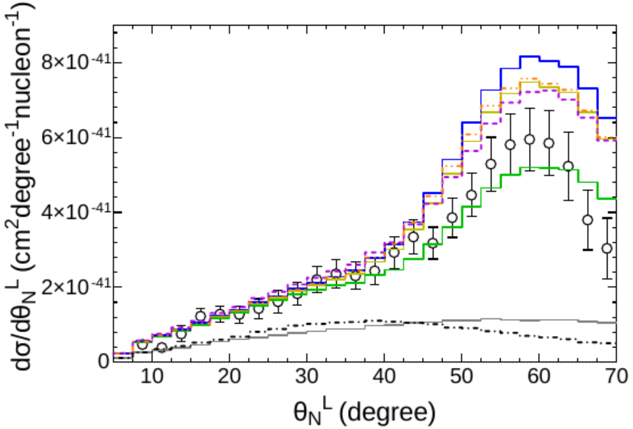
Franco-Patino et al. (2022), https://doi.org/10.1103/PhysRevD.106.113005









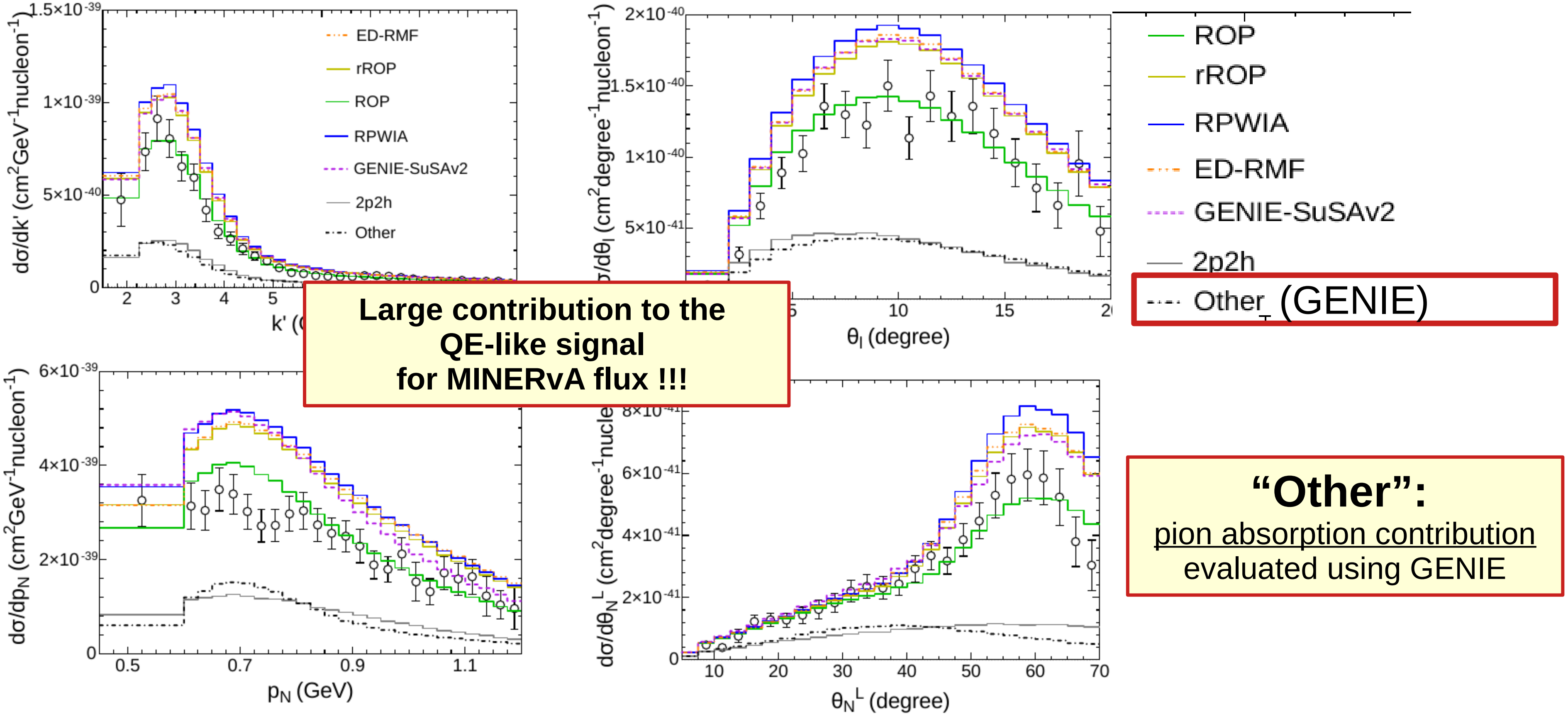


"Other":

pion absorption contribution evaluated using GENIE

MINERνA no-pion ν_μ-^{12}C cross section

Franco-Patino et al. (2022), https://doi.org/10.1103/PhysRevD.106.113005



Large contribution to the QE-like signal for MINERvA flux !!!

“Other”:

pion absorption contribution evaluated using GENIE

Important Note:

From here to the end I am simply and openly sharing my doubts and ignorance with you.

+ Delta propagator in the MEC 2p2h: Full vs real ??

