Finanziato dall'Unione europea NextGenerationEU | Ministero dell'Università e della Ricerca | Italiadomani PIANO NAZIONALE DI RIPRESA E RESILIENZA | UNIVERSITÀ DEGLI STUDI FIRENZE DIPARTIMENTO DI FISICA E ASTRONOMIA

Quantum kinetic theory and spin hydrodynamics for spin-$1/2$ and spin-1 particles

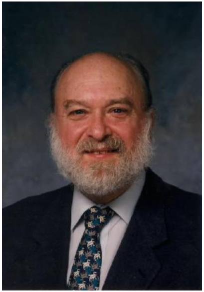
David Wagner

Spin and quantum features of QCD plasma
20.09.2024

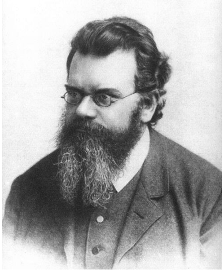
INFN FIRENZE

Introduction to the framework

What is quantum kinetic theory?

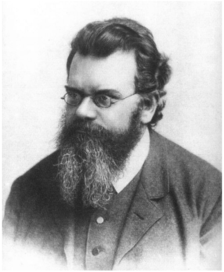


What is ~~quantum~~ kinetic theory?



What is ~~quantum~~ kinetic theory?

- Nonequilibrium statistical description of a dilute gas
  - Nonequilibrium: Describes process of thermalization selfconsistently, concept of temperature emerges from dynamics
  - Statistical: Quantity of interest: single-particle distribution function
  - Dilute: Main assumption: System consists of essentially pointlike free particles scattering via short-range interactions



What is ~~quantum~~ kinetic theory?

Ihr solltet mein Papier lesen!

L. Boltzmann, Sitz.-Ber. Akad. Wiss. Wien (II) 66, 275–370 (1872)
english translation: The Kinetic Theory of Gases, 262-349 (2003)

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  - Statistical: Quantity of interest: single-particle distribution function
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Connection to macroscopic currents

- Kinetic theory is an effective microscopic description
  - Provides (infinitely) more information than macroscopic approaches, such as thermo- and hydrodynamics
  - Can be used to extract information about any macroscopic current of interest

Kinetic representation of currents

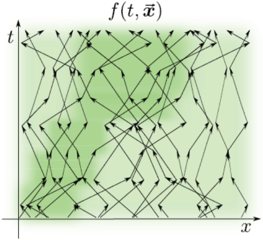
$$N^{\mu}(t,\mathbf{x}) = \int \mathrm{d}K\, k^{\mu} f(t,\mathbf{x},\mathbf{k}) \ ,$$

$$T^{\mu\nu}(t,\mathbf{x}) = \int \mathrm{d}K\, k^{\mu} k^{\nu} f(t,\mathbf{x},\mathbf{k}) \ ,$$

$$S^{\mu}(t,\mathbf{x}) = -\int \mathrm{d}K\, k^{\mu} f(t,\mathbf{x},\mathbf{k}) \left[\ln f(t,\mathbf{x},\mathbf{k}) - 1\right]$$

$\mathrm{d}K := \mathrm{d}^3k/[(2\pi\hbar)^3 k^0]$

Evolution of the distribution function



L. Rezzolla, O. Zanotti, 978-0-19-174650-5 (2013)

Boltzmann equation (classical version)

$$\left(\frac{\partial}{\partial t}+\frac{\mathbf{k}}{m}\cdot\boldsymbol{\nabla}+\mathbf{F}\cdot\boldsymbol{\nabla}_{\mathbf{k}}\right)f(\mathbf{x},\mathbf{k})=C[f]$$

- Left-hand side: Advection through $(\mathbf{x},\mathbf{k})$-phase space
- Right-hand side: Collision term
  - Depends on higher-order distribution functions, e.g. $f_2(\{\mathbf{x}_1,\mathbf{k}_1\};\{\mathbf{x}_2,\mathbf{k}_2\})$
  - Has to be truncated ($\rightarrow$ BBGKY hierarchy)
  - *Stoßzahlansatz*: Replace $f_2(\{\mathbf{x}_1,\mathbf{k}_1\};\{\mathbf{x}_2,\mathbf{k}_2\})\rightarrow f(\mathbf{x}_1,\mathbf{k}_1)f(\mathbf{x}_2,\mathbf{k}_2)$

Collision term

$$C[f]=\frac{1}{2}\int \mathrm{d}K_1\,\mathrm{d}K_2\,\mathrm{d}K'\mathcal{W}\left(f_1f_2-ff'\right)$$

Connecting to quantum theory

- How to translate these ideas to quantum mechanics?
  - Try to build on the conserved currents and find $W(x,k,t)$ such that

$$\mathrm{Tr}\left[\widehat{\rho}(\hat{\mathbf{x}},\hat{\mathbf{p}},t)\,\widehat{A}(\hat{\mathbf{x}},\hat{\mathbf{p}})\right] = \int \mathrm{d}^3x \int \frac{\mathrm{d}^3k}{(2\pi\hbar)^3} A(\mathbf{x},\mathbf{k}) W(\mathbf{x},\mathbf{k},t)$$

- Choice "closest" to classical kinetic theory: **Wigner function**

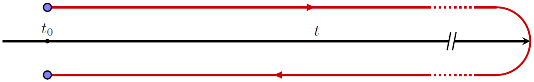
$$W(\mathbf{x},\mathbf{k},t) = \int \mathrm{d}^3v\, e^{-\frac{i}{\hbar}\mathbf{k}\cdot\mathbf{v}} \left\langle \mathbf{x}+\frac{\mathbf{v}}{2}\right|\widehat{\rho}\left|\mathbf{x}-\frac{\mathbf{v}}{2}\right\rangle$$

E. P. Wigner, Phys. Rev. 40, 749-760 (1932)
H.-W. Lee, Physics Reports 259, 147-211 (1995)

- Price to pay: Wigner function is not positive semidefinite
- Relativistic field-theoretical version (scalar field):

$$W(x,k) = \int \mathrm{d}^4v\, e^{-\frac{i}{\hbar}k\cdot v} \left\langle \widehat{\phi}^\dagger\left(x+\frac{v}{2}\right)\widehat{\phi}\left(x-\frac{v}{2}\right)\right\rangle \equiv G^<(x,k)$$

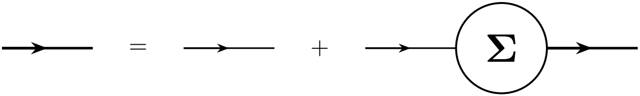
Equations of motion



$$G = \begin{pmatrix} G^{++} & G^{+-} \\ G^{-+} & G^{--} \end{pmatrix} = \begin{pmatrix} G^{F} & G^{<} \\ G^{>} & G^{\bar{F}} \end{pmatrix}$$

- Real-time QFT: Expectation values of operators can be represented as time evolution along a closed-time path
- EoM: (contour-ordered) Dyson-Schwinger equation

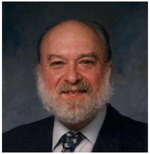
$$G_0^{-1} G^{AB}(x_1, x_2) = -ic^{AB} \delta^{(4)}(x_1 - x_2) + i \int \mathrm{d}^4 x' \Sigma^{AC}(x_1, x') c_{CD} G^{DB}(x', x_2)$$



- Wigner transform: $-i\hbar \partial_1^\mu G^{AB}(x_1, x_2) \to \left(k^\mu - \frac{i\hbar}{2}\partial^\mu\right) G^{AB}(x, k)$

Approximations

- $\hbar$-gradient expansion: Assume that the two-point functions are sufficiently localized in central coordinate $x = (x_1 + x_2)/2$
  - *Notion of a particle should make sense!*
- Allows to approximate memory integrals



$$\int \mathrm{d}^4 x' G(x_1, x') \Sigma(x', x_2) \longrightarrow G(x,k)\Sigma(x,k) - \frac{i\hbar}{2}\left\{G(x,k), \Sigma(x,k)\right\}_{\mathrm{PB}}$$

(Gradient-expanded) Kadanoff-Baym equations

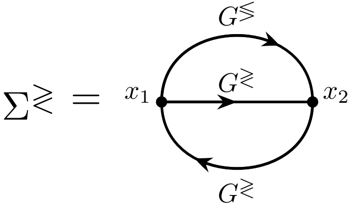
$$G_0^{-1} G^<(x,k) = \frac{i}{2}\left[\Sigma^>(x,k)G^<(x,k) - \Sigma^<(x,k)G^>(x,k)\right] + \frac{\hbar}{4}\left[\left\{\Sigma^>(x,k), G^<(x,k)\right\}_{\mathrm{PB}} - \left\{\Sigma^<(x,k), G^>(x,k)\right\}_{\mathrm{PB}}\right]$$

L. P. Kadanoff, G. Baym, ISBN 9780429493218 (1989)

$$\{f,g\}_{\mathrm{PB}} := (\partial_\mu f)(\partial_k^\mu g) - (\partial_k^\mu f)(\partial_\mu g)$$

Example: Scalar field

- Approximate self-energy to lowest nontrivial order
- Separate real and imaginary parts of KB equations



Quantum kinetic equations (lowest order)

$$\left(k^2 - m^2\right) G^{<}(x,k) = 0 \quad \Longrightarrow \quad G^{<}(x,k) = 2\pi\hbar^2 \delta(k^2 - m^2) f(x,k) \;,$$

$$k \cdot \partial f(x,k) = \frac{1}{2} \int \mathrm{d}K_1 \, \mathrm{d}K_2 \, \mathrm{d}K' \, (2\pi\hbar)^4 \delta^{(4)}(k_1 + k_2 - k - k') \times \frac{|M|^2}{16} \left(f_1 f_2 \widetilde{f} \widetilde{f}' - \widetilde{f}_1 \widetilde{f}_2 f f' \right)$$

"Tell me something new!"

