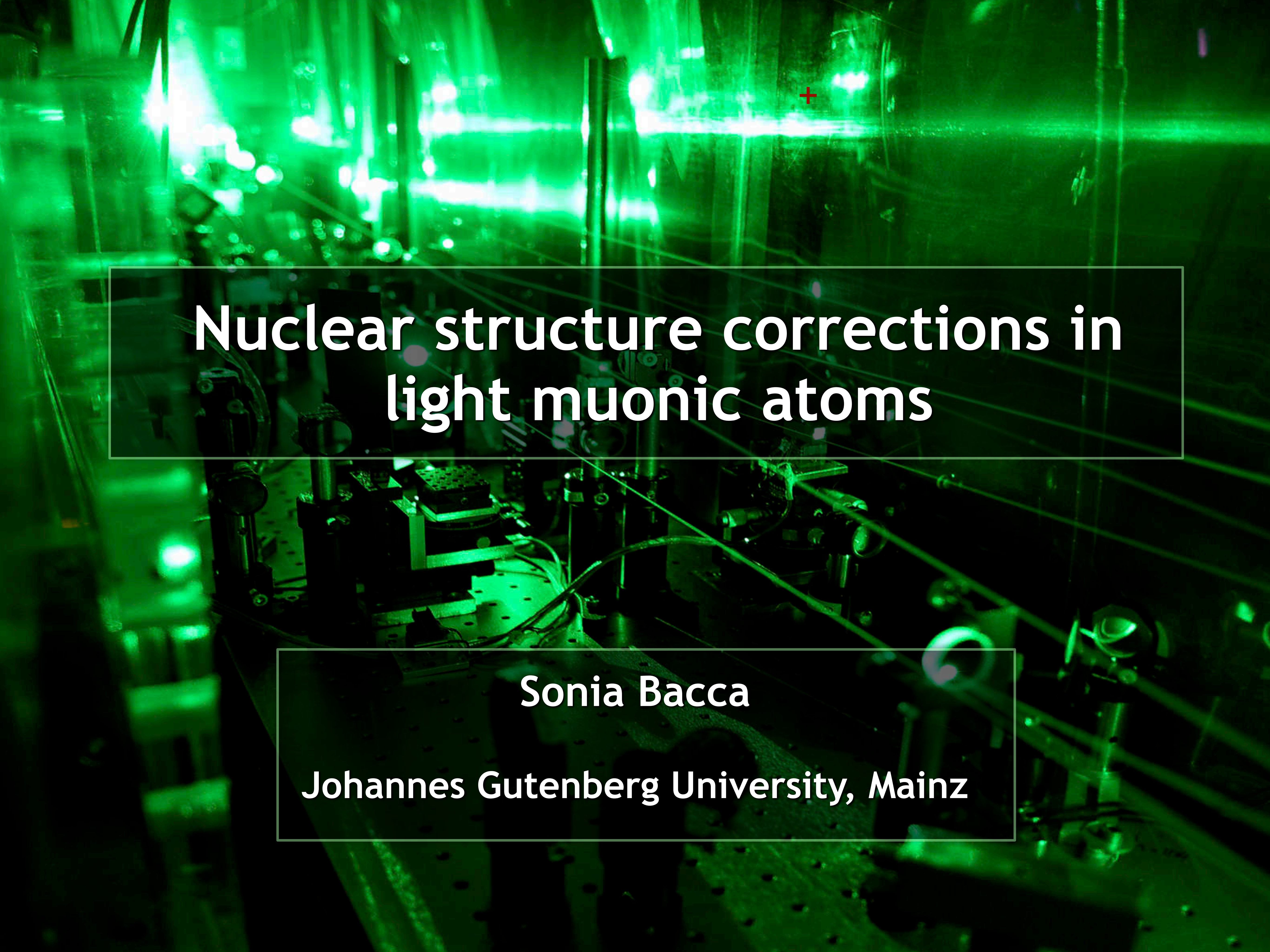


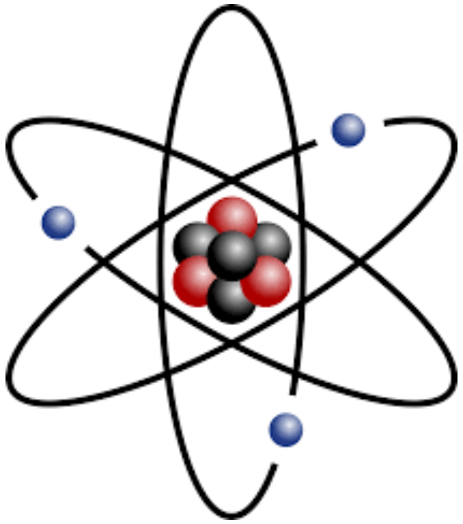
Nuclear structure corrections in light muonic atoms

Sonia Bacca

Johannes Gutenberg University, Mainz

What are muonic atoms?

Exotic atoms



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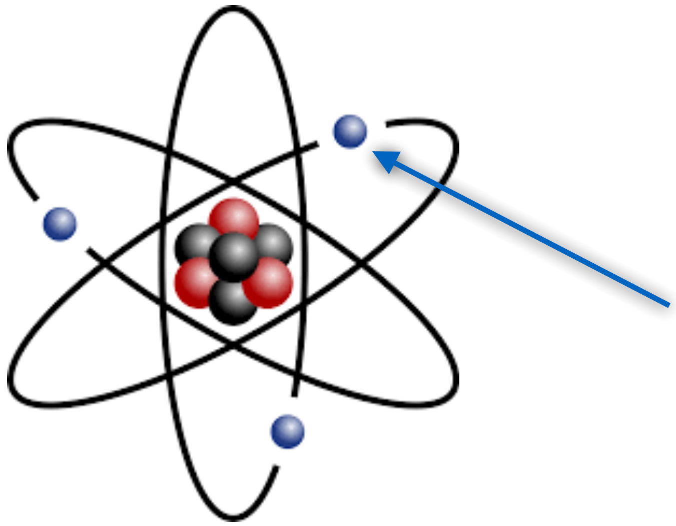

Electron replaced
with a muon

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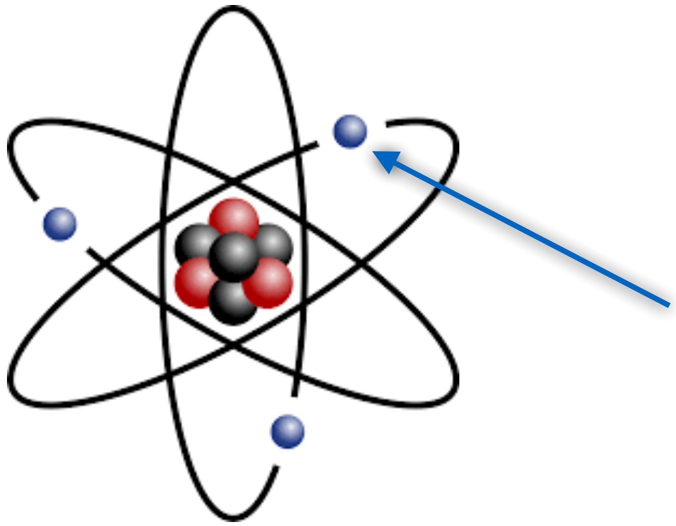
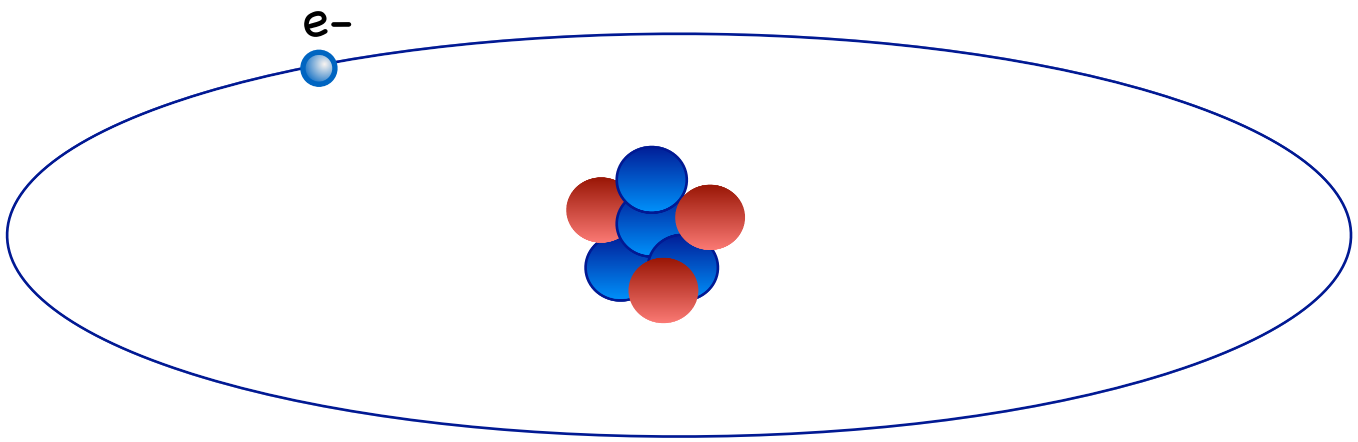


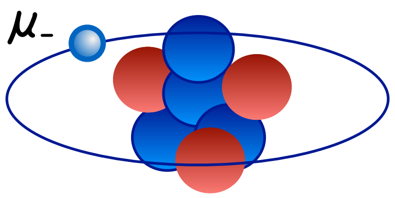
Electron replaced with a muon

Hydrogen-like systems

Ordinary atoms



Muonic atoms



muon more sensitive to the nucleus

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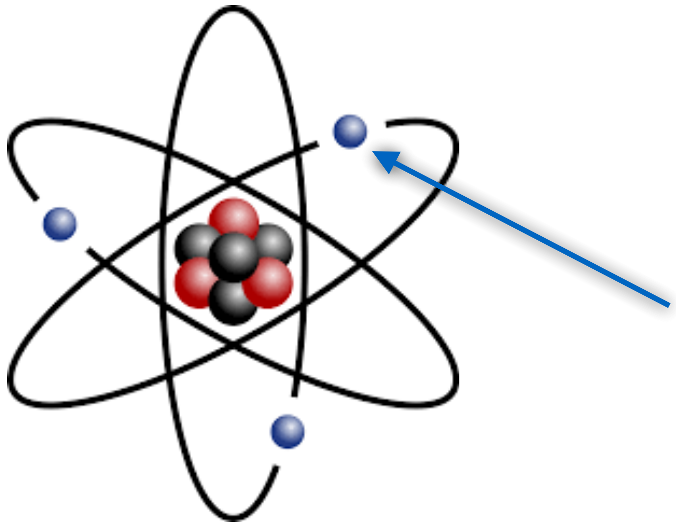

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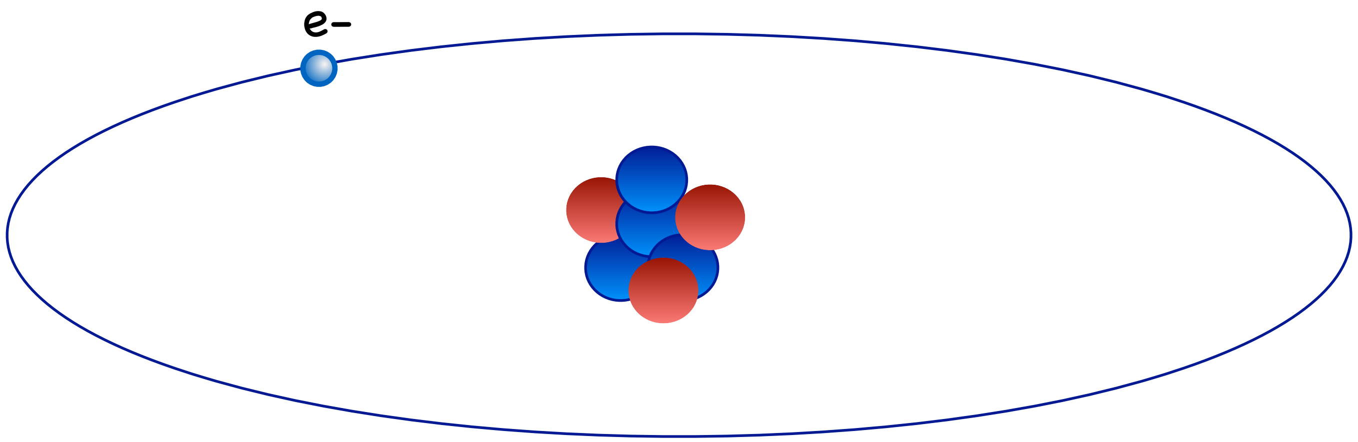

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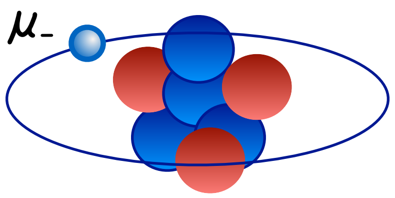
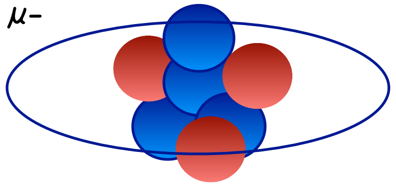


muon more sensitive to the nucleus

Can be used as a precision probe for the nucleus

Extracting the charge radius

Strong experimental program by the CREMA collaboration to extract the nuclear charge radius from the Lamb shift measurement in **muonic atoms**



$$\Delta E_{2S-2P} = \delta_{\rm QED} + \frac{m_r^4 (Z\alpha)^4}{12} \langle r_c^2 \rangle + \delta_{\rm TPE}$$

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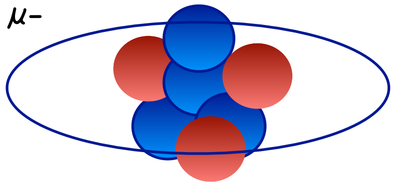

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$\uparrow$

what is measured

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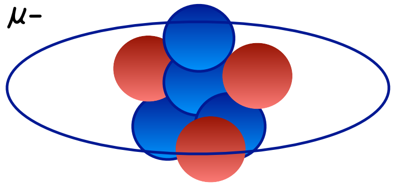

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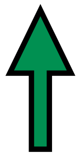

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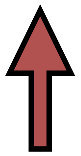

what you want to extract

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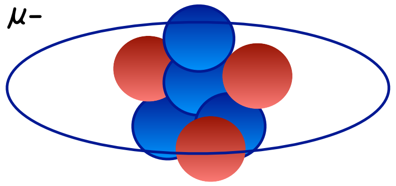

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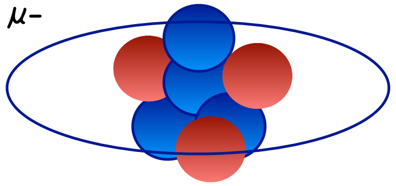

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well known

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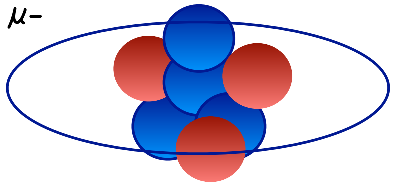

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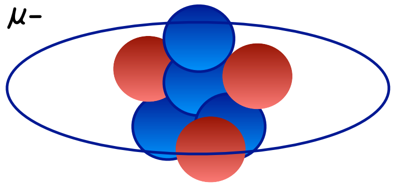

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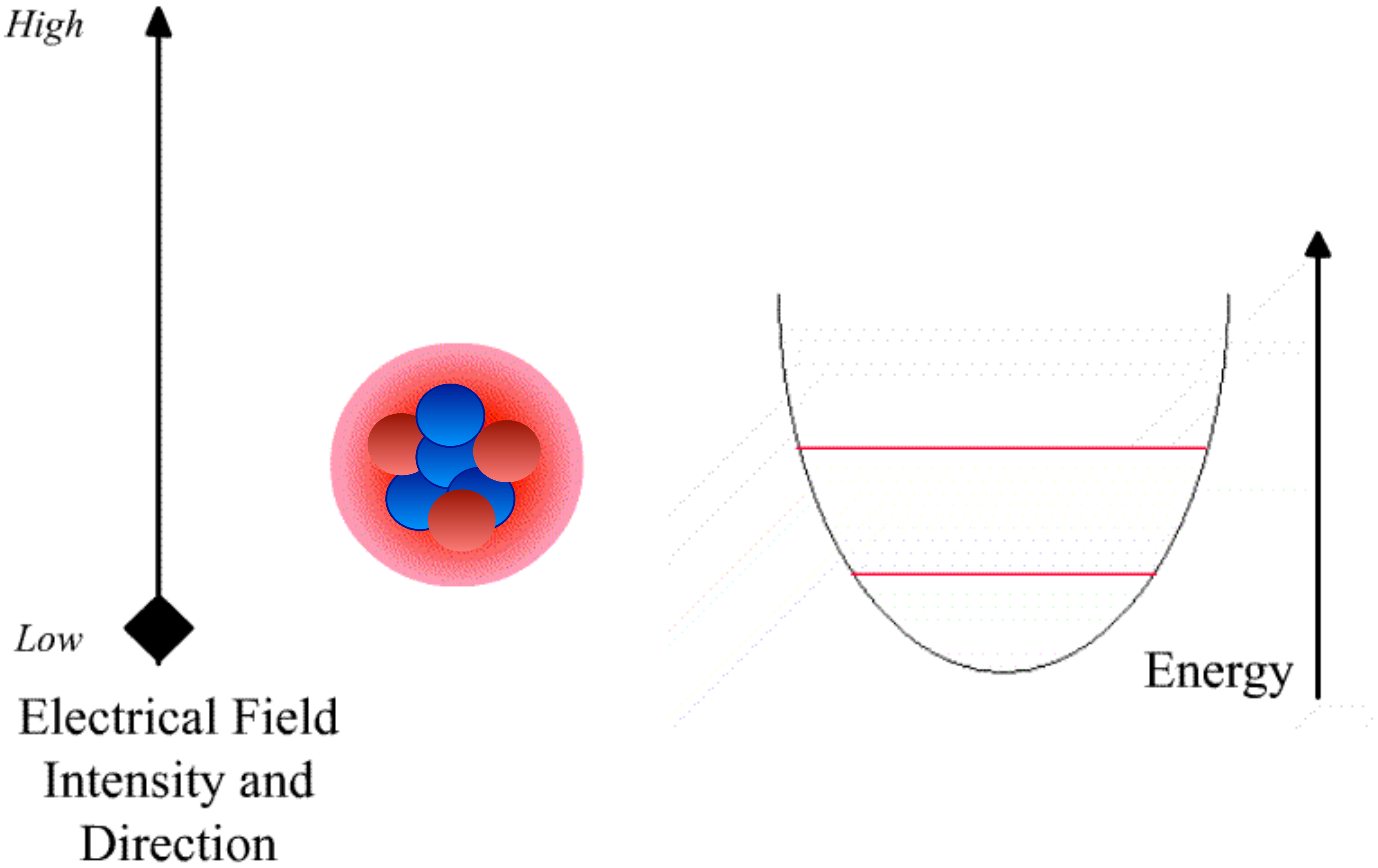
well known

not well known

- μD $\longrightarrow$ results released in 2016
- μ^4He$^+$ $\longrightarrow$ results released in 2021
- μ^3He$^+$ $\longrightarrow$ results released in 2023
- μ^3H $\longrightarrow$ not feasible
- μ^6Li^{2+} $\longrightarrow$ future plan for QUARTET
- μ^7Li^{2+} $\longrightarrow$ future plan for QUARTET

