Applications of quantum kinetic theory to heavy ion collisions

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(**Spin and quantum features of QCD plasma**, ECT*, Sept. 20, 2024)

Relativistic kinetic theory

- Kinetic theory describes the statistical dynamics of **dilute** quasi-particles **out of equilibrium** in **phase space**.

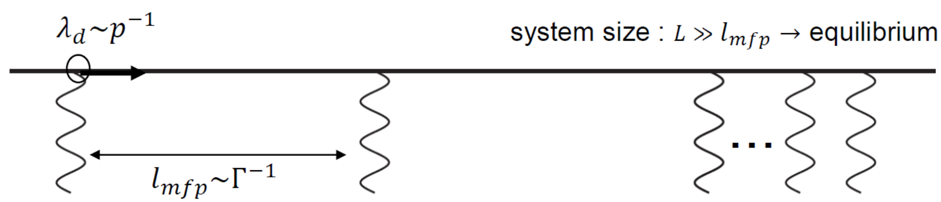
❖ Boltzmann (Vlasov) Eq. : $q^\mu \Delta_\mu f(q, X) = q^\mu \mathcal{C}_\mu[f], \quad \Delta_\mu = \partial_\mu + F_{\nu\mu}\frac{\partial}{\partial q^\nu}.$

(on-shell: $q^2 = m^2$) ⟹ $\partial_\mu J^\mu = 0, \quad \partial_\mu T^{\mu\nu} = F^{\nu\rho} J_\rho.$

moments : $J^\mu(X) = \int \frac{d^3\mathbf{q}}{(2\pi)^3}\frac{q^\mu}{E_q} f(q, X), \;\; T^{\mu\nu}(X) = \int \frac{d^3\mathbf{q}}{(2\pi)^3}\frac{q^\mu q^\nu}{E_q} f(q, X).$

❖ Valid for weak coupling : mean free path ≫ de Broglie wavelength

⟹ In principle not applicable to sQGP



✓ Directly in connection to underlying QFT

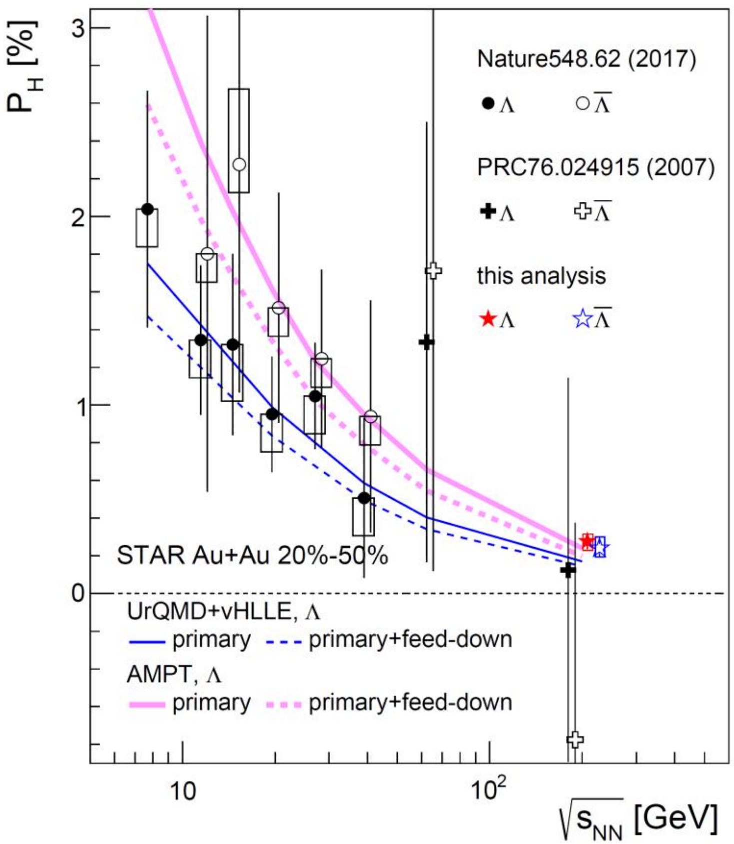
✓ Phase space info. is known : particle spectra

✓ Derivation of hydrodynamics by coarse graining

❖ "Quantum" kinetic theory (QKT) : semi-classical corrections (gradient exp.)

Global Λ polarization in HIC

- The large AM generated in HIC could induce spin polarization of the QGP via spin-orbit interaction. (relativistic Barnett effect) Z.-T. Liang and X.-N. Wang, PRL. 94, 102301 (2005)
- Global polarization of Λ hyperons :



L. Adamczyk et al. (STAR), Nature 548, 62 (2017)

❖ In global equilibrium : $\partial_\mu \beta_\nu + \partial_\nu \beta_\mu = 0$

$$\mathcal{P}^\mu(\boldsymbol{p}) = \frac{1}{8m}\epsilon^{\mu\nu\rho\sigma} p_\nu \frac{\int d\Sigma \cdot p\,\omega_{\rho\sigma} f_p^{(0)}(1-f_p^{(0)})}{\int d\Sigma \cdot p f_p^{(0)}},$$

$$\omega_{\mu\nu} = \frac{1}{2}(\partial_\mu \beta_\nu - \partial_\nu \beta_\mu).$$

thermal vorticity $(\beta^\mu \equiv u^\mu / T)$

F. Becattini, et al., Ann. Phys. 338, 32 (2013)

R. Fang, et al., PRC 94, 024904 (2016)

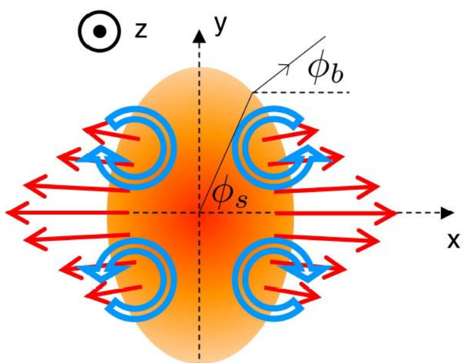
❖ Indication of strong (kinetic) vorticity :

$$P_{\Lambda(\bar{\Lambda})} \simeq \frac{1}{2}\frac{\omega}{T} \pm \frac{\mu_\Lambda B}{T} \Longrightarrow \omega \sim 10^{22}\,\mathrm{s}^{-1}$$

F. Becattini et al., PRC95, 054902 (2017)

❑ Local vorticity :

transverse expansion :
longitudinal vorticity
& polarization

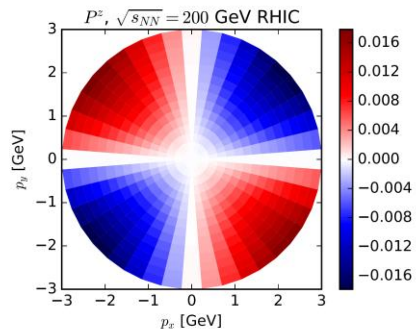


Why do we need the spin transport theory?

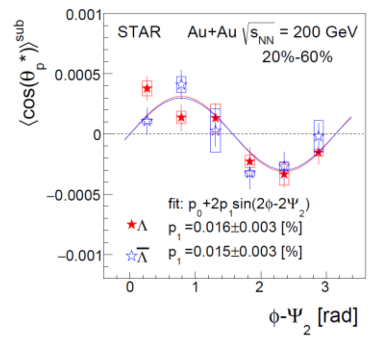
- Sign problem :

spin harmonics :

$$\frac{dP^z}{2\pi d\phi} = f_{z,0} + 2f_{z,2}\sin(2\phi)$$



v.s.



(same structure, opposite signs!)

F. Becattini, I. Karpenko, PRL 120, 012302 (2018). $f^{\rm th}_{z,2} < 0$ $f^{\rm exp}_{z,2} > 0$ J. Adam et al. (STAR, PRL. 123, 132301 (2019).

- How to understand the **dynamical spin polarization** beyond global equilibrium?
- Relativistic angular momentum (canonical) :

$$M_C^{\lambda\mu\nu} = M_S^{\lambda\mu\nu} + M_O^{\lambda\mu\nu},$$

$$M_S^{\lambda\mu\nu} = -\frac{1}{2}\epsilon^{\lambda\mu\nu\rho}\bar{\psi}\gamma_\rho\gamma_5\psi = -\frac{1}{2}\epsilon^{\lambda\mu\nu\rho}J_{5\rho},$$

$$M_O^{\lambda\mu\nu} = \frac{i}{2}\bar{\psi}\gamma^\lambda\left(x^\mu\overleftrightarrow{\partial}^\nu - x^\nu\overleftrightarrow{\partial}^\mu\right)\psi = x^\mu T_C^{\lambda\nu} - x^\nu T_C^{\lambda\mu}$$

spin-orbit int.

spin, orbit:

$$\Rightarrow\quad -\frac{\hbar}{2}\epsilon^{\lambda\mu\nu\rho}\partial_\lambda J_{5\rho} + 2T_A^{\mu\nu} = 0$$

- Spin polarization spectrum : $\mathcal{P}^\mu(\mathbf{p}) = \frac{\int d\Sigma\cdot p\mathcal{J}_5^\mu(p,X)}{2m\int d\Sigma\cdot\mathcal{N}(p,X)}\Big|_{p_0=\epsilon_{\mathbf{p}}}$

(from Pauli–Lubanski pseudovector)

axial-charge current density in phase space

❖ QKT could provide the phase-space info. for spin transport !

Outline

- Basics of quantum kinetic theory (QKT) : (see also the talk by Valet)
  - Massless fermions : chiral kinetic theory (CKT)
  - Massive fermions

QKT Review : Y. Hidaka S. Pu, Q, Wang, DY, PPNP 127 (2022) 103989

- Applications & extension :
  - Spin polarization (see also the talk by Pu)
  - Self-energy corrections & aCVE
  - Inclusion of color dof & spin alignment
  - Kinetic theory for quarkonium

I will highlight some unsettled issues; derivation of spin hydro etc. will be skipped

(see the talk by Wagner)

CKT with collisions

- The non-QFT approach: adding **Berry curvature** as the quantum correction

  D. T. Son & N. Yamamoto, PRL. 109, 181602 (2012)

  M. Stephanov & Y. Yin, PRL. 109, 162001 (2012)

- QFT derivation : **Wigner functions** (Kadanoff-Baym eq) + **gradient exp**.
- Chiral kinetic eq. (CKE) : J.-W. Chen et al., PRL110, 262301 (2013); Y. Hidaka, S. Pu, DY, PRD 95, 091901 (2017), PRD 97, 016004 (2018); A. Huang et al., PRD 98, 036010 (2018)

$$0=\delta\left(p^2-\hbar\chi\frac{B\cdot q}{q\cdot n}\right)\left[\left(q\cdot\Delta+\frac{\hbar\chi S^{\mu\nu}_{(n)}E_\mu}{q\cdot n}\Delta_\nu+\hbar\chi S^{\mu\nu}_{(n)}(\partial_\mu F_{\rho\nu})\partial^\rho_q\right)f_\chi-\hat{C}[f_\chi]\right],$$

$\chi=+,-$ for $f_\chi=f_R,f_L$.

spin tensor : $S^{\mu\nu}_{(n)}=\frac{\epsilon^{\mu\nu\alpha\beta}q_\alpha n_\beta}{2q\cdot n},\quad E^\mu=n_\nu F^{\mu\nu},\quad B^\mu=\frac{\epsilon^{\mu\nu\alpha\beta}}{2}n_\nu F_{\alpha\beta}.$