Including interesting stuff

Including electromagnetic fields

- Standard definition of Wigner function is not gauge invariant due to fields at different positions
  - Remedy: Include gauge link $U(x_1, x_2) = \exp\left[-\frac{i}{\hbar}(x_1 - x_2)\cdot\int_{-1/2}^{1/2}\mathrm{d}t A(x_1 + x_2 + t(x_1 - x_2))\right]$

Wigner function with EM fields

$$G^<(x,k) = \int \mathrm{d}^4 v\, e^{-\frac{i}{\hbar}k\cdot v}\left\langle \hat{\phi}^\dagger\left(x+\frac{v}{2}\right) U\left(x+\frac{v}{2}, x-\frac{v}{2}\right)\hat{\phi}\left(x-\frac{v}{2}\right)\right\rangle$$

- Main effect: Wigner representation of momentum changes

$$-i\hbar\partial_1^\mu G^< \to \left\{k^\mu - \frac{\hbar}{2} j_1(\Delta) F^{\mu\nu}\partial_{k,\nu} - \frac{i\hbar}{2}\left[\partial^\mu - j_0(\Delta)F^{\mu\nu}\partial_{k,\nu}\right]\right\} G^<(x,k)$$

D. Vasak, M. Gyulassy, H. T. Elze, Annals Phys. 173, 462-492 (1987)

$\Delta := (\hbar/2)\partial\cdot\partial_k$

Including spin

- Case of nonzero spin: Wigner function becomes **matrix-valued**
  $\rightarrow$ Additional components encode spin degrees of freedom

Wigner function (spin $1/2$)

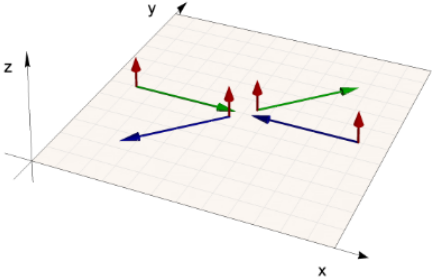
$$G^{<} = \frac{1}{4}\left(\mathcal{F} + i\gamma_5\mathcal{P} + \not{\mathcal{V}} + \gamma_5\not{\mathcal{A}} + \frac{i}{4}[\gamma_\mu, \gamma_\nu]\mathcal{S}^{\mu\nu}\right)$$

- Underlying equations (Dirac, Proca, $\cdots$) can be solved perturbatively in $\hbar$ expansion
  $\rightarrow$ Gradient contributions appear!
    - $\mathcal{V}^\mu \sim -\frac{\hbar}{2m}\partial_\nu\mathcal{S}^{\nu\mu}$, $k_\mu\mathcal{S}^{\mu\nu} \sim -\frac{\hbar}{2}\partial^\nu\mathcal{F}$, $\mathcal{P} \sim -\frac{\hbar}{4m^2}\epsilon^{\mu\nu\alpha\beta}k_\mu\partial_\nu\mathcal{S}_{\alpha\beta}$, $\cdots$
  - Responsible for a lot of the interesting transport phenomena

H. T. Elze, M. Gyulassy, D. Vasak, Nucl. Phys. B 276, 706-728 (1986)
D. Vasak, M. Gyulassy, H. T. Elze, Annals Phys. 173, 462-492 (1987)
S. Mrowczynski, U. W. Heinz, Annals Phys. 229, 1-54 (1994)
Y. A. Markov, M. A. Markova, Theor. Math. Phys. 108, 977-991 (1996)
Y. A. Markov, M. A. Markova, Theor. Math. Phys. 111, 601-612 (1997)

Going nonlocal: Collisions (with spin)

Angular momentum and collisions

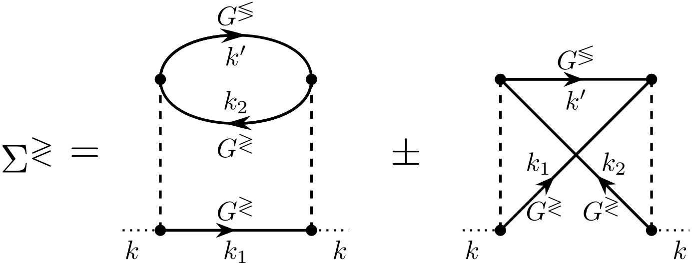


W. Florkowski, A. Kumar, R. Ryblewski, Prog. Part. Nucl. Phys. 108, 103709 (2019)

- Assume that collisions take place in a point
  - → Total orbital angular momentum vanishes
  - → Spin is conserved on its own
  - → No exchange of spin and orbital angular momenta
- Collisions must be **nonlocal** for spin equilibration!

Collisions from QKT

$$k \cdot \partial G^{<}(x,k) = \frac{1}{2}\left[\Sigma^{<}(x,k)G^{>}(x,k) - \Sigma^{<}(x,k)G^{<}(x,k)\right]$$



- Collisions determined by self-energies
- Crucial quantum enhancement: *All internal lines have to be evaluated to order* $\mathcal{O}(\hbar)$*!*
  - $\longrightarrow$ Introduces gradient corrections <u>inside the collision integral!</u>

Collisions from QKT

DW, NW, DHR, Phys.Rev.D 106 (2022) 11, 116021

Boltzmann equation with collisions

$$\mathfrak{f}(x,k,\mathfrak{s}) = \delta(k^2-m^2)f(x,k,\mathfrak{s}) := \frac{1}{2}\left[\mathcal{F}(x,k) - \mathfrak{s}\cdot\mathcal{A}(x,k)\right]$$

$$\begin{aligned} k\cdot\partial f(x,k,\mathfrak{s}) = \frac{1}{2}\int & \mathrm{d}\Gamma_1 \mathrm{d}\Gamma_2 \mathrm{d}\Gamma' \mathrm{d}\bar{S}(k)\delta^{(4)}(k_1+k_2-k-k')\mathcal{W} \\ & \times\left[f(x+\Delta_1-\Delta,k_1,\mathfrak{s}_1)f(x+\Delta_2-\Delta,k_2,\mathfrak{s}_2)\right. \\ & \left. -f(x,k,\bar{\mathfrak{s}})f(x+\Delta'-\Delta,k',\mathfrak{s}')\right] \end{aligned}$$

$$\mathrm{d}\Gamma := 2\mathrm{d}^4k\,\delta(k^2-m^2)\mathrm{d}S(k)$$

Collisions from QKT

DW, NW, DHR, Phys.Rev.D 106 (2022) 11, 116021

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$$\times\left[f(x+\Delta_1-\Delta,k_1,\mathfrak{s}_1)f(x+\Delta_2-\Delta,k_2,\mathfrak{s}_2)\right.$$
$$\left.-f(x,k,\bar{\mathfrak{s}})f(x+\Delta'-\Delta,k',\mathfrak{s}')\right]$$

- Contributions inside the collision term have gradient corrections

$$f(x,k,\mathfrak{s}) + \Delta^\mu\partial_\mu f(x,k,\mathfrak{s}) \approx f(x+\Delta,k,\mathfrak{s})$$

- A (momentum- and spin-dependent) **spacetime shift** Δ^μ enters
  $\rightarrow$ Particles do not scatter at the same spacetime point!
- This enables a conversion of orbital and spin angular momenta

$\mathrm{d}\Gamma := 2\mathrm{d}^4k\,\delta(k^2-m^2)\mathrm{d}S(k)$

Collisions from QKT

More on collisional QKT:

S. Mrowczynski, U. W. Heinz, Annals Phys. 229, 1-54 (1994)

K. Morawetz, P. Lipavsky, V. Spicka, N.-H. Kwong, Phys. Rev. C 59, 3052–3059 (1999)

D.-L. Yang, K. Hattori, Y. Hidaka, Yoshimasa JHEP 07, 070 (2020)

K. Hattori, Y. Hidaka, N. Yamamoto, D.-L. Yang, JHEP 02, 001 (2021)

N. Weickgenannt, E. Speranza, X.-L. Sheng, Q. Wang, D. H. Rischke, Phys. Rev. D 104, 016022 (2021)

X.-L. Sheng, N. Weickgenannt, E. Speranza, D. H. Rischke, Q. Wang, Phys. Rev. D 104, 016029 (2021)

J. Hu, 2110.12339 (2021)

S. Lin, Phys. Rev. D 105, 076017 (2022)

S. Fang, S. Pu, D.-L. Yang, Phys. Rev. D 106, 016002 (2022)

Y. Hidaka, S. Pu, Q. Wang, D.-L. Yang, Prog. Part. Nucl. Phys. 127, 103989 (2022)

DW, N. Weickgenannt, D. H. Rischke, Phys. Rev. D 106, 116021 (2022)

X.-L. Sheng, Q. Wang, D. H. Rischke, Phys. Rev. D 106, L111901 (2022)

DW, N. Weickgenannt, E. Speranza, Phys. Rev. D 108, 116017 (2023)

N. Yamamoto, D.-L. Yang, Phys. Rev. D 109, 056010 (2024)

$x, k)]$

$)\mathcal{W}$

$\mathfrak{s}_2)$

ions

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$\mathrm{d}\Gamma := 2\mathrm{d}^4k\delta(k^2-m^2)\mathrm{d}S(k)$

Finding equilibrium

- Local-equilibrium distribution function makes local collision term vanish (without spacetime shifts Δ^μ)
- Has to depend on the **collisional invariants**
  - $\rightarrow$ Charge, four-momentum and total angular momentum

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Local-equilibrium distribution function