+ In medium modifications of the delta in the SPP contribution

+ In medium modifications of the delta in the MEC 2p2h contribution

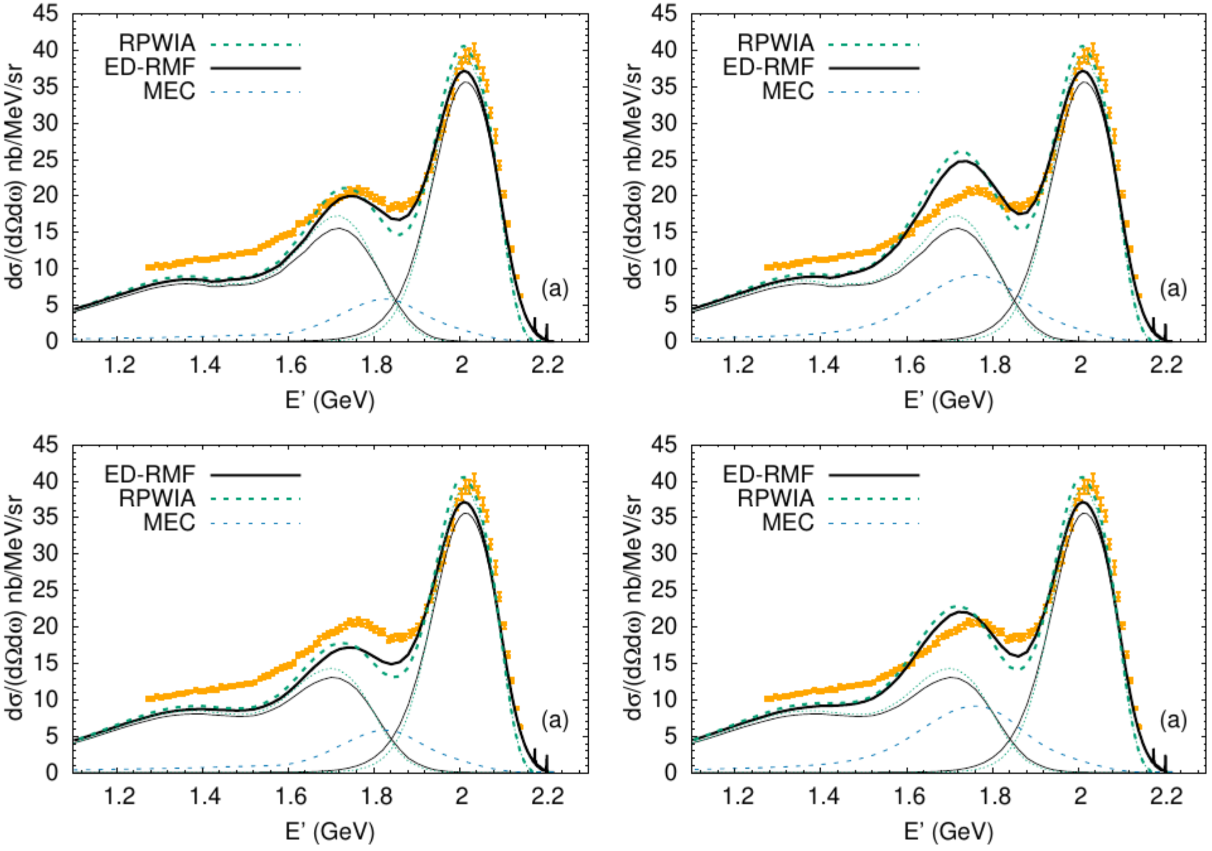
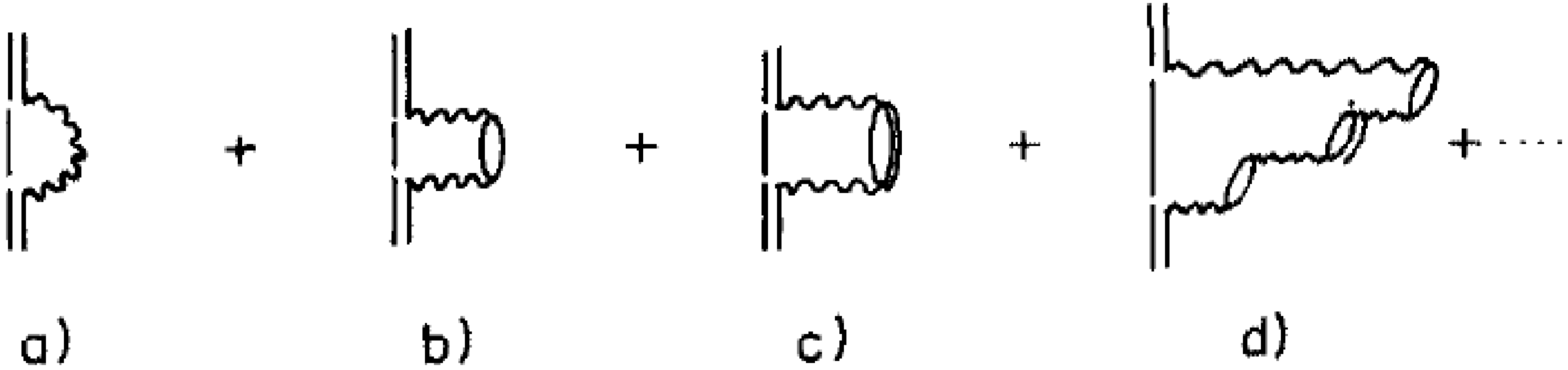


FIG. 1: Left panels are the results with only the real part of the delta propagator in the MEC contribution. Right panels are the results with the full delta propagator in the MEC. Top panels are the results without medium-modification in the delta-decay width in the SPP contribution. Bottom panels are the results including medium-modification in the delta-decay width in the SPP (see text for details). The QE response is the same in the four panels.