(const.) frame vector n^μ : choice of the spin basis

❖ f_χ is frame dependent : the full CKT is frame independent

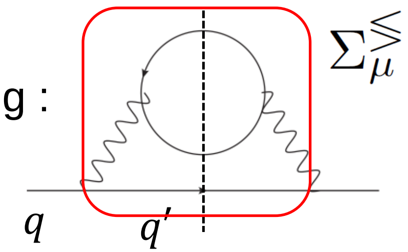
- Quantum corrections on the collision term :

$$\hat{C}[f_\chi]=q^\mu\left(\Sigma^<_\mu(1-f_\chi)-\Sigma^>_\mu f_\chi\right)+\frac{\hbar\chi S^{\mu\nu}_{(n)}E_\mu}{q\cdot n}\left(\Sigma^<_\nu(1-f_\chi)-\Sigma^>_\nu f_\chi\right)+\hbar\chi S^{\mu\nu}_{(n)}\left((1-f_\chi)\Delta_\mu\Sigma^<_\nu-f_\chi\Delta_\mu\Sigma^>_\nu\right)$$

$q\cdot\Sigma^{\lessgtr}\propto$ emission & absorption rate

may also include hbar corrections

e.g., 2-2 scattering :



Wigner functions

- Beyond the moments : $J^\mu_\chi = 2\int \frac{d^4q}{(2\pi)^4}\mathcal{W}^{<\mu}_\chi, \quad T^{\mu\nu}_\chi = \int \frac{d^4q}{(2\pi)^4}(\mathcal{W}^{<\mu}_\chi q^\nu + \mathcal{W}^{<\nu}_\chi q^\mu).$

❖ Wigner functions: ⇒ $J^\mu_V = J^\mu_R + J^\mu_L, \quad J^\mu_5 = J^\mu_R - J^\mu_L.$

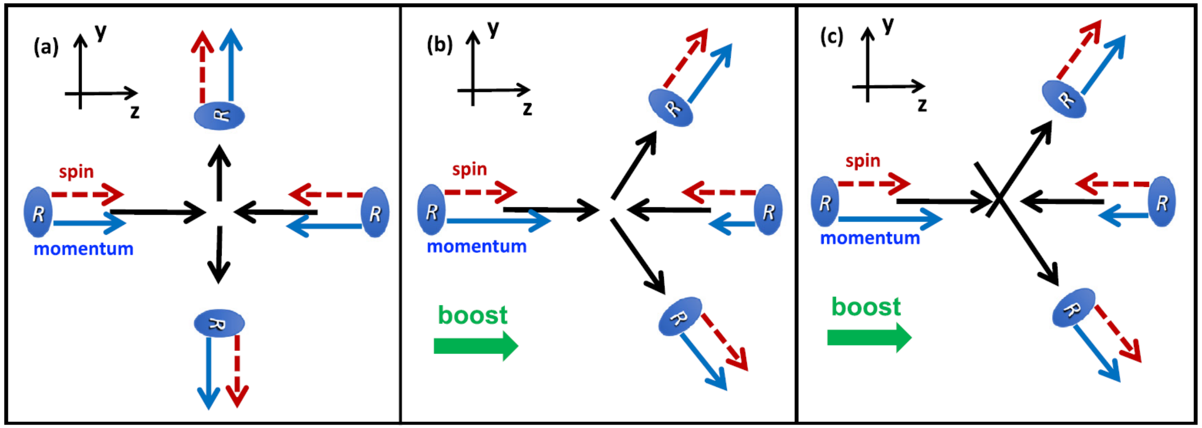
$$\mathcal{W}^{<\mu}_\chi = 2\pi\mathrm{sgn}(q_0)\left[\delta(q^2)\left(q^\mu + \hbar\chi S^{\mu\nu}_{(n)}\Delta_\nu\right)f_\chi - \hbar\chi S^{\mu\nu}_{(n)}\left(\Sigma^<_\nu(1-f_\chi) - \Sigma^> f_\chi\right) + \frac{\hbar\chi}{2}\delta'(q^2)\epsilon^{\mu\nu\alpha\beta}q_\nu F_{\alpha\beta}f_\chi\right]$$

$$\int_q \mathrm{CKE} \to \partial_\mu J^\mu_\chi = \frac{-\hbar\chi}{4\pi^2}E\cdot B$$

side-jump :
spin locking + AM cons.

⇒ spin-orbit int.

J.-Y. Chen et al., PRL. 113, 182302 (2014)



- AM cons. in global equilibrium (const. T, μ) :

$$J^\mu_5 = N_5 u^\mu + \mathcal{O}(\hbar),$$

$$T^{\mu\nu}_A = \sum_\chi \int \frac{d^4q}{(2\pi)^4}(\mathcal{W}^{<\mu}_\chi q^\nu - \mathcal{W}^{<\nu}_\chi q^\mu)$$

$$= \frac{\hbar}{2}N_5\left(\omega^\mu u^\nu - \omega^\nu u^\mu\right).$$

AM cons. ⇒ spin: $-\frac{\hbar}{2}\epsilon^{\lambda\mu\nu\rho}\partial_\lambda J_{5\rho}$ + orbit: $2T^{\mu\nu}_A$ $= 0$

$$-\frac{\hbar}{2}\epsilon^{\lambda\mu\nu\rho}\partial_\lambda J_{5\rho} + 2T^{\mu\nu}_A = 0$$

DY, PRD 98 (2018) 7, 076019

QKT for relativistic massive fermions

- Spin is no longer locked by momenta of massive particles.
- Power counting : spin dof. is induced by gradients and thus subleading
- **Axial kinetic theory (AKT)** : scalar/axial-vector kinetic eqs. (SKE/AKE)

K. Hattori, Y. Hidaka, DY, PRD 100 (2019), 096011
DY, K. Hattori, Y. Hidaka, JHEP 20, 070 (2020)
N. Weickgenannt et al., PRD 100 (2019) 5, 056018
J.-H. Gao, Z.-T. Liang, PRD 100 (2019) 5, 056021

➢ SKE : $p\cdot\Delta f_V = \mathcal{C}[f_V],\ \ \Delta_\mu = \partial_\mu + F_{\nu\mu}\partial_p^\nu.$

standard Boltzmann (Vlasov) eq.

➢ AKE : $p\cdot\Delta\tilde{a}^\mu + F^{\nu\mu}\tilde{a}_\nu + \hbar\mathcal{Q}^\mu[f_V] = \hat{L}^{\mu\nu}\tilde{a}_\nu + \hbar\hat{H}^{\mu\nu}\partial_\nu f_V$ (entangled f_V & $\tilde{a}^\mu$)

($\tilde{a}^\mu \equiv a^\mu f_A$: effective spin four vector)
($\hbar$: gradient corrections in phase space)

$$\begin{cases} p\cdot a = p^2 - m^2 \\ m\to 0 : a^\mu \to p^\mu\,,\ \ f_{V/A} \to (f_R \pm f_L)/2 \end{cases}$$

- **Matrix valued spin dependent distributions (MVSD)** :

N. Weickgenannt et al., PRL127, 052301 (2021)
N. Weickgenannt et al., PRD 104 (1) (2021) 016022
X.-L. Sheng et al., PRD 104 (1) (2021) 016029
D. Wagner et al., PRD 106 (2022) 11, 116021

$$\delta(p^2-m^2)\,p\cdot\partial f(x,p,\mathfrak{s}) = \delta(p^2-m^2)\,\mathfrak{C}_{\text{on-shell}}[f]$$

➢ Non-local collisions from the space-time shift : $f(x+\hbar\Delta, p, \mathfrak{s})$

(valid up to $\hbar$: physically equivalent to the quantum corrections in AKT)

❑ To spin pol. spectra :

$$\mathcal{J}_5^\mu(\boldsymbol{p},x) \propto \int dp_0 \mathcal{A}^\mu(p,x) \begin{cases} \mathcal{A}^\mu = 2\pi\big(\delta(p^2-m^2)\tilde{a}^\mu + \hbar\delta'(p^2-m^2)\tilde{F}^{\mu\nu}p_\nu f_V\big) \ \text{for}\ n^\mu = p^\mu/m \\ \text{or} \qquad \text{(dynamical)} \qquad \text{(non-dynamical)} \\ \mathcal{A}^\mu = 4\pi\delta(p^2-m^2)m\int dS(p)\mathfrak{s}^\mu f(x,p,\mathfrak{s}) \end{cases}$$

❑ AKT : smooth connection to CKT; MVSD : more convenient for deriving spin hydro.

Spin polarization in local equilibrium

- Local equilibrium : $\hat{C}[f^{\rm eq}_\chi]=0 \Rightarrow \mathcal{W}^{{\rm eq}<\mu}_\chi$ Y. Hidaka, S. Pu, DY, PRD 97, 016004 (2018) S. Fang, S. Pu, DY, PRD 106, 016002 (2022)

$(q\cdot\Delta+\hbar\chi\tilde{\Delta})(f^{\rm eq}_\chi+\delta f_\chi)=\hat{C}[\delta f_\chi] \Rightarrow \delta\mathcal{W}^{<\mu}_\chi$ (non-eq corrections : dissipative terms)

➢ Polarization :

$$\mathcal{J}^\mu_5=\mathcal{J}^\mu_{\rm thermal}+\mathcal{J}^\mu_{\rm shear}+\mathcal{J}^\mu_{\rm accT}+\mathcal{J}^\mu_{\rm chemical}+\mathcal{J}^\mu_{\rm EB},$$ (+ dissipative terms)

$$\mathcal{J}^\mu_{\rm thermal} = a\frac{1}{2}\epsilon^{\mu\nu\alpha\beta}p_\nu\omega_{\alpha\beta}, \Rightarrow J^\mu_5=\sigma_{5\omega}\omega^\mu$$

$$\mathcal{J}^\mu_{\rm shear} = -a\frac{1}{(u\cdot p)T}\epsilon^{\mu\nu\alpha\beta}p_\alpha u_\beta p^\sigma\pi_{\sigma\nu},$$

$$\mathcal{J}^\mu_{\rm accT} = -a\frac{1}{2T}\epsilon^{\mu\nu\alpha\beta}p_\nu u_\alpha\left(u\cdot\partial u_\beta-\frac{1}{T}\partial_\beta T\right),$$

$$a=4\pi\hbar\,{\rm sign}(u\cdot p)\delta(p^2)f^{(0)}_V\left(1-f^{(0)}_V\right).$$

C. Yi, S. Pu, DY, PRC 104, 064901(2021)

$$\mathcal{J}^\mu_{\rm chemical} = a\frac{1}{(u\cdot p)}\epsilon^{\mu\nu\alpha\beta}p_\alpha u_\beta\partial_\nu\frac{\mu}{T},$$

(“naïve” extension to massive fermions : $\delta(p^2)\rightarrow\delta(p^2-m^2)$)

$$\mathcal{J}^\mu_{\rm EB} = a\frac{1}{(u\cdot p)T}\epsilon^{\mu\nu\alpha\beta}p_\alpha u_\beta E_\nu+a\frac{B^\mu}{T}, \Rightarrow J^\mu_5=\sigma_{5B}B^\mu$$

- Generalization to the massive case was also derived from the linear response theory and statistical field theory. S. Liu and Y. Yin, PRD 104, 054043 (2021) S. Liu, Y. Yin, JHEP 07, 188 (2021)

(The same and similar results are found for arbitrary mass)

$u^\mu \leftrightarrow \hat{t}^\mu$

F. Becattini, M. Buzzegoli, A. Palermo, PLB 820,136519 (2021)

Non-equilibrium corrections

- Dissipative terms : mostly of $\mathcal{O}(\partial^2)$. e.g., $\tau_{\rm R}\pi^{\mu\nu}\omega_\nu$, $\tau_{\rm R}^{-1}\sim Te^4\ln e^{-1}$.