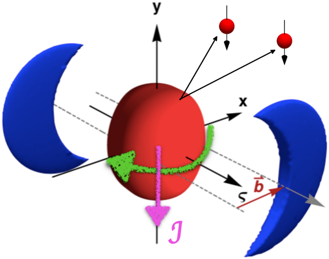
$$f_{\text{eq}}(x,k,\mathfrak{s}) = \exp\left(\alpha_0 - \beta_0 E_{\mathbf{k}} + \frac{\hbar}{2}\Omega_{0,\mu\nu}\Sigma_{\mathfrak{s}}^{\mu\nu}\right)$$

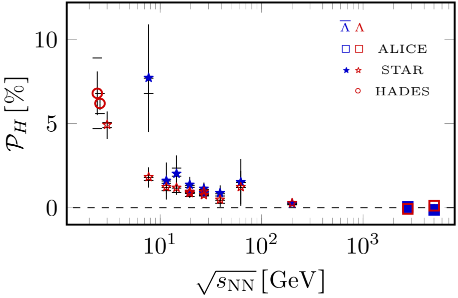
- Necessary conditions on Lagrange multipliers for a vanishing **nonlocal** collision term: $\partial^\mu \alpha_0 = 0$, $\partial^{(\mu}(\beta_0 u^{\nu)}) = 0$, $\Omega_0^{\mu\nu} = -\frac{1}{2}\partial^{[\mu}(\beta_0 u^{\nu]})$
- Same conditions as for **global** equilibrium, where $k \cdot \partial f_{\text{eq}} = 0$

$\Sigma_{\mathfrak{s}}^{\mu\nu} := -\frac{1}{m}\epsilon^{\mu\nu\alpha\beta}k_\alpha \mathfrak{s}_\beta$, $E_{\mathbf{k}} := k \cdot u$

Polarization & spin hydrodynamics

Global Λ-Polarization

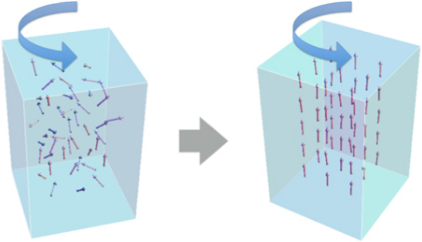




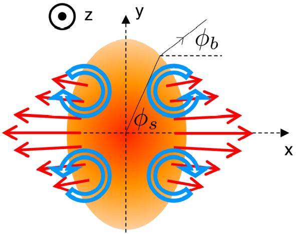
- "Global": Integrated polarization along the direction of orbital angular momentum
- Can be explained by assuming spins in equilibrium

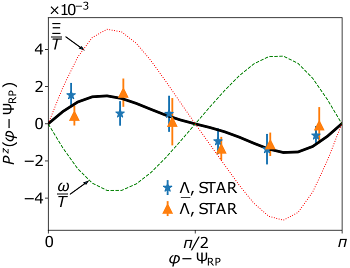
→ "Polarization through rotation"

- ▶ Analogous to Barnett effect



Local Λ-Polarization

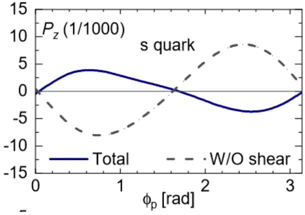




F. Becattini, M. Buzzegoli, G. Inghirami, I. Karpenko, A. Palermo, PRL 127, 272302 (2021)

- "Local": Angle-dependent polarization along beam-direction
- Could only be explained recently by incorporating shear effects

→ Simple picture of equilibrated spins not complete



B. Fu, S. Y. F. Liu, L. Pang, H. Song, Y. Yin, Phys. Rev. Lett. 127, 142301 (2021)

Open questions

Pauli-Lubanski vector in (global) equilibrium

$$S^\mu_\varpi = -\epsilon^{\mu\nu\alpha\beta} k_\nu \frac{\int \mathrm{d}\Sigma_\lambda k^\lambda f_0(1-f_0)\varpi_{\alpha\beta}}{8m \int \mathrm{d}\Sigma_\lambda k^\lambda f_0}$$

- Traditional approaches to computing the polarization
  - assume equilibrated spin degrees of freedom
  - neglect dissipative terms

$\varpi_{\mu\nu} := \frac{1}{2}[\partial_\mu (u_\nu/T) - \partial_\nu (u_\mu/T)]$

Open questions

Pauli-Lubanski vector in (global) equilibrium

$$S^\mu_\varpi = -\epsilon^{\mu\nu\alpha\beta} k_\nu \frac{\int \mathrm{d}\Sigma_\lambda k^\lambda f_0(1-f_0)\varpi_{\alpha\beta}}{8m \int \mathrm{d}\Sigma_\lambda k^\lambda f_0}$$

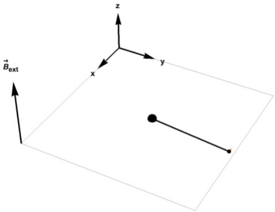
- Traditional approaches to computing the polarization
  - assume equilibrated spin degrees of freedom
  - neglect dissipative terms
- Not clear so far:
  - (I) How fast do spin degrees of freedom equilibrate?
  - (II) How do dissipative effects influence polarization?

→ Can be answered through spin hydrodynamics from quantum kinetic theory

$\varpi_{\mu\nu} := \frac{1}{2}[\partial_\mu (u_\nu/T) - \partial_\nu (u_\mu/T)]$

(I) Timescales: Magnetic resonance imaging (MRI)

- MRI: Large constant B-field in z-direction and short-lived alternating field in x, y-plane
- Identify materials by relaxation times T_1, T_2



https://en.wikipedia.org/wiki/Bloch_equations

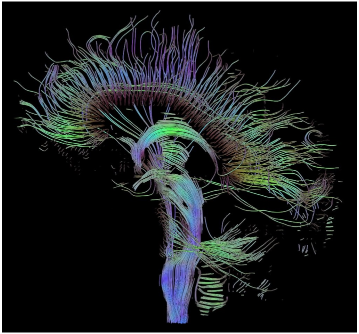
Bloch equations

$$T_2 \dot{M}_{x,y} + M_{x,y} = \mu_2 \left(\mathbf{M} \times \mathbf{B}\right)_{x,y} \ ,$$
$$T_1 \dot{M}_z + M_z = \mu_1 \left(\mathbf{M} \times \mathbf{B}\right)_z + M_0 \ .$$

$\mu_1 := T_1 \frac{gq}{2m}$, $\mu_2 := T_2 \frac{gq}{2m}$

(II) Dissipation: Diffusion-MRI

- Apply additional $\vec{B}$-gradients to make Larmor frequencies position-dependent
- Allows to additionally track diffusion of molecules



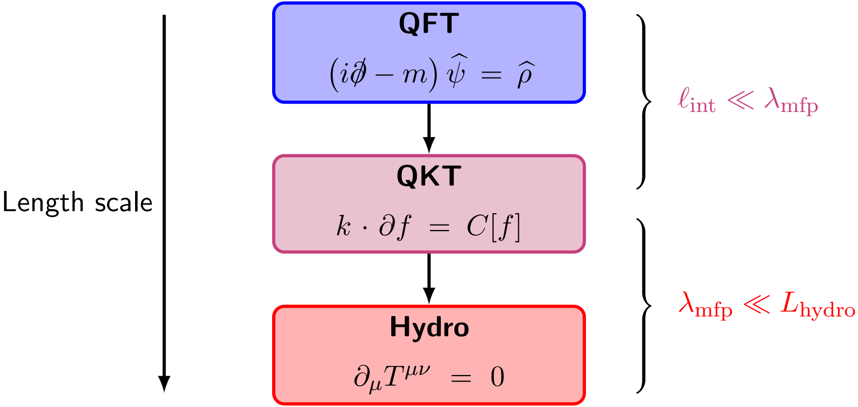
https://en.wikipedia.org/wiki/Diffusion_MRI

Bloch-Torrey equations

$$T_2 \dot{M}_{x,y} + M_{x,y} = \mu_2 \left(\mathbf{M} \times \mathbf{B}\right)_{x,y} + D\Delta M_{x,y} \ ,$$
$$T_1 \dot{M}_z + M_z = \mu_1 \left(\mathbf{M} \times \mathbf{B}\right)_z + D\Delta M_z + M_0 \ .$$

$\mu_1 := T_1 \frac{gq}{2m}$, $\mu_2 := T_2 \frac{gq}{2m}$

Spin hydrodynamics: Procedure



Spin hydrodynamics: Ideal case

Conservation equations

$$\partial_\mu T^{(\mu\nu)} = 0 + \mathcal{O}(\hbar^2)\ ,$$
$$\partial_\lambda S^{\lambda\mu\nu} = \frac{1}{\hbar} T^{[\nu\mu]} + \mathcal{O}(\hbar^2)\ .$$

- No backreaction of spin on fluid evolution, fluid profile serves as input for spin potential
  - Simplest configuration: fluid at rest, $u^\mu = \text{const}$, $T = \text{const}$
- Ideal fluid (kinetic language): Assume local equilibrium, i.e., set $f = f_{\text{eq}}$
  - Express $S^{\lambda\mu\nu}$ and $T^{[\nu\mu]}$ in terms of $\kappa_0^\mu := \Omega_0^{\nu\mu} u_\nu$ and $\omega_0^\mu := \frac{1}{2}\epsilon^{\mu\nu\alpha\beta} u_\nu \Omega_{0,\alpha\beta}$

Form of $T^{[\mu\nu]}$

$$T^{[\mu\nu]} = -\hbar^2 \Gamma^{(\kappa)} \left(u^{[\mu} \kappa_0^{\nu]} - u_\alpha \varpi^{\alpha[\nu} u^{\mu]} \right) + \hbar^2 \Gamma^{(\omega)} \left(\epsilon^{\mu\nu\alpha\beta} u_\alpha \omega_{0,\beta} - \varpi^{\langle\mu\rangle\langle\nu\rangle} \right)$$

