Constraining the strength of the vector interaction

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Outline:

- Motivation: why is the vector interaction (G_V/g_V) important?
- Major goal of the exercise
- The formalism of our approach, namely the NJL model
- Results
- Conclusion

Motivation

Why is the vector interaction (G_V/g_V) important?

Importance of vector interaction

1. In a dense environment, it becomes necessary to consider a non-zero vector interaction due to its direct coupling with the number density operator.

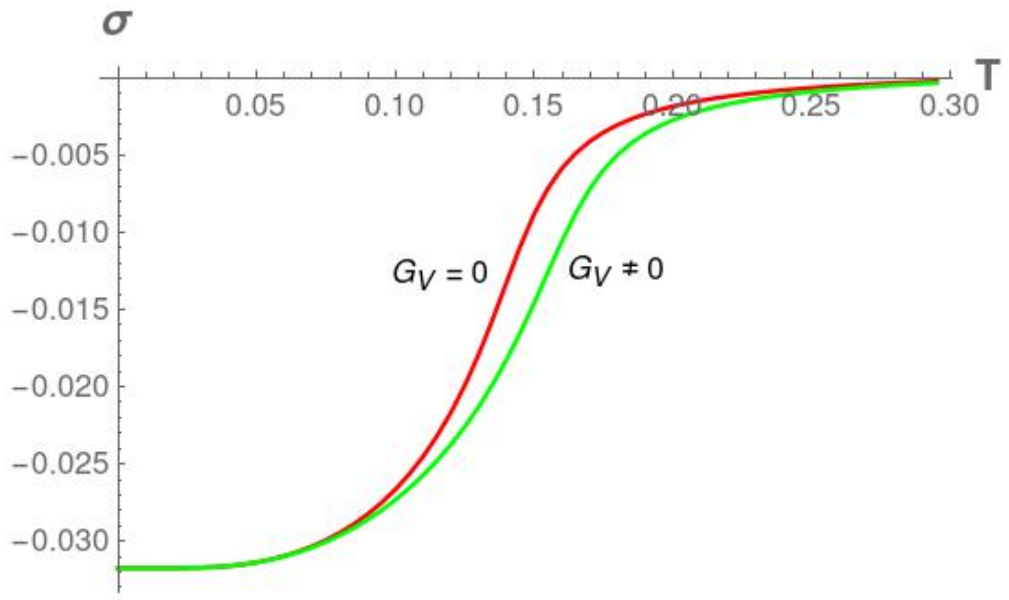
→ A typtical example: $(\bar{q}\gamma^{\mu}q)^2$

→ The zeroth component corresponds to the density operator: $(q_i^{\dagger}q_i)^2$

→ The non-zero density environment induces a non-zero vector interaction.

