Spin alignment phenomena from the medium modified spectral functions in QCD matter

Shuai Liu

Hunan University

In collaboration: Feng Li

Spin and quantum features of QCD plasma, Sep 16-20, 2024, Trento, Italy

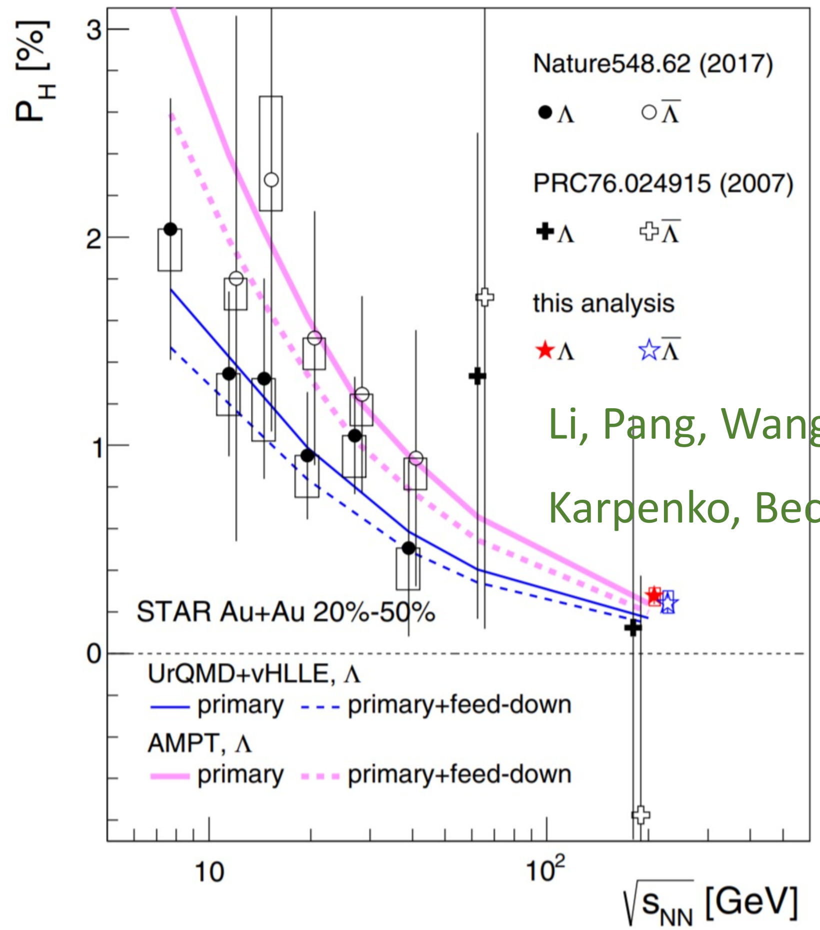
Based on work, Li and Liu, arXiv: 2206.11890 and Liu & Rapp series research 2018-2022

Outline

1) Spin observables in heavy-ion collisions
2) Theory for tensor polarization and spin alignment
3) How to produce a large enough spin alignment?
4) How to generate the rich behaviors of the spin alignment in a model?
5) Summary and outlooks