In-medium modification of the resonance properties

In-medium modification of the resonance properties:
Microscopic approach

Oset and Salcedo,
https://doi.org/10.1016/0375-9474(87)90185-0



In-medium modification of the resonance properties:
Microscopic approach

Delta propagator:

$$S_{\Delta,\alpha\beta} = \frac{-(\not{K}_\Delta + M_\Delta)}{K_\Delta^2 - M_N^2 + iM_\Delta\Gamma_{\text{width}}}\left(g_{\alpha\beta} - \frac{1}{3}\gamma_\alpha\gamma_\beta - \frac{2}{3M_\Delta^2}K_{\Delta,\alpha}K_{\Delta,\beta} - \frac{2}{3M_\Delta}(\gamma_\alpha K_{\Delta,\beta} - K_{\Delta,\alpha}\gamma_\beta)\right)$$

Replace the **free decay width** by an ***in-medium*** *one*:

$$\Gamma_{\text{width}}^{\text{free}} \longrightarrow \Gamma_{\text{width}}^{\text{in-medium}} = \Gamma_{\text{Pauli}} - 2\Im(\Sigma_\Delta)\,, \quad M_\Delta^{\text{free}} \longrightarrow M_\Delta^{\text{in-medium}} = M_\Delta^{\text{free}} + \Re(\Sigma_\Delta)\,.$$

+ Γ_{Pauli}: some nucleons from Δ-decay are Pauli blocked (the Δ-decay width decreases).

+ The parametrization of $\Im(\Sigma_\Delta)$ and $\Re(\Sigma_\Delta)$ is given in terms of the nuclear density ρ:

$$\begin{aligned} -\Im(\Sigma_\Delta) &= C_{QE}\,(\rho/\rho_0)^\alpha + C_{A2}\,(\rho/\rho_0)^\beta + C_{A3}\,(\rho/\rho_0)^\gamma\,, \\ \Re(\Sigma_\Delta) &= 40\text{ MeV}\,(\rho/\rho_0)\,. \end{aligned}$$

Oset and Salcedo, https://doi.org/10.1016/0375-9474(87)90185-0

In-medium modification of the resonance properties:

Microscopic approach

$$-\Im(\Sigma_\Delta) = C_{QE}\,(\rho/\rho_0)^\alpha + C_{A2}\,(\rho/\rho_0)^\beta + C_{A3}\,(\rho/\rho_0)^\gamma$$

Each contribution corresponds to a different process:

- QE $\Longrightarrow \Delta N \to \pi NN$ (still one pion in the final state)
- A2 $\Longrightarrow \Delta N \to NN$ (no pions in the final state)
- A3 $\Longrightarrow \Delta NN \to NNN$ (no pions in the final state)

Oset and Salcedo, https://doi.org/10.1016/0375-9474(87)90185-0

SCENARIO A
All these processes make the delta width wider. Fine with that.
1. Modify the width of the delta in your MEC 2p2h.
2. Modify the width of the delta in your SPP model.
3. Should include the contributions from the new channels that were opened: piNN, NNN; otherwise will underestimate the inclusive cross section.

Note that you do not need to include *again* the NN because that's MEC 2p2h, which was already included.

(Of course, we could solve the wave eq. with real potentials for the pion and nucleons. If we do this, the cross section, generally, decreases.)

4. If you're modeling an inclusive process you're done.

5. If you're doing MC, then, in the cascade there should exist the possibility for pion absorption and emission of NN and NNN, and pion knocking out a couple of nucleons, piNN.

So the final state is NN+N, NNN+N and piNN+N.

SCENARIO B
All these processes make the delta width wider. Fine with that.
1. Modify the width of the delta in your MEC 2p2h.
2. Modify the width of the delta in your SPP model.
3. We don't know how to model the new channels that were opened: piNN and NNN. So we decide to ignore the contribution from these terms (in the width of the delta)...

4. If you're modeling an inclusive process you're done.

5. If we're doing MC then, in the cascade there should exist the possibility for pion absorption and emission of NN and NNN, and pion knocking out a couple of nucleons, piNN.

So, as in A, the final state is NN+N, NNN+N and piNN+N.

Since in this approach we started with more pions than in scenario A, and no piNN or NNN, will we end up with the same predictions as in approach A…??

By the way...