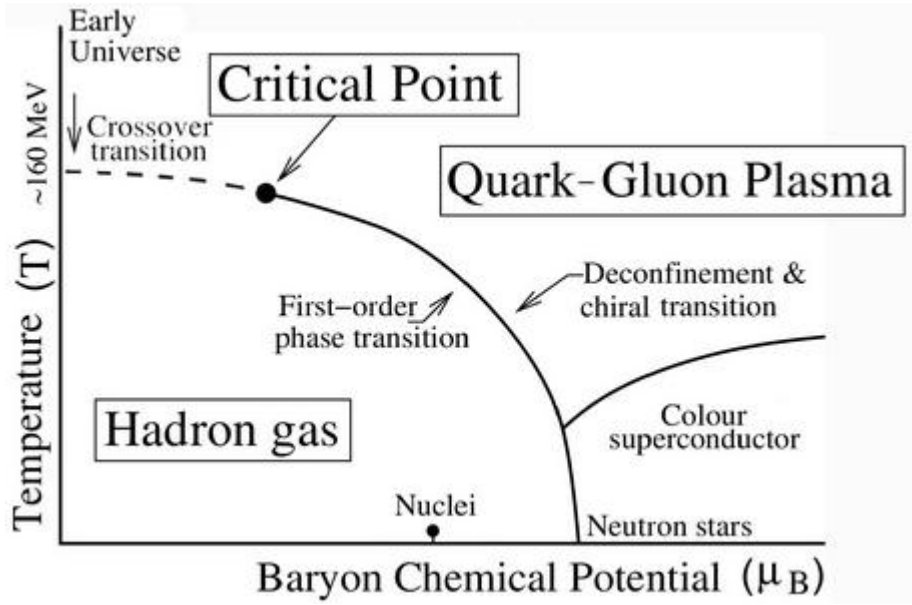
Importance of vector interaction

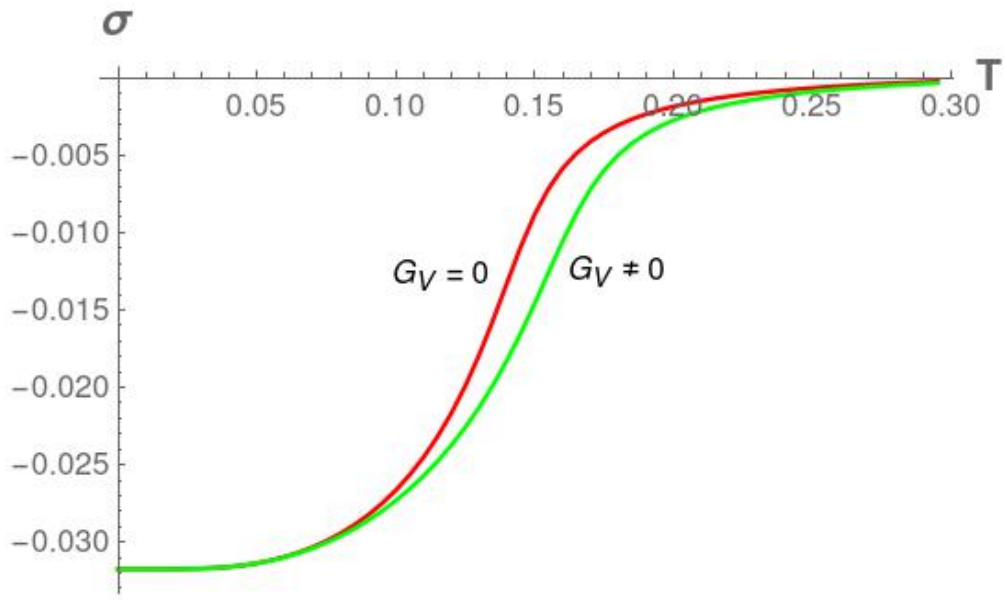
2. It affects the restoration of the chiral symmetry at non-zero μ_B:



Importance of vector interaction

3. It directly influences the location of the critical endpoint (CEP) and the curvature of the chiral crossover line at zero μ_B:





4. It can also impact the baryon numberdensity and the pressure of the system and thus plays an important role in obtaining acceptable equation of state (EOS) for astrophysical objects like neutron stars.

Strenght of the vector interaction

- The strenght of the vector interaction (G_V/g_V) is not a well-known quantity:

  Induced by the medium and thus cannot be fixed using vector meson properties in the vacuum. This is in contrast to the scalar interaction.

  ➡ It strength is varied in units of the scalar interaction.

- In fact, its sign is not universally agreed upon: either positive or negative

  ➡ Implications

Existing knowledge on vector interaction

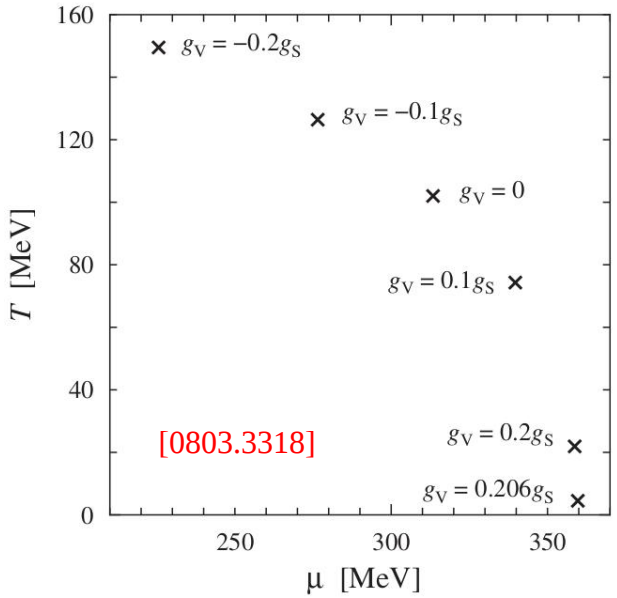
- Multiple efforts have been put in the past to understand the vector interaction and its implications for QCD.

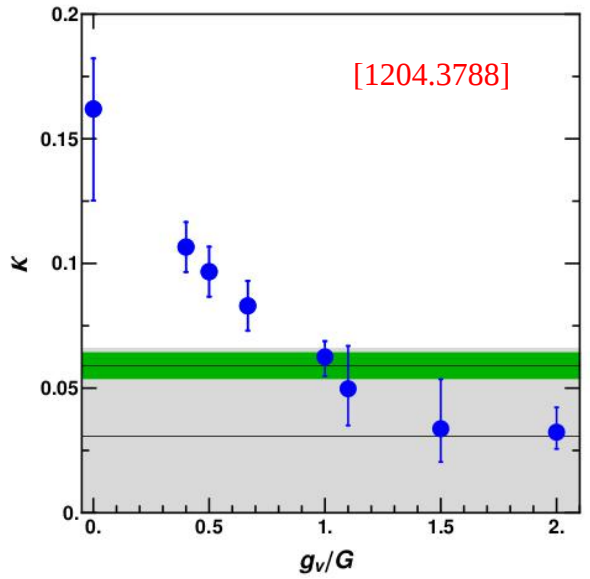
➡ Either by playing with the strength of the interaction parameter

[K. Fukushima, arXiv:0803.3318]

➡ Or by comparing the curvature calculated by LQCD studies

[N. M. Bratovic et al., arXiv:1204.3788]





- Other relevant refs:

[G. A. Contrera et al., arXiv:1207.4890]
[J. Steinheimer et al., arXiv:1401.4051]
[A. V. Friesen et al., arXiv:1412.6872]

Goal

What we want to achieve?

