Electromagnetic decays of some 'onia'

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Outline

- Decay law: recall and link to energy distribution and non-exponential nature
- Multichannel decay law
- Electromagnetic decay of the 2P state of the H-atom
- Other e.m. decays as outlooks

Basic definitions

Let $|S\rangle$ be an unstable state prepared at t = 0.

Survival probabilty ammplitude at t > 0:

$$a(t) = \langle S | e^{-iHt} | S \rangle \qquad (\hbar = 1)$$

Survival probability: $p(t) = |a(t)|^2$

Rep. Prog. Phys., Vol. 41, 1978. Printed in Great Britain

Decay theory of unstable quantum systems

L FONDA, G C GHIRARDI and A RIMINI

Deviations from the exp. law at short times

Taylor expansion of the amplitude:

$$a(t) = \langle S|e^{-iHt}|S\rangle = 1 - it\langle S|H|S\rangle - \frac{t^2}{2}\langle S|H^2|S\rangle + \ldots$$

$$a^*(t) = \langle S|e^{-iHt}|S\rangle = 1 + it\langle S|H|S\rangle - \frac{t^2}{2}\langle S|H^2|S\rangle + \ldots$$

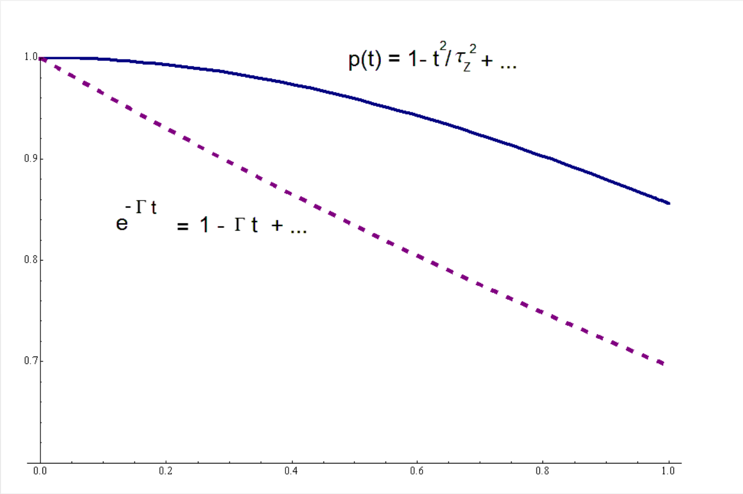
p(t) decreases quadratically (not linearly); no exp. decay for short times.
τ_Z is the `Zeno time´.

It follows:

$$p(t) = |a(t)|^2 = a^*(t)a(t) = 1 - t^2\left(\langle S|H^2|S\rangle - \langle S|H|S\rangle^2\right) + \ldots = 1 - \frac{t^2}{\tau_Z^2} + \ldots$$

where $\tau_Z = \dfrac{1}{\sqrt{\langle S|H^2|S\rangle - \langle S|H|S\rangle^2}}$.

Note: the quadratic behavior holds
for any quantum transition, not only for decays.



Time evolution and energy distribution

The unstable state $|S\rangle$ is not an eigenstate of the Hamiltonian H.

Let $d_S(E)$ be the energy distribution of the unstable state $|S\rangle$.

Normalization holds: $\int_{-\infty}^{+\infty} d_S(E) dE = 1$

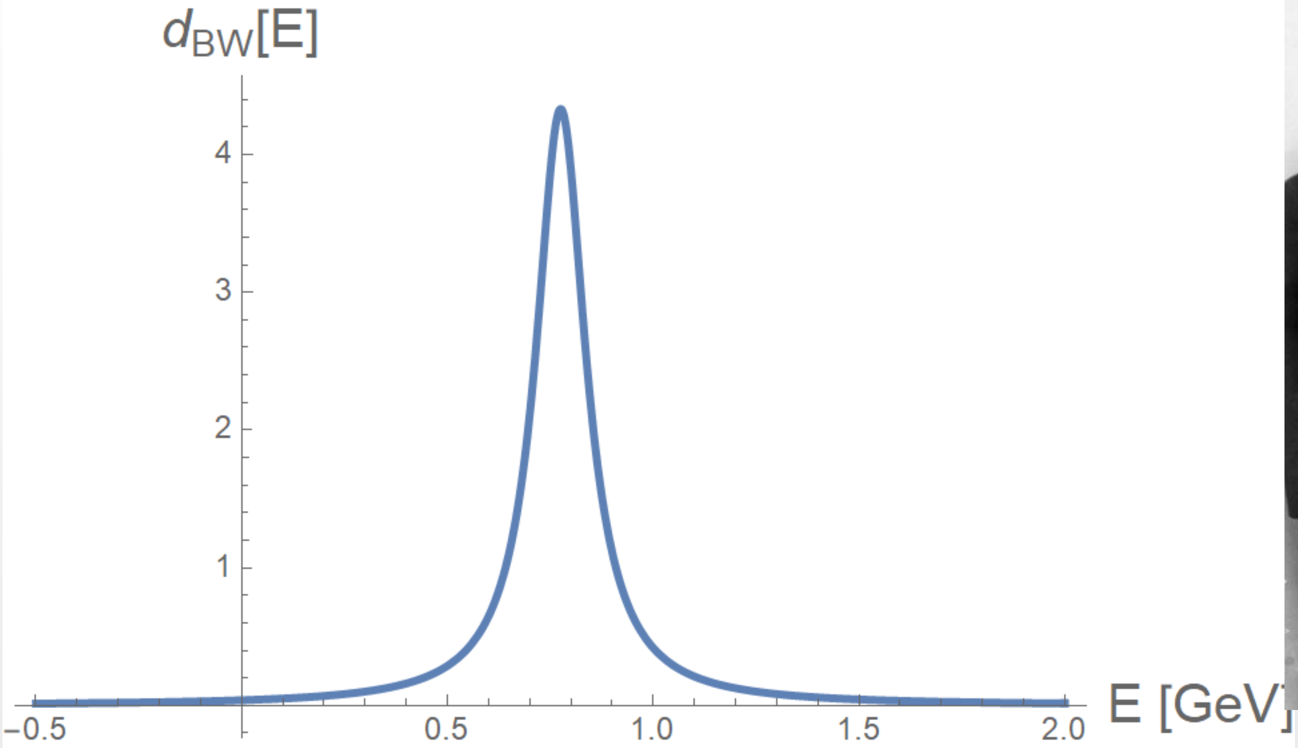
$$p(t) = \left| \int_{E_{th,1}}^{\infty} dE d_S(E) e^{-\frac{i}{\hbar}Et} \right|^2$$

In stable limit : $d_S(E) = \delta(E - M) \rightarrow a(t) = e^{-iMt} \rightarrow p(t) = 1$

Payley and Wiener (1934) theorem: P(t) is not exponential at large time if a left-threshold is present

Breit-Wigner distribution

$$d_S^{\mathrm{BW}}(E) = \frac{\Gamma}{2\pi} \frac{1}{(E-M)^2 + \frac{\Gamma^2}{4}}$$



Rho-meson as example.

BW extends from –inf to +inf. There is no left threshold.

BW: properties

BW-distribution:

$$d_S^{\mathrm{BW}}(E) = \frac{\Gamma}{2\pi} \frac{1}{(E-M)^2 + \frac{\Gamma^2}{4}}$$

Pole:

$$z_{pole}^{\mathrm{BW}} = M - i\Gamma/2$$

BW-propagator:

$$G_S^{\mathrm{BW}}(E) = \frac{1}{E - M + i\Gamma/2 + i\varepsilon}$$

Link prop-dist:

$$d_S^{\mathrm{BW}}(E) = -\frac{1}{\pi}\,\mathrm{Im}[G_S^{\mathrm{BW}}(E)] = \frac{\Gamma}{2\pi} \frac{1}{(E-M)^2 + \frac{\Gamma^2}{4}}$$

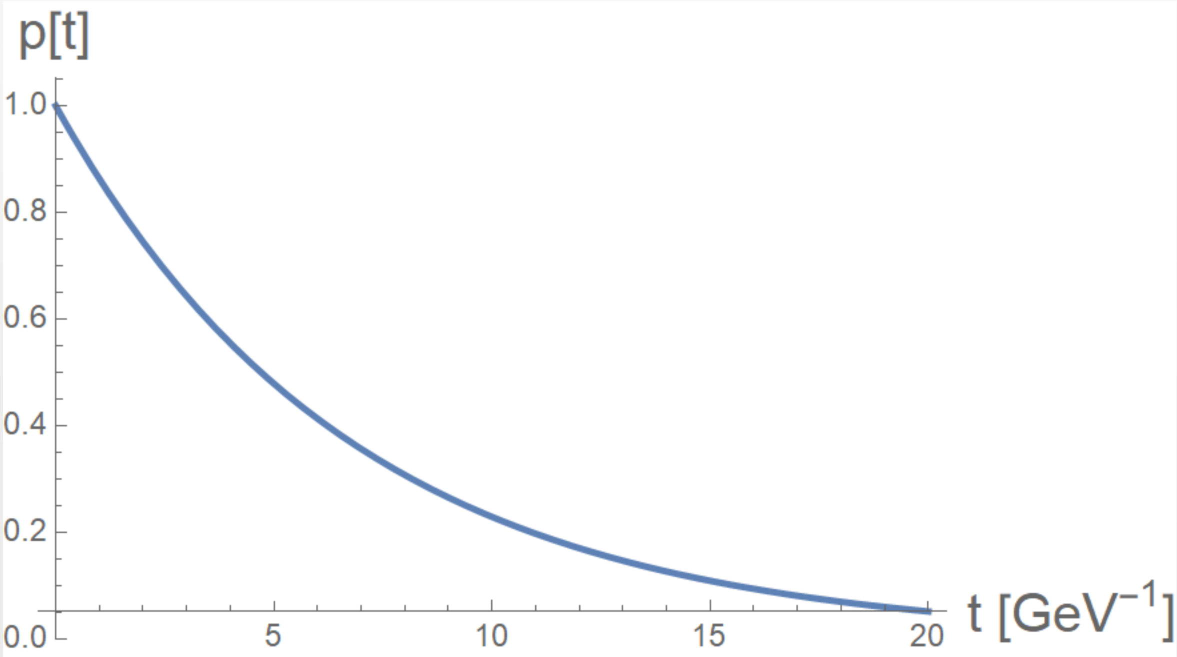
Normalization:
(important for prob. interpretation)

$$\int_{-\infty}^{+\infty} d_S^{\mathrm{BW}}(E)\,dE = 1$$

BW corresponds to exp. decay

$$a_S^{\mathrm{BW}}(t) = \frac{i}{2\pi}\int_{-\infty}^{+\infty} \mathrm{dE} G_S^{\mathrm{BW}}(E) e^{-iEt} = \int_{E_{th}}^{+\infty} \mathrm{dE} d_S^{\mathrm{BW}}(E) = e^{-iMt-\Gamma t/2}$$

$$p^{\mathrm{BW}}(t) = \left|a_S^{\mathrm{BW}}(t)\right|^2 = e^{-\Gamma t}$$



Going beyond Breit-Wigner:

We have seen that via the Breit-Wigner distribution:

$$d_S(E) = \frac{\Gamma}{2\pi} \frac{1}{(E-M)^2 + \Gamma^2/4} \rightarrow a(t) = e^{-iM_0 t - \Gamma t/2} \rightarrow p(t) = e^{-\Gamma t}.$$

The Breit-Wigner energy distribution cannot be exact.

Two physical conditions for a realistic $d_S(E)$ are:

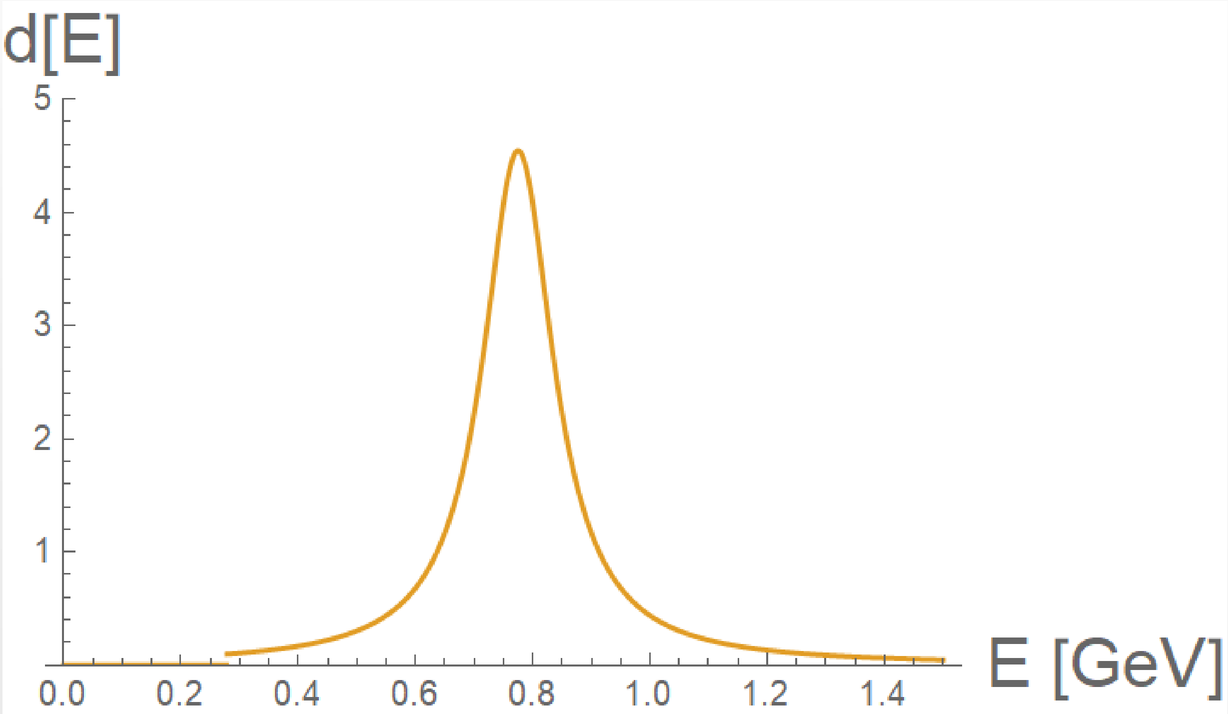
1) Minimal energy: $d_S(E) = 0 \text{ for } E < E_{\min}$

2) Mean energy finite: $\langle E \rangle = \int_{-\infty}^{+\infty} d_S(E) E dE = \int_{E_{\min}}^{+\infty} d_S(E) E dE < \infty$

How to introudce a threshold?
BW with threshold (naive approach)

$$d_S(E) = N\frac{\Gamma}{2\pi}\frac{1}{(E-M)^2+\frac{\Gamma^2}{4}}\theta(E-E_{th})$$

N is needed because the normalization is lost!

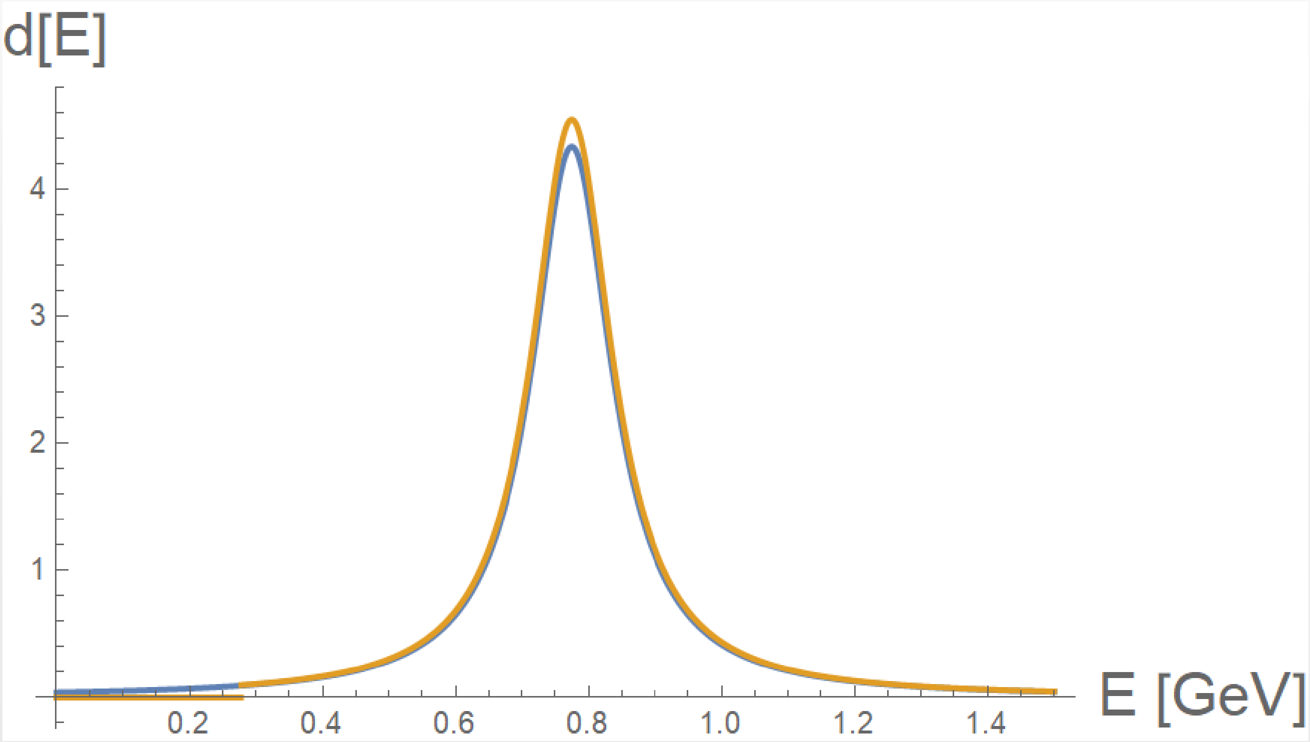


BW with threshold (naive treatment)/2

$$d_S(E) = N \frac{\Gamma}{2\pi} \frac{1}{(E-M)^2 + \frac{\Gamma^2}{4}} \theta(E - E_{th})$$