Spin equations of motion in a fluid at rest

- Project equations of motion for spin tensor to obtain evolution equations for the components of the spin potential

Equations of motion for the spin potential

$$\tau_\kappa \dot{\kappa}_0^{\langle\mu\rangle} + \kappa_0^\mu = \mu_\kappa \epsilon^{\mu\nu\alpha\beta} u_\nu \nabla_\alpha \omega_{0,\beta} \ ,$$

$$\tau_\omega \dot{\omega}_0^{\langle\mu\rangle} + \omega_0^\mu = -\mu_\omega \epsilon^{\mu\nu\alpha\beta} u_\nu \nabla_\alpha \kappa_{0,\beta} \ .$$

- Go to fluid rest frame, $\kappa_0^\mu \equiv (0, \boldsymbol{\kappa})$, $\omega_0^\mu \equiv (0, \boldsymbol{\omega})$

$$\tau_\kappa \dot{\boldsymbol{\kappa}} + \boldsymbol{\kappa} = \mu_\kappa \boldsymbol{\nabla} \times \boldsymbol{\omega} \ ,$$

$$\tau_\omega \dot{\boldsymbol{\omega}} + \boldsymbol{\omega} = -\mu_\omega \boldsymbol{\nabla} \times \boldsymbol{\kappa} \ ,$$

- Relaxation times τ_κ, τ_ω determined by **nonlocal** collisions

$\tau_\kappa := \frac{I_{31}}{2m^2\Gamma^{(\kappa)}}$, $\tau_\omega := \frac{I_{30}-I_{31}}{4m^2\Gamma^{(\omega)}}$, $\mu_\kappa := \frac{\tau_\kappa}{2}$, $\mu_\omega := \frac{I_{31}}{4m^2\Gamma^{(\omega)}}$

Spin equations of motion in a fluid at rest

- Project equations of motion for spin tensor to obtain evolution equations for the components of the spin potential

Equations of motion for the spin potential

$$\tau_\kappa \dot{\kappa}_0^{\langle\mu\rangle} + \kappa_0^\mu = \mu_\kappa \epsilon^{\mu\nu\alpha\beta} u_\nu \nabla_\alpha \omega_{0,\beta} \ ,$$
$$\tau_\omega \dot{\omega}_0^{\langle\mu\rangle} + \omega_0^\mu = -\mu_\omega \epsilon^{\mu\nu\alpha\beta} u_\nu \nabla_\alpha \kappa_{0,\beta} \ .$$

- Go to fl

Let $\tau_\kappa \to 0$ for (boring) version of Bloch equations:

$$\tau_\omega \dot{\boldsymbol{\omega}} + \boldsymbol{\omega} = 0$$

- Relaxation times τ_κ, τ_ω determined by **nonlocal** collisions

$\tau_\kappa := \frac{I_{31}}{2m^2\Gamma^{(\kappa)}}, \quad \tau_\omega := \frac{I_{30}-I_{31}}{4m^2\Gamma^{(\omega)}}, \quad \mu_\kappa := \frac{\tau_\kappa}{2}, \quad \mu_\omega := \frac{I_{31}}{4m^2\Gamma^{(\omega)}}$

Spin waves

- $\boldsymbol{\kappa}$ and $\boldsymbol{\omega}$ follow coupled relaxation equations
  $\longrightarrow$ Disentangle longitudinal and transverse components

Longitudinal components: Decay

$$\tau_\kappa \frac{\mathrm{d}}{\mathrm{d}t}\left(\boldsymbol{\nabla}\cdot\boldsymbol{\kappa}\right) = -\boldsymbol{\nabla}\cdot\boldsymbol{\kappa}\;,$$
$$\tau_\omega \frac{\mathrm{d}}{\mathrm{d}t}\left(\boldsymbol{\nabla}\cdot\boldsymbol{\omega}\right) = -\boldsymbol{\nabla}\cdot\boldsymbol{\omega}\;,$$

Transverse components: Damped waves

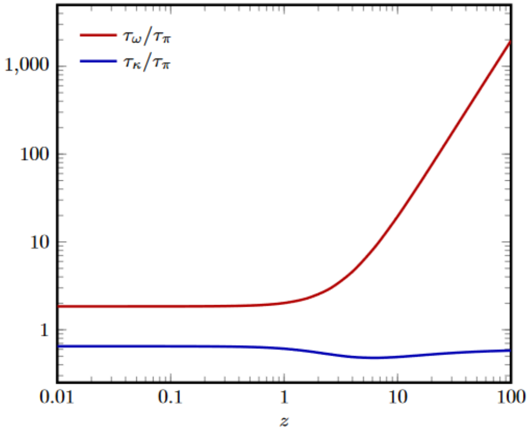
$$\ddot{\boldsymbol{\kappa}} + a\dot{\boldsymbol{\kappa}} + b\boldsymbol{\kappa} - c_{\mathfrak{s}}^2\Delta\boldsymbol{\kappa} = 0\;,$$
$$\ddot{\boldsymbol{\omega}} + a\dot{\boldsymbol{\omega}} + b\boldsymbol{\omega} - c_{\mathfrak{s}}^2\Delta\boldsymbol{\omega} = 0\;,$$

V. E. Ambrus, R. Ryblewski, R. Singh, Phys. Rev. D 106, 014018(2022)
J. Hu, Z. Xu, Phys. Rev. D 107, 016010 (2023)
DW, M. Shokri, D. H. Rischke, 2405.00533 (2024)

$a := \frac{\tau_\kappa+\tau_\omega}{\tau_\kappa\tau_\omega}\;,\qquad b := \frac{1}{\tau_\kappa\tau_\omega}\;,\qquad c_{\mathfrak{s}}^2 := \frac{\mu_\kappa\mu_\omega}{\tau_\kappa\tau_\omega}.$

Spin relaxation timescales

- Relaxation time depends on interaction model, can potentially get large
  - → Spin relaxation can be a slow process, dynamics cannot be neglected!



DW, M. Shokri, D. H. Rischke, 2405.00533 (2024)

Spin relaxation timescales

- Relaxation time depends on interaction model, can potentially get large
  - → Spin relaxation can be a slow process, dynamics cannot be neglected!

More on spin relaxation:

Y. Hidaka, M. Hongo, M. A. Stephanov, H.-U. Yee, Phys. Rev. C 109, 054909 (2024)

M. Hongo, X.-G. Huang, M. Kaminski, M. Stephanov, H.-U. Yee, JHEP 08, 263 (2022)

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A. Ayala, D. de la Cruz, L. A. Hernández, J. Salinas, Phys. Rev. D 102, 056019 (2020)

A. Ayala, D. de la Cruz, S. Hernández-Ortíz, L. A. Hernández, J. Salinas, Phys. Lett. B 801, 135169 (2020)

J. I. Kapusta, E. Rrapaj, S. Rudaz, Phys. Rev. C 101, 024907 (2020)

J. Hu, Phys. Rev. D 105, 096021 (2022)

J. Hu, Z. Xu, Phys. Rev. D 107, 016010 (2023)

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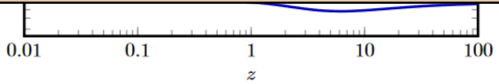
S. Lin, H. Tang, 2406.17632 (2024)

G. Torrieri, D. Montenegro, Phys. Rev. D 107, 076010 (2023)



DW, M. Shokri, D. H. Rischke, 2405.00533 (2024)

Beyond equilibrium: Moment method

- Split distribution function $f = f_{\text{eq}} + \delta f$
- Perform moment expansion including spin degrees of freedom

Irreducible moments

$$\rho_r^{\mu_1 \cdots \mu_\ell}(x) \quad := \quad \int \mathrm{d}\Gamma E_{\mathbf{k}}^r k^{\langle \mu_1} \cdots k^{\mu_\ell \rangle} \delta f(x, k, \mathfrak{s})$$

$$\tau_r^{\mu, \mu_1 \cdots \mu_\ell}(x) \quad := \quad \int \mathrm{d}\Gamma \mathfrak{s}^\mu E_{\mathbf{k}}^r k^{\langle \mu_1} \cdots k^{\mu_\ell \rangle} \delta f(x, k, \mathfrak{s})$$

- Equations of motion can be derived from Boltzmann equation
- Knowing the evolution of all moments is equivalent to solving the Boltzmann equation

$k^{\langle \mu_1} \cdots k^{\mu_\ell \rangle} := \Delta^{\mu_1 \cdots \mu_\ell}_{\nu_1 \cdots \nu_\ell} k^{\nu_1} \cdots k^{\nu_\ell}$

Beyond equilibrium: Moment method

- Split distribution function $f = f_{\text{eq}} + \delta f$
- Perform moment expansion including spin degrees of freedom

Irreducible moments

Standard dissipation

$$\rho_r^{\mu_1 \cdots \mu_\ell}(x) \quad := \quad \int \mathrm{d}\Gamma E_{\mathbf{k}}^r k^{\langle \mu_1} \cdots k^{\mu_\ell \rangle} \delta f(x,k,\mathfrak{s})$$

$$\tau_r^{\mu,\mu_1 \cdots \mu_\ell}(x) \quad := \quad \int \mathrm{d}\Gamma \mathfrak{s}^\mu E_{\mathbf{k}}^r k^{\langle \mu_1} \cdots k^{\mu_\ell \rangle} \delta f(x,k,\mathfrak{s})$$

Spin dissipation

- Equations of motion can be derived from Boltzmann equation
- Knowing the evolution of all moments is equivalent to solving the Boltzmann equation

---

$k^{\langle \mu_1} \cdots k^{\mu_\ell \rangle} := \Delta^{\mu_1 \cdots \mu_\ell}_{\nu_1 \cdots \nu_\ell} k^{\nu_1} \cdots k^{\nu_\ell}$

