Gauge invariant thermodynamics of rotating magnetized systems

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K. Fukushima, K. Hattori and KM (in prep.)

Spin and quantum features of QCD plasma

Sep 16 – 20, 2024
ECT*
Europe/Rome timezone

Now happening: Discussion (Aula Renzo Leonardi) *4:30 PM - 6:00 PM*

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- Overview
- Timetable
- Contribution List
- Registration
- Participant List

Contact

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Spin and quantum features of QCD plasma

The recent measurement of the spin polarization of particles produced in nuclear collisions has opened a new frontier for the study of strong-interaction matter under extreme conditions. Future experimental efforts will measure spin observables with unprecedented precision. On the theoretical front, there is rapid progress in discovering new effects that polarize spin. However, the understanding of these new effects as well as their implementation in dynamical frameworks such as quantum hydrodynamics and kinetic theory are still under development. The goal of the workshop is to gather experts from both theory and experiment to determine the state-of-the-art knowledge in the field, to exchange ideas and methods, and to initiate new developments.

Spin and quantum features of QCD plasma

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Spin hydrodynamics

Spin kinetic theory

Spin polarization

Spin alignment

S

theory and experiment to determine the state-of-the-art knowledge in the field, to exchange ideas and methods, and to initiate new developments.

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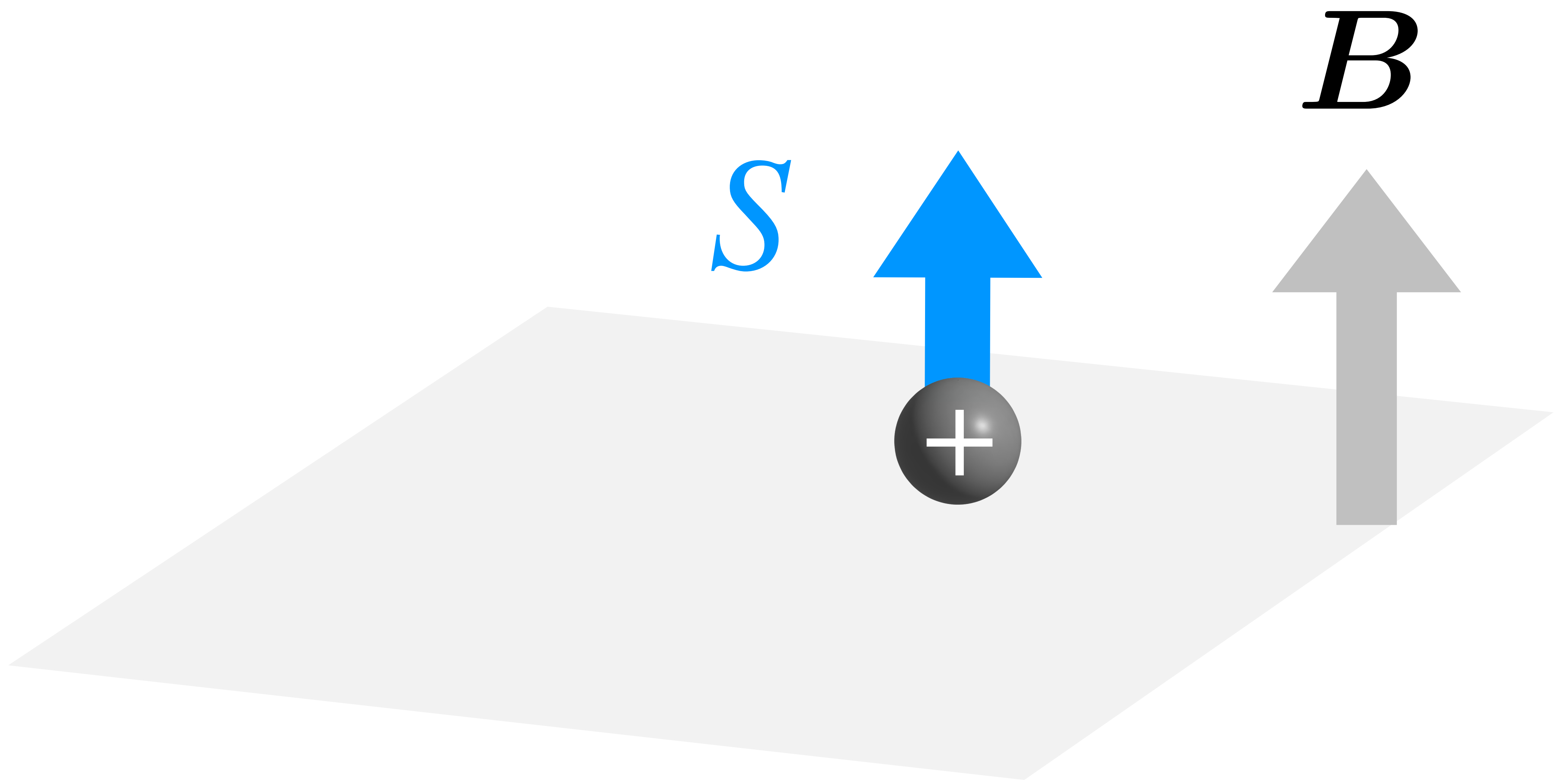
$$J = S + L$$

theory and experiment to determine the state-of-the-art knowledge in the field, to exchange ideas and methods, and to initiate new developments.

What I Will Tell Today

$$\left[\mathrm{i}\gamma^{\mu}(\partial_{\mu} + \mathrm{i}A_{\mu}) + m\right]\psi = 0$$



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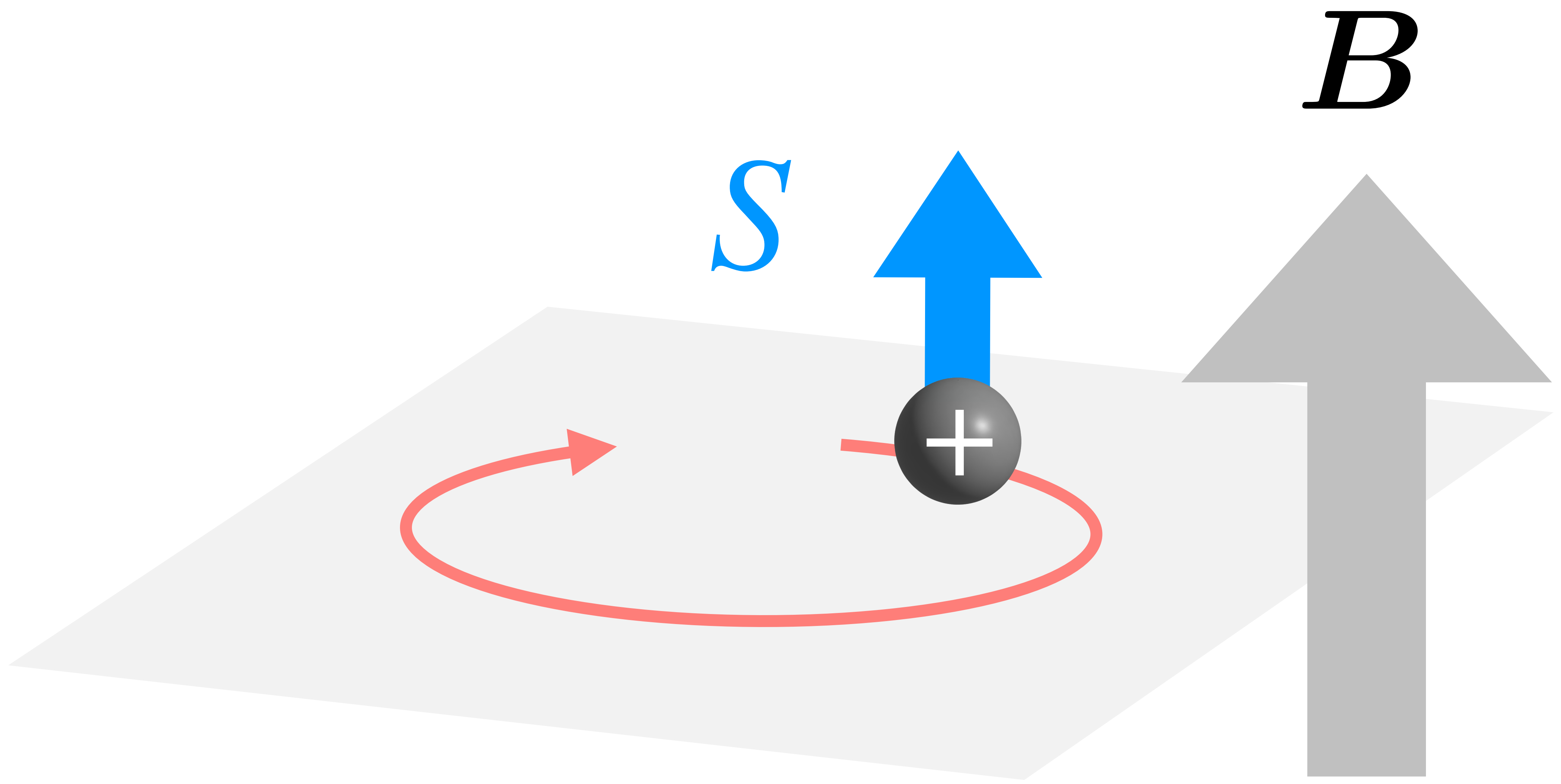