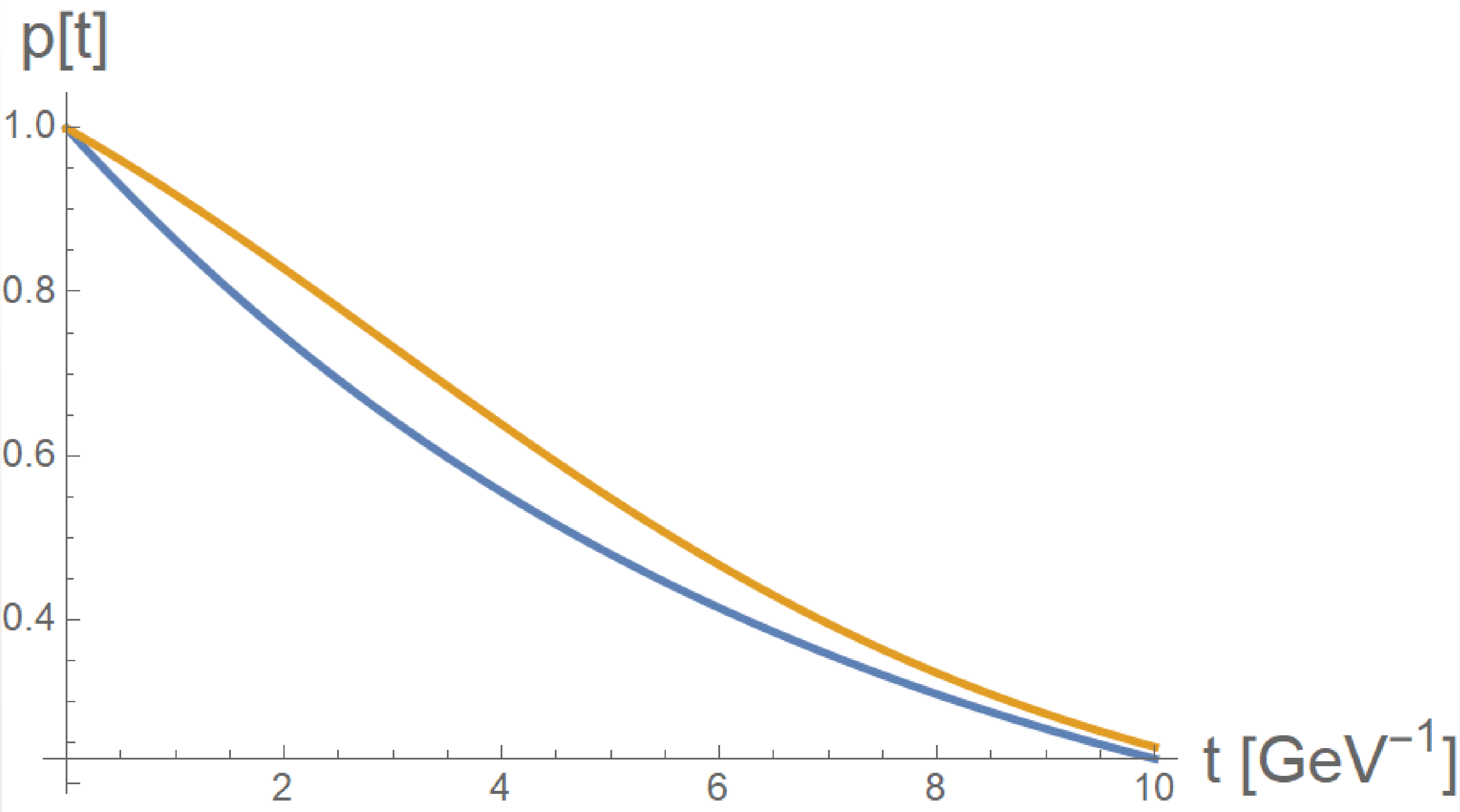
N is needed because the **normalization is lost**!

For the ρ meson, we get $N = 1.05$



<E>=lnΛ=∞

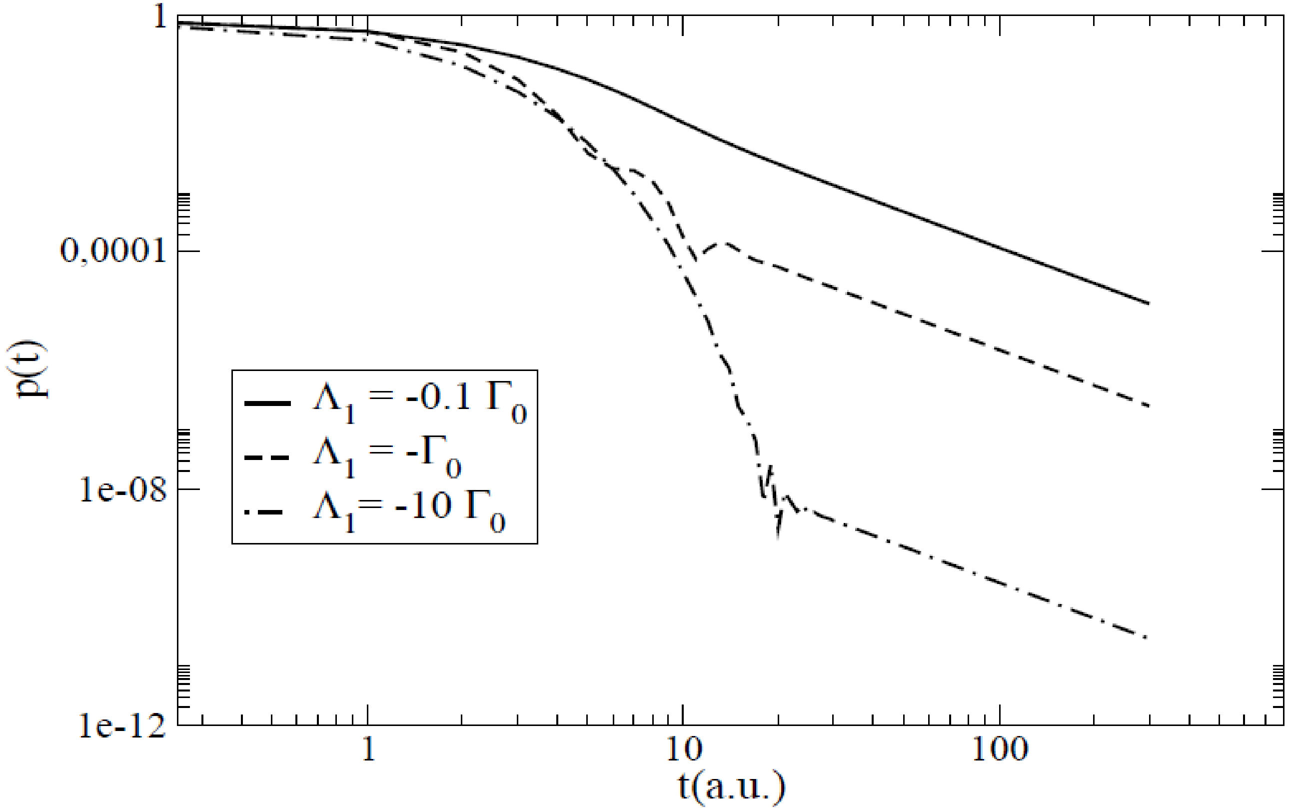
Time evolution



Blue: plain BW, yellow: BW with threshold (naive)

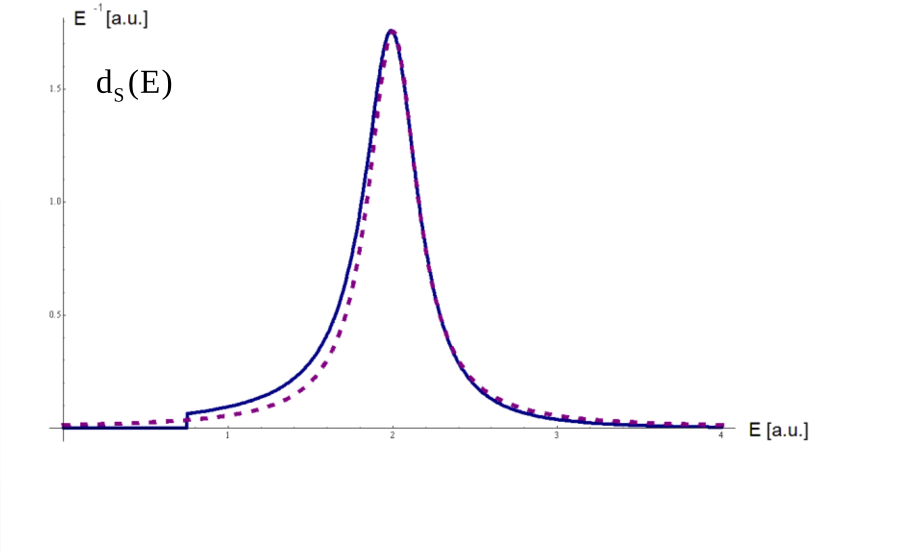
<E>=lnΛ=∞

Single left-threshold at long times



F.G. and G. Pagliara, [arXiv:1204.1896 [nucl-th]].

Second example: threshold plus form factor

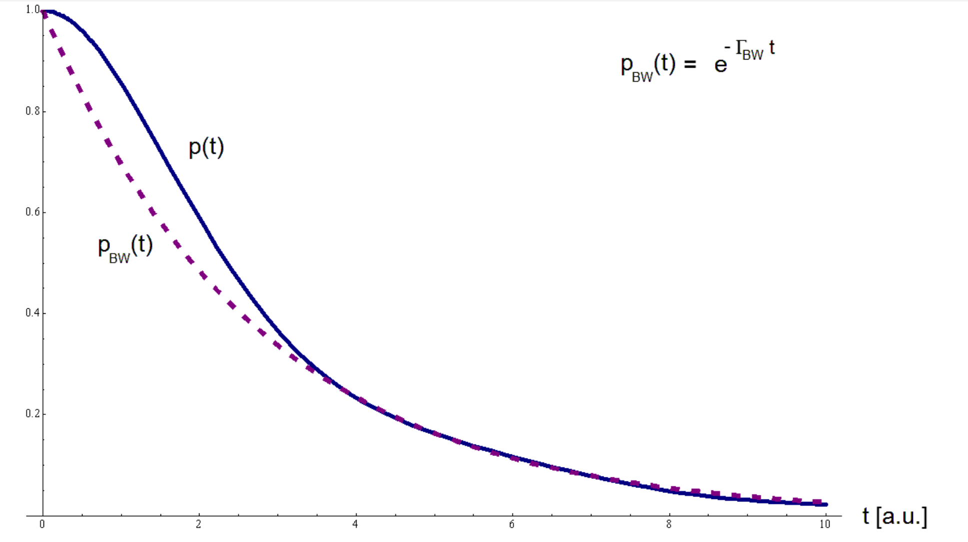


$$M_0 = 2;\ E_{min} = 0.75;\ \Gamma = 0.4; \Lambda = 3$$

$$d_S(E) = N_0 \frac{\Gamma}{2\pi} \frac{e^{-(E^2-E_0^2)/\Lambda^2}\theta(E-E_{min})}{(E-M_0)^2+\Gamma^2/4}$$

$$d_{BW}(E) = \frac{\Gamma_{BW}}{2\pi} \frac{1}{(E-M_0)^2+\Gamma_{BW}{}^2/4}$$

Γ_{BW}, such that $d_{BW}(M_0) = d_S(M_0)$



$$a(t) = \int_{-\infty}^{+\infty} d_S(E) e^{-iEt} dE; \quad p(t) = |a(t)|^2$$

$$p_{BW}(t) = e^{-\Gamma_{BW} t}$$

Experimental confirmation of non-exponential decay: short times

Uniwersytet Jana Kochanowskiego w Kielcach

Cold Na atoms in a optical potential

NATURE | VOL 387 | 5 JUNE 1997

Experimental evidence for non-exponential decay in quantum tunnelling

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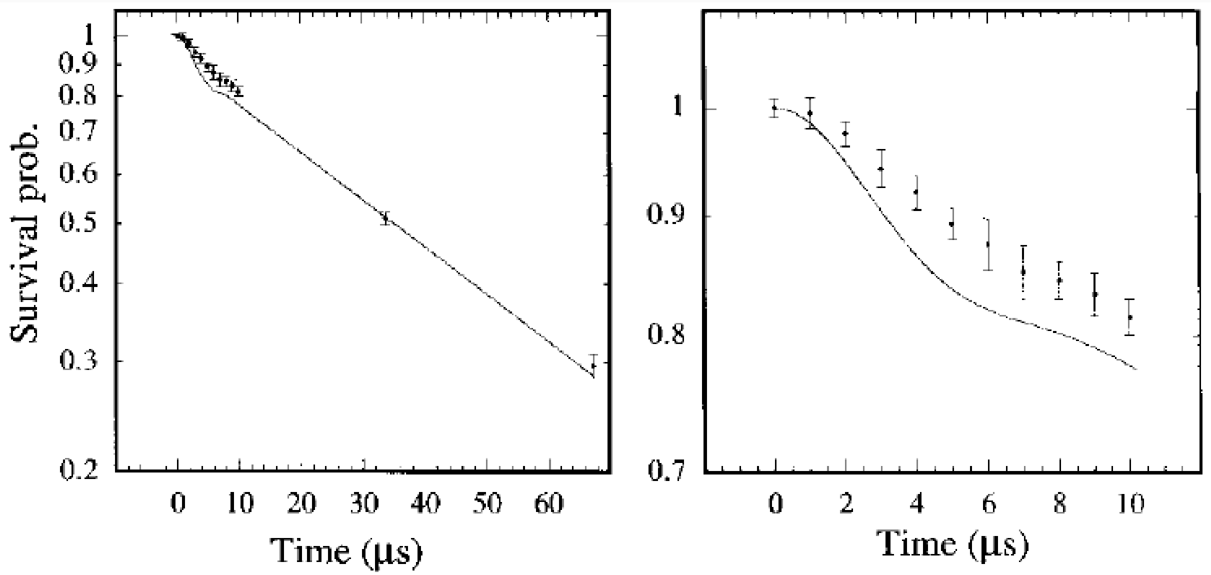
An exponential decay law is the universal hallmark of unstable systems and is observed in all fields of science. This law is not, however, fully consistent with quantum mechanics and deviations from exponential decay have been predicted for short as well as long times[1–8]. Such deviations have not hitherto been observed experimentally. Here we present experimental evidence for short-time deviation from exponential decay in a quantum tunnelling experiment. Our system consists of ultra-cold sodium atoms that are trapped in an accelerating periodic optical potential created by a standing wave of light. Atoms can escape the wells by quantum tunnelling, and the number that remain can be measured as a function of interaction time for a fixed value of the well depth and acceleration. We observe that for short times the survival probability is initially constant before developing the characteristics of exponential decay. The conceptual simplicity of the experiment enables a detailed comparison with theoretical predictions.



Experimental confirmation of non-exponential decay: long times

PRL 96, 163601 (2006) PHYSICAL REVIEW LETTERS week ending 28 APRIL 2006

Violation of the Exponential-Decay Law at Long Times

C. Rothe, S. I. Hintschich, and A. P. Monkman
Department of Physics, University of Durham, Durham, DH1 3LE, United Kingdom
(Received 4 July 2005; published 26 April 2006)

First-principles quantum mechanical calculations show that the exponential-decay law for any metastable state is only an approximation and predict an asymptotically algebraic contribution to the decay for sufficiently long times. In this Letter, we measure the luminescence decays of many dissolved organic materials after pulsed laser excitation over more than 20 lifetimes and obtain the first experimental proof of the turnover into the nonexponential decay regime. As theoretically expected, the strength of the nonexponential contributions scales with the energetic width of the excited state density distribution

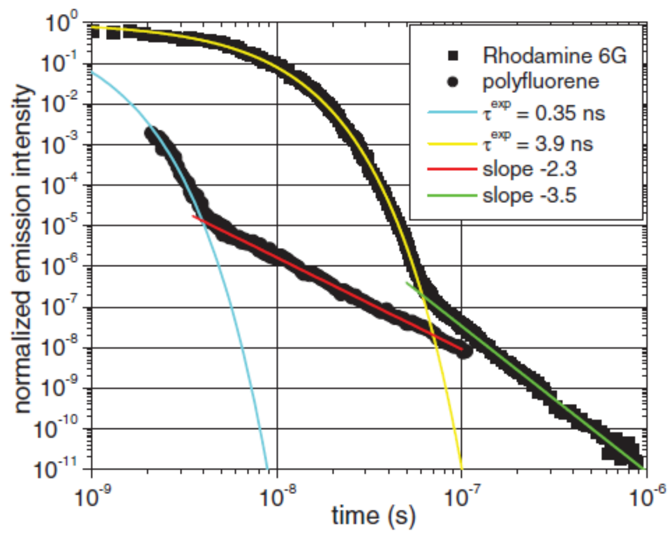


FIG. 2 (color). Corresponding double logarithmic fluorescence decays of the emissions shown in Fig. 1. Exponential and power law regions are indicated by solid lines and the emission intensity at time zero has been normalized.