Resumming (spin) hydrodynamics: IReD

DW, A. Palermo, V. E. Ambruș, Phys. Rev. D **106**, 016013 (2022)
DW, 2409.07143 (2024)

- Basic idea: Power-counting scheme to second order in
  - Knudsen number $\mathrm{Kn} := \lambda_{\mathrm{mfp}}/L_{\mathrm{hydro}}$
  - inverse Reynolds numbers $\mathrm{Re}^{-1} \sim \delta f / f_{\mathrm{eq}}$
- Derive asymptotic (Navier-Stokes) relations to close the system

Asymptotic matching (example)

$$\rho_r^{\mu\nu} = \eta_r \sigma^{\mu\nu} + \mathcal{O}(\mathrm{Kn}\mathrm{Re}^{-1}) = \frac{\eta_r}{\eta_0}\pi^{\mu\nu} + \mathcal{O}(\mathrm{Kn}\mathrm{Re}^{-1})$$

- The same procedure can be done for the moments $\tau_r^{\mu,\mu_1\cdots\mu_\ell}$
- Many moments can be related to ω_0^μ and κ_0^μ
  - No need to introduce more dynamical quantities
- Exception: tensor-valued moments $t_r^{\mu\nu} := \tau_{r,\alpha,\beta}{}^{\langle\mu}\epsilon^{\nu\rangle\alpha\beta\rho}u_\rho$
  - Additional dynamical quantity $\mathfrak{t}^{\mu\nu}$ is needed, $S^{\lambda\mu\nu} \sim \mathfrak{t}^{\lambda[\mu}u^{\nu]}$

Dissipative spin hydrodynamics

DW, 2409.07143 (2024)

$$
\begin{aligned}
\tau_\omega \dot{\omega}_0^{\langle\mu\rangle} + \omega_0^\mu = {}& -\beta_0 \omega^\mu + \delta_{\omega\omega}\omega_0^\mu\theta + \lambda_{\omega\omega}\sigma^{\mu\nu}\omega_{0,\nu} + \lambda_{\omega\mathfrak{t}}\mathfrak{t}^{\mu\nu}\omega_\nu \\
& + \epsilon^{\mu\nu\alpha\beta}u_\nu \left(\ell_{\omega\kappa}\nabla_\alpha\kappa_{0,\beta} - \tau_\omega \dot{u}_\alpha \kappa_{0,\beta} + \lambda_{\omega\kappa} I_\alpha \kappa_{0,\beta}\right. \\
& \qquad \left. + \ell_{\omega n}\nabla_\alpha n_\beta + \tau_{\omega n}\dot{u}_\alpha n_\beta + \lambda_{\omega n} I_\alpha n_\beta\right) \\
\tau_\kappa \dot{\kappa}_0^{\langle\mu\rangle} + \kappa_0^\mu = {}& -\beta_0 \dot{u}^\mu + \mathfrak{b} I^\mu + \delta_{\kappa\kappa}\kappa_0^\mu\theta + \left(\lambda_{\kappa\kappa}\sigma^{\mu\nu} + \frac{\tau_\kappa}{2}\omega^{\mu\nu}\right)\kappa_{0,\nu} \\
& + \epsilon^{\mu\nu\alpha\beta}u_\nu \left(\frac{\tau_\kappa}{2}\nabla_\alpha\omega_{0,\beta} + \tau_\kappa \dot{u}_\alpha\omega_{0,\beta} + \lambda_{\kappa\omega} I_\alpha \omega_{0,\beta}\right) \\
& + \mathfrak{t}^{\mu\nu}\left(\tau_{\kappa\mathfrak{t}}\dot{u}_\nu + \lambda_{\kappa\mathfrak{t}} I_\nu\right) + \ell_{\kappa\mathfrak{t}}\Delta^\mu_\lambda \nabla_\nu \mathfrak{t}^{\nu\lambda} \\
& + \ell_{\kappa n}\dot{n}^{\langle\mu\rangle} + \delta_{\kappa n} n^\mu \theta + \left(\lambda_{\kappa n}\sigma^{\mu\nu} + \tau_{\kappa n}\omega^{\mu\nu}\right) n_\nu \\
\tau_\mathfrak{t} \dot{\mathfrak{t}}^{\langle\mu\nu\rangle} + \mathfrak{t}^{\mu\nu} = {}& \mathfrak{d}\beta_0\sigma^{\mu\nu} + \delta_{\mathfrak{t}\mathfrak{t}}\mathfrak{t}^{\mu\nu}\theta + \lambda_{\mathfrak{t}\mathfrak{t}}\mathfrak{t}_\lambda{}^{\langle\mu}\sigma^{\nu\rangle\lambda} + \frac{5}{3}\tau_\mathfrak{t}\mathfrak{t}_\lambda{}^{\langle\mu}\omega^{\nu\rangle\lambda} + \ell_{\mathfrak{t}\kappa}\nabla^{\langle\mu}\kappa_0^{\nu\rangle} \\
& + \lambda_{\mathfrak{t}\kappa} I^{\langle\mu}\kappa_0^{\nu\rangle} + \tau_{\mathfrak{t}\omega}\omega^{\langle\mu}\omega_0^{\nu\rangle} + \lambda_{\mathfrak{t}\omega}\sigma_\lambda{}^{\langle\mu}\epsilon^{\nu\rangle\lambda\alpha\beta}u_\alpha\omega_{0,\beta} \\
& + \ell_{\mathfrak{t}n}\nabla^{\langle\mu}n^{\nu\rangle} + \tau_{\mathfrak{t}n}\dot{u}^{\langle\mu}n^{\nu\rangle} + \lambda_{\mathfrak{t}n} I^{\langle\mu}n^{\nu\rangle}
\end{aligned}
$$

Dissipative spin hydrodynamics (simpler)

DW, 2409.07143 (2024)

$$\tau_\omega \dot{\omega}_0^{\langle\mu\rangle} + \omega_0^\mu = -\beta_0 \omega^\mu + \delta_{\omega\omega}\omega_0^\mu \theta + \lambda_{\omega\omega}\sigma^{\mu\nu}\omega_{0,\nu} + \lambda_{\omega\mathfrak{t}}\mathfrak{t}^{\mu\nu}\omega_\nu + \epsilon^{\mu\nu\alpha\beta}u_\nu\left(\ell_{\omega\kappa}\nabla_\alpha \kappa_{0,\beta} - \tau_\omega \dot{u}_\alpha \kappa_{0,\beta} + \lambda_{\omega\kappa}I_\alpha \kappa_{0,\beta}\right)$$

$$\tau_\kappa \dot{\kappa}_0^{\langle\mu\rangle} + \kappa_0^\mu = -\beta_0 \dot{u}^\mu + \mathfrak{b}I^\mu + \delta_{\kappa\kappa}\kappa_0^\mu\theta + \left(\lambda_{\kappa\kappa}\sigma^{\mu\nu} + \frac{\tau_\kappa}{2}\omega^{\mu\nu}\right)\kappa_{0,\nu} + \epsilon^{\mu\nu\alpha\beta}u_\nu\left(\frac{\tau_\kappa}{2}\nabla_\alpha\omega_{0,\beta} + \tau_\kappa \dot{u}_\alpha \omega_{0,\beta} + \lambda_{\kappa\omega}I_\alpha\omega_{0,\beta}\right) + \mathfrak{t}^{\mu\nu}\left(\tau_{\kappa\mathfrak{t}}\dot{u}_\nu + \lambda_{\kappa\mathfrak{t}}I_\nu\right) + \ell_{\kappa\mathfrak{t}}\Delta^\mu_\lambda \nabla_\nu \mathfrak{t}^{\nu\lambda}$$

$$\tau_\mathfrak{t}\dot{\mathfrak{t}}^{\langle\mu\nu\rangle} + \mathfrak{t}^{\mu\nu} = \mathfrak{d}\beta_0\sigma^{\mu\nu} + \delta_{\mathfrak{t}\mathfrak{t}}\mathfrak{t}^{\mu\nu}\theta + \lambda_{\mathfrak{t}\mathfrak{t}}\mathfrak{t}_\lambda{}^{\langle\mu}\sigma^{\nu\rangle\lambda} + \frac{5}{3}\tau_\mathfrak{t}\mathfrak{t}_\lambda{}^{\langle\mu}\omega^{\nu\rangle\lambda} + \ell_{\mathfrak{t}\kappa}\nabla^{\langle\mu}\kappa_0^{\nu\rangle} + \lambda_{\mathfrak{t}\kappa}I^{\langle\mu}\kappa_0^{\nu\rangle} + \tau_{\mathfrak{t}\omega}\omega^{\langle\mu}\omega_0^{\nu\rangle} + \lambda_{\mathfrak{t}\omega}\sigma_\lambda{}^{\langle\mu}\epsilon^{\nu\rangle\lambda\alpha\beta}u_\alpha\omega_{0,\beta}$$

$I^\mu := \nabla^\mu \alpha_0$

Dissipative spin hydrodynamics (simpler)