(via Chapman-Enskog expansion) $|\boldsymbol{q}|\gg\lambda_{\rm mfp}\gg\partial$

e.g. Y. Hidaka, S. Pu, DY, PRD 97, 016004 (2018)
Y. Hidaka, DY, PRD 98, 016012 (2018)
S. Shi, C. Gale, S. Jeon, PRC 103, 044906 (2021)
S. Banerjee et al., arXiv:2405.05089

- Peculiar coupling independent corrections (for shear & spin-Hall) :

$$\mathcal{W}^{<\mu}_{\chi}\propto S^{\mu\nu}_{(u)}\mathcal{C}_\nu[\delta f_\chi]\;:\;\; \text{e.g.,}\,\mathcal{C}_\nu[\delta f_\chi]\sim-\frac{q_\nu\delta f_\chi}{q\cdot u\tau_{\rm R}},\qquad \delta f_\chi\propto\tau_{\rm R}\beta q^\rho q^\sigma\pi_{\rho\sigma}\partial_{q\cdot u}f^{\rm eq}_\chi$$

❖ The coupling cancelation is not subject to the relaxation-time form

S. Lin, Z. Wang, JHEP 12 (2022) 030.
arXiv:2406.10003.
S. Fang, S. Pu, arXiv:2408.09877

$$\delta\mathcal{J}^\mu_5=2\pi\delta(p^2)S^{\mu\nu}_{(u)}\big(g_1(|\boldsymbol{p}|)\pi_{\nu\rho}p^\rho+g_2(|\boldsymbol{p}|)\partial_\nu(\mu/T)\big)$$

- Massive fermions : collision term vanishes in global equilibrium

(for both AKT & MVSD)

Z. Wang, X. Guo, P. Zhuang, Eur. Phys. J. C 81, 799 (2021)
N. Weickgenannt et al., PRL127, 052301 (2021)

- Local-equilibrium (e.g. with shear) WF for massive fermions? collision term does not vanish with the extended WF of massless fermions!

The gap btw massless & massive?

Z. Wang, PRD106, 076011 (2022)
N. Weickgenannt et al., PRD 106, L091901 (2022)

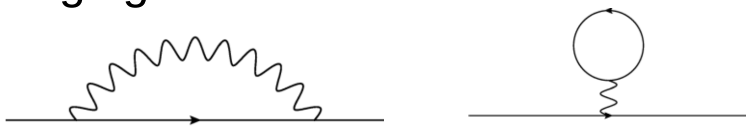
- For phenomenology : near-equilibrium WFs (in QGP or HG?) + hydro

Fully non-equilibrium QKT simulations? e.g. chiral transport model based on CKT

S. Liu, Y. Sun, C. M. Ko, PRL 125, 062301 (2020)

QKT with the "self-energy" corrections

- We mostly consider $\Sigma_{\mu}^{\lessgtr}$ related to $\mathrm{Im}(\Sigma_{\mu}^{\mathrm{r/a}})$ for scattering.
- How about $\bar{\Sigma}_{\mu}=\mathrm{Re}(\Sigma_{\mu}^{\mathrm{r/a}})$?
  - changing onshell condition : $q^2=0\rightarrow q^2=2q\cdot\bar{\Sigma}=m_{\mathrm{th}}^2$ ⟹ $\partial_{\mu}\bar{\Sigma}_{\nu}\sim F_{\mu\nu}^{\mathrm{eff}}$



J. Blaizot, E. Iancu, Phys.Rept. 359 (2002) 355-528

- CKT up to $\mathcal{O}(\hbar)$, $\mathcal{O}(\bar{\Sigma}_{\chi})$, and $\mathcal{O}(\Sigma_{\chi}^{\lessgtr})$: with the replacements

$$q^{\mu}\rightarrow\tilde{q}^{\mu}=q^{\mu}-\bar{\Sigma}_{\chi}^{\mu},\ \ \Delta_{\rho}\rightarrow\tilde{\Delta}_{\rho}=\Delta_{\rho}+(\Delta_{\nu}\bar{\Sigma}_{\chi\rho})\partial_q^{\nu}-(\partial_{q\nu}\bar{\Sigma}_{\chi\rho})\partial^{\nu}.$$

N. Yamamoto, DY ,PRD 109, 056010 (2024)

- Extension to AKT ⟹ $\mathcal{O}(g^2\partial)$ corrections on spin pol. even for the vorticity contribution in global equilibrium!

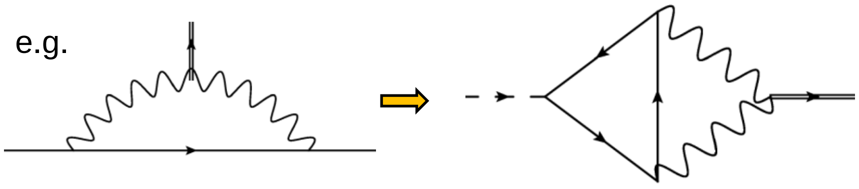
S. Fang, S. Pu, DY, PRD 109, 034034 (2024).

- Related to the radioactive correction on aCVE?

S. Golkar, D. T. Son, JHEP 02, 169 (2012).
D.-F. Hou, H. Liu, H.-c. Ren, PRD 86, 121703 (2012).

$$J_5^{\mu}=N_cN_f\frac{T^2}{6}\left(1+\frac{g^2(N_c^2-1)}{8\pi^2N_c}\right)\omega^{\mu}$$

e.g.



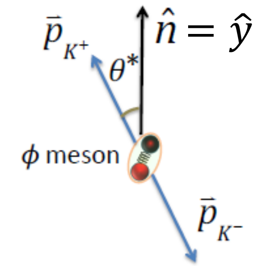
S. Fang, S. Pu, DY, in progress

Spin alignment of vector mesons

- Production of the decay daughter w.r.t the quantization axis :

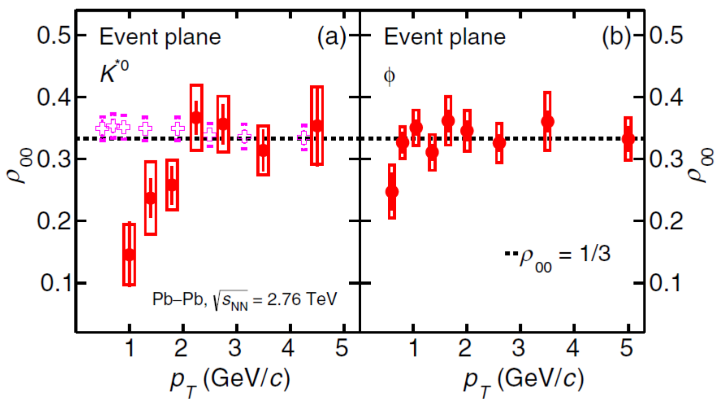
$$\frac{dN}{d\cos\theta^*} \propto [1-\rho_{00}+\cos^2\theta^*(3\rho_{00}-1)]$$

$$\rho_{00} = \frac{1-\langle \mathcal{P}_q^y \mathcal{P}_{\bar{q}}^y \rangle}{3+\langle \mathcal{P}_q^y \mathcal{P}_{\bar{q}}^y \rangle}$$

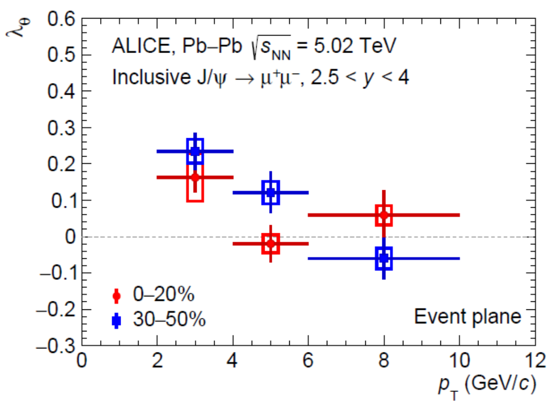


Z.-T. Liang and X.-N. Wang, PLB 629, 20 (2005)

$\rho_{00} \neq 1/3$: spin alignment



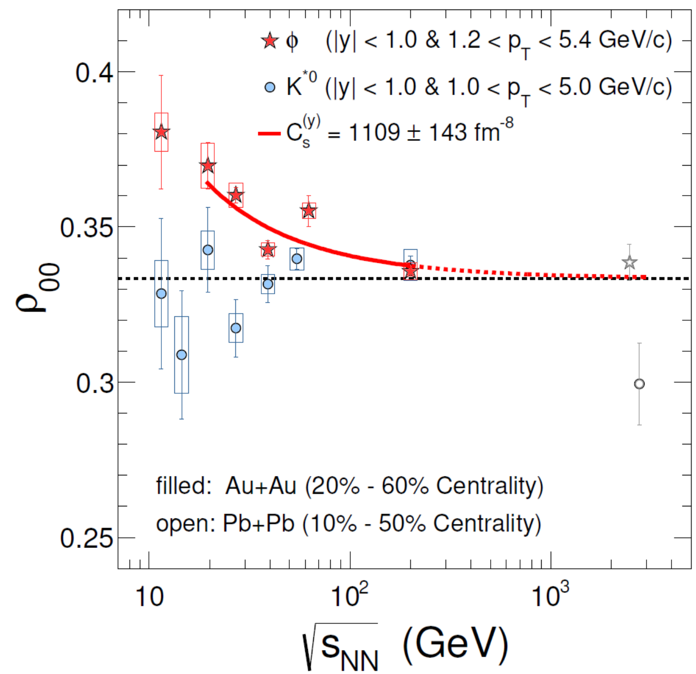
S. Acharya et al. (ALICE), PRL.125, 012301 (2020)



S. Acharya et al. , PRL 131,042303 (2023)

$$\lambda_\theta = \frac{1-3\rho_{00}}{1+\rho_{00}} > 0$$

$$\Rightarrow \rho_{00} < \frac{1}{3}$$



M.S. Abdallah et al. (STAR), Nature 614 (2023) 7947, 244-248

From spin correlations to spin alignment

- Spin alignment puzzle : the deviation of ρ_{00} from 1/3 is unexpectedly large
  e.g. $\rho_{00} \approx \frac{1}{3} - \left(\frac{\omega}{T}\right)^2, \ \frac{\omega}{T} \sim 0.1\%$ at LHC energy. (from Λ polarization)

(see also the talks by Tang & Sheng)

- Spin alignment is led by spin correlations : $\langle \mathcal{P}_q^i \mathcal{P}_{\bar{q}}^i \rangle \neq \langle \mathcal{P}_q^i \rangle \langle \mathcal{P}_{\bar{q}}^i \rangle$
  ⇒ $\rho_{00} \neq 1/3$ with $\langle \mathcal{P}_{q/\bar{q}}^i \rangle = 0$ is possible
  spin polarization of Λ could be unaffected
  (the sources for spin alignment may be fluctuating)
- Spin quantization axis needs not be parallel to the spin polarization (or correlation)
- Anisotropic spin correlation is needed :