TPE like polarizability



$$\mathbf{P} = \alpha_{\mathbf{E1}} \mathbf{E}$$

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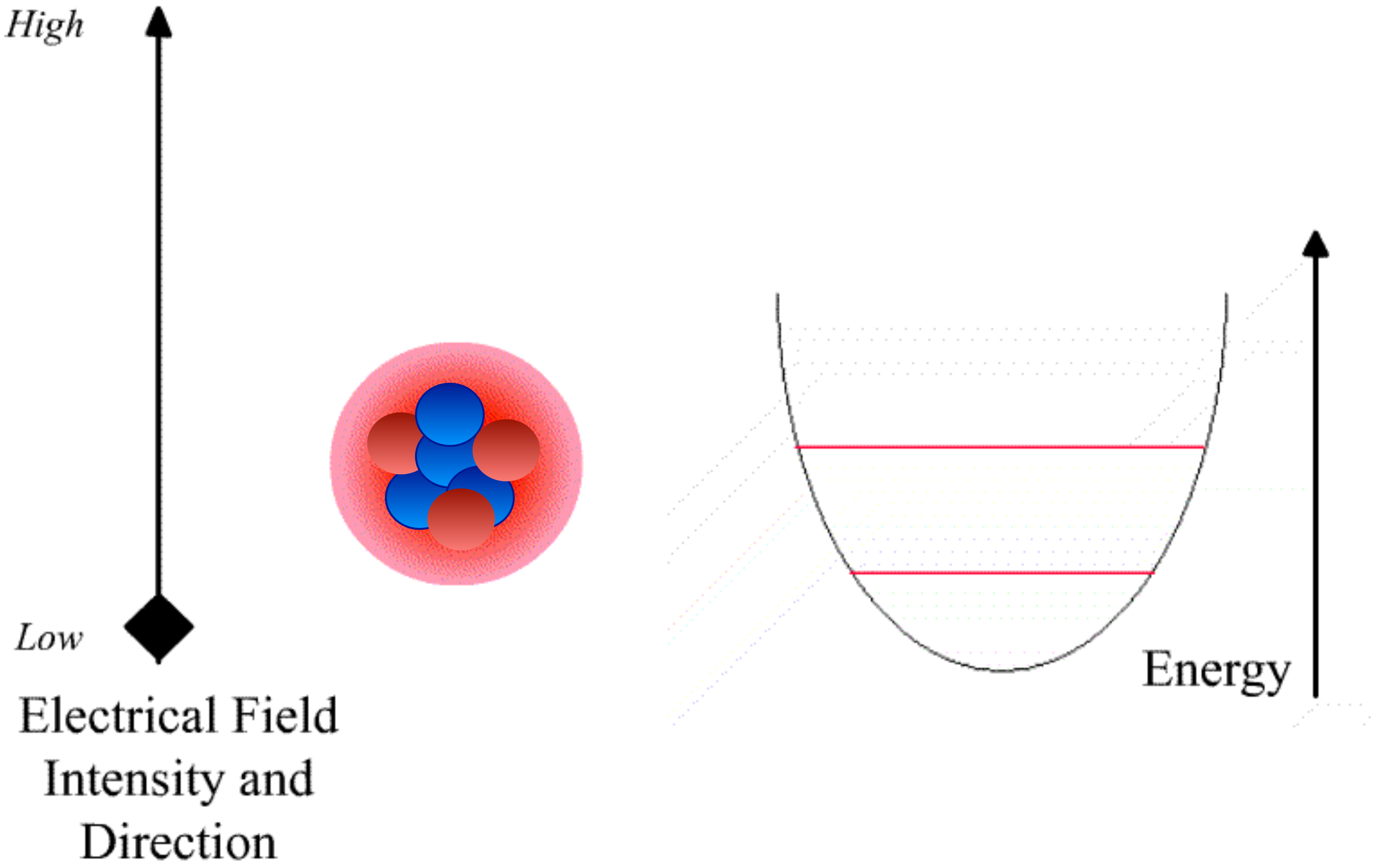

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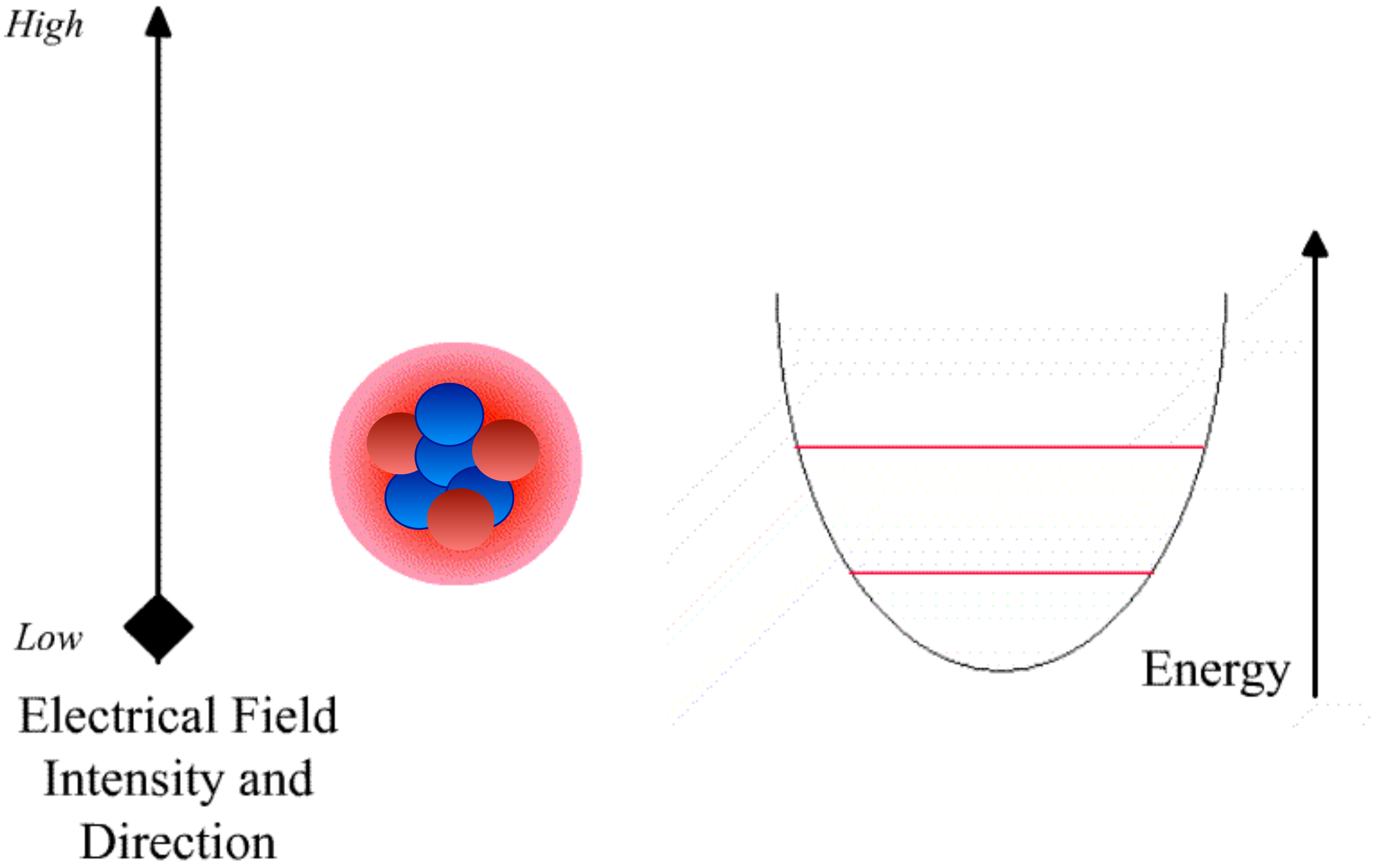

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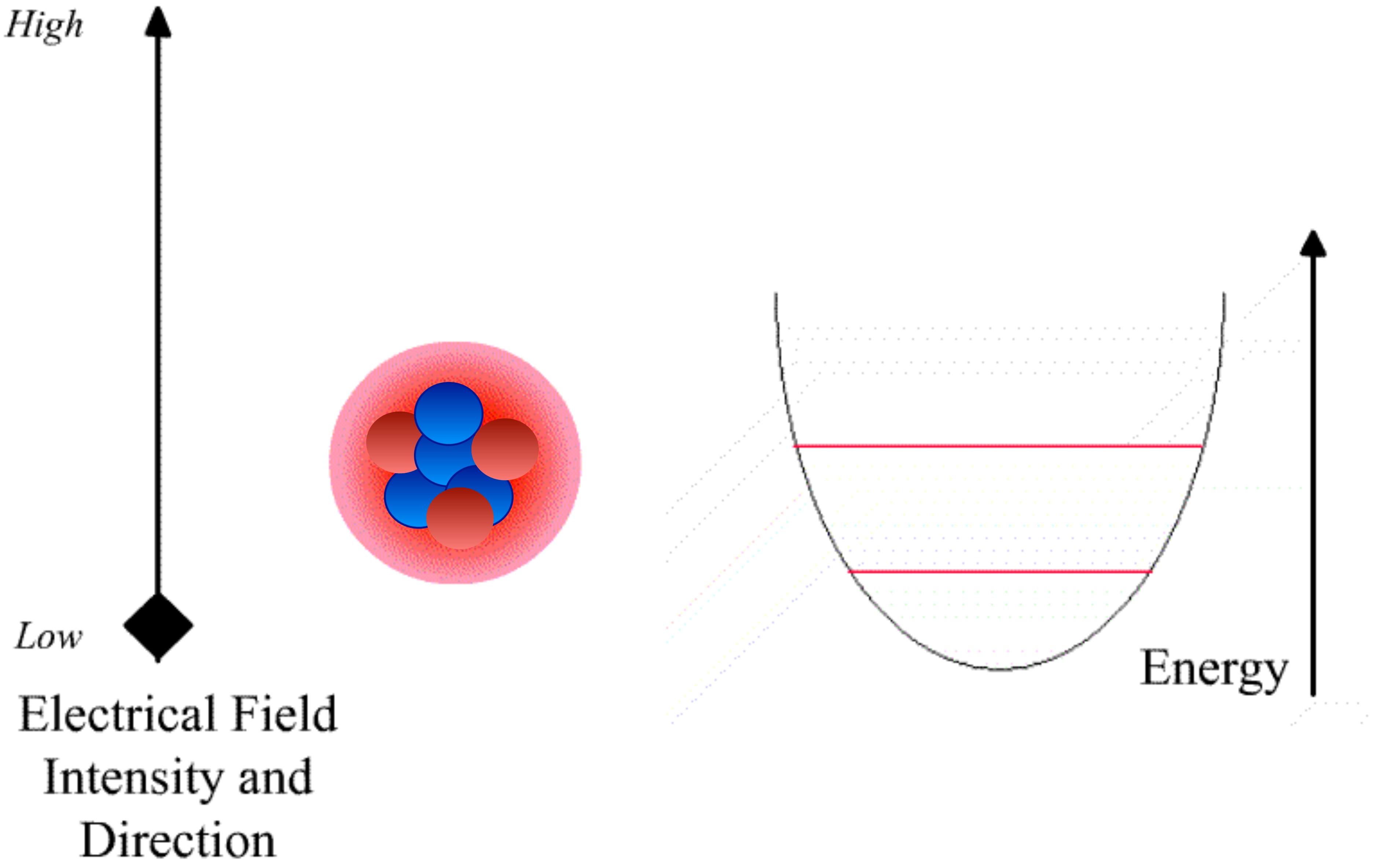

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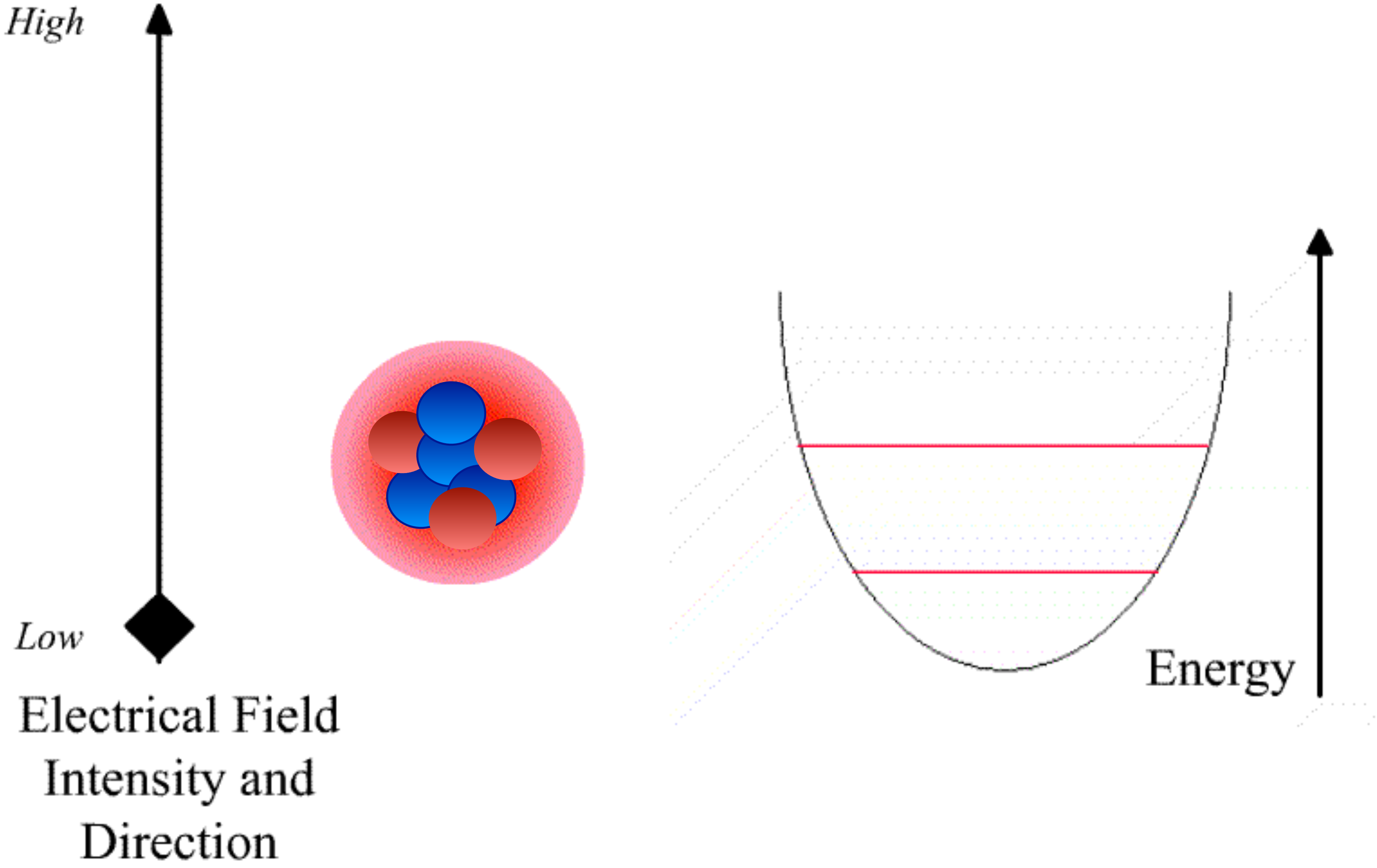
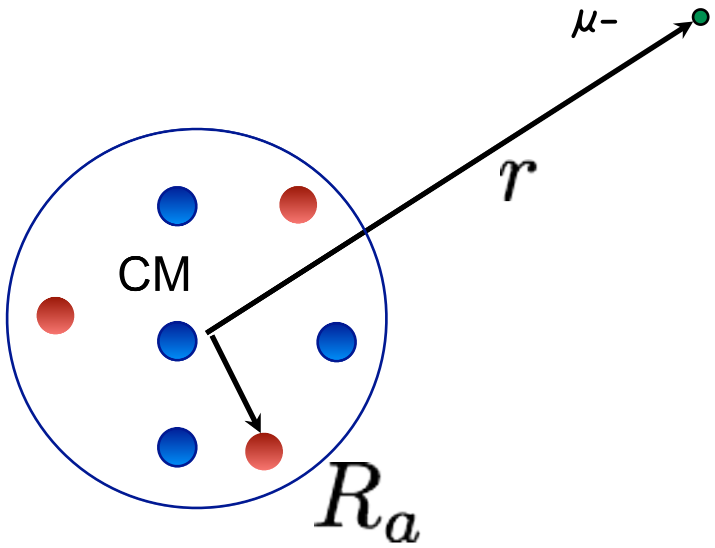


$$\mathbf{P} = \alpha_{\mathbf{E1}} \mathbf{E}$$

Theoretical derivation of TPE

$$H = H_N + H_\mu + \Delta V$$

$$H_\mu = \frac{p^2}{2m_r} - \frac{Z\alpha}{r}$$



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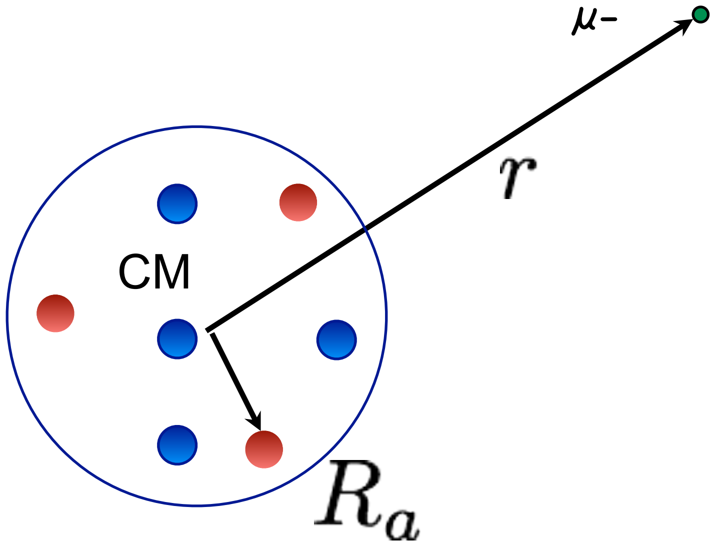

Perturbative potential: correction to the bulk Coulomb

$$\Delta V = \sum_a^Z \alpha \left(\frac{1}{r} - \frac{1}{|\vec{r} - \vec{R}_a|} \right)$$

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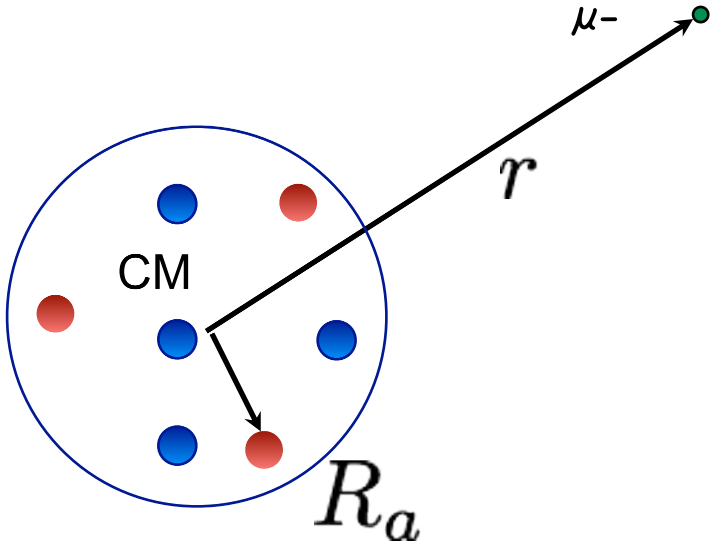
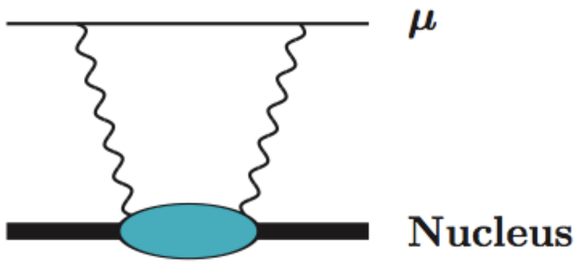


Perturbative potential: correction to the bulk Coulomb

$$\Delta V = \sum_a^Z \alpha \left(\frac{1}{r} - \frac{1}{|\vec{r} - \vec{R}_a|} \right)$$

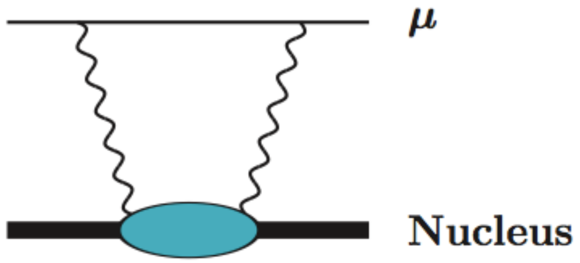
Using perturbation theory at second order one obtains the expression for TPE up to order $(Z\alpha)^5$