Confirmation of: L. A. Khalfin. 1957. 1957 (Engl. trans. Zh.Eksp.Teor.Fiz.,33,1371)

Considerations/1

- No other short- or long-time deviation from the exp. law was ‘directly’seen in unstable states.

- Verification of the two aforementioned works (Reizen + Rothe) would be needed.

- The measurement of deviations in simple natural systems (elementary particles, nuclei, atoms) would be a great achievement.

Considerations/2

- The ‘brute force’ threshold and high-energy behavior can be good as a first approximation, but it is just an ‘ad hoc’ modification of Breit-Wigner.

- How to properly describe the theory of decay?

- Which is a suitable energy distribution for e.m. decays?

General non-relativistic approach

Propagator

$$G_S(E) = \frac{1}{E - M + \Pi(E) + i\varepsilon}$$

Self-energy (or loop)
E_{th} is the threshold energy

$$\Pi(E) = -\int_{E_{th}}^{\infty} \frac{1}{\pi} \frac{\mathrm{Im}\,\Pi(E')}{E - E' + i\varepsilon} dE'$$

Energy dependent 'decay width'

$$\Gamma(E) = 2\,\mathrm{Im}\,\Pi(E)$$

Energy distribution (or spectral function)

$$d_S(E) = -\frac{1}{\pi}\,\mathrm{Im}[G_S(E)] = \frac{1}{\pi} \frac{\mathrm{Im}\,\Pi(E)}{(E - M + \mathrm{Re}\,\Pi(E))^2 + (\mathrm{Im}\,\Pi(E))^2}$$

Link between propagator and distribution

The propagator can be expressed as (H being the full Hamiltonian)

$$G_S(E) = \langle S| \frac{1}{E - H + i\varepsilon} |S\rangle = \frac{1}{E - M + \Pi(E) + i\varepsilon} = \int_{E_{th}}^{+\infty} \mathrm{dE}' \frac{d_S(E')}{E - E' + i\varepsilon}$$

out of which

$$d_S(E) = -\frac{1}{\pi} \mathrm{Im}[G_S(E)]$$

Normalization ok!!

$$\int_{E_{th}}^{\infty} \mathrm{dE}\, d_S(E) = 1$$

Symmetries in Science XVIII IOP Publishing
Journal of Physics: Conference Series **1612** (2020) 012012 doi:10.1088/1742-6596/1612/1/012012

The Lee model: a tool to study decays

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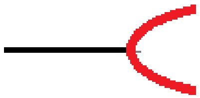
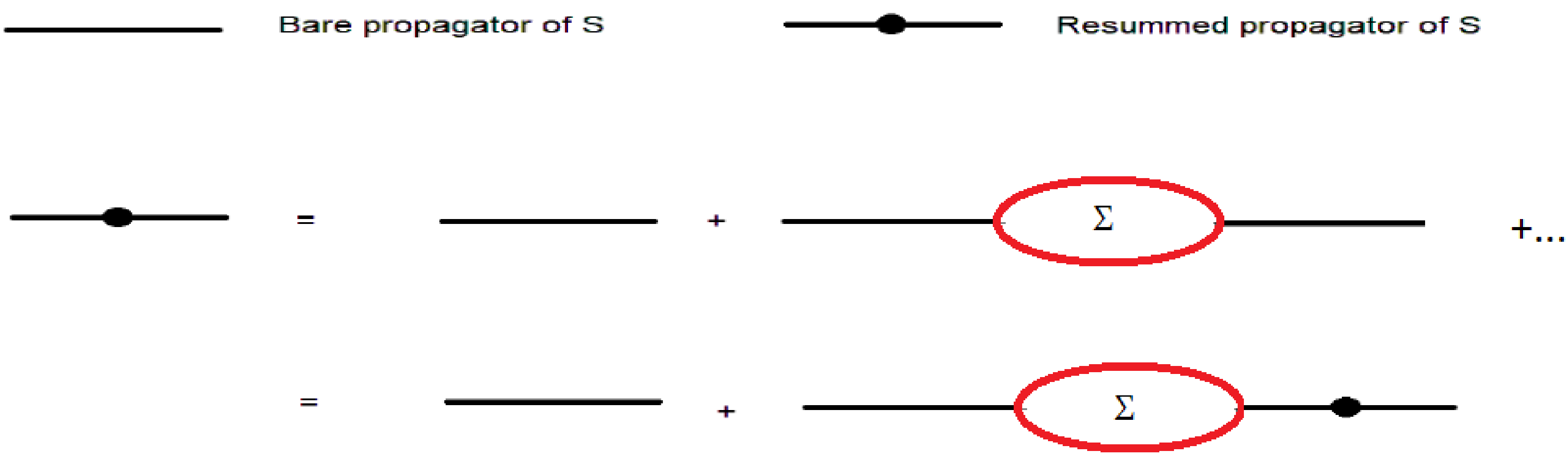
[arXiv:2001.07781 [hep-ph]]

Pictorial representation

Decay



Propagator of S



Time-evolution (general)

$$a_S(t) = Z e^{-i z_{pole} t} + \ldots,$$

The dots describe short- and long-time deviations from the exponential decay

The pole:

$$z_{pole} - M + \Pi_{II}(z_{pole}) = 0\ ,$$

where II refers to the second Riemann sheet. Then:

$$z_{pole} = M_{pole} - i\,\frac{\Gamma_{pole}}{2}\ .$$

Multichannel decay law

Physics Letters B 831 (2022) 137200

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Multichannel decay law

Francesco Giacosa [a,b,*]

$w_1(t)$ is the probability that the decay has occurred in the first channel between (0,t)

$$\sum_{i=1}^{N} w_i = 1 - p(t)$$

ACTA PHYSICA POLONICA A No. 3 Vol. **142** (2022)

Proceedings of the 4th Jagiellonian Symposium on Advances in Particle Physics and Medicine

Multichannel Decay: Alternative Derivation of the *i*-th Channel Decay Probability

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A simple question

Next, a simple question might be asked: Which is the probability that the decay of the unstable state occurs between $t = 0$ and the time t? The answer is trivial, since the probability that the decay has actually occurred, denoted as $w(t)$, must be

$$w(t) = 1 - p(t)$$

Similarly, the quantity $h(t) = w'(t) = -p'(t)$ is the probability decay density, with $h(t)dt$ being the probability that the decay occurs between t and $t + dt$.

$$h(t) = w'(t) = -p'(t)$$

A difficult question

Then, a natural, but less easy question is the following: *How to calculate, in a general fashion, the probability, denoted as* $w_i(t)$, *that the decay occurs in the* i*-th channel between* 0 *and* t?

How to calculate the probabilities $w_i(t)$????

N decay channels: formal aspects

$$\Gamma_i(E)$$

$i = 1, ..., N$ enumerates the decay channels

$$E_{th,1} \leq E_{th,2} \leq ... \leq E_{th,N}$$

$$\Pi(E) = \sum_{i=1}^{N} \Pi_i(E)\, ,\, \Gamma_i(E) = 2\,\mathrm{Im}\,\Pi_i(E)$$

partial BW widths are $\Gamma_i = \Gamma_i(M)$

$$\Gamma = \Gamma(M) = \sum_{i=1}^{N} \Gamma_i$$

Breit-Wigner limit

$$d_S(E) = \frac{\Gamma}{2\pi} \frac{1}{(E-M)^2 + \Gamma^2/4}$$

$$\Gamma = \sum_{i=1}^{N} \Gamma_i$$

$$p(t) = e^{-\frac{\Gamma}{\hbar}t}$$

$$w_i(t) = \frac{\Gamma_i}{\Gamma} w(t) = \frac{\Gamma_i}{\Gamma}\left(1 - e^{-\frac{\Gamma}{\hbar}t}\right)$$

$$h_i(t) = \frac{\Gamma_i}{\Gamma} h(t) = \frac{\Gamma_i}{\Gamma} e^{-\frac{\Gamma}{\hbar}t}$$

$$\frac{w_i(t)}{w_j(t)} = \frac{h_i(t)}{h_j(t)} = \frac{\Gamma_i}{\Gamma_j} = const.$$

Partial decay probabilities

$$\Gamma_i(E) = 2g_i^2 \frac{\sqrt{E-E_{th,i}}}{E^2+\Lambda^2}$$

2108.07838 [quant-ph]

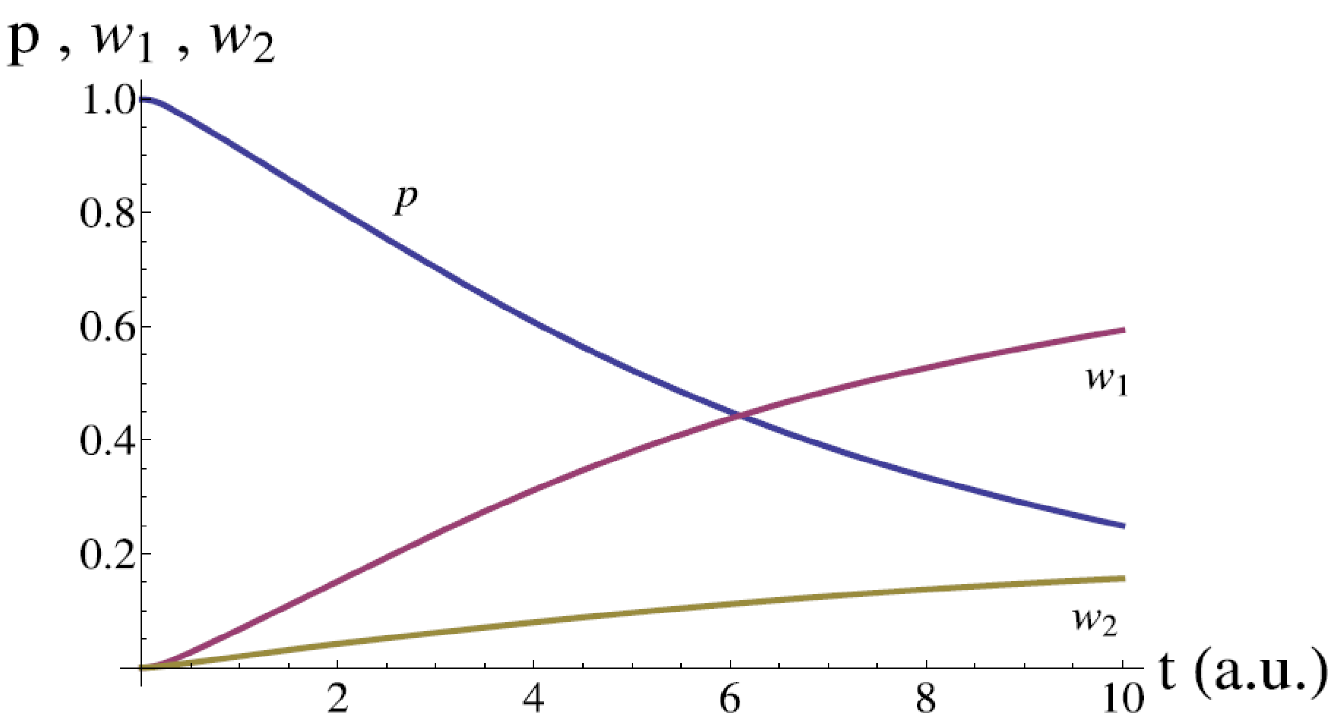

Fig. 1. The survival probability $p(t)$ of Eq. (1) and the decay probabilities $w_1(t)$ and $w_2(t)$ of Eq. (14) are plotted as function of t. The constraint $p+w_1+w_2=1$ holds. Note, t is expressed in a.u. of $[M^-1]$.

$g_1/\sqrt{M}=1$, $g_2/\sqrt{M}=0.6$ $\quad E_{th,1}/M = 1/10$, $E_{th,2}/M = 1/2$, $\Lambda/M = 4$,

Ratio of partial decay probabilities (not a constant)

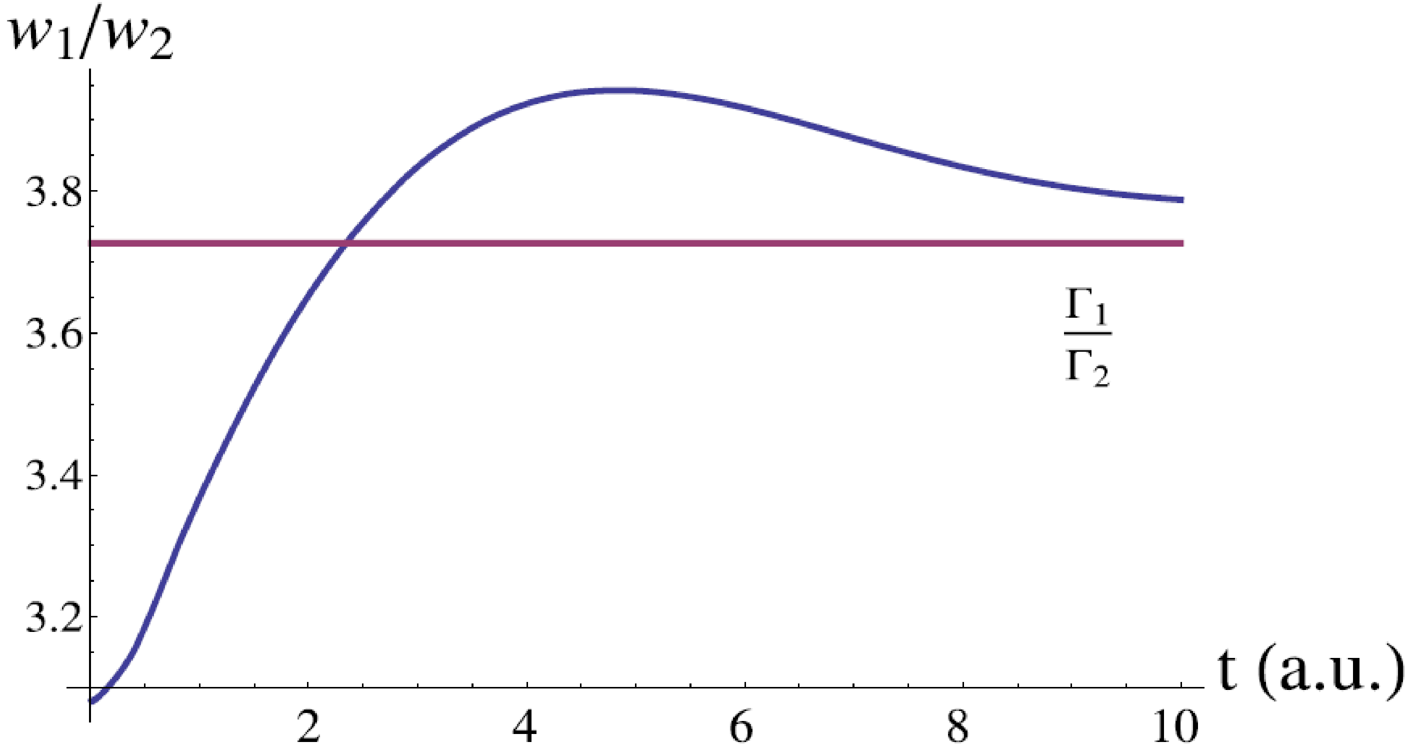

Fig. 2. The ratio w_1/w_2 is plotted as function of t. The straight line corresponds to the BW limit Γ_1/Γ_2, see Eq. (19).

2108.07838 [quant-ph]

Partial probability decay densities

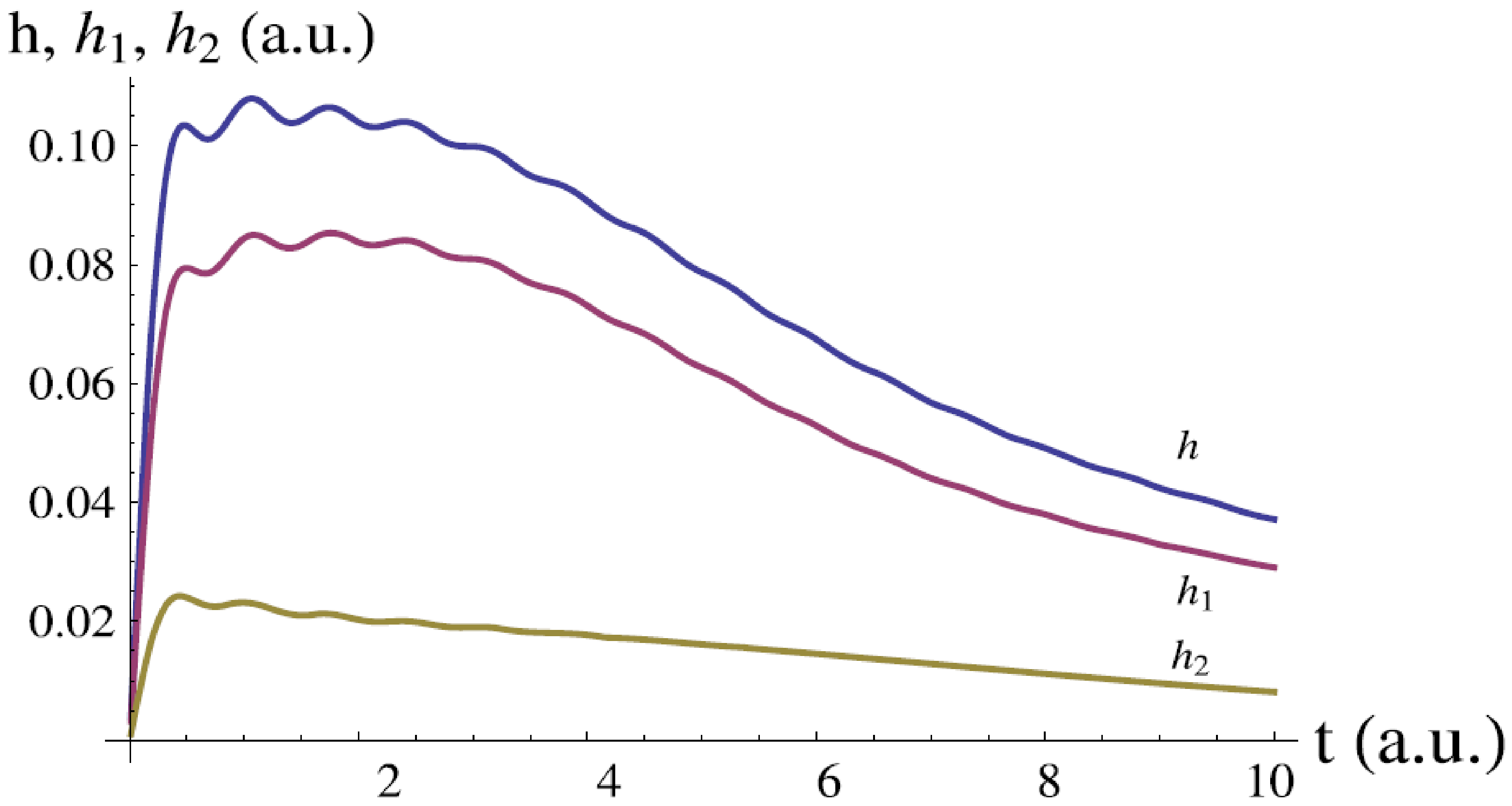

Fig. 3. The quantity $h(t) = w'(t) = -p'(t)$ as well as $h_i(t) = w_i'(t)$ is plotted. The equality $h(t) = h_1(t) + h_2(t)$ holds. Note, h and h_i are in units of $[M]$.