How do we constrain the vector interaction?

- Both CEP and curvatures are affected by the vector interaction, as already discussed.

- We choose curvature (**not CEP!**) to decide on its strength, which has also been previously exploited.

- More precise and controlled lattice data provides narrower bound.

- In addition, we investigate the effects of flavour independent and dependent vector interactions. Also the effect of strangeness neutrality. **Note:** We assume no restrictions on the sign of the vector interaction.

- As a model prediction we estimate the behaviour of the curvature as a function of strangeness chemical potential (if I get time).

Formalism

Which method do we use to serve our purpose?

Formalism

- We work with a 2+1 flavour NJL Lagrangian,

$$\mathcal{L} = \bar{q}\left(i\partial\!\!\!/ - \hat{m} + \gamma^0\hat{\mu}\right)q + G_S\sum_{a=0}^{8}\left[(\bar{q}\lambda_a q)^2 + (\bar{q}i\gamma^5\lambda_a q)^2\right]$$
$$- 8K\left[\det(\bar{q}P_R q) + \det(\bar{q}P_L q)\right]$$
$$- \begin{cases} G_V(\bar{q}\gamma^\mu q)^2 & \longrightarrow \text{Model-I} \\ g_V\sum_{a=0}^{8}\left[(\bar{q}\gamma^\mu\lambda_a q)^2 + (\bar{q}i\gamma^\mu\gamma^5\lambda_a q)^2\right] & \longrightarrow \text{Model-II} \end{cases}$$

where, $q^{\mathrm{T}} = (u, d, s)$, $\hat{m} = \mathrm{diag}(m_u, m_d, m_s)$ and $\hat{\mu} = \mathrm{diag}(\mu_u, \mu_d, \mu_s)$

- Model-I: flavour independent vector interaction
- Model-II: flavour dependent vector interaction

Thermodynamic potential

- The thermodynamic potential is $\Omega(T,\mu) = \Omega_{\rm cond} + \Omega_{\rm vac} + \Omega_{\rm med}$

where, $\Omega_{\rm cond} = 2G_S \sum_i \sigma_i^2 - 4K \prod_i \sigma_i - \begin{cases} G_V \left(\sum_i n_i\right)^2 & \longrightarrow \text{Model-I} \\ g_V \sum_i n_i^2 & \longrightarrow \text{Model-II} \end{cases}$,

[K. Masuda et al., arXiv:1212.6803]
[K. Fukushima, arXiv:0803.3318]

$$\Omega_{\rm vac} = -2N_c \sum_i \int_\Lambda \frac{d^3p}{(2\pi)^3} E_i(p) \quad \text{and}$$

$$\Omega_{\rm med} = -2N_c T \sum_i \int_0^\infty \frac{\mathrm{d}^3p}{(2\pi)^3} \left(\ln\left[1 + e^{-(E_i(p)+\tilde{\mu}_i)/T}\right] + \ln\left[1 + e^{-(E_i(p)-\tilde{\mu}_i)/T}\right]\right).$$

with $E_i = \sqrt{p^2 + M_i^2}$, $\sigma_i = \langle \bar{q}_i q_i \rangle$ is the quark condensate and $n_i = \langle q_i^\dagger q_i \rangle$ is the number density.

$$M_i = m_i - 4G_S\sigma_i - 2K\sigma_j\sigma_k$$

Effective mass

$$\tilde{\mu}_i = \begin{cases} \mu_i - 2G_V \sum_j n_j & \longrightarrow \text{Model-I (mixing)} \\ \mu_i - 2g_V n_i & \longrightarrow \text{Model-II} \end{cases}$$

Effective chemical potential

Gap equatons and parameters

- We solve the gap equations:

$$\frac{\partial\Omega}{\partial\sigma_u} = \frac{\partial\Omega}{\partial\sigma_d} = \frac{\partial\Omega}{\partial\sigma_s} = 0$$

- The chemical potentials of the system:

$$\mu_u = \mu_d = \frac{1}{3}\mu_B,$$
$$\mu_s = \frac{1}{3}\mu_B - \mu_S.$$

- The parameters of the model:

	Λ (MeV)	$G_S\Lambda^2$	$K\Lambda^5$	m_l(MeV)	m_s(MeV)
Set I	631.4	1.835	9.29	5.5	135.7
Set II	602.3	1.835	12.36	5.5	140.7

How to calculate the curvature coefficients?

For small chemical potential (μ_X), the pseudocritical line can be Taylor expanded at the lowest order in μ_X^2, where one defines the line with the following ansatz:

$$\frac{T_{pc}(\mu_X)}{T_{pc}(0)} = 1 - \kappa_2^X \left(\frac{\mu_X}{T_{pc}(0)}\right)^2 - \kappa_4^X \left(\frac{\mu_X}{T_{pc}(0)}\right)^4 .$$

Here, μ_X corresponds to chemical potential associated with various charges like baryon charge B, electric charge Q, and strangeness S.