The delta is not the only “problem”, there are other resonances there and an important non-resonant background.

https://doi.org/10.48550/arXiv.2310.18056

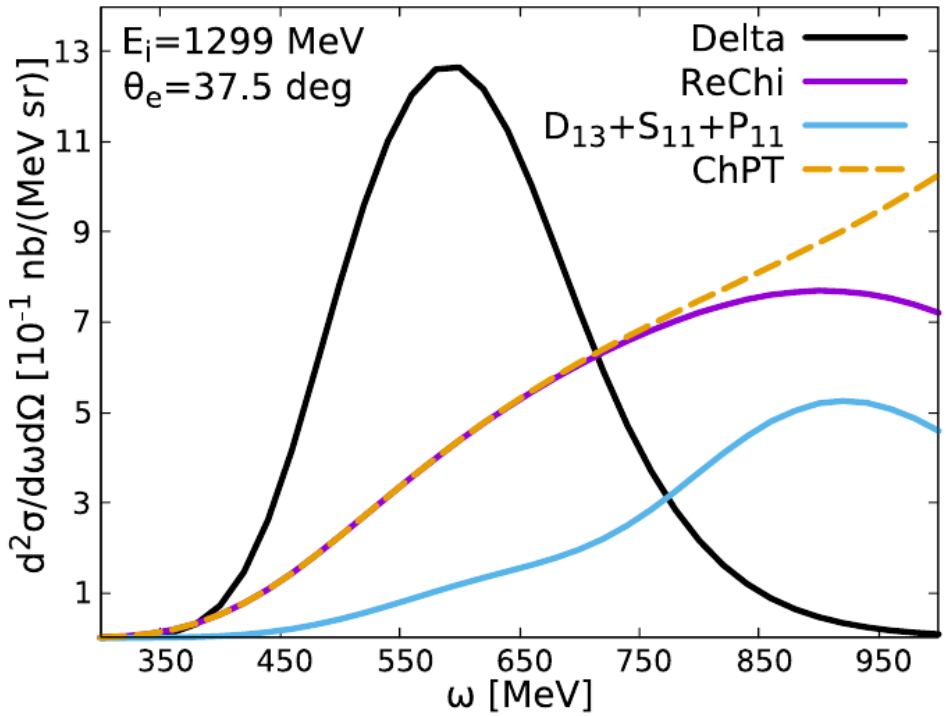


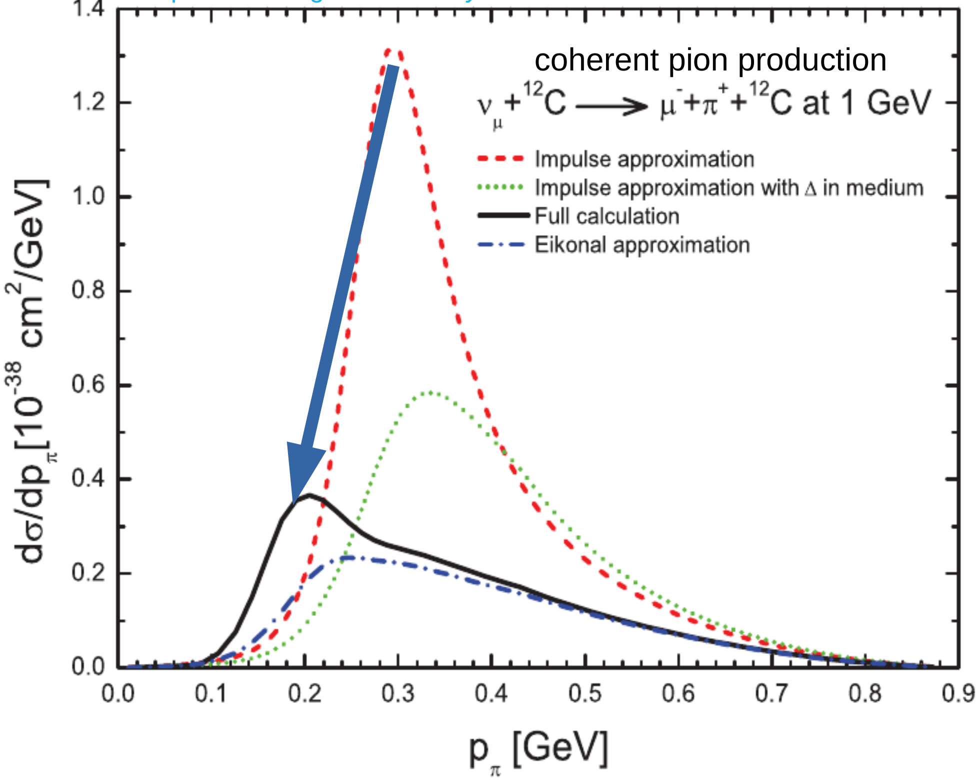
FIG. 4. Inclusive $^{12}C(e, e')$ cross section using different pieces of the current operator for a specific lepton kinematics. The black line is for the Δ contribution, the purple line is for *reggeized* background, and the blue line is the contribution from the other three resonances. The dashed orange line is the ChPT background but without Regge. Calculations were performed within the RPWIA approach.

In-medium modification of the resonance properties

An energy-dependent optical potential fit to **elastic pion-nucleus scattering** data should (by construction) account for the loss of pions

why not???

Alvarez-Ruso et al. (2007)
http://dx.doi.org/10.1103/PhysRevC.75.055501



Can we do something else/different???