Theoretical derivation of TPE

- Non relativistic term

Take non-relativistic kinetic energy in muon propagator neglecting the Coulomb force in the intermediate state



$$P \simeq \frac{m_r^3 (Z\alpha)^5}{12} \sqrt{\frac{2m_r}{\omega}} \left[|\boldsymbol{R} - \boldsymbol{R}'|^2 - \frac{\sqrt{2m_r\omega}}{4} |\boldsymbol{R} - \boldsymbol{R}'|^3 + \frac{m_r\omega}{10} |\boldsymbol{R} - \boldsymbol{R}'|^4 \right]$$

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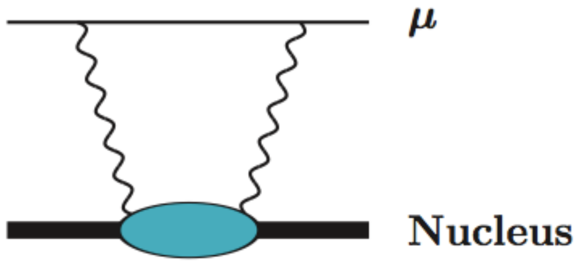

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- ★ $|\boldsymbol{R} - \boldsymbol{R}'|$ “virtual” distance traveled by the proton between the two-photon exchange
- ★ Uncertainty principle $|\boldsymbol{R} - \boldsymbol{R}'| \sim \frac{1}{\sqrt{2m_N\omega}}$
- ★ $\eta = \sqrt{2m_r\omega}|\boldsymbol{R} - \boldsymbol{R}'| \sim \sqrt{\frac{m_r}{m_N}}$

expand the muon matrix elements in powers of η up to the second order

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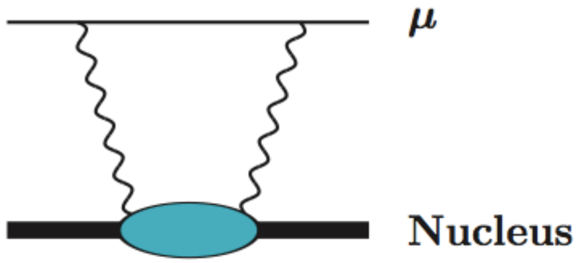

$$P \simeq \frac{m_r^3 (Z\alpha)^5}{12}\sqrt{\frac{2m_r}{\omega}}\left[\underset{\overset{\uparrow}{\delta^{(0)}}}{|\boldsymbol{R}-\boldsymbol{R}'|^2} - \underset{\overset{\uparrow}{\delta^{(1)}}}{\frac{\sqrt{2m_r\omega}}{4}|\boldsymbol{R}-\boldsymbol{R}'|^3} + \underset{\overset{\nearrow}{\delta^{(2)}}}{\frac{m_r\omega}{10}|\boldsymbol{R}-\boldsymbol{R}'|^4}\right]$$

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expand the muon matrix elements in powers of η up to the second order

Theoretical derivation of TPE

- Non relativistic term

★ $\delta^{(0)} \propto |\boldsymbol{R} - \boldsymbol{R}'|^2$

dominant term, related to the energy-weighted integral

$$\delta^{(0)}_{D1} = -\frac{2\pi m_r^3}{9}(Z\alpha)^5 \int_{\omega_{\rm th}}^{\infty} d\omega \sqrt{\frac{2m_r}{\omega}} S_{D_1}(\omega)$$

of the dipole response function

$$S_{D1}(\omega) = \frac{1}{2J_0+1} \sum\!\!\!\!\!\!\!\int_{N \neq N_0} |\langle NJ||\hat{D}_1||N_0 J_0\rangle|^2 \delta(\omega - \omega_N)$$

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★ $\delta^{(1)} \propto |\boldsymbol{R} - \boldsymbol{R}'|^3$ Related to Zemach moment elastic contribution

$$\delta^{(1)}_{Z3} = \frac{\pi}{3} m_r (Z\alpha)^2 \phi^2(0) \iint d^3R d^3R' |\boldsymbol{R} - \boldsymbol{R}'|^3 \rho_0^p(\boldsymbol{R}) \rho_0^p(\boldsymbol{R}')$$

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★ $\delta^{(2)} \propto |\boldsymbol{R}-\boldsymbol{R}'|^4$

leads to energy-weighted integrals of three different response functions

$S_{R^2}(\omega), S_Q(\omega), S_{D1D3}(\omega)$

Theoretical derivation of TPE

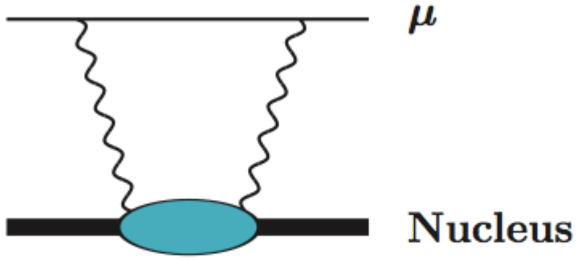
- Coulomb term

Consider the Coulomb force in the intermediate states

Naively $\delta_C^{(0)} \sim (Z\alpha)^6$, actually logarithmically enhanced

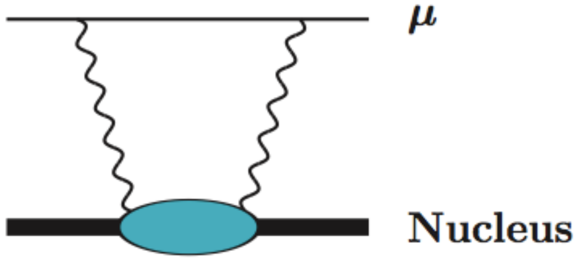
$\delta_C^{(0)} \sim (Z\alpha)^5 \log(Z\alpha)$ Friar (1977), Pachucki (2011)

Related to the dipole response function



Theoretical derivation of TPE

Coulomb term



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Related to the dipole response function

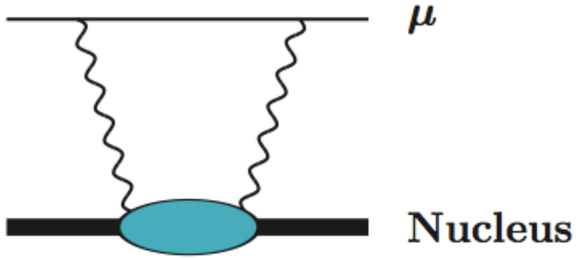
Relativistic terms

Take the relativistic kinetic energy in muon propagator
Related to the dipole response function

$$\delta_{L(T)}^{(0)} = \frac{2m_r^3}{9}(Z\alpha)^5 \int_{\omega_{\rm th}}^{\infty} d\omega\, K_{L(T)}\left(\frac{\omega}{m_r}\right)\, S_{D_1}(\omega)$$

Theoretical derivation of TPE

Coulomb term



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Finite nucleon-size corrections

Consider finite nucleon-size by including their charge distributions and obtain terms, e.g.,

$$\delta_{R1}^{(1)} = -8\pi m_r (Z\alpha)^2 \phi^2(0) \int\int d^3R d^3R' |\boldsymbol{R}-\boldsymbol{R}'| \left[\frac{2}{\beta^2}\rho_0^{pp}(\boldsymbol{R},\boldsymbol{R}') - \lambda\rho_0^{np}(\boldsymbol{R},\boldsymbol{R}')\right]$$

Theoretical derivation of TPE

$$\delta_{\rm TPE} = \delta^A_{\rm Zem} + \delta^n_{\rm Zem} + \delta^A_{\rm pol} + \delta^n_{\rm pol}$$

Theoretical derivation of TPE

$$\delta_{\rm TPE} = \delta^{A}_{\rm Zem} + \delta^{n}_{\rm Zem} + \delta^{A}_{\rm pol} + \delta^{n}_{\rm pol}$$

$$\begin{aligned}\delta^{A}_{\rm pol} = \delta^{(0)}_{D1} + \delta^{(1)}_{R3} + \delta^{(1)}_{Z3} + \delta^{(2)}_{R^2} + \delta^{(2)}_{Q} + \delta^{(2)}_{D1D3} + \delta^{(0)}_{C} \\ + \delta^{(0)}_{L} + \delta^{(0)}_{T} + \delta^{(0)}_{M} + \delta^{(1)}_{R1} + \delta^{(1)}_{Z1} + \delta^{(2)}_{NS}\end{aligned}$$

Theoretical derivation of TPE

$$\delta_{\rm TPE} = \delta^A_{\rm Zem} + \delta^n_{\rm Zem} + \delta^A_{\rm pol} + \delta^n_{\rm pol}$$

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$$\delta^A_{\rm Zem} = -\delta^{(1)}_{Z3} - \delta^{(1)}_{Z1}$$

Theoretical derivation of TPE

$$\delta_{\mathrm{TPE}} = \delta^{A}_{\mathrm{Zem}} + \delta^{n}_{\mathrm{Zem}} + \delta^{A}_{\mathrm{pol}} + \delta^{n}_{\mathrm{pol}}$$

$$\delta^{A}_{\mathrm{pol}} = \delta^{(0)}_{D1} + \delta^{(1)}_{R3} + \cancel{\delta^{(1)}_{Z3}} + \delta^{(2)}_{R^2} + \delta^{(2)}_{Q} + \delta^{(2)}_{D1D3} + \delta^{(0)}_{C}$$
$$+\delta^{(0)}_{L} + \delta^{(0)}_{T} + \delta^{(0)}_{M} + \delta^{(1)}_{R1} + \cancel{\delta^{(1)}_{Z1}} + \delta^{(2)}_{NS}$$

$$\delta^{A}_{\mathrm{Zem}} = -\cancel{\delta^{(1)}_{Z3}} - \cancel{\delta^{(1)}_{Z1}}$$

Friar an Payne ('97)

A matter of precision

The uncertainty of the extracted radius depends on the precision of the TPE

$$\Delta E_{2S-2P} = \delta_{\mathrm{QED}} + \mathcal{A}_{\mathrm{OPE}} \langle r_c^2 \rangle + \delta_{\mathrm{TPE}}$$

Even though, roughly: 95% 4% 1%

The uncertainty on TPE exceeds the experimental precision, hence reducing uncertainties is important

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Uncertainties comparison

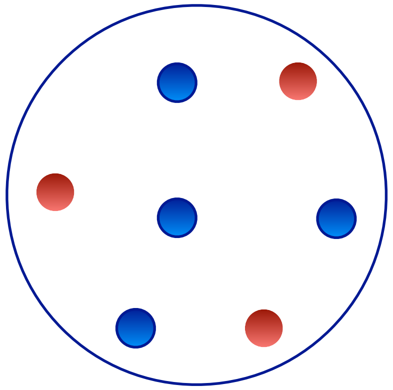
Atom	$\Delta E_{2S\text{-}2P}$	$\Delta\delta_{\mathrm{TPE}}$
μ^2H	0.003 meV	0.03 meV
μ^3He$^+$	0.08 meV	1 meV
μ^4He$^+$	0.06 meV	0.6 meV
$\mu\,^{6,7}$Li^{++}	0.7 meV	4 meV

Ab Initio Nuclear Theory

- Solve the Schrödinger equation for few-nucleons

$$H_N|\Psi_{NJ}\rangle = E_{NJ}|\Psi_{NJ}\rangle$$

using numerical methods that allow to assign uncertainties



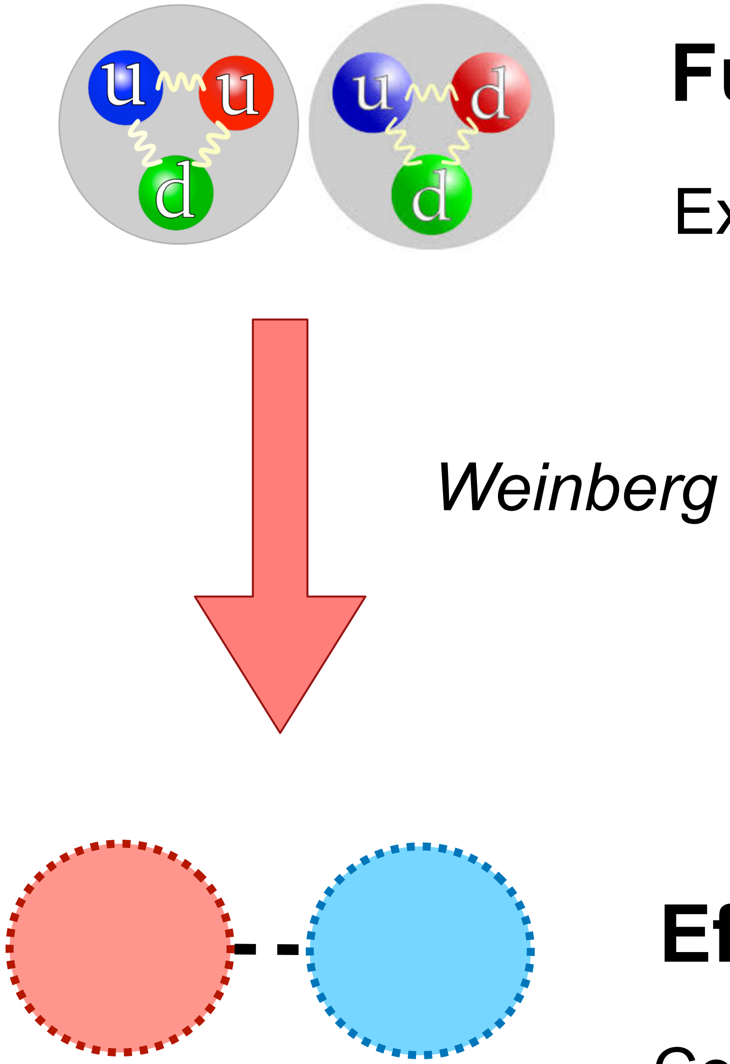
- Starting from a nuclear Hamiltonian

$$H_N = T + V$$

V Phenomenology or Chiral Effective Field Theory

There will also be an uncertainty due to the modelling of the nuclear Hamiltonian

Chiral Effective Field Theory



Weinberg

Fundamental: Quarks/gluons

Explicit and spontaneously broken **chiral symmetry**

Effective: Nucleons/pions

Construct the most general theory compatible with explicit and spontaneous **chiral symmetry breaking.** **Low-energy constants** encapsulate the non-resolved high energy physics