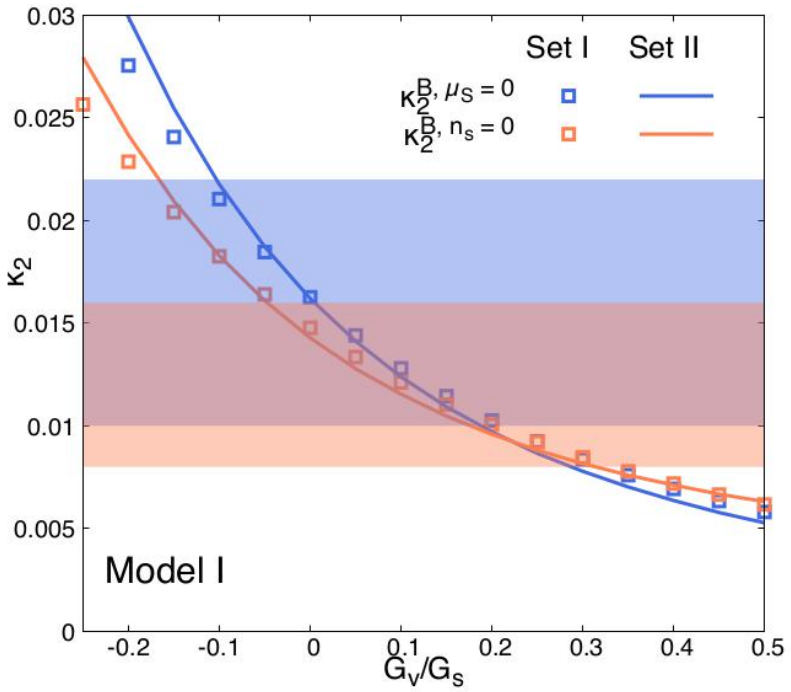
➡ Methods utilised: Taylor expansion/imaginary chem pot
They agree with each other.

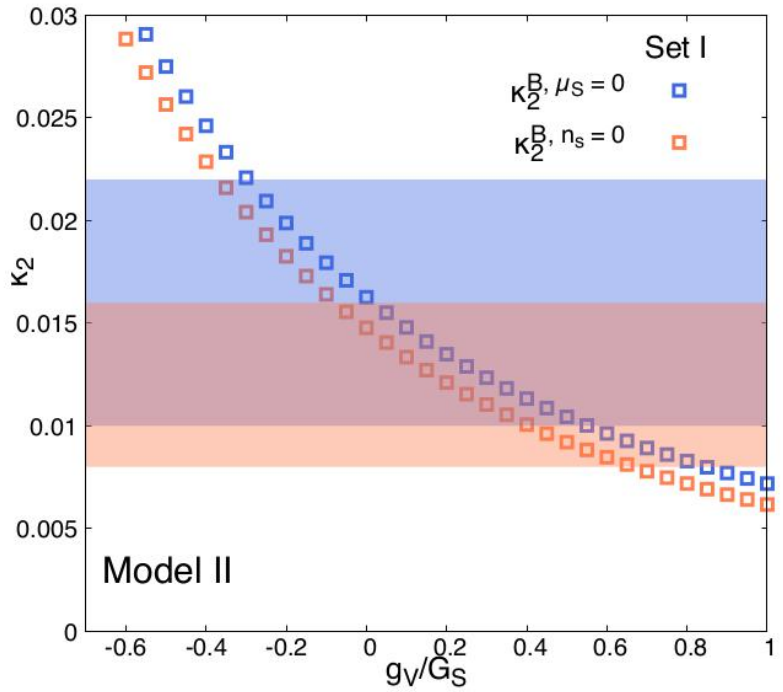
Relevant LQCD refs:

[R. Bellwied et al., arXiv:1507.07510]
[C. Bonati et al., arXiv:1805.02960]
[A. Bazavov et al., arXiv:1812.08235]

Results

Curvature comparison (k_2)





	Λ (MeV)	$G_S\Lambda^2$	$K\Lambda^5$	m_l(MeV)	m_s(MeV)
Set I	631.4	1.835	9.29	5.5	135.7
Set II	602.3	1.835	12.36	5.5	140.7

- Constraining vector interaction:

LQCD data:

[A. Bazavov et al., PLB 795, 15 (2019)]

$$\frac{T_{pc}(\mu_X)}{T_{pc}(0)} = 1 - \kappa_2^X \left(\frac{\mu_X}{T_{pc}(0)}\right)^2 - \kappa_4^X \left(\frac{\mu_X}{T_{pc}(0)}\right)^4$$