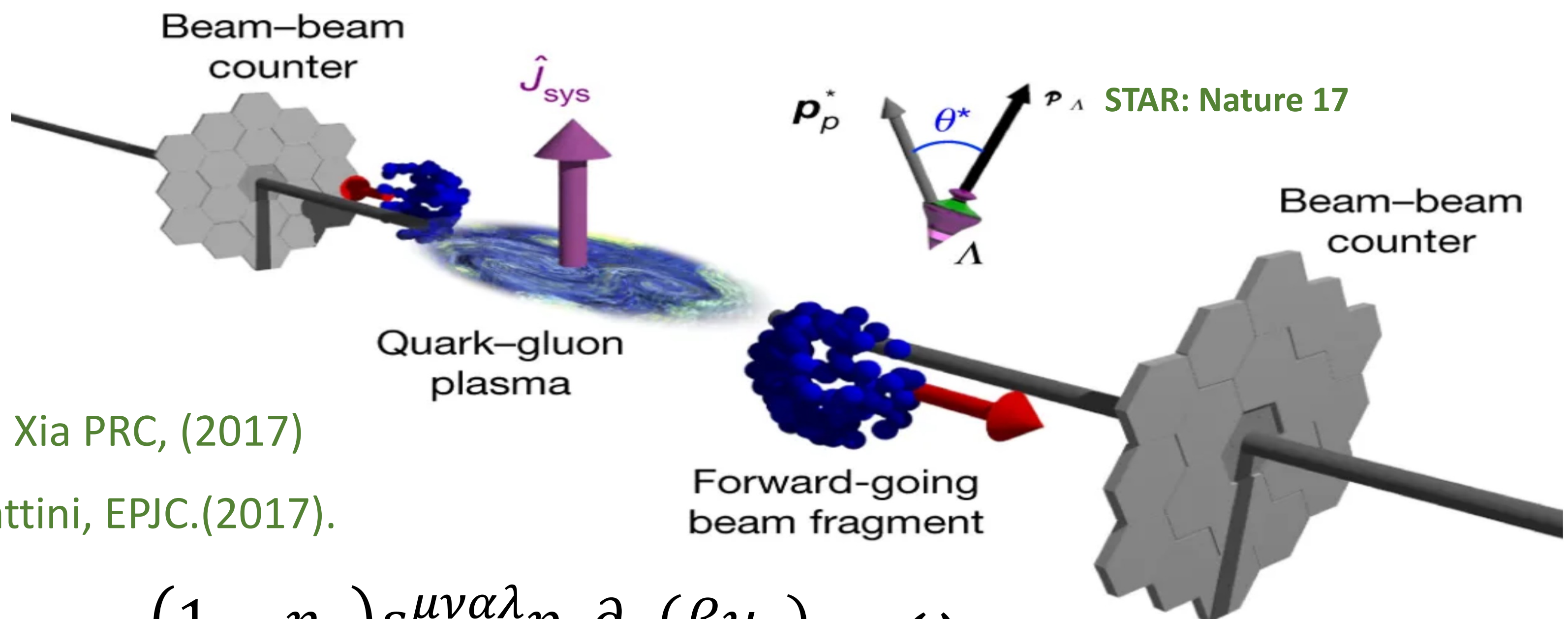
Vorticity Induced Polarization

STAR PRC 2018



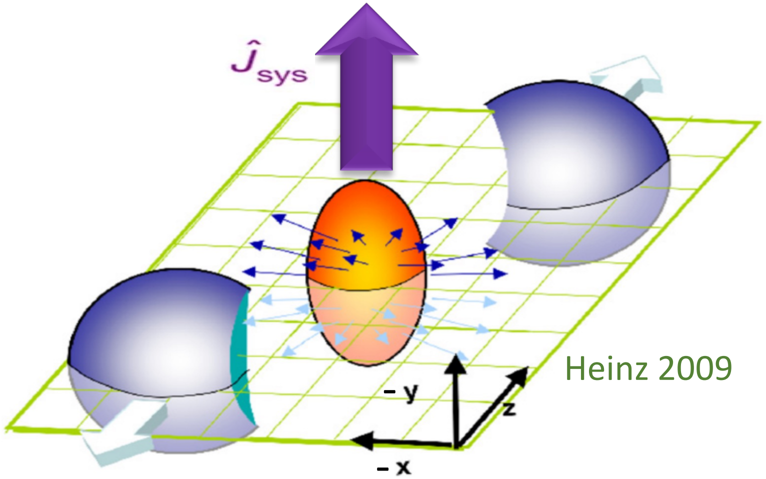
Li, Pang, Wang, Xia PRC, (2017)

Karpenko, Becattini, EPJC.(2017).