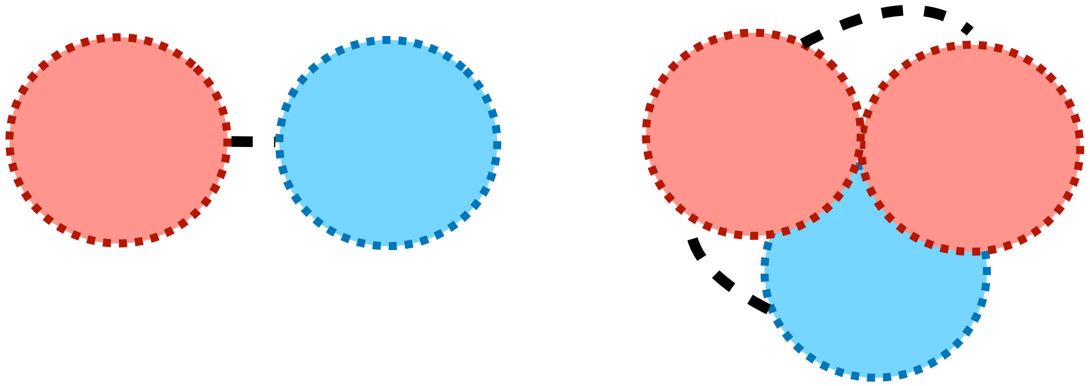
Chiral Effective Field Theory

Systematic expansion in powers of Q/Λ

$$V = V_{\mathrm{LO}} + V_{\mathrm{NLO}} + V_{\mathrm{NNLO}} \dots$$

Three-nucleon forces appear naturally and consistently with two-nucleon forces

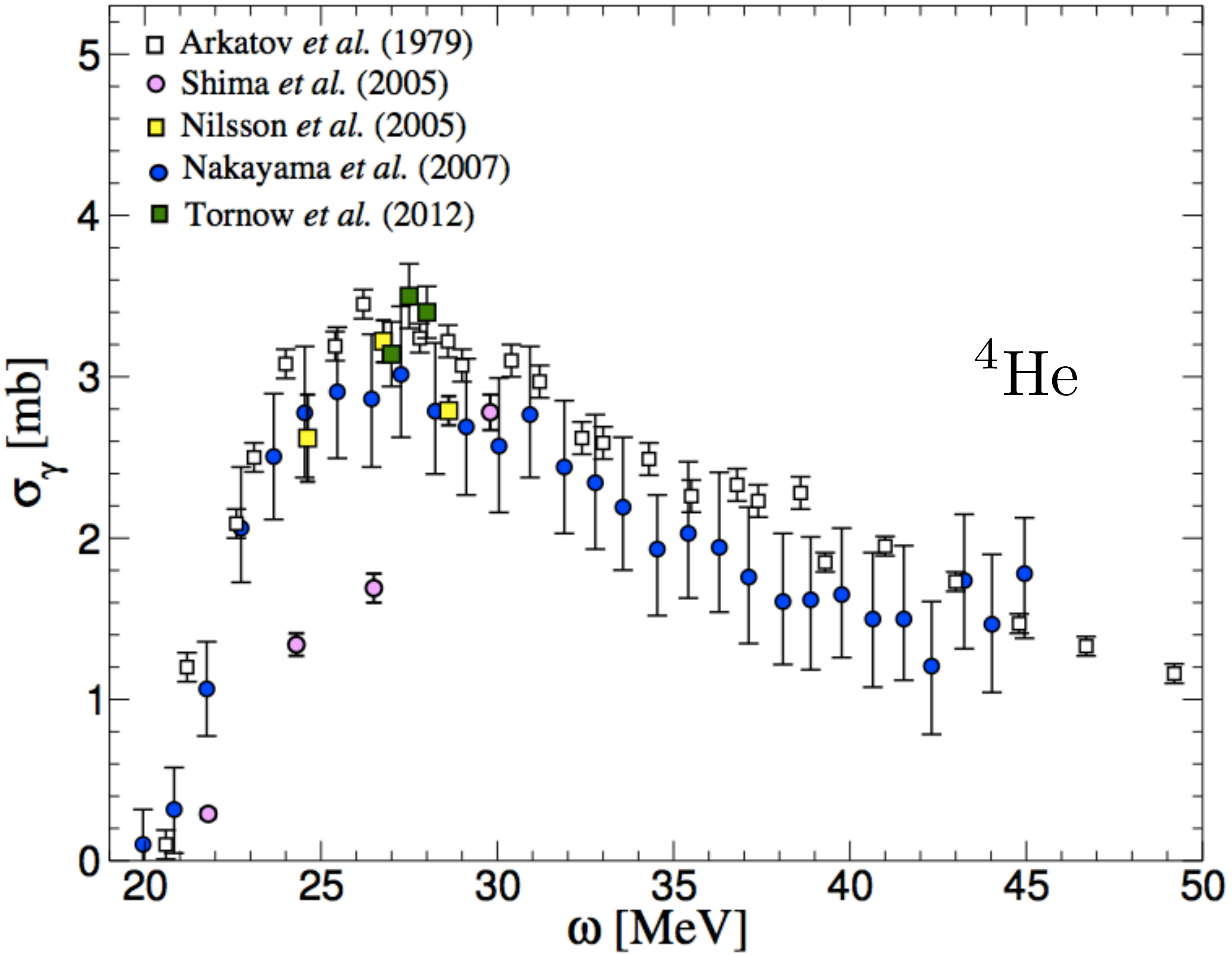
$$V = V_{NN} + V_{3N} + \dots$$



Few-body methods

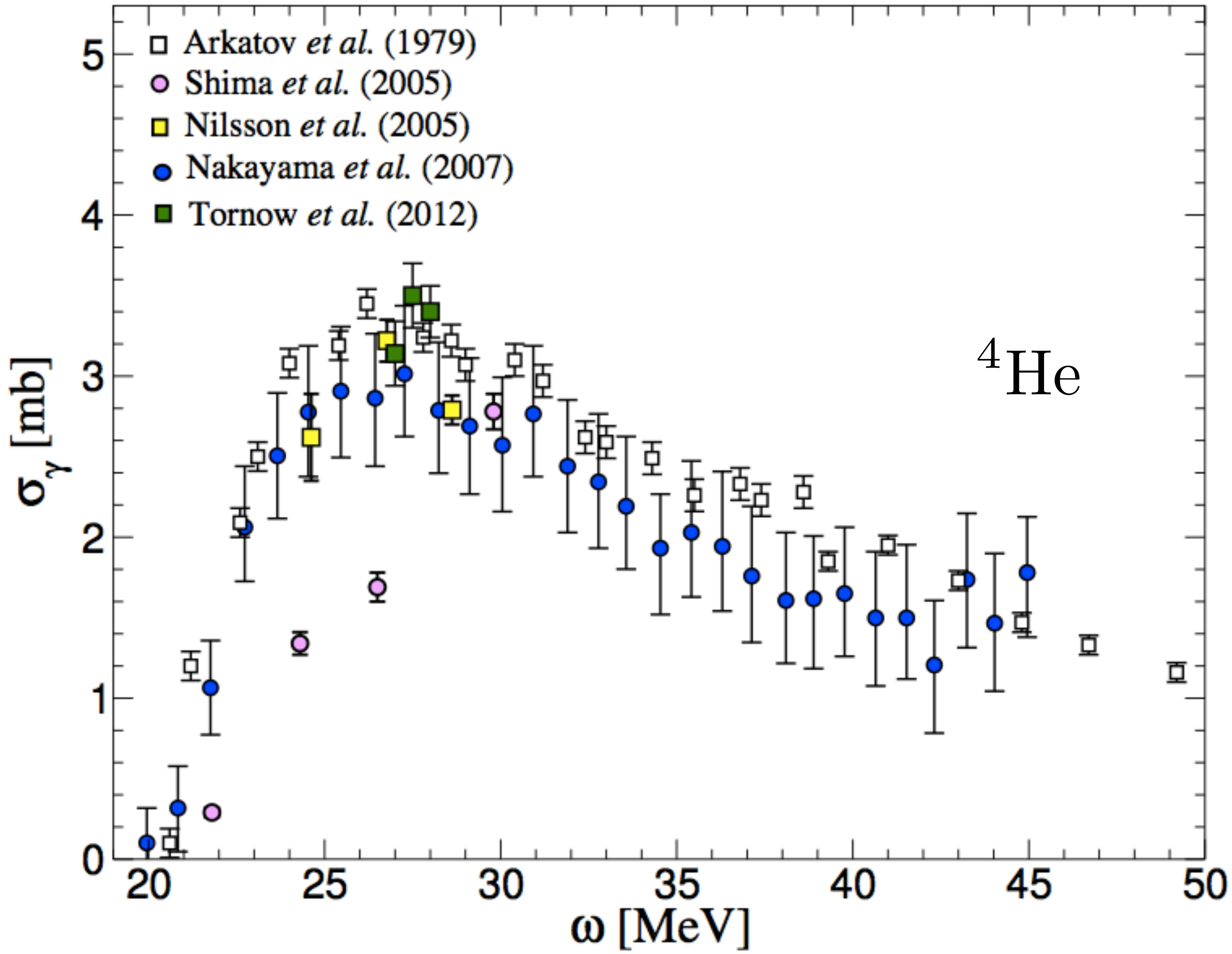
Lorentz Integral Transform
and
Hyperspherical Harmonics expansion

An example



SB and Saori Pastore, Journal of Physics G.: Nucl. Part. Phys. **41**, 123002 (2014)

An example



$$\delta_{D1}^{(0)} \rightarrow S_{\mathrm{D1}}(\omega)$$

$$S_{\mathrm{D1}}(\omega) = \frac{9}{16\pi^3 \alpha \omega Z^2} \sigma_\gamma(\omega)$$

SB and Saori Pastore, Journal of Physics G.: Nucl. Part. Phys. **41**, 123002 (2014)

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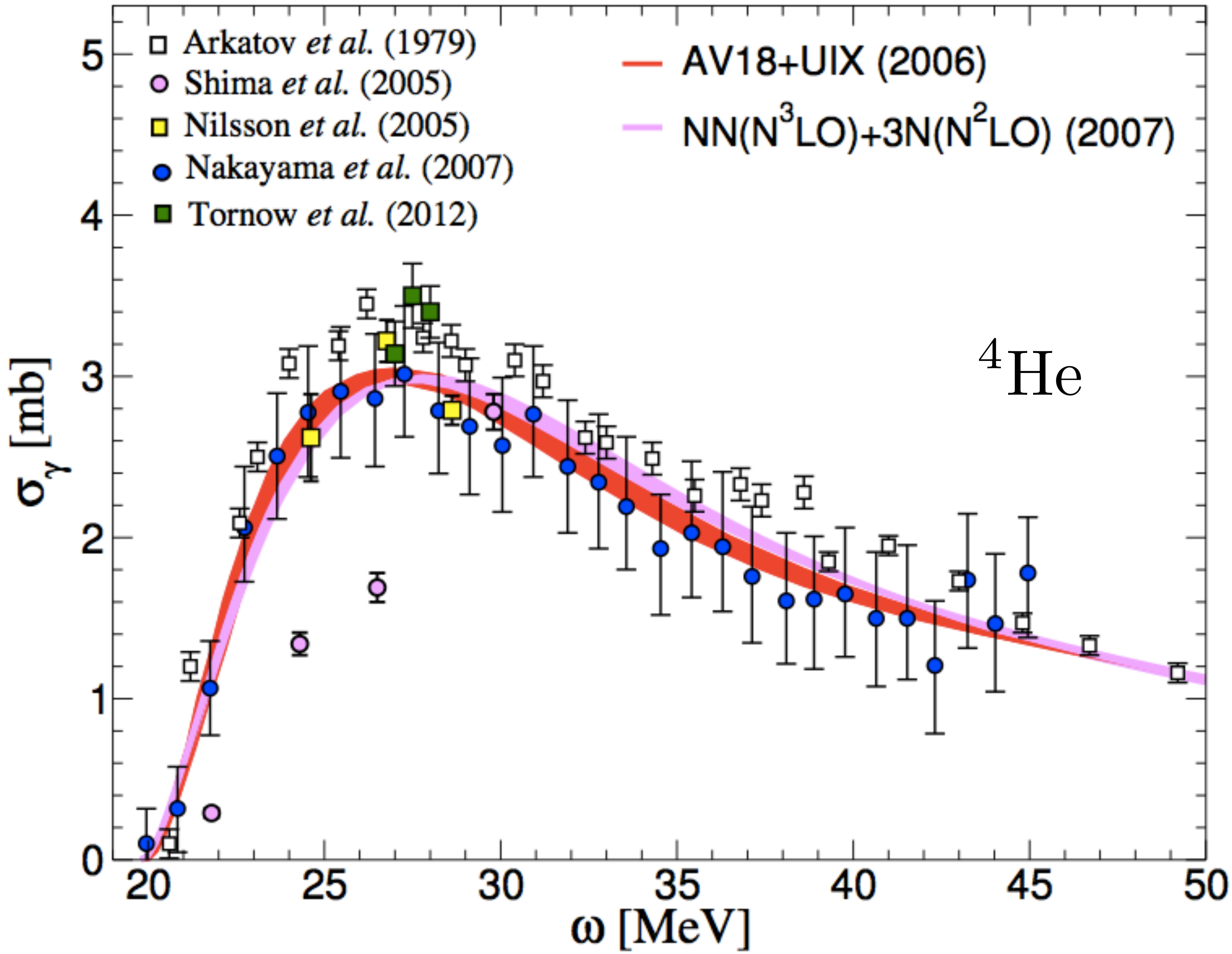

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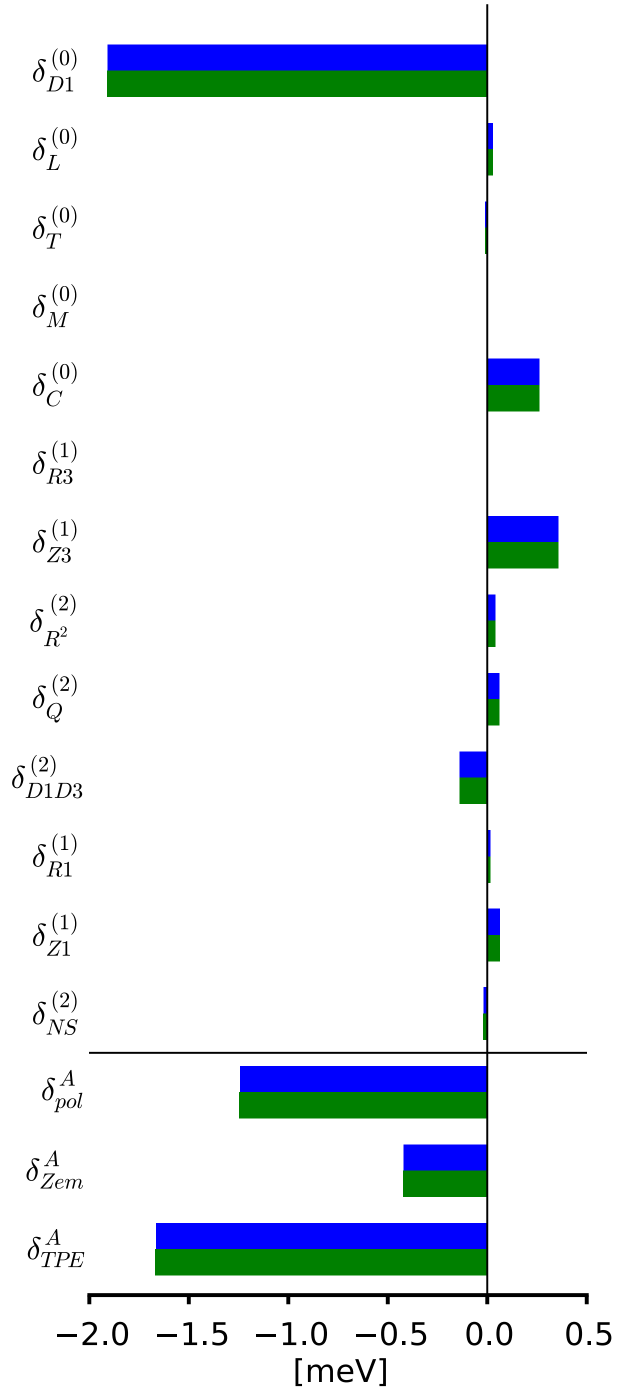
SB and Saori Pastore, Journal of Physics G.: Nucl. Part. Phys. **41**, 123002 (2014)

Use these technology to analyze muonic atoms

Muonic Deuterium



	AV18	χEFT
$\delta_{D1}^{(0)}$	-1.907	-1.912
$\delta_{L}^{(0)}$	0.029	0.029
$\delta_{T}^{(0)}$	-0.012	-0.012
$\delta_{M}^{(0)}$	0.003	0.003
$\delta_{C}^{(0)}$	0.262	0.262
$\delta_{R3}^{(1)}$	-	-
$\delta_{Z3}^{(1)}$	0.357	0.359
$\delta_{R^2}^{(2)}$	0.042	0.041
$\delta_{Q}^{(2)}$	0.061	0.061
$\delta_{D1D3}^{(2)}$	-0.139	-0.139
$\delta_{R1}^{(1)}$	0.017	0.017
$\delta_{Z1}^{(1)}$	0.064	0.064
$\delta_{NS}^{(2)}$	-0.020	-0.021
δ_{pol}^{A}	-1.243	-1.248
δ_{Zem}^{A}	-0.421	-0.423
δ_{TPE}^{A}	-1.664	-1.671

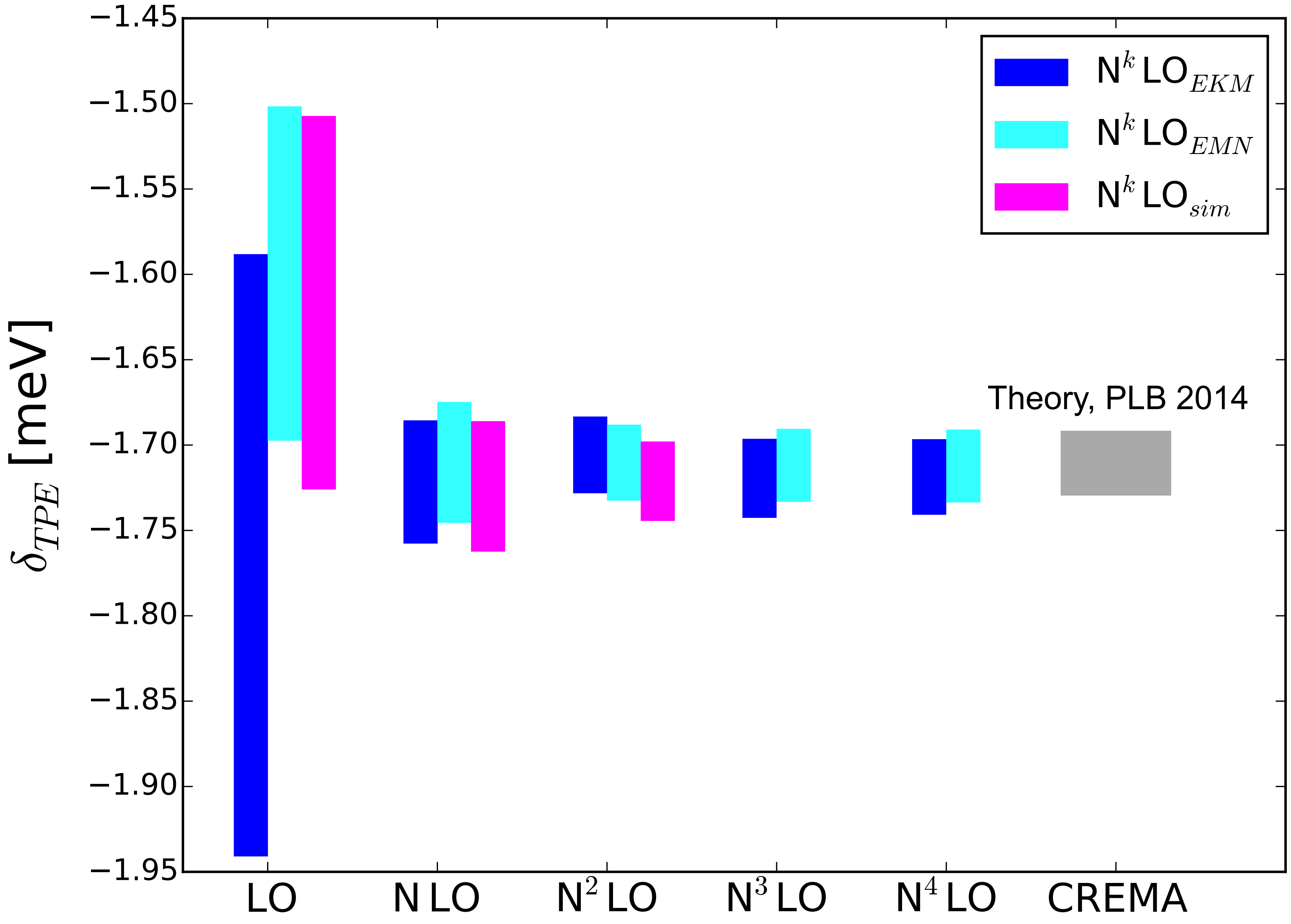
Hernandez et al, Phys. Lett. B **736**, 344 (2014)

AV18 in agreement with Pachucki (2011)+ Pachucki, Wienczek (2015)

Order-by-order chiral expansion

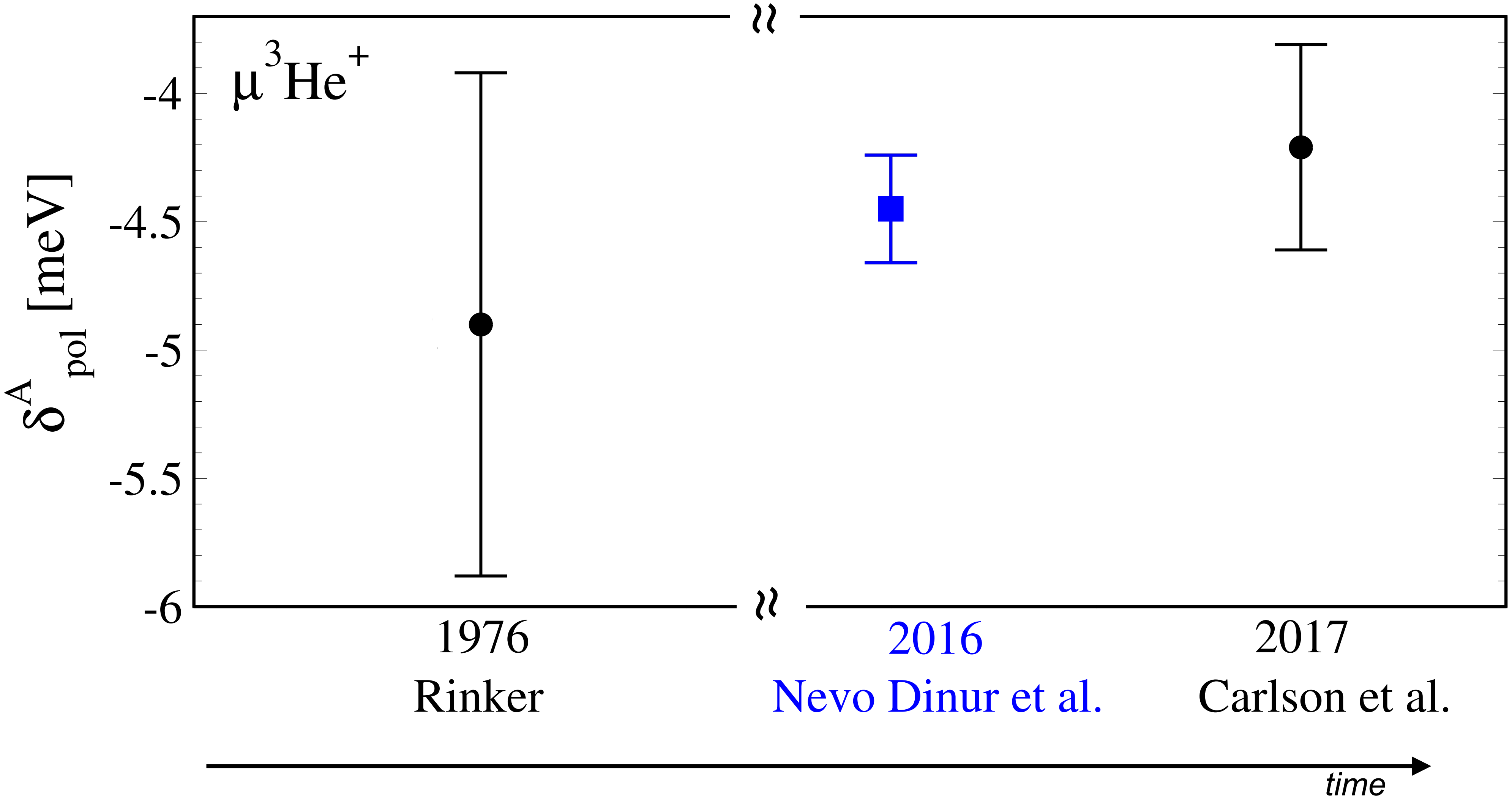
Statistical and systematic uncertainty analysis

O.J. Hernandez et al, Phys. Lett. B **778**, 377 (2018)



Impact of ab initio theory

- Reduction of Uncertainties -



C.Ji et al., JPG: Part. Nucl. **45,** 093002 (2018)