2108.07838 [quant-ph]

Ratio of partial probability decay densities

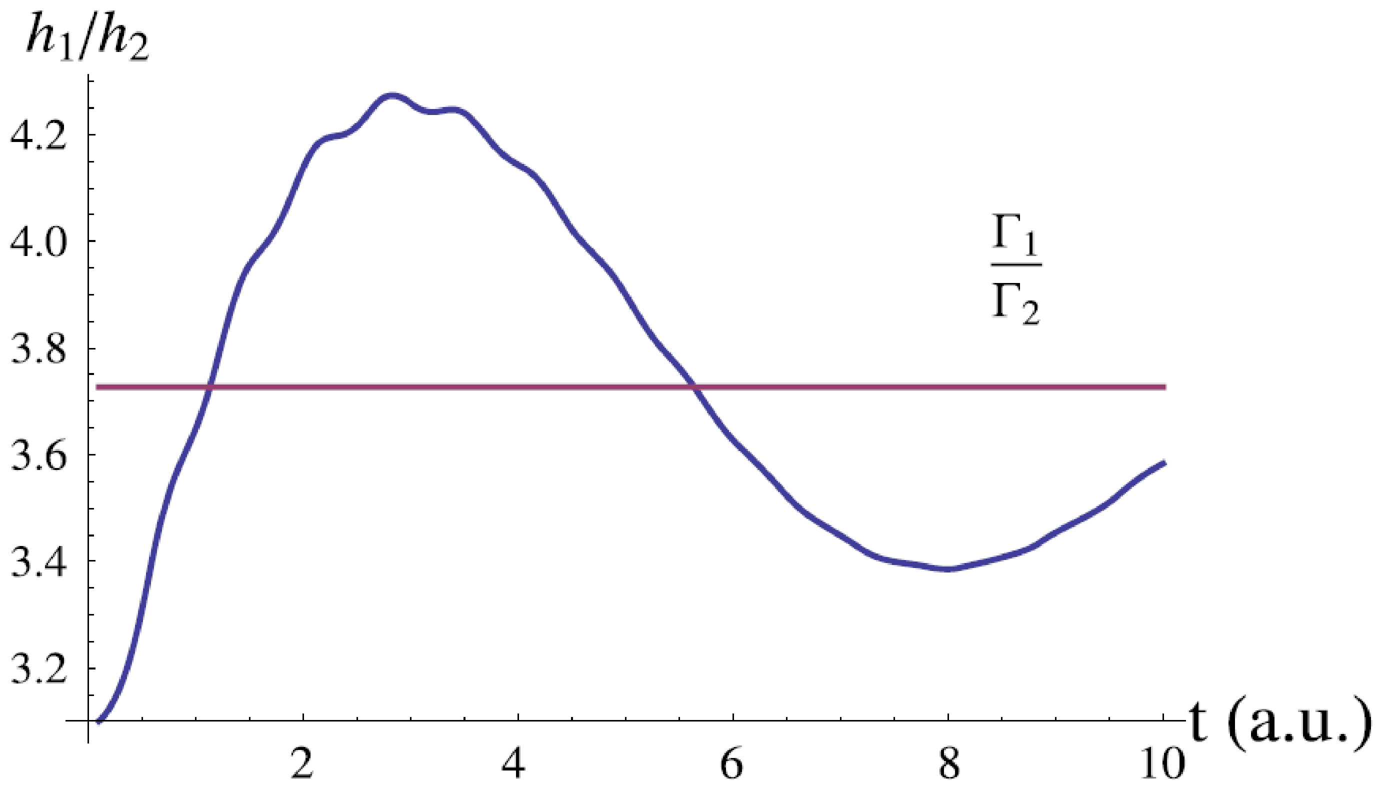

Fig. 4. Ratio h_1/h_2 as function of t. The straight line corresponds to the BW limit Γ_1/Γ_2, see Eq. (19). For the time intervals where $h_1/h_2 > \Gamma_1/\Gamma_2$, the decay in the first channel is enhanced (the opposite is true for $h_1/h_2 < \Gamma_1/\Gamma_2$).

2108.07838 [quant-ph]

2P-1S transition of H-atom

See also:

Giacosa and K. Kyziol,
Nonexponential decay law of the 2P-1S transition of the H-atom,
[arXiv:2408.06905 [quant-ph]].

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27 April 1998

PHYSICS LETTERS A

Physics Letters A 241 (1998) 139–144

Temporal behavior and quantum Zeno time of an excited state of the hydrogen atom

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Communicated by P.R. Holland

$$\mathrm{Im}[\Pi(E)] = \pi\chi\Lambda \frac{\frac{E-E_{th}}{\Lambda}}{\left(1+\left(\frac{E-E_{th}}{\Lambda}\right)^2\right)^4} \vartheta(E-E_{th}) \ ,$$

$$\chi = \frac{2}{\pi}\left(\frac{2}{3}\right)^9 \alpha^3 \simeq 6.43509\times 10^{-9}, \ \Lambda = \frac{3}{2}\alpha m_e \simeq 5593.41\,\mathrm{eV} \ .$$

Decay width and lifetime of 2P level of H-atom

$$\Gamma = \frac{1}{\tau} = \frac{3}{2}\left(\frac{2}{3}\right)^9 \frac{m_e \alpha^5}{\left(1+\left(\frac{\alpha}{4}\right)^2\right)^4} = 4.12582 \times 10^{-7}\,\mathrm{eV}$$

$$\tau \simeq 2.42376 \times 10^6\,\mathrm{eV}^{-1} = 1.59535 \times 10^{-9}\,\mathrm{s}$$

Spectral function: analytic expression

$$\mathrm{Im}\left[\Pi(E)\right] = \frac{\gamma\sqrt{E-E_{th}}}{\left(1+\frac{(E-E_{th})^2}{\Lambda^2}\right)^2}$$

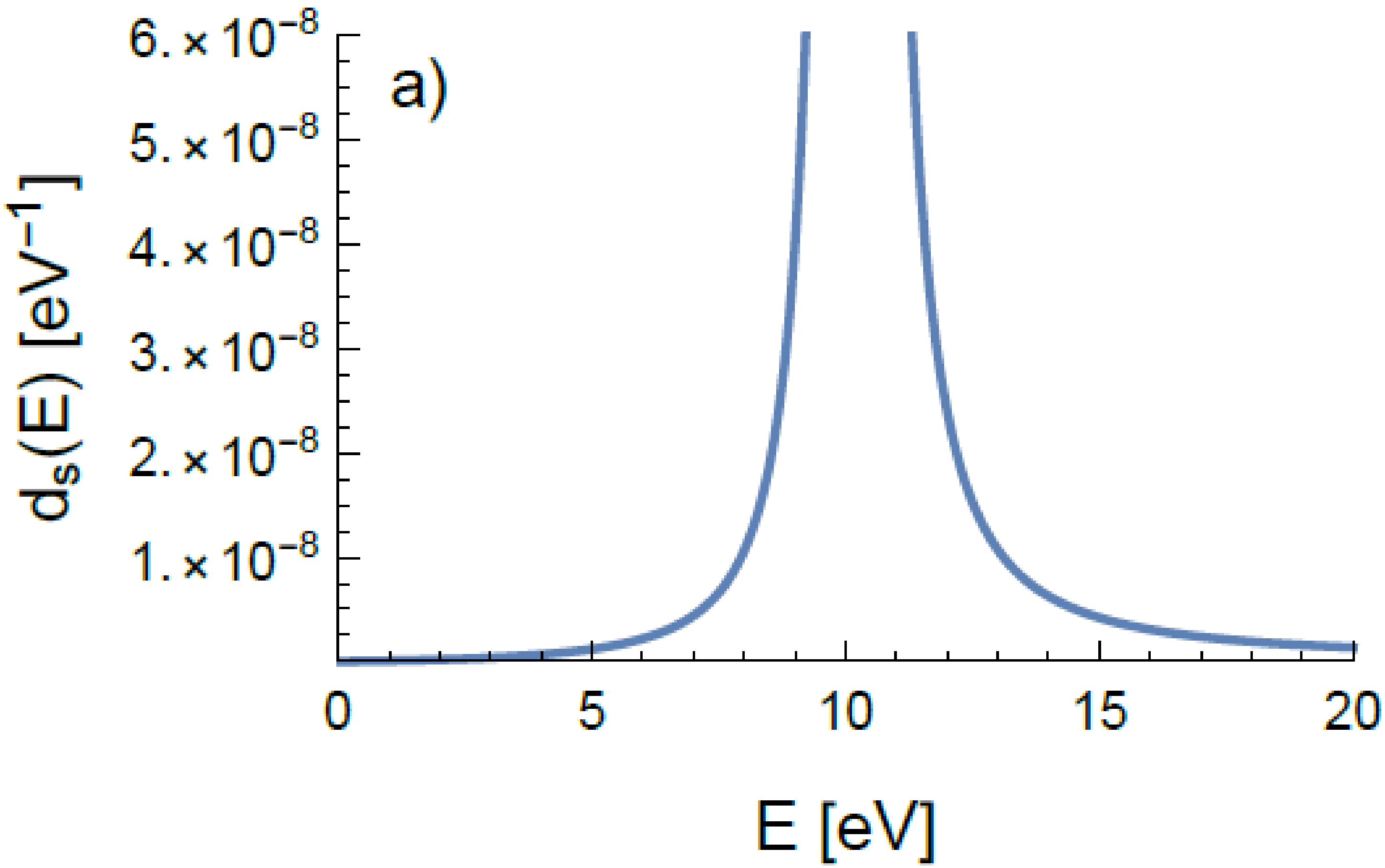
$$\frac{1}{\chi\Lambda}\mathrm{Re}[\Pi(E)] - C = -\frac{2\frac{E-E_{th}}{\Lambda}\ln\left(\frac{E-E_{th}}{\Lambda}\right)+\pi\left(\frac{E-E_{th}}{\Lambda}\right)^2}{2\left(1+\left(\frac{E-E_{th}}{\Lambda}\right)^2\right)^4} - \frac{2\frac{E-E_{th}}{\Lambda}+\pi\left(\frac{E-E_{th}}{\Lambda}\right)^2}{4\left(1+\left(\frac{E-E_{th}}{\Lambda}\right)^2\right)^3} - \frac{4\frac{E-E_{th}}{\Lambda}+3\pi\left(\frac{E-E_{th}}{\Lambda}\right)^2}{16\left(1+\left(\frac{E-E_{th}}{\Lambda}\right)^2\right)^2} + \frac{15\pi-16\frac{E-E_{th}}{\Lambda}}{96\left(1+\left(\frac{E-E_{th}}{\Lambda}\right)^2\right)}$$

2408.06905 [quant-ph]

$$d_S(E) = -\frac{1}{\pi}\mathrm{Im}[G_S(E)] = \frac{1}{\pi}\frac{\mathrm{Im}\,\Pi(E)}{(E-M+\mathrm{Re}\,\Pi(E))^2+(\mathrm{Im}\,\Pi(E))^2}$$

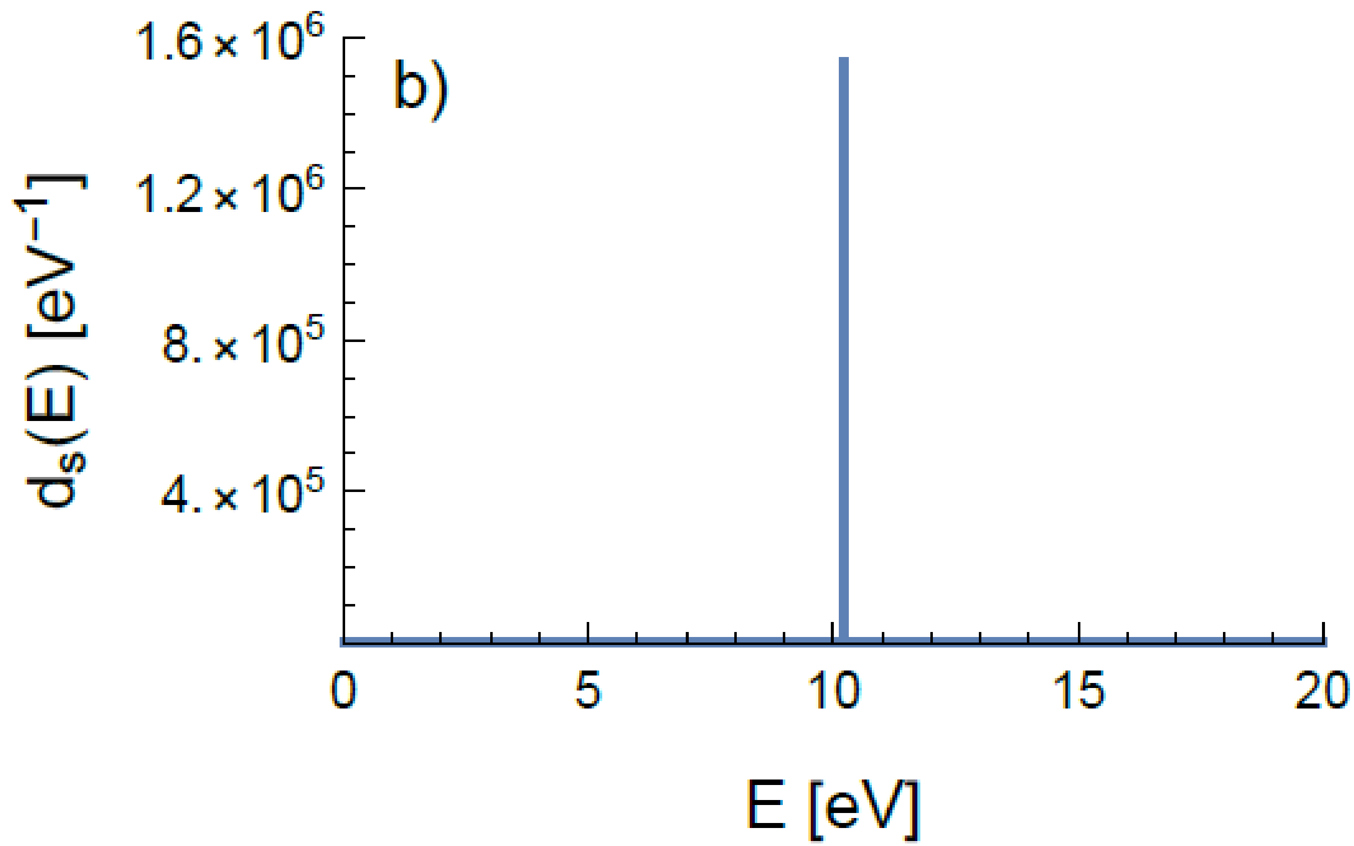
$$M = \frac{3}{8}\alpha^2 m_e \simeq 10.2043\,\mathrm{eV}$$