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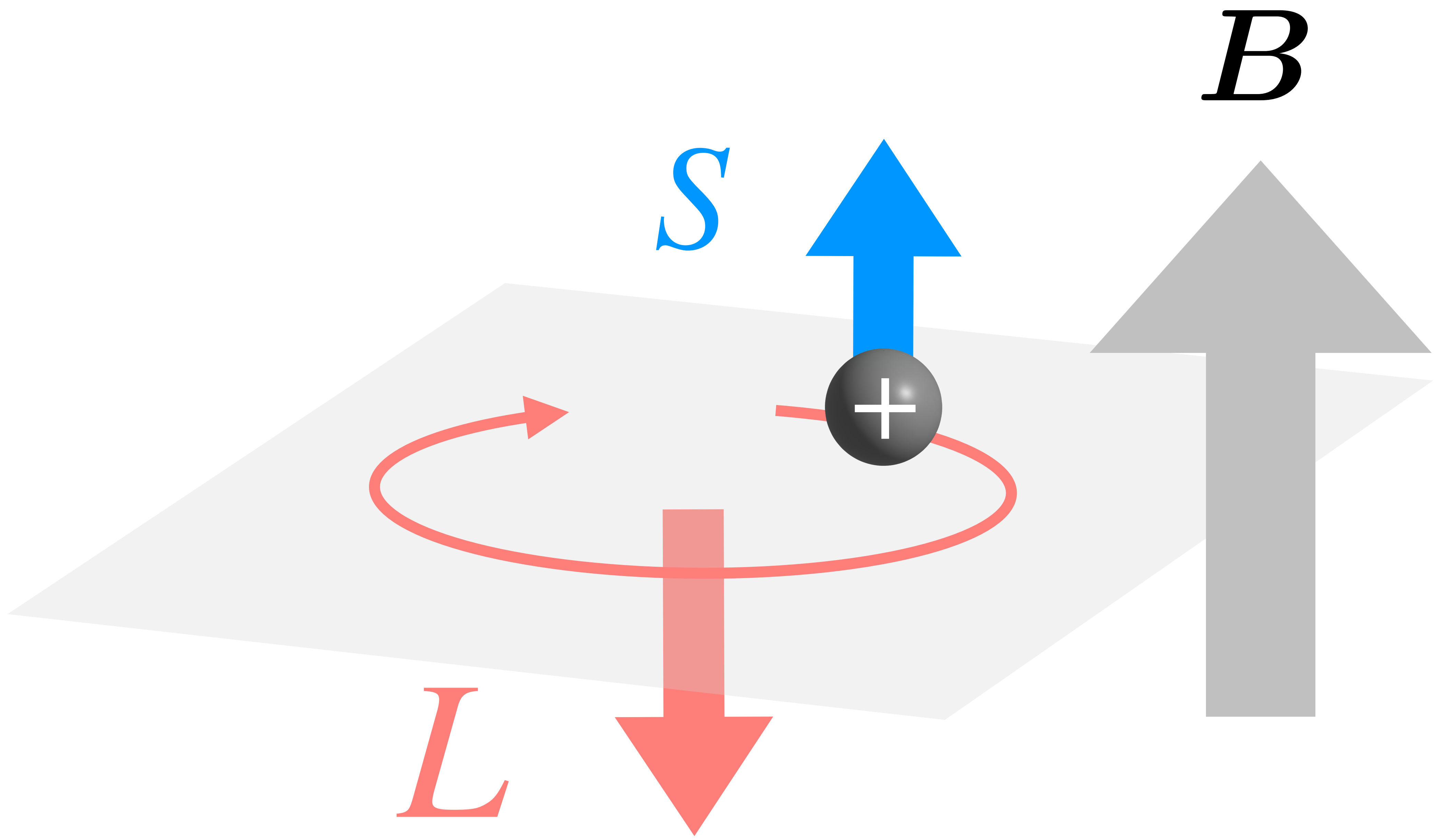

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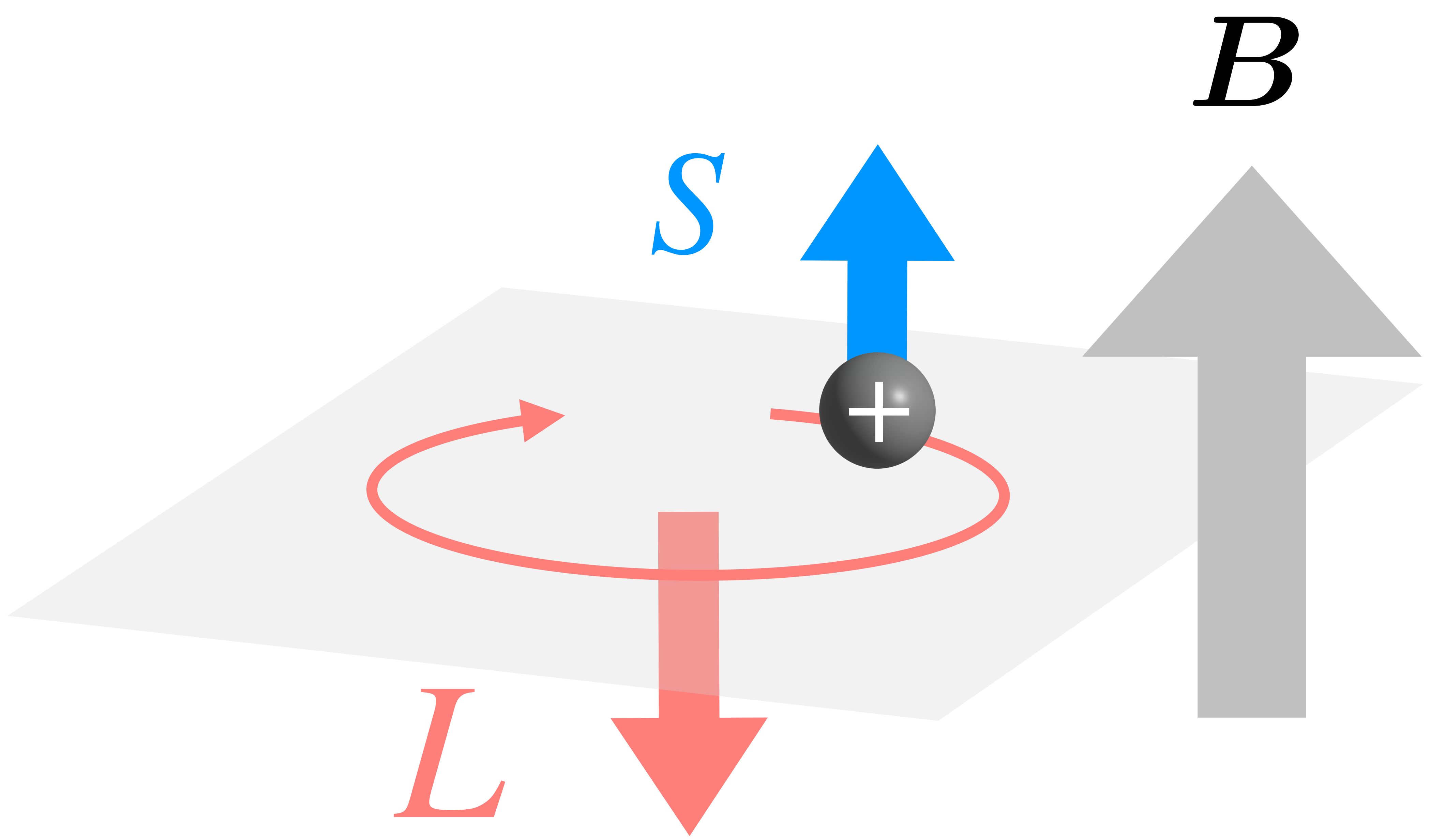