DW, 2409.07143 (2024)

$$\tau_\omega \dot{\omega}_0^{\langle\mu\rangle} + \omega_0^\mu = \ldots$$

$$\tau_\kappa \dot{\kappa}_0^{\langle\mu\rangle} + \kappa_0^\mu = \ldots {}_{0,\nu}$$

$$+ \epsilon^{\mu\nu\alpha\beta} u_\nu \left(\frac{\ldots}{2} \nabla_\alpha \omega_{0,\beta} + \tau_\kappa u_\alpha \omega_{0,\beta} + \lambda_{\kappa\omega} I_\alpha \omega_{0,\beta} \right)$$

$$+ \mathfrak{t}^{\mu\nu} \left(\tau_{\kappa\mathfrak{t}} \dot{u}_\nu + \lambda_{\kappa\mathfrak{t}} I_\nu \right) + \ell_{\kappa\mathfrak{t}} \Delta^\mu_\lambda \nabla_\nu \mathfrak{t}^{\nu\lambda}$$

$$\tau_\mathfrak{t} \dot{\mathfrak{t}}^{\langle\mu\nu\rangle} + \mathfrak{t}^{\mu\nu} = \mathfrak{d}\beta_0 \sigma^{\mu\nu} + \delta_{\mathfrak{t}\mathfrak{t}} \mathfrak{t}^{\mu\nu}\theta + \lambda_{\mathfrak{t}\mathfrak{t}} \mathfrak{t}_\lambda{}^{\langle\mu}\sigma^{\nu\rangle\lambda} + \frac{5}{3}\tau_\mathfrak{t} \mathfrak{t}_\lambda{}^{\langle\mu}\omega^{\nu\rangle\lambda} + \ell_{\mathfrak{t}\kappa} \nabla^{\langle\mu}\kappa_0^{\nu\rangle}$$

$$+ \lambda_{\mathfrak{t}\kappa} I^{\langle\mu}\kappa_0^{\nu\rangle} + \tau_{\mathfrak{t}\omega} \omega^{\langle\mu}\omega_0^{\nu\rangle} + \lambda_{\mathfrak{t}\omega} \sigma_\lambda{}^{\langle\mu}\epsilon^{\nu\rangle\lambda\alpha\beta} u_\alpha \omega_{0,\beta}$$

Without fluid gradients:

$$\tau_\omega \dot{\omega}_0^{\langle\mu\rangle} + \omega_0^\mu = \ell_{\omega\kappa} \epsilon^{\mu\nu\alpha\beta} u_\nu \nabla_\alpha \kappa_{0,\beta}$$

$$\tau_\kappa \dot{\kappa}_0^{\langle\mu\rangle} + \kappa_0^\mu = \frac{\tau_\kappa}{2} \epsilon^{\mu\nu\alpha\beta} u_\nu \nabla_\alpha \omega_{0,\beta} + \ell_{\kappa\mathfrak{t}} \Delta^\mu_\lambda \nabla_\nu \mathfrak{t}^{\nu\lambda}$$

$$\tau_\mathfrak{t} \dot{\mathfrak{t}}^{\langle\mu\nu\rangle} + \mathfrak{t}^{\mu\nu} = \ell_{\mathfrak{t}\kappa} \nabla^{\langle\mu}\kappa_0^{\nu\rangle}$$

$I^\mu := \nabla^\mu \alpha_0$

Dissipative spin hydrodynamics (simpler)

DW, 2409.07143 (2024)

$\tau_\omega \dot{\omega}_0^{\langle\mu\rangle} + \omega_0^\mu$

$\tau_\kappa \dot{\kappa}_0^{\langle\mu\rangle} + \kappa_0^\mu$

$+ \epsilon^{\mu\nu\alpha\beta} u_\nu \left(\frac{\quad}{2} \nabla_\alpha \omega_{0,\beta} + \tau_\kappa u_\alpha \omega_{0,\beta} + \lambda_{\kappa\omega} I_\alpha \omega_{0,\beta} \right)$

$\tau_{\mathfrak{t}} \dot{\mathfrak{t}}^{\langle\mu\nu\rangle}$

Without fluid gradients:

$$\tau_\omega \dot{\omega}_0^{\langle\mu\rangle} + \omega_0^\mu = \ell_{\omega\kappa} \epsilon^{\mu\nu\alpha\beta} u_\nu \nabla_\alpha \kappa_{0,\beta}$$

$$\tau_\kappa \dot{\kappa}_0^{\langle\mu\rangle} + \kappa_0^\mu = \frac{\tau_\kappa}{2} \epsilon^{\mu\nu\alpha\beta} u_\nu \nabla_\alpha \omega_{0,\beta} + \ell_{\kappa\mathfrak{t}} \Delta^\mu_\lambda \nabla_\nu \mathfrak{t}^{\nu\lambda}$$

$$\tau_{\mathfrak{t}} \dot{\mathfrak{t}}^{\langle\mu\nu\rangle} + \mathfrak{t}^{\mu\nu} = \ell_{\mathfrak{t}\kappa} \nabla^{\langle\mu} \kappa_0^{\nu\rangle}$$

Let $\tau_{\mathfrak{t}} \to 0$ to get Bloch–Torrey-type contributions:

$$\tau_\omega \dot{\omega}_0^{\langle\mu\rangle} + \omega_0^\mu = \ell_{\omega\kappa} \epsilon^{\mu\nu\alpha\beta} u_\nu \nabla_\alpha \kappa_{0,\beta}$$

$$\tau_\kappa \dot{\kappa}_0^{\langle\mu\rangle} + \kappa_0^\mu = \frac{\tau_\kappa}{2} \epsilon^{\mu\nu\alpha\beta} u_\nu \nabla_\alpha \omega_{0,\beta} + \frac{\ell_{\kappa\mathfrak{t}} \ell_{\mathfrak{t}\kappa}}{2} \Delta^\mu_\lambda \Delta \kappa_0^\lambda + \frac{\ell_{\kappa\mathfrak{t}} \ell_{\mathfrak{t}\kappa}}{6} \nabla^\mu \nabla_\lambda \kappa_0^\lambda$$

$I^\mu := \nabla^\mu \alpha_0$

Relaxation times and first-order coefficients

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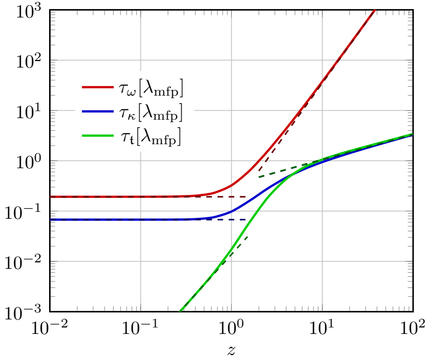

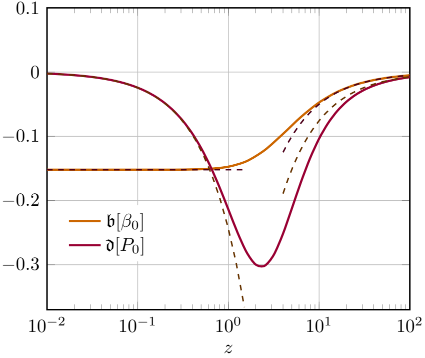

- τ_ω grows with z^2 compared to τ_κ and $\tau_{\mathfrak{t}}$
- $\tau_{\mathfrak{t}}$ vanishes for $z \to 0$

Relaxation times and first-order coefficients

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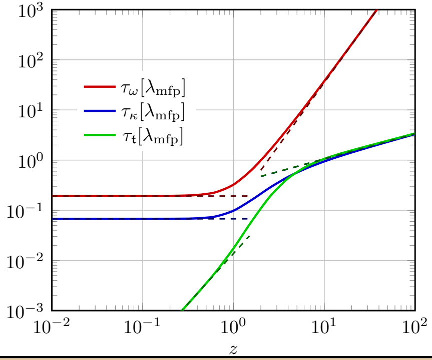

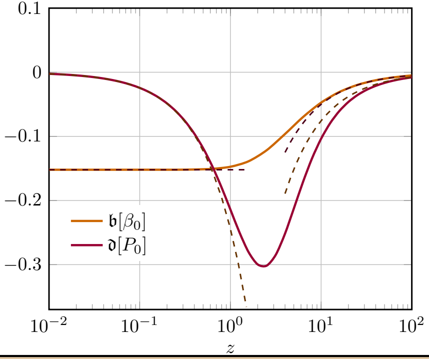

	$\tau_\omega[\lambda_{\mathsf{mfp}}]$	$\tau_\kappa[\lambda_{\mathsf{mfp}}]$	$\tau_{\mathfrak{t}}[\lambda_{\mathsf{mfp}}]$	$\mathfrak{b}[\beta_0]$	$\mathfrak{d}[P_0]$
$z \to 0$	$5/26$	$5/74$	$4z^2/291$	$-45/296$	$-592z/2425$
$z \to \infty$	$\sqrt{\pi}z^{5/2}/16$	$3\sqrt{\pi}\sqrt{z}/16$	$60\sqrt{\pi}\sqrt{z}/317$	$-1/(2z)$	$-240/(317z)$