$$\mu_{B,S}/T_{pc}(0) \leq 1.0$$

$G_V \left(\sum_i n_i\right)^2 \rightarrow$ Model-I

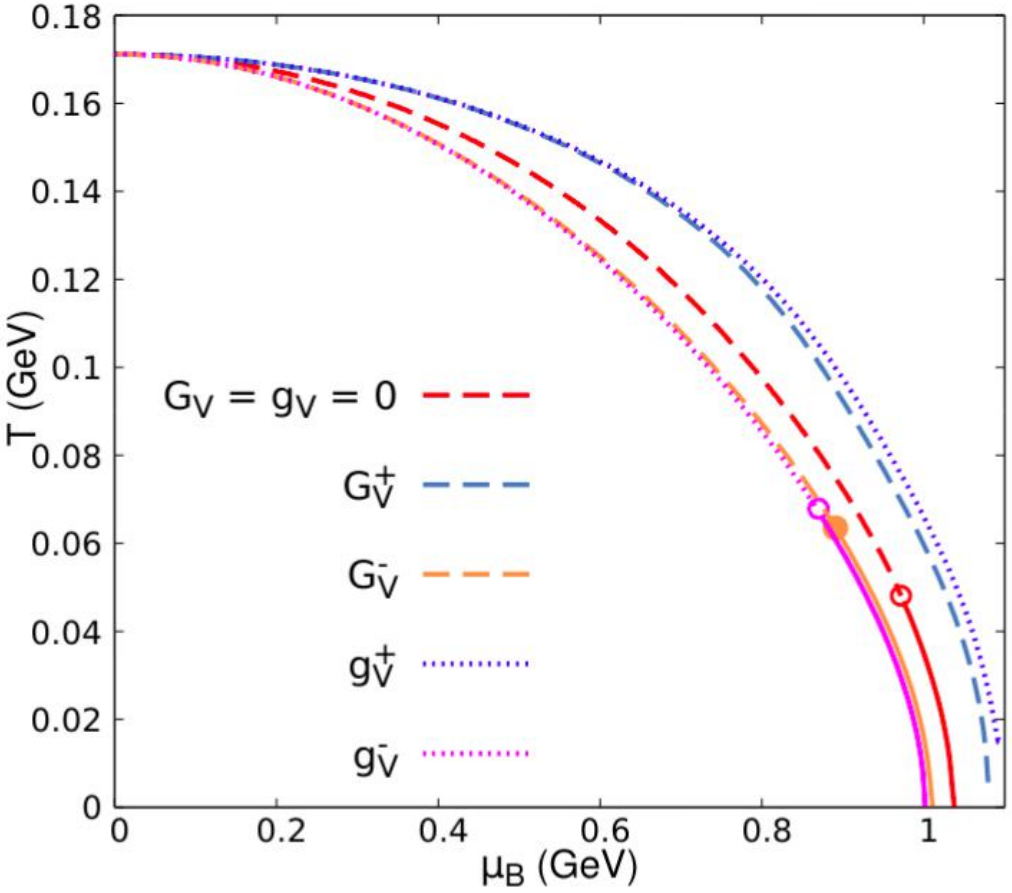
$g_V \sum_i n_i^2 \rightarrow$ Model-II

$$\tilde{\mu}_i = \begin{cases} \mu_i - 2G_V \sum_j n_j \rightarrow \text{Model-I (mixing)} \\ \mu_i - 2g_V n_i \rightarrow \text{Model-II} \end{cases}$$

	$\kappa_2^B\ (\mu_S = 0)$	$\kappa_2^S(\mu_B = 0)$	$\kappa_2^{B,n_S=0}$
LQCD	0.016(6)	0.017(5)	0.012(4)
G_V/G_s	$[-0.116621, 0.212351]$	NA	$[-0.0382045, 0.333958]$
g_V/G_s	$[-0.296349, 0.552566]$	NA	$[-0.0765385, 0.667442]$

Range of G_V/G_S and g_V/G_S constrained from the variation of κ_2.