$$J = S + L < 0 \quad \text{for} \quad B \to \infty$$

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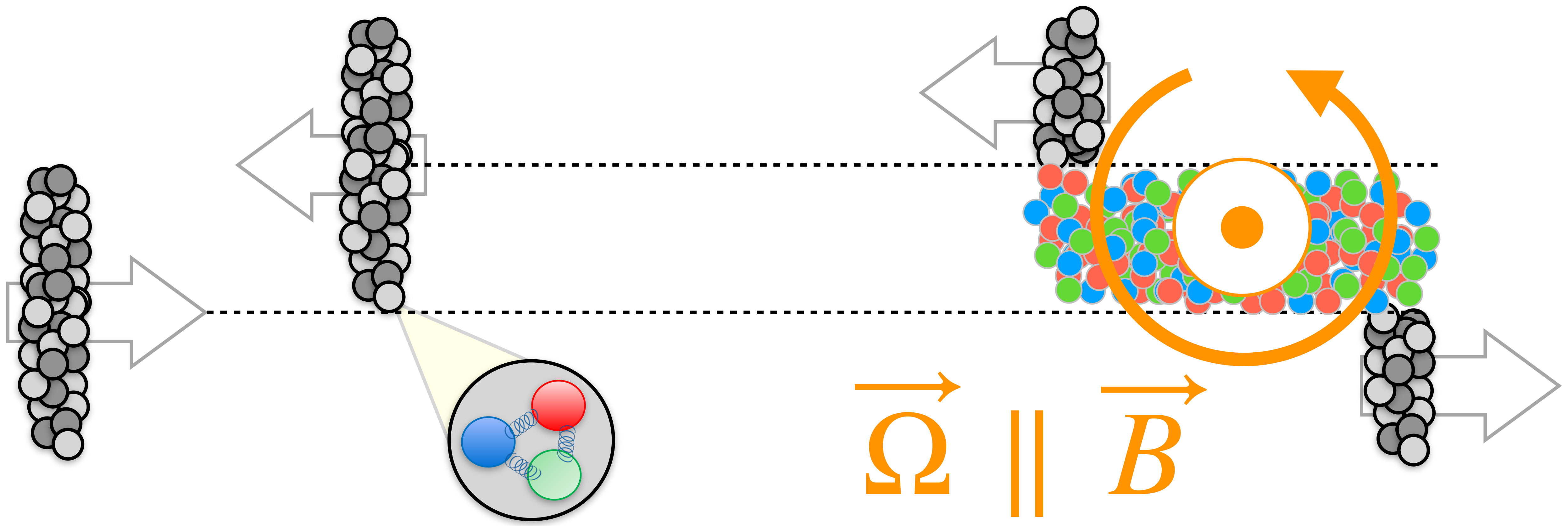
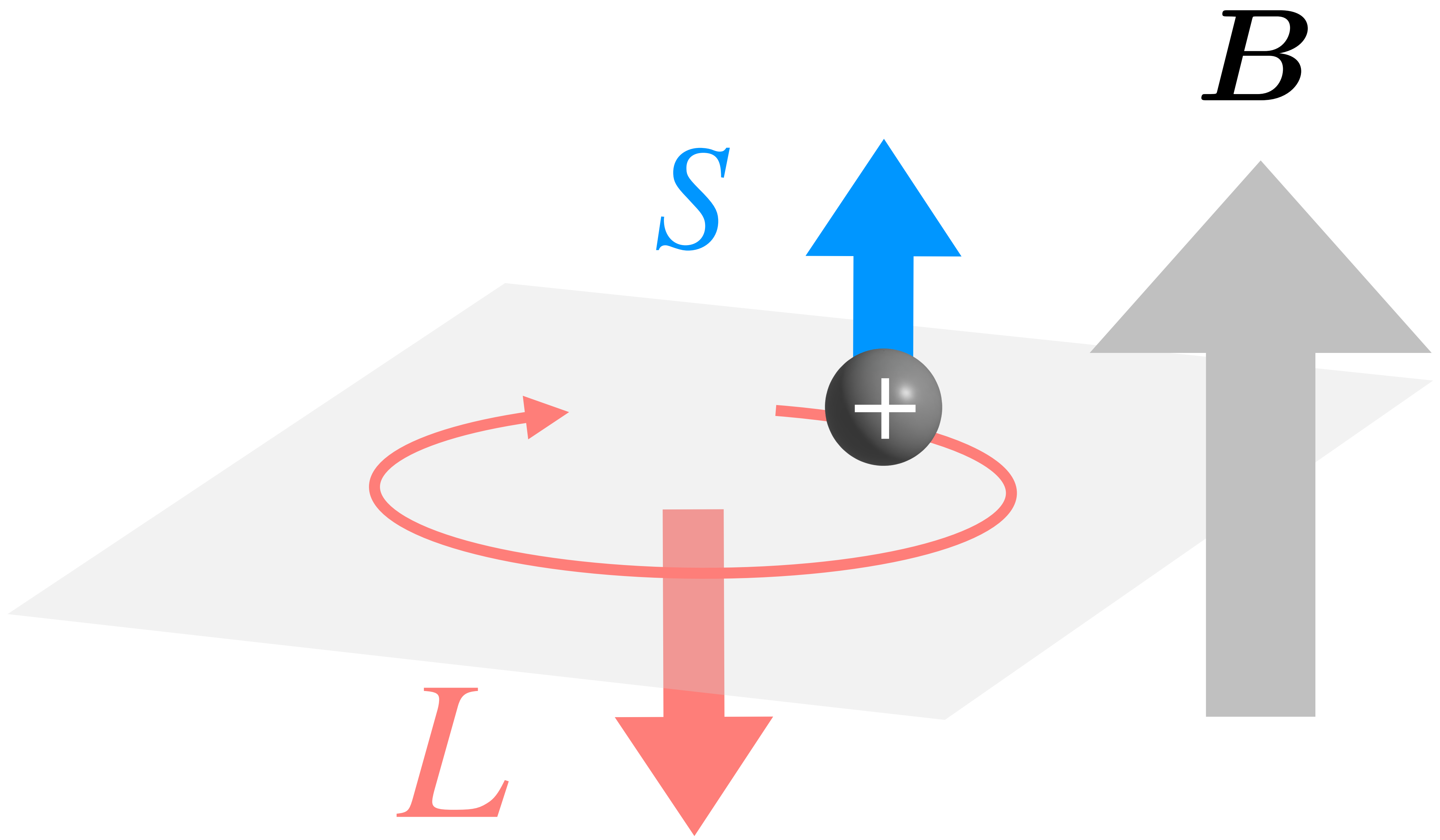




$$J = S + L < 0 \quad \text{for} \quad B \to \infty$$

thermodynamics of magneto-vortical matter

Partition Function

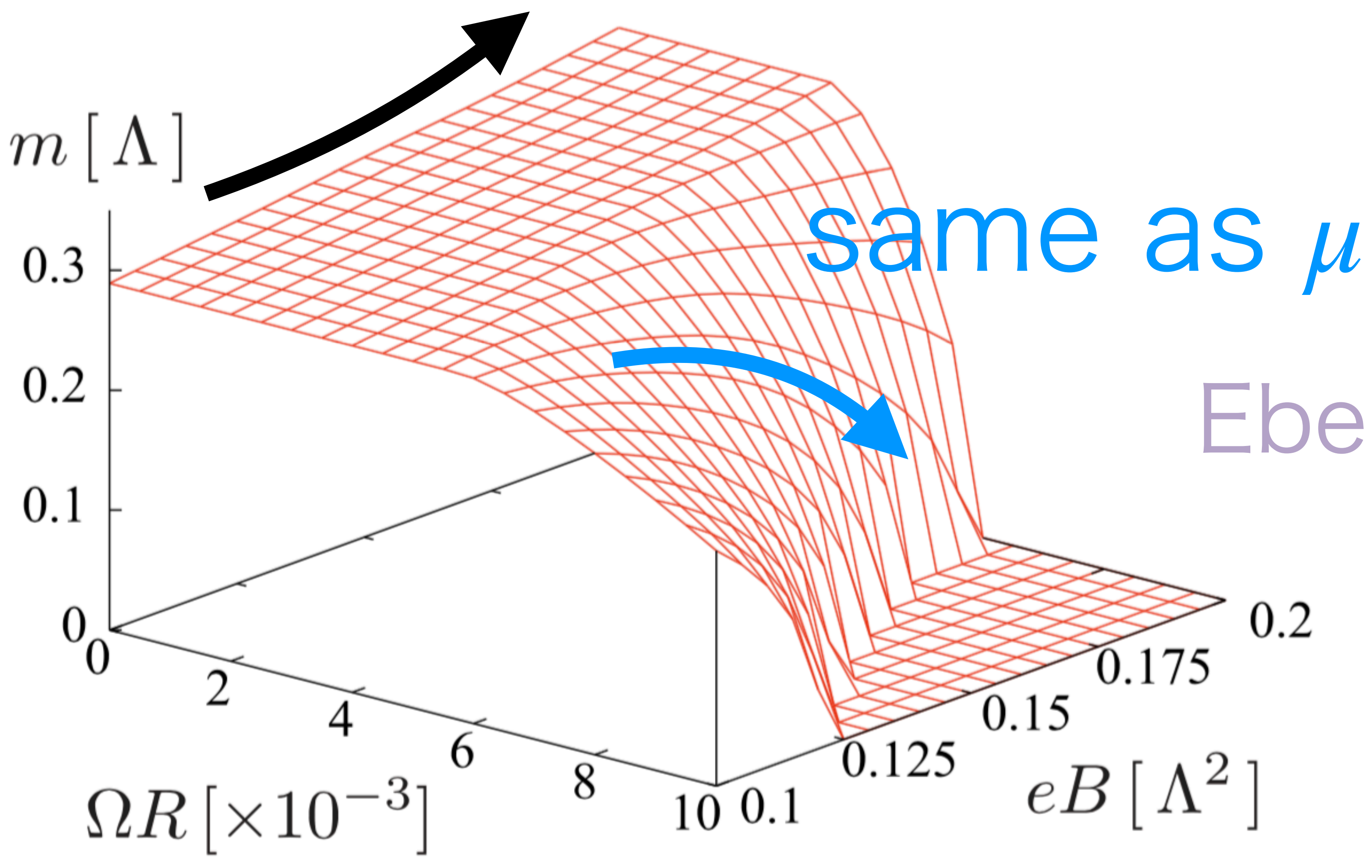
Landau-Lifshitz (1958)   Vilenkin (1979)