STAR: Nature 17

$$\mathcal{P} = \frac{(1-n_f)\varepsilon^{\mu\nu\alpha\lambda}p_\nu\partial_\alpha(\beta u_\lambda)}{4m} \approx \frac{\boldsymbol{\omega}}{2T} \sim 1\%$$

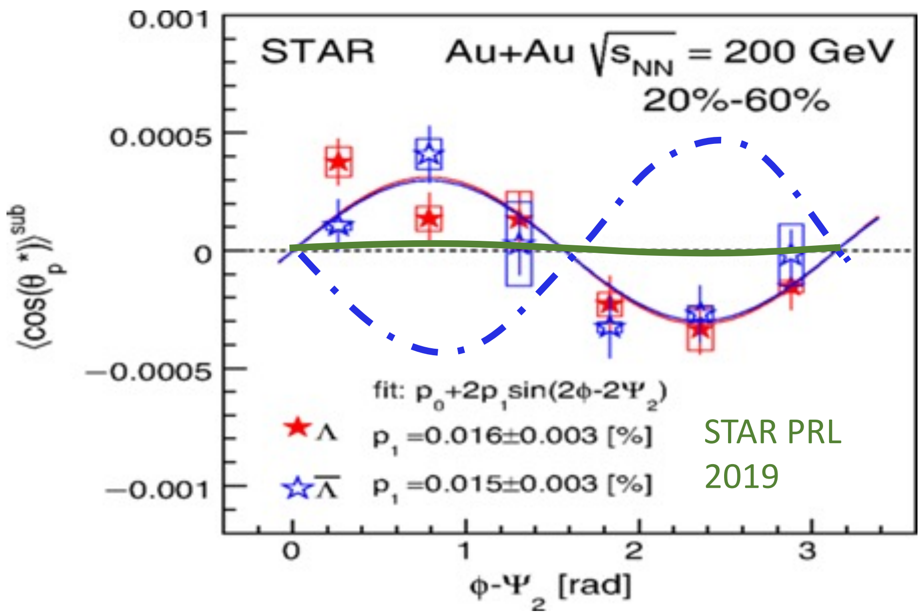


Heinz 2009

- ❖Rotation, vorticity, most vortical fluid
- ❖Vorticities can lead to polarization

Local Polarization and Shear Induced Polarization

Local polarization sign puzzles



Sketches of theoretical curves

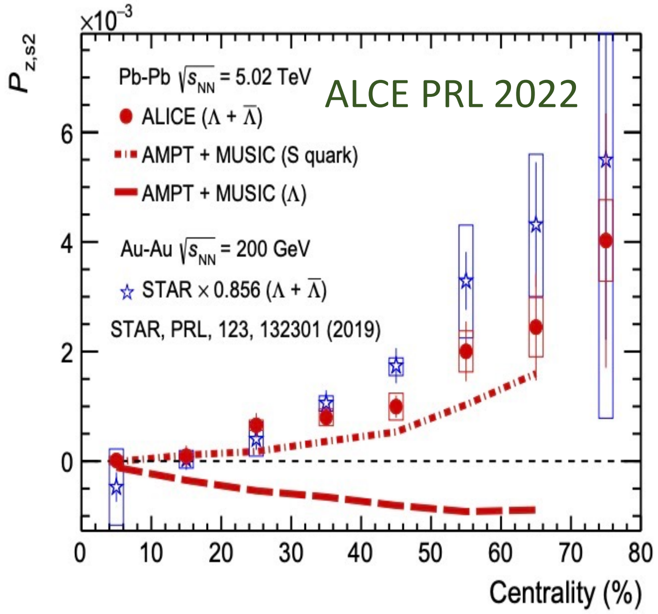
Voloshin, EPJ 2018
Becattini, Karpenko, PRL 2018

❖Previous theory

$$\mathcal{P} = \frac{(1-n_f)\epsilon^{\mu\nu\alpha\lambda}p_\nu\partial_\alpha(\beta u_\lambda)}{4m}$$

❖Sign is opposite!