X.-L. Sheng et al., PRD 109, 036004, (2024)
A. Kumar, B. Müller, DY, PRD 108, 016020 (2023)

$$\rho_{00}(q) = \frac{1 - \mathrm{Tr_c} \langle \hat{\mathcal{P}}_q^y(\boldsymbol{q}/2) \hat{\mathcal{P}}_{\bar{q}}^y(\boldsymbol{q}/2) \rangle_{\boldsymbol{q}=0}}{3 - \sum_{i=x,y,z} \mathrm{Tr_c} \langle \hat{\mathcal{P}}_q^i(\boldsymbol{q}/2) \hat{\mathcal{P}}_{\bar{q}}^i(\boldsymbol{q}/2) \rangle_{\boldsymbol{q}=0}}$$

(quark model & kinetic equation of vector mesons in the non-relativistic limit)

$$\xrightarrow[|\langle \mathcal{P}_q^i \mathcal{P}_{\bar{q}}^i \rangle| \ll 1]{} \quad \rho_{00} \approx \frac{1}{3} + \frac{1}{9} \mathrm{Tr_c} \left(\langle \hat{\mathcal{P}}_q^x \hat{\mathcal{P}}_{\bar{q}}^x \rangle + \langle \hat{\mathcal{P}}_q^z \hat{\mathcal{P}}_{\bar{q}}^z \rangle - 2 \langle \hat{\mathcal{P}}_q^y \hat{\mathcal{P}}_{\bar{q}}^y \rangle \right)$$

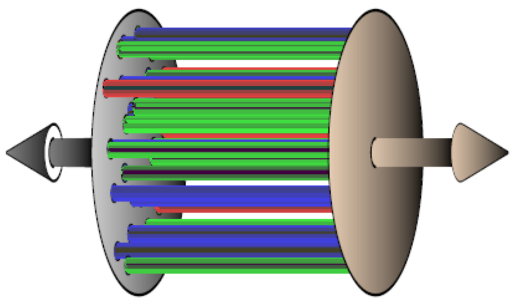
$\rho_{00} = 1/3$ when $\langle \mathcal{P}_q^j \mathcal{P}_{\bar{q}}^j \rangle \neq 0$ is isotropic.

- ❑ Fluctuating color fields from soft gluons in QGP or even the early phase?

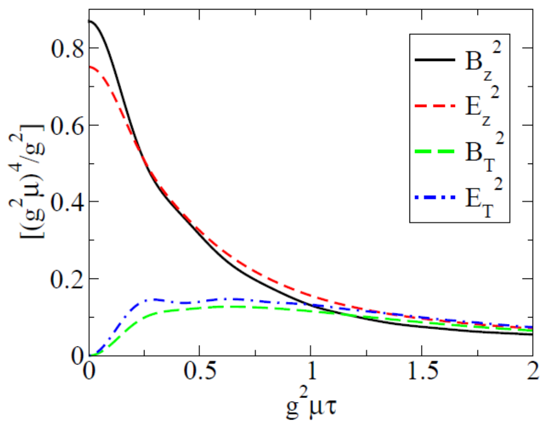
  Extension of QKT to include color dof.

Chromo-electromagnetic fields in HIC

- Color flux tubes in the glasma phase from color glass condensate : **longitudinal** chromo-EM fields in early times.

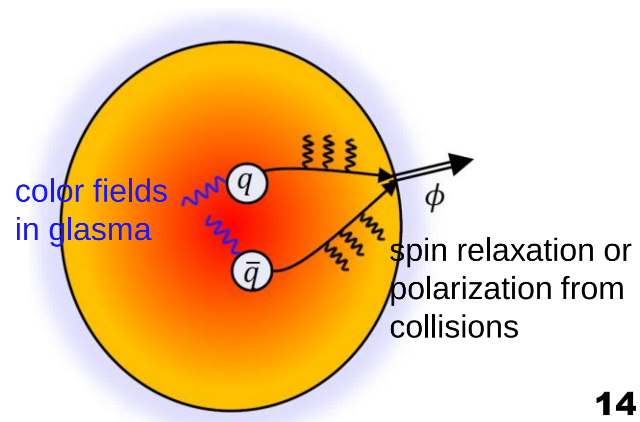


review: F. Gelis, E. Iancu, J. Jalilian-Marian, R. Venugopalan, Ann.Rev.Nucl.Part.Sci.60:463-489,2010



T. Lappi, PLB 643 (2006) 11-16

❖ Why glasma fields ?
(1) intrinsic saturation scale Q_s~1-2 GeV$\gg \omega$
(2) fluctuating
(3) intrinsic anisotropy



WFs and AKE with source terms

- Incorporation of background color fields into WFs and kinetic theory.
- Color decomposition : $O = \boxed{O^s} I + O^a t^a$

U. W. Heinz, Phys. Rev. Lett. 51, 351 (1983)
H. T. Elze, M. Gyulassy, D. Vasak, Nucl. Phys. B276, 706 (1986).

Spin polarization: $\mathcal{P}^{\mu}(\boldsymbol{p}) = \dfrac{\int d\Sigma\cdot p\,\boxed{\mathrm{Tr_c}\mathcal{A}^{\mu}(\boldsymbol{p},X)}}{2m\int d\Sigma\cdot p\,(2\epsilon_{\boldsymbol{p}})^{-1} f_V^{\mathrm{s}}(\boldsymbol{p},X)} = \dfrac{\int d\Sigma\cdot p\,\mathcal{A}^{s\mu}(\boldsymbol{p},X)}{2m\int d\Sigma\cdot p\,(2\epsilon_{\boldsymbol{p}})^{-1} f_V^{\mathrm{s}}(\boldsymbol{p},X)}$

- SKE, AKE, WFs are decomposed into color-singlet & octet components.
- Perturbatively, we may rewrite $f_V^a,\ \tilde{a}^{a\mu}$ in terms of $f_V^s,\ \tilde{a}^{s\mu}$.

M. Asakawa, S. A. Bass, B. Muller, PRL. 96, 252301 (2006)

- Color singlet SKE & AKE : $0 = p\cdot\partial f_V^{\mathrm{s}}(p,X) - \partial_p^{\kappa}\mathscr{D}_{\kappa}[f_V^{\mathrm{s}}]$, $\sim g^2 >$ collosions$\sim g^4$: anomalous viscosity

diffusion

DY, JHEP 06, 140 (2022)
B. Müller, DY, PRD 105, L011901 (2022)
X.-L. Luo, J.-H. Gao, JHEP 11 (2021) 115

$$0 = p\cdot\partial\tilde{a}^{s\mu}(p,X) - \partial_p^{\kappa}\mathscr{D}_{\kappa}[\tilde{a}^{s\mu}] + \hbar\partial_p^{\kappa}\left(\mathscr{A}_{\kappa}^{\mu}[f_V^{\mathrm{s}}]\right),$$

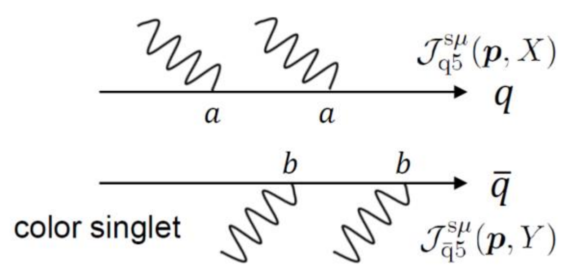
source : dynamical spin polarization

diffusion : $E_{\lambda}^a(X)E_{\rho}^a(X'), B_{\lambda}^a(X)B_{\rho}^a(X')$

source : $B_{\lambda}^a(X)E_{\rho}^a(X')$

also $\propto B(X)E(X')$

- Full WF : $\mathcal{A}^{s\mu}(\boldsymbol{p},X) \propto \tilde{a}^{s\mu} + \boxed{\hbar\mathcal{A}^{\mu}_{\mathrm{nondy}}[f_V^{\mathrm{s}}]}$



No parity violation :

❖ Spin pol. of Λ (from a single strange) : $\langle\mathcal{A}_q^{s\mu}\rangle \approx 0$

❖ Spin correlation of Λ & $\bar{\Lambda}$: $\langle\mathcal{A}_q^{s\mu}\mathcal{A}_{\bar{q}}^{s\nu}\rangle \neq 0$

$$\sim\langle EBEB\rangle > 0$$

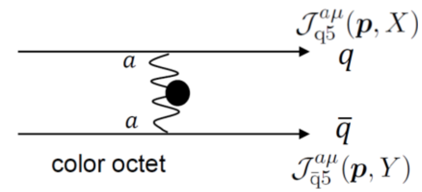
4-field correlations $\propto g^4$

A. Kumar, B. Müller, DY, PRD 107, 076025 (2023)

Spin alignment from glasma

- Spin correlations for spin alignment :

$$\mathrm{Tr_c}\langle \hat{\mathcal{P}}^i_q(\boldsymbol{p})\hat{\mathcal{P}}^i_{\bar{q}}(\boldsymbol{p})\rangle \approx \frac{4\int d\Sigma_X\cdot p\big(\langle \mathcal{A}^{si}_q(\boldsymbol{p},X)\mathcal{A}^{si}_{\bar{q}}(\boldsymbol{p},X)\rangle + \boxed{\langle \mathcal{A}^{ai}_q(\boldsymbol{p},X)\mathcal{A}^{ai}_{\bar{q}}(\boldsymbol{p},X)\rangle}/(2N_c)\big)}{\int d\Sigma_X\cdot p f^{s}_{\mathrm{V}q}(\boldsymbol{p},X) f^{s}_{\mathrm{V}\bar{q}}(\boldsymbol{p},X)}$$



2-field correlations $\propto g^2$

- Dynamical spin polarization from glasma fields : A. Kumar, B. Müller, DY, PRD 108, 016020 (2023)

suppressed

$$\tilde{a}^{a\mu}(\boldsymbol{p},x) \approx -\frac{\hbar g}{2} e^{-(t_{\mathrm{f}}-t_{\mathrm{i}})/\tau^{\mathrm{o}}_{\mathrm{R}}}\left(B^{a\mu}(t_{\mathrm{i}})\partial_{\epsilon_{\boldsymbol{p}}} f^{\mathrm{s}}_{\mathrm{V}}(\epsilon_{\boldsymbol{p}},t_{\mathrm{i}}) - B^{a\mu}(t_{\mathrm{f}})\partial_{\epsilon_{\boldsymbol{p}}} f^{\mathrm{s}}_{\mathrm{V}}(\epsilon_{\boldsymbol{p}},t_{\mathrm{f}})\right)$$

❖ spin correlation : $\langle \mathcal{P}^z_q \mathcal{P}^z_{\bar{q}}\rangle \sim \boxed{\dfrac{Q_s^2}{m_q m_{\bar{q}}}} e^{-2(t_{\mathrm{f}}-t_{\mathrm{i}})/\tau^{\mathrm{o}}_{\mathrm{R}}}$

❖ Order-of-magnitude estimation (for ϕ) : $\rho_{00} \sim \dfrac{1}{3 + 10\, e^{-2(t_{\mathrm{f}}-t_{\mathrm{i}})/\tau^{\mathrm{o}}_{\mathrm{R}}}} < \dfrac{1}{3}$

$Q_s \approx 1 \sim 2\ \mathrm{GeV}$

glasma effect relaxation effect

❖ Heavy-quark approx. : $\tau^{\mathrm{o}}_{\mathrm{R}} \approx \left(\dfrac{g^2 C_2(F) m_D^2 T}{6\pi m^2}\ln g\right)^{-1} \approx 5\,\mathrm{fm/c}$ ⟹ $\rho_{00} \approx 0.24$

M. Hongo et al., JHEP 08, 263 (2022)

(probably consistent with exp. but model dependent)

❑ Caveat : QKT is in general not valid for light quarks in sQGP.