Impact of ab initio theory

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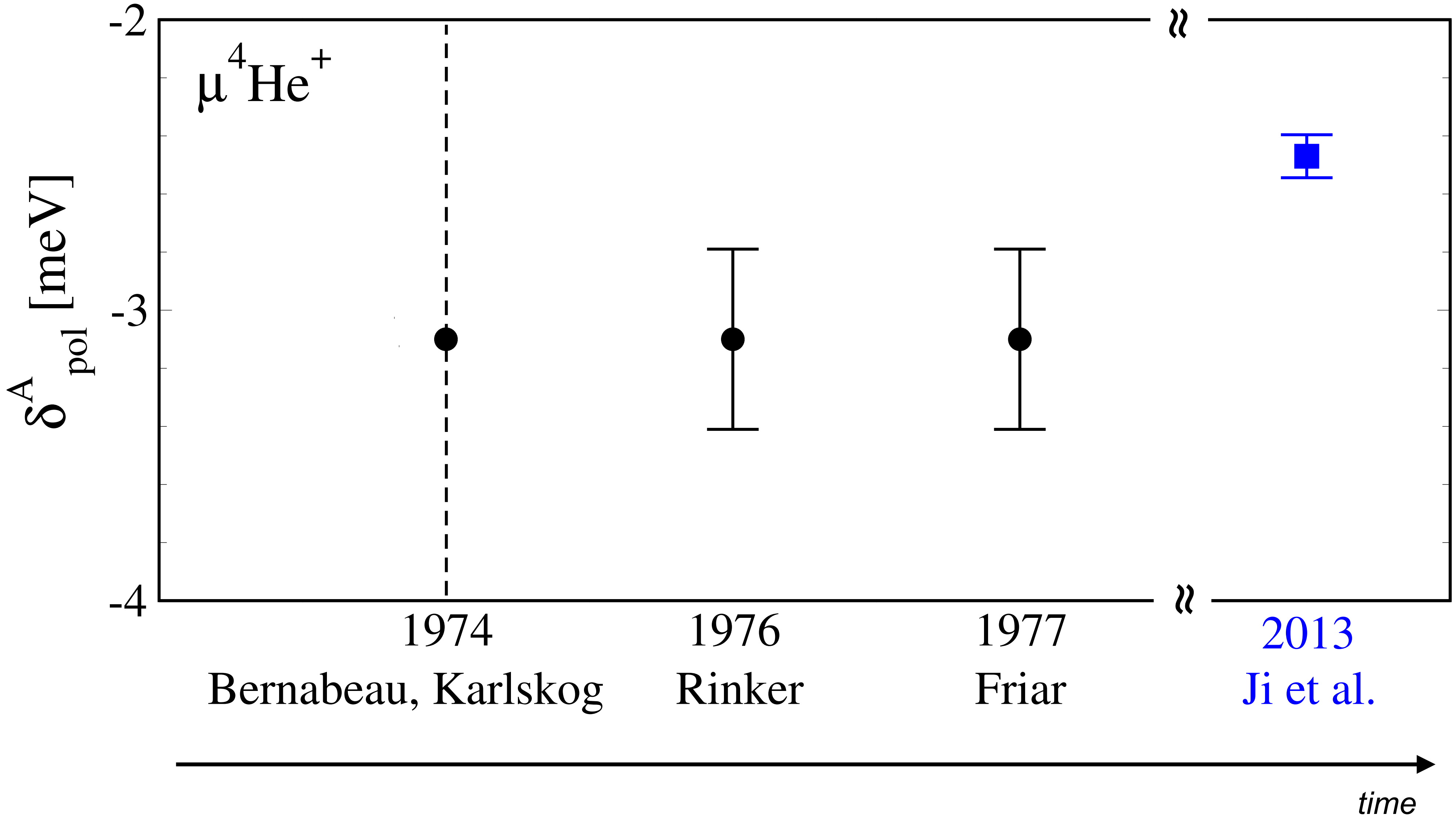
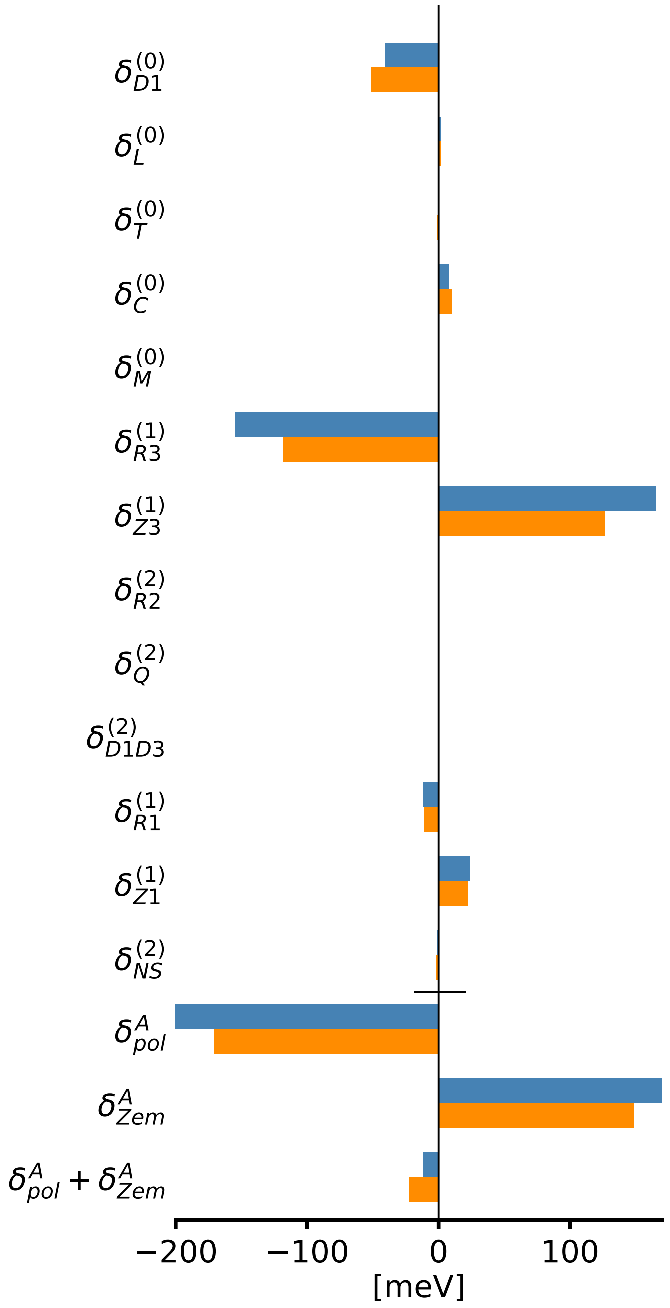


C.Ji et al., JPG: Part. Nucl. **45,** 093002 (2018)

Muonic Lithium

Li Muli, SB, Poggialini, SciPost (2020)

With AV4’ potential, in meV



	μ-^{6}Li^{2+}	μ-^{7}Li^{2+}
$\delta_{D1}^{(0)}$	-41.0	-51.0
$\delta_{L}^{(0)}$	1.66	2.04
$\delta_{T}^{(0)}$	-0.75	-0.92
$\delta_{C}^{(0)}$	7.85	9.89
$\delta_{M}^{(0)}$	-	-
$\delta_{R3}^{(1)}$	-155.0	-118.0
$\delta_{Z3}^{(1)}$	165.4	126.5
$\delta_{R2}^{(2)}$	-	-
$\delta_{Q}^{(2)}$	-	-
$\delta_{D1D3}^{(2)}$	-	-
$\delta_{R1}^{(1)}$	-12.0	-11.0
$\delta_{Z1}^{(1)}$	23.47	22.03
$\delta_{NS}^{(2)}$	-1.41	-1.75
δ_{pol}^{A}	-200.65	-170.74
δ_{Zem}^{A}	188.87	148.53
$\delta_{pol}^{A} + \delta_{Zem}^{A}$	-11.78	-22.21

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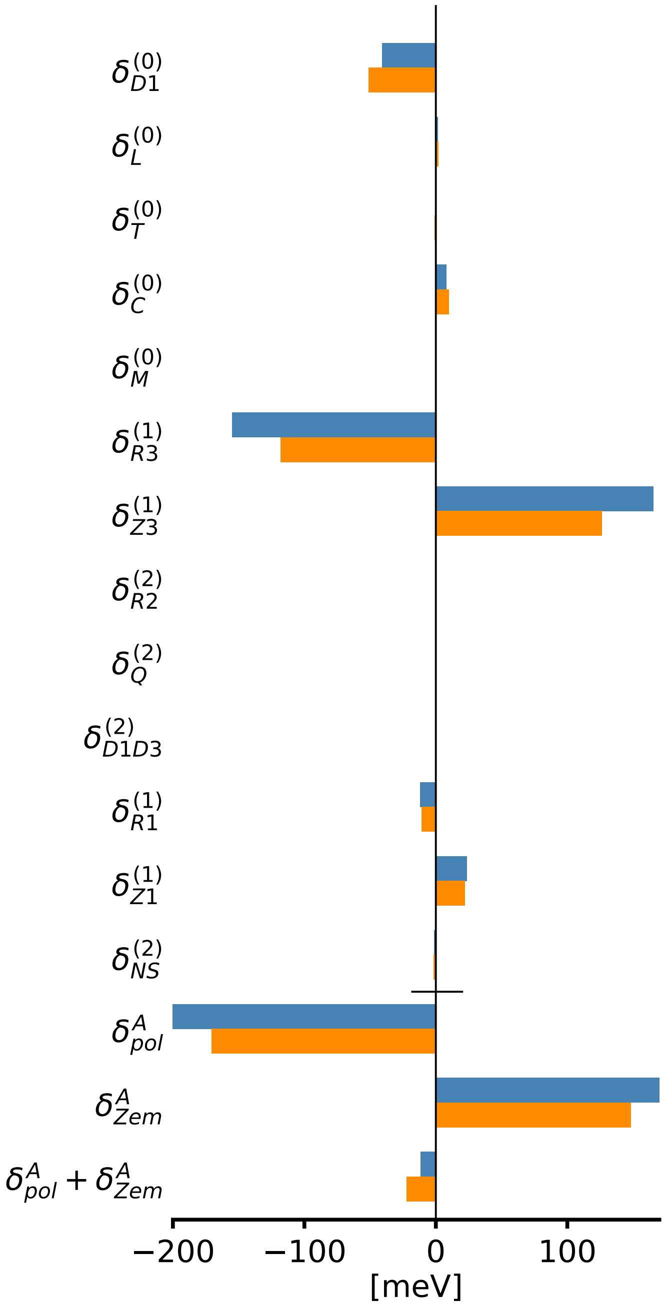

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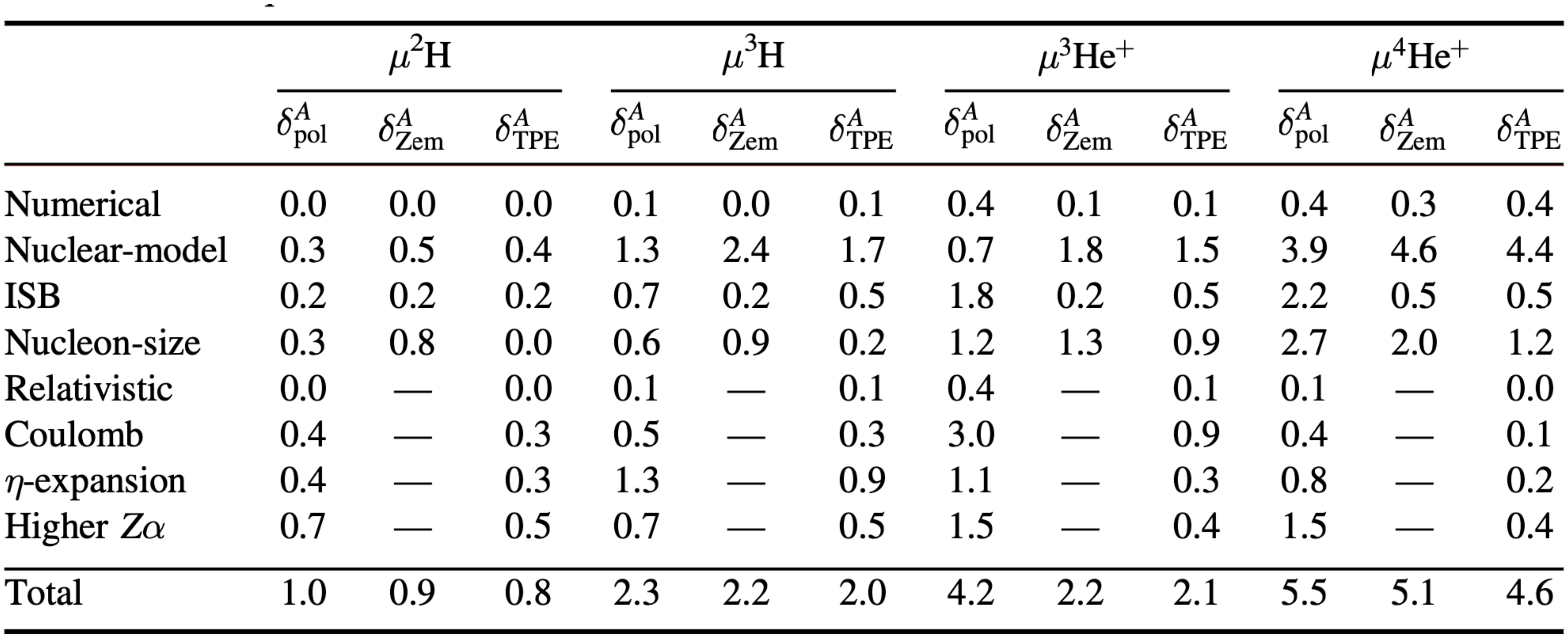
-15 ± 4 -21 ± 4 Drake et al., PRA (1985)

Uncertainty quantification

C.Ji et al., JPG: Part. Nucl. **45,** 093002 (2018)

Relative % error



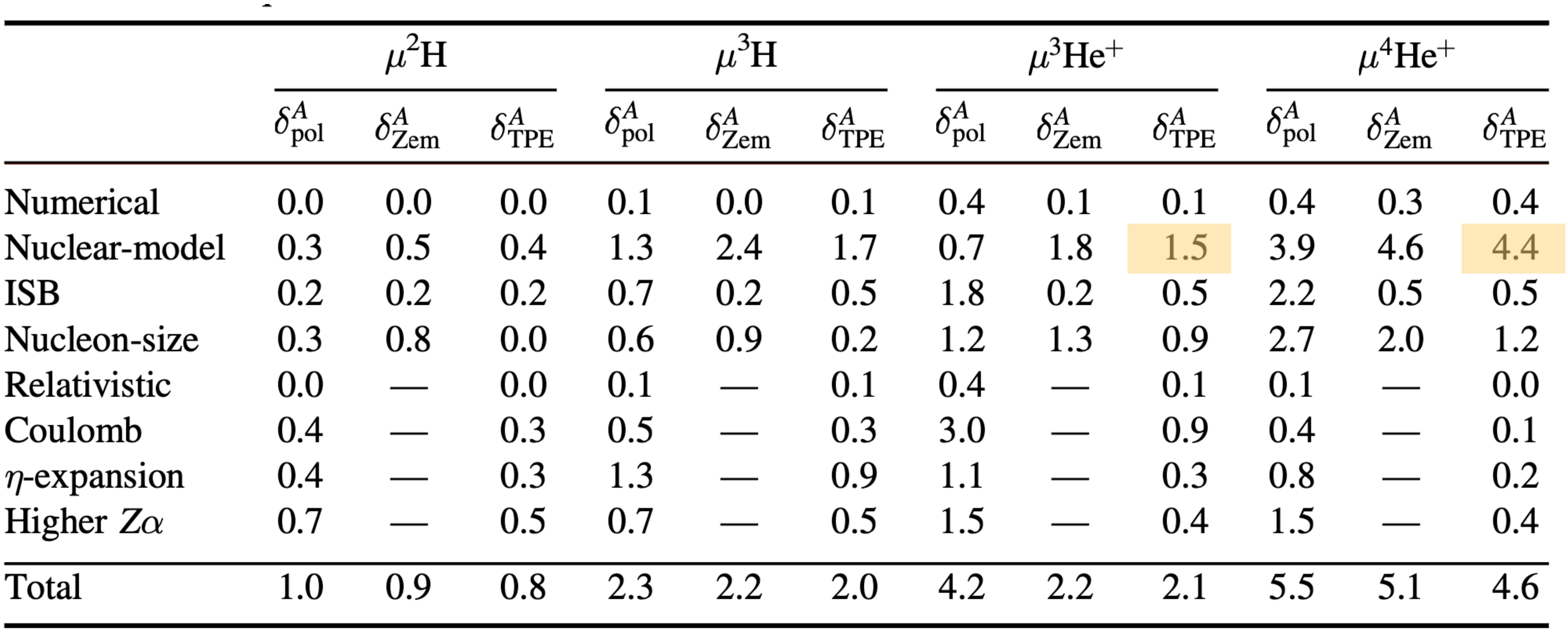
	μ^2H			μ^3H			μ^3He$^+$			μ^4He$^+$		
	$\delta^A_{\rm pol}$	$\delta^A_{\rm Zem}$	$\delta^A_{\rm TPE}$	$\delta^A_{\rm pol}$	$\delta^A_{\rm Zem}$	$\delta^A_{\rm TPE}$	$\delta^A_{\rm pol}$	$\delta^A_{\rm Zem}$	$\delta^A_{\rm TPE}$	$\delta^A_{\rm pol}$	$\delta^A_{\rm Zem}$	$\delta^A_{\rm TPE}$
Numerical	0.0	0.0	0.0	0.1	0.0	0.1	0.4	0.1	0.1	0.4	0.3	0.4
Nuclear-model	0.3	0.5	0.4	1.3	2.4	1.7	0.7	1.8	1.5	3.9	4.6	4.4
ISB	0.2	0.2	0.2	0.7	0.2	0.5	1.8	0.2	0.5	2.2	0.5	0.5
Nucleon-size	0.3	0.8	0.0	0.6	0.9	0.2	1.2	1.3	0.9	2.7	2.0	1.2
Relativistic	0.0	—	0.0	0.1	—	0.1	0.4	—	0.1	0.1	—	0.0
Coulomb	0.4	—	0.3	0.5	—	0.3	3.0	—	0.9	0.4	—	0.1
η-expansion	0.4	—	0.3	1.3	—	0.9	1.1	—	0.3	0.8	—	0.2
Higher $Z\alpha$	0.7	—	0.5	0.7	—	0.5	1.5	—	0.4	1.5	—	0.4
Total	1.0	0.9	0.8	2.3	2.2	2.0	4.2	2.2	2.1	5.5	5.1	4.6

Nuclear model error is dominating for ^{3,4}He and we have not performed yet an order-by-order analysis in chiral EFT

Uncertainty quantification

C.Ji et al., JPG: Part. Nucl. **45,** 093002 (2018)