In-medium modification of the resonance properties

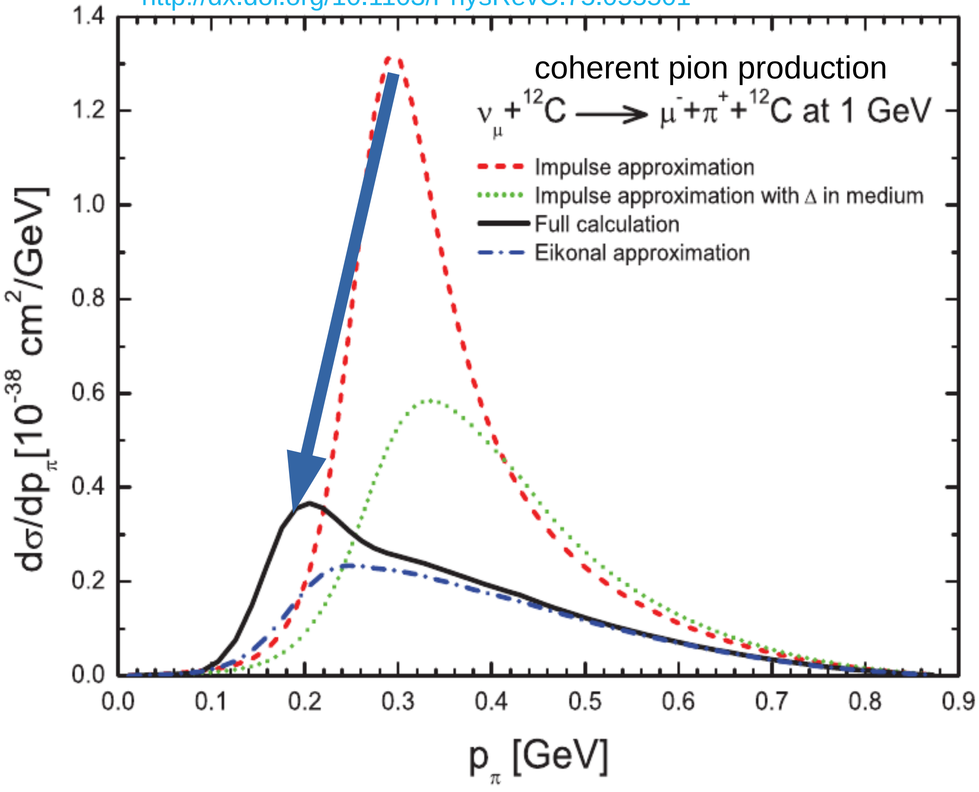
An energy-dependent optical potential fit to **elastic pion-nucleus scattering** data should (by construction) account for the loss of pions

why not???

It would provide solid predictions for 1π production with **minimum nuclear uncertainties.** It would correspond to the process in which the pion only interacts elastically (it passes through the cascade without interactions)

Useful to benchmark cascade models. Analogously to the idea proposed for the QE case in Nikolakopoulos et al. (2022) https://doi.org/10.1103/PhysRevC.105.054603

Alvarez-Ruso et al. (2007) http://dx.doi.org/10.1103/PhysRevC.75.055501



In-medium modification of the resonance properties

Li, Wright and Benhold (1993), https://doi.org/10.1103/PhysRevC.48.816

There is no delta medium modification in this model, just an optical potential fit to elastic pion-nucleus scattering.

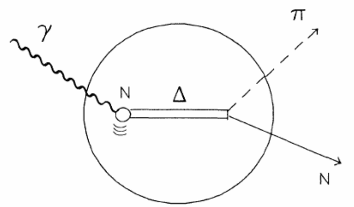


FIG. 1. Diagram of the reaction $A(\gamma, \pi N)B$ in the Δ region. The background Born terms are not shown.

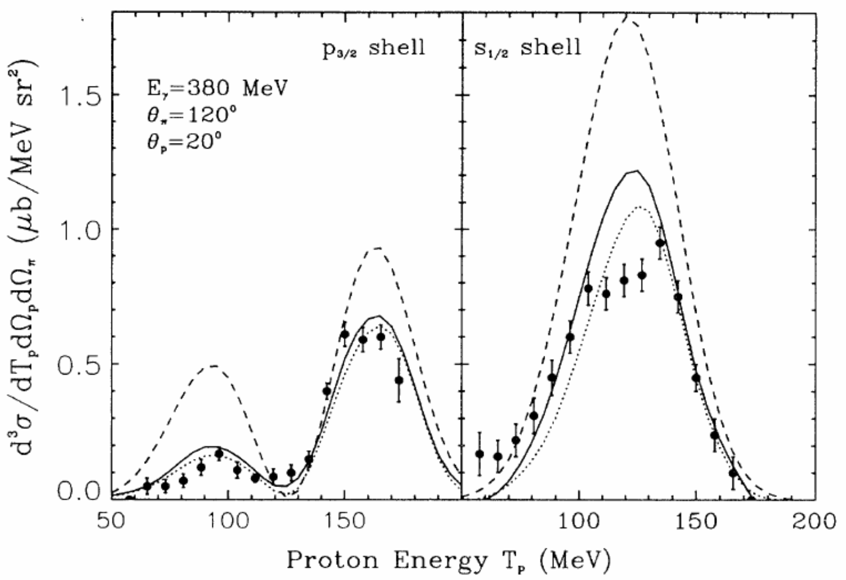
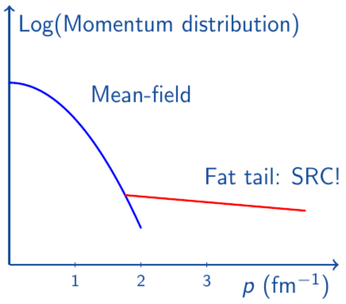


FIG. 2. Proton energy dependence of the triple coincidence cross section from $p_{3/2}$ and $s_{1/2}$ shell neutrons in $^{12}C(\gamma, \pi^- p)^{11}C$ for fixed E_γ, θ_π, and θ_p. Theoretical curves are calculated in PWIA (dashed line), local DWIA (dotted line), and nonlocal DWIA (solid line). Data are taken from Ref. [13].