$$Z = \mathrm{tr}\left[\mathrm{e}^{-\beta(H-\Omega\mathcal{J})}\right]$$

rotation $\simeq$ finite density

Chen-Fukushima-Huang-Mameda (2016)

NJL model under $\vec{\Omega} \parallel \vec{B}$



same as μ (inverse MC)

Ebert et al. (2016)

Choice of Angular Momenta

$$Z = \mathrm{tr}\left[\mathrm{e}^{-\beta(H-\Omega\mathcal{J})}\right] = \det\left[-\mathrm{i}\gamma^i D_i + m - \gamma^0\Omega(L+S)\right]$$

Chen-Fukushima-Huang-Mameda (2016)

$$L_{\mathrm{can}} = xp_y - yp_x$$

conserved AM

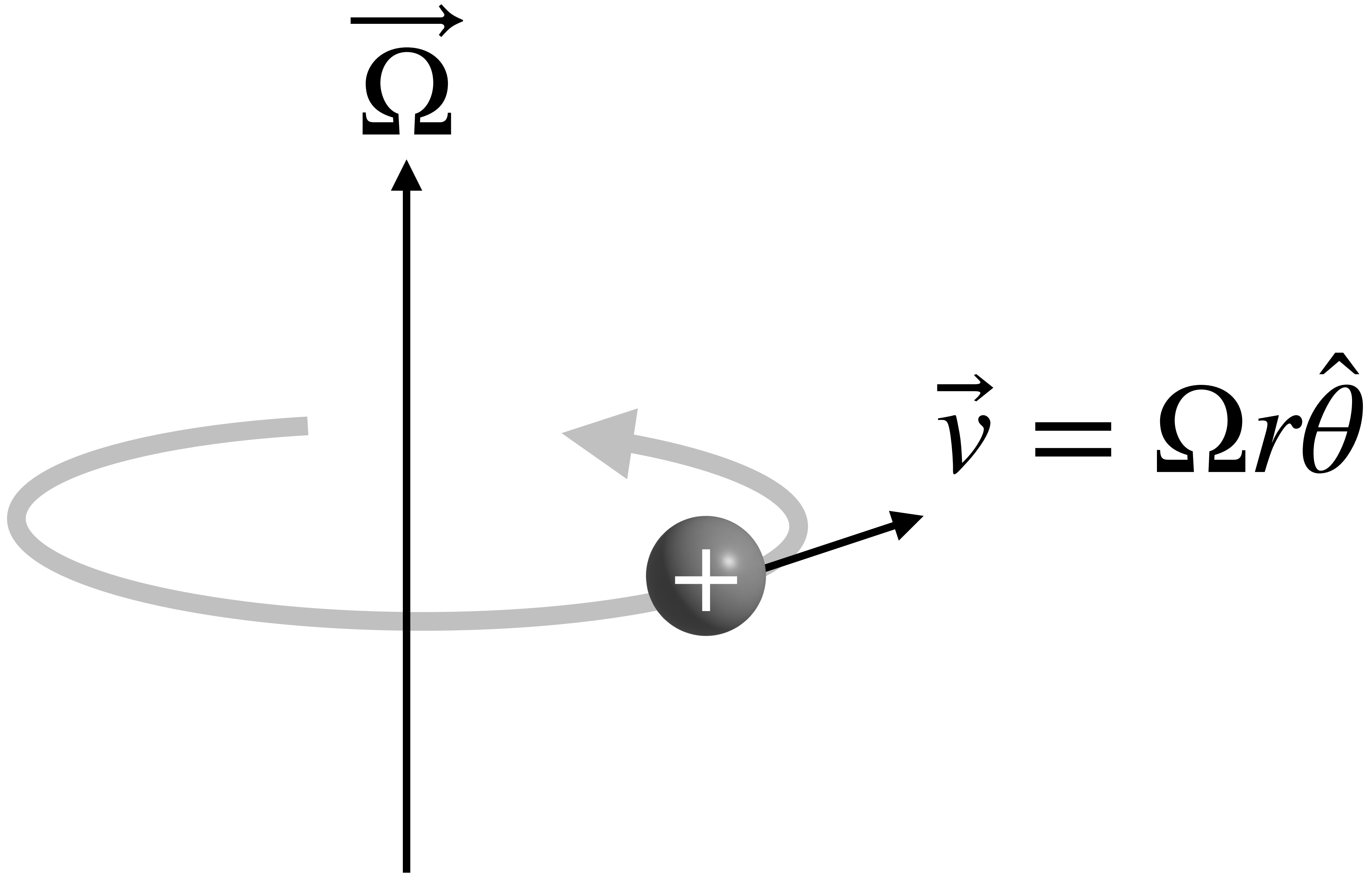
Fukushima-Hattori-Mameda (in prep.)