Spectral function: plot/1



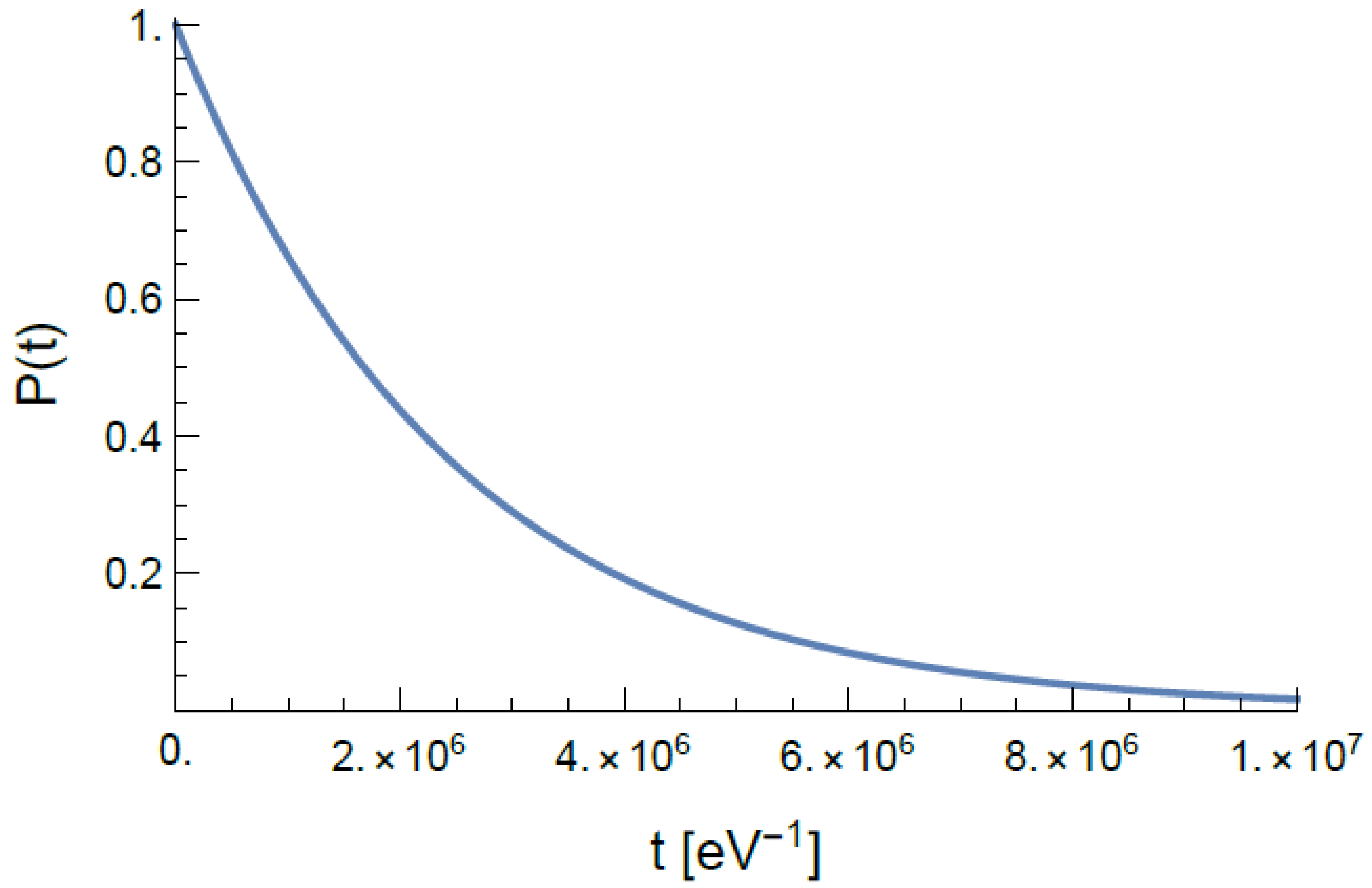
2408.06905 [quant-ph]

Spectral function/plot 2



2408.06905 [quant-ph]

Survival probability P(t)



2408.06905 [quant-ph]

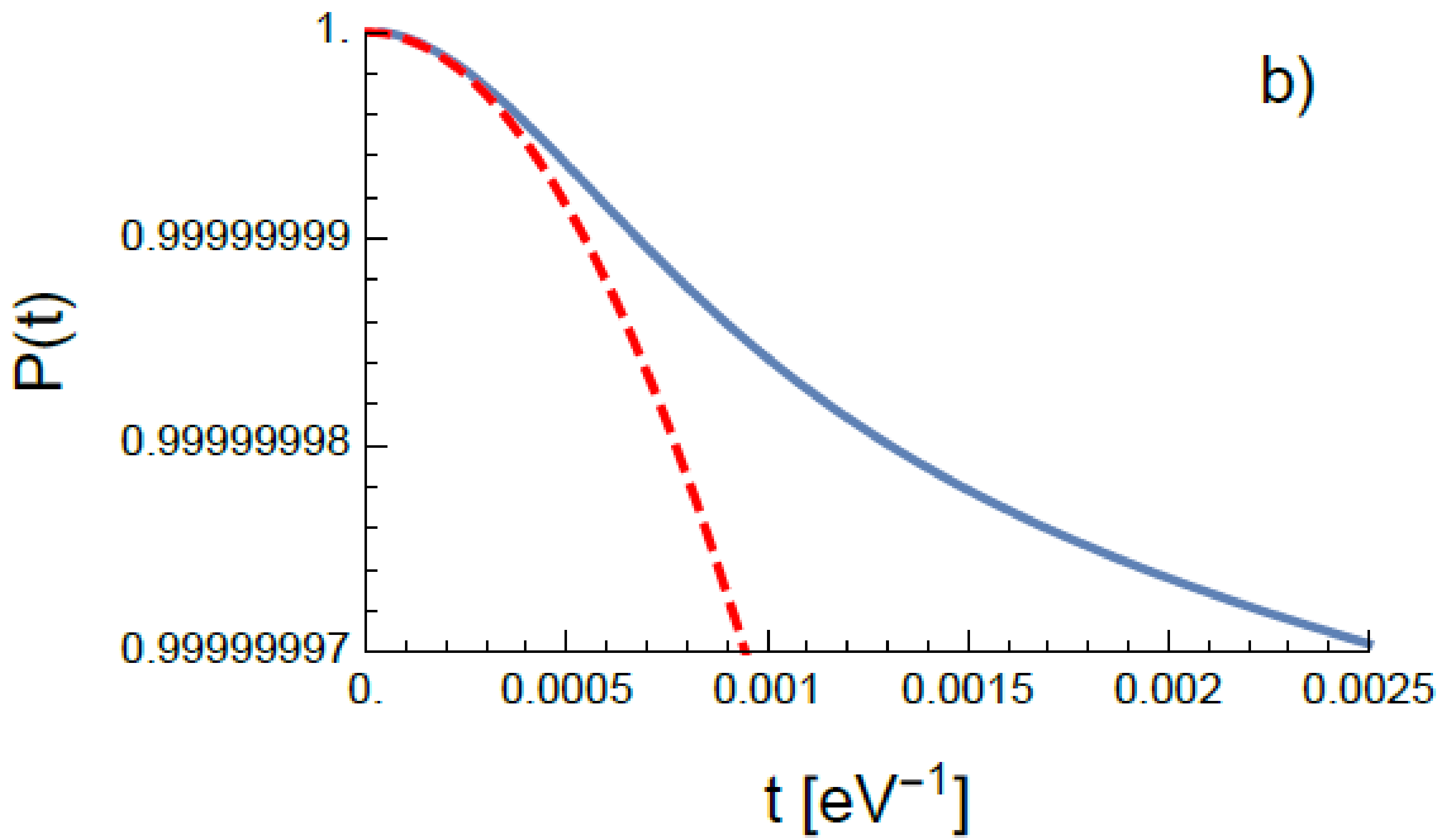
Survival probability at very short times/1

$$P(t) \simeq 1 - \frac{1}{2}\frac{\mathrm{d}^2 P(t)}{\mathrm{d}t^2}\bigg|_{t=0} t^2 + \ldots = 1 - \frac{t^2}{\tau_Z^2} + \ldots \, .$$

$$A(t) = 1 - it\,\langle E\rangle - \frac{t^2}{2}\,\langle E^2\rangle + \ldots$$

$$\tau_Z = \sqrt{\frac{1}{\langle E^2\rangle - \langle E\rangle^2}} = \frac{1}{\sigma_E} \simeq 5.45911\,\mathrm{eV}^{-1} = 3.59325 \times 10^{-15}\,\mathrm{s}\ .$$

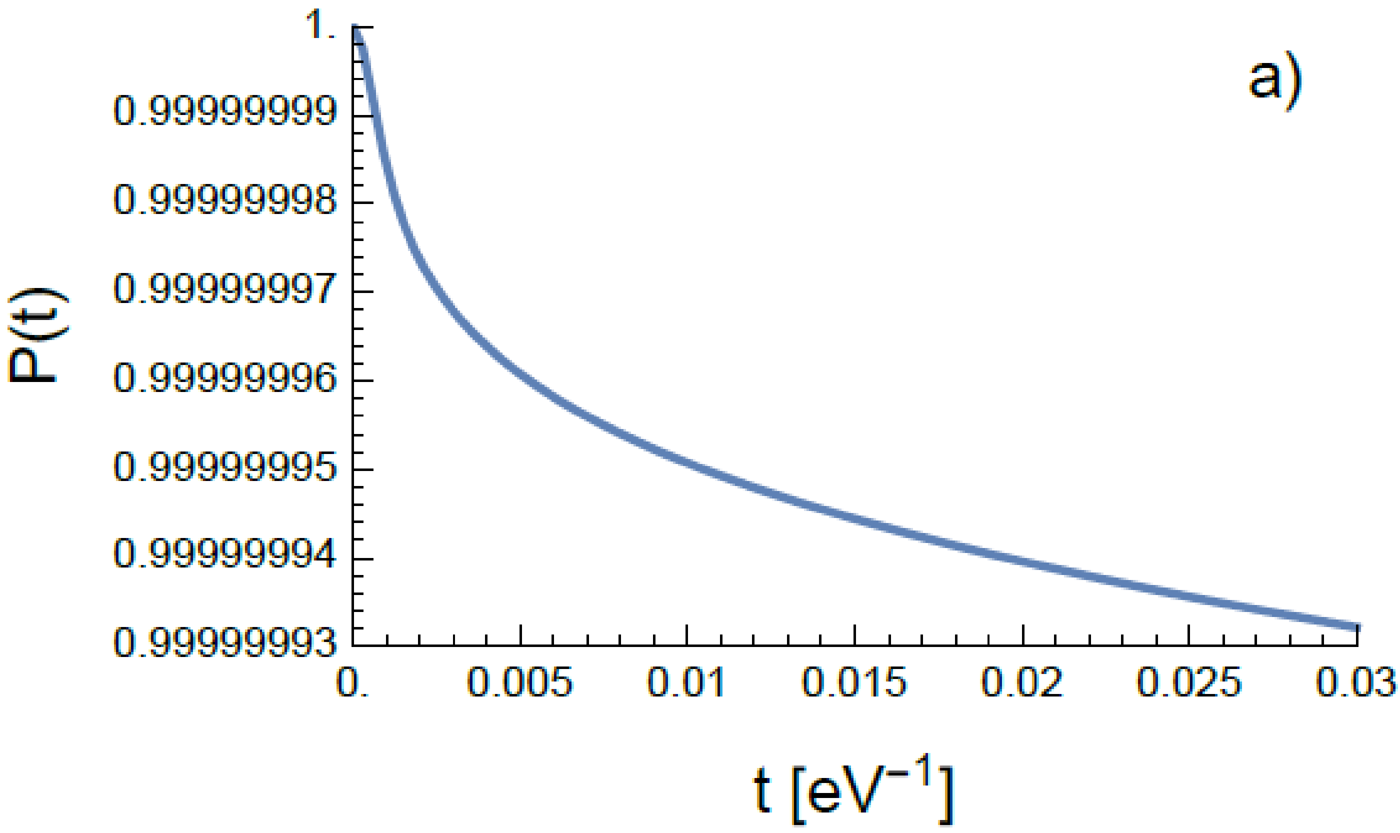
Survival probability at very short times/2



It is important to stress that in the present case, the Zeno time τ_Z is actually much longer than the non-exponential region in general and the quadratic region in particular.

2408.06905 [quant-ph]

Survival probability at short times: anti-Zeno



2408.06905 [quant-ph]

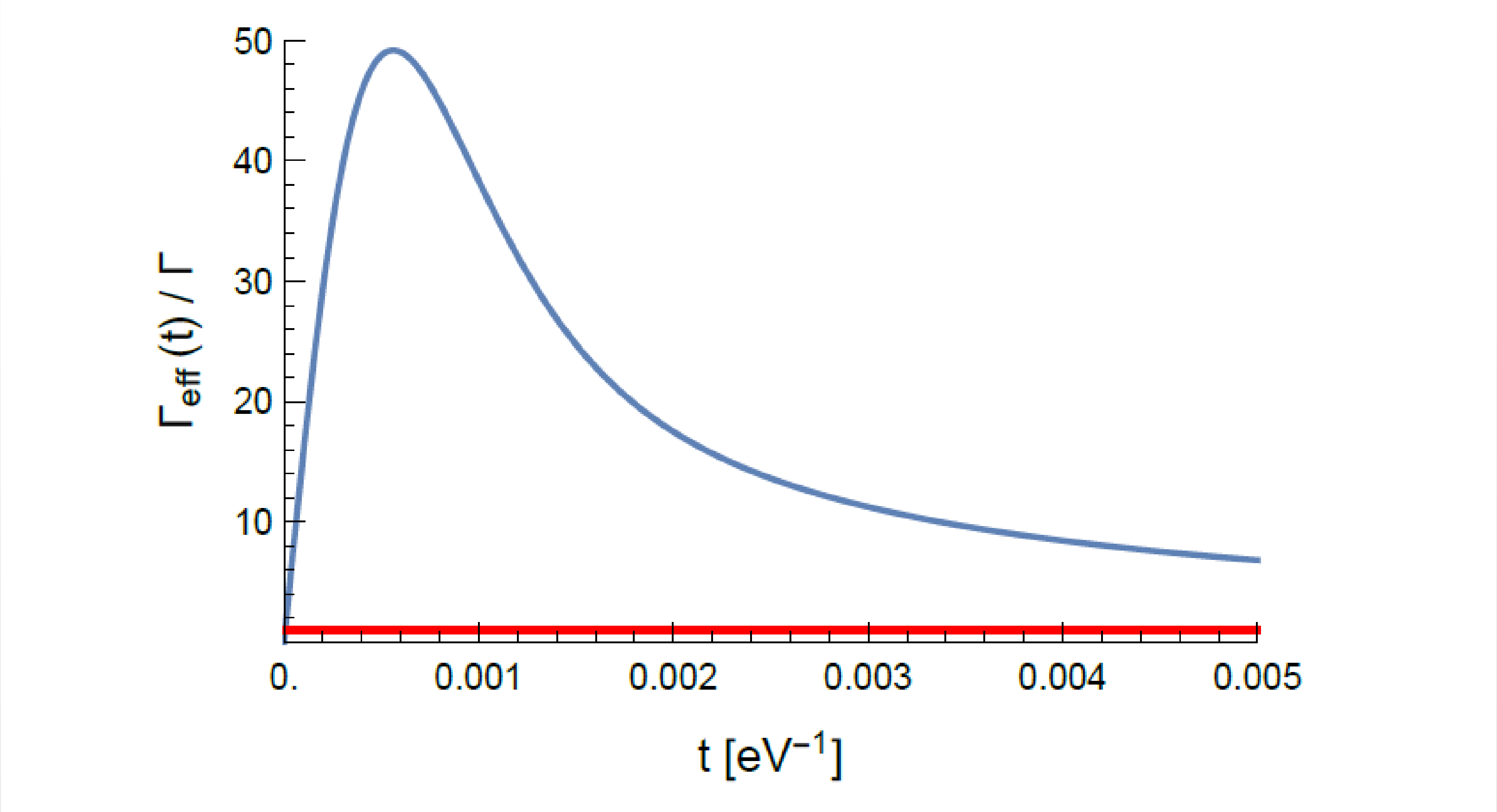
“Effective” decay width/1

$$\Gamma_{\text{eff}}(t) = -\frac{\mathrm{d}P(t)}{\mathrm{d}t}\frac{1}{P(t)} \,.$$

Table 1: Selected numerical values of the effective decay width within the anti-Zeno domain together with the corresponding times.