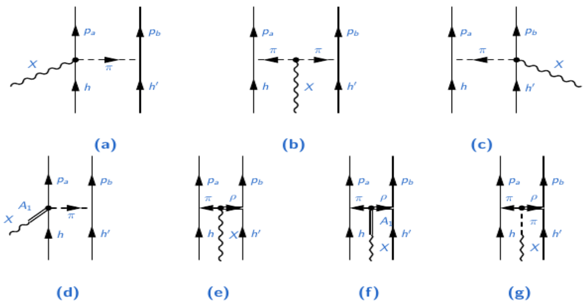
Two-nucleon knockout processes

Two mechanisms give rise to the emission of two nucleons (apart from FSI):

Short-range correlations

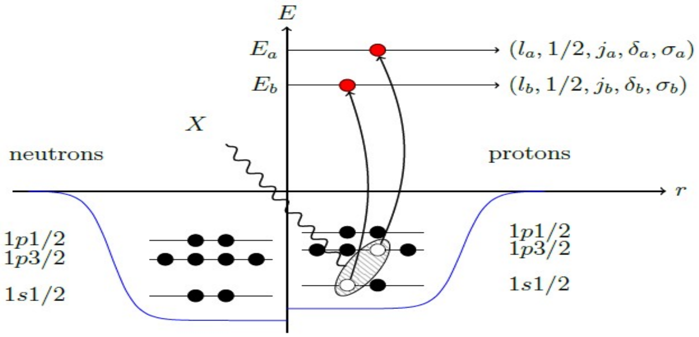


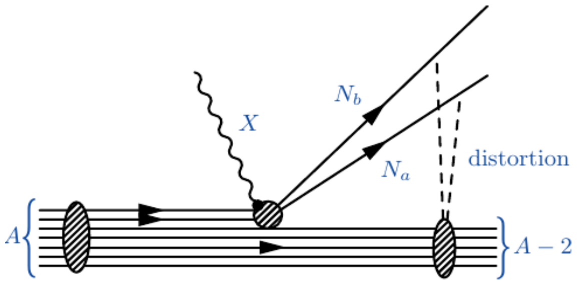
Meson-exchange currents



Images from T. Van Cuyck's PhD Thesis

The same **mean-field** model is used to describe the **bound and scattered nucleons**:

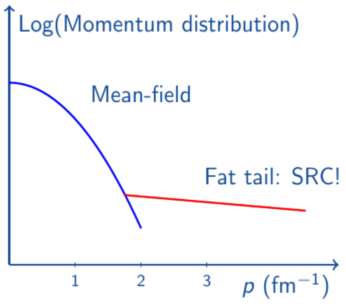




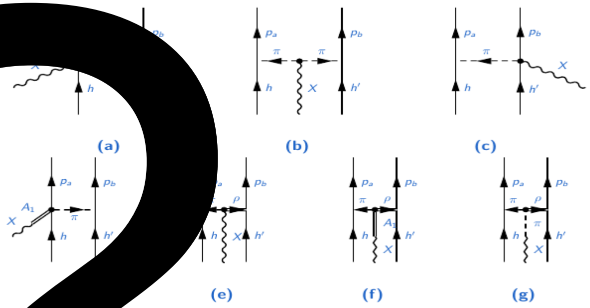
Two-nucleon knockout processes

Two mechanisms give rise to the emission of two nucleons (apart from FSI):

Short-range correlations

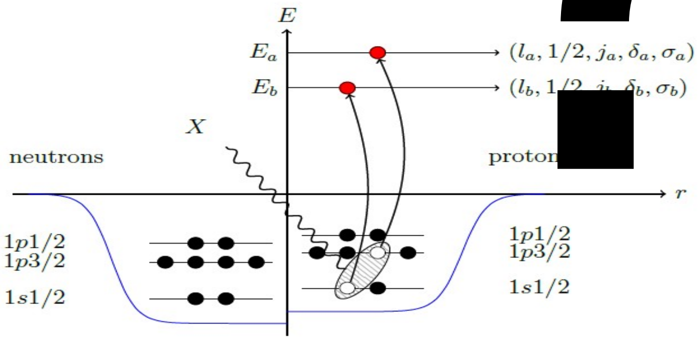


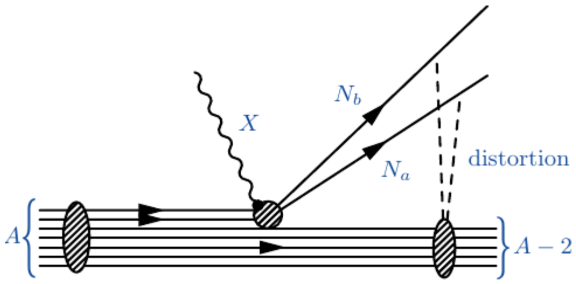
Meson-exchange currents



Images from T. Van Cuyck's PhD Thesis

The same **mean-field** model is used to describe the [illegible]**nd and scattered nucleons**:





Final remarks

+ Compare the predictions from different approaches is useful to estimate uncertainties. **But,** we should work with models that have passed the electron-scattering test.