$$L_{\mathrm{kin}} = x\Pi_y - y\Pi_x$$

$$\Pi_i = p_i - eA_i$$

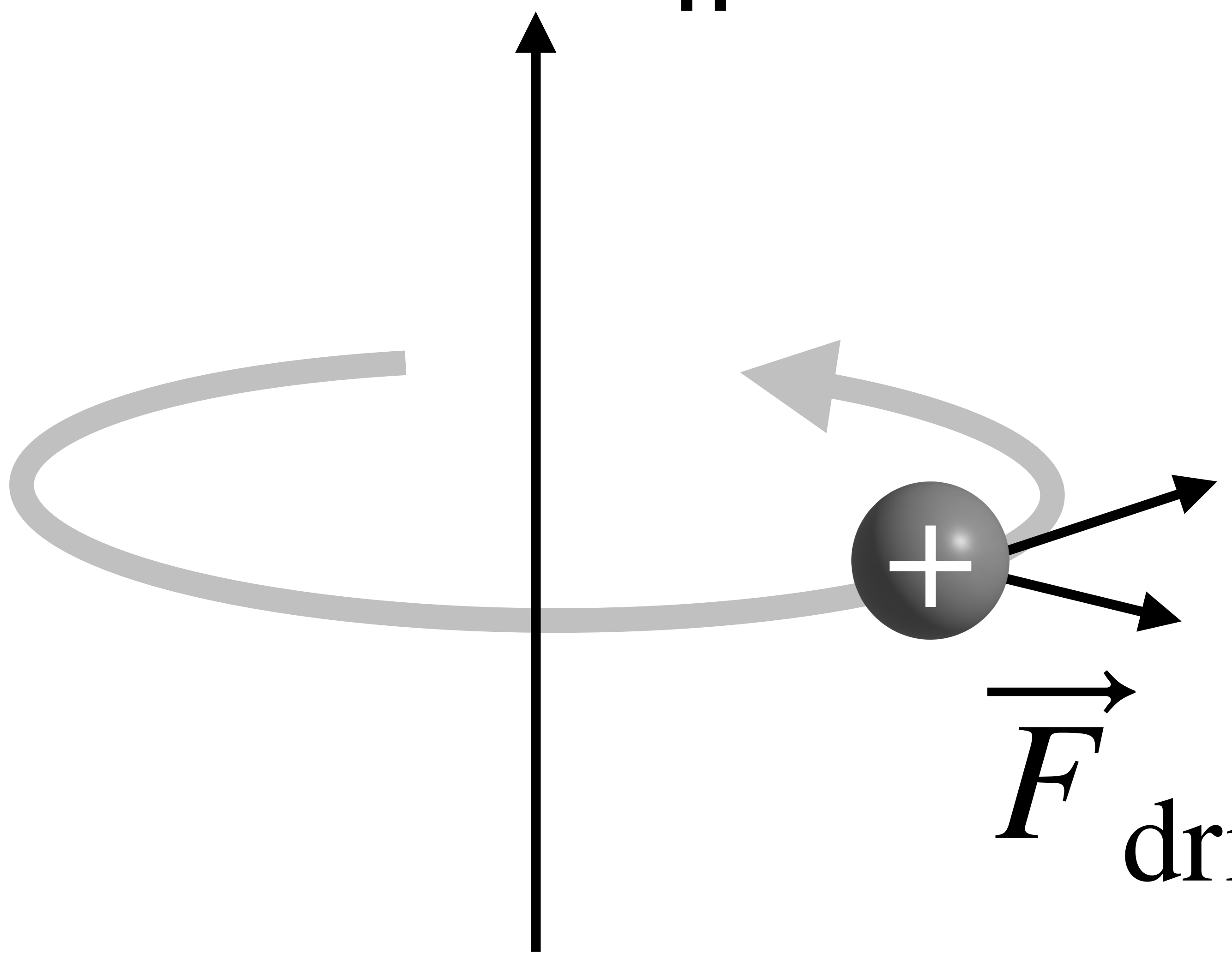
gauge invariant AM

Classical Interpretation



Classical Interpretation

$$\vec{\Omega} \parallel \vec{B}$$



$$\vec{v} = \Omega r \hat{\theta}$$

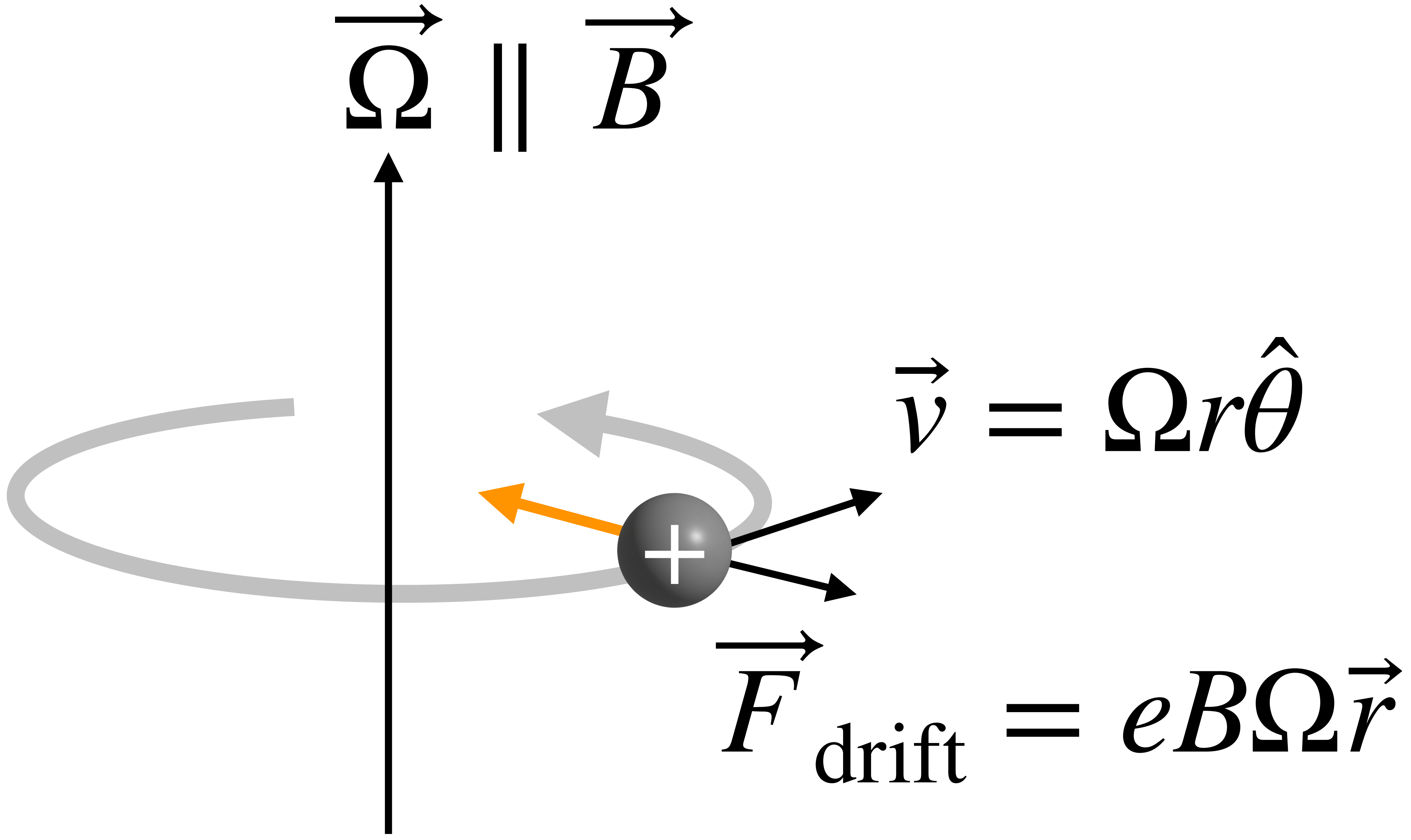
$$\vec{F}_{\text{drift}} = eB\Omega\vec{r}$$

no longer circular

$$H - \Omega L_{\text{can}}$$

unstable

Classical Interpretation



$$e\vec{E} = -eB\Omega\vec{r}$$
$$= -\vec{\nabla}[\Omega(L_{\text{can}} - L_{\text{kin}})]$$

$$H + \Omega(L_{\text{can}} - L_{\text{kin}}) - \Omega L_{\text{can}} = H - \Omega L_{\text{kin}}$$ stable

cf. Buzzegoli (2020)

gauge invariance ⟷ thermodynamic stability

Almost Solved?

$$\mathcal{J} = \int_{\boldsymbol{x}} \psi^\dagger (L + S) \psi$$

$$L = x\Pi_y - y\Pi_x$$

gauge invariant AM

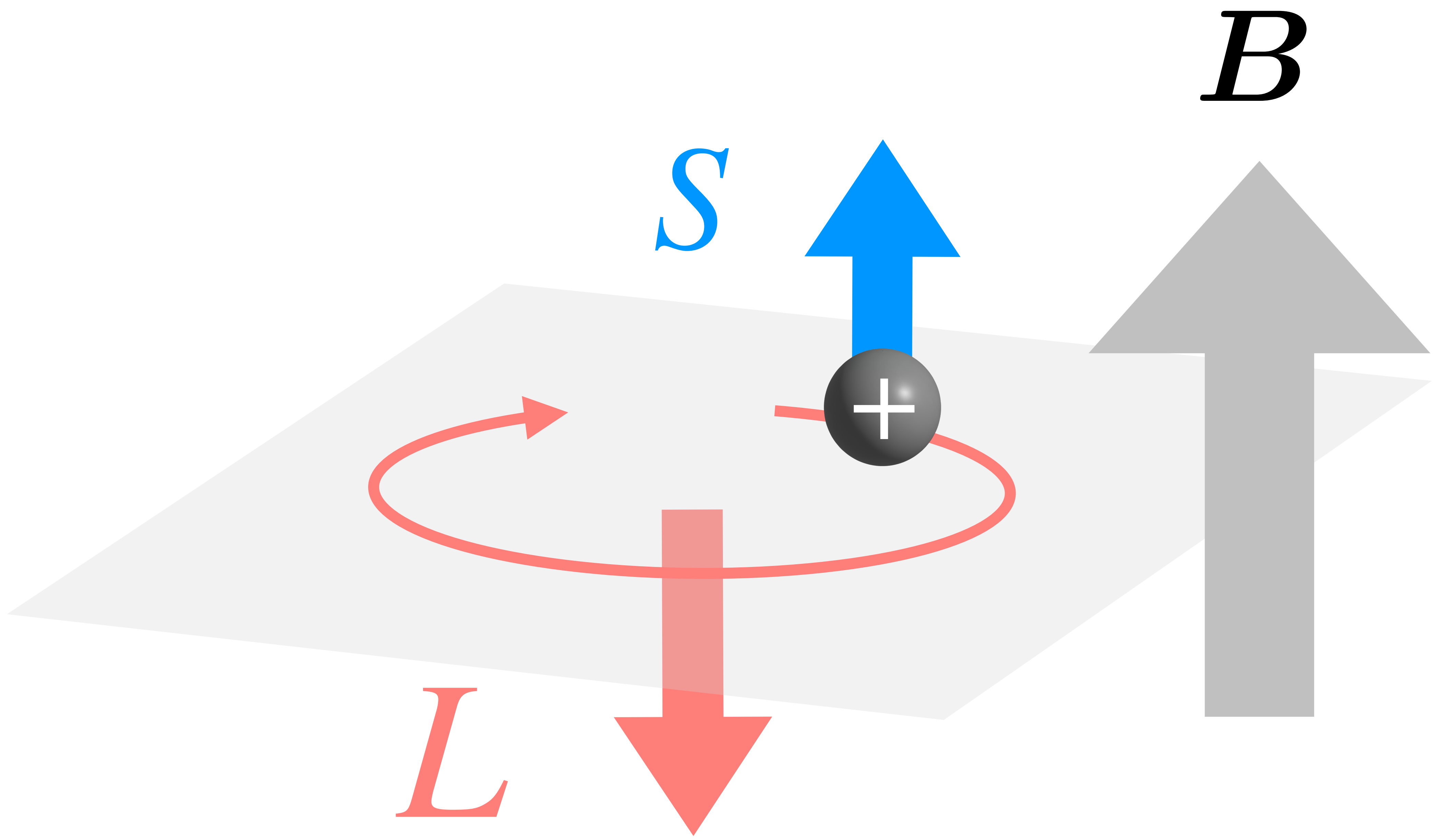
Dirac fermion under $\overrightarrow{B}$

$$Z = \det\left[-\mathrm{i}\gamma^i D_i + m - \gamma^0 \Omega (L + S)\right]$$

How to diagonalize this?

