Theoretical Progress on Spin Alignment in Heavy-ion Collisions

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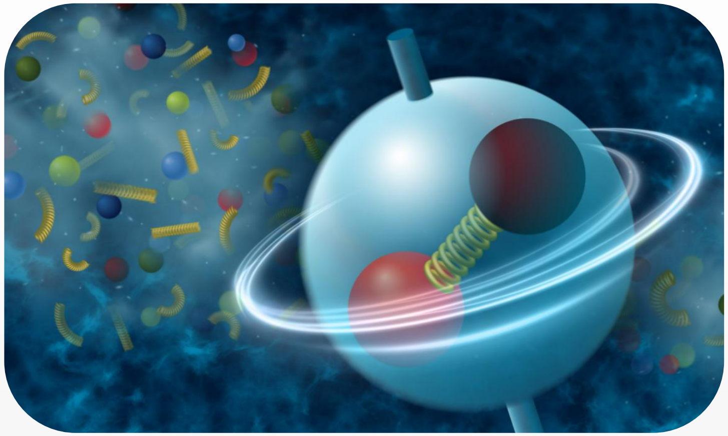
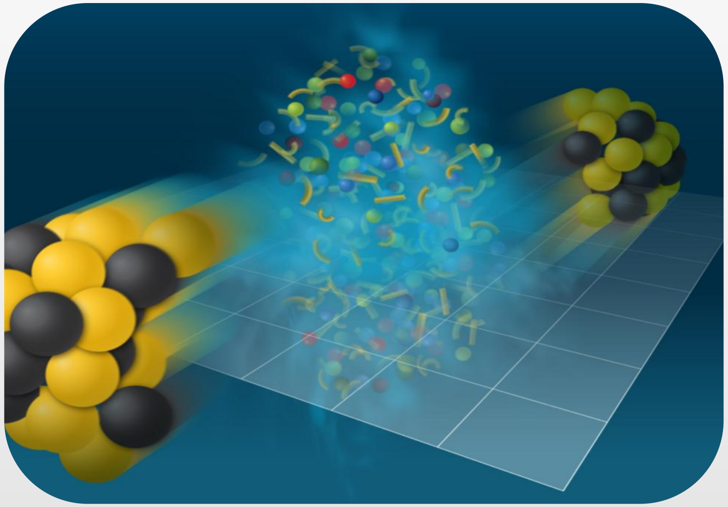
Spin and quantum features of QCD plasma

Sep. 16-20, 2024





www.bnl.gov/newsroom/news.php?a=120967

Outline

- Introduction

  Non-relativistic quark coalescence with spin

- Strong-force field fluctuation

  Spin kinetic theory -> relativistic quark coalescence with spin

- Other theoretical studies for vector meson’s spin alignment

- Outlook and summary

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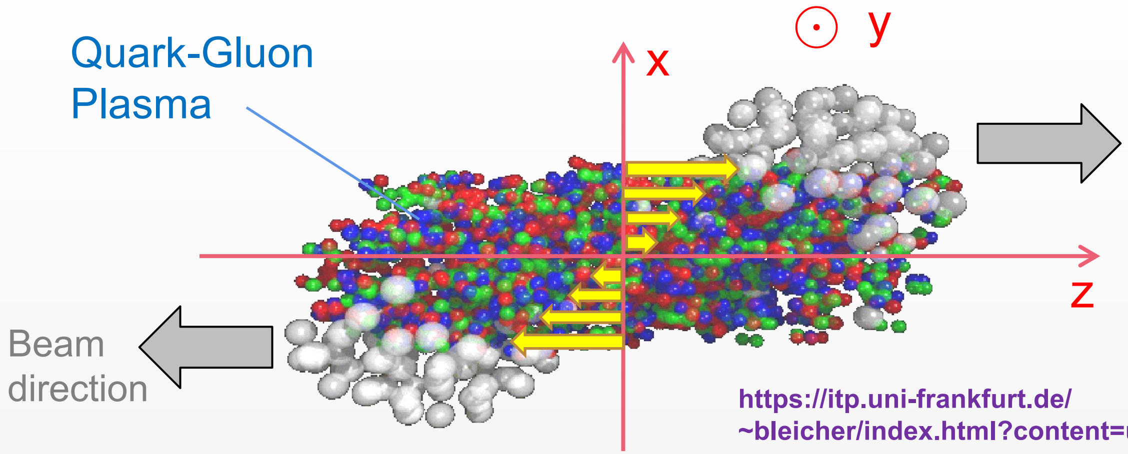
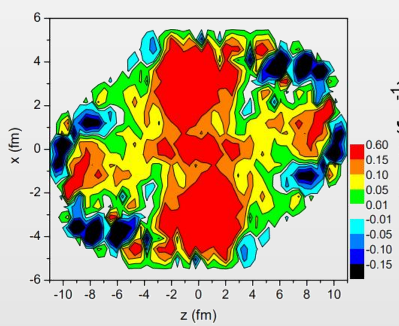
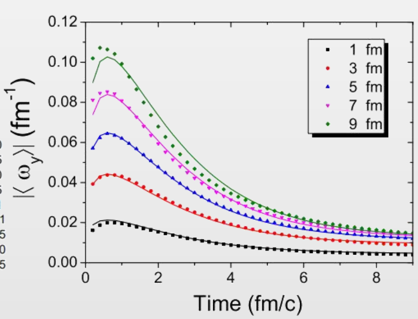
Relativistic heavy-ion collisions generate **strongly interacting matter with vorticity and magnetic fields**



https://itp.uni-frankfurt.de/~bleicher/index.html?content=urqmd

Vorticity fields $\omega \sim 10^{21} s^{-1}$

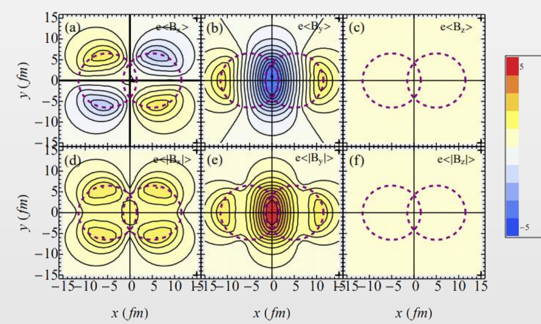


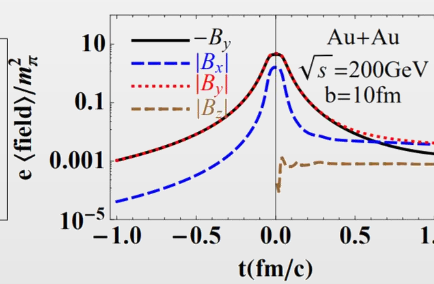


F. Becattini, L. Csernai, D.J. Wang, PRC 88, 034905 (2013); PRC 93, 069901 (2016)

Y. Jiang, Z.-W. Lin, J. Liao, PRC 94, 044910 (2016); PRC 95,049904 (2017)

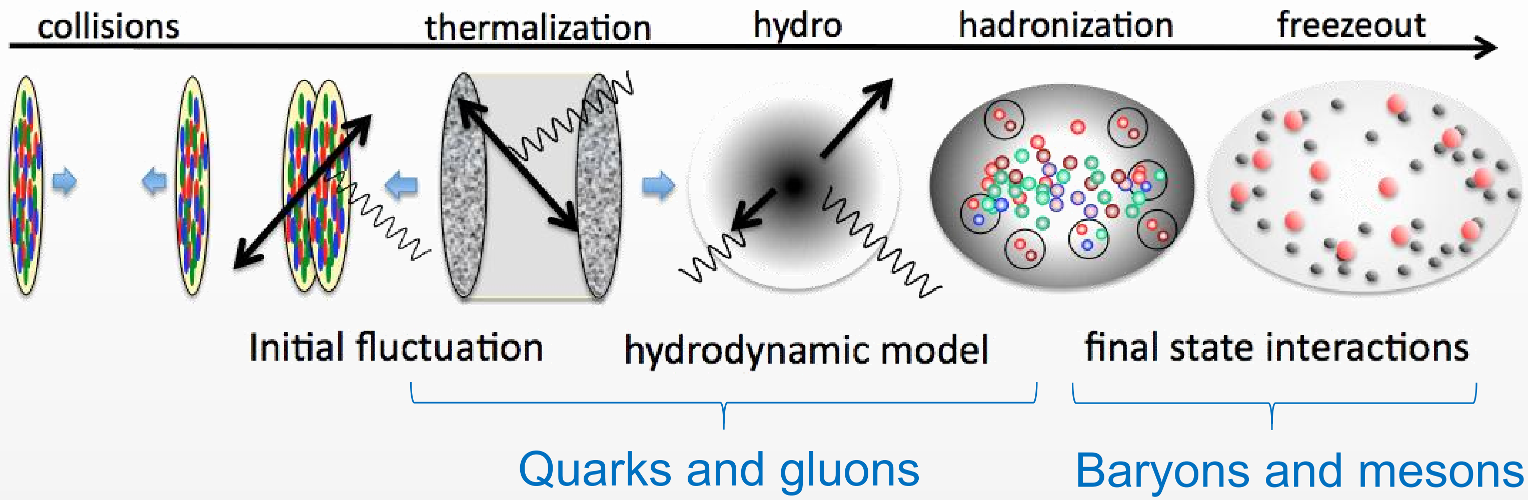
Magnetic fields $B \sim 10^{18}$ Gauss



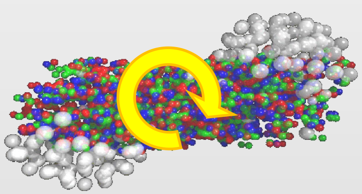


W.-T. Deng, X.-G. Huang, PRC 85, 044907 (2012).

Heavy-ion collisions



Initial orbital angular momentum → Vorticity field, Magnetic field → Polarized quark/gluon →

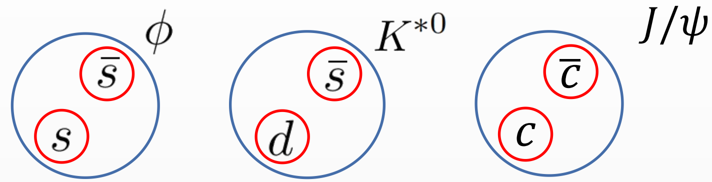


Spin polarization for spin-1/2 or spin-3/2 baryons, Λ, Σ^0, Δ^{++}, Ω^-, ...

Spin alignment for vector mesons, ϕ, K^{*0}, ρ^0, ...

Spin alignment

- Spin alignment for a vector meson ($J^P = 1^-$) is 00-element ρ_{00} of its normalized spin density matrix, probability of spin-0 state, $\rho_{00} = 1/3$ if no polarization



$$\rho_{rs}^{S=1} = \begin{pmatrix} \rho_{+1,+1} & \rho_{+1,0} & \rho_{+1,-1} \\ \rho_{0,+1} & \rho_{00} & \rho_{0,-1} \\ \rho_{-1,+1} & \rho_{-1,0} & \rho_{-1,-1} \end{pmatrix} = \frac{1}{3} + \frac{1}{2} P_i \Sigma_i + T_{ij} \Sigma_{ij}$$

P_i: Vector polarization (3 components, not measurable)

T_{ij}: Tensor polarization (5 components, measurable)

- Measured through polar angle distribution of decay products

Processes	Examples	Polar angle distribution $W(\theta)$	Spin is converted to
Strong p-wave decay	$K^{*0} \to K^+ + \pi^-$ $\phi \to K^+ + K^-$	$\frac{3}{4}\left[1 - \rho_{00} + (3\rho_{00} - 1)\cos^2\theta\right]$	OAM
Dilepton decay	$J/\psi \to \mu^+ + \mu^-$	$\frac{3}{8}\left[1 + \rho_{00} + (1 - 3\rho_{00})\cos^2\theta\right]$	Spin

K. Schilling, P. Seyboth, G. E. Wolf, NPB 15, 397 (1970) [Erratum-ibid. B 18, 332 (1970)].
P. Faccioli, C. Lourenco, J. Seixas, H. K. Wohri, EPJC 69, 657-673 (2010)

Experiment results

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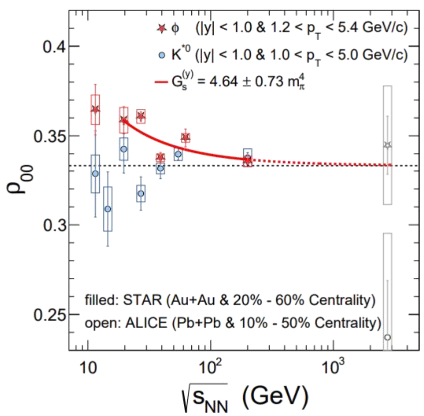
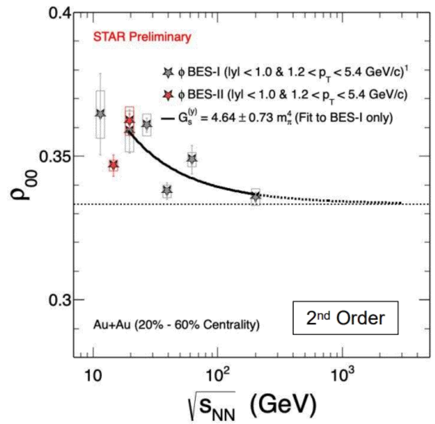
Global spin alignment of ϕ and K^{*0} @ RHIC

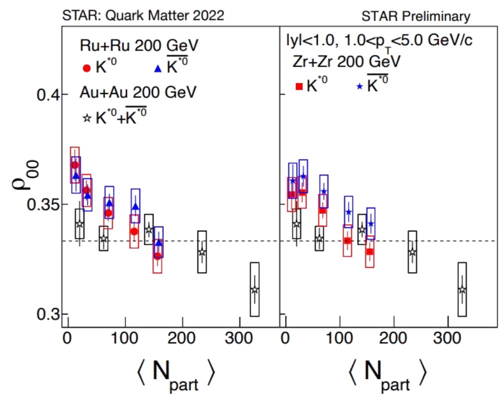


STAR, Nature 614, 244 (2023)

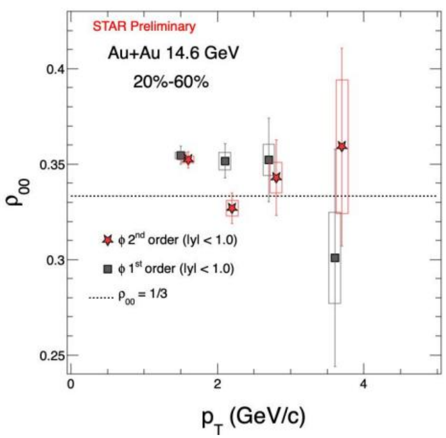
ϕ @ RHIC (BES II)

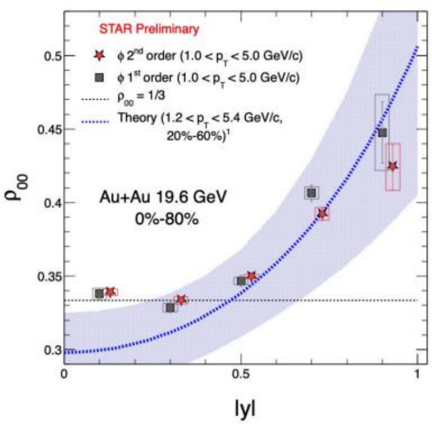


STAR Preliminary @ QM2023

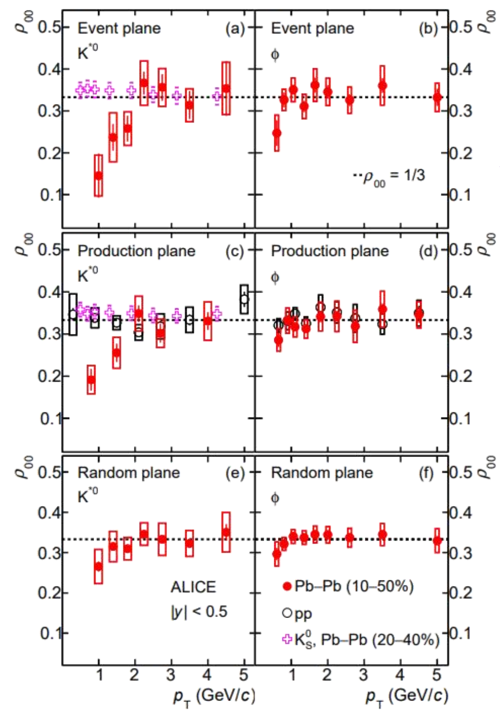


STAR Preliminary @ QM2022





STAR Preliminary @ QM2023

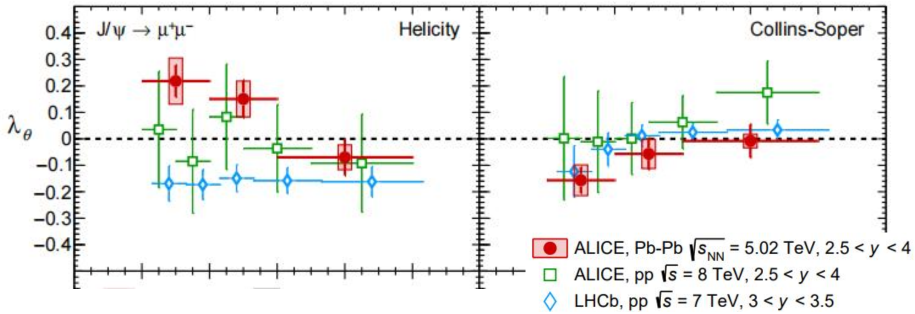


ϕ and K^{*0} @ LHC

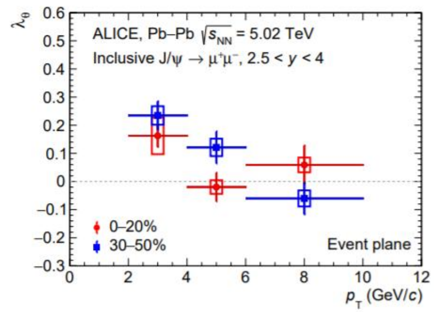
ALICE, PRL 125, 012301 (2020)