Relative % error



	μ^2H			μ^3H			μ^3He$^+$			μ^4He$^+$		
	$\delta^A_{\rm pol}$	$\delta^A_{\rm Zem}$	$\delta^A_{\rm TPE}$	$\delta^A_{\rm pol}$	$\delta^A_{\rm Zem}$	$\delta^A_{\rm TPE}$	$\delta^A_{\rm pol}$	$\delta^A_{\rm Zem}$	$\delta^A_{\rm TPE}$	$\delta^A_{\rm pol}$	$\delta^A_{\rm Zem}$	$\delta^A_{\rm TPE}$
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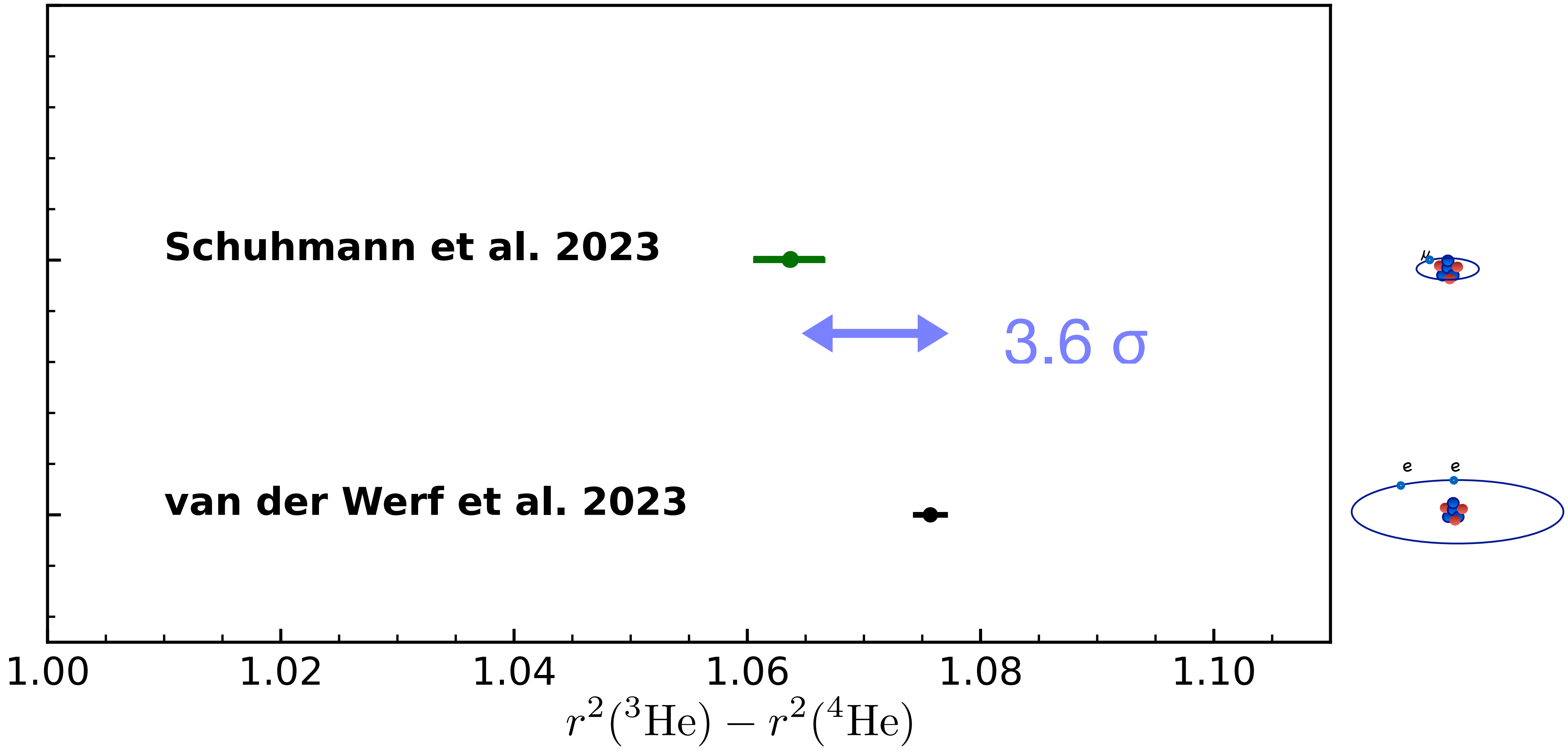
The Helium Isotope Shift puzzle



The Helium Isotope Shift puzzle

Li Muli, Richardson, SB, arXiv:2401.13424

Nuclear structure corrections to ordinary atoms

$$\delta^{A}_{\mathrm{TPE},\,e} = -\frac{2}{3} m (Z\alpha)^2 \phi^2_{nS}\ \tilde{\alpha}_{\mathrm{pol,e}}$$

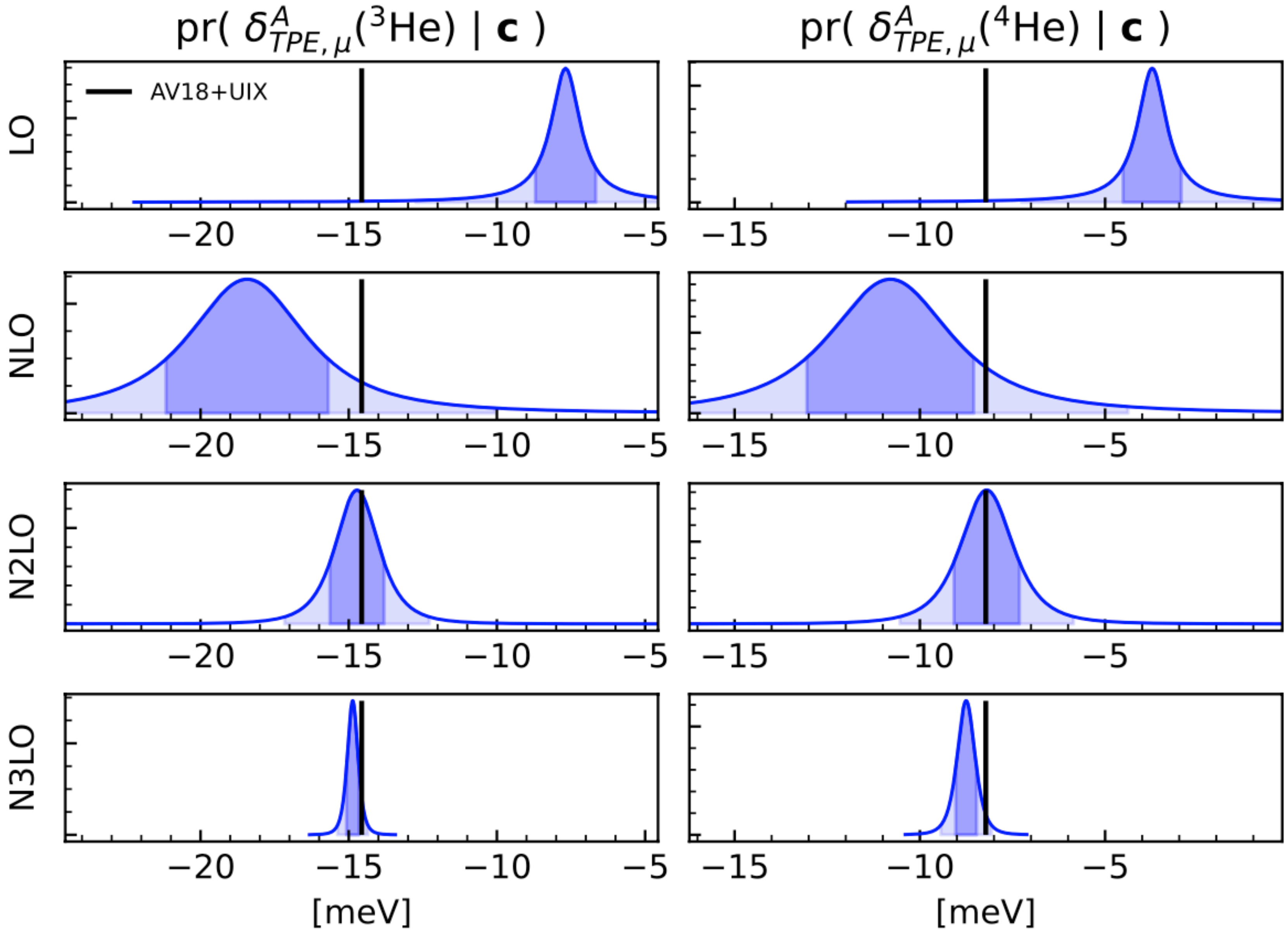
$$\tilde{\alpha}_{\mathrm{pol,e}} = \sum_{N\neq 0} |\langle N|\,\mathbf{D}\,|0\rangle|^2 \left[\frac{19}{6\omega_N} + \frac{5\ln(2\omega_N/m)}{\omega_N}\right]$$

In meV	^{3}He	^{4}He
Our at N3LO	3.514(68)	1.909(96)
Pachucki, Moro (2007)	3.560(360)	2.070(200)

The Helium Isotope Shift puzzle

Li Muli, Richardson, SB, arXiv:2401.13424

Bayesian uncertainty quantification $\delta = \delta_{\rm ref} \sum_{n=0}^{\infty} c_n (Q/\Lambda)^n$



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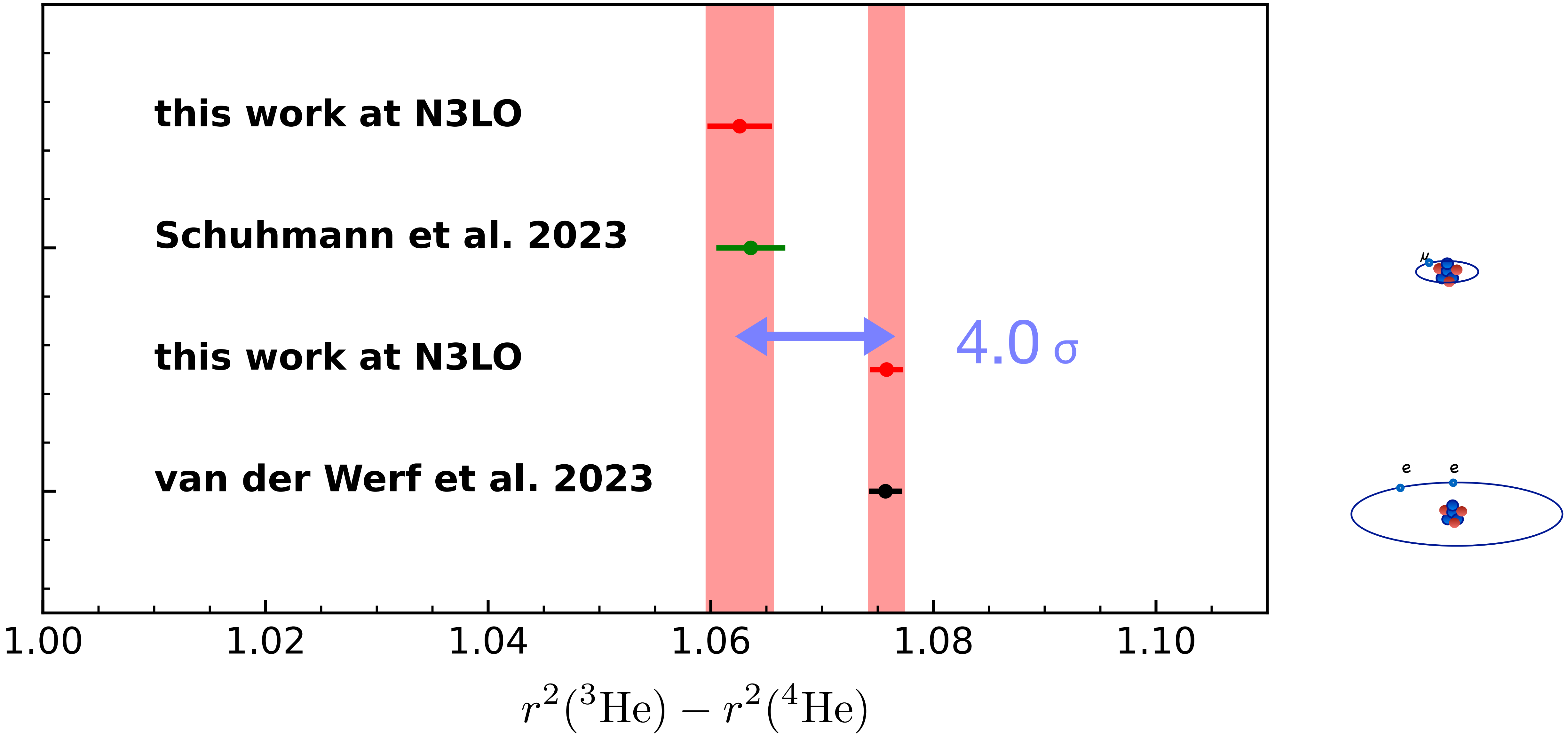
Li Muli, Richardson, SB, arXiv:2401.13424



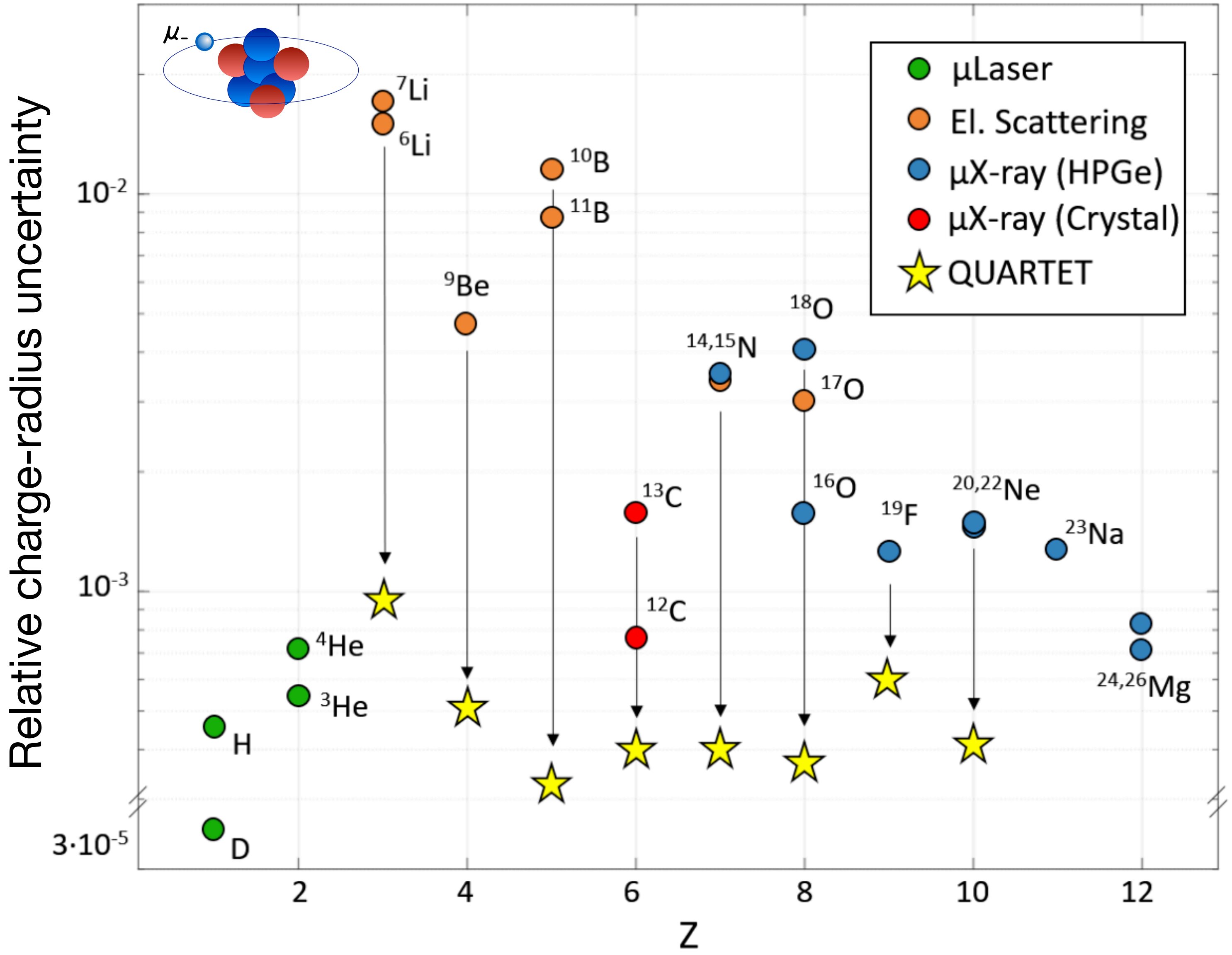
Nuclear structure does not solve the puzzle, it rather slightly enhances it

Outlook

- Ab-initio nuclear theory has allowed a strong reduction of uncertainties.

- Future: How can nuclear theory be useful and how shall we prioritize?
  Radii & TPE for p-shell nuclei, HFS for s-shell and p-shell nuclei

Credit: Quartet collaboration



Thanks to my collaborators

N.Barnea, O.J. Hernandez, C.Ji, S. Li Muli, N. Nevo Dinur, T. Richardson, A. Poggialini

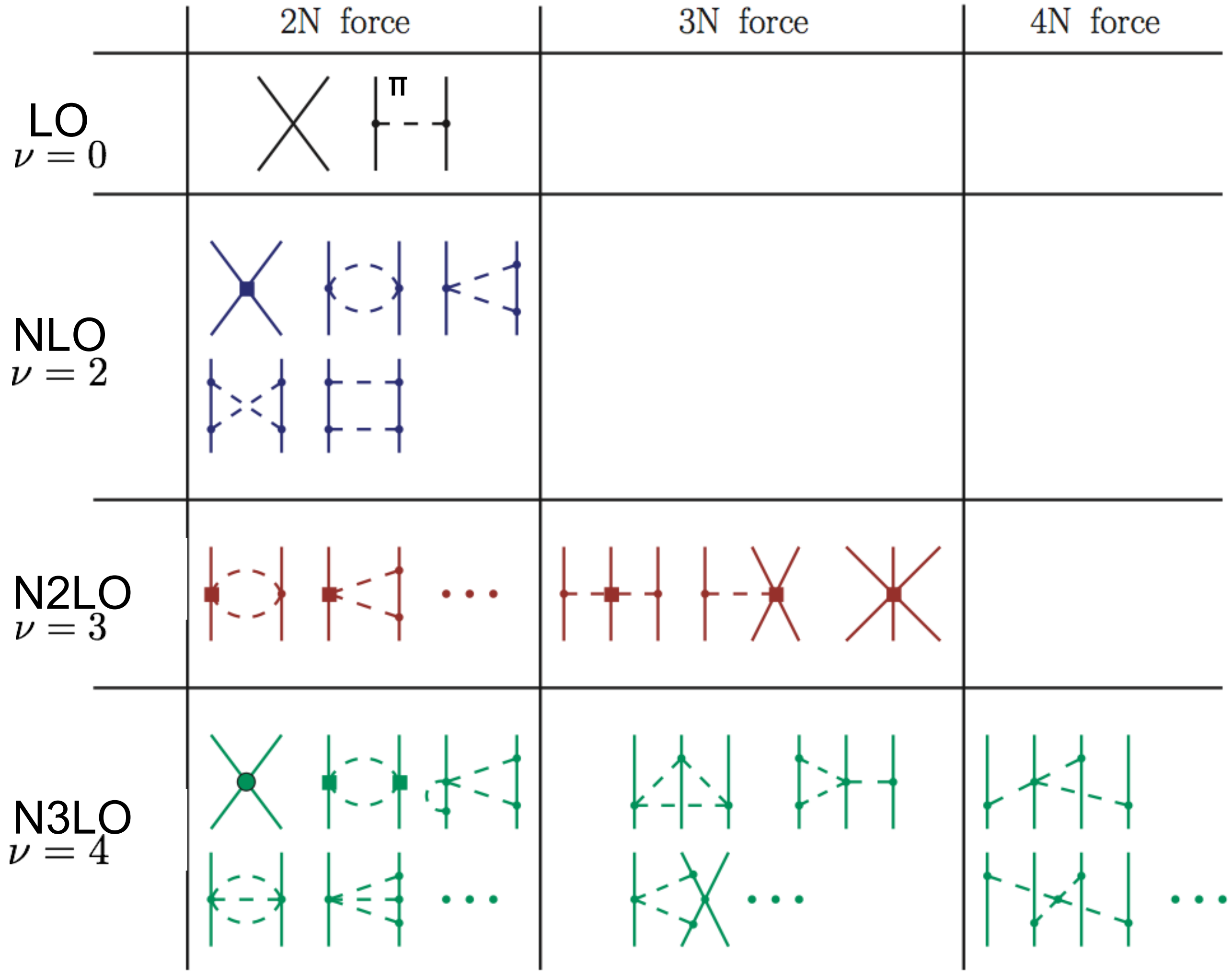
THANK YOU

Backup Slides

Nuclear Hamiltonians

- Chiral effective filed theory

Systematic expansion $\mathcal{L} = \sum_{\nu} c_{\nu} \left(\frac{Q}{\Lambda_b} \right)^{\nu}$