Phase diagram

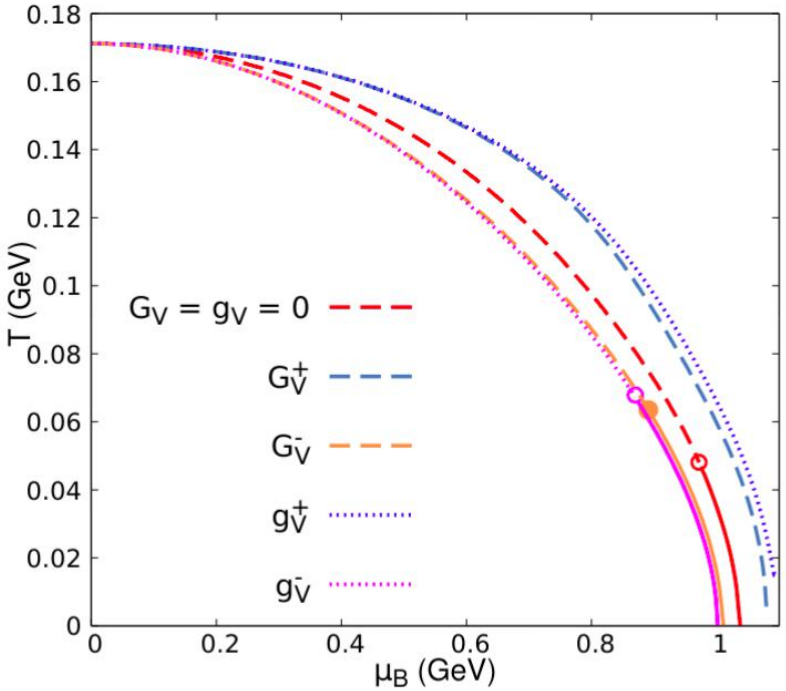


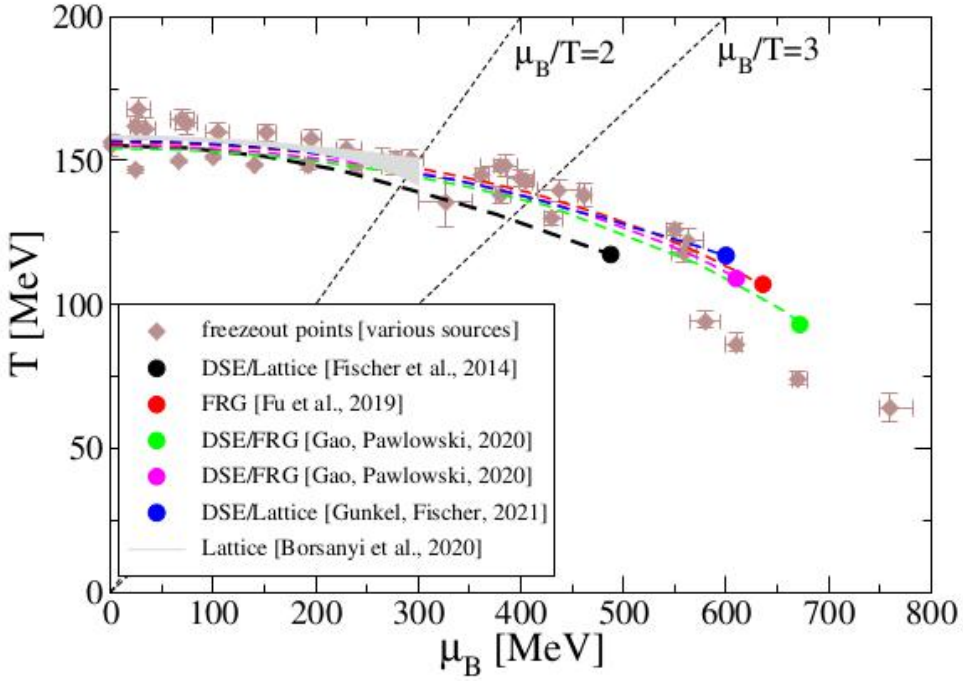
- The CEP exists only for attractive interactions.

G_V, g_V	μ_B^{CEP} (GeV)	T^{CEP} (GeV)
$G_V = g_V = 0$	0.972	0.0481
G_V^-	0.891	0.0635
G_V^+	NA	NA
g_V^-	0.870	0.0679
g_V^+	NA	NA

Locatino of the critical endpoint (CEP) for different values of G_V and g_V.