Experiment results

Spin alignment of J/ψ @ LHC



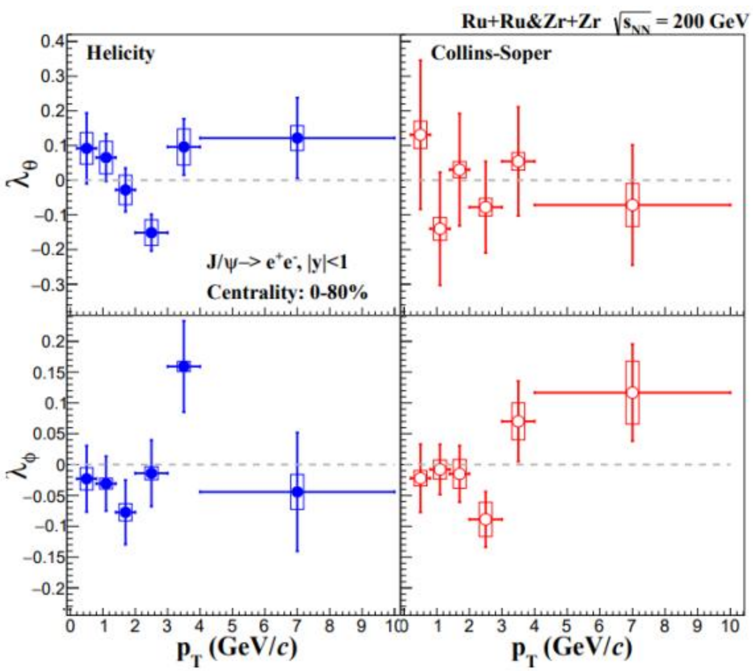
ALICE, PLB 815, 136146 (2021)



ALICE, PRL 131, 042303

$$\lambda_\theta = \frac{1-3\rho_{00}}{1+\rho_{00}} \approx -\frac{9}{4}\left(\rho_{00}-\frac{1}{3}\right)$$

$$\rho_{00} < \frac{1}{3}$$

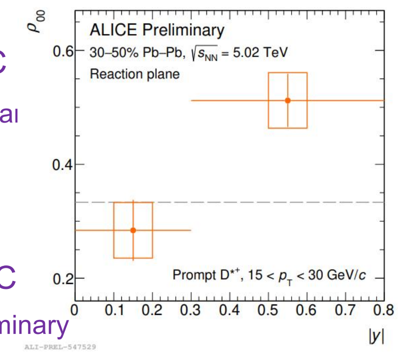


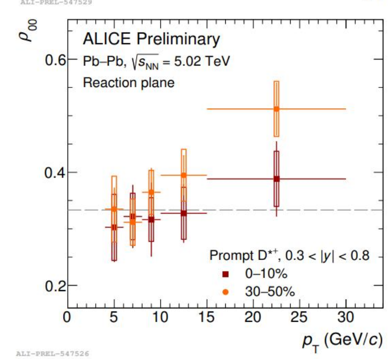
J/ψ @ RHIC

STAR Preliminary @ QM2023

D^{*+} @ LHC

ALICE Preliminary @ Spin2023





More experiment results

Talk by Aihong Tang, Thursday 9:30

Quark coalescence

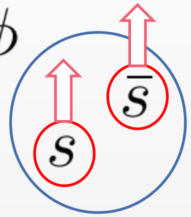
Spin Alignment of Vector Mesons in Non-central $A+A$ Collisions

Zuo-Tang Liang[1] and Xin-Nian Wang[2,1]
[1] *Department of Physics, Shandong University, Jinan, Shandong 250100, China*
'uclear Science Division, MS 70R0319, Lawrence Berkeley National Laboratory, Berkeley, California 947
(Dated: November 5, 2018)

- Non-relativistic coalescence model   Z.-T. Liang, X.-N. Wang, PLB 629, 20 (2005).

Quark spin state $|S, S_z\rangle$ → Vector meosn spin state $|S, S_z\rangle$

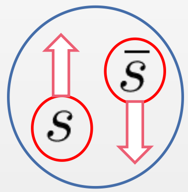
ϕ



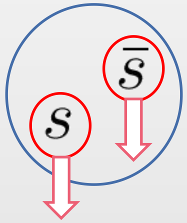
$$\left|\frac{1}{2}, +\frac{1}{2}\right\rangle \qquad \left|\frac{1}{2}, -\frac{1}{2}\right\rangle$$

$$|1, +1\rangle = \left|\frac{1}{2}, +\frac{1}{2}\right\rangle \left|\frac{1}{2}, +\frac{1}{2}\right\rangle$$

$$|1, 0\rangle = \frac{1}{\sqrt{2}}\left(\left|\frac{1}{2}, +\frac{1}{2}\right\rangle \left|\frac{1}{2}, -\frac{1}{2}\right\rangle + \left|\frac{1}{2}, -\frac{1}{2}\right\rangle \left|\frac{1}{2}, +\frac{1}{2}\right\rangle\right)$$



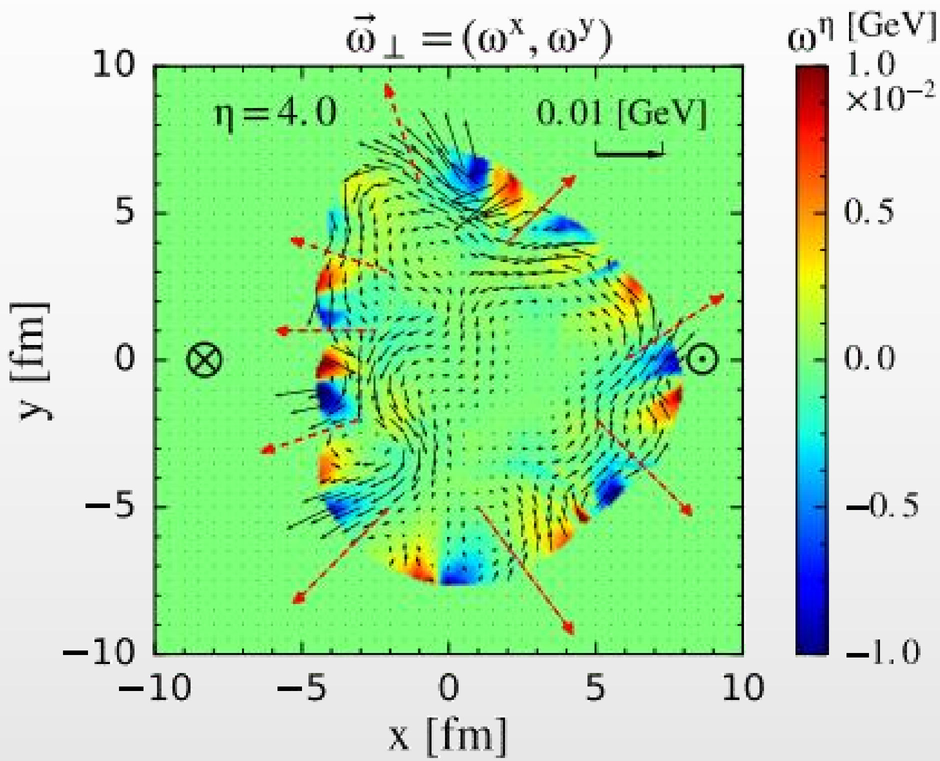
$$|1, -1\rangle = \left|\frac{1}{2}, -\frac{1}{2}\right\rangle \left|\frac{1}{2}, -\frac{1}{2}\right\rangle$$



$$\hat{\rho}^q = \frac{1}{2}\begin{pmatrix} 1+P_q & 0 \\ 0 & 1-P_q \end{pmatrix} \quad \Rightarrow \quad \rho_{00}^{V\,(\text{rec})} = \frac{1 - P_q P_{\bar{q}}}{3 + P_q P_{\bar{q}}} \approx \frac{1}{3} - \frac{4}{9} P_q P_{\bar{q}}$$

Local polarization

- Local vortical structure in heavy-ion collisions



- Quark polarization is a function of phase space coordinate
- Possible polarizations in all spatial directions

$$\left(P_q^x, P_q^y, P_q^z\right)(\mathbf{x}, \mathbf{p})$$

$$\begin{pmatrix} 1 + P_q & 0 \\ 0 & 1 - P_q \end{pmatrix}$$

$$\Downarrow$$

$$\begin{pmatrix} 1 + P_q^z(\mathbf{x},\mathbf{p}) & P_q^x(\mathbf{x},\mathbf{p}) - iP_q^y(\mathbf{x},\mathbf{p}) \\ P_q^x(\mathbf{x},\mathbf{p}) + iP_q^y(\mathbf{x},\mathbf{p}) & 1 - P_q^z(\mathbf{x},\mathbf{p}) \end{pmatrix}$$

L.-G. Pang, H. Petersen, Q. Wang, X.-N. Wang, PRL 117, 192301

Quark coalescence

- Vector meson's tensor polarization
  (spin quantization direction is set to z-direction)

$$\rho_{00}^{V}(\vec{x},\vec{p}) \approx \frac{1}{3} - \frac{2}{3}\langle P_{q_1}^z P_{\bar{q}_2}^z \rangle_V + \frac{2}{9}\left\langle \vec{P}_{q_1}\cdot \vec{P}_{\bar{q}_2}\right\rangle_V ,$$

Difference between z-z correlation and x-x (y-y) correlation

$$\mathrm{Re}\rho_{1,-1}^{V}(\vec{x},\vec{p}) \approx \frac{1}{3}\langle P_{q_1}^x P_{\bar{q}_2}^x - P_{q_1}^y P_{\bar{q}_2}^y \rangle_V ,$$

Difference between x-x correlation and y-y correlation

$$-\mathrm{Im}\rho_{1,-1}^{V}(\vec{x},\vec{p}) \approx \frac{1}{3}\langle P_{q_1}^x P_{\bar{q}_2}^y + P_{q_1}^y P_{\bar{q}_2}^x \rangle_V ,$$

x-y correlation

$$\mathrm{Re}\left[\rho_{1,0} - \rho_{-1,0}\right](\vec{x},\vec{p}) \approx \frac{\sqrt{2}}{3}\langle P_{q_1}^z P_{\bar{q}_2}^x + P_{q_1}^x P_{\bar{q}_2}^z \rangle_V ,$$

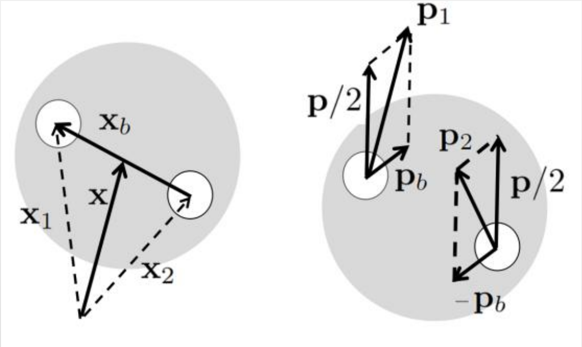
x-z correlation

$$-\mathrm{Im}\left[\rho_{1,0} + \rho_{-1,0}\right](\vec{x},\vec{p}) \approx \frac{\sqrt{2}}{3}\langle P_{q_1}^y P_{\bar{q}_2}^z + P_{q_1}^z P_{\bar{q}_2}^y \rangle_V$$

y-z correlation

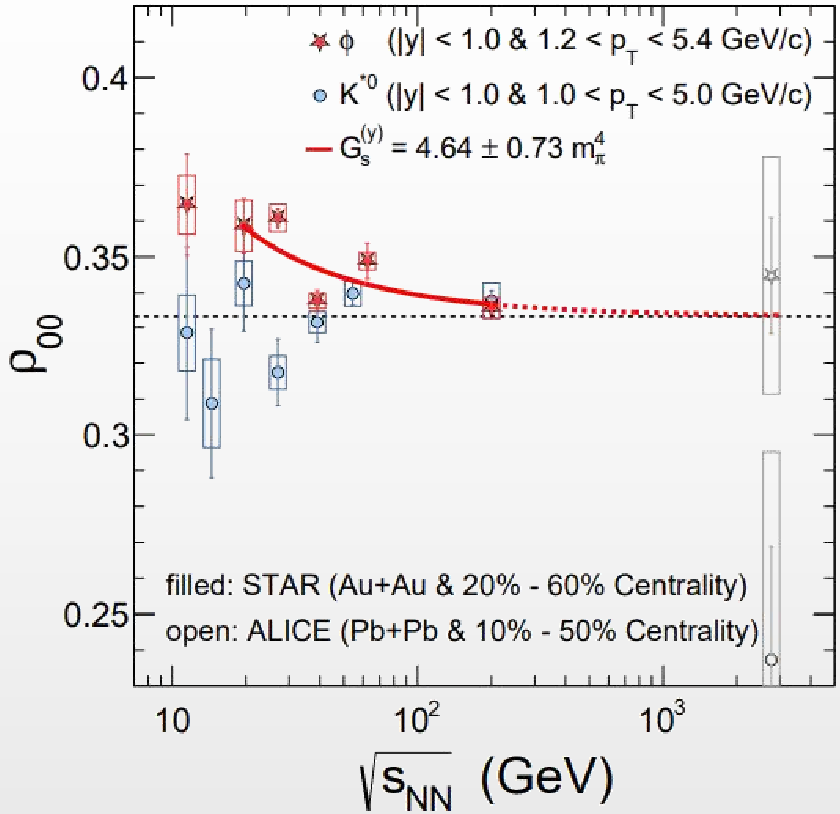
- Average over constituent $q/\bar{q}$'s relative position and relative momentum

$$\left\langle P_{q_1}^i P_{\bar{q}_2}^j \right\rangle_V \equiv \frac{1}{\pi^3}\int d^3\vec{x}_b d^3\vec{p}_b \exp\left(-\frac{\vec{p}_b^{\,2}}{a_V^2} - a_V^2 \vec{x}_b^{\,2}\right) \times P_{q_1}^i(\vec{x}_1,\vec{p}_1) P_{\bar{q}_2}^j(\vec{x}_2,\vec{p}_2) .$$



X.-L. Xia, H. Li, X.-G. Huang, H.-Z. Huang, PLB 817, 136325 (2021)
XLS, Q. Wang, X.-N. Wang, Phys.Rev.D 102 (2020) 5, 056013
J.-H. Chen, Z.-T. Liang, Y.-G. Ma, XLS, Q. Wang, arXiv:2407.06480

But ...



STAR, Nature 614, 244 (2023)

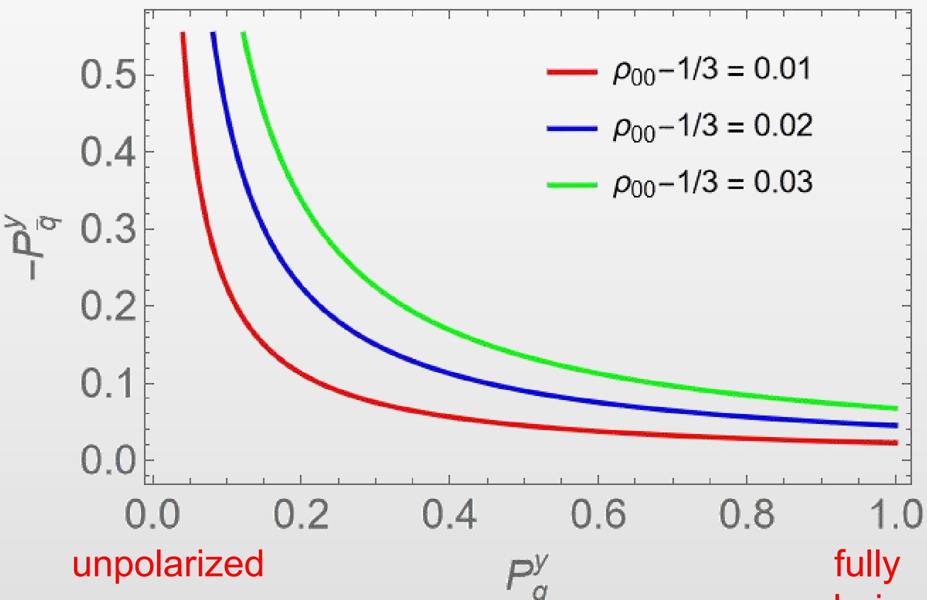
Experiment

$$\rho_{00} - \frac{1}{3} \approx 0.01 - 0.03$$

$$\approx -\frac{4}{9}\langle P_q^y P_{\bar{q}}^y \rangle + \frac{2}{9}\left[\langle P_q^x P_{\bar{q}}^x \rangle + \langle P_q^z P_{\bar{q}}^z \rangle\right]$$

Theory

Quark polarization?



unpolarized

fully polarized

Brief summary

- Spin alignment (tensor polarization) ⟺ spin-spin correlation
- Anything that can polarize quark/antiquark ⟺ spin alignment

- In order to explain spin alignment of ϕ meson observed in experiments, we need
  - Large quark or antiquark polariztion
  - Significant quark-antiquark spin correlation

Spin alignment

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Cannot explain large positive deviation from 1/3

$$\rho_{00} \approx \frac{1}{3} + c_{\text{hydro}} + c_{\text{EM}} + c_F + c_A + c_h + c_{\text{strong}}$$

Hydrodynamic gradient (vorticity, acceleraction, shear tensor, second order) [1-13]