How about heavy quarks or quarkonia?

Quarkonium in heavy ion collisions

- Quarkonium is a useful probe for the quark gluon plasma (QGP) in heavy ion collisions. e.g., J/ψ suppression
- Heavy-quark potential : Vacuum : $V(r) = -\frac{A}{r} + Br$

Finite T :

static screening

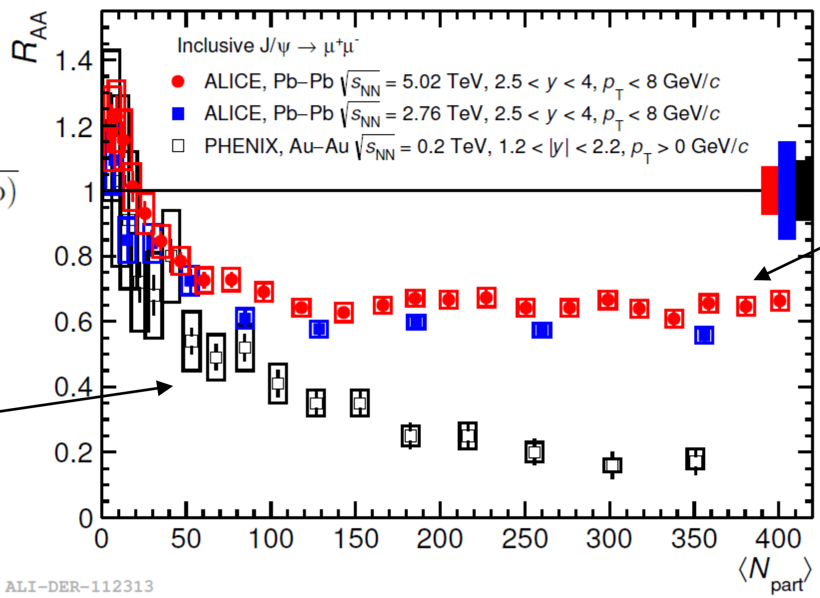
T. Matsui & H. Satz, PLB 178 (1986) 416.
F. Karsch, M. T. Mehr and H. Satz, Z. Phys. C 37 (1988) 617

$$V(r, t \to \infty) = -\frac{A}{r} e^{-m_D r} - i\phi(m_D r)$$

dynamical dissociation

M. Laine et al., JHEP 03 (2007) 054.
A. Beraudo, J. P. Blaizot, C. Ratti, Nucl. Phys. A 806 (2008) 312.

$$R_{AA} = \frac{d^2N/dp_T d\eta(A+A)}{N_{coll} d^2N/dp_T d\eta(p+p)}$$



J/ψ suppression: screening +dissociation

recombination

R. L. Thews, M. Schroedter, J. Rafelski, PRC 63 (2001) 054905

Transport theory for quarkonia

- How to construct the transport theory for quarkonia (in QGP) from the first principle?

❖ Quarkonia (or heavy quarks) are dilute : open quantum system (OQS)

Total system= subsystem + environment $H = H_S + H_E + \boxed{H_I}$ weakly coupled due to separation of scales

X. Yao, Int. J. Mod. Phys. A 36 (2021) 2130010

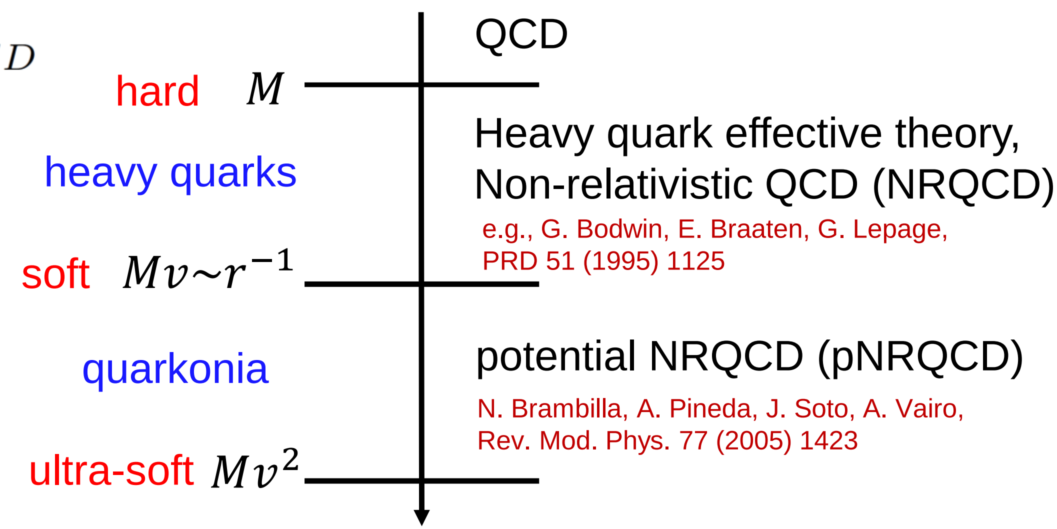
Y. Akamatsu, Prog. Part. Nucl. Phys. 123 (2022) 103932

❖ Separation of scales : non-relativistic effective fields theories (NREFT)

$$M \gg Mv \gg Mv^2, T, \Lambda_{QCD}$$

M : heavy quark mass

v : relative velocity



➢ OQS + pNRQCD ⟹ kinetic theory for quarkonia

Evolution of the density matrix

- Tracking the evolution of the density matrix (unitary → time reversible):

$$\frac{d\rho^{(\mathrm{int})}(t)}{dt} = -i[H_I^{(\mathrm{int})}(t), \rho^{(\mathrm{int})}(t)] \qquad H_I = \sum_\alpha O_\alpha^{(S)} \otimes O_\alpha^{(E)}$$

→ weakly coupled due to M^{-1} or $r\sim(Mv)^{-1}$ suppression

- Tracing over the environment (non-unitary → time irreversible):

$$\rho^{(\mathrm{int})}(t_i) = \rho_S^{(\mathrm{int})}(t_i) \otimes \rho_E^{(\mathrm{int})}(t_i), \quad \rho_S^{(\mathrm{int})}(t) = \mathrm{Tr}_E[\rho^{(\mathrm{int})}(t)].$$

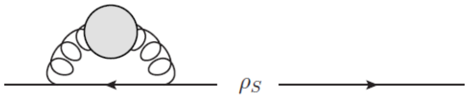
→ a matrix in both color & spin spaces

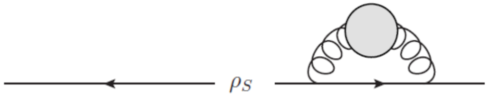
$\approx \rho_T(0)$: static thermal equilibrium medium

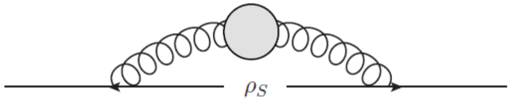
⇨ master eq :

$$\rho_S^{(\mathrm{int})}(t) = \rho_S^{(\mathrm{int})}(0) - \int_0^t dt_1 \int_0^t dt_2 \frac{\mathrm{sign}(t_1 - t_2)}{2} D_{\alpha\beta}(t_1, t_2) \left[O_\alpha^{(S)}(t_1) O_\beta^{(S)}(t_2), \rho_S^{(\mathrm{int})}(0) \right]$$
$$+ \int_0^t dt_1 \int_0^t dt_2 D_{\alpha\beta}(t_1, t_2) \left(O_\beta^{(S)}(t_2) \rho_S^{(\mathrm{int})}(0) O_\alpha^{(S)}(t_1) - \frac{1}{2} \left\{ O_\alpha^{(S)}(t_1) O_\beta^{(S)}(t_2), \rho_S^{(\mathrm{int})}(0) \right\} \right),$$

environment correlation function : $D_{\alpha\beta}(t_1, t_2) = \mathrm{Tr}_E \left(O_\alpha^{(E)}(t_1) O_\beta^{(E)}(t_2) \rho_T(0) \right).$







From OQS to kinetic equations

- Master eq. :

  - Quantum optical limit (low T) → Boltzmann eq.
    $\tau_R \gg \tau_E \sim T^{-1} \& \tau_R \gg \tau_S \sim 1/(Mv^2)$ Wigner transform
    pol indep. : X. Yao, T. Mehen, PRD 99 (2019) 096028; JHEP 02 (2021) 062.
    pol dep. : DY, X. Yao, arXiv:2405.20280.
  - Quantum Brownian motion (high T) → Langevin/Fokker-Planck eq.
    $\tau_R \gg \tau_E \ \& \ \tau_S \gg \tau_E \longrightarrow$ Lindblad eq.

- Markovian approximation (coarse graining such that $\tau_R \gg t \gg \tau_E$) : the time-difference eq. becomes a differential eq. :

recombination dissociation

$$\frac{\partial}{\partial t} f_\lambda(\boldsymbol{x},\boldsymbol{k},t) + \frac{\boldsymbol{k}}{2M}\cdot\nabla_{\boldsymbol{x}} f_\lambda(\boldsymbol{x},\boldsymbol{k},t) = \boxed{\mathcal{C}^+_\lambda(\boldsymbol{x},\boldsymbol{k},t)[f^{(8)}_{Q\bar{Q}\lambda}]} - \boxed{\mathcal{C}^-_\lambda(\boldsymbol{x},\boldsymbol{k},t)[f_\lambda]}\,,$$

$\lambda = \pm 1, 0.$

polarization (vector mesons)

color singlet-octet transitions

e.g., $J/\psi + g \leftrightarrow c + \bar{c}$

- Collision terms : pol. indep. $\sim r^i r^j \underbrace{\langle E^a_i(t) E^a_j(0)\rangle}$, pol. dep. $\sim \frac{\epsilon^{*i}_\lambda \epsilon^j_\lambda}{M^2} \underbrace{\langle B^a_i(t) B^a_j(0)\rangle}$

non-perturbative : dressed by adjoint Wilson lines in the real time contour

❖ One also needs to track $f^{(8)}_{Q\bar{Q}} \sim f_Q f_{\bar{Q}}$ → solving coupled Boltzmann eqs.

X. Yao et al., JHEP 01 (2021) 046

❖ Spin dependence from $f^{(8)}_{Q\bar{Q}\lambda}$? glasma phase?

Summary

❖ Summary

✓ QKT for relativistic fermions is constructed from Wigner functions, which incorporates the chiral anomaly and spin-orbit interaction.

✓ QKT could be used to study dynamical spin polarization in HIC :

➢ Equilibrium & non-equilibrium corrections from interactions (in QGP or HG?).

➢ Self-energy corrections : in connection to radioactive corrections on aCVE?

➢ The gap btw massless & massive fermions in local equilibrium?

✓ Applications to spin alignment :

➢ Spin correlations of quarks and antiquarks dynamically generated from color fields (from the glasma state).