Nuclear Hamiltonians

- Chiral effective filed theory

Systematic expansion

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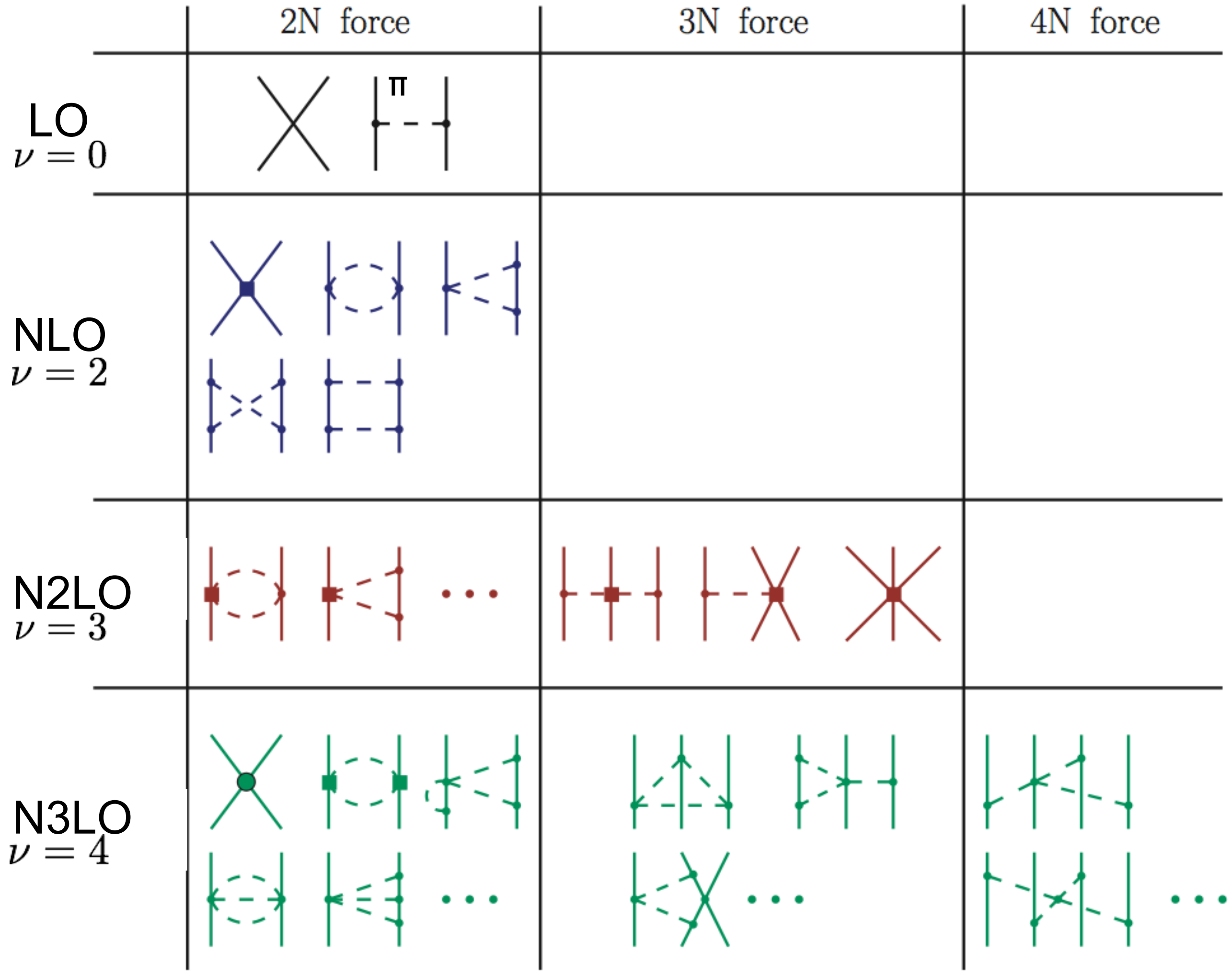
Details of short distance physics not resolved, but captured in low energy constants (LEC)



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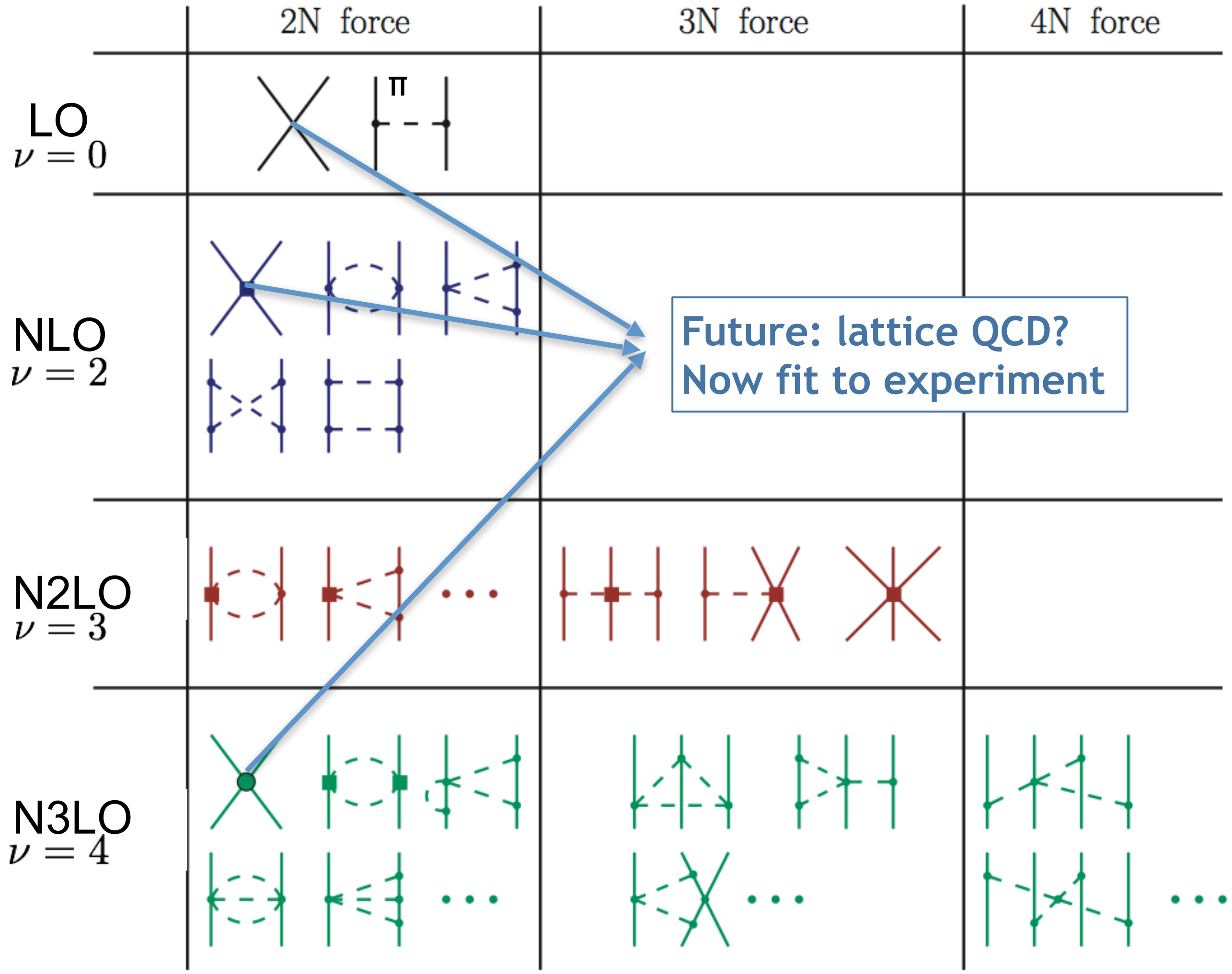

Future: lattice QCD?
Now fit to experiment

Nuclear Hamiltonians

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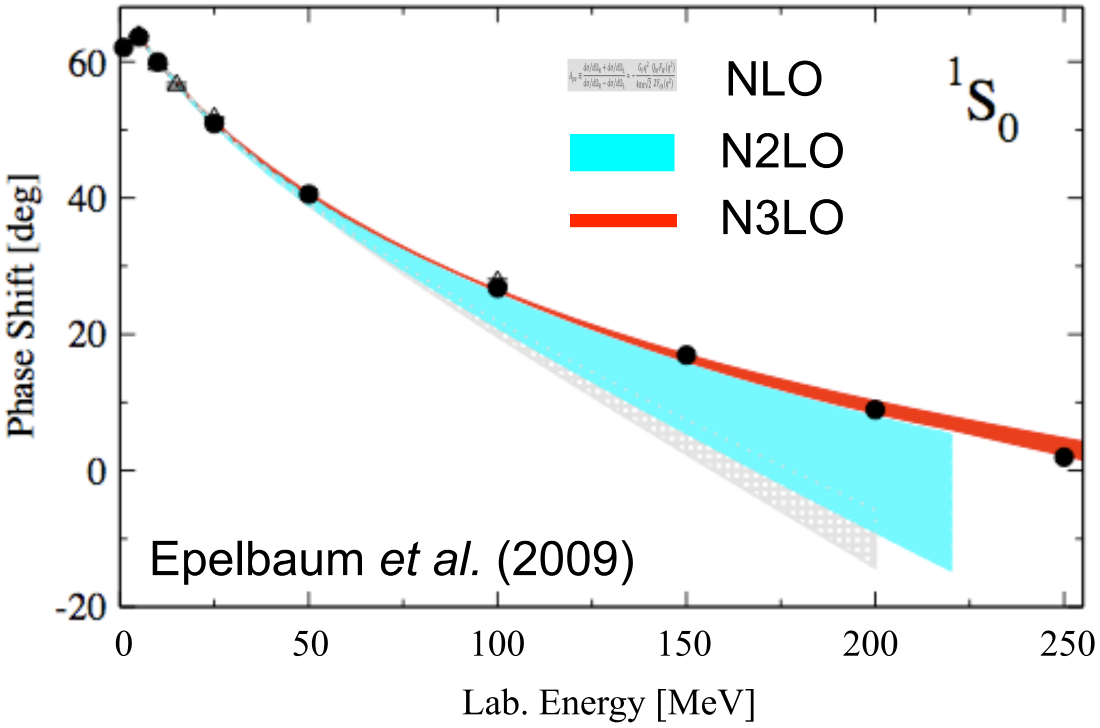
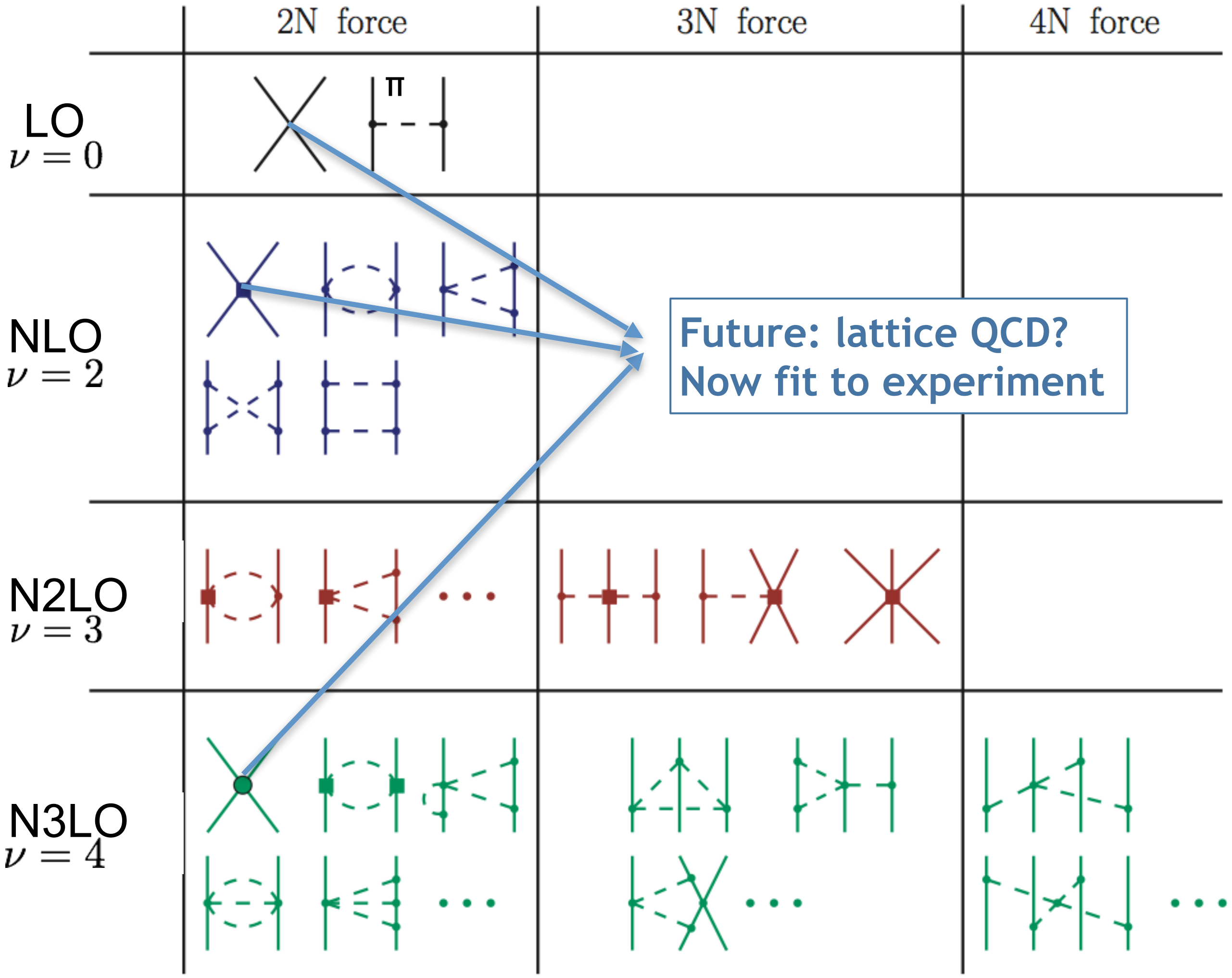
LEC fit to experiment - NN sector -





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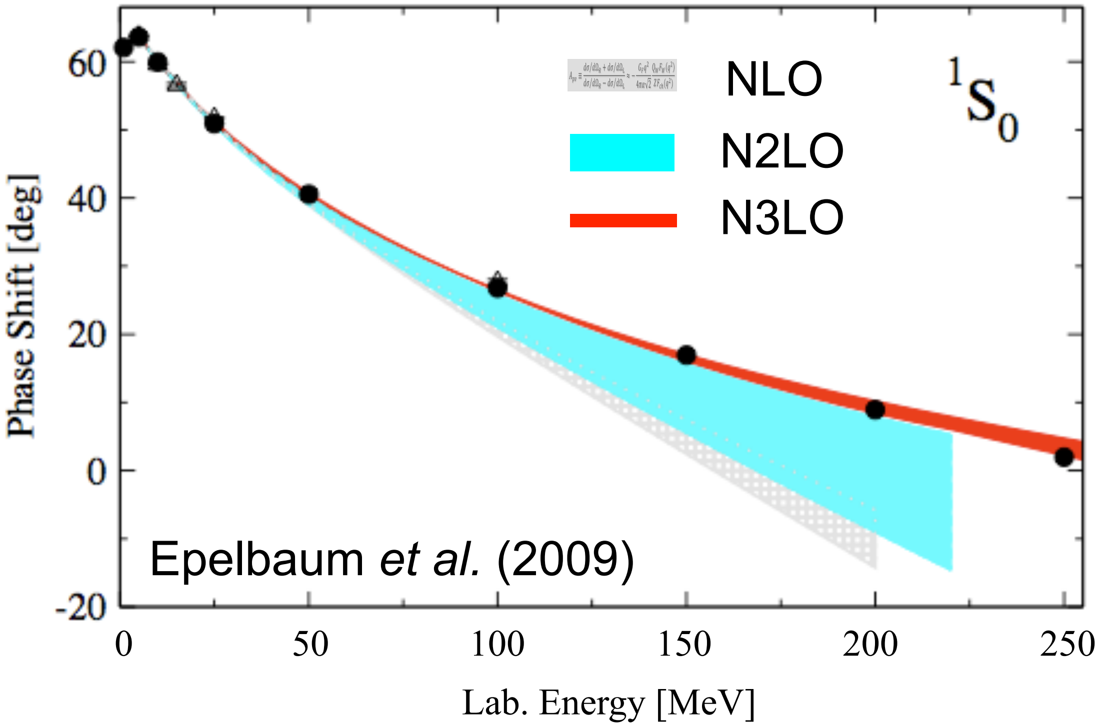
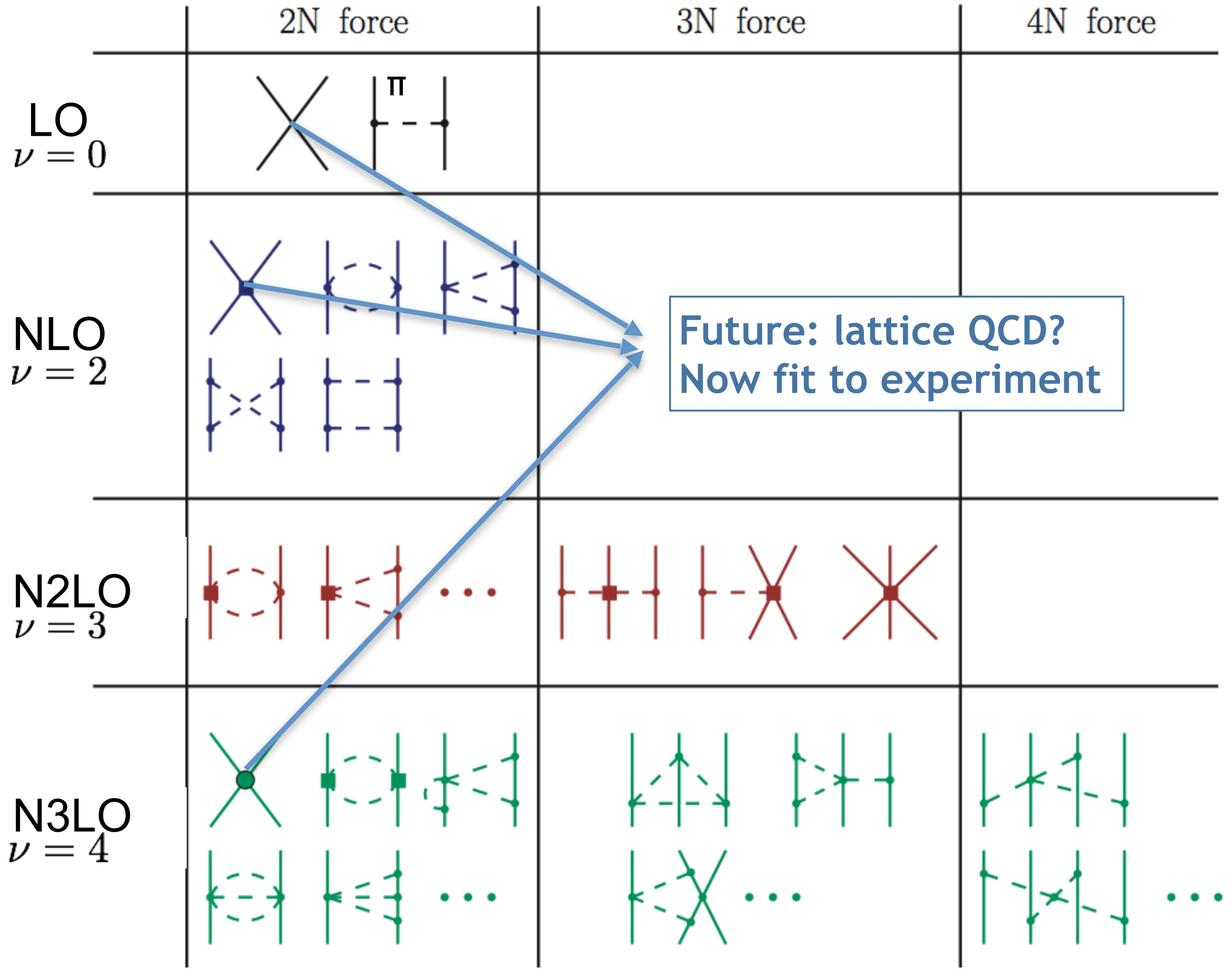




Future: lattice QCD?
Now fit to experiment

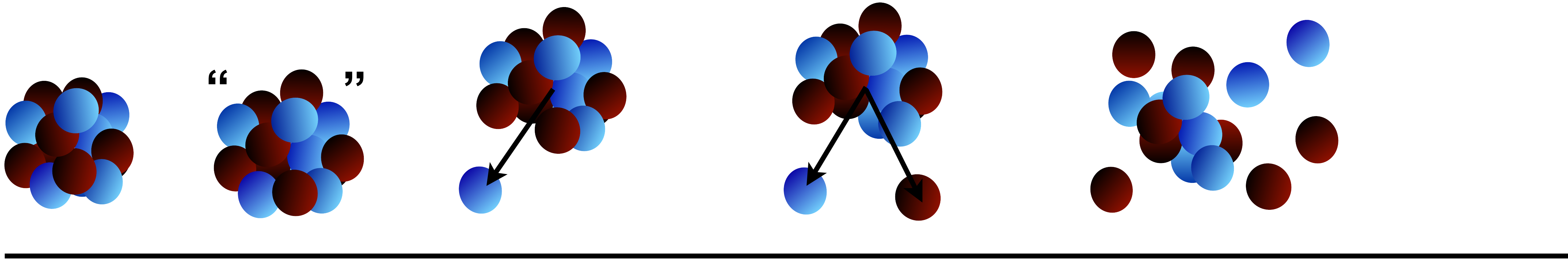
- Traditional hamiltonians

Exploit all other symmetries (e.g. translational, rotational invariance) but the chiral; use some ansatz for short range physics;
Fit NN phase shifts

The continuum problem

$$R(\omega) = \not\!\!\sum_f \left| \left\langle \psi_f \middle| J^\mu \middle| \psi_0 \right\rangle \right|^2 \delta(E_f - E_0 - \omega)$$

Depending on E_f, many channels involved



bound excited state

2-body break-up

3-body break-up

...