Electromagnetic fields [3,4,14,15]

Fragmentation [1]

Anomalous color field [16]

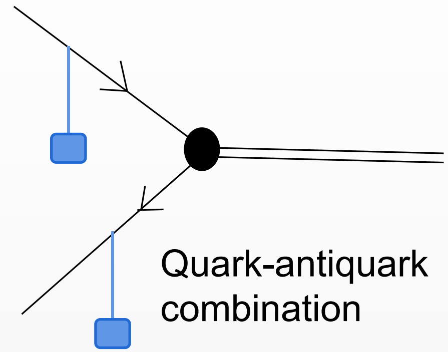
Helicity polarization [17]

Anisotropic strong force [4,18-21]

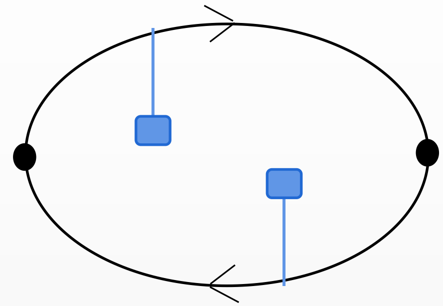
[1] Z.-T. Liang, X.-N. Wang, PLB 629, 20 (2005)
[2] F. Becattini, L. Csernai, D.-J. Wang, PRC 88, 034905 (2013)
[3] Y.-G. Yang, R.-H. Fang, Q. Wang, X.-N. Wang, PRC 97, 034917 (2018)
[4] XLS, L. Oliva, Q. Wang, PRD 101, 096005 (2020)
[5] X.-L. Xia, H. Li, X.-G. Huang, H.-Z. Huang, PLB 817, 136325 (2021)
[6] F. Li, S. Liu, arXiv: 2206.11890
[7] D. Wagner, N. Weickgenannt, E. Speranza, PRR 5, 013187 (2023)
[8] M. Wei, M. Huang, CPC 47, 104105 (2023)
[9] P. H. D. Moura, K. J. Goncalves, G. Torrieri, PRD 108, 034032 (2023)
[10] A. Kumar, P. Gubler, D.-L. Yang, arXiv:2312.16900
[11] S. Fang, S. Pu, D.-L. Yang, arXiv:2311.15197.
[12] W.-B. Dong, Y.-L. Yin, XLS, S.-Z. Yang, Q. Wang, arXiv:2311.18400.
[13] F. Sun, J. Shao, R. Wen, K. Xu, M. Huang, arXiv: 2402.16595.
[14] XLS, S.-Y. Yang, Y.-L. Zou, D. Hou, arXiv:2209.01872
[15] Y.-Q. Zhao, XLS, S.-W. Li, D. Hou, arXiv: 2403.07468
[16] B. Muller, D.-L. Yang, PRD 105, 1 (2022).
[17] J.-H. Gao, PRD 104, 076016 (2021)
[18] XLS, L. Oliva, Z.-T. Liang, Q. Wang, X.-N. Wang, PRL 131, 042304 (2023); PRD 109, 036004 (2024)
[19] A. Kumar, B. Muller, D.-L. Yang, PRD 108, 016020 (2023)
[20] XLS, S. Pu, Q. Wang, PRC 108, 054902 (2023).
[21] XLS, Y.-Q. Zhao, S.-W. Li, F. Becattini, D. Hou, arXiv: 2403.07522

Theoretical approaches

- **Coalescence model with spin**
  - Quark/antiquark polarized by external field
  - Non-equilibrium process described by kinetic theory



Quark-antiquark combination



Vector meson self-energy

Z.-T. Liang, X.-N. Wang, PLB 629, 20 (2005).
XLS, Q. Wang, X.-N. Wang PRD 102, 056013 (2020).
X.-L. Xia, H. Li, X.-G. Huang, H.-Z. Huang, PLB 817, 136325 (2021).
A. Kumar, B. Mueller, D.-L. Yang, PRD 108, 016020 (2023).
XLS, L.Oliva, Z.-T.Liang, Q.Wang, X.-N.Wang, PRL 131, 042304 (2023); PRD 109, 036004 (2024).
XLS, S. Pu, Q. Wang, PRC 108, 054902 (2023).

- **Spectral function method**
  - Splitting between spectral functions of longitudinal and transverse modes due to external fields or motion relative to a thermal background, calculated by QFT, NJL model, holographic model...
  - Meson at thermodynamical equilibrium

XLS, S.-Y. Yang, Y.-L. Zou, D. Hou, arXiv: 2209.01872.
A. Kumar, B. Mueller, D.-L. Yang, PRD 108, 016020 (2023).
M. Wei, M. Huang, CPC 47, 104105 (2023).
W.-B. Dong, Y.-L. Yin, XLS, S.-Z. Yang, Q. Wang, arXiv:2311.18400.
XLS, Y.-Q. Zhao, S.-W. Li, F. Becattini, D. Hou, arXiv:2403.07522
Y.-Q. Zhao, XLS, S.-W. Li, D. Hou, arXiv:2403.07468

- **Spin kinetic equation**

D. Wagner, N. Weickgenannt, E. Speranza, PRR 5, 013187 (2023)
S. Fang, S. Pu, D.-L. Yang, PRD 109, 034034 (2024)
Y.-L. Yin, W.-B. Dong, J.-Y. Pang, S. Pu, Q. Wang, arXiv:2402.03672

- **Linear response theory**

F. Li, S. Liu, arXiv: 2206.11890
W.-B. Dong, Y.-L. Yin, XLS, S.-Z. Yang, Q. Wang, arXiv:2311.18400.

Outline

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- Introduction

  Non-relativistic quark coalescence with spin

- Strong-force field fluctuation

  Spin kinetic theory -> relativistic quark coalescence with spin

- Other theoretical studies for vector meson's spin alignment

- Outlook and summary

Green function and MVSD

- Wigner function expressed in terms of
  matrix valued spin-dependent distributions (MVSD)

$$G^<_{\mu\nu}(x,p) = \int d^4y\, e^{ip\cdot y/\hbar} \langle A^\dagger_\nu(x_2) A_\mu(x_1)\rangle$$

$$A^\mu_V(x) = \sum_{\lambda=0,\pm1} \int \frac{d^3\mathbf{p}}{(2\pi\hbar)^3} \frac{1}{2E^V_\mathbf{p}} \times \left[\epsilon^\mu(\lambda,\mathbf{p}) a_V(\lambda,\mathbf{p}) e^{-ip\cdot x/\hbar} + \epsilon^{*\mu}(\lambda,\mathbf{p}) a^\dagger_V(\lambda,\mathbf{p}) e^{ip\cdot x/\hbar}\right]$$

$$G^<_{\mu\nu}(x,p) = 2\pi\hbar \sum_{\lambda_1,\lambda_2} \delta\left(p^2 - m_V^2\right) \times \left\{\theta(p^0)\epsilon_\mu(\lambda_1,\mathbf{p})\,\epsilon^*_\nu(\lambda_2,\mathbf{p})\, f_{\lambda_1\lambda_2}(x,\mathbf{p}) + \theta(-p^0)\epsilon^*_\mu(\lambda_1,-\mathbf{p})\,\epsilon_\nu(\lambda_2,-\mathbf{p}) \times \left[\delta_{\lambda_2\lambda_1} + f_{\lambda_2\lambda_1}(x,-\mathbf{p})\right]\right\},$$

polarization vector for a meson with spin λ

creation/anihilation operator
a_V, $b_V^\dagger$ if meson is not self-conjugate

- MVSD for vector meson

$$f_{\lambda_1\lambda_2}(x,\mathbf{p}) \equiv \int \frac{d^4u}{2(2\pi\hbar)^3} \delta(p\cdot u) e^{-iu\cdot x/\hbar} \left\langle a^\dagger_V\left(\lambda_2, \mathbf{p} - \frac{\mathbf{u}}{2}\right) a_V\left(\lambda_1, \mathbf{p} + \frac{\mathbf{u}}{2}\right)\right\rangle$$

$$= 2E^V_\mathbf{p} \int \frac{dp^0}{2\pi\hbar} \theta(p^0) \epsilon^{*\mu}(\lambda_1,\mathbf{p}) \epsilon^\nu(\lambda_2,\mathbf{p}) G^<_{\mu\nu}(x,p)$$

Relation to Wigner function

$$= 3 f(x,\mathbf{p}) \rho_{\lambda_1\lambda_2}(x,\mathbf{p})$$

Relation to spin-averaged distribution and normalized density matrix

$$f(x,\mathbf{p}) \equiv \frac{1}{3} \sum_{\lambda=0,\pm1} f_{\lambda\lambda}(x,\mathbf{p}), \qquad \sum_{\lambda=0,\pm1} \rho_{\lambda\lambda}(x,\mathbf{p}) = 1$$

Kadanoff-Baym equation

- With help of Schwinger-Keldysh (closed-time path) formalism, we derive Kadanoff-Baym equation at leading order in spatial gradient

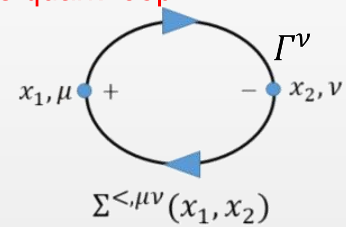
P. Martin, J. S.Schwinger, PR 115 (1959) 1342.
L. P. Kadanoff and G. Baym, Quantum Statistical Mechanics (Benjamin, New York, 1962).
L.V. Keldysh, Zh. Eksp. Teor. Fiz. 47 (1964) 1515.

$$L^{\mu}_{\eta} G^{<,\mu\eta}(x,p)$$

$$= -\frac{i\hbar}{2}\int d^4x' \left[G^{<,\mu}{}_{\alpha}(x_1,x')\Sigma^{>,\alpha\nu}(x',x_2) - G^{>,\mu}{}_{\alpha}(x_1,x')\Sigma^{<,\alpha\nu}(x',x_2)\right]$$

Green functions and self-energy on the closed-time path contour

Leading order: one quark-loop



$$L^{\mu}_{\eta} \equiv -g^{\mu}_{\eta}\left(p^2 - m_V^2\right) + p^{\mu}p_{\eta} + i\hbar\left[g^{\mu}_{\eta}p\cdot\partial_x - \frac{1}{2}\left(p_{\eta}\partial^{\mu}_{x} + p^{\mu}\partial^{x}_{\eta}\right)\right]$$

- Comparing Kadanoff-Baym equation with its Hermitian conjugate, we are able to derive

Boltzmann equation $\quad p\cdot\partial_x G^{<,\mu\nu} - \frac{1}{4}(p^{\mu}\partial^{x}_{\eta}G^{<,\eta\nu} + p^{\nu}\partial^{x}_{\eta}G^{<,\mu\eta}) = \cdots$

Mass-shell condition $\quad -(p^2 - m_V^2)G^{<,\mu\nu} + (p^{\mu}p_{\eta}G^{<,\eta\nu} + p^{\nu}p_{\eta}G^{<,\mu\eta}) = \cdots$

Boltzmann equation

- Dyson-Schwinger equation
  - ⇒ Kadanoff-Baym equation for Wigner function
  - ⇒ Matrix-form Boltzmann equation

XLS, L.Oliva, Z.-T.Liang, Q.Wang, X.-N.Wang, PRD 109, 036004 (2024).

$$k \cdot \partial_x f^V_{\lambda_1\lambda_2}(x, \mathbf{k}) = \frac{1}{8}\left[\epsilon^*_\mu(\lambda_1, \mathbf{k})\epsilon_\nu(\lambda_2, \mathbf{k})\mathcal{C}^{\mu\nu}_{\text{coal}}(x, \mathbf{k}) - f^V_{\lambda_1\lambda_2}(x, \mathbf{k})\mathcal{C}_{\text{diss}}(x, \mathbf{k})\right]$$

Dilute gas limit

$$f_q \sim f_{\bar{q}} \sim f_V \ll 1$$

Meson polarization vectors

Coalescence

$$q + \bar{q} \to V$$

Dissociation (independent from quark distributions)

$$V \to q + \bar{q}$$

- Contribution from coalescence

$$\begin{aligned}\mathcal{C}^{\mu\nu}_{\text{coal}}(x, \mathbf{k}) = &\int \frac{d^3\mathbf{p}'}{(2\pi\hbar)^2}\frac{1}{E^{\bar{q}}_{\mathbf{p}'}E^q_{\mathbf{k}-\mathbf{p}'}}\delta\left(E^V_{\mathbf{k}} - E^{\bar{q}}_{\mathbf{p}'} - E^q_{\mathbf{k}-\mathbf{p}'}\right)\\ &\times \text{Tr}\left\{\Gamma^\nu(p' \cdot \gamma - m_{\bar{q}})\left[1 + \gamma_5\gamma \cdot P^{\bar{q}}(x, \mathbf{p}')\right]\right.\\ &\left.\times\Gamma^\mu\left[(k - p')\cdot\gamma + m_q\right]\left[1 + \gamma_5\gamma \cdot P^q(x, \mathbf{k} - \mathbf{p}')\right]\right\}\\ &\times f_{\bar{q}}(x, \mathbf{p}')f_q(x, \mathbf{k} - \mathbf{p}'),\end{aligned}$$

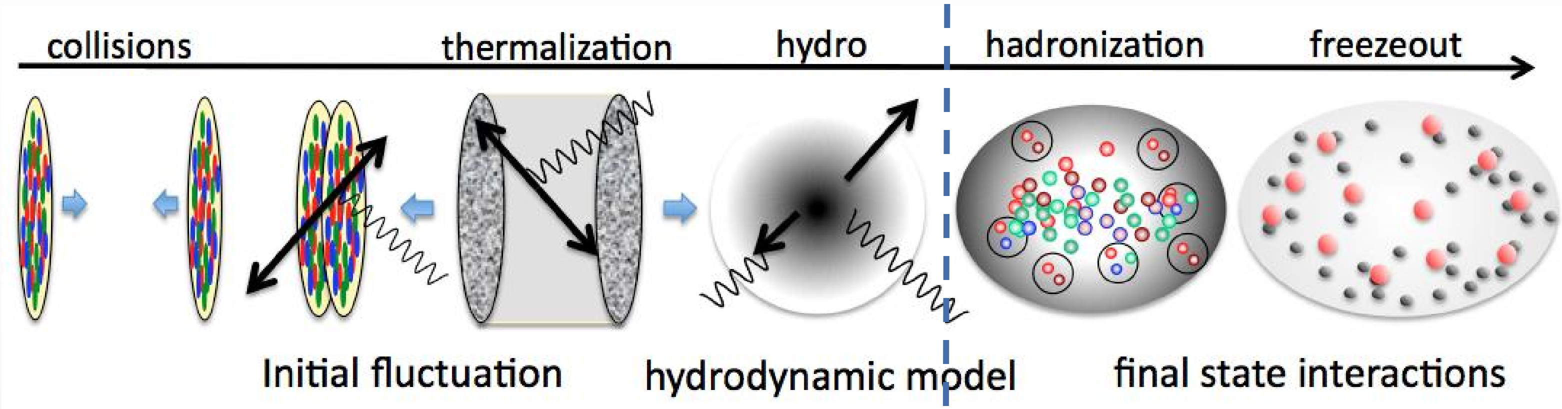
Energy conservation (all particles are on their normal mass shells)

Quark-antiquark-meson vertex

Polarizations of quark/antiquark

unpolarized quark/antiquark distributions

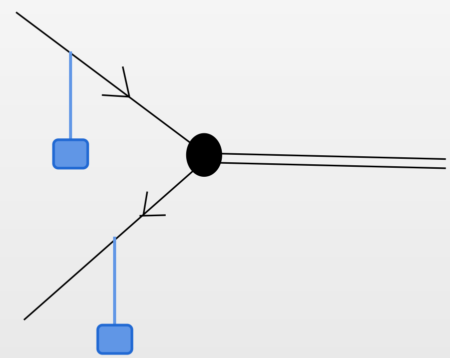
Spin alignment



No vector meson $f^V_{\lambda_1\lambda_2} = 0$ t_0

- Neglecting space-derivatives and assuming that $f^V_{\lambda_1\lambda_2} = 0$ before hadronization stage t_0, we obtain formal solution

$$f^V_{\lambda_1\lambda_2}(x,\mathbf{k}) \sim \frac{1-\exp[-\mathcal{C}_{\text{diss}}(x,\mathbf{k})\Delta t]}{\mathcal{C}_{\text{diss}}(x,\mathbf{k})}\left[\epsilon^*_\mu(\lambda_1,\mathbf{k})\epsilon_\nu(\lambda_2,\mathbf{k})\mathcal{C}^{\mu\nu}_{\text{coal}}(x,\mathbf{k})\right]$$