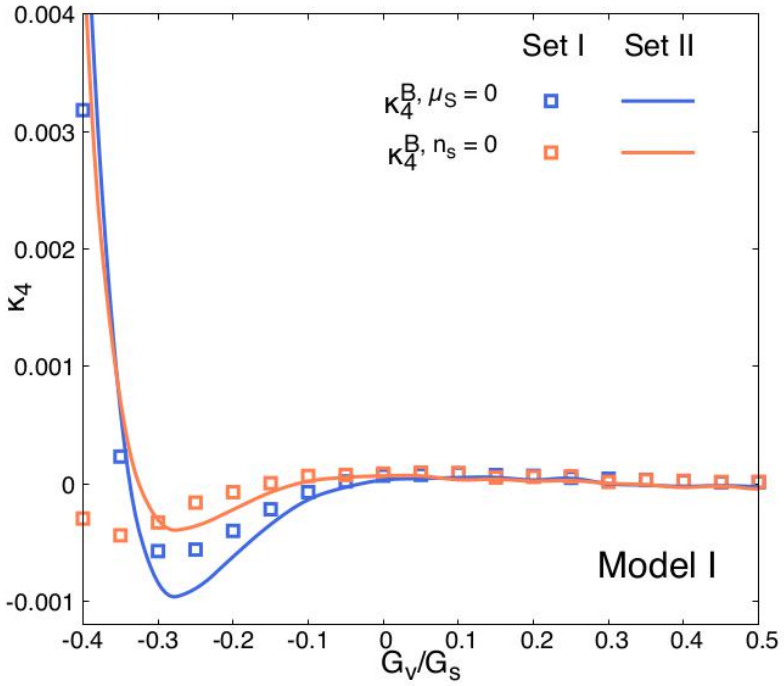
Phase diagram

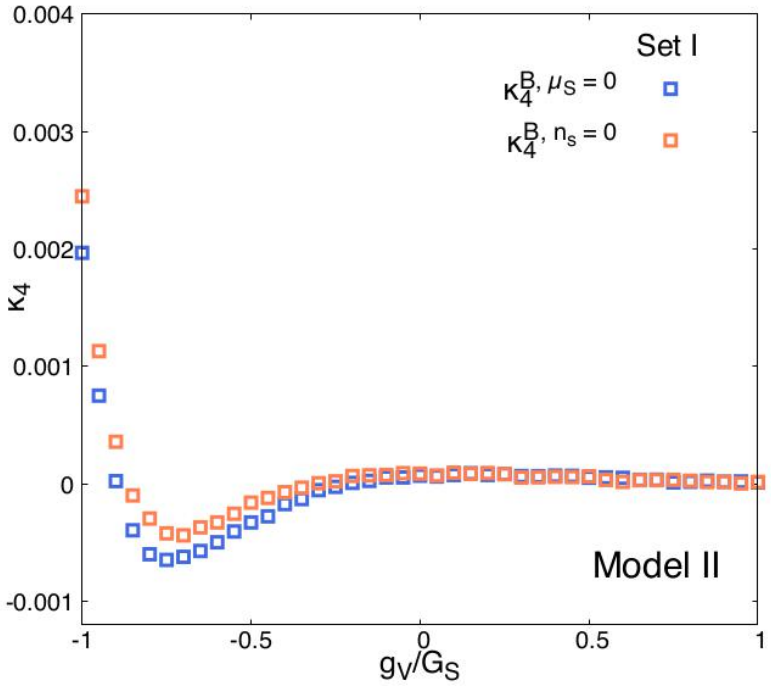




- In comparison with other existing studies.

Curvature coefficient: k_4





$$\frac{T_{pc}(\mu_X)}{T_{pc}(0)} = 1 - \kappa_2^X \left(\frac{\mu_X}{T_{pc}(0)}\right)^2 - \kappa_4^X \left(\frac{\mu_X}{T_{pc}(0)}\right)^4$$

$$\mu_{B,S}/T_{pc}(0) \leq 1.0$$

- Lattice QCD estimates for k_4 are consistently zero within the error bars (**0.001(7)**), as small μ_B/T limit, the parametric form has a weaker dependence on k_4.

- This coefficient provides limited constraints on the coupling strengths G_V and g_V.

Conclusion

- We used improved LQCD data on the curvature (k_2) of the chiral crossover line to constrain vecotr interaction. Improved bound can be obtained.

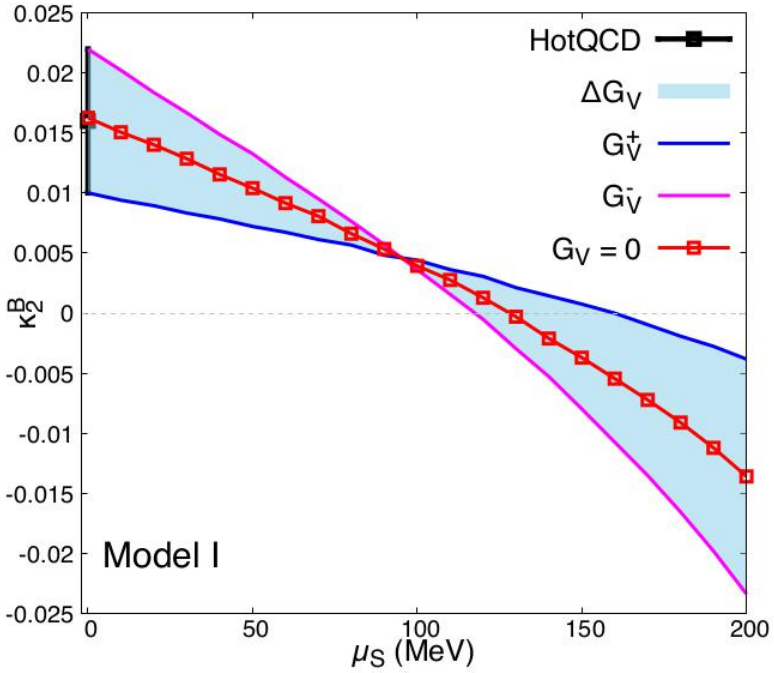
- We perform our analysis in both zero strangeness and strangeness neutral conditions. The results are consistent with each other.

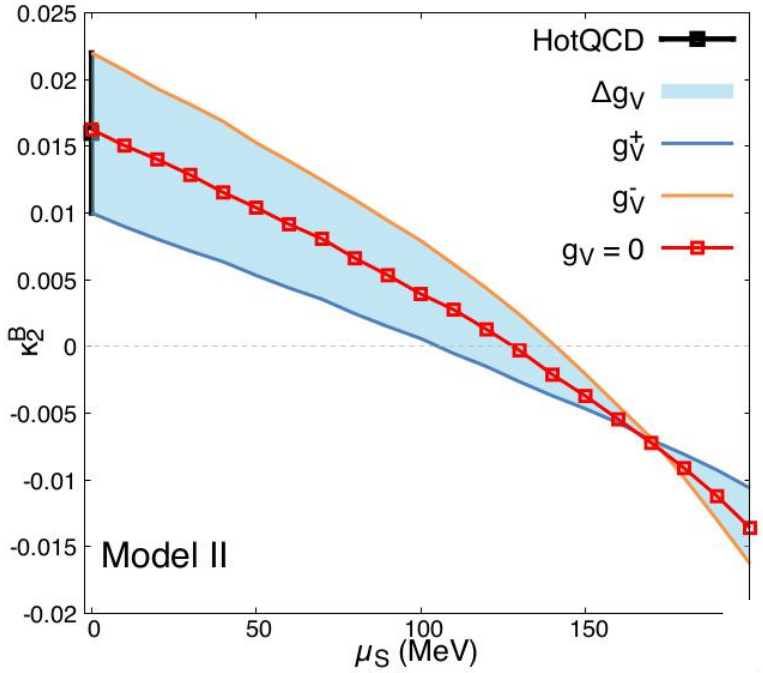
- With the constrained vector interaction we obtained the phase diagram along with critical end point. In the model, it is present only for attractive interaction.