$\Gamma_{\text{eff}}(t)/\Gamma$	Time in eV^{-1}	Time in s
2	0.02130	1.40183×10^{-17}
1.1	0.06242	4.10857×10^{-17}
1.01	0.08234	5.41941×10^{-17}

“Effective” decay width/2



2408.06905 [quant-ph]

Survival amplitude at late times/1

$$z_{pole} = M - i\frac{\Gamma}{2} = M - \frac{i}{2\tau}$$

$$A(t) = -\frac{2i\,\mathrm{Im}[\Pi(z_{pole})]e^{-iz_{pole}t}}{z_{pole} - M + \mathrm{Re}[\Pi(z_{pole})] - i\,\mathrm{Im}[\Pi(z_{pole})]} - \frac{\chi}{M^2}\,t^{-2}$$

$$t_{\text{turn-over}} \simeq 3.03297 \times 10^8\,\mathrm{eV}^{-1} = 1.99634 \times 10^{-7}\,\mathrm{s} \simeq 125.1\,\tau$$

Survival amplitude at late times/2

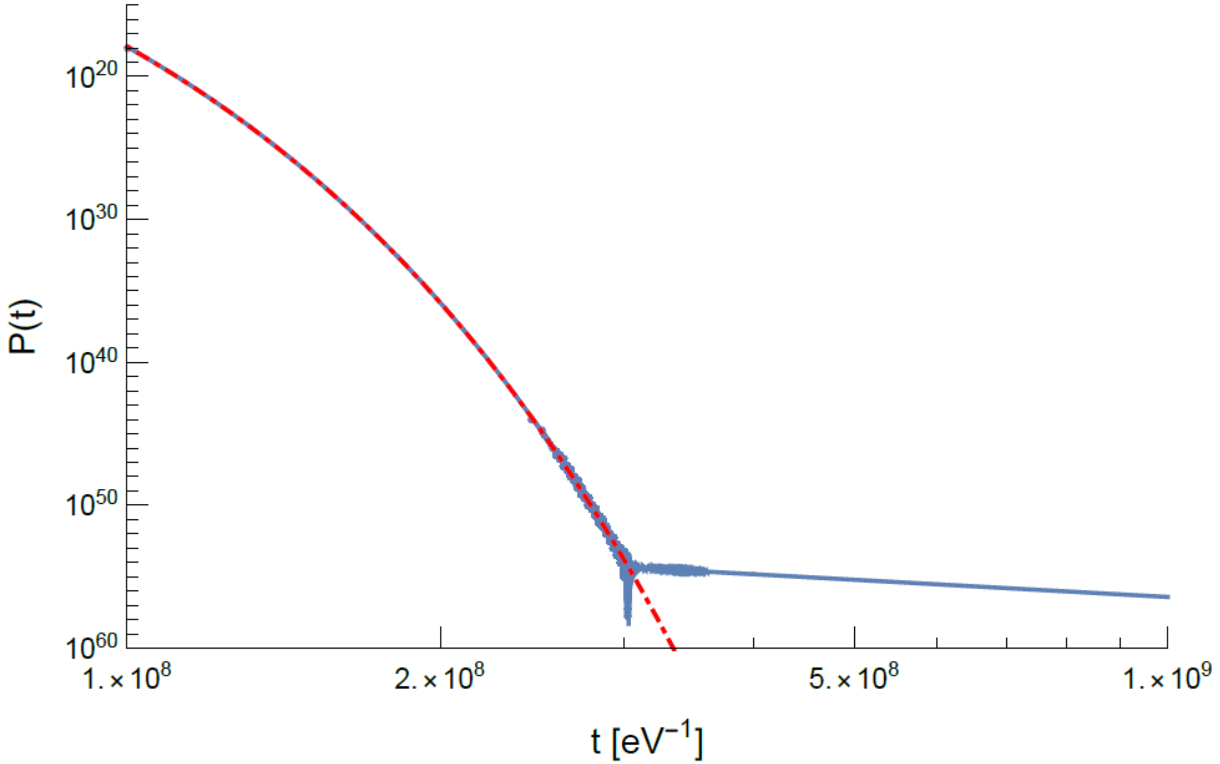


Figure 5: Survival probability at long times in log-log form. The red curve corresponds to purely exponential decay. An interesting feature is given by the fast oscillations close to the turn-over time.

2408.06905 [quant-ph]

Discussions and outlooks

- Para-positronium into two photons (talk of M. Piotrowska)
- Quarkonia (e.g. η_c meson, goes to $\gamma\gamma$ but also to many other channels)
- Other systems where to study the decay law???

Thanks!

BW with threshold **properly done**

We assume that:

$$\mathrm{Im}\,\Pi(E) = \begin{cases} \frac{\Gamma}{2} \text{ for } E \in (E_{th}, \Lambda) \\ 0 \text{ otherwise} \end{cases}$$

<E>=lnΛ=∞

In the limit $\Lambda \to \infty$ and by using one subtraction we get:

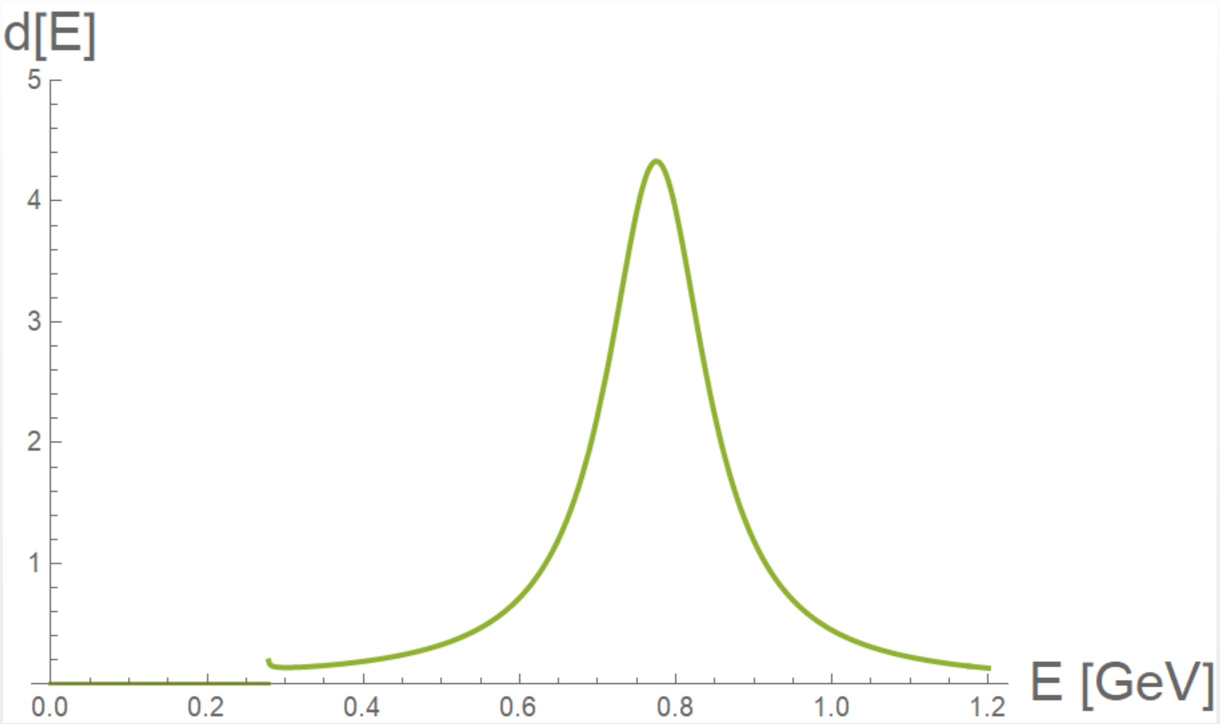
$$\Pi(E) = \frac{\Gamma}{2\pi} \ln\left(\frac{-E_{th} + M}{E_{th} - E}\right)$$

Then:

$$d_S(E) = \frac{\Gamma}{2\pi} \frac{1}{\left[E - M + \frac{\Gamma}{2\pi}\ln\left(\frac{M - E_{th}}{E_{th} - E}\right)\right]^2 + \frac{\Gamma^2}{4}} \theta(E - E_{th})$$

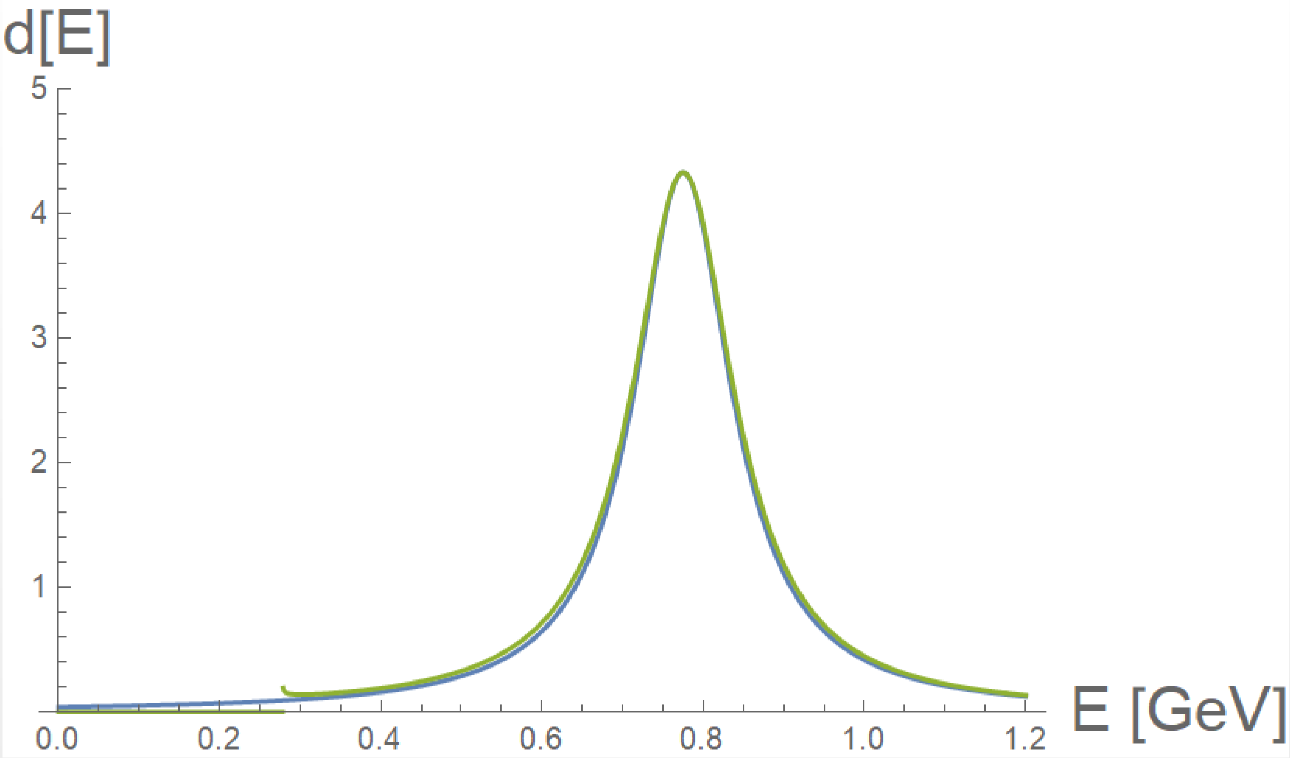
This is actually the correct Breit-Wigner with threshold!
Correctly normalized to unity, no need of an extra N…but somewhat not handy

BW with threshold **properly done**



Correct BW with threshold

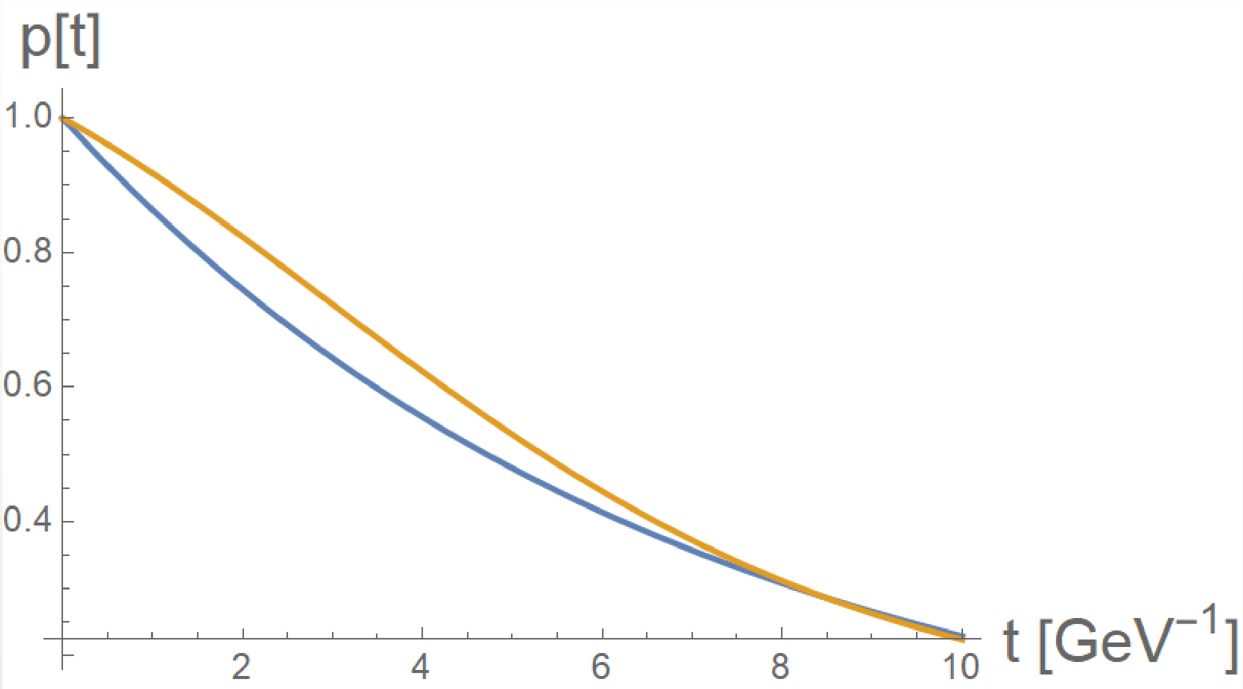
BW with threshold **properly done**



Comparison with plain BW: indeed very similar around the peak!

Yet: unphysical behavior at threshold!

BW with threshold **(properly done)** and time-evolution



Blue: BW, yellow: BW with threshold (properly done)

<E>=lnΛ=∞

Sill distribution

Eur. Phys. J. A (2021) 57:336
https://doi.org/10.1140/epja/s10050-021-00641-2

THE EUROPEAN PHYSICAL JOURNAL A

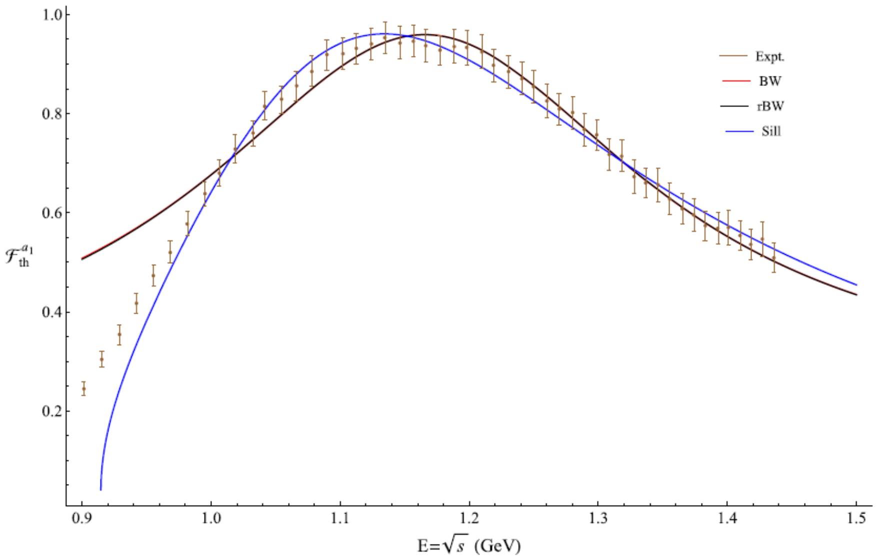
Regular Article - Theoretical Physics

A simple alternative to the relativistic Breit–Wigner distribution

Francesco Giacosa[1,2], Anna Okopińska[1], Vanamali Shastry[1,a]

Uniwersytet Jana Kochanowskiego w Kielcach

Example: a_1(1230) meson



Distribution	M (MeV)	Γ (MeV)	χ^2/d.o.f	$\sqrt{s_{pole}}$(MeV)
Nonrelativistic BW	1165.6 ± 1.2	415 ± 15	4.31	1166 − i 208
Relativistic BW	1146.5 ± 1.6	424 ± 16	4.25	1165 − i 209
Sill	1181.3 ± 3.4	539 ± 27	3.52	1046 − i 250

Applications in e.g.

EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

ALICE

CERN

CERN-EP-2023-106
30 May 2023

Accessing the strong interaction between Λ baryons and charged kaons with the femtoscopy technique at the LHC

ALICE Collaboration*

Ξ(1620) $I(J^P) = \frac{1}{2}(?^?)$ Status: ∗
J, P need confirmation.

OMITTED FROM SUMMARY TABLE
What little evidence there is consists of weak signals in the $\Xi\pi$ channel. A number of other experiments (e.g., BORENSTEIN 72 and HASSALL 81) have looked for but not seen any effect.