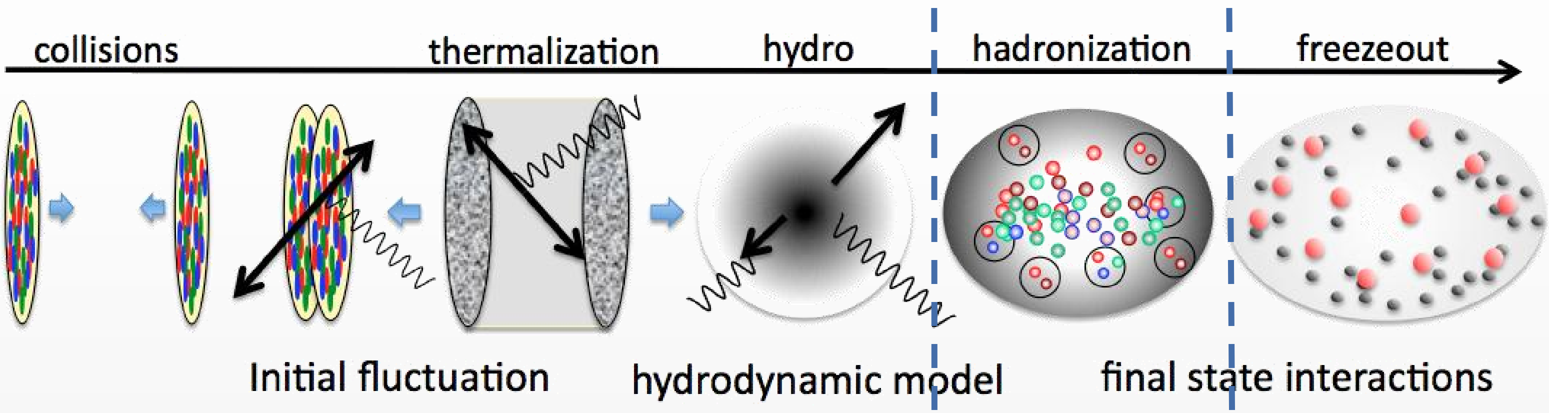
$$\Delta t = t - t_0$$

- Spin alignment only depend on coalescence process

$$\rho_{00} \equiv \frac{f^V_{00}}{f^V_{+1,+1}+f^V_{00}+f^V_{-1,-1}} = \frac{\epsilon^*_\mu(0,\mathbf{k})\epsilon_\nu(0,\mathbf{k})\mathcal{C}^{\mu\nu}_{\text{coal}}(x,\mathbf{k})}{\sum_{\lambda=0,\pm1}\epsilon^*_\mu(\lambda,\mathbf{k})\epsilon_\nu(\lambda,\mathbf{k})\mathcal{C}^{\mu\nu}_{\text{coal}}(x,\mathbf{k})}$$

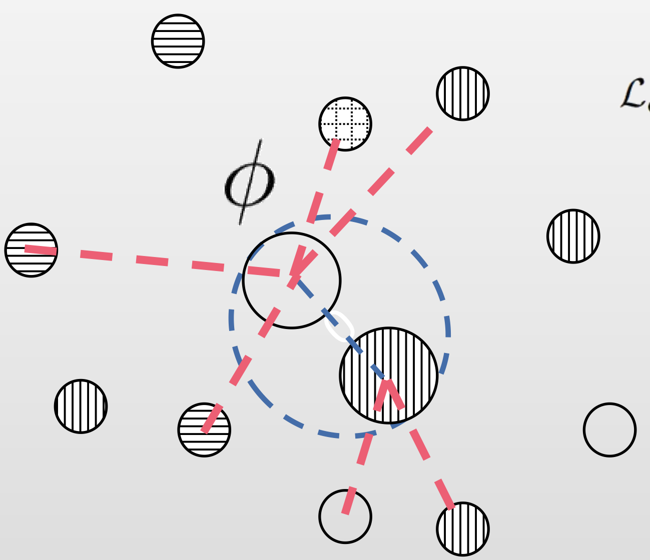
XLS, L.Oliva, Z.-T.Liang, Q.Wang, X.-N.Wang, PRD 109, 036004 (2024)

Quark-meson model



Decreasing *T*

Theories:	QCD	Quark-meson model	Hadron gas model



Quark effective mass

Dirac field $(u,d,s)^T$

$$\mathcal{L}_{\rm eff}(x) = \overline{\psi}(x)\left[i\partial\cdot\gamma - (m_0 + g_\sigma\sigma) - g_V\gamma\cdot V\right]\psi(x) + \frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2\right) + \frac{1}{2}m_V^2 V_\mu V^\mu - \frac{1}{4}V^{\mu\nu}V_{\mu\nu}$$

Vector meson field

$$\begin{pmatrix} \frac{\omega+\rho^0}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & \frac{\omega-\rho^0}{\sqrt{2}} & K^{*0} \\ K^{*-} & \overline{K}^{*0} & \phi \end{pmatrix} \qquad V_{\mu\nu} \equiv \partial_\mu V_\nu - \partial_\nu V_\mu$$

Short wave-length: quantum fields (particles)

Long wave-length: classical fields

Quark polarization

- Polarizations of strange quark/antiquark in a thermal equilibrium system

$$P_s^\mu(x,\mathbf{p}) \approx \frac{1}{4m_s}\epsilon^{\mu\nu\alpha\beta}p_\nu\left[\omega_{\rho\sigma} + \frac{Q_s}{(u\cdot p)T}F_{\rho\sigma} + \frac{g_\phi}{(u\cdot p)T}F^\phi_{\rho\sigma}\right]$$

$$P_{\bar{s}}^\mu(x,\mathbf{p}) \approx \frac{1}{4m_s}\epsilon^{\mu\nu\alpha\beta}p_\nu\left[\omega_{\rho\sigma} - \frac{Q_s}{(u\cdot p)T}F_{\rho\sigma} - \frac{g_\phi}{(u\cdot p)T}F^\phi_{\rho\sigma}\right]$$

F.Becattini, V.Chandra, L.Del Zanna, E.Grossi, Annals Phys. 338, 32 (2013)

Y.-G. Yang, R.-H. Fang, Q. Wang, and X.-N. Wang, Phys.Rev.C 97, 3 (2018).

XLS, L.Oliva, Q.Wang, PRD 101, 096005 (2020);

XLS, L.Oliva, Z.-T.Liang, Q.Wang, X.-N.Wang, PRL 131, 042304 (2023); PRD 109, 036004 (2024).

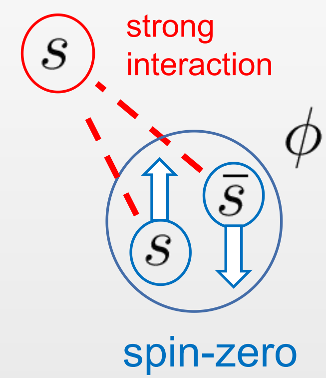
thermal vorticity field (rotation and acceleration)

classical electromagnetic field

$$\frac{e^2}{4\pi} \sim \frac{1}{137}$$

vector ϕ field (long wave-length components)

$$\frac{g_\phi^2}{4\pi} \sim \mathcal{O}(1) \gg \frac{e^2}{4\pi}$$



- Vector ϕ field has been used to explain the difference between polarizations of Λ and $\overline{\Lambda}$

L.P.Csernai, J.I.Kapusta, T.Welle, PRC 99, 021901 (2019)

Spin alignment

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- Spin alignment of the ϕ meson in its rest frame measuring along the direction of ϵ_0

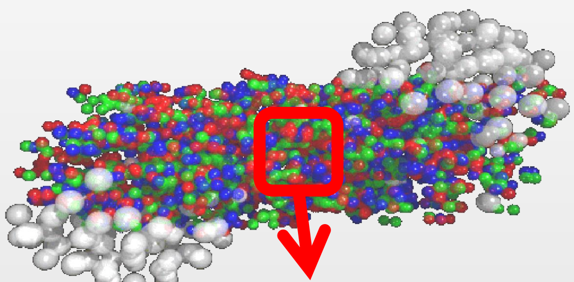
$$\rho_{00} \approx \frac{1}{3} - \frac{4g_\phi^2}{m_\phi^2 T_h^2} C_1 \left[\frac{1}{3}\mathbf{B}'_\phi \cdot \mathbf{B}'_\phi - (\epsilon_0 \cdot \mathbf{B}'_\phi)^2\right] - \frac{4g_\phi^2}{m_\phi^2 T_h^2} C_2 \left[\frac{1}{3}\mathbf{E}'_\phi \cdot \mathbf{E}'_\phi - (\epsilon_0 \cdot \mathbf{E}'_\phi)^2\right]$$

$$C_1 = \frac{8m_s^4 + 16m_s^2 m_\phi^2 + 3m_\phi^4}{120 m_s^2 (m_\phi^2 + 2m_s^2)},$$
$$C_2 = \frac{8m_s^4 - 14m_s^2 m_\phi^2 + 3m_\phi^4}{120 m_s^2 (m_\phi^2 + 2m_s^2)}.$$

Temperature at hadronization time

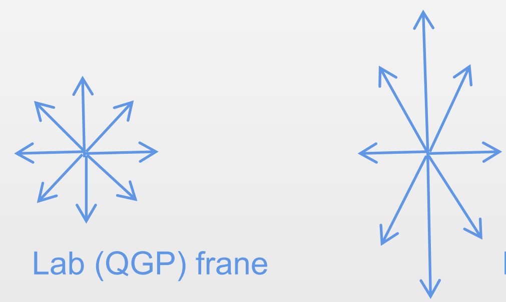
Vector ϕ field: mean value is zero, but can incorporate large fluctuations

- Spin alignment measures anisotropy of fluctuations in meson's rest frame



$$\left\langle g_\phi^2 \mathbf{B}_\phi^i \mathbf{B}_\phi^j / T_{\mathrm{h}}^2 \right\rangle = \left\langle g_\phi^2 \mathbf{E}_\phi^i \mathbf{E}_\phi^j / T_{\mathrm{h}}^2 \right\rangle$$
$$= F^2 \delta^{ij} + \Delta \hat{\mathbf{a}}^i \hat{\mathbf{a}}^j$$

Isotropic — Anisotropy of QGP



Motion-induced anisotropy

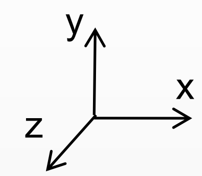
$$\rho_{00}^y - \frac{1}{3} \propto \frac{1}{3}\mathbf{p}\cdot\mathbf{p} - p_y^2$$

XLS, L.Oliva, Z.-T.Liang, Q.Wang, X.-N.Wang, PRL 131, 042304 (2023); PRD 109, 036004 (2024).

Fluctuation-induced ρ_{00}

For example, contribution from $\mathbf{B}'_\phi$ to spin alignment along *y*-direction

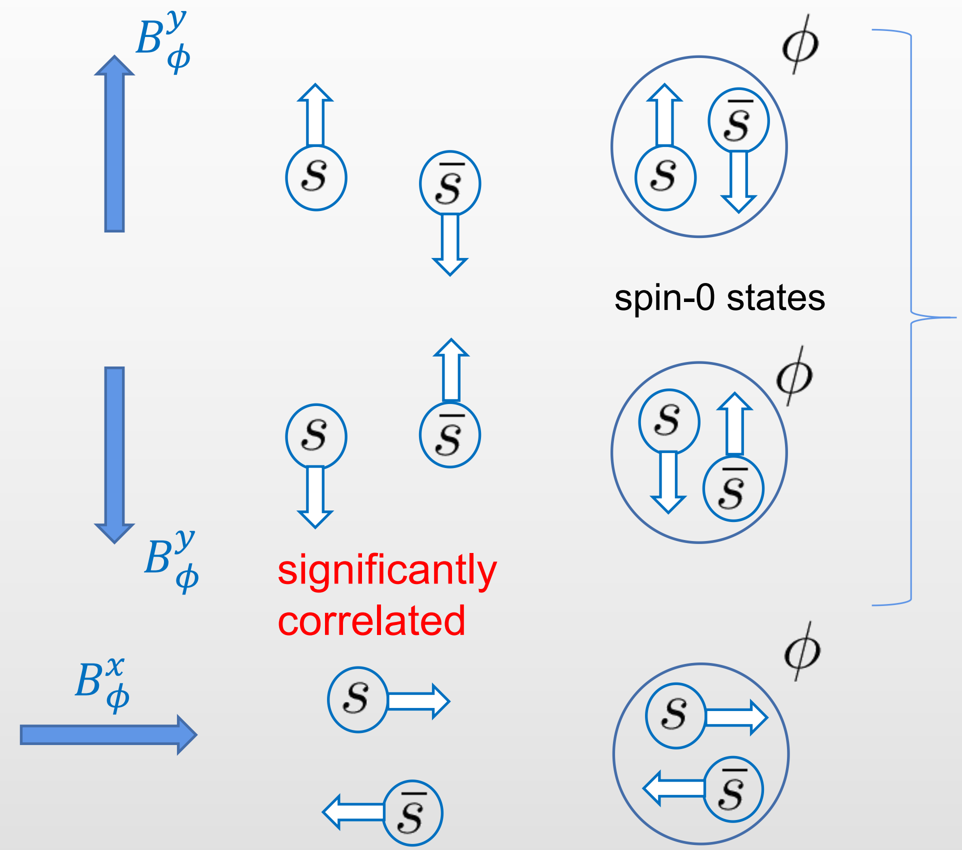
$$\propto (B'_{\phi,y})^2 - \frac{(B'_{\phi,x})^2 + (B'_{\phi,z})^2}{2}$$



Case 1

Case 2

Case 3



$$\langle B_\phi^y \rangle = 0,$$

$$\langle \left(B_\phi^y\right)^2 \rangle \neq 0$$

$$\rho_{00}^y > 1/3$$

$$\rho_{00}^x > 1/3$$

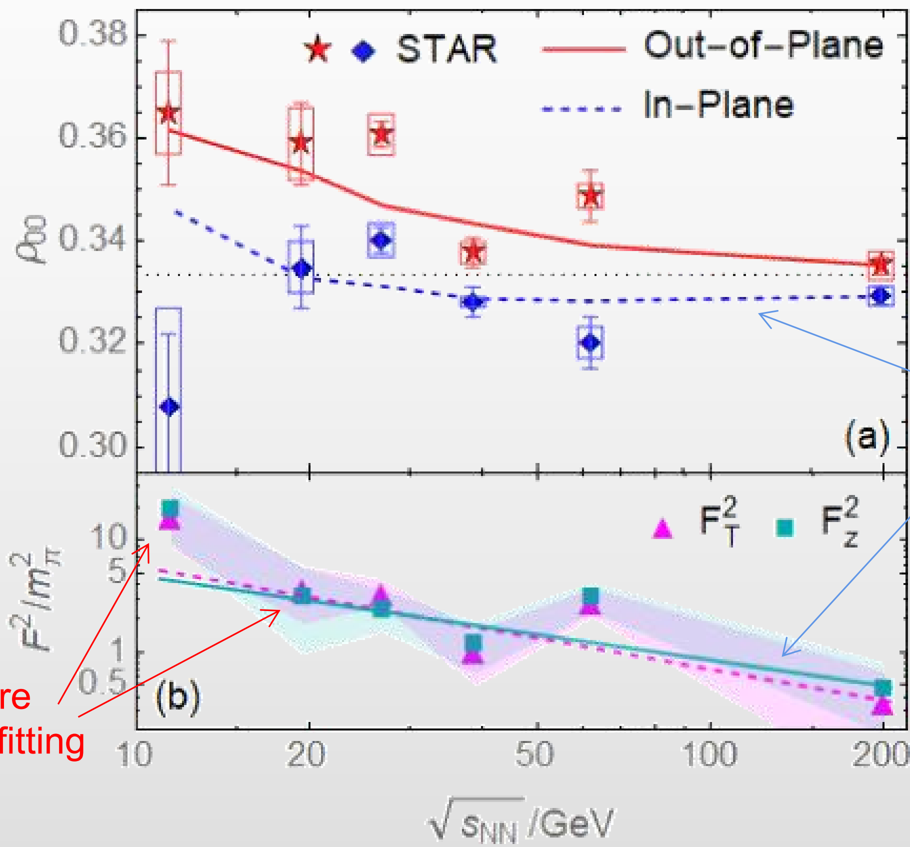
$$\rho_{00}^y, \rho_{00}^z < 1/3$$

$$\rho_{00}^x + \rho_{00}^y + \rho_{00}^z = 1$$

Fitting experiment datas

- Taking fluctuations of transverse and longitudinal fields as two independent parameters.

$$\left\langle (g_\phi \mathbf{B}^\phi_{x,y}/T_\mathrm{h})^2 \right\rangle = \left\langle (g_\phi \mathbf{E}^\phi_{x,y}/T_\mathrm{h})^2 \right\rangle \equiv F_T^2 \qquad \left\langle (g_\phi \mathbf{B}^\phi_{z}/T_\mathrm{h})^2 \right\rangle = \left\langle (g_\phi \mathbf{E}^\phi_{z}/T_\mathrm{h})^2 \right\rangle \equiv F_z^2$$



Parameters are evaluated by fitting STAR data

Energy-dependent parameters fitted by

$$\ln(F_T^2/m_\pi^2) = 3.90 - 0.924 \ln \sqrt{s_\mathrm{NN}}$$
$$\ln(F_z^2/m_\pi^2) = 3.33 - 0.760 \ln \sqrt{s_\mathrm{NN}}$$

$$F_T^2 \approx F_z^2$$

STAR, Nature 614, 244 (2023)

XLS, L.Oliva, Z.-T.Liang, Q.Wang, X.-N.Wang, PRL 131, 042304 (2023)