- From the fourth order of the curvature coefficents (k_4) we cannot really put constraints on vector interactions.

Thank you

k_2 as a function of μ_S

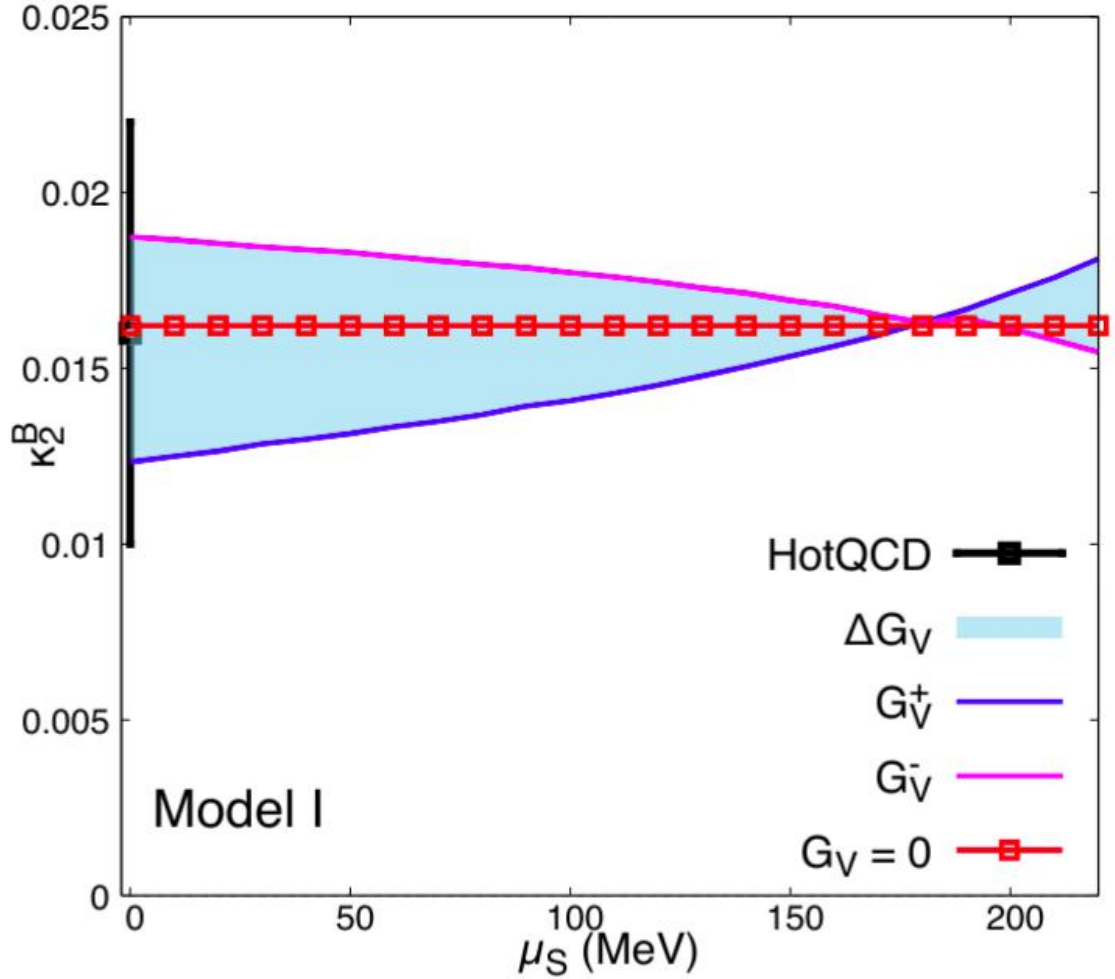




$$G_V \left(\sum_i n_i\right)^2 \longrightarrow \text{Model-I}$$
$$g_V \sum_i n_i^2 \longrightarrow \text{Model-II}$$

$$\tilde{\mu}_i = \begin{cases} \mu_i - 2G_V \sum_j n_j \longrightarrow \text{Model-I (mixing)} \\ \mu_i - 2g_V n_i \longrightarrow \text{Model-II} \end{cases}$$

- It decreases for both models and for both repulsive and attractive interactions


0.025
0.02
0.015
0.01
0.005
0
κ_2^B
0
50
100
150
200
μ_S (MeV)
HotQCD
ΔG_V
G_V^+
G_V^-
$G_V = 0$
Model I

Curvature coefficient: k_4