+ When can we consider that the process can be modeled with the cascade (NO INTERFERENCES) and when we have to model the process at the amplitude level (INTERFERENCES)?

+ Everything beyond 1p-1h and one-body currents is extremely difficult. More people working and thinking about:
  ++ 2N knockout channels: MEC, SRC, interferences...
  ++ pion production: pion FSI, medium modification of the pion-production operator

+ There is no experimental data on electron-induced pion production on the nucleus (in the energy region of interest to neutrino experiments). These data very much are needed.

+ The theory or modeling is far from being mature enough, for example, compared to the modeling of the QE channel.
But even further from being implemented in a MC event generator, due to high computational cost and high dimensionality of the phase space.

Just in case slides

https://doi.org/10.48550/arXiv.2310.18056

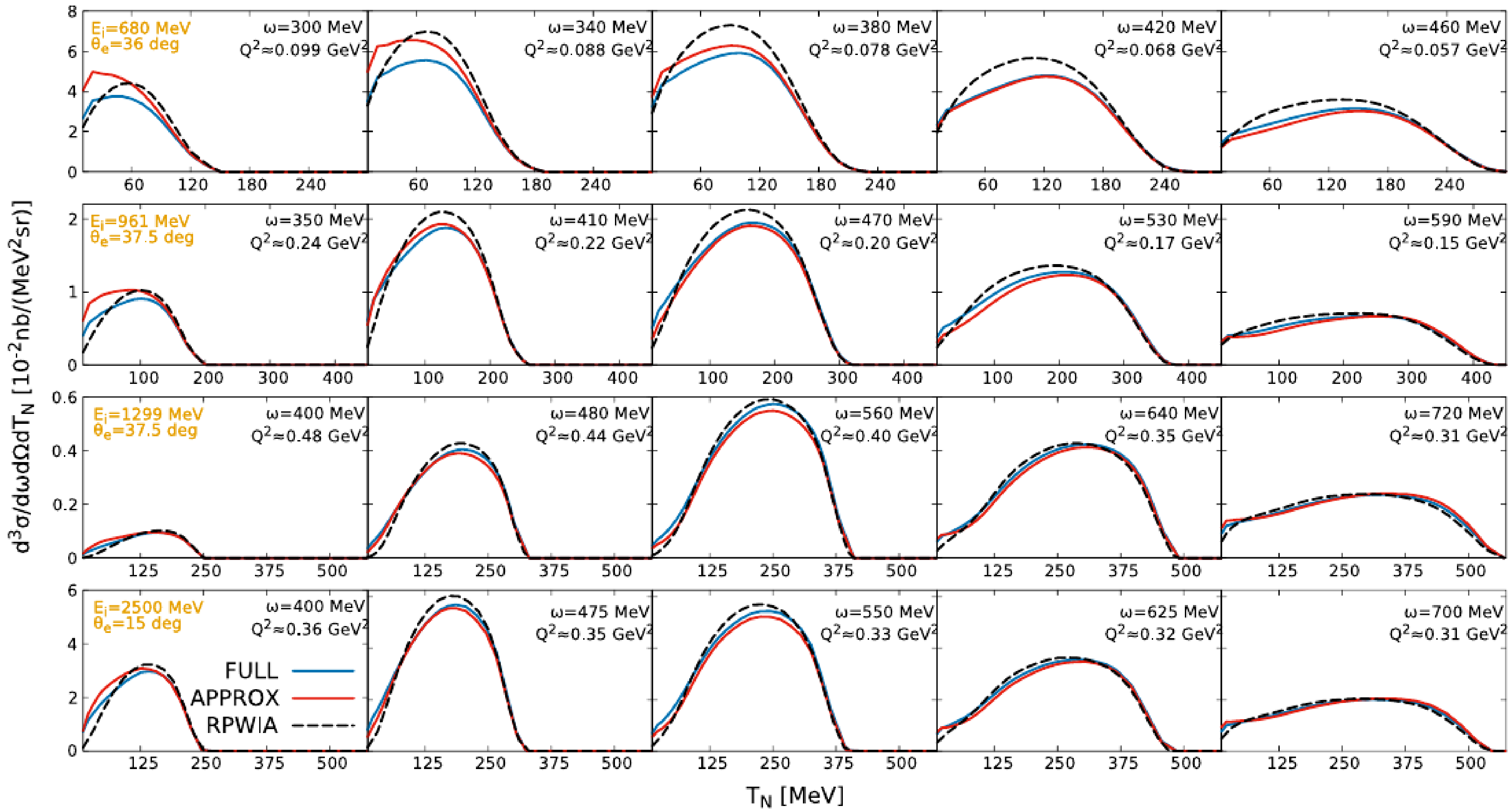


FIG. 7. SPP contribution to the $^{12}C(e, e')$ semi-inclusive cross section for different kinematics. Plots show RPWIA and ED-RMF (with and without asymptotic approximation) treatments for the final nucleon. Each row is for a different kinematic, and the energy transfer ω is fixed for each panel.