Model predictions

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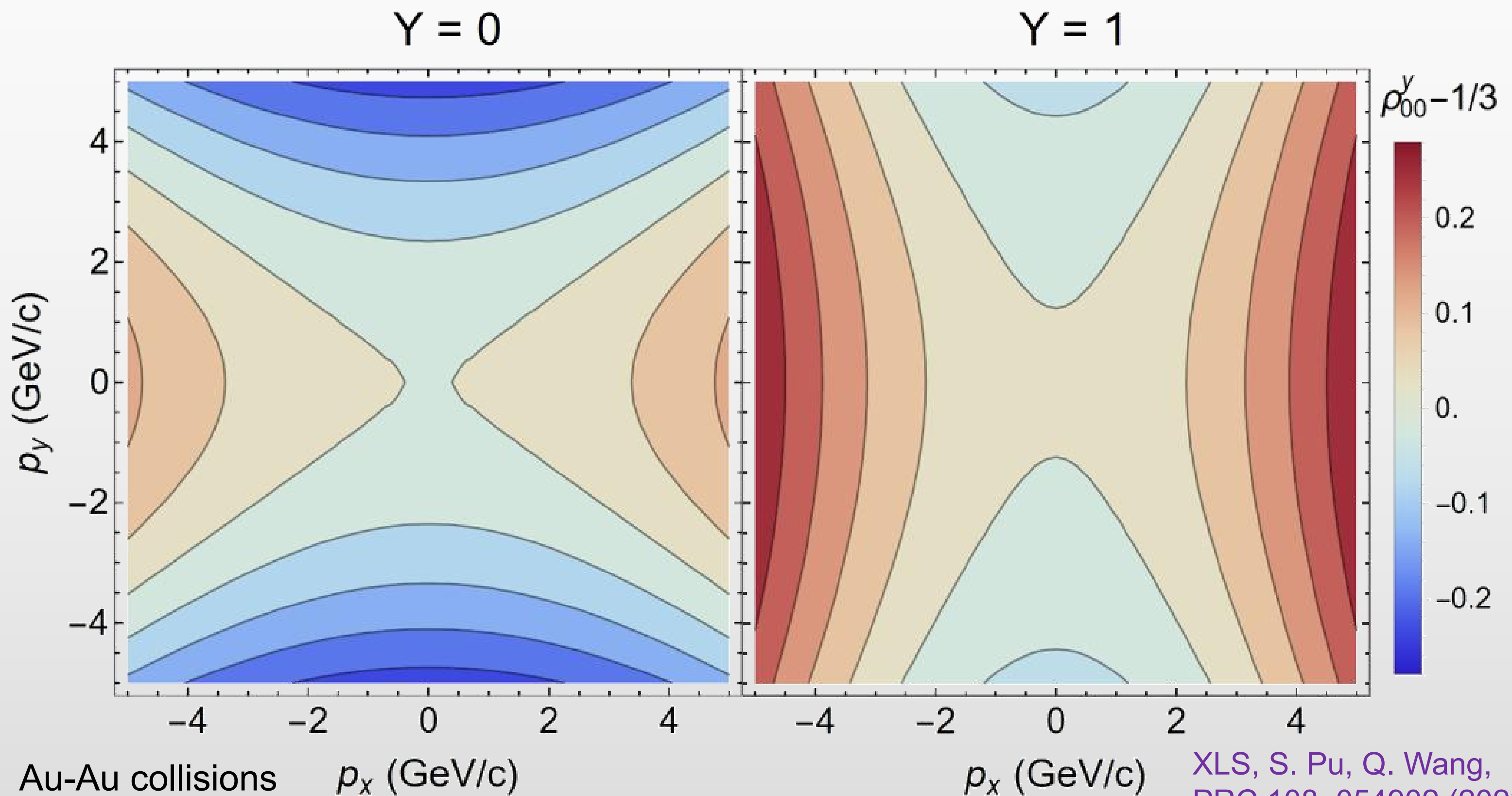
Dominant!

Fluctuations in lab frame $\left\langle g_\phi^2 \mathbf{B}_\phi^i \mathbf{B}_\phi^j / T_\mathrm{h}^2 \right\rangle = \left\langle g_\phi^2 \mathbf{E}_\phi^i \mathbf{E}_\phi^j / T_\mathrm{h}^2 \right\rangle = F^2 \delta^{ij} + \Delta \hat{\mathbf{a}}^i \hat{\mathbf{a}}^j$

In momentum direction $\delta\rho_{00}^h \propto \mathbf{p}^2$ → $\delta\rho_{00}^y \propto \frac{1}{2} p_T^2 \left[3\cos(2\varphi) - 1\right] + \left(p_T^2 + m_\phi^2\right) \sinh^2 Y$



Au-Au collisions at 200 GeV/A

XLS, S. Pu, Q. Wang, PRC 108, 054902 (2023).

Model predictions

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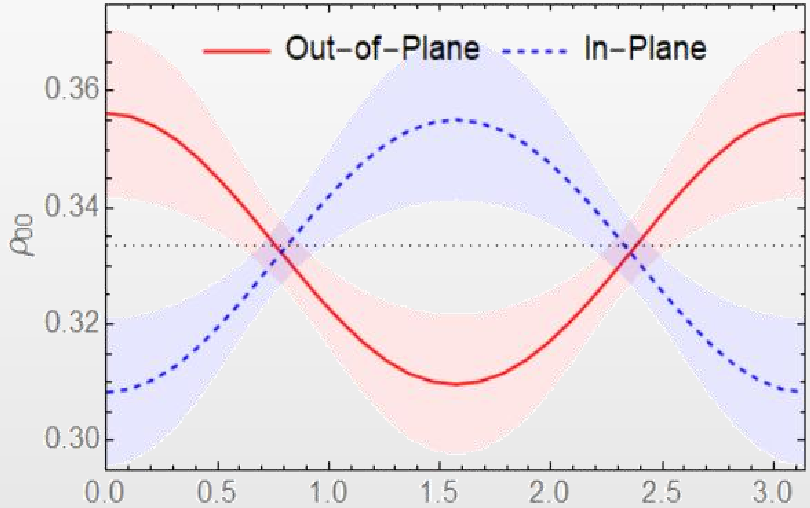
Dominant!

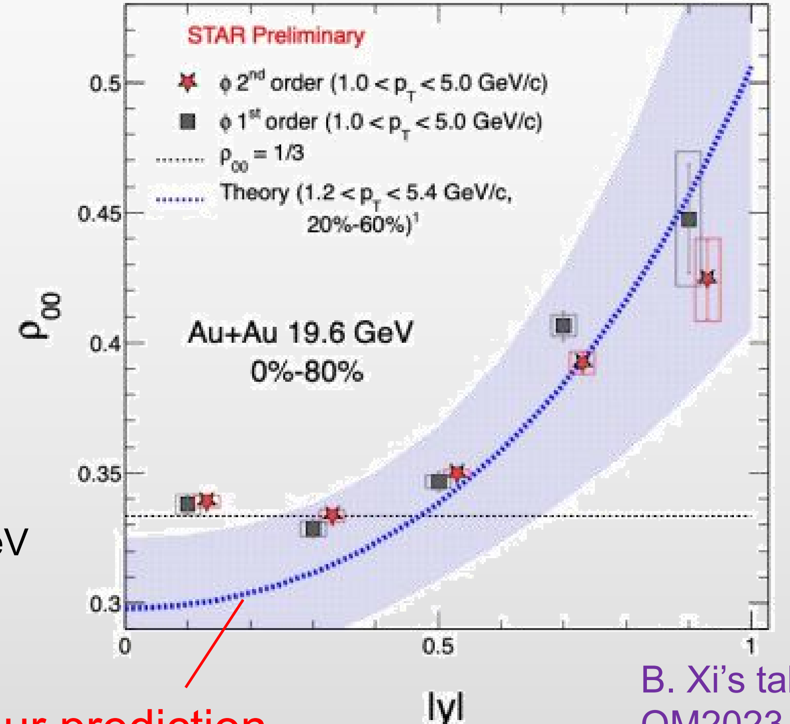
Fluctuations in lab frame $\left\langle g_\phi^2 \mathbf{B}_\phi^i \mathbf{B}_\phi^j / T_\mathrm{h}^2 \right\rangle = \left\langle g_\phi^2 \mathbf{E}_\phi^i \mathbf{E}_\phi^j / T_\mathrm{h}^2 \right\rangle = F^2 \delta^{ij} + \Delta \hat{\mathbf{a}}^i \hat{\mathbf{a}}^j$

$$\delta\rho_{00}^y \propto \frac{1}{2} p_T^2 \left[3\cos(2\varphi) - 1\right] + \left(p_T^2 + m_\phi^2\right) \sinh^2 Y$$

- Predictions for azimuthal angle dependence and rapidity dependence



Au-Au collisions at 200 GeV/A $\quad 1.2 < p_T < 5.4$ GeV



our prediction

XLS, L.Oliva, Z.-T.Liang, Q.Wang, X.-N.Wang, PRL 131, 042304 (2023)

XLS, S. Pu, Q. Wang, PRC 108, 054902 (2023).

B. Xi's talk in QM2023

Outline

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- Introduction

  Non-relativistic quark coalescence with spin

- Strong-force field fluctuation

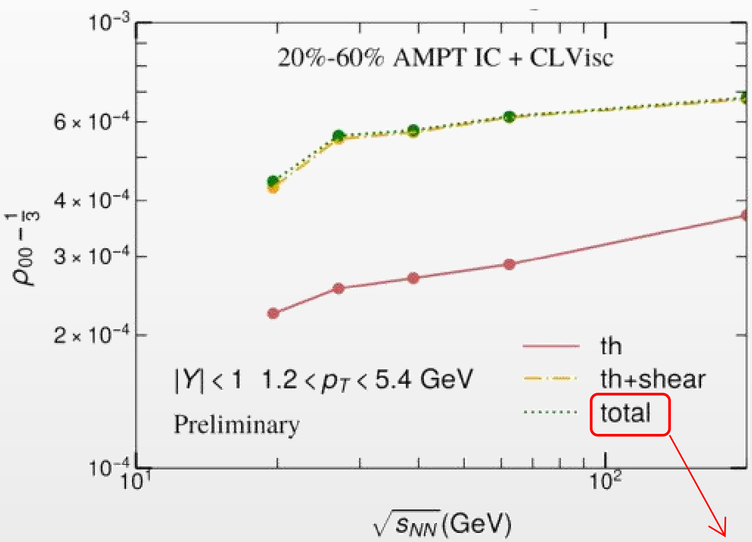
  Spin kinetic theory -> relativistic quark coalescence with spin

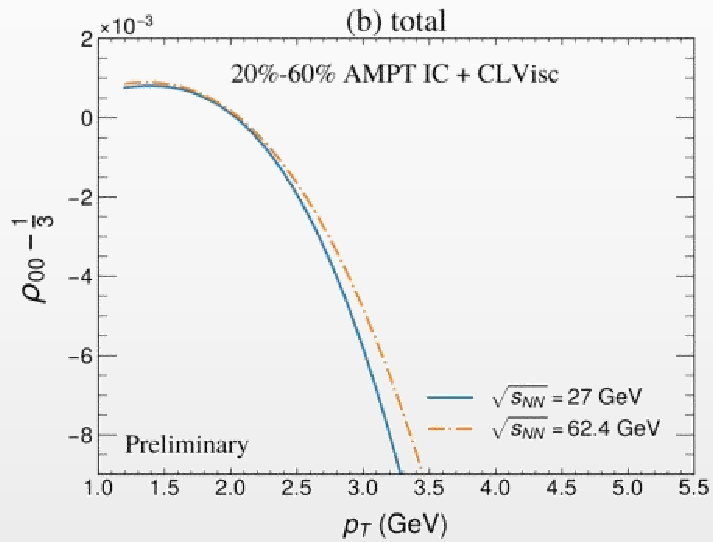
- Other theoretical studies for vector meson’s spin alignment

- Outlook and summary

- Spin alignment induced by various hydrodynamical gradient terms

Talk by Cong Yi (易聪) on Chirality 2023 (Beijing)





Including: quadrotic terms of thermal vorticity, shear viscous tensor, fluid acceleration, gradient of chemical potential

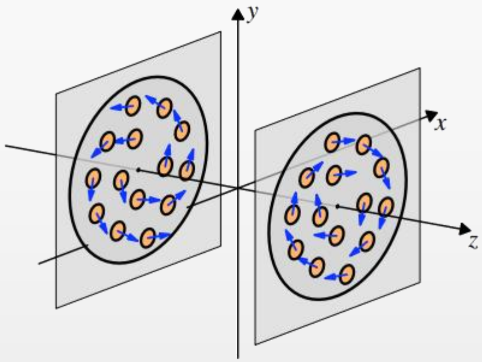
- ϕ meson with $p_T < 2$ GeV ($p_T > 2$ GeV) have $\rho_{00} > 1/3$ ($\rho_{00} < 1/3$);
- For mesons with $p_T > 3.5$ GeV , contributions from hydrodynamical gradient terms becomes significant (~ -0.01)

Local vorticity

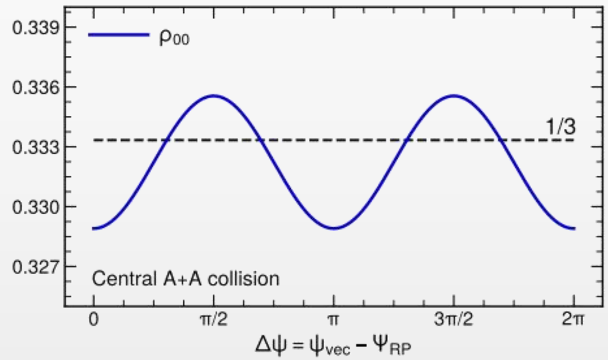
- Quark's polarization and ϕ meson's spin alignment induced by local vorticity field in central collisions

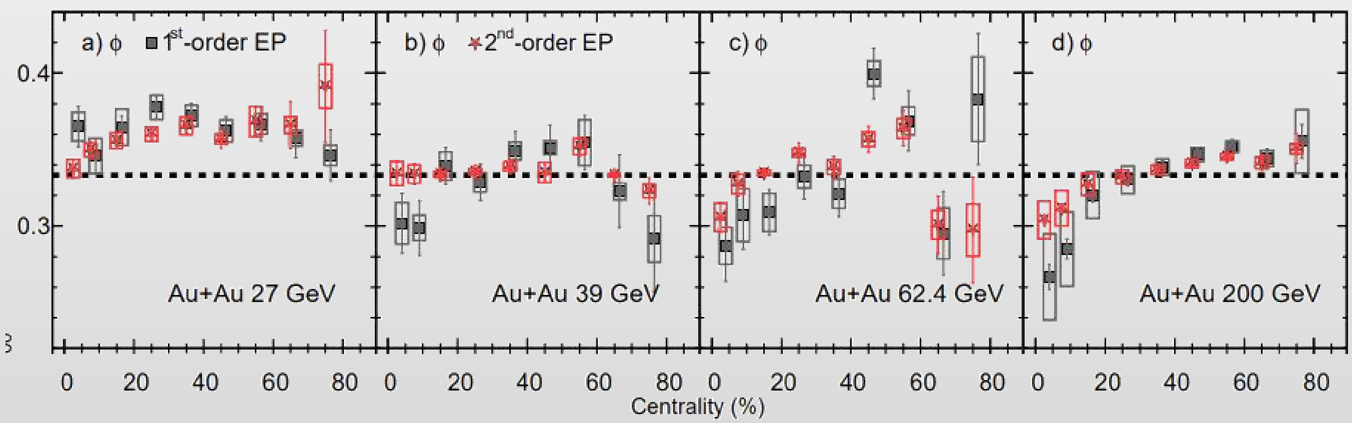
  X.-L. Xia, H. Li, X.-G. Huang, H.-Z. Huang, PLB 817, 136325 (2021)

  - Ring structure of local vorticity



  - Significant azimuthal angle dependence, average value<1/3





STAR, Nature 614, 244 (2023)

Spin alignment of rho mesons

The spin alignment of rho mesons in a pion gas

Yi-Liang Yin,[1] Wen-Bo Dong,[1] Jin-Yi Pang,[2] Shi Pu,[1] and Qun Wang[1,3]

PRC 110, 024905 (2024)

- Kadanoff-Baym equation on the closed-time path

⬇

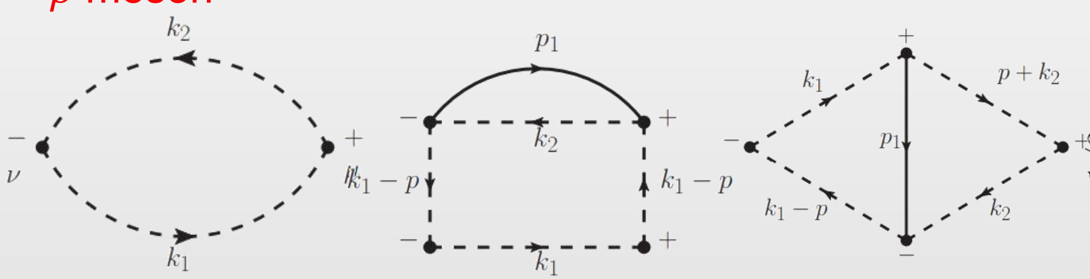
- Generalized Boltzmann equation with spin

$$\frac{p}{E_p^\rho} \cdot \partial_x f_{\lambda_1\lambda_2}(x, \mathbf{p}) = C_{\mathrm{coal/diss}} + C_{\mathrm{scat}}$$

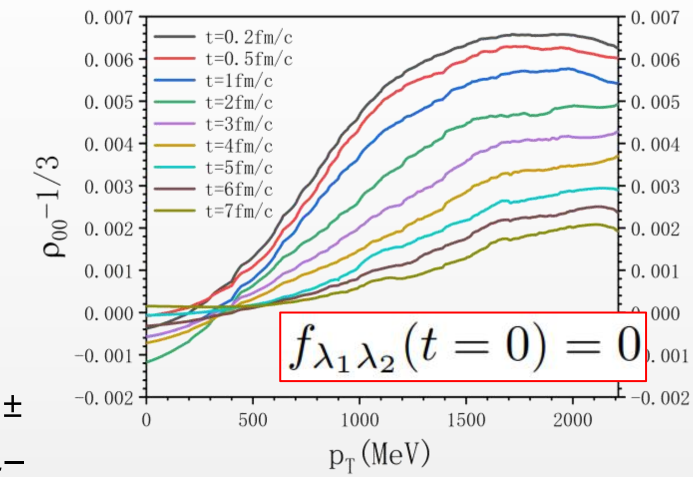
$$\mathcal{L}_{\mathrm{int}} = i g_{\rho\pi\pi} A^\mu \left(\phi^\dagger \partial_\mu \phi - \phi \partial_\mu \phi^\dagger\right)$$