Second-order coefficients for ω_0^μ and κ_0^μ

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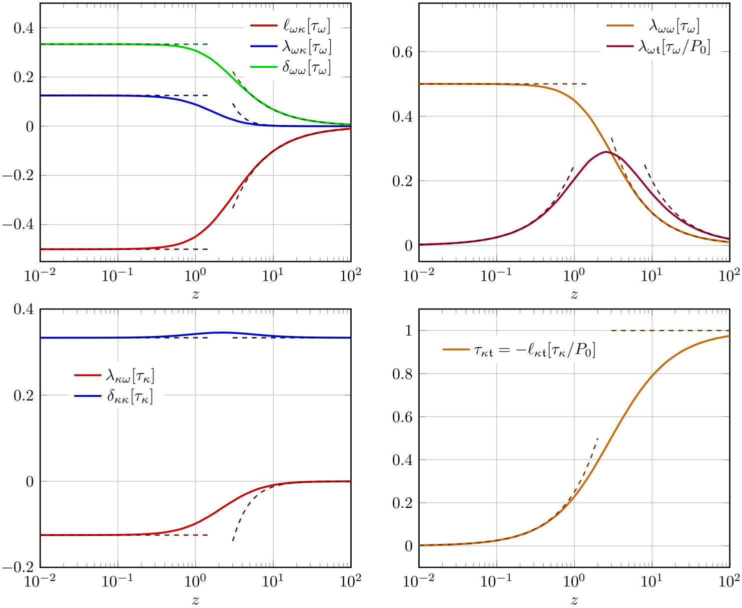
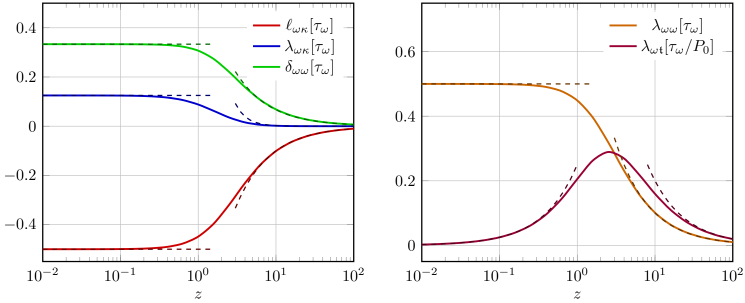
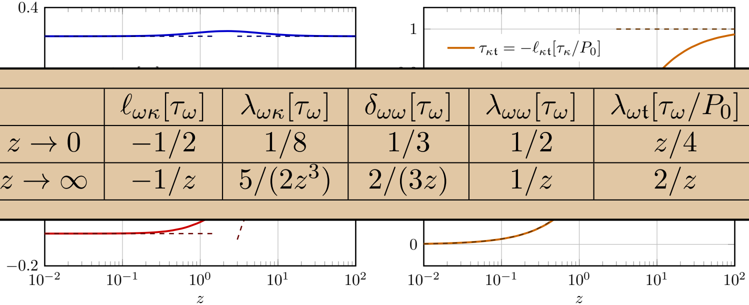


Second-order coefficients for ω_0^μ and κ_0^μ

DW, 2409.07143 (2024)

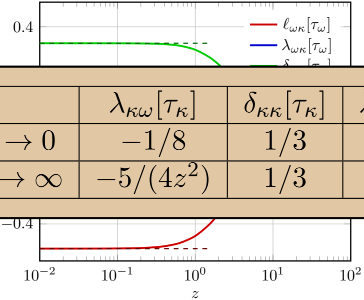


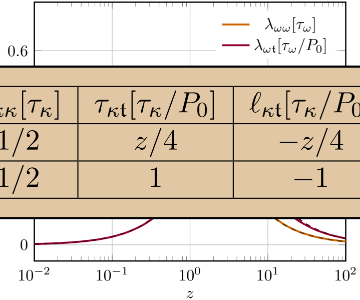


	$\ell_{\omega\kappa}[\tau_\omega]$	$\lambda_{\omega\kappa}[\tau_\omega]$	$\delta_{\omega\omega}[\tau_\omega]$	$\lambda_{\omega\omega}[\tau_\omega]$	$\lambda_{\omega\mathfrak{t}}[\tau_\omega/P_0]$
$z \to 0$	$-1/2$	$1/8$	$1/3$	$1/2$	$z/4$
$z \to \infty$	$-1/z$	$5/(2z^3)$	$2/(3z)$	$1/z$	$2/z$

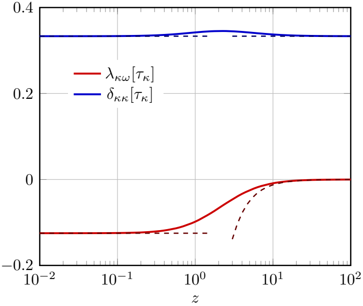
Second-order coefficients for ω_0^μ and κ_0^μ

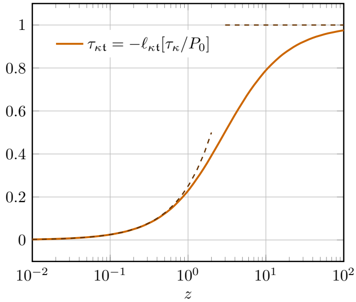
DW, 2409.07143 (2024)





	$\lambda_{\kappa\omega}[\tau_\kappa]$	$\delta_{\kappa\kappa}[\tau_\kappa]$	$\lambda_{\kappa\kappa}[\tau_\kappa]$	$\tau_{\kappa t}[\tau_\kappa/P_0]$	$\ell_{\kappa t}[\tau_\kappa/P_0]$
$z \to 0$	$-1/8$	$1/3$	$1/2$	$z/4$	$-z/4$
$z \to \infty$	$-5/(4z^2)$	$1/3$	$1/2$	1	-1





Second-order coefficients for $\mathfrak{t}^{\mu\nu}$

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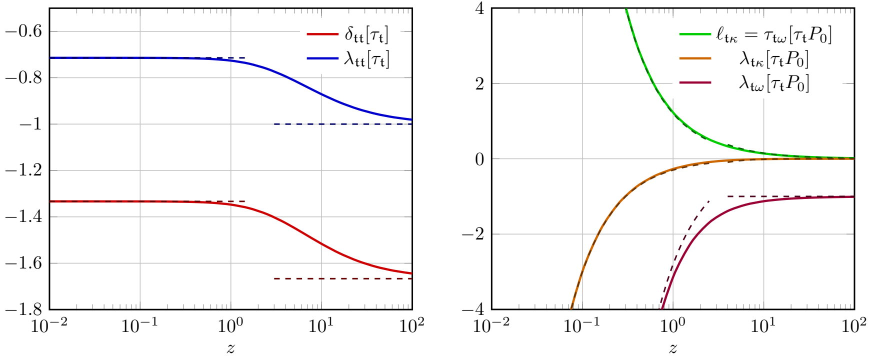
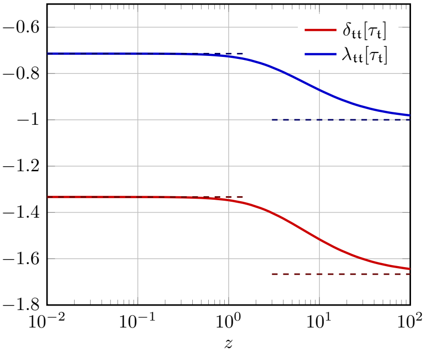
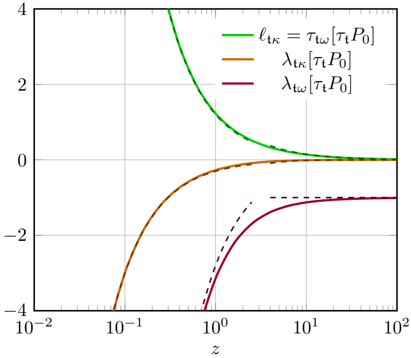


- Some coefficients diverge for $z \to 0$ in units of $\tau_\mathfrak{t}$, but since $\tau_\mathfrak{t} \sim z^2$ they actually vanish
- Equation becomes trivial in the ultrarelativistic case, $\mathfrak{t}^{\mu\nu} = 0$
  - Spin dynamics solely described by ω_0^μ and κ_0^μ

Second-order coefficients for $\mathfrak{t}^{\mu\nu}$

DW, 2409.07143 (2024)





	$\delta_{\mathfrak{tt}}[\tau_\mathfrak{t}]$	$\lambda_{\mathfrak{tt}}[\tau_\mathfrak{t}]$	$\ell_{\mathfrak{t}\kappa}[\tau_\mathfrak{t} P_0]$	$\lambda_{\mathfrak{t}\kappa}[\tau_\mathfrak{t} P_0]$	$\tau_{\mathfrak{t}\omega}[\tau_\mathfrak{t} P_0]$	$\lambda_{\mathfrak{t}\omega}[\tau_\mathfrak{t} P_0]$
$z \to 0$	$-4/3$	$-5/7$	$6/(5z)$	$-3/(10z)$	$6/(5z)$	$-14/(5z)$
$z \to \infty$	$-5/3$	-1	$3/(2z)$	$-3/(2z^2)$	$3/(2z)$	-1

Summary and outlook

- Quantum kinetic theory is a versatile effective microscopic theory
  - At local equilibrium: provides a basis for ideal (spin) hydrodynamics
  - Near local equilibrium: provides a basis for dissipative (spin) hydrodynamics
  - Away from equilibrium: can be studied as a full-fledged nonequilibrium transport theory
- Future applications & developments
  - Study of spin hydro from QKT in various setups, in particular polarization dynamics
    - Allows to provide numerical answers to points (I) and (II) in the context of heavy-ion collisions
  - Numerical implementation of kinetic equations to first order in $\hbar$

Appendix

Conserved currents in QKT

Conserved currents

$$\frac{1}{2}T^{(\mu\nu)} = \int \mathrm{d}\Gamma k^\mu k^\nu f \;,$$

$$S^{\lambda\mu\nu} = \frac{1}{2m}\int \mathrm{d}\Gamma k^\lambda \epsilon^{\mu\nu\alpha\beta} k_\alpha \mathfrak{s}_\beta f \;.$$

$$T^{[\mu\nu]} = \frac{1}{2}\int [\mathrm{d}\Gamma]\widetilde{\mathcal{W}}\Delta^{[\mu} k^{\nu]}\left(f_1 f_2 - f f'\right)$$

Conservation laws

$$\int \mathrm{d}\Gamma k^\mu C[f] = 0$$

$$\frac{\hbar}{2m}\int \mathrm{d}\Gamma \epsilon^{\mu\nu\alpha\beta} k_\alpha \mathfrak{s}_\beta C[f] = \frac{\hbar}{m}\int \frac{\mathrm{d}^4 k}{(2\pi\hbar)^4} k^{[\mu}\mathcal{D}_{\mathcal{V}}^{\nu]}$$

$[\mathrm{d}\Gamma] \coloneqq \mathrm{d}\Gamma_1\,\mathrm{d}\Gamma_2\,\mathrm{d}\Gamma\,\mathrm{d}\Gamma'$