Ξ(1620) MASS

VALUE (MeV)	EVTS	DOCUMENT ID	TECN	COMMENT
≈ 1620 OUR ESTIMATE				

[arXiv:2106.03749 [hep-ph]].

https://doi.org/10.1140/epja/s10050-021-00641-2 PHYSICAL JOURNAL A Check for updates

Regular Article - Theoretical Physics

A simple alternative to the relativistic Breit–Wigner distribution

Francesco Giacosa[1,2], Anna Okopińska[1], Vanamali Shastry[1,a]

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See F.G., V. Shastry and A. Okopinska[arXiv:2106.03749 [hep-ph]].
and also [arXiv:2310.06346 [hep-ph]] for the non-rel limit

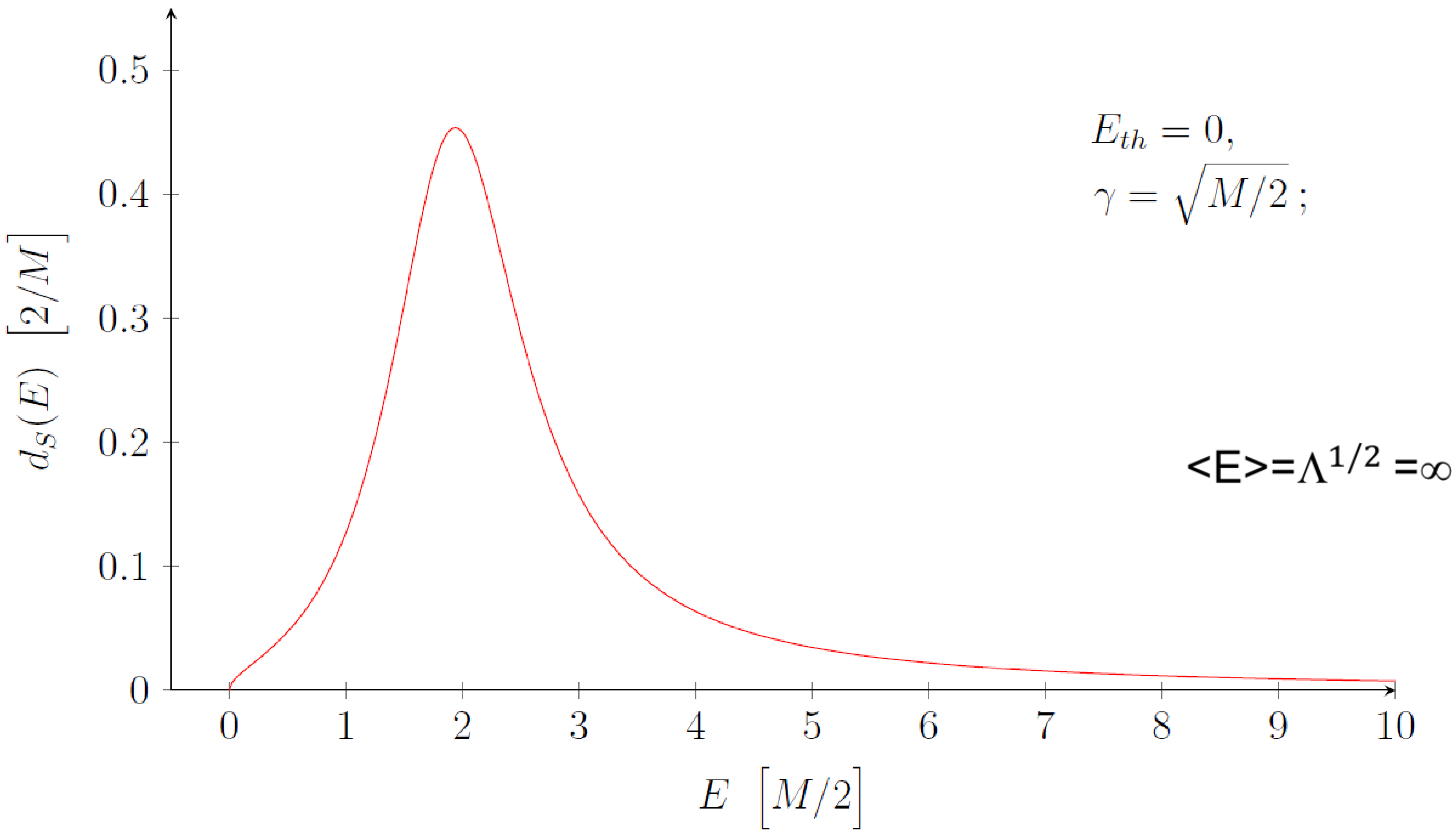
$$\mathrm{Im}[\Pi(E)] = \frac{\gamma}{2}\sqrt{E - E_{th}}\,\theta(E - E_{th})$$

$$\Pi(E) = \frac{i\gamma\sqrt{E - E_{th}}}{2}$$

$$G_S(E) = \frac{1}{E - M + \frac{i\gamma\sqrt{E-E_{th}}}{2} + i\epsilon}$$

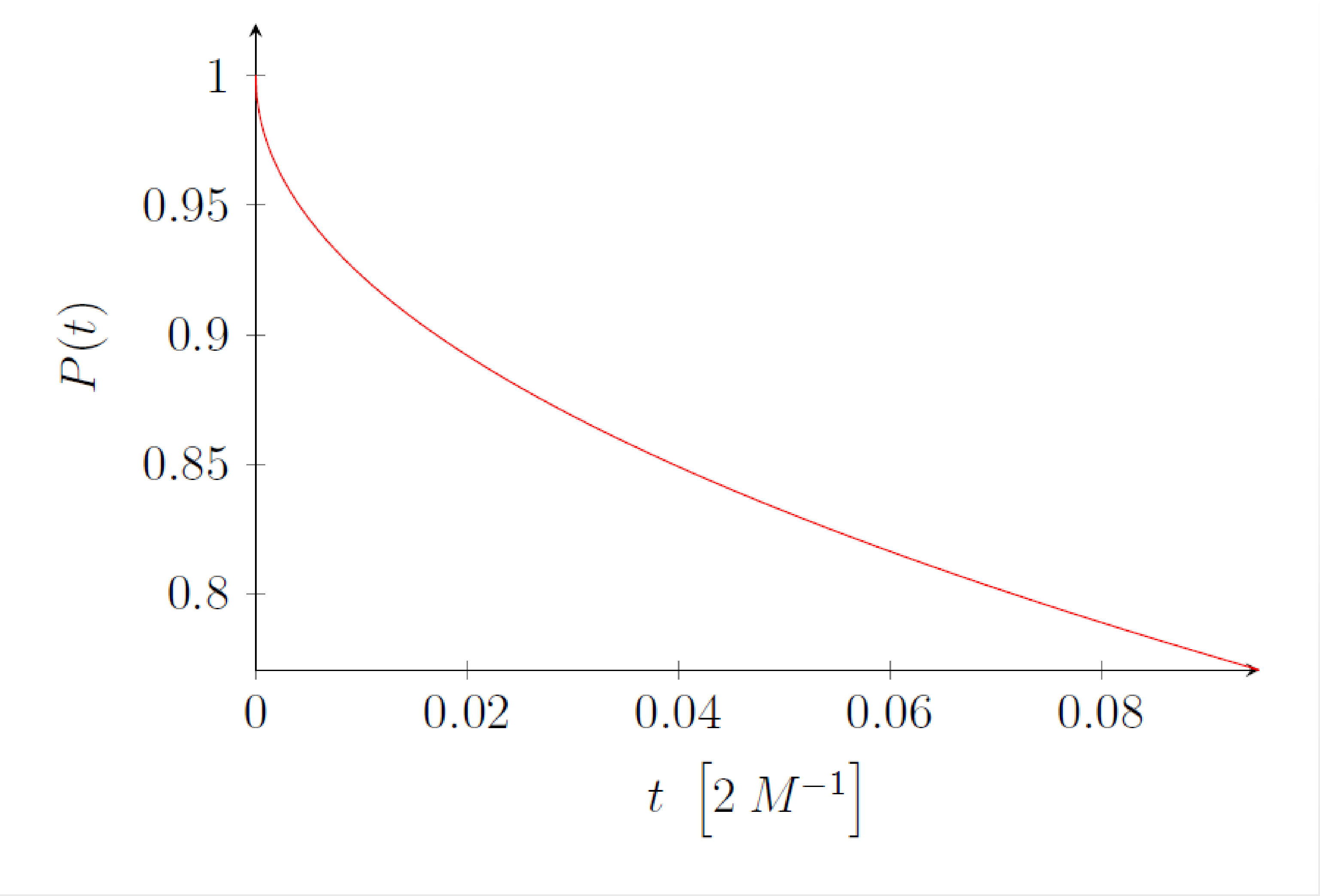
$$d_S(E) = \frac{\gamma\sqrt{E - E_{th}}}{2\pi}\frac{1}{(E - M)^2 + \frac{1}{4}(\gamma\sqrt{E - E_{th}})^2}$$

Non-rel. Sill distribution: 'nice' threshold behavior



K. Kyzioł, bachelor thesis, UJK Kielce, 2024

Plot of the survival probability nonrel Sill



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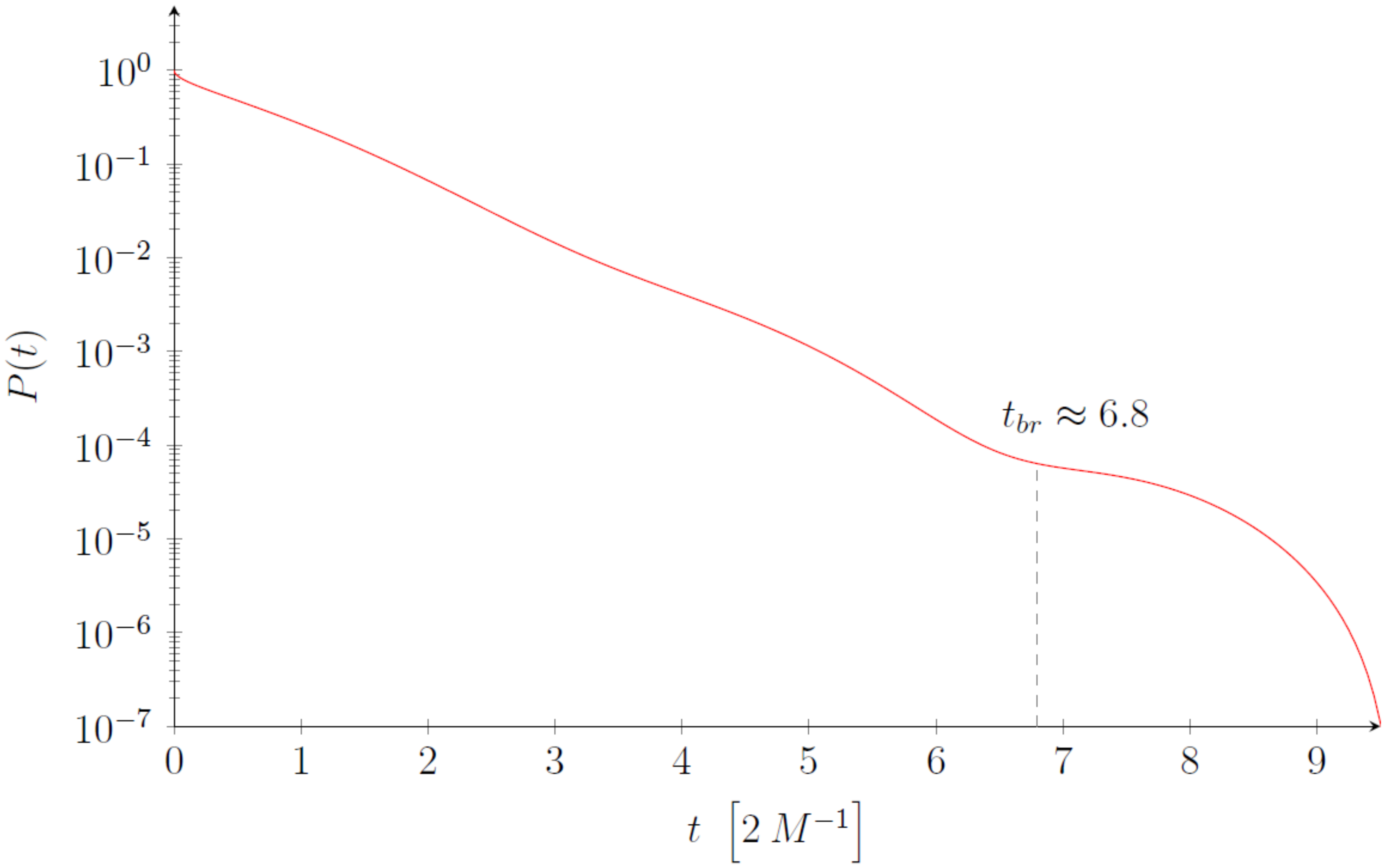
$<E>=\Lambda^{1/2}=\infty$

Log-plot



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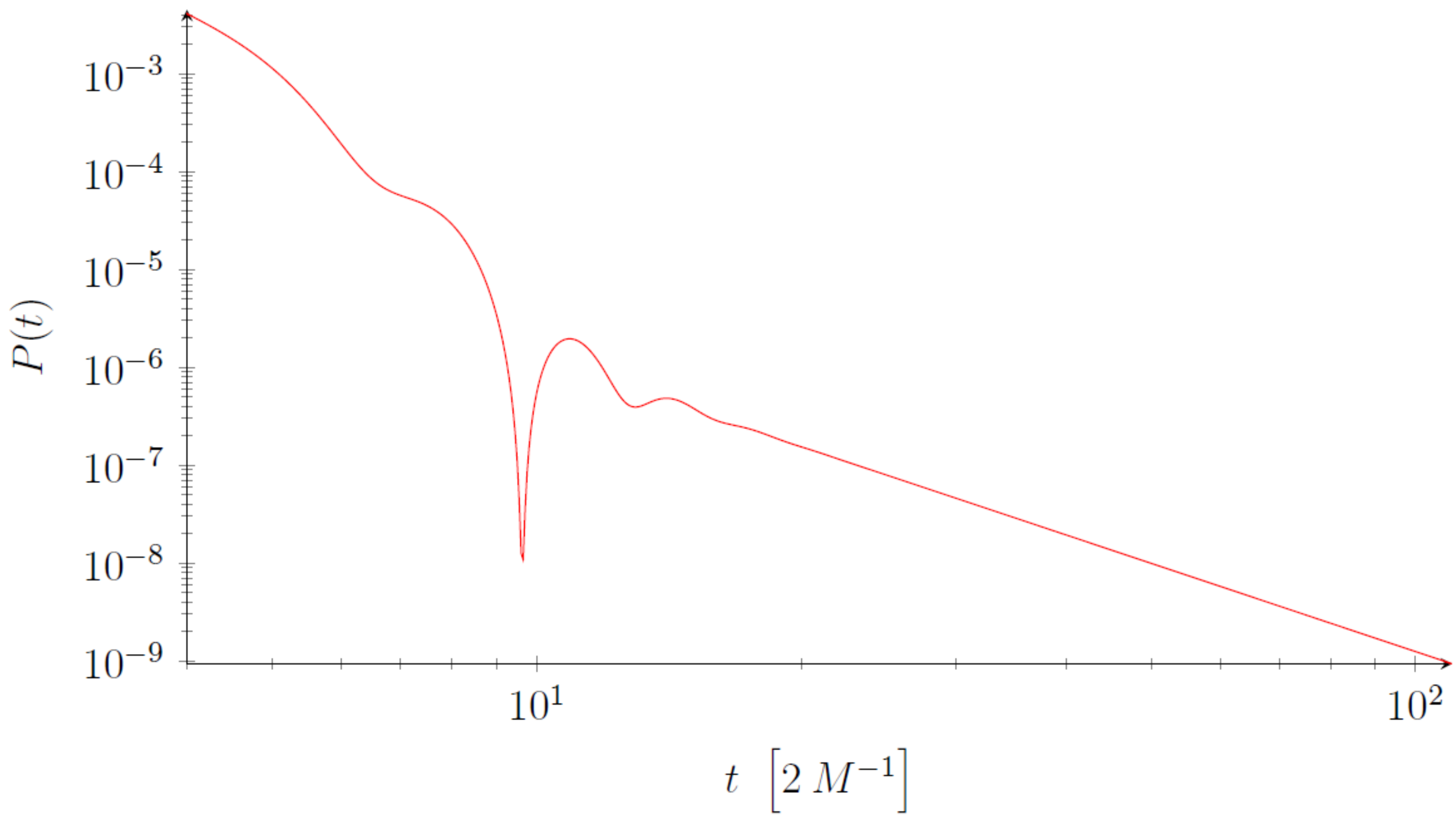
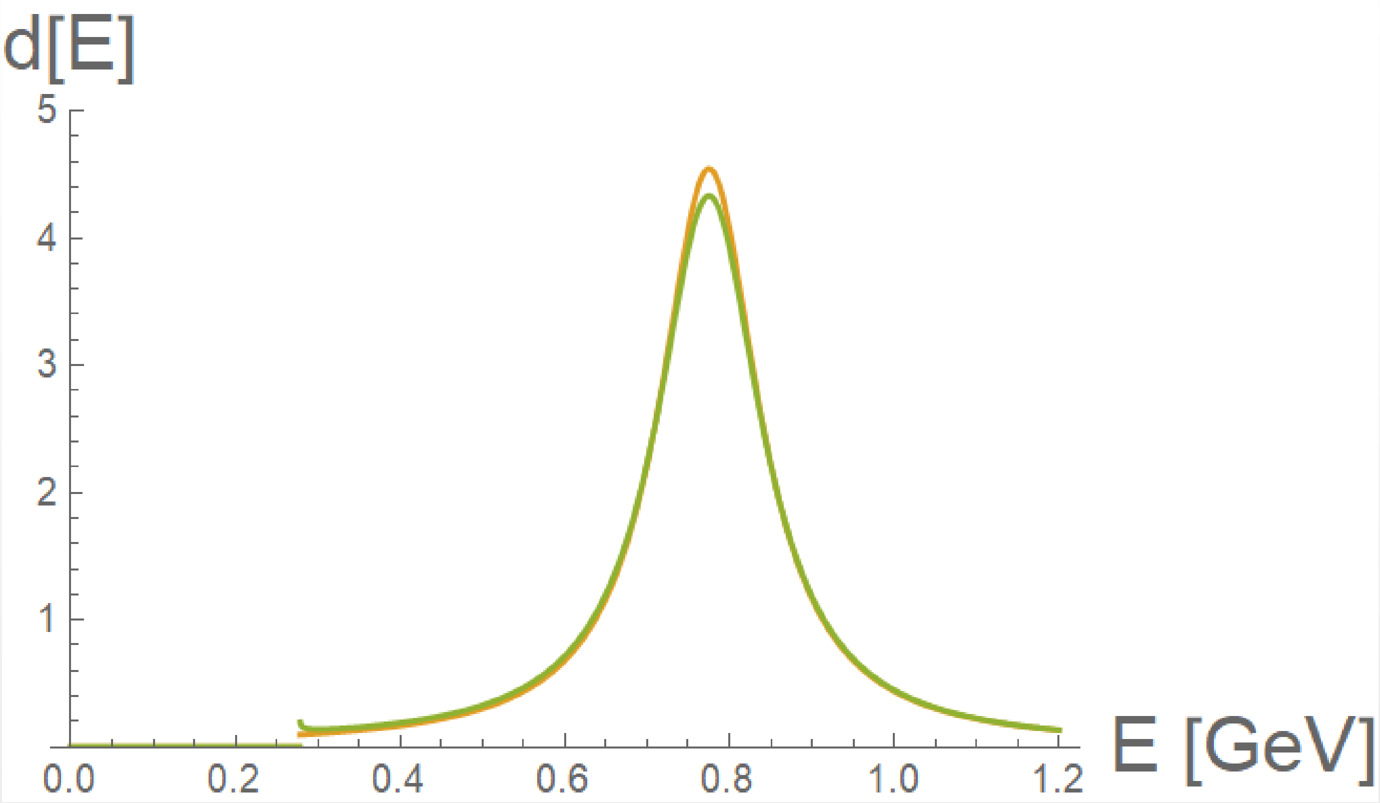
Log-Log-Plot