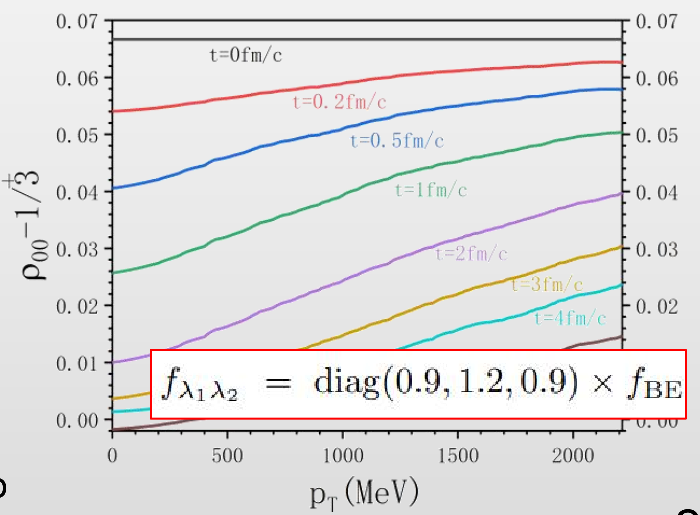
ρ meson $\quad$ π meson

$$\rho^0 \leftrightarrow \pi^+ + \pi^-$$
$$\rho^0 + \pi^\pm \rightarrow \rho^0 + \pi^\pm$$
$$\rho^0 + \rho^0 \leftrightarrow \pi^+ + \pi^-$$



Leading order $\qquad$ Next-to-leading order





- ➢ $\rho\pi\pi$ interaction generates a very small spin alignment
- ➢ A sizable initial spin alignment decreases fastly towards zero

Shear-stress tensor

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Generating Tensor Polarization from Shear Stress

David Wagner,[1,2] Nora Weickgenannt,[1,3] and Enrico Speranza[4]

PRR 5, 013187 (2023)

$$W^{\mu\nu}(x,k) := -\frac{2}{(2\pi\hbar)^4\hbar}\int d^4y\, e^{-ik\cdot y/\hbar}\langle : V^{\dagger\mu}(x+y/2)\,V^{\nu}(x-y/2):\rangle$$

↓

$$k\cdot\partial f(x,k,\mathfrak{s}) = \mathfrak{C}[f]$$

Generalized Boltzmann equation with spin

↓ Method of moment expansion

Talk by David Wagner, Friday 11:00

Deviation from local equilibrium distribution

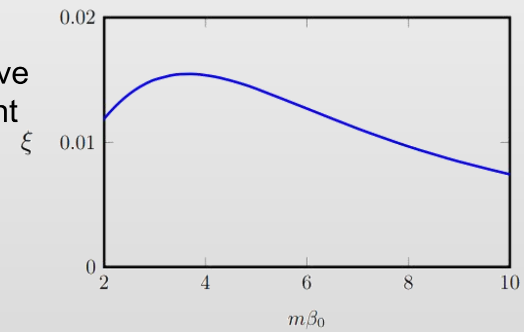
$$\delta f_{\mathbf{k}\mathfrak{s}} = f_{0\mathbf{k}}\left(-\frac{3}{m^2}\mathcal{H}^{(0,0)}_{\mathbf{k}0}\Pi + \mathcal{H}^{(0,2)}_{\mathbf{k}0}k_{\langle\mu}k_{\nu\rangle}\pi^{\mu\nu} + \xi\beta_0\mathcal{H}^{(2,0)}_{\mathbf{k}1}\mathfrak{s}_\alpha\mathfrak{s}_\beta K^{\alpha\beta}_{\mu\nu}\Xi^{\mu\nu}_{\rho\sigma}\pi^{\rho\sigma}\right)$$

$$\rho_{00}(k) = \frac{1}{3} - \frac{4}{15}\frac{\int d\Sigma_\alpha k^\alpha \xi \beta_0 f_{0\mathbf{k}}\mathcal{H}^{(2,0)}_{\mathbf{k}1}\epsilon^{(0)}_\alpha\epsilon^{(0)}_\beta K^{\alpha\beta}_{\mu\nu}\Xi^{\mu\nu}_{\rho\sigma}\pi^{\rho\sigma}}{\int d\Sigma_\alpha k^\alpha f_{0\mathbf{k}}\left(1 - 3\mathcal{H}^{(0,0)}_{\mathbf{k}0}\Pi/m^2 + \mathcal{H}^{(0,2)}_{\mathbf{k}0}\pi^{\mu\nu}k_{\langle\mu}k_{\nu\rangle}\right)}$$

Shear stress tensor

Bose distribution

Dissipative coefficient



- Dissipative effect
- Sign and magnitude waiting for numerical simulation

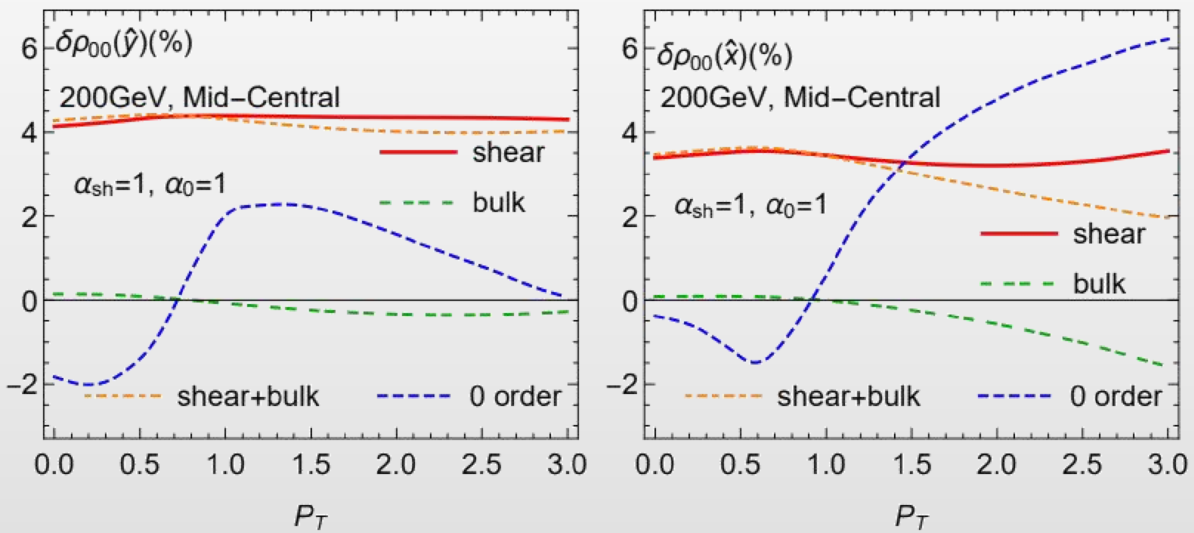
- Kubo formula for Wigner function as linear response to shear tensor

$$W^{\mu\nu}_{+(1)} = \varepsilon_{\boldsymbol{p}} \lim_{\omega,q\to 0} \frac{\partial}{\partial\omega}[-\mathrm{Im}G^{\mu\nu\lambda\gamma}_{R+}(\omega,\boldsymbol{q},\boldsymbol{p})]\xi_{\lambda\gamma}$$

$$G^{\mu\nu\lambda\gamma}_{R}(t-t',\boldsymbol{x},\boldsymbol{z},\boldsymbol{y}) \equiv \int \frac{d\omega}{2\pi}\frac{d^3\boldsymbol{q}}{(2\pi)^3}\frac{d^3\boldsymbol{p}}{(2\pi)^3}e^{-i\omega\cdot(t-t')} \times e^{i\boldsymbol{q}\cdot(\boldsymbol{x}-\boldsymbol{z})}e^{i\boldsymbol{p}\cdot\boldsymbol{y}}G^{\mu\nu\lambda\gamma}_{R}(\omega,\boldsymbol{q},\boldsymbol{p})$$
$$= (-i)\Theta(t-t')\langle[V^\mu(t,\boldsymbol{x}^-)V^\nu(t,\boldsymbol{x}^+),T^{\lambda\gamma}(t',\boldsymbol{z})]\rangle,$$

$$\xi_{\lambda\gamma} \approx \sigma_{\lambda\gamma} + \left[\frac{1}{3}\bar{\Delta}_{\lambda\gamma} + c_s^2 u_\lambda u_\gamma\right]\theta$$

Talk by Shuai Liu, Thursday 11:30



F. Li, S. Liu, arXiv: 2206.11890

Shear-stress tensor

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- Kubo formula for the linear response to shear tensor

$$\left\langle \hat{O}(x)\right\rangle = \left\langle \hat{O}\right\rangle_{\mathrm{LE}} + \partial_\mu \beta_\nu(x) \lim_{K^\mu\to 0} \frac{\partial}{\partial K_0} \ \mathrm{Im}\left[iT(x)\int_{-\infty}^{t} d^4x' \left\langle\left[\hat{O}(x), \hat{T}^{\mu\nu}(x')\right]\right\rangle_{\mathrm{LE}} e^{-iK\cdot(x'-x)}\right]$$

- Wigner operator and energy-stress tensor for a vector field

$$\hat{G}^{\mu\nu}_{<}(x,p) = \int d^4y e^{ip\cdot y} \hat{A}^\nu\left(x-\frac{y}{2}\right)\hat{A}^\mu\left(x+\frac{y}{2}\right)$$

$$\hat{T}^{\mu\nu} = \hat{F}^{\mu}{}_{\alpha}\hat{F}^{\alpha\nu} + m_V^2 \hat{A}^\mu \hat{A}^\nu - g^{\mu\nu}\left(-\frac{1}{4}\hat{F}_{\rho\eta}\hat{F}^{\rho\eta} + \frac{1}{2}m_V^2 \hat{A}_\rho \hat{A}^\rho\right)$$

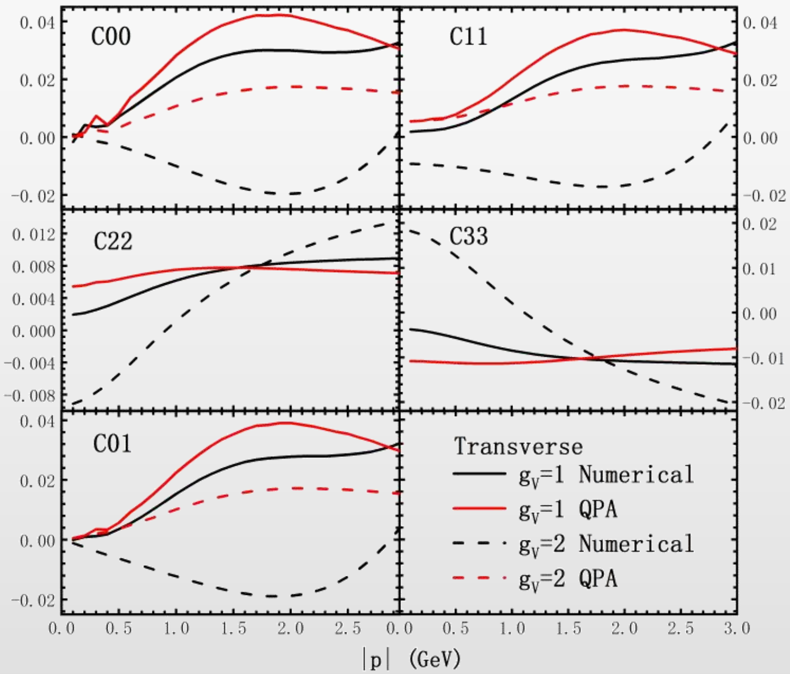
- Spin alignment

$$\delta\rho_{00} = \frac{L^{\mu\nu}(p_{\mathrm{on}})\int_0^{+\infty} dp_0 \left[G^{<}_{\mu\nu}(x,p) + \delta G^{<}_{\mu\nu}(x,p)\right]}{-\Delta^{\mu\nu}(p_{\mathrm{on}})\int_0^{+\infty} dp_0 \left[G^{<}_{\mu\nu}(x,p) + \delta G^{<}_{\mu\nu}(x,p)\right]} = \delta\rho_{00}^{(\xi=0)}(\mathbf{p}) + \xi_{\mu\nu}C^{\mu\nu}(\mathbf{p})$$

Linear response to shear tensor

Calculated with a quark-meson interaction

$C^{\mu\nu} \sim 10^{-2}$



W.-B. Dong, Y.-L. Yin, XLS, S.-Z. Yang, Q. Wang, PRD 109, 056002 (2024)

Second order gradient

- Second order hydrodynamic gradient term: $\omega\omega$, aa, $a\omega$, $\partial\omega$, ∂a

$$\rho_{00}(q) = \frac{\int d\Sigma_X \cdot q \left[1 + \frac{\hbar^2}{8}\frac{(q^y)^2}{M^4}\not{\partial}\right] f_V(q,X)}{\int d\Sigma_X \cdot q \left[3 + \frac{\hbar^2}{8}\left(\frac{|\boldsymbol{q}|^2}{M^4}\right)\not{\partial}\right] f_V(q,X)}$$

$$\not{\partial} = \left(3(\partial_t)^2 - (\partial^i)^2 - \frac{4}{|\boldsymbol{q}|^2}(q^i\partial^i)\, q^\alpha \partial_\alpha\right).$$

$$\rho_{00}(q) = \frac{1}{3} - \frac{\hbar^2}{12M^2}\frac{\int d\Sigma_X \cdot q\,(2(\partial^y)^2 - (\partial^x)^2 - (\partial^z)^2) f_V(q,X)}{\int d\Sigma_X \cdot q\, f_V(q,X)}.$$

Talk by Di-Lun Yang, Friday 9:30

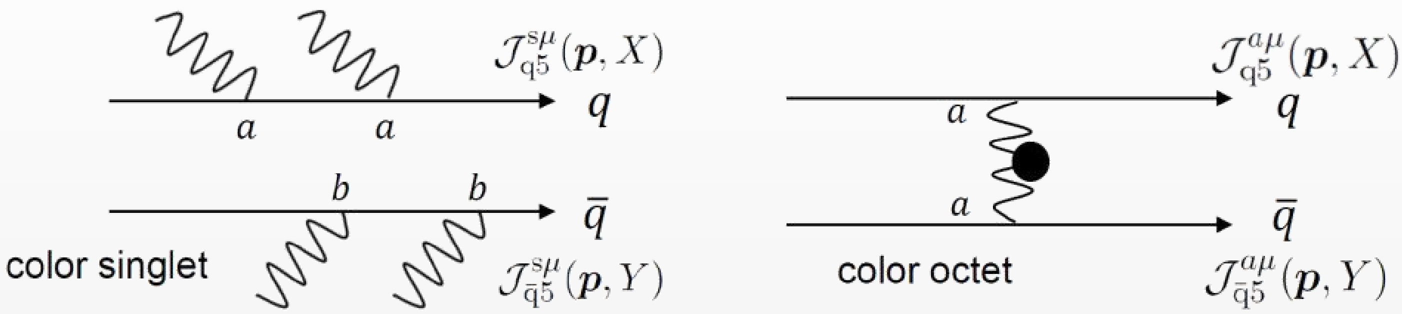
$$\delta\Theta_{00}\big|_{\partial\omega}(x,k) = -\frac{1}{9}(1+n_B)\Bigg\{\left[\frac{3\gamma_k(\gamma_k-1)}{2m(\gamma_k+1)}(v^y)^2 - \frac{\gamma_k{}^2+1}{2m\gamma_k}\right] \times \boldsymbol{v}\cdot(\nabla\times\boldsymbol{\omega}) + \frac{3}{m}(v^y)(\nabla\times\boldsymbol{\omega})^y\Bigg\},$$

Talk by Zhong-Hua Zhang, Thursday 11:00

$$\delta\Theta_{00}\big|_{\partial a}(x,k) = \frac{1}{9}(1+n_B)\Bigg\{\left[\frac{\gamma_k{}^2-3}{2m\gamma_k} - \frac{3\gamma_k}{2m}(v^y)^2\right]\nabla\cdot\boldsymbol{a} + \left[\frac{3\gamma_k{}^2(\gamma_k-1)}{2m(\gamma_k+1)^2}(v^y)^2 - \frac{\gamma_k}{2m}\right](\boldsymbol{v}\cdot\nabla)(\boldsymbol{a}\cdot\boldsymbol{v}) + \frac{3}{m\gamma_k}\partial_y a^y + \frac{3\gamma_k}{m(\gamma_k+1)}(\boldsymbol{v}\cdot\nabla)(a^y v^y) + \frac{3(\gamma_k-1)}{2m(\gamma_k+1)}(v^y\partial_y)(\boldsymbol{a}\cdot\boldsymbol{v})\Bigg\}$$

Z.-H. Zhang, X.-G. Huang, F. Becattini, XLS, in preparation

Spin-spin correlation induced by color field



$$\rho_{00} - \frac{1}{3} \approx \frac{-\hbar^2 g^2 e^{-2X_0^{\mathrm{eq}}/\tau_{\mathrm{R}}^{\mathrm{o}}} \int d\Sigma_X \cdot q \Pi_B(\boldsymbol{X}) (\partial_{\epsilon_{\boldsymbol{q}/2}} \tilde{f}_V(\epsilon_{\boldsymbol{q}/2}, 0))^2}{72 N_c^2 m^2 \int d\Sigma_X \cdot q f^{\mathrm{th}}_{\mathrm{V}q}(\epsilon_{\boldsymbol{q}/2}) f^{\mathrm{th}}_{\mathrm{V}\bar{q}}(\epsilon_{\boldsymbol{q}/2})}$$