Back to Quantum Mechanics

$$L = x\Pi_y - y\Pi_x = -\left(2a^\dagger a + 1\right) + [\text{off-diagonal}]$$



$$\langle J \rangle = \langle S \rangle + \langle L \rangle$$
$$= +1/2 - (2n+1) < 0$$

Thermodynamics

Fukushima-Hattori-Mameda (in prep.)

$$Z = \det\left[-\mathrm{i}\gamma^i D_i + m - \gamma^0 \Omega(L + S)\right]$$

Not calculable analytically, except for the LLL limit

Thermodynamics

Fukushima-Hattori-Mameda (in prep.)

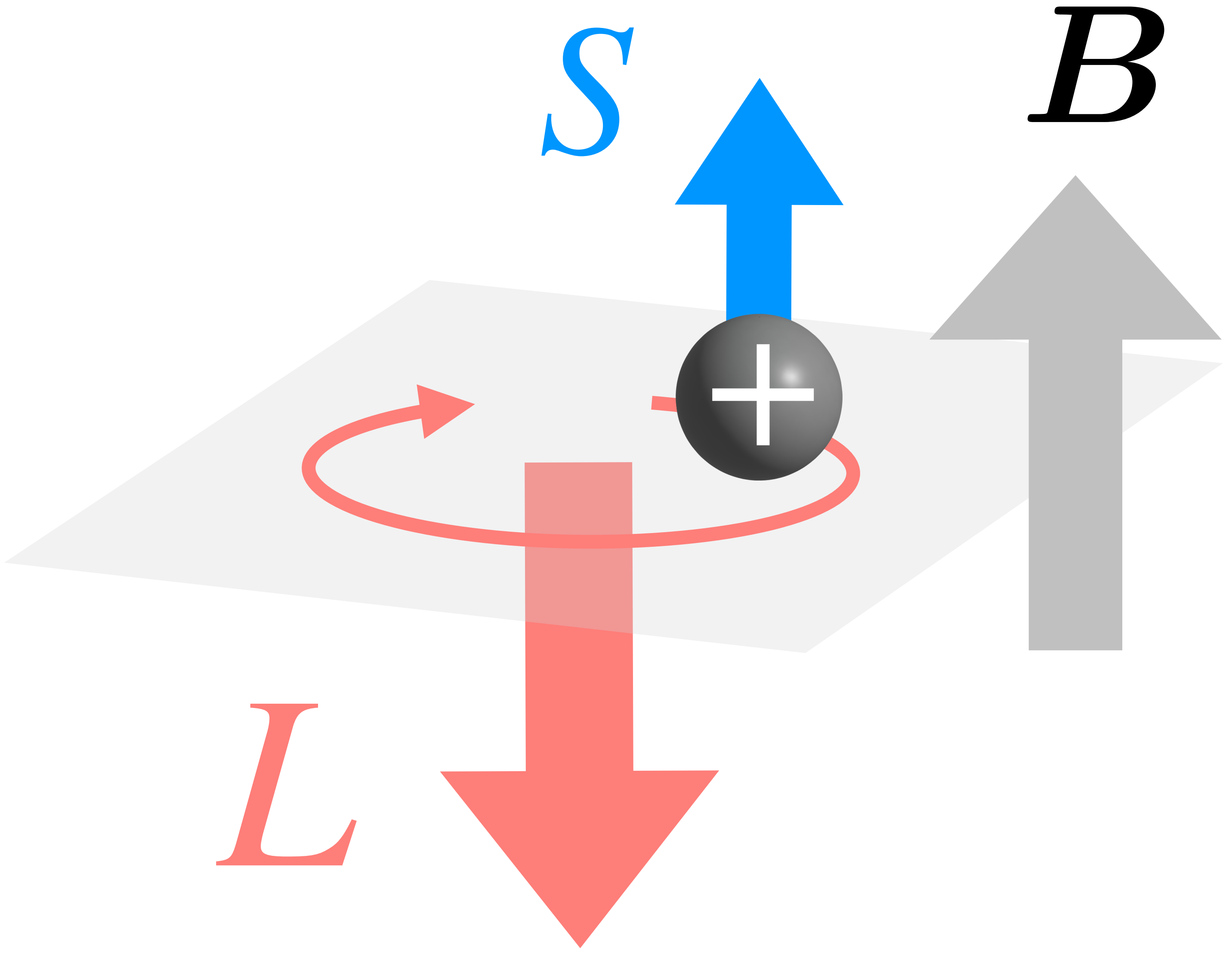
$$Z = \det\left[-\mathrm{i}\gamma^i D_i + m - \gamma^0 \underline{\Omega(L+S)}\right] \quad \nu = -\Omega/2 \text{ (LLL)}$$

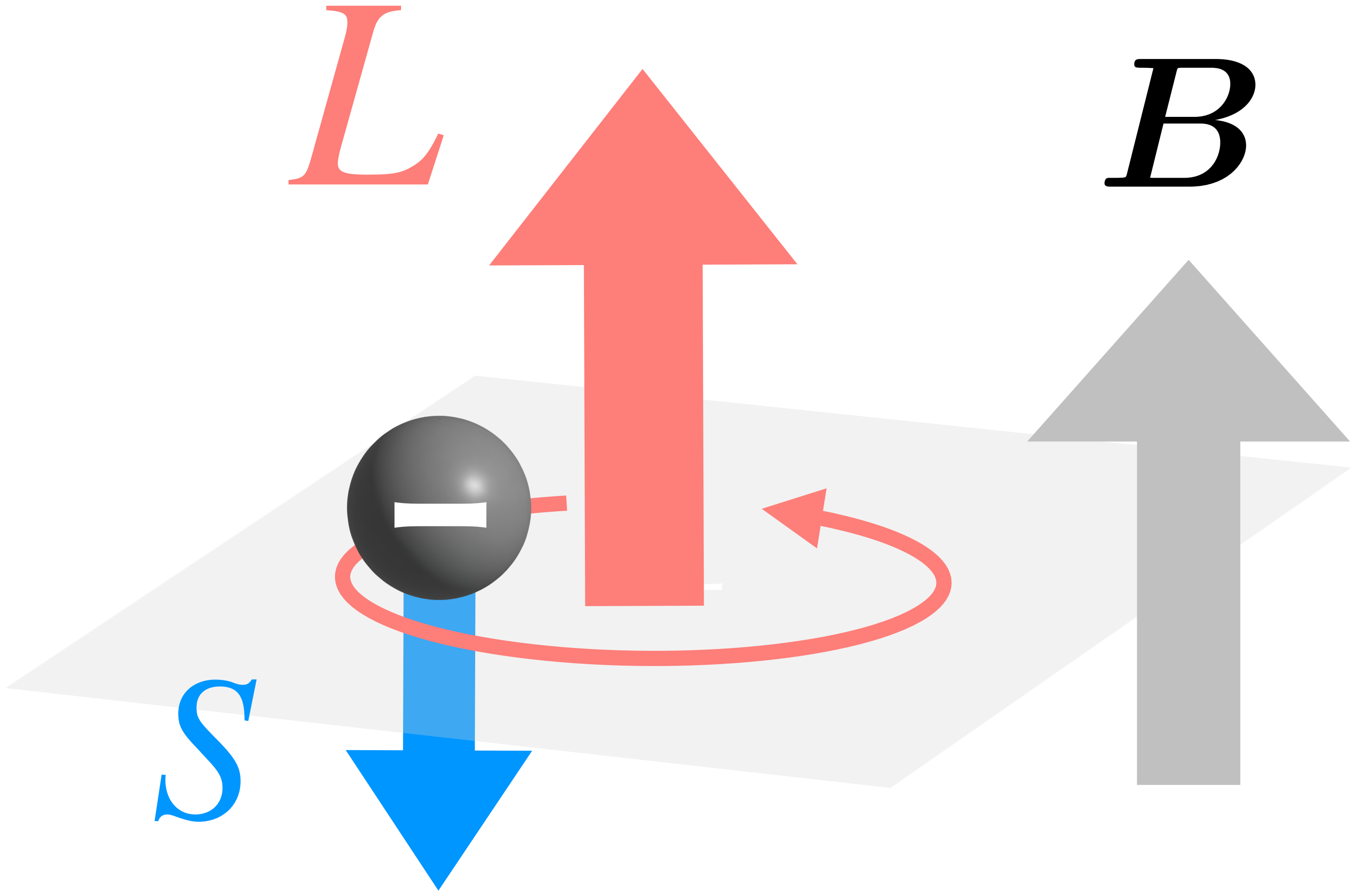
Not calculable analytically, except for the LLL limit

$$P = \frac{eB}{2\pi}\int \frac{\mathrm{d}p_z}{2\pi}\left[\epsilon + T\sum_{\eta=\pm}\ln\left(1+\mathrm{e}^{-\beta(\epsilon-\eta\nu)}\right)\right]$$

massless limit $\rho = \frac{\partial P_{\mathrm{LLL}}}{\partial \nu} = -\frac{eB\Omega}{4\pi^2}$ (T-independent)