➢ QKT for vector quarkonia from OPS+NREFT

e.g. $$\rho_{\lambda\lambda}(\boldsymbol{k}) = \frac{\int d\Sigma_x \cdot k\, f_\lambda(x, \boldsymbol{k}, t)}{\int d\Sigma_x \cdot k \sum_{\lambda'=\pm1,0} f_{\lambda'}(x, \boldsymbol{k}, t)},$$

ρ_{00}: spin alignment

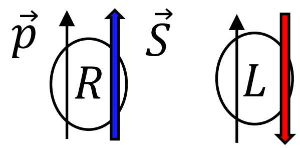
$\rho_{11} - \rho_{-1-1}$: spin polarization

Thank you!

Quantum transport in chiral matter

- Weyl fermions : (massless fermions)

  chirality=helicity



$$\mathbf{J}_V = \mathbf{J}_R + \mathbf{J}_L$$

$$\mathbf{J}_5 = \mathbf{J}_R - \mathbf{J}_L \quad \text{(~spin current)}$$

- Chiral anomaly : $\partial_\mu J^\mu_{R/L} = \pm \frac{\mathbf{E}\cdot\mathbf{B}}{4\pi^2} \Rightarrow \partial_\mu J^\mu_5 = \frac{\mathbf{E}\cdot\mathbf{B}}{2\pi^2}$

  S. Adler, J. Bell, R. Jackiw, 69

  K. Fujikawa, 79

- Quantum transport (in chiral matter) :

$\mathcal{P}$-odd $\mathcal{T}$-even

Chiral magnetic effect (CME) : $\mathbf{J}_V = \boxed{\frac{1}{2\pi^2}\mu_5}\mathbf{B}$

Chiral separation effect (CSE) : $\mathbf{J}_5 = \frac{1}{2\pi^2}\mu_V \mathbf{B}$

Chiral vortical effect (CVE) : $\mathbf{J}_V = \frac{1}{\pi^2}\mu_5\mu_V\,\boldsymbol{\omega}$

kinetic vorticity : $\boldsymbol{\omega} = \frac{1}{2}\nabla\times\boldsymbol{v}$

A. Vilenkin, PRD 20, 1807 (1979). PRD 22, 3080 (1980)

K. Fukushima, D. Kharzeev, H. Warringa, PRD78, 074033 (2008)

D. T. Son, P. Surowka, PRL 103, 191601 (2009)

K. Landsteiner, E. Megias, F. Pena-Benitez, PRL107,021601(2011)

$$\boxed{\mathbf{J}_5 = \left(\frac{\mu_V^2+\mu_5^2}{2\pi^2} + \frac{T^2}{6}\right)\boldsymbol{\omega}}$$

axial CVE (aCVE)

❖ Non-equilibrium chiral transport : chiral kinetic theory (CKT)

e.g., $(q\cdot\Delta + \hbar\tilde{\Delta})f_R = q\cdot C[f_R] + \hbar\tilde{C}[f_R] \Rightarrow \partial_\mu J^\mu_R = -\frac{\hbar}{4\pi^2}E\cdot B \sim O(\partial^2)$

(for right-handed chiral fermions) (chiral anomaly)

Axial kinetic theory (AKT)

- QKT for massive fermions (e.g. for strange quarks)?
- Spin is no longer locked by chirality : a new dynamical dof
- 4 dynamical variables : $f_V, \quad \tilde{a}^\mu \equiv a^\mu f_A$
- Spin four vector a^μ: $\begin{cases} p\cdot a = p^2 - m^2 \\ m \to 0 : a^\mu \to p^\mu \, , \; f_{V/A} \to (f_R \pm f_L)/2 \end{cases}$

K. Hattori, Y. Hidaka, DY, PRD 100 (2019), 096011.
See also N. Weickgenannt, PRD 100 (2019), 056018.
J.-H. Gao & Z.-T. Liang, , PRD100 (2019), 056021.
Z. Wang, et al., PRD 100 (2019), 014015.

❖ power counting : $f_V \sim \mathcal{O}(\hbar^0), \quad f_A \sim \mathcal{O}(\hbar), \quad \Sigma^{\lessgtr} = \Sigma^{\lessgtr}_F + \Sigma^{\lessgtr}_{V\mu}\gamma^\mu + \hbar\Sigma^{\lessgtr}_{A\mu}\gamma^5\gamma^\mu + \frac{\hbar}{2}\Sigma^{\lessgtr}_{T\mu\nu}\sigma^{\mu\nu} + i\hbar^2\Sigma^{\lessgtr}_P\gamma^5.$

DY, K. Hattori, Y. Hidaka, JHEP 20, 070 (2020)

- Kinetic eqs. for n^μ =const., $F_{\mu\nu} = 0 \;\; (p^2 = m^2)$:

$$p\cdot\partial f_V = -p_\mu\widehat{\Sigma^\mu_V f_V} - m\widehat{\Sigma_F f_V},$$ scalar kinetic equation (SKE) $\quad \widehat{XY} = X^{>}Y^{<} - X^{<}Y^{>}$

$$p\cdot\partial\tilde{a}^\mu = -p_\nu\widehat{\Sigma^\nu_V\tilde{a}^\mu} - m\widehat{\Sigma_F\tilde{a}^\mu} + \hbar\delta\mathcal{C}^\mu[f_V],$$ axial kinetic equation (AKE)

$$\delta\mathcal{C}^\mu[f_V] = p^\mu p_\nu\widehat{\Sigma^\nu_A f_V} - p^\mu S^{\rho\nu}_{m(n)}(\partial_\rho\widehat{\Sigma_{V\nu})f_V} + \frac{m}{2}\epsilon^{\mu\nu\rho\sigma}p_\nu\widehat{\Sigma_{T\rho\sigma}f_V} - m^2\widehat{\Sigma^\mu_A f_V}$$
$$+m\left[S^{\mu\nu}_{m(n)}(\partial_\nu\widehat{\Sigma_F)f_V} + \frac{\epsilon^{\mu\nu\rho\sigma}(p_\rho + mn_\rho)}{2(p\cdot n + m)}(\partial_\sigma\widehat{\Sigma_{V\nu})f_V}\right], \quad S^{\mu\nu}_{m(n)} = \frac{\epsilon^{\mu\nu\alpha\beta}q_\alpha n_\beta}{2(q\cdot n + m)}.$$

❖ Wigner function: $\mathcal{A}^{<\mu} = 2\pi\mathrm{sgn}(q_0)\left[\delta(p^2 - m^2)\left(\tilde{a}^\mu + \hbar S^{\mu\nu}_{m(n)}(\Delta_\nu f_V + \widehat{\Sigma_{V\nu}f_V})\right) + \hbar\widetilde{F}^{\mu\nu}p_\nu\delta'(p^2 - m^2)f_V\right], \quad J^\mu_5 = 4\int_q \mathcal{A}^{<\mu}.$

(axial component for spin)

Extended phase space and non-local collisions

- An alternative way to construct the QKT : introducing an distribution function with extended phase space

$$\mathfrak{f}(x,p,\mathfrak{s}) \equiv \frac{1}{2}\left[\bar{\mathcal{F}}(x,p) - \mathfrak{s}\cdot\mathcal{A}(x,p)\right]$$

$$\bar{\mathcal{F}} = \int dS(p)\, \mathfrak{f}(x,p,\mathfrak{s}) \ ,$$

$$\mathcal{A}^\mu = \int dS(p)\, \mathfrak{s}^\mu \mathfrak{f}(x,p,\mathfrak{s}) \ .$$

N. Weickgenannt et al., PRL127, 052301 (2021)
N. Weickgenannt et al., PRD 104 (1) (2021) 016022
X.-L. Sheng et al., PRD 104 (1) (2021) 016029

- On-shell Boltzmann equation :

$$\delta(p^2-m^2)\, p\cdot\partial f(x,p,\mathfrak{s}) = \delta(p^2-m^2)\, \mathfrak{C}_{\text{on-shell}}[f]$$

- Non-local collisions : $\tilde{\mathfrak{C}}_{\text{on-shell}}[f] = \int d\Gamma_1 d\Gamma_2 d\Gamma'\, \widetilde{\mathcal{W}}\,[f(x+\Delta_1,p_1,\mathfrak{s}_1)$

$$\times f(x+\Delta_2,p_2,\mathfrak{s}_2) - f(x+\Delta,p,\mathfrak{s}) f(x+\Delta',p',\mathfrak{s}')]$$

$$+ \int d\Gamma_2\, dS_1(p)\, \mathfrak{W} f(x+\Delta_1,p,\mathfrak{s}_1) f(x+\Delta_2,p_2,\mathfrak{s}_2) \ ,$$

$$\Delta^\mu \equiv -\frac{\hbar}{2m(p\cdot\hat{t}+m)}\epsilon^{\mu\nu\alpha\beta} p_\nu \hat{t}_\alpha \mathfrak{s}_\beta \ , \quad \hat{t}^\mu = (1,0)$$

Axial kinetic theory with color fields

- Incorporation of background color fields into Wigner functions and kinetic equations.
- Color decomposition : $O = O^s I + O^a t^a$

U. W. Heinz, Phys. Rev. Lett. 51, 351 (1983)
H. T. Elze, M. Gyulassy, D. Vasak, Nucl. Phys. B276, 706 (1986).

e.g., $\mathcal{A}^\mu(\boldsymbol{p},x) = \mathcal{A}^{s\mu}(\boldsymbol{p},x)I + \mathcal{A}^{a\mu}(\boldsymbol{p},x)\,t^a,\ \ f_V(\boldsymbol{p},x) = f_{\rm V}^{\rm s}(\boldsymbol{p},x)I + f_{\rm V}^a(\boldsymbol{p},x)\,t^a,$

$\tilde{a}^\mu(\boldsymbol{p},x) = \tilde{a}^{s\mu}(\boldsymbol{p},x)I + \tilde{a}^{a\mu}(\boldsymbol{p},x)\,t^a.$

- Kinetic equations :

DY, JHEP 06, 140 (2022)
B. Müller, DY, PRD 105, L011901 (2022)
X.-L. Luo, J.-H. Gao, JHEP 11,115 (2021)

SKEs : $$p^\rho\left(\partial_\rho f_{\rm V}^{\rm s} + \frac{g}{2N_c}F^a_{\nu\rho}\partial_p^\nu f_{\rm V}^a\right) = \mathcal{C}_{\rm s},\quad p^\rho\left(\partial_\rho f_{\rm V}^a + gF^a_{\nu\rho}\partial_p^\nu f_{\rm V}^{\rm s} + \frac{d^{bca}}{2}gF^b_{\nu\rho}\partial_p^\nu f_{\rm V}^c\right) = \mathcal{C}_{\rm o}^a,$$

diffusion & relaxation

dynamical spin polarization

AKEs : $$p^\rho\partial_\rho\tilde{a}^{s\mu} + \frac{g}{2N_c}\left(p^\rho F^a_{\nu\rho}\partial_p^\nu\tilde{a}^{a\mu} + F^{a\nu\mu}\tilde{a}^a_\nu\right) - \frac{\hbar}{4N_c}\epsilon^{\mu\nu\rho\sigma}p_\rho(\partial_\sigma gF^a_{\beta\nu})\partial_p^\beta f_{\rm V}^a = \mathcal{C}_{\rm s}^\mu,$$

$$p^\rho\partial_\rho\tilde{a}^{a\mu} + g\left(p^\rho F^a_{\nu\rho}\partial_p^\nu\tilde{a}^{s\mu} + F^{a\nu\mu}\tilde{a}^{\rm s}_\nu\right) + \frac{d^{bca}}{2}g\left(p^\rho F^b_{\nu\rho}\partial_p^\nu\tilde{a}^{c\mu} + F^{b\nu\mu}\tilde{a}^c_\nu\right) - \frac{\hbar}{2}\epsilon^{\mu\nu\rho\sigma}p_\rho(\partial_\sigma gF^a_{\beta\nu})\partial_p^\beta f_{\rm V}^{\rm s} = \mathcal{C}_{\rm o}^{a\mu}.$$