Talk by Di-Lun Yang, Friday 9:30

$$\Pi_B(\boldsymbol{X}) = \langle B^{ax}_{\mathrm{r}}(0, \boldsymbol{X}) B^{ax}_{\mathrm{r}}(0, \boldsymbol{X}) \rangle + \langle B^{az}_{\mathrm{r}}(0, \boldsymbol{X}) B^{az}_{\mathrm{r}}(0, \boldsymbol{X}) \rangle - 2 \langle B^{ay}_{\mathrm{r}}(0, \boldsymbol{X}) B^{ay}_{\mathrm{r}}(0, \boldsymbol{X}) \rangle$$

	small-P_T	large-P_T	central	non-central
glasma	$\rho_{00}^{\phi,J/\psi} < 1/3$	$\rho_{00}^{\phi,J/\psi} \lesssim 1/3$	$\rho_{00}^{\phi,J/\psi} < 1/3$	$\rho_{00}^{\phi,J/\psi} \lesssim 1/3$
effective potential	$\lvert\rho_{00}^{\phi,J/\psi} - 1/3\rvert \gtrsim 0$	$\lvert\rho_{00}^{\phi,J/\psi} - 1/3\rvert > 0$	$\rho_{00}^{\phi,J/\psi} < 1/3$	$\rho_{00}^{\phi,J/\psi} > 1/3$

A. Kumar, B. Mueller, D.-L. Yang, PRD 108, 016020 (2023)
B. Mueller, D.-L. Yang, PRD 105, L011901

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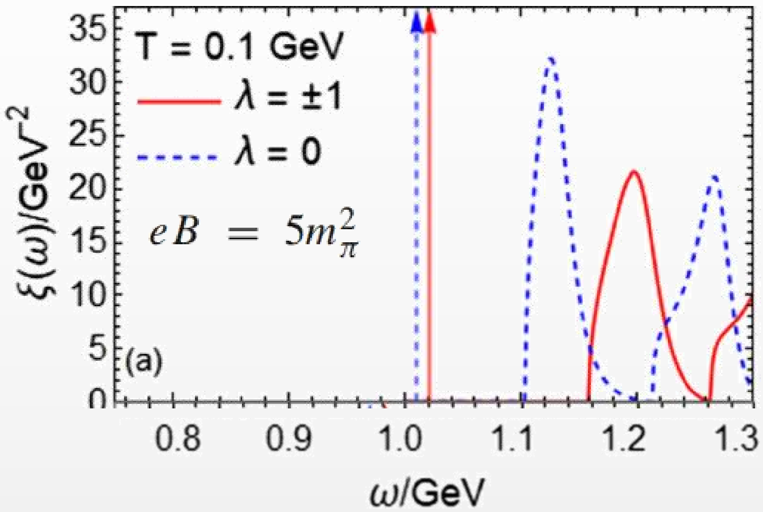
- Nonzero magnetic field: ϕ meson spectral function



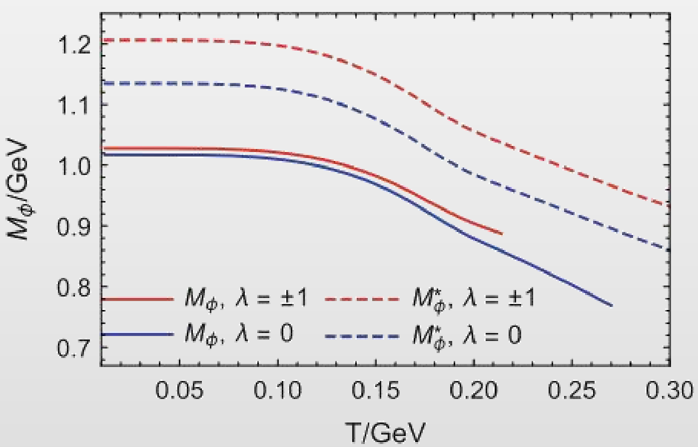
$$f_\lambda(\mathbf{p}) = \int_0^\infty d\omega \frac{2\omega}{e^{\omega/T}-1}\rho_\lambda(\omega.\mathbf{p})$$

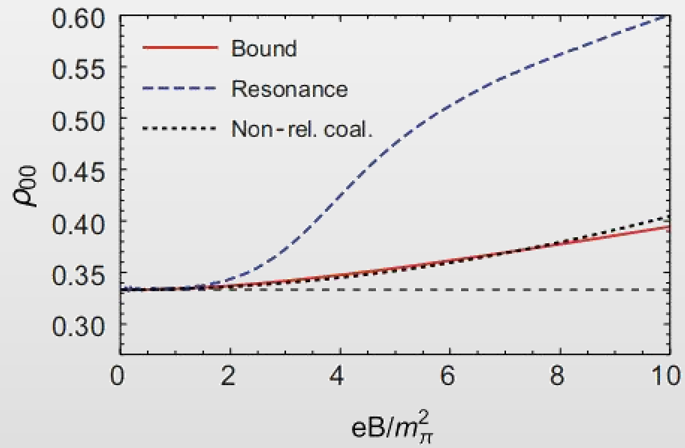
Spectral function for spin-λ state

$$\rho_\lambda(\omega.\mathbf{p}) = \frac{1}{\pi}\mathrm{Im}\frac{1}{p^2 - m_V^2 - \Sigma_\lambda}$$

Magnetic field induce splitting between spectral functions

→ Spin alignment in direction of magnetic field

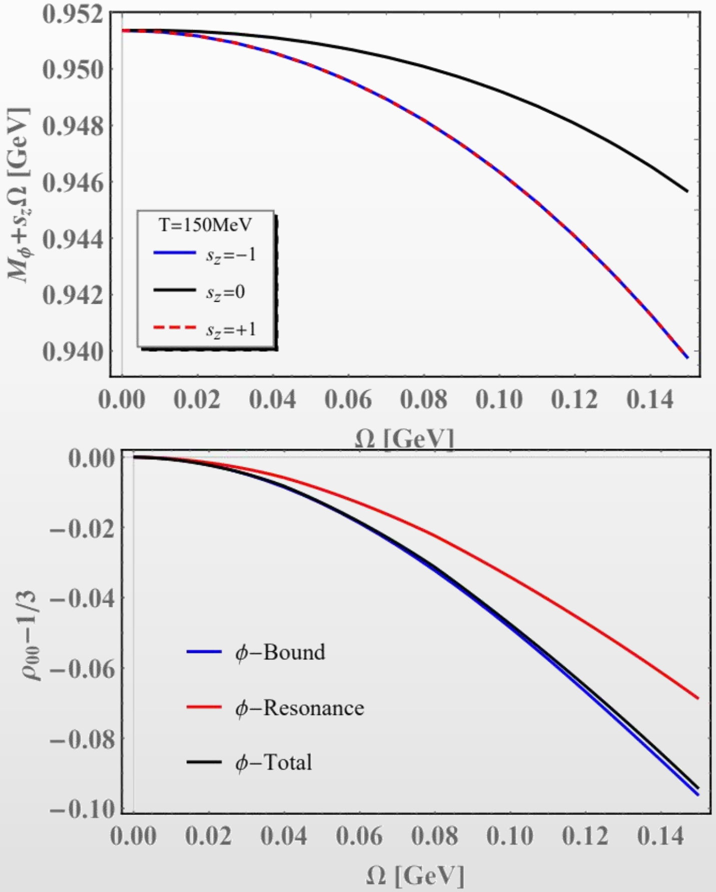




XLS, S.-Y. Yang, Y.-L. Zou, D. Hou, Eur. Phys. J. C 84, 299 (2024)

NJL model calculation

- Nonzero vorticity field: ϕ meson mass as a function of angular velocity

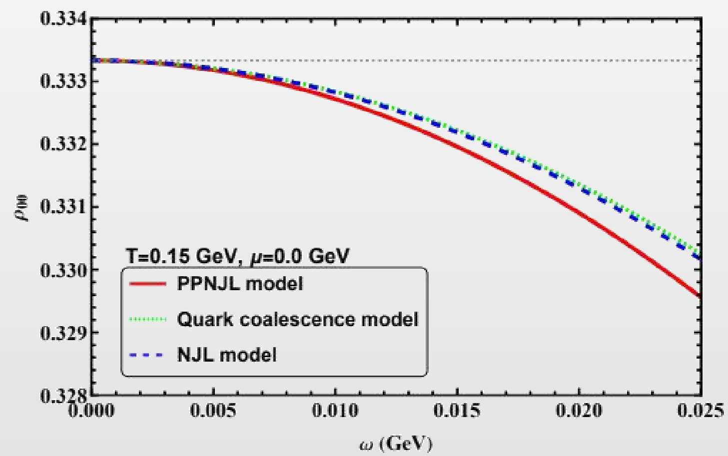


M. Wei, M. Huang, CPC 47, 104105 (2023)

Rotation induce a mass splitting

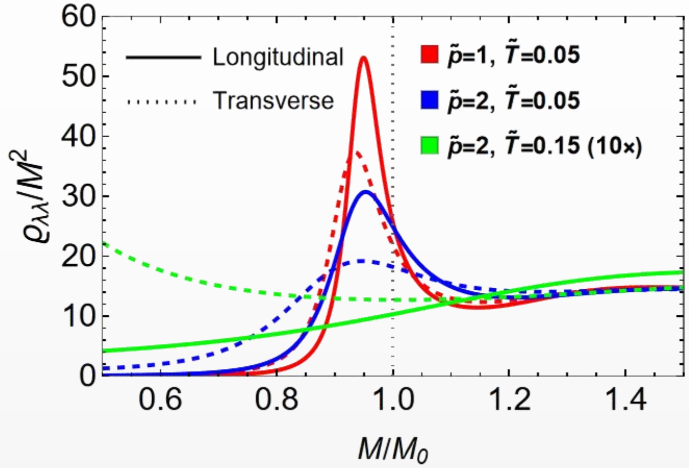
→ Spin alignment in direction of rotation

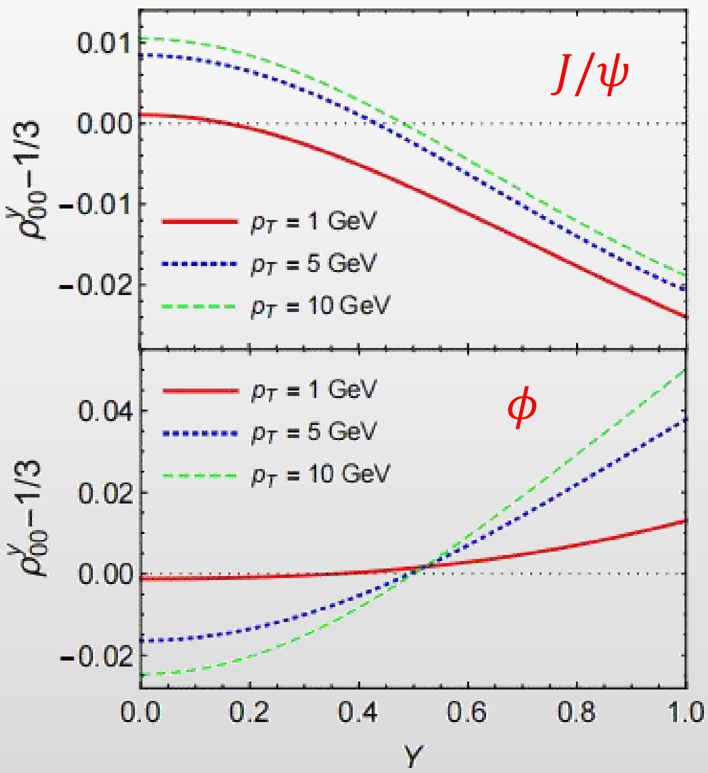
Qualitatively agree with non-relativistic coalescence model



F. Sun, J. Shao, R. Wen, K. Xu, M. Huang, PRD 109, 116017 (2024)

Holographic model

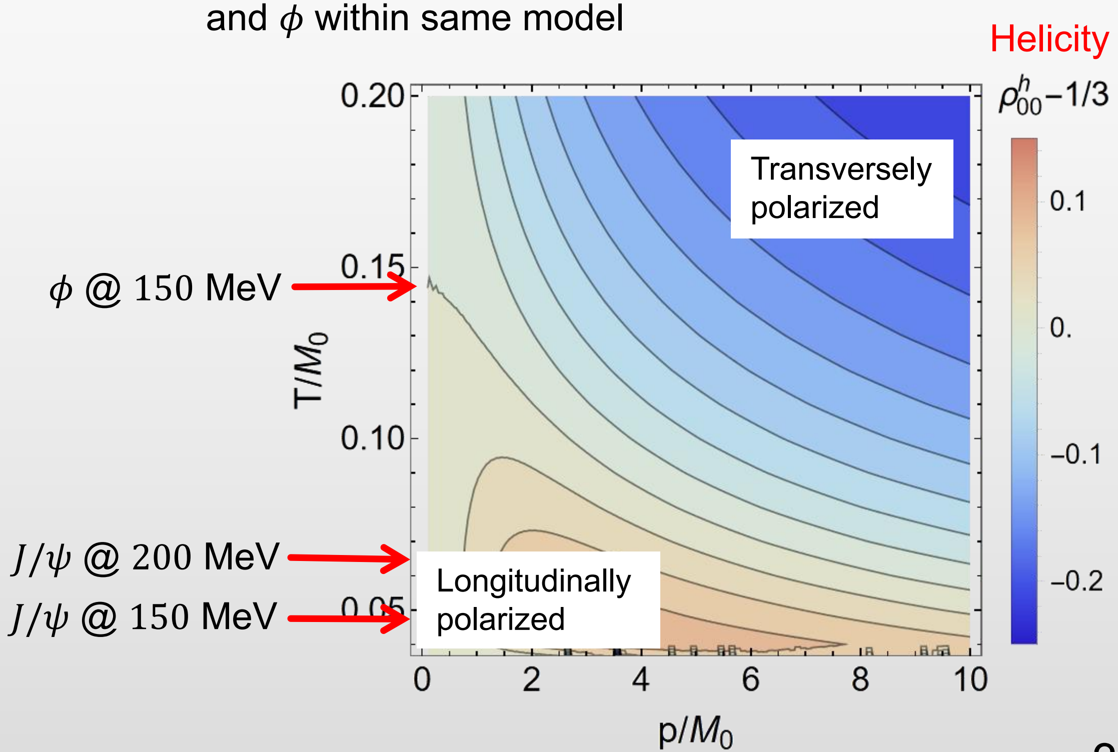




- Spin alignment induced by motion in a holographic model

  XLS, Y.-Q. Zhao, S.-W. Li, F. Becattini, D. Hou, arXiv:2403.07522 (accepted by PRD)

  - Strongly coupled system
  - Opposite behaviours of J/ψ and ϕ within same model



Outline

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- Introduction

  Non-relativistic quark coalescence with spin

- Strong-force field fluctuation

  Spin kinetic theory -> relativistic quark coalescence with spin

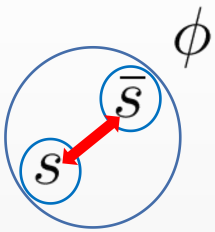
- Other theoretical studies for vector meson's spin alignment

- Outlook and summary

Λ-$\bar{\Lambda}$ spin correlation

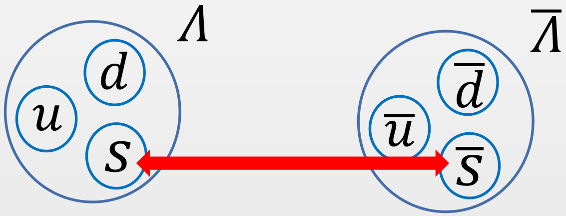
- Vector meson's spin alignment
  - → Spin correlation between constituent quark and antiquark

$$\rho_{00}^{V}(\vec{x},\vec{p}) \approx \frac{1}{3} - \frac{2}{3}\left\langle P_{q_1}^{z} P_{\bar{q}_2}^{z} \right\rangle_V + \frac{2}{9}\left\langle \vec{P}_{q_1} \cdot \vec{P}_{\bar{q}_2} \right\rangle_V$$



Short-range spin correlation

- Λ's spin polarization
  - → Spin polarization of constituent s quark
- Λ-$\bar{\Lambda}$ spin correlation
  - → Spin correlation between s and $\bar{s}$



Long-range spin correlation

- If strong force fluctuation can induce a short-range spin correlation, it may also contribute to long-range spin correlation, which may be verified by Λ-$\bar{\Lambda}$ spin correlation

- Transform of density matrix when changing spin quantization direction

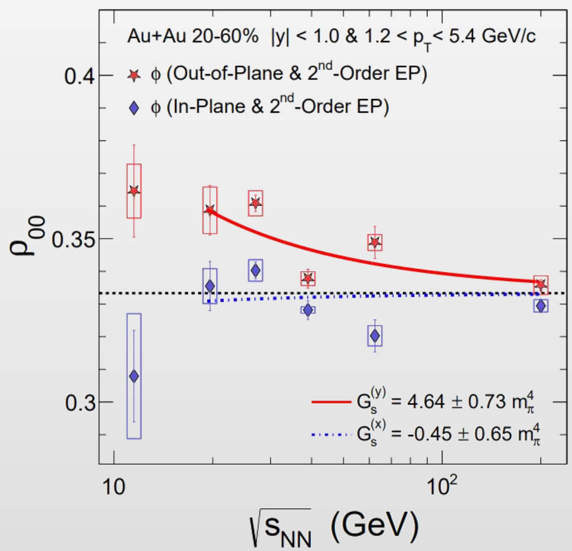
XLS, S. Pu ..., in preparation

$$\rho^{\mathbf{n}'}_{\lambda\lambda'} = \sum_{\lambda_1,\lambda_2=0,\pm1} \left[D^1_{\lambda_1\lambda}(\theta,\varphi)\right]^* D^1_{\lambda_2\lambda'}(\theta,\varphi)\rho^{\mathbf{n}}_{\lambda_1\lambda_2} \qquad \rho^{\mathbf{n}}_{11} + \rho^{\mathbf{n}}_{00} + \rho^{\mathbf{n}}_{-1,-1} = 1$$