It Should Be Negative

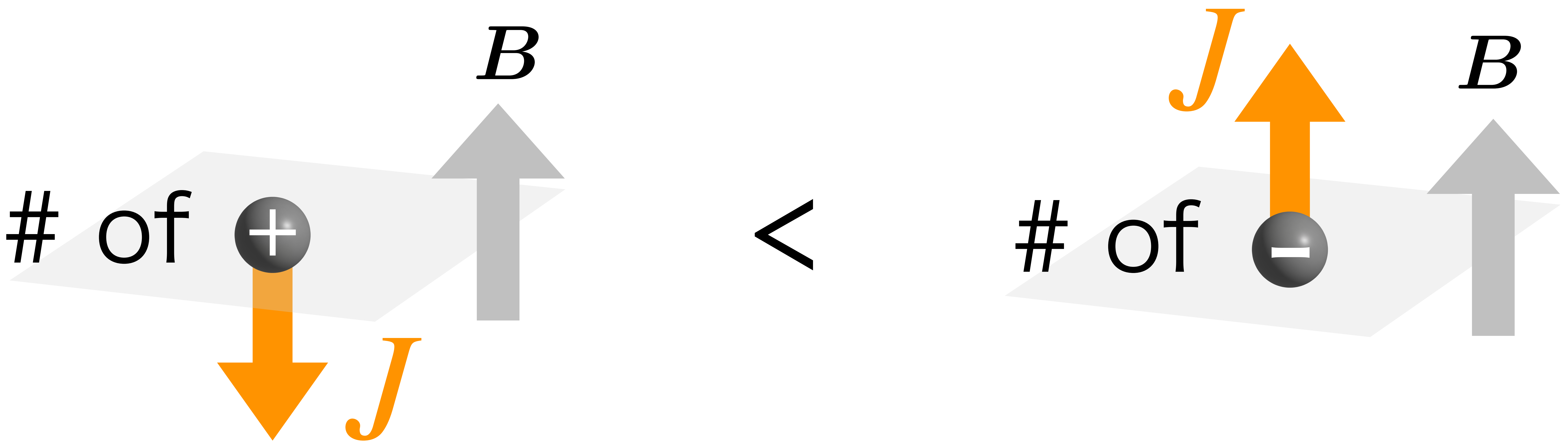




It Should Be Negative

vorticity coupling $E = E_0 - \Omega J$

It Should Be Negative



vorticity coupling $E = E_0 - \Omega J$

Comparisons

Fukushima-Hattori-Mameda (in prep.)
partition function (LLL)

$$\rho = \frac{eB\Omega}{4\pi^2} - \frac{eB\Omega}{2\pi^2}$$

spin orbital

Ebihara-Fukushima-Mameda (2017)
partition function (LLL)
incorrect

$$\rho = \frac{eB\Omega}{4\pi^2} + (\text{divergence w.r.t. AM})$$

due to $\vec{F}_{\text{drift}} = eB\Omega\vec{r}$

Hattori-Yin (2016)
linear responce (LLL)
incorrect

$$\rho = \frac{eB\Omega}{4\pi^2}$$

wrong calculation

Yang et. al (2020) Mameda(2023)
chiral kinetic theory
correct

$$\rho = \frac{eB\Omega}{4\pi^2}$$

no Landau level formed by weak B

Relation to Chiral Anomaly

charge
$$\rho = \frac{\partial P_{\mathrm{LLL}}}{\partial \mu} = \frac{eB\Omega}{4\pi^2} - \frac{eB\Omega}{2\pi^2} + \frac{eB\mu}{2\pi^2}$$

spin orbital

angular momentum
$$J = \frac{\partial P_{\mathrm{LLL}}}{\partial \Omega} = \frac{eB\mu}{4\pi^2} - \frac{eB\mu}{2\pi^2} + \frac{eB\Omega}{8\pi^2}$$

$$\frac{\partial \rho}{\partial \Omega} = \frac{\partial J}{\partial \mu} = \frac{\partial^2 P_{\mathrm{LLL}}}{\partial \mu\, \partial \Omega}$$

→ same coefficients shared

$$\frac{eB}{4\pi^2} \qquad -\frac{eB}{2\pi^2}$$

Relation to Chiral Anomaly

charge

$$\rho = \frac{\partial P_{\mathrm{LLL}}}{\partial \mu} = \underline{\frac{eB\Omega}{4\pi^2}} \underline{- \frac{eB\Omega}{2\pi^2} + \frac{eB\mu}{2\pi^2}}$$

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angular momentum

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$$= S = j^5_{\mathrm{CSE}}/2$$

$$\frac{\partial \rho}{\partial \Omega} = \frac{\partial J}{\partial \mu} = \frac{\partial^2 P_{\mathrm{LLL}}}{\partial \mu \, \partial \Omega}$$

→ Since j^5_{CSE} is anomaly-related, so is ρ

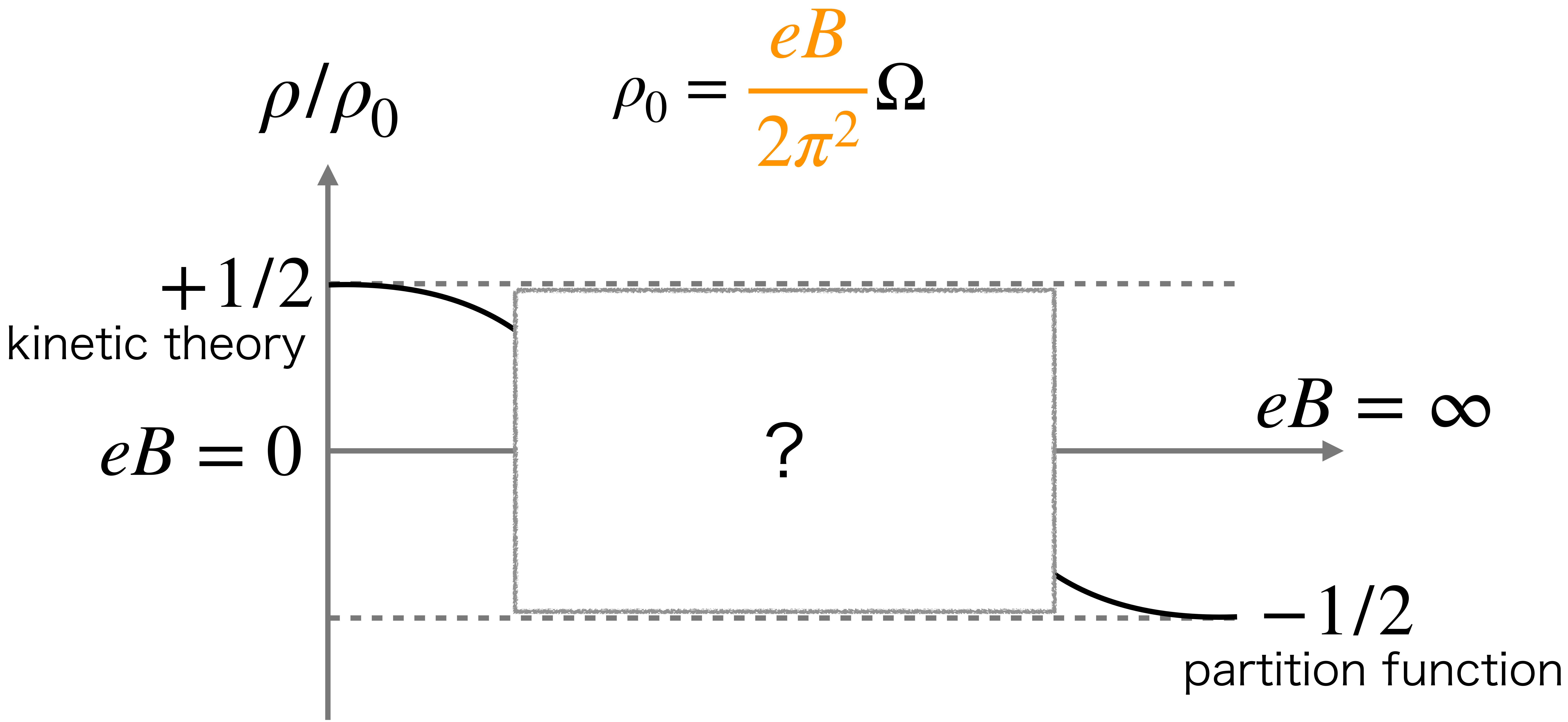
cf. Yang-Yamamoto (2021)