$$\ln P(t) \approx \ln(C \cdot t^{-3}) = \ln C - 3\ln t \quad .$$



K. Kyzioł, bachelor thesis, UJK Kielce, 2024

BW with threshold **properly done**



Comparision with ‘naive’ BW with threshold

Relativistic Sill

Let us consider a resonance with mass M decaying into twoparticles:

$$E_{th} = m_1 + m_2 = \sqrt{s_{th}} \qquad s_{th} = E_{th}^2$$

We **assume** that:

$$\mathrm{Im}\,\Pi(s) = \sqrt{s - s_{th}}\tilde{\Gamma}\theta(s - s_{th})$$

$$\Gamma M = \tilde{\Gamma}\sqrt{M^2 - E_{th}^2}$$

Decay width as function of the energy:

$$\Gamma(s) = \frac{\sqrt{s - s_{th}}}{\sqrt{s}}\tilde{\Gamma}$$

Note, it saturates for large s

Relativsitic Sill

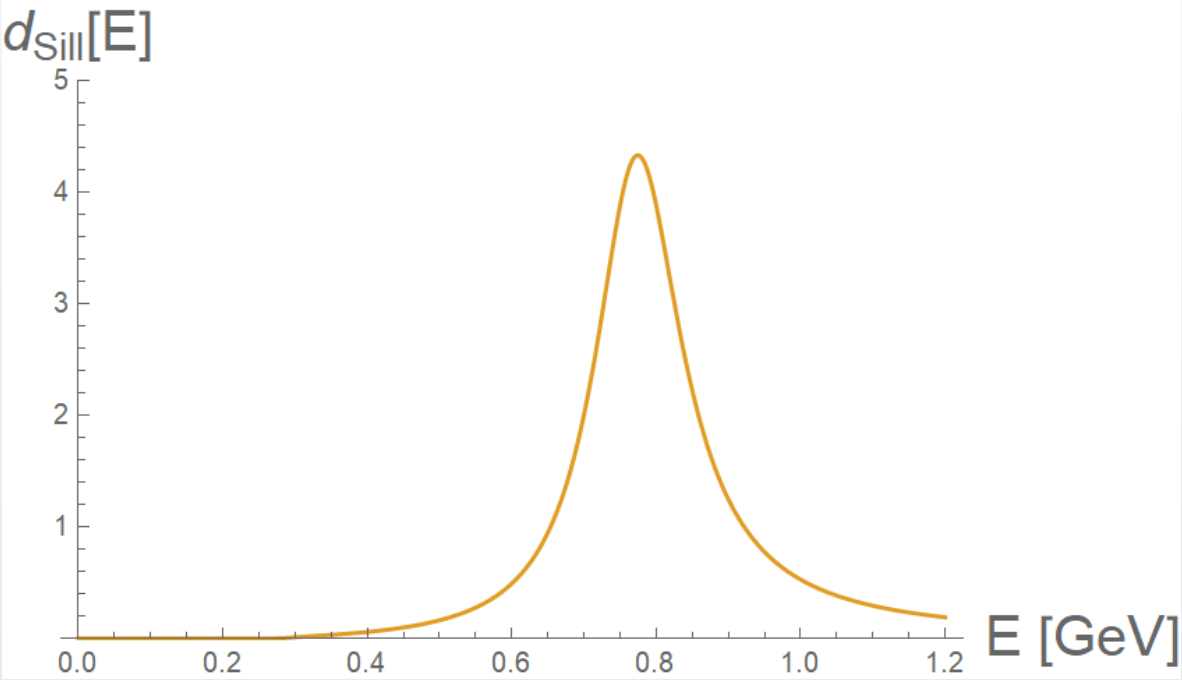
$$\Pi(s) = i\tilde{\Gamma}\sqrt{s - s_{th}}$$

$$G_S(s) = \frac{1}{s - M^2 + i\tilde{\Gamma}\sqrt{s - s_{th}} + i\varepsilon}$$

$$\begin{aligned} d_S(s) &= -\frac{1}{\pi}\,\mathrm{Im}\,\frac{1}{s - M^2 + i\tilde{\Gamma}\sqrt{s - s_{th}} + i\varepsilon} \\ &= \frac{1}{\pi}\frac{\sqrt{s - s_{th}}\,\tilde{\Gamma}}{(s - M^2)^2 + (\sqrt{s - s_{th}}\,\tilde{\Gamma})^2}\theta(s - s_{th}) \end{aligned}$$

Relativistic Sill

$$d_S(E) = d_S^{\text{Sill}}(E) = \frac{2E}{\pi} \frac{\sqrt{E^2 - E_{th}^2}\,\tilde{\Gamma}}{(E^2 - M^2)^2 + \left(\sqrt{E^2 - E_{th}^2}\,\tilde{\Gamma}\right)^2} \theta(E - E_{th})$$



Sill for the rho-meson

Comments

$$s_{pole} = M^2 - \frac{\tilde{\Gamma}^2}{2} - i\sqrt{(M^2 - s_{th})\tilde{\Gamma}^2 + \frac{\tilde{\Gamma}^4}{4}}\ .$$

Note, for $\tilde{\Gamma}^2$ sufficiently smaller than $M^2 - s_{th}$, the pole of s can be approximated as

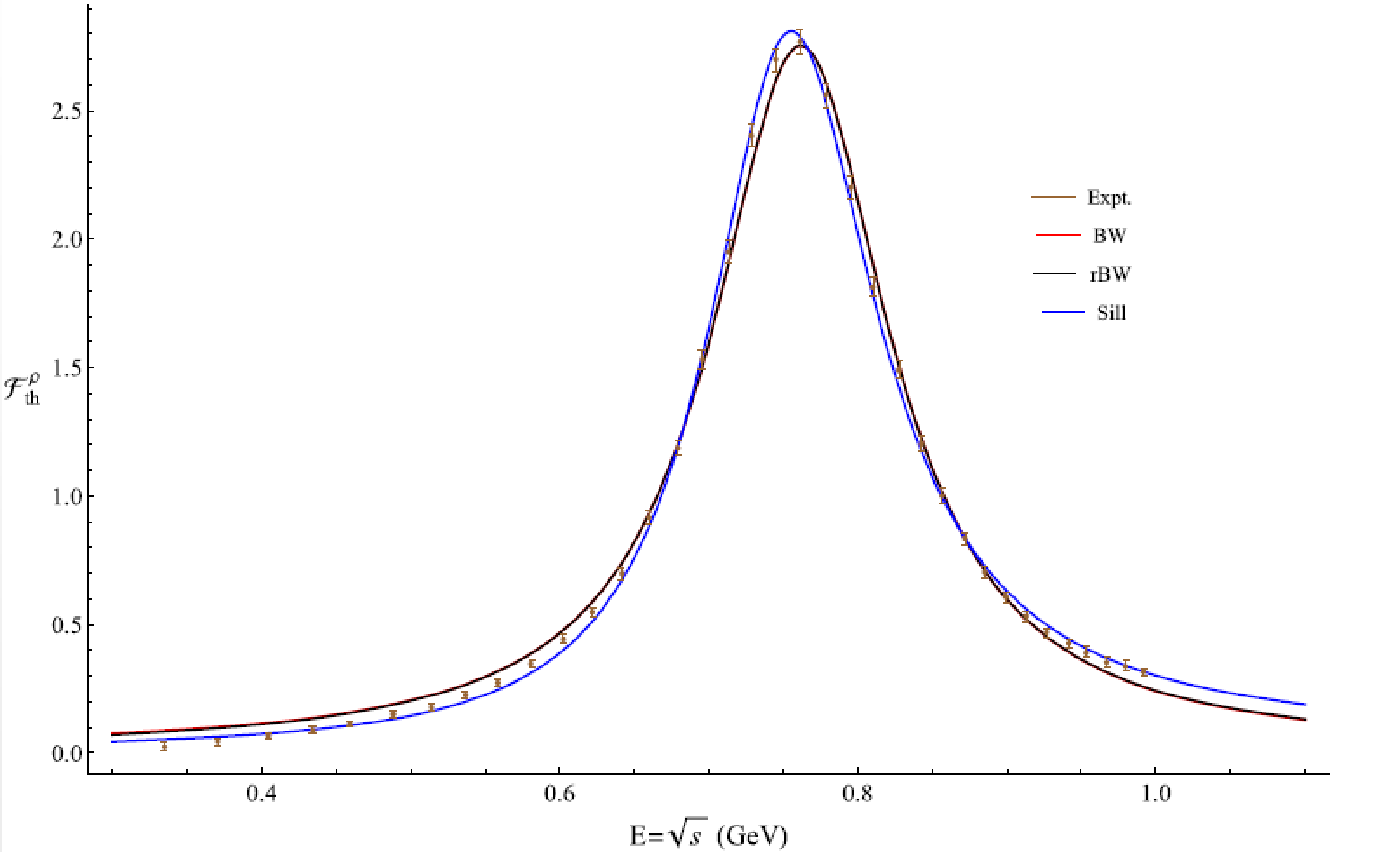
$$s_{pole} \simeq M^2 - i\sqrt{(M^2 - s_{th})}\tilde{\Gamma} = M^2 - i\,M\,\Gamma\ ,$$

The normalization

$$\int_{E_{th}}^{+\infty} \mathrm{dE}\, d_S^{\mathrm{Sill}}(E) = 1$$

for any E_{th}, M, and $\tilde{\Gamma}$ is a consequence of the proper treatment of the real part of the loop

Rho meson



Aleph data for tau decay

Distribution	M (MeV)	Γ (MeV)	χ^2/d.o.f	$\sqrt{s_{pole}}$(MeV)
Nonrelativistic BW	761.64 ± 0.32	144.6 ± 1.3	10.16	$761.6 - i\,72.3$
Relativistic BW	758.1 ± 0.33	145.2 ± 1.3	9.42	$761.5 - i\,72.3$
Sill	755.82 ± 0.33	137.3 ± 1.1	3.52	$751.7 - i\,68.6$

More than a single channel

The extension to the N channels is straightforward:

$$G_S(s) = \frac{1}{s - M^2 + i\sum_{k=1}^{N} \tilde{\Gamma}_k \sqrt{s - s_{k,th}} + i\varepsilon}$$

with

$$\tilde{\Gamma}_k = \Gamma_k \frac{M}{\sqrt{M^2 - E_{k,th}^2}} \text{ and}$$

$$s_{1,th} = E_{1,th}^2 \leq s_{2,th} \leq \ldots \leq_{N,th} = E_{N,th}^2 \,.$$

$$d_s^k(s) = \frac{1}{\pi} \frac{\sqrt{s - s_{\mathrm{th,k}}}\;\tilde{\Gamma}_k}{(s - M^2 - \sum_{i=1}^{Q} \sqrt{s_{\mathrm{th,i}} - s}\;\tilde{\Gamma}_i)^2 + \sum_{i=Q+1}^{N} (\sqrt{s - s_{\mathrm{th,i}}}\;\tilde{\Gamma}_i)^2} \theta(s - s_{\mathrm{th,k}})$$

where, $s_{\mathrm{th,k}}$ is the k^{th} threshold, and the integer Q is such that, for all $i < Q$, $s_{th,i} < s_{th,k}$

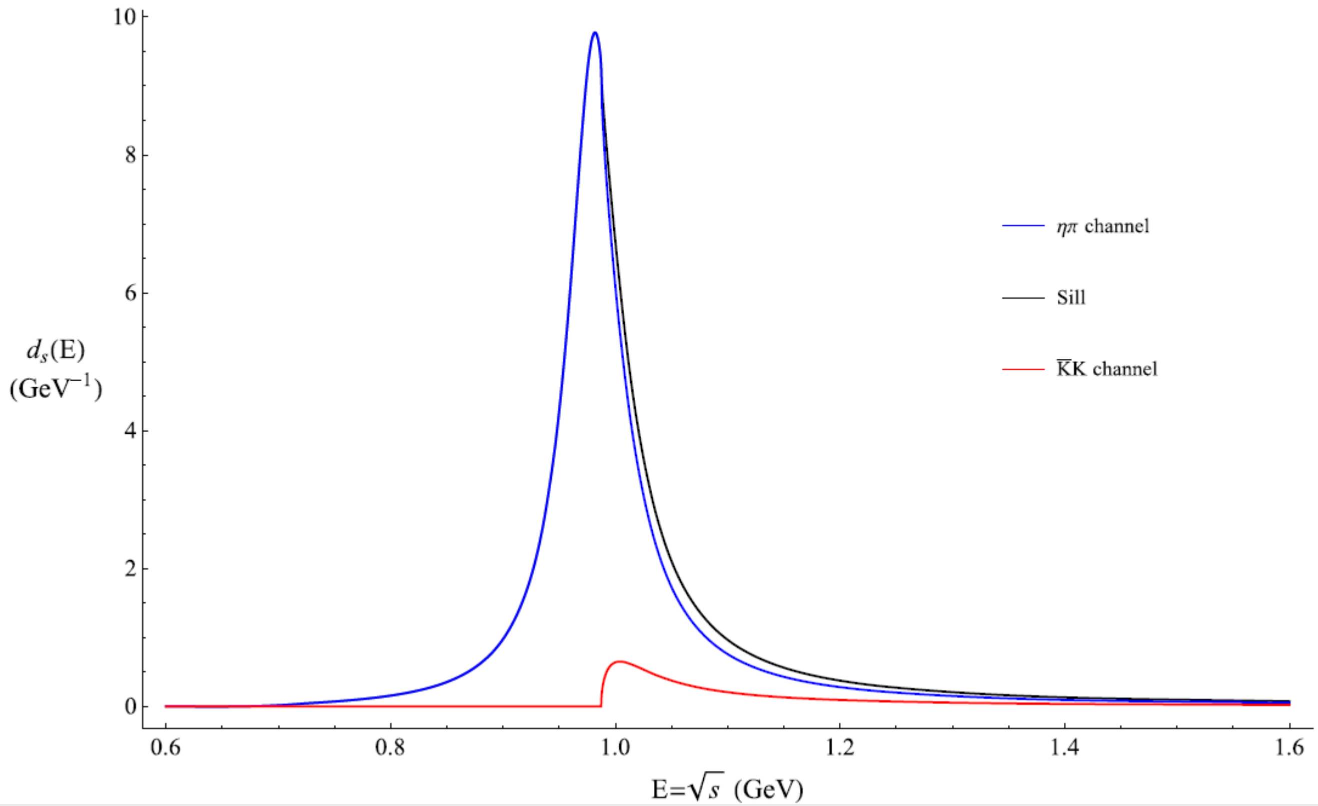
Two-channel case

$$G_S(s) = \frac{1}{s - M^2 + i\tilde{\Gamma}_1\sqrt{s - s_{1,th}} + i\tilde{\Gamma}_2\sqrt{s - s_{2,th}} + i\varepsilon},$$

$$d_S(s) = -\frac{1}{\pi}\,\mathrm{Im}[G_S(s)] = \begin{cases} \frac{1}{\pi}\,\frac{\tilde{\Gamma}_1\sqrt{s-s_{1,th}}+\tilde{\Gamma}_2\sqrt{s-s_{2,th}}}{(s-M^2)^2+(\tilde{\Gamma}_1\sqrt{s-s_{1,th}}+\tilde{\Gamma}_2\sqrt{s-s_{2,th}})^2} & \text{for } s > s_{2,th} \\ \frac{1}{\pi}\,\frac{\tilde{\Gamma}_1\sqrt{s-s_{1,th}}}{\left(s-M^2-\tilde{\Gamma}_2\sqrt{s_{2,th}-s}\right)^2+(\tilde{\Gamma}_1\sqrt{s-s_{1,th}})^2} & \text{for } s_{1,th} \leq s \leq s_{2,th} \\ 0 & \text{for } s < s_{1,th} \end{cases}$$

a0(980) example

Fig. 8 The Sill distribution of the $a_0(980)$ and the $\eta\pi$ and $\bar{K}K$ channels. The non-BW form due to the KK threshold is evident



Other recent Sill application

PHYSICAL REVIEW D **106,** 094009 (2022)

XYZ spectroscopy at electron-hadron facilities. II. Semi-inclusive processes with pion exchange

D. Winney,[1,2,*] A. Pilloni,[3,4,†] V. Mathieu,[5,‡] A. N. Hiller Blin,[6,7] M. Albaladejo,[8] W. A. Smith,[9,10] and A. Szczepaniak[9,10,11]

(Joint Physics Analysis Center)

description of the πp mass distribution in the Δ mass region:

$$d_{\Delta\to\pi p}(M^2) = \frac{1}{\pi}\frac{\rho(M^2)\tilde{\Gamma}_\Delta}{[M^2 - m_\Delta^2]^2 + [\rho(M^2)\tilde{\Gamma}_\Delta]^2}, \quad (39)$$

with $\rho(M^2) = \sqrt{M^2 - M_{\min}^2}$ and $\tilde{\Gamma}_\Delta = \Gamma_\Delta m_\Delta / \rho(m_\Delta^2)$. Interestingly, this function is normalized across the mass