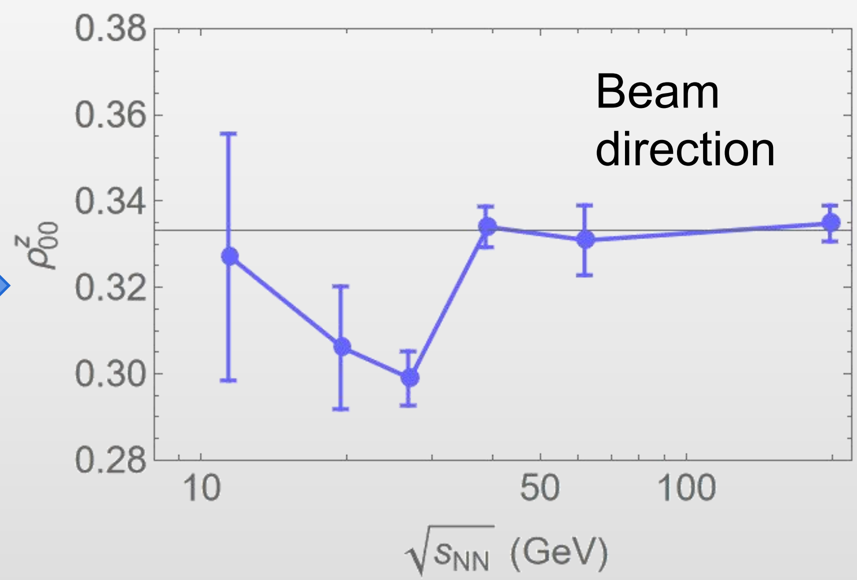
$$\rho^{\hat{\mathbf{x}}}_{00} = \frac{1}{2}\left(\rho^{\hat{\mathbf{z}}}_{11} + \rho^{\hat{\mathbf{z}}}_{-1,-1} - \rho^{\hat{\mathbf{z}}}_{1,-1} - \rho^{\hat{\mathbf{z}}}_{-1,1}\right)$$

$$\rho^{\hat{\mathbf{y}}}_{00} = \frac{1}{2}\left(\rho^{\hat{\mathbf{z}}}_{11} + \rho^{\hat{\mathbf{z}}}_{-1,-1} + \rho^{\hat{\mathbf{z}}}_{1,-1} + \rho^{\hat{\mathbf{z}}}_{-1,1}\right)$$

$$\rho^{\hat{\mathbf{x}}}_{00} + \rho^{\hat{\mathbf{y}}}_{00} + \rho^{\hat{\mathbf{z}}}_{00} = 1$$



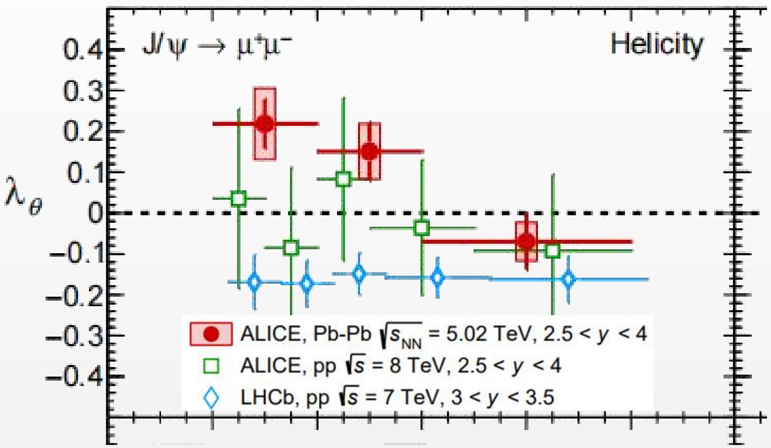
STAR, Nature 614, 244 (2023)

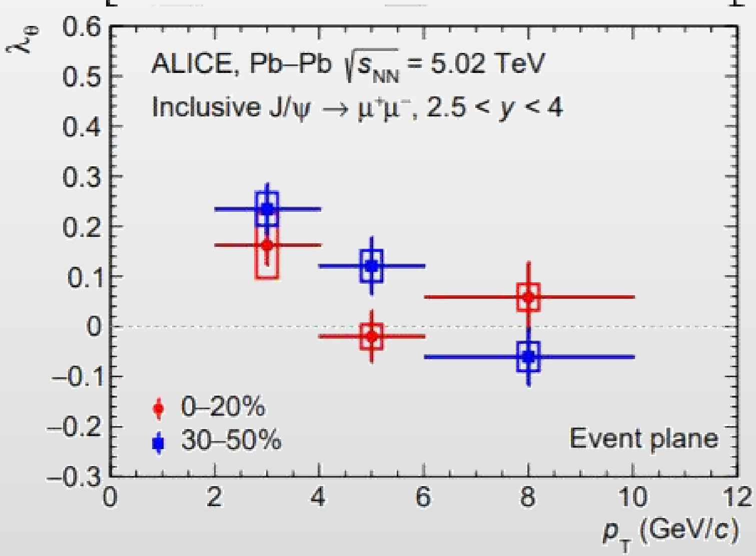


Normalization condition

- Prediction for J/ψ's spin alignment in in-plane direction

XLS, S. Pu ..., in preparation



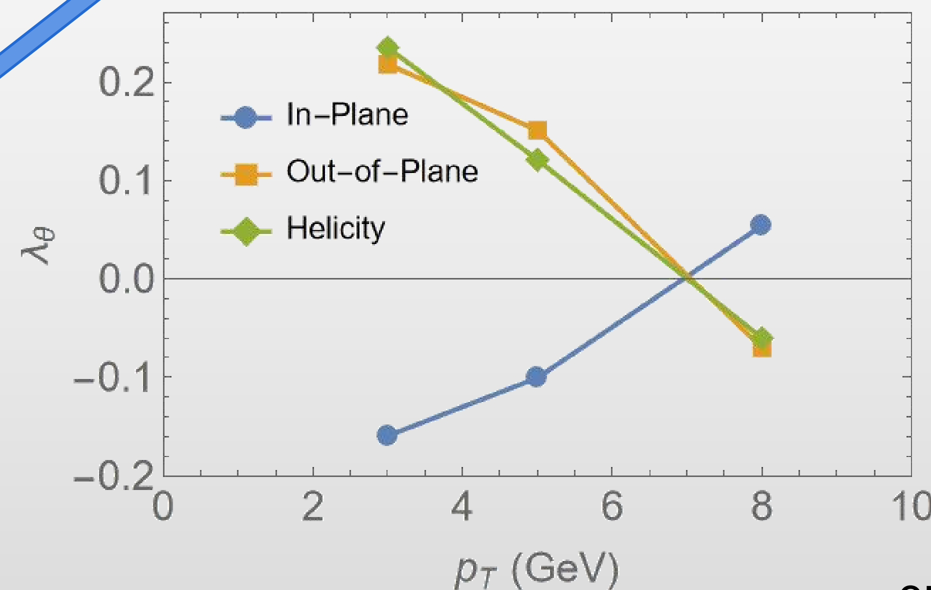


$$\lambda_\theta = \frac{1 - 3\rho_{00}}{1 + \rho_{00}}$$

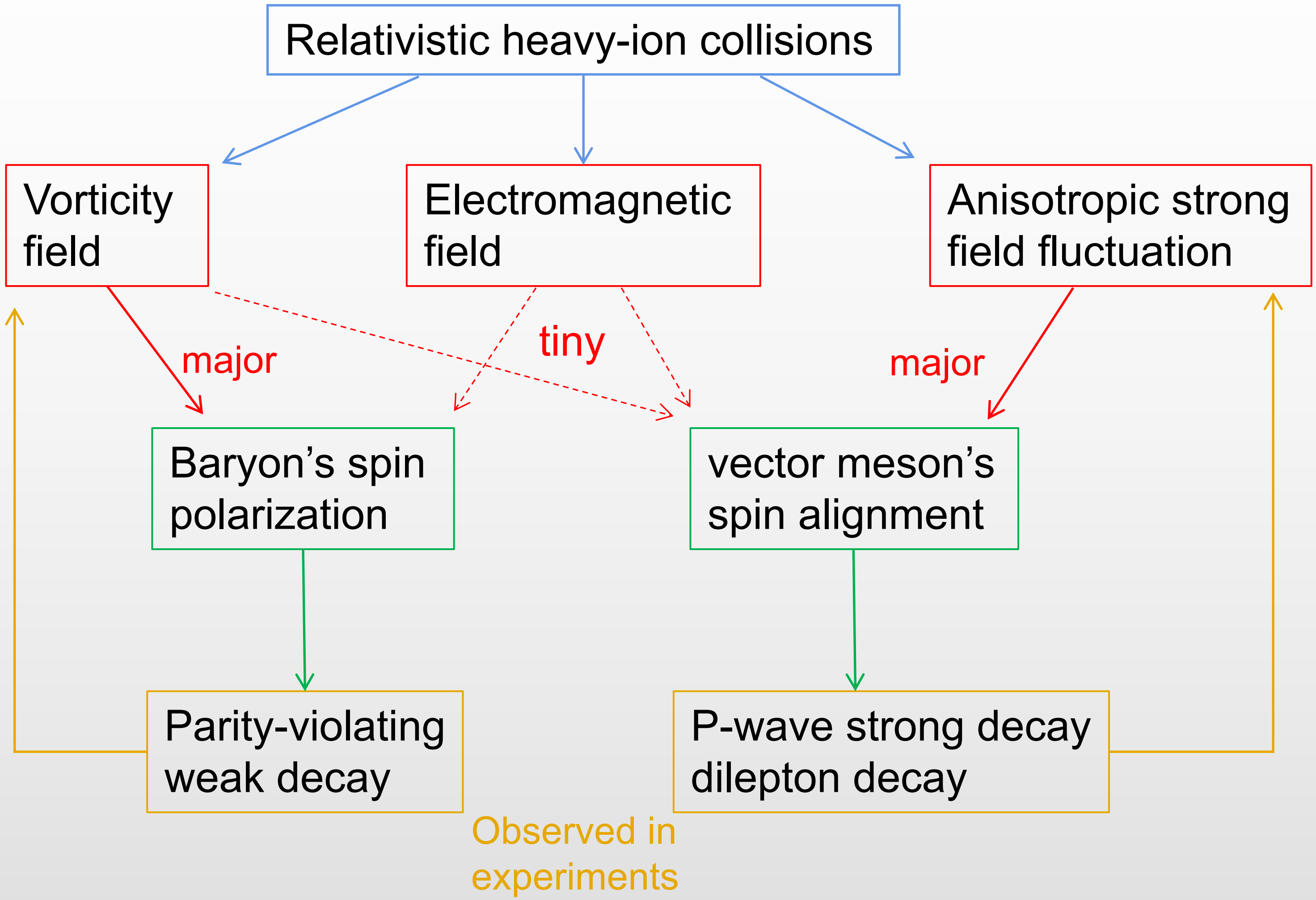
$$\rho_{00}^{\hat{\mathbf{z}}}$$

$$\rho_{00}^{\hat{\mathbf{y}}}$$

$$\rho_{00}^{\hat{\mathbf{x}}} + \rho_{00}^{\hat{\mathbf{y}}} + \rho_{00}^{\hat{\mathbf{z}}} = 1$$



Summary


Relativistic heavy-ion collisions
Vorticity field
Electromagnetic field
Anisotropic strong field fluctuation
major
tiny
major
Baryon's spin polarization
vector meson's spin alignment
Parity-violating weak decay
P-wave strong decay dilepton decay
Observed in experiments

Summary

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- Spin alignment measures anisotropy of strong field fluctuations in meson's rest frame.
- Dominate contribution to anisotropy may be motion of meson relative to background
- Predictions for momentum dependence of spin alignment need to be tested by more experiment results
- More theoretical efforts needed for J/ψ, K^{*0}, D^{*+}, ρ^0 ...

Thanks for your attention!

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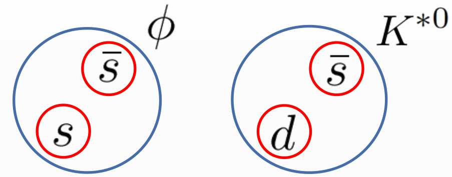
- Through strong p-wave decay

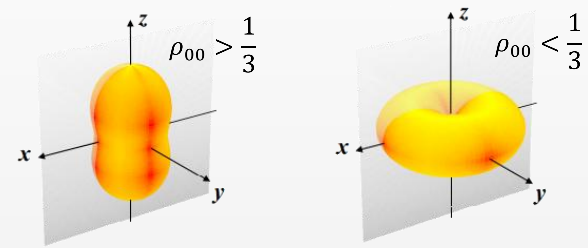
$$K^{*0} \rightarrow K^+ + \pi^-, \quad (\sim 100\%)$$

$$\phi \rightarrow K^+ + K^-, \quad (\sim 49\%).$$



J^P 1^- 0^- 0^- branch ratio

Meson's spin is converted to OAM



- Angular distribution of daughter particle

$$\begin{aligned}\frac{dN}{d\Omega} = & \frac{3}{8\pi}\left[(1-\rho_{00}) + (3\rho_{00}-1)\cos^2\theta\right. \\ & -2\mathrm{Re}\rho_{-1,1}\sin^2\theta\cos(2\phi) - 2\mathrm{Im}\rho_{-1,1}\sin^2\theta\sin(2\phi) \\ & +\sqrt{2}\mathrm{Re}(\rho_{-1,0}-\rho_{01})\sin(2\theta)\cos\phi \\ & \left.+\sqrt{2}\mathrm{Im}(\rho_{-1,0}-\rho_{01})\sin(2\theta)\sin\phi\right]\end{aligned}$$

$$\begin{aligned}\rho_{00} &= \frac{1}{3} - T_{33} \\ \mathrm{Re}\rho_{-1,1} &= \frac{1}{2}(T_{11}-T_{22}) \\ \mathrm{Im}\rho_{-1,1} &= T_{12} \\ \frac{\sqrt{2}}{2}\mathrm{Re}(\rho_{-1,0}-\rho_{01}) &= -T_{13} \\ \frac{\sqrt{2}}{2}\mathrm{Im}(\rho_{-1,0}-\rho_{01}) &= -T_{23}\end{aligned}$$

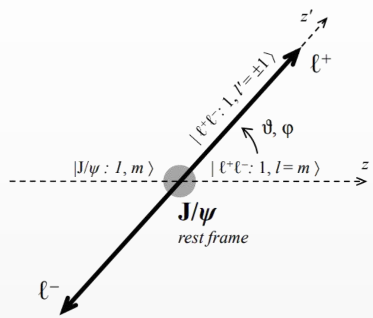
K. Schilling, P. Seyboth, G. E. Wolf, NPB 15, 397 (1970) [Erratum-ibid. B 18, 332 (1970)].
Y.G. Yang, R.-H. Fang, Q. Wang, X.-N. Wang, PRC 97, 034917 (2018)

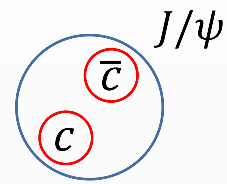
How to measure

- Through dilepton decay

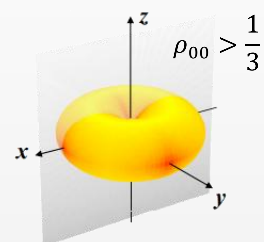
$$J/\psi \rightarrow \mu^+ + \mu^-$$

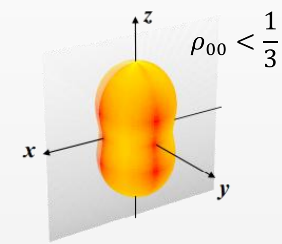
Meson's spin is converted to spin of dilepton





- Angular distribution of daughter particle





$$\frac{dN}{d\Omega} = \frac{3}{16\pi}\Big[(1+\rho_{00}) + (1-3\rho_{00})\cos^2\theta$$
$$+2\mathrm{Re}\rho_{-1,1}\sin^2\theta\cos 2\phi - 2\mathrm{Im}\rho_{-1,1}\sin^2\theta\sin 2\phi$$
$$-\sqrt{2}\mathrm{Re}(\rho_{-1,0}-\rho_{10})\sin 2\theta\cos\phi$$
$$+\sqrt{2}\mathrm{Im}(\rho_{-1,0}-\rho_{10})\sin 2\theta\sin\phi\Big]$$

J/ψ "polarization" parameter

$$\lambda_\theta = \frac{1-3\rho_{00}}{1+\rho_{00}} \approx -\frac{9}{4}\left(\rho_{00}-\frac{1}{3}\right)$$

$$\lambda_\varphi = \frac{2\mathrm{Re}\rho_{1,-1}}{1+\rho_{00}}, \quad \lambda_{\theta\varphi} = \frac{\sqrt{2}\mathrm{Re}(\rho_{01}-\rho_{0,-1})}{1+\rho_{00}},$$
$$\lambda_\varphi^\perp = \frac{-2\mathrm{Im}\rho_{1,-1}}{1+\rho_{00}}, \quad \lambda_{\theta\varphi}^\perp = \frac{\sqrt{2}\mathrm{Im}(\rho_{01}+\rho_{0,-1})}{1+\rho_{00}}.$$

P. Faccioli, C. Lourenco, J. Seixas, H. K. Wohri, EPJC 69, 657-673 (2010)