Axial Wigner functions : $$\mathcal{A}^{s\mu}(\boldsymbol{p},x) = \frac{1}{2\epsilon_{\boldsymbol{p}}}\left[\tilde{a}^{s\mu} - \frac{\hbar}{4N_c}\tilde{F}^{a\mu\nu}\left(\partial_{p\nu}f_{\rm V}^a - \frac{\epsilon_{\boldsymbol{p}}}{2}\partial_{p_\perp\nu}(f_{\rm V}^a/\epsilon_{\boldsymbol{p}})\right)\right]_{p_0=\epsilon_{\boldsymbol{p}}},$$

$$\mathcal{A}^{a\mu}(\boldsymbol{p},x) = \frac{1}{2\epsilon_{\boldsymbol{p}}}\left[\tilde{a}^{a\mu} - \frac{\hbar}{2}\tilde{F}^{a\mu\nu}\left(\partial_{p\nu}f_{\rm V}^{\rm s} - \frac{\epsilon_{\boldsymbol{p}}}{2}\partial_{p_\perp\nu}(f_{\rm V}^{\rm s}/\epsilon_{\boldsymbol{p}})\right)\right]_{p_0=\epsilon_{\boldsymbol{p}}}.$$

dynamical (w/ memory effect)

non-dynamical (w/o memory effect)

Spin polarization: $$\mathcal{P}^\mu(\boldsymbol{p}) = \frac{\int d\Sigma\cdot p\,{\rm Tr}_c\mathcal{A}^\mu(\boldsymbol{p},x)}{2m\int d\Sigma\cdot p\,(2\epsilon_{\boldsymbol{p}})^{-1}f_{\rm V}^{\rm s}(\boldsymbol{p},x)} = \frac{\int d\Sigma\cdot p\,\mathcal{A}^{s\mu}(\boldsymbol{p},x)}{2m\int d\Sigma\cdot p\,(2\epsilon_{\boldsymbol{p}})^{-1}f_{\rm V}^{\rm s}(\boldsymbol{p},x)}.$$

AKE with the diffusion and source term

- SKE : $0 = p\cdot\partial f^{\mathrm{s}}_{V}(p,X) - \partial^{\kappa}_{p}\mathscr{D}_{\kappa}[f^{\mathrm{s}}_{V}]\big),$ M. Asakawa, S. A. Bass, B. Muller, PRL. 96, 252301 (2006)

diffusion ⇨ anomalous shear viscosity

$$\mathscr{D}_{\kappa}[O] = \bar{C}_2\int^{p}_{k,X'} p^{\lambda}p^{\rho}\underbrace{F^{a}_{\kappa\lambda}(X)F^{a}_{\alpha\rho}(X')}\partial^{\alpha}_{p}O(p,X'),$$

$$\propto \text{e.g.}, E^{a}_{\lambda}(X)U^{ab}(X,X')E^{b}_{\rho}(X'), B^{a}_{\lambda}(X)U^{ab}(X,X')B^{b}_{\rho}(X')$$

$$\int^{p}_{k,X'} \equiv \int d^4k\int\frac{d^4X'}{(2\pi)^4}e^{ik\cdot(X'-X)}\big(\pi\delta(p\cdot k)+iPV(1/p\cdot k)\big).$$

- AKE: $0 = p\cdot\partial\tilde{a}^{\mathrm{s}\mu}(p,X) - \partial^{\kappa}_{p}\mathscr{D}_{\kappa}[\tilde{a}^{s\mu}] + \hbar\partial^{\kappa}_{p}\big(\mathscr{A}^{\mu}_{\kappa}[f^{\mathrm{s}}_{V}]\big),$

diffusion source : dynamical spin polarization

$$\mathscr{A}^{\mu}_{\kappa}[O] = \frac{\bar{C}_2}{2}\epsilon^{\mu\nu\rho\sigma}\int^{p}_{k,X'} p^{\lambda}p_{\rho}\Big(\partial_{X'\sigma}\underbrace{\big(F^{a}_{\kappa\lambda}(X)F^{a}_{\alpha\nu}(X')\big)} + \partial_{X\sigma}\big(F^{a}_{\kappa\nu}(X)F^{a}_{\alpha\lambda}(X')\big)\Big)\partial^{\alpha}_{p}O(p,X').$$

$$\propto \text{e.g.}, E^{a}_{\lambda}(X)U^{ab}(X,X')B^{b}_{\nu}(X')$$

❖ No global parity violation : vanishing spin polarization but nonzero spin correlation

$$\langle\mathcal{A}^{\mu}_{\mathrm{s}}\rangle \approx 0, \quad \langle\mathcal{A}^{\mu}_{\mathrm{s}}\mathcal{A}^{\nu}_{\mathrm{s}}\rangle \neq 0.$$

Spin alignment with momentum dependent

- How to retrieve the momentum dep.? boosting the color fields to the lab frame

$$B_{\mathrm{r}}^{ai} = \gamma(B^{ai} + \epsilon^{ijk} v_j E_k^a) - (\gamma - 1)\boldsymbol{B}^a \cdot \hat{\boldsymbol{v}}\hat{v}^i, \quad v^i = q^i/\sqrt{|\boldsymbol{q}|^2 + M^2} \text{ and } \hat{v}^i = v^i/|\boldsymbol{v}|.$$

- Momentum-dep. analysis (qualitative) : mid-rapidity, small-momentum region

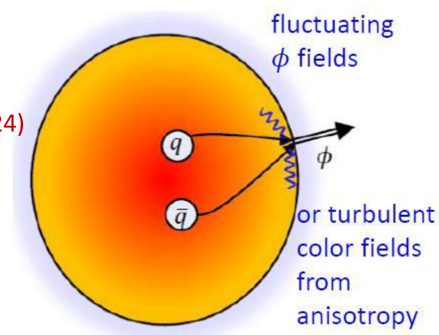
glasma : $\rho_{00} - \frac{1}{3} \propto (v_x^2 - 2v_y^2 - 1) \int d\Sigma_X \cdot q\langle B^{az}(0,\boldsymbol{x}) B^{az}(0,\boldsymbol{x})\rangle$

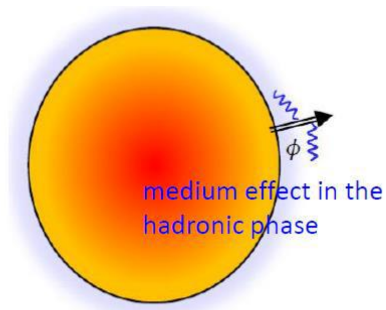
isotropic BFs : $\rho_{00} - \frac{1}{3} \propto (v_x^2 - 2v_y^2) \int d\Sigma_X \cdot q\langle F^a(x) F^a(x)\rangle, \quad B^{ai} = E^{ai} = F^a.$

	small-P_T	large-P_T	central	non-central
glasma	$\rho_{00}^{\phi,J/\psi} < 1/3$	$\rho_{00}^{\phi,J/\psi} \lesssim 1/3$	$\rho_{00}^{\phi,J/\psi} < 1/3$	$\rho_{00}^{\phi,J/\psi} \lesssim 1/3$
effective potential	$\lvert\rho_{00}^{\phi,J/\psi} - 1/3\rvert \gtrsim 0$	$\lvert\rho_{00}^{\phi,J/\psi} - 1/3\rvert > 0$	$\rho_{00}^{\phi,J/\psi} < 1/3$	$\rho_{00}^{\phi,J/\psi} > 1/3$

- Other proposals :

X.-L. Sheng et al., PRD 109, 036004, (2024)
PRL 131, 042304 (2023)
B. Müller, DY, PRD 105, L011901 (2022)
DY, JHEP 06, 140 (2022)

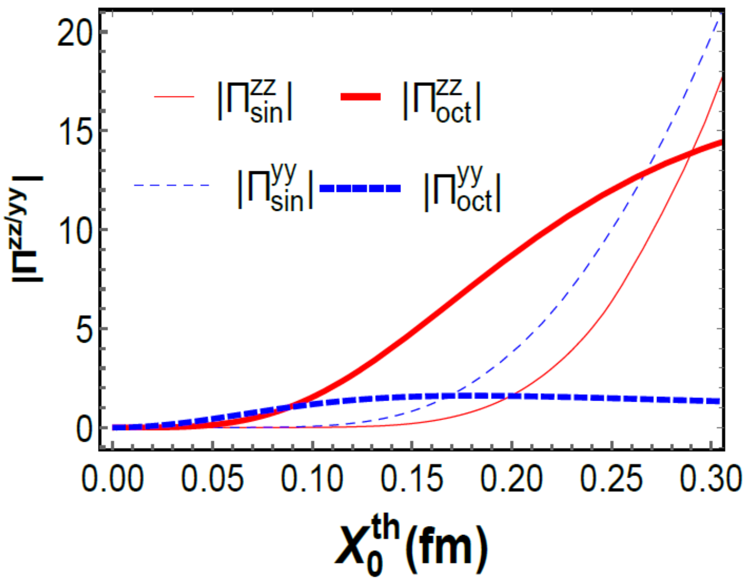




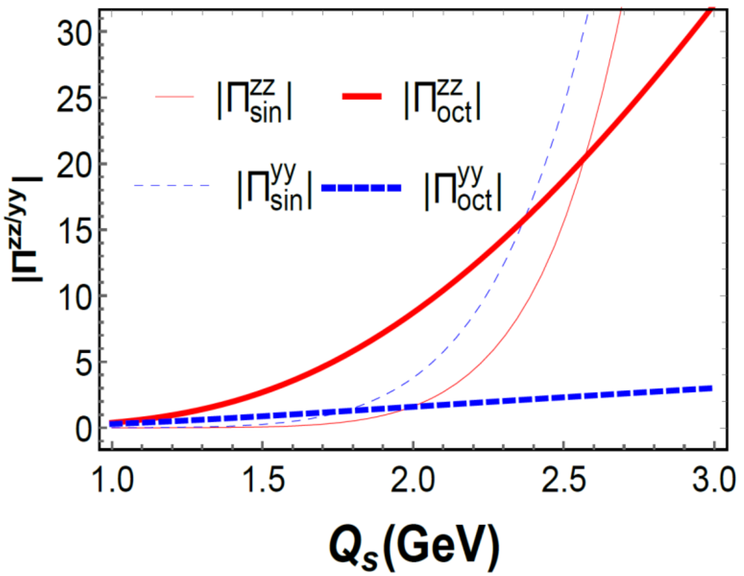
D. Wagner, N. Weickgenannt, E. Speranza,PRR 5, 013187 (2023)
F. Li, S. Liu, arXiv:2206.11890
A. Kumar, Philipp Gubler, DY, PRD 109, 054038 (2024)