Polarization observables in kinetic theory

Vector Polarization (Pauli-Lubanski Pseudovector)

$$S^\mu(k) := \mathrm{Tr}\left[\hat{S}^\mu\, \hat{\rho}(k)\right] = \frac{1}{N(k)} \int \mathrm{d}\Sigma_\lambda k^\lambda \int \mathrm{d}S(k) \mathfrak{s}^\mu f(x, k, \mathfrak{s})$$

Tensor Polarization

$$\rho_{00}(k) = \frac{1}{3} - \sqrt{\frac{2}{3}} \epsilon^{(0)}_\mu(k) \epsilon^{(0)}_\nu(k) \Theta^{\mu\nu}(k)$$

$$\Theta^{\mu\nu}(k) := \frac{1}{2}\sqrt{\frac{3}{2}} \mathrm{Tr}\left[\left(\hat{S}^{(\mu}\hat{S}^{\nu)} + \frac{4}{3} K^{\mu\nu}\right) \hat{\rho}(k)\right]$$

$$= \frac{1}{2}\sqrt{\frac{3}{2}} \frac{1}{N(k)} \int \mathrm{d}\Sigma_\lambda k^\lambda \int \mathrm{d}S(k) K^{\mu\nu}_{\alpha\beta} \mathfrak{s}^\alpha \mathfrak{s}^\beta f(x, k, \mathfrak{s})$$

$N(k) := \int \mathrm{d}\Sigma_\gamma k^\gamma \int \mathrm{d}S(k) f(x, k, \mathfrak{s})$, $\quad \hat{S}^\mu := -(1/2m)\epsilon^{\mu\nu\alpha\beta} \hat{J}_{\nu\alpha} \hat{P}_\beta$

Polarization in spin hydrodynamics

Local Polarization

$$S_0^\mu = \frac{2\sigma^2\hbar}{N(k)m}\int \mathrm{d}\Sigma_\lambda k^\lambda \left(u^\mu \omega_0^\nu k_\nu - E_\mathbf{k}\omega_0^\mu + \epsilon^{\mu\nu\alpha\beta}u_\nu k_\alpha \kappa_{0,\beta}\right) f_0 \widetilde{f}_0$$

$$\delta S^\mu = -\frac{2\sigma}{N(k)}\int \mathrm{d}\Sigma_\lambda k^\lambda K^{\mu\gamma}\Xi_{\gamma\alpha} f_0 \widetilde{f}_0 \times \left(\mathfrak{x}_n \epsilon^{\alpha\beta\rho\sigma}u_\beta k_\rho n_\sigma + \mathfrak{x}_\mathfrak{t} \mathfrak{t}_\rho{}^{\langle\beta}\epsilon^{\gamma\rangle\alpha\sigma\rho}u_\sigma k_{\langle\beta}k_{\gamma\rangle}\right)$$

Global Polarization

$$\overline{S}_0^\mu = -\frac{2\sigma^2\hbar}{\overline{N}m}\int \mathrm{d}\Sigma_\lambda \left(J_{21}u^\mu\omega_0^\lambda + J_{20}\omega_0^\mu u^\lambda + J_{21}\epsilon^{\mu\nu\lambda\beta}u_\nu \kappa_{0,\beta}\right)$$

$$\delta\overline{S}^\mu = \frac{\sigma}{\overline{N}}\frac{1}{2}\int \mathrm{d}\Sigma_\lambda\, B_0 \epsilon^{\mu\lambda\alpha\beta}u_\alpha n_\beta$$

$\mathfrak{x}_n := \frac{1}{2}\sum_n \mathcal{H}_{\mathbf{k}n}^{(1,1)} \frac{\mathfrak{b}_n^{(1)}}{\varkappa}$, $\mathfrak{x}_\mathfrak{t} := \frac{2}{3}\sum_n \mathcal{H}_{\mathbf{k}n}^{(1,2)} \frac{\mathfrak{d}_n}{\mathfrak{d}_0}$

Nonlocal collisions

DW, NW, ES, 2306.05936 (2023)

Spacetime shifts

$$\Delta^\mu \quad := \quad -\frac{i\hbar}{8m}\frac{m^4}{\mathcal{W}} M^{\gamma_1\gamma_2\delta_1\delta_2} M^{\zeta_1\zeta_2\eta_1\eta_2} h_{1,\gamma_1\eta_1} h_{2,\gamma_2\eta_2} h'_{\zeta_2\delta_2}[h,\gamma^\mu]_{\zeta_1\delta_1}$$

- Depend on the transfer-matrix elements

$$\langle 11'|\,\widehat{t}\,|22'\rangle = \bar{u}_{1,\alpha}\bar{u}_{1',\beta}u_{2,\gamma}u_{2',\delta}M^{\alpha\beta\gamma\delta}$$

- Manifestly covariant
  - $\rightarrow$ no "no-jump" frame

$h := \frac{1}{4}(\mathbb{1} + \gamma_5 \not{s})(\not{k} + m)$

Moment equations: Spin-rank 1

- Same procedure as for the moments of spin-rank 0

Moment equation for $\ell = 2$

$$\dot{\tau}_r^{\langle\mu\rangle,\nu\lambda} - \mathfrak{C}_{r-1}^{\langle\mu\rangle,\nu\lambda} = \cdots$$

- Navier-Stokes limit: $\mathfrak{C}_{r-1}^{\langle\mu\rangle,\nu\lambda} = 0$
- Contains local and nonlocal contributions
  - $\mathfrak{C}_{r,\text{local}}^{\langle\mu\rangle,\nu\lambda} \sim \tau_r^{\mu,\nu\lambda}$
  - $\mathfrak{C}_{r,\text{nonlocal}}^{\langle\mu\rangle,\nu\lambda} \sim \sigma_\rho{}^{\langle\nu}\epsilon^{\lambda\rangle\mu\alpha\rho}u_\alpha$
- Leads to shear-induced polarization, coefficient independent of total cross-section
- Magnitude not yet clear

N. Weickgenannt, DW, E. Speranza, D. H. Rischke, Phys. Rev. D 106, L091901 (2022)
N. Weickgenannt, DW, E. Speranza, D. H. Rischke, Phys. Rev. D 106, 096014 (2022)

Moment equations: Spin-rank 2

Moment equation for $\ell = 0$

$$\dot{\psi}_r^{\langle\mu\nu\rangle} - \mathfrak{C}_{r-1}^{\langle\mu\nu\rangle} = -\frac{\theta}{3}\left[(r+2)\psi_r^{\langle\mu\nu\rangle} - (r-1)m^2\psi_{r-2}^{\langle\mu\nu\rangle}\right]\theta + r\psi_{r-1}^{\langle\mu\nu\rangle,\alpha}\dot{u}_\alpha$$
$$-\Delta^{\mu\nu}_{\alpha\beta}\nabla_\gamma\psi_{r-1}^{\alpha\beta,\gamma} + (r-1)\psi_{r-2}^{\langle\mu\nu\rangle,\alpha\beta}\sigma_{\alpha\beta}$$

- No dependence on equilibrium quantities appears because moments of spin-rank 2 do not appear in any conserved current
- Nonetheless, they are important to e.g. describe **tensor polarization** of spin-1 particles

  DW, NW, ES, 2207.01111 (2022)

  - Main argument: $\mathfrak{C}_{r-1}^{\langle\mu\nu\rangle} \simeq A_r\psi_r^{\mu\nu} + B_r\pi^{\mu\nu}$ leads to $\psi_r^{\mu\nu} \sim \pi^{\mu\nu}$ in the Navier-Stokes limit

$\Delta^{\mu\nu}_{\alpha\beta} := (\Delta^{(\mu}_\alpha\Delta^{\nu)}_\beta)/2 - (1/3)\Delta^{\mu\nu}\Delta_{\alpha\beta}$

Alignment of ϕ-mesons

- Spin-1 particles feature tensor polarization ($\hat{=}$ alignment)
- Intuition: "Polarization counts the difference between spin-projections $+1$ and -1, alignment counts spin-projection 0"
  - Larger than expected
  - Many theoretical developments
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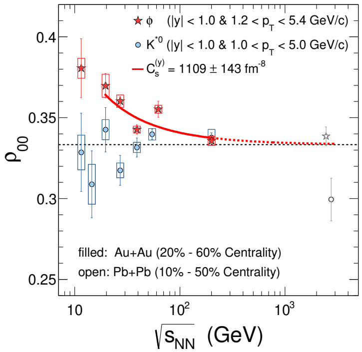
    J.-H. Chen, Z.-T. Liang, Y.-G. Ma, X.-L. Sheng, Q. Wang, 2407.06480



STAR collaboration, Nature 614, 244-248 (2023)

Alignment in QKT

- Spin-1 Wigner function has 16 components, 9 of which are independent

$$W^{\mu\nu} = E^{\mu\nu} f_E + \frac{k^{(\mu}}{2k} F_S^{\nu)} + F_K^{\mu\nu} + K^{\mu\nu} f_K + i\frac{k^{[\mu}}{2k} F_A^{\nu]} + i\epsilon^{\mu\nu\alpha\beta} \frac{k_\alpha}{m} G_\beta$$

  - $f_K \longrightarrow$ particle-number density
  - $G^\mu \longrightarrow$ polarization
  - $F_K^{\mu\nu} \longrightarrow$ tensor polarization/ alignment
- Several approaches on the market (incomplete list)
  - Coalescence
    Y.-G. Yang, R.-H. Fang, Q. Wang, X.-N. Wang, Phys. Rev. C, 97, 034917 (2018)
  - Strong force fields
    X.-L. Sheng, L. Oliva, Z.-T. Liang, Q. Wang, X.-N. Wang, Phys. Rev. Lett. 131, 042304 (2023)
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  - Holography
    Y.-Q. Zhao, X.-L. Sheng, S.-W. Li, D. Hou,2403.07468
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