Summary

- ✓ reformulate gauge-invariant and stable thermodynamics
- ✓ Magnetovortical charge sign-inverted by cyclotron motion
- ✓ The charge is anomaly-related
- ✓ applicability to

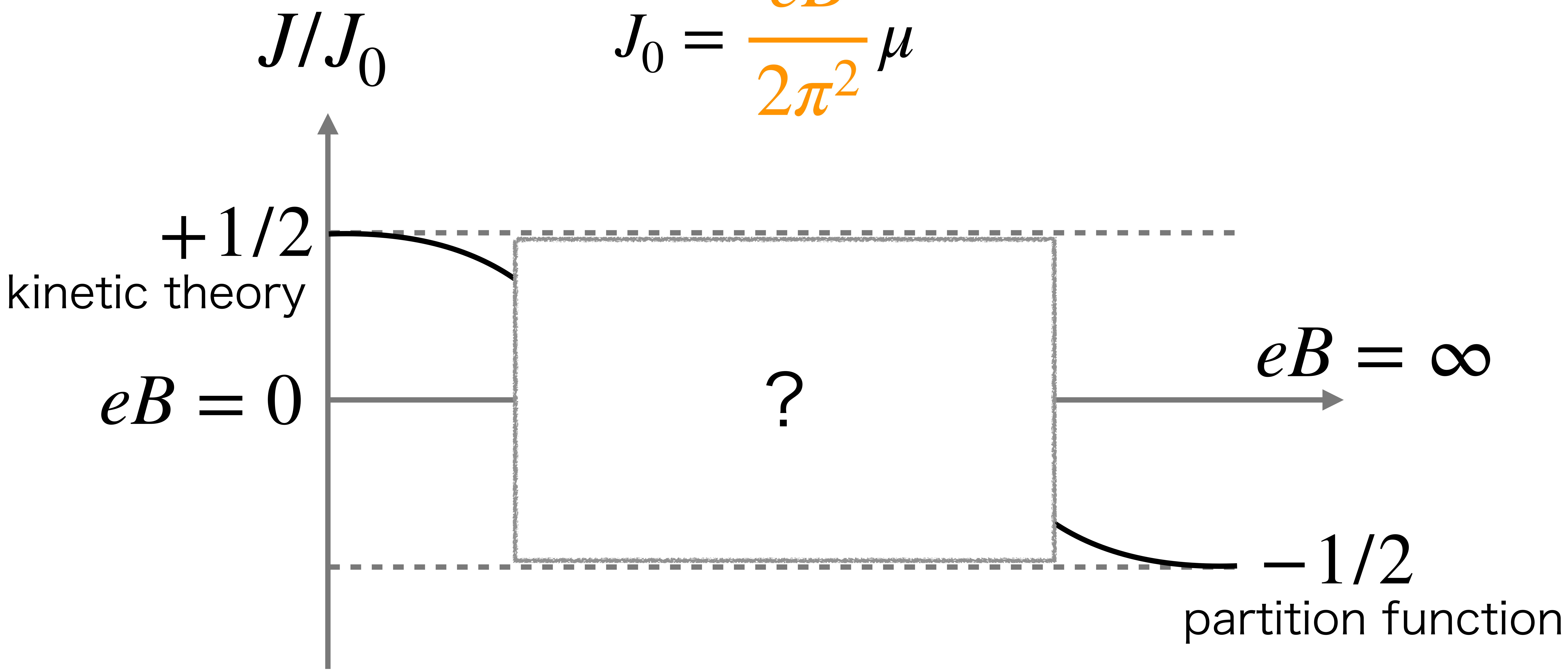
  HIC : spin polarization under strong B
  cold atoms : quantum simulator
  (nonrelativistic Hamiltonian can be diagonalized)

Charge Density



Total Angular Momentum

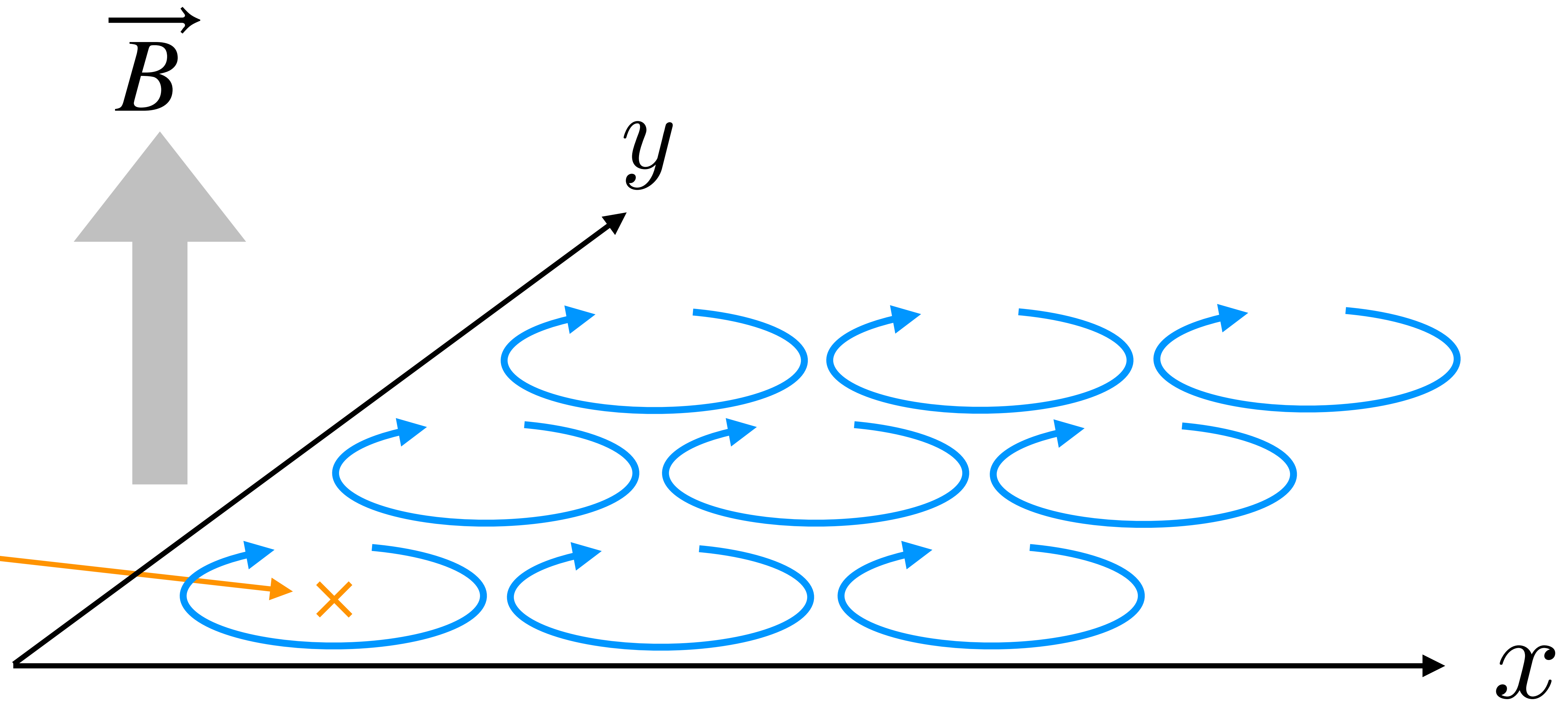
$$J_0 = \frac{eB}{2\pi^2}\mu$$



Landau Level Basis

kinetic momentum $\vec{\Pi} = \vec{p} - e\vec{A}$

guiding center $\vec{X}$



$$a = \frac{1}{\sqrt{2eB}}(\Pi_x + \mathrm{i}\Pi_y) \qquad b = \sqrt{\frac{eB}{2}}(X - \mathrm{i}Y)$$

Landau level basis $|\, n\, , m\, \rangle \propto (a^\dagger)^n\, (b^\dagger)^m\, |\, 0{,}0 \rangle$

kinetic energy

$$\vec{\Pi}^2 = eB(2a^\dagger a + 1)$$

distance from origin

$$\vec{X}^2 = (2b^\dagger b + 1)/eB$$