Singlet v.s. octet

- More sophisticated analysis : $\mathrm{Tr_c}\langle\hat{\mathcal{P}}^i_q(\boldsymbol{q}/2)\hat{\mathcal{P}}^i_{\bar{q}}(\boldsymbol{q}/2)\rangle = \Pi^{ii}_{\mathrm{oct}} + \Pi^{ii}_{\mathrm{sin}} + \Pi^{ii}_{\mathrm{EM}}$ (suppressed)



$Q_s = 2\ \mathrm{GeV}$



$X_0^{\mathrm{th}} = 0.2\ \mathrm{fm}$

⇨ $Q_s = 2\ \mathrm{GeV}$ & $X_0^{\mathrm{th}} = 0.2\ \mathrm{fm}$ for $|\Pi^{zz}_{\mathrm{oct}}| > |\Pi^{ii}_{\mathrm{sin}}|$

Effective Lagrangian in pNRQCD

- Spin-independent case (with multipole expansion, small $r \sim (Mv)^{-1}$) :

$$\mathcal{L} = S^\dagger(\boldsymbol{R},\boldsymbol{r},t)(i\partial_0 - \mathcal{H}_s)S(\boldsymbol{R},\boldsymbol{r},t) + O^{a\dagger}(\boldsymbol{R},\boldsymbol{r},t)(iD_0 - \mathcal{H}_o)O^a(\boldsymbol{R},\boldsymbol{r},t)$$

kinetic terms & static potentials

$$+ V_A\sqrt{\frac{T_F}{N_c}}\Big(S^\dagger(\boldsymbol{R},\boldsymbol{r},t)\boldsymbol{r}\cdot g\boldsymbol{E}^a(\boldsymbol{R},t)O^a(\boldsymbol{R},\boldsymbol{r},t) + O^{a\dagger}(\boldsymbol{R},\boldsymbol{r},t)\boldsymbol{r}\cdot g\boldsymbol{E}^a(\boldsymbol{R},t)S(\boldsymbol{R},\boldsymbol{r},t)\Big)$$

$$+ V_B d^{abc}O^{a\dagger}(\boldsymbol{R},\boldsymbol{r},t)\boldsymbol{r}\cdot g\boldsymbol{E}^b(\boldsymbol{R},t)O^c(\boldsymbol{R},\boldsymbol{r},t) + \mathcal{O}(r^2)\,,$$

color singlet-octet transitions

$\text{e.g.,}\ J/\psi + g \leftrightarrow c + \bar{c}$

- Spin-dependent case (the transition terms) :

(pseudo) scalar-scalar transitions　　vector-vector transitions

$$\mathcal{L}_t = V_A\sqrt{\frac{T_F}{N_c}}\Big(S_1^\dagger(\boldsymbol{R},\boldsymbol{r},t)\boldsymbol{r}\cdot g\boldsymbol{E}^a(\boldsymbol{R},t)O_1^a(\boldsymbol{R},\boldsymbol{r},t) + \sum_\lambda S_{\lambda i}^\dagger(\boldsymbol{R},\boldsymbol{r},t)\boldsymbol{r}\cdot g\boldsymbol{E}^a(\boldsymbol{R},t)O_{\lambda i}^a(\boldsymbol{R},\boldsymbol{r},t) + \text{h.c.}\Big)$$

$$+\frac{c_4}{M}V_A^s\sqrt{\frac{T_F}{N_c}}\sum_\lambda\Big[S_1^\dagger(\boldsymbol{R},\boldsymbol{r},t)gB_i^a(\boldsymbol{R},t)O_{\lambda i}^a(\boldsymbol{R},\boldsymbol{r},t) + S_{\lambda i}^\dagger(\boldsymbol{R},\boldsymbol{r},t)gB_i^a(\boldsymbol{R},t)O_1^a(\boldsymbol{R},\boldsymbol{r},t) + \text{h.c.}\Big]$$

pseudo scalar-vector transitions :
vector mesons polarized by B fields

Dissociation for vector quarkonia

- Dissociation term :

E of J/ψ ; E of $Q\bar{Q}$; E of gluons

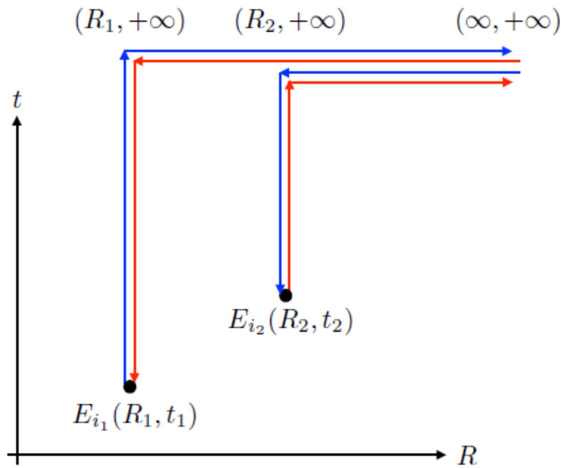
$$\mathcal{C}_\lambda^-(\boldsymbol{x},\boldsymbol{k},t)[f_\lambda] = \int \frac{\mathrm{d}^3p_{\mathrm{cm}}}{(2\pi)^3}\frac{\mathrm{d}^3p_{\mathrm{rel}}}{(2\pi)^3}\int \mathrm{d}^4q\,\delta(E_k^\lambda - E_p - q_0)\delta^3(\boldsymbol{k}-\boldsymbol{p}_{\mathrm{cm}}-\boldsymbol{q})|\mathcal{M}_d|^2 f_\lambda(\boldsymbol{x},\boldsymbol{k},t)\,,$$

$$|\mathcal{M}_d|^2 = \frac{V_A^2 T_F}{N_c}\tilde{g}_{ij}^{E++}(q)\langle\psi^\lambda|r_i|\Psi^\lambda_{\boldsymbol{p}_{\mathrm{rel}}}\rangle\langle\Psi^\lambda_{\boldsymbol{p}_{\mathrm{rel}}}|r_j|\psi^\lambda\rangle + \frac{(c_4V_A^s)^2T_F}{M^2N_c}\tilde{g}_{ij}^{B++}(q)\varepsilon^*_{\lambda i}\varepsilon_{\lambda j}|\langle\psi^\lambda|\Psi_{\boldsymbol{p}_{\mathrm{rel}}}\rangle|^2\,,$$

field-field correlators : $\tilde{g}_{ij}^{V++}(q) = \int \mathrm{d}\delta t\mathrm{d}^3\delta R\, e^{iq_0\delta t - i\boldsymbol{q}\cdot\delta\boldsymbol{R}} g_{ij'}^{V++}(t_1,t_2,\boldsymbol{R}_1,\boldsymbol{R}_2)\,,\quad V=E,B,$

$$\delta t = t_1 - t_2,\quad \delta\boldsymbol{R} = \boldsymbol{R}_1 - \boldsymbol{R}_2,$$

$$g_{ij}^{V++}(t) = \mathrm{Tr}_E\Big\{gV_i^a(\boldsymbol{R},t)W^{ac}[(\boldsymbol{R},t),(\boldsymbol{R},\infty)W^{cb}[(\boldsymbol{R},\infty),(\boldsymbol{R},0)]gV_j^b(\boldsymbol{R},0)\rho_T(0)\Big\}.$$

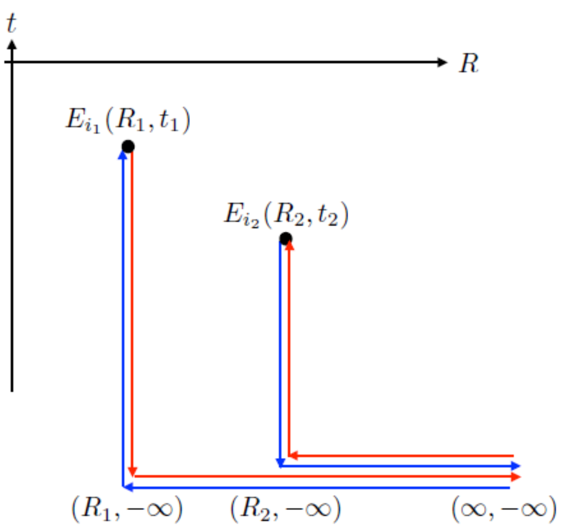


Recombination for vector quarkonia

- Recombination term :

$$\mathcal{C}_\lambda^+(\boldsymbol{x},\boldsymbol{k},t)[f_{Q\bar{Q}}^{(8)},f_{Q\bar{Q}\lambda}^{(8)}]=\int\frac{\mathrm{d}^3p_{\mathrm{cm}}}{(2\pi)^3}\frac{\mathrm{d}^3p_{\mathrm{rel}}}{(2\pi)^3}\int\mathrm{d}^4q\,\delta(E_k^\lambda-E_p+q_0)\delta^3(\boldsymbol{k}-\boldsymbol{p}_{\mathrm{cm}}+\boldsymbol{q})$$
$$\times\Big(|\mathcal{M}_{r,e}|^2f_{Q\bar{Q}\lambda}^{(8)}(\boldsymbol{x},\boldsymbol{p}_{\mathrm{cm}},\boldsymbol{x}_0,\boldsymbol{p}_{\mathrm{rel}},t)+|\mathcal{M}_{r,b}|^2f_{Q\bar{Q}}^{(8)}(\boldsymbol{x},\boldsymbol{p}_{\mathrm{cm}},\boldsymbol{x}_0,\boldsymbol{p}_{\mathrm{rel}},t)\Big)\,,$$

$$|\mathcal{M}_{r,e}|^2=\frac{V_A^2T_F}{N_c}\tilde{g}_{ji}^{E--}(q)\langle\psi^\lambda|r_j|\Psi_{\boldsymbol{p}_{\mathrm{rel}}}^\lambda\rangle\langle\Psi_{\boldsymbol{p}_{\mathrm{rel}}}^\lambda|r_i|\psi^\lambda\rangle\,,$$
$$|\mathcal{M}_{r,b}|^2=\frac{(c_4V_A^s)^2T_F}{M^2N_c}\tilde{g}_{ji}^{B--}(q)\varepsilon_{\lambda i}^*\varepsilon_{\lambda j}|\langle\psi^\lambda|\Psi_{\boldsymbol{p}_{\mathrm{rel}}}\rangle|^2\,.$$



field-field correlators :

$$\tilde{g}_{ij}^{V--}(q)=\int\mathrm{d}\delta t\mathrm{d}^3\delta R\,e^{-iq_0\delta t+i\boldsymbol{q}\cdot\delta\boldsymbol{R}}[g_{ij}^{V--}(t_2,t_1,\boldsymbol{R}_2,\boldsymbol{R}_1)]^{aa}\,,$$

$$[g_{ji}^{V--}(t_2,t_1,\boldsymbol{R}_2,\boldsymbol{R}_1)]^{a_2a_1}\equiv\mathrm{Tr}_E\Big\{W^{a_2b}[(\boldsymbol{R}_1,-\infty),(\boldsymbol{R}_2,-\infty)]$$
$$\times W^{bc}[(\boldsymbol{R}_2,-\infty),(\boldsymbol{R}_2,t_2)]gV_j^c(\boldsymbol{R}_2,t_2)gV_i^d(\boldsymbol{R}_1,t_1)W^{da_1}[(\boldsymbol{R}_1,t_1),(\boldsymbol{R}_1,-\infty)]\rho_T(0)\Big\}\,.$$