In-medium modification of the resonance properties:
Microscopic approach

C. Praet PhD Thesis (2009), Ghent University
https://biblio.ugent.be/publication/734583

$$\Delta\Gamma = \widetilde{\Gamma} - \Gamma,$$

$$\widetilde{\Gamma} = \Gamma_{\text{Pauli}} - 2\,\Im(\Sigma_\Delta),$$

$$-\Im(\Sigma_\Delta) = C_{QE}\left(\frac{\rho}{\rho_0}\right)^{\alpha} + C_{A2}\left(\frac{\rho}{\rho_0}\right)^{\beta} + C_{A3}\left(\frac{\rho}{\rho_0}\right)^{\gamma},$$

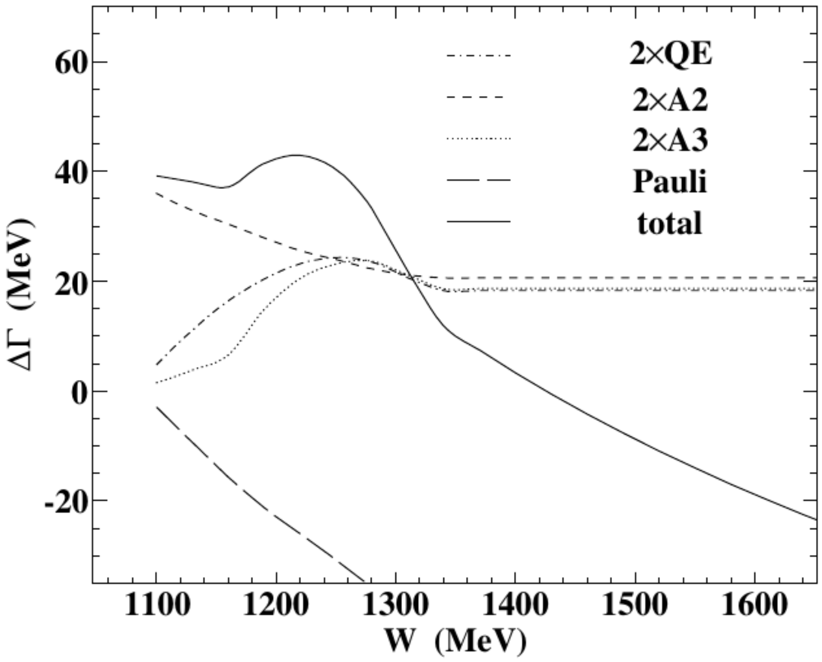


Figure 3.10: Overview of medium corrections to the free Δ width, using the parameterizations in Refs. [29, 176] for $\rho = 0.75\rho_0$.

Nuclear Physics A468 (1987) 631-652

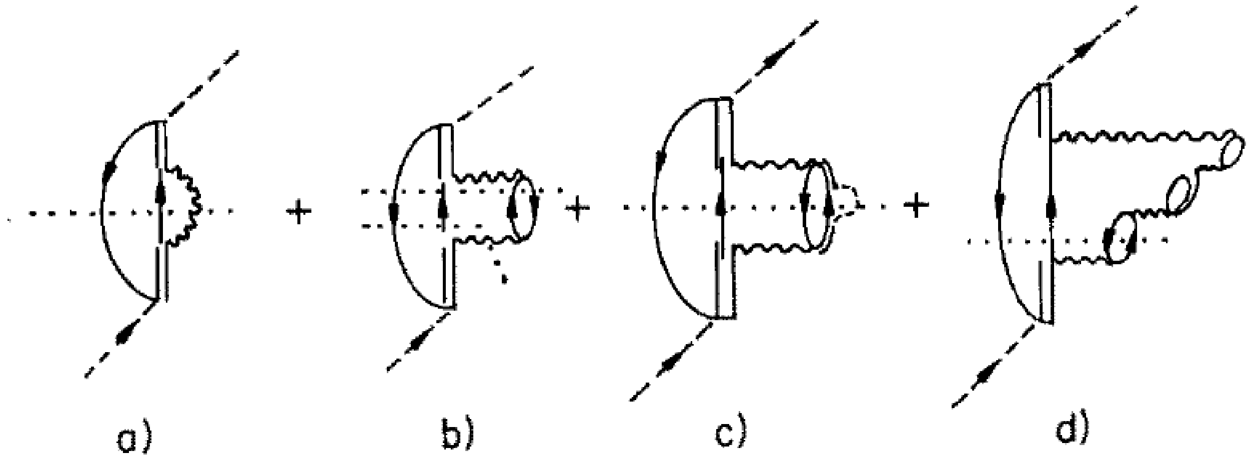


Fig. 3. Corresponding diagrams for the pion–nucleus optical potential through Δh excitation which include the Δ self-energy pieces of fig. 2. The dotted lines indicate analytical cuts which provide sources of imaginary part to the optical potential.