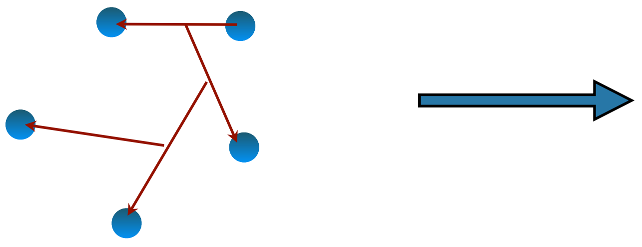
A-body break-up

$$\boxed{(H - E_0 - \sigma + i\Gamma) \mid \tilde{\psi}\rangle = J_\mu \mid \psi_0\rangle}$$

Lorentz Integral Transform: Efros, *et al.*, JPG.: Nucl.Part.Phys. **34** (2007) R459

Hyperspherical Harmonics

$$|\psi(\vec{r}_1,\vec{r}_2,\ldots,\vec{r}_A)\rangle = |\varphi(\vec{R}_{CM})\Psi(\vec{\eta}_1,\vec{\eta}_2,\ldots,\vec{\eta}_{A-1})\rangle$$



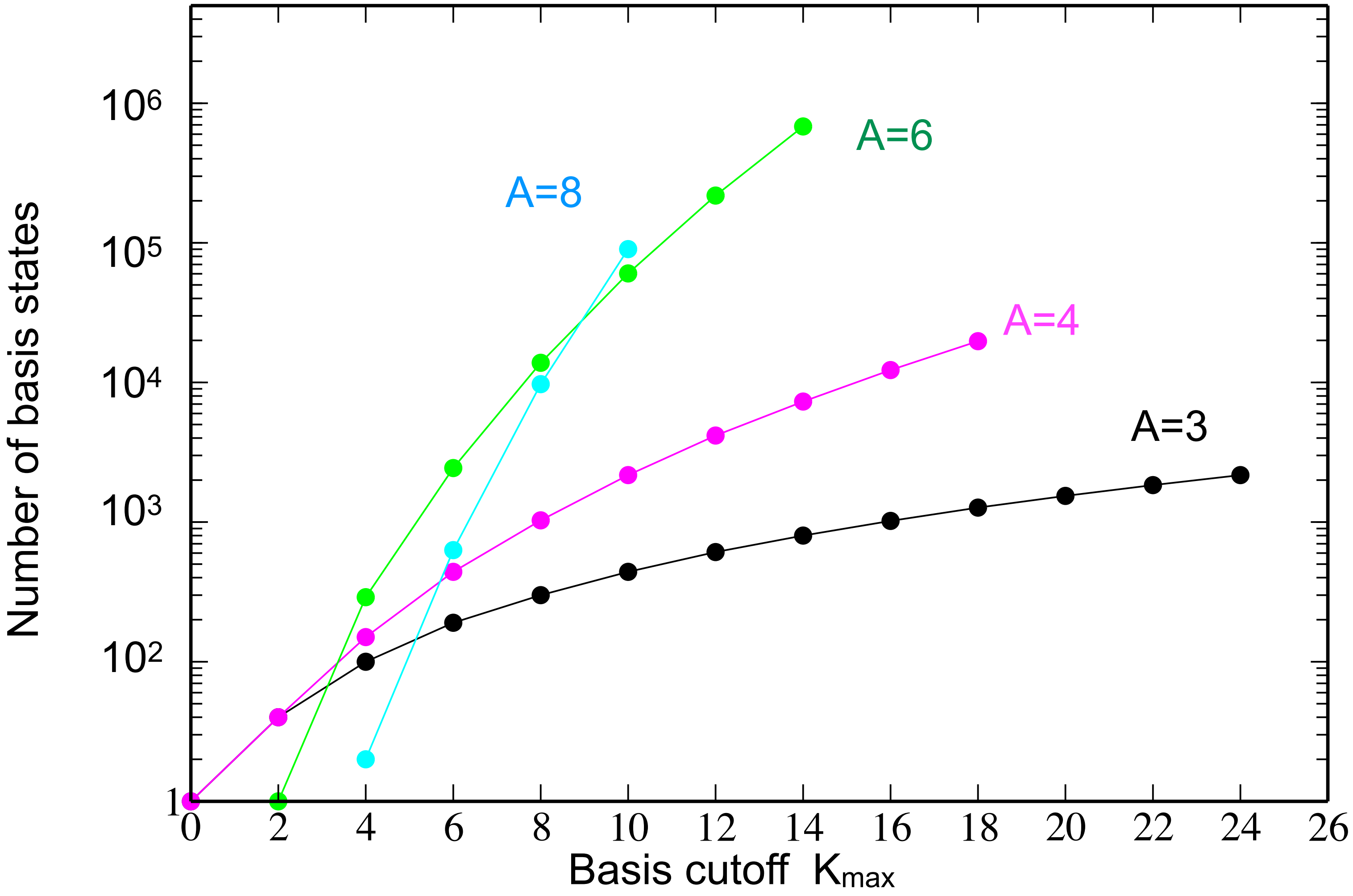
Recursive definition of hyper-spherical coordinates

$\vec{\eta}_0 = \sqrt{A}\vec{R}_{CM}$ $\quad \vec{\eta}_1, \ldots, \vec{\eta}_{A-1}$

$\rho, \Omega \qquad \rho^2 = \sum_{i=1}^{A} r_i^2 = \sum_{i=1}^{A-1} \eta_i^2$

$$\Psi = \sum_{[K],\nu}^{K_{max},\nu_{max}} c_\nu^{[K]} e^{-\rho/2} \rho^{n/2} L_\nu^n(\frac{\rho}{b}) [\mathcal{Y}_{[K]}^{\mu}(\Omega) \chi_{ST}^{\bar{\mu}}]_{JT}^{a}$$

Hyperspherical Harmonics

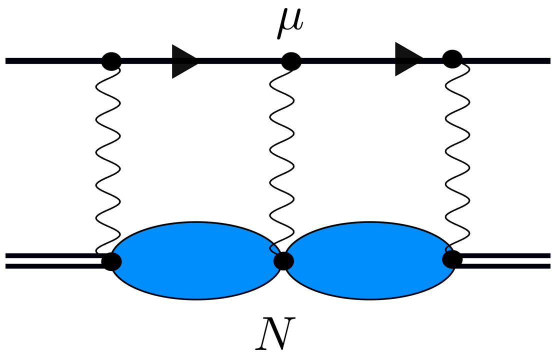


Exact method

Bad computational scaling

Higher order corrections in α

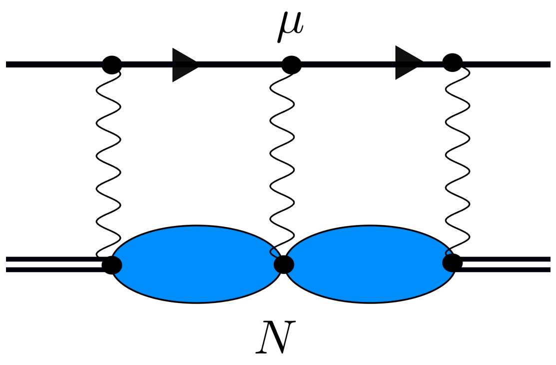
Three-photon exchange



Pachucki et al., Phys. Rev. A **97** 062511 (2018)

$(Z\alpha)^6$ correction, negligible?

Higher order corrections in α



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Pachucki et al., Phys. Rev. A **97** 062511 (2018)

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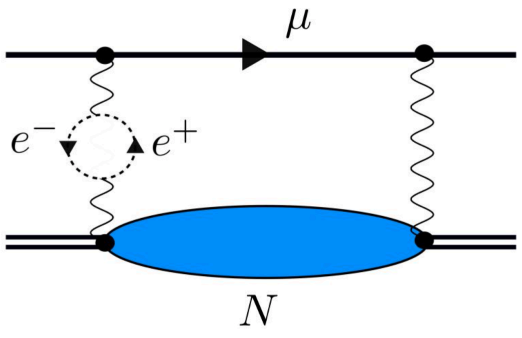


Vacuum polarization

One the many α^6 corrections, supposedly the largest

Kalinowski, Phys. Rev. A **99** 030501 (2019)

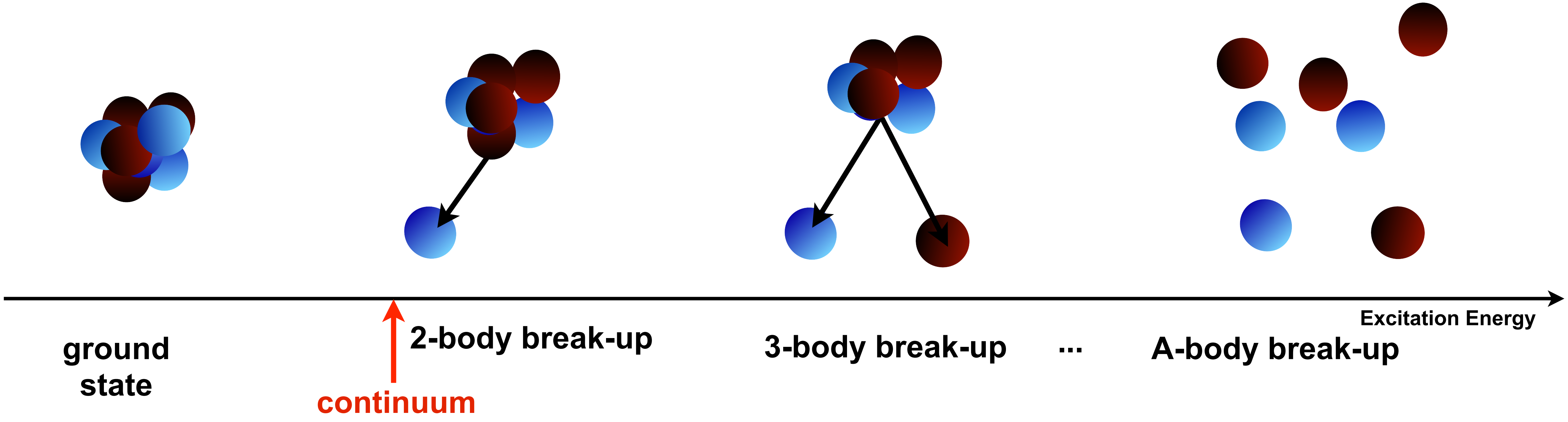
$$\delta_{\rm TPE} = -1.750^{+14}_{-16}\ {\rm meV}$$ Theory

$$\delta_{\rm TPE} = -1.7638(68)\ {\rm meV}$$ Exp

Consistent within 1σ

Lorentz integral transform method

Efros, *et al.*, JPG.: Nucl.Part.Phys. **34** (2007) R459



Excitation Energy

ground state

continuum

2-body break-up

3-body break-up

...

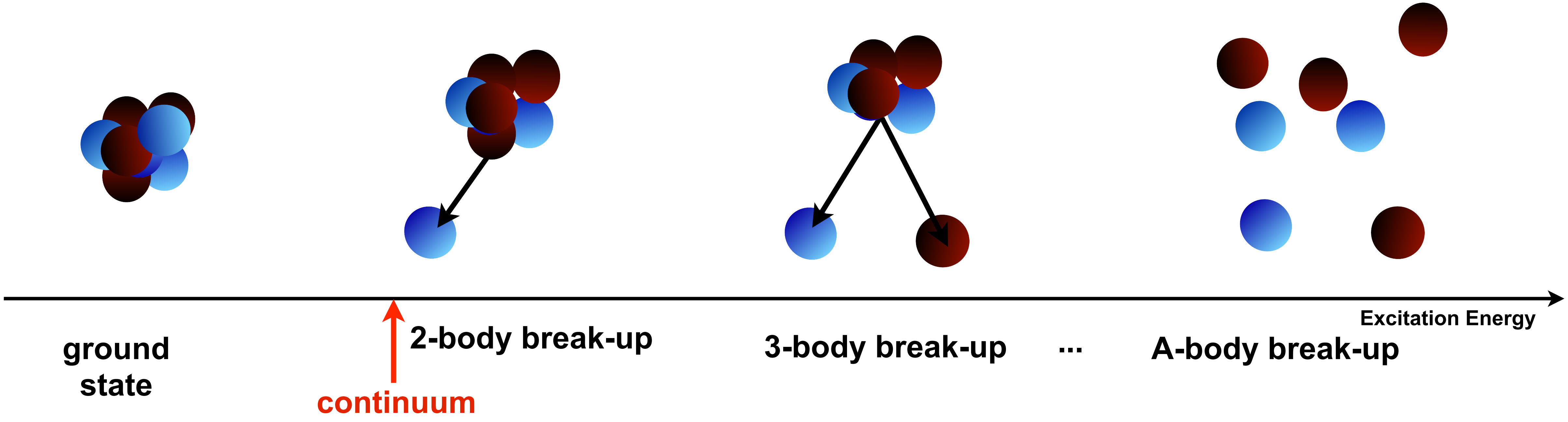
A-body break-up

$$S(\omega) \to |\langle NJ||\hat{O}||N_0 J_0\rangle|^2$$

Exact knowledge limited in energy and mass number

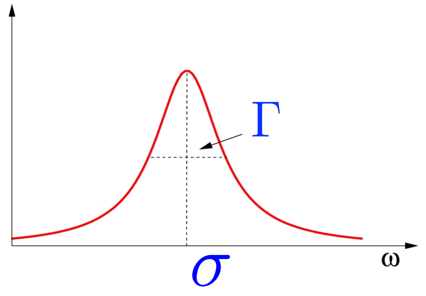
Lorentz integral transform method

Efros, *et al.*, JPG.: Nucl.Part.Phys. **34** (2007) R459



$$S(\omega) \to |\langle NJ||\hat{O}||N_0J_0\rangle|^2 \quad \Longleftrightarrow \quad L(\sigma,\Gamma) = \frac{\Gamma}{\pi}\int d\omega \frac{S(\omega)}{(\omega-\sigma)^2+\Gamma^2} = \langle\tilde{\psi}|\tilde{\psi}\rangle$$

Exact knowledge limited in energy and mass number



$$\Rightarrow \quad \boxed{(H - E_0 - \sigma + i\Gamma)\,|\,\tilde{\psi}\rangle = \Theta\,|\,\psi_0\rangle}$$

Reduce the continuum problem to a bound-state-like equation

Impact of ab initio theory

- Reduction of Uncertainties -

Atom	$\Delta E_{2S\text{-}2P}$	$\Delta\delta_{TPE}$	$\Delta\delta_{TPE}$ from ab initio NT
μ^2H	0.003 meV	0.03 meV	0.02 meV
μ^3He$^+$	0.08 meV	1 meV	0.3 meV
μ^4He$^+$	0.06 meV	0.6 meV	0.4 meV
$\mu\,^{6,7}$Li^{++}	0.7 meV	4 meV	?

Muonic Lithium

$$\delta^{(0)}_{\mathrm{D1}} \propto \int_0^\infty d\omega \sqrt{\frac{2m_r}{\omega}} S_{\mathrm{D1}}(\omega)$$

$$\delta^{(0)}_{\mathrm{C}} \propto \int_0^\infty d\omega \frac{m_r}{\omega} \ln \frac{2(Z\alpha)^2 m_r}{\omega} S_{\mathrm{D1}}(\omega)$$

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A. Poggialini,
S.B,

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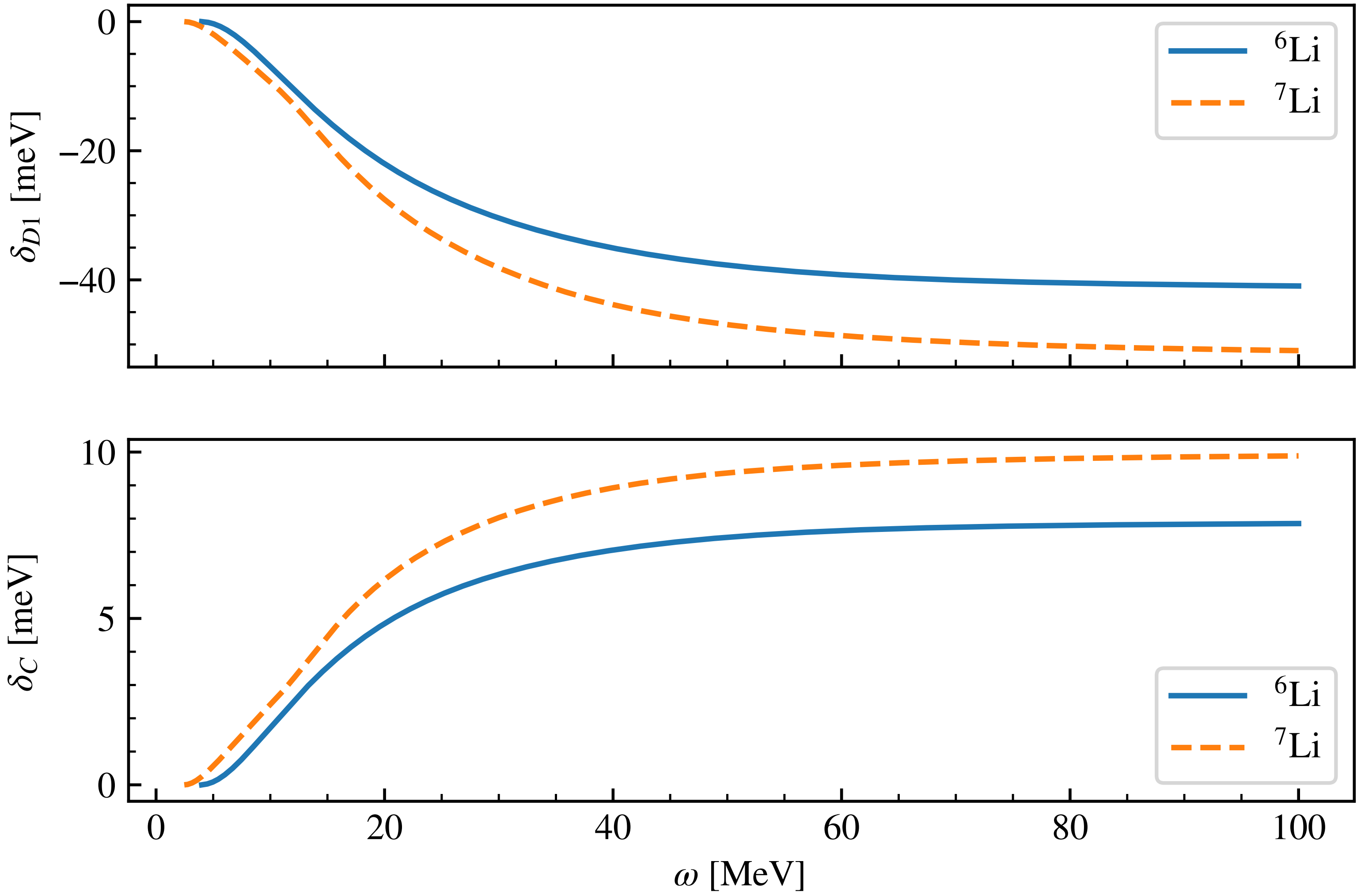

With AV4’

Semi realistic

potential