Progress



Liu, Yin, JHEP, 2021;Becatinni, Buzzegoli, Palermo, PLB, 2021

❖Shear-Induced Polarization

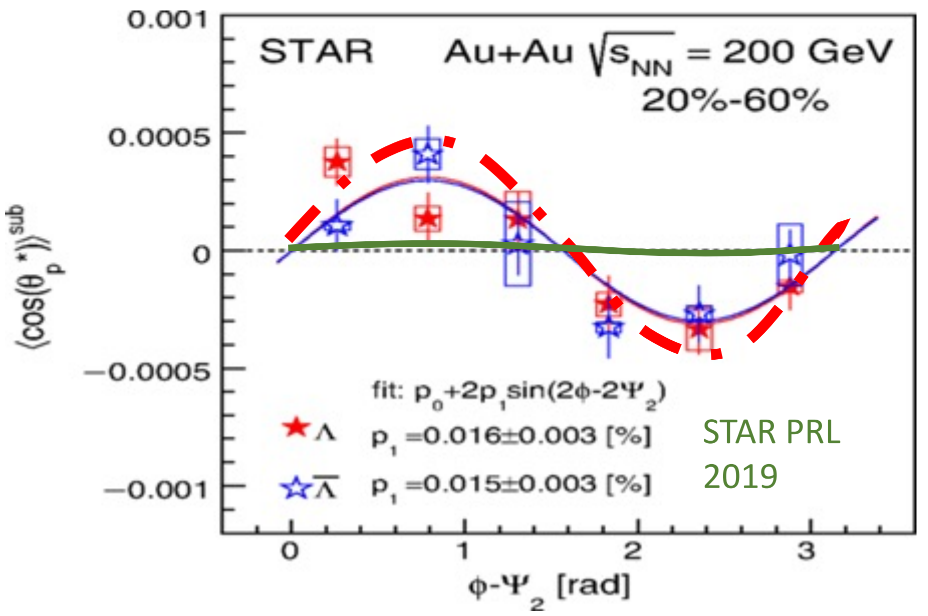
$$-n_0(1-n_0)\frac{1}{\varepsilon_0}\epsilon^{\mu\nu\alpha\rho}u_\nu\, p_\rho p^\lambda\partial_{(\alpha}(\beta u)_{\lambda)}$$

❖Right sign, large phenomenologically

Fu, Liu, Pang, Song, Yin, PRL, 2021;
Becatinni, Buzzegoli, Palermo, Inghirami,Karpenko PRL, 2021

Local Polarization and Shear-induced Polarization

Local polarization sign puzzle



Sketches of theoretical curves

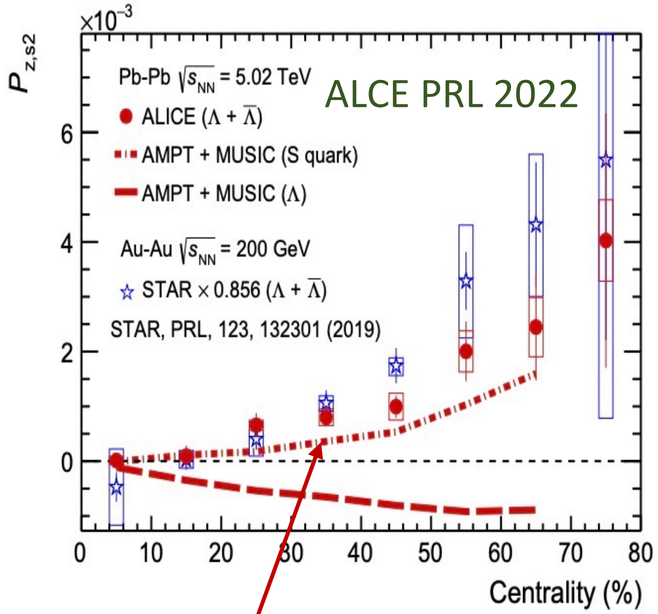
Voloshin, EPJ 2018
Becattini, Karpenko, PRL 2018

❖Previous theory

$$\mathcal{P} = \frac{(1-n_f)\epsilon^{\mu\nu\alpha\lambda}p_\nu\partial_\alpha(\beta u_\lambda)}{4m}$$

❖Sign is opposite!

Progress



Liu, Yin, JHEP, 2021;Becatinni, Buzzegoli, Palermo, PLB, 2021

❖Shear-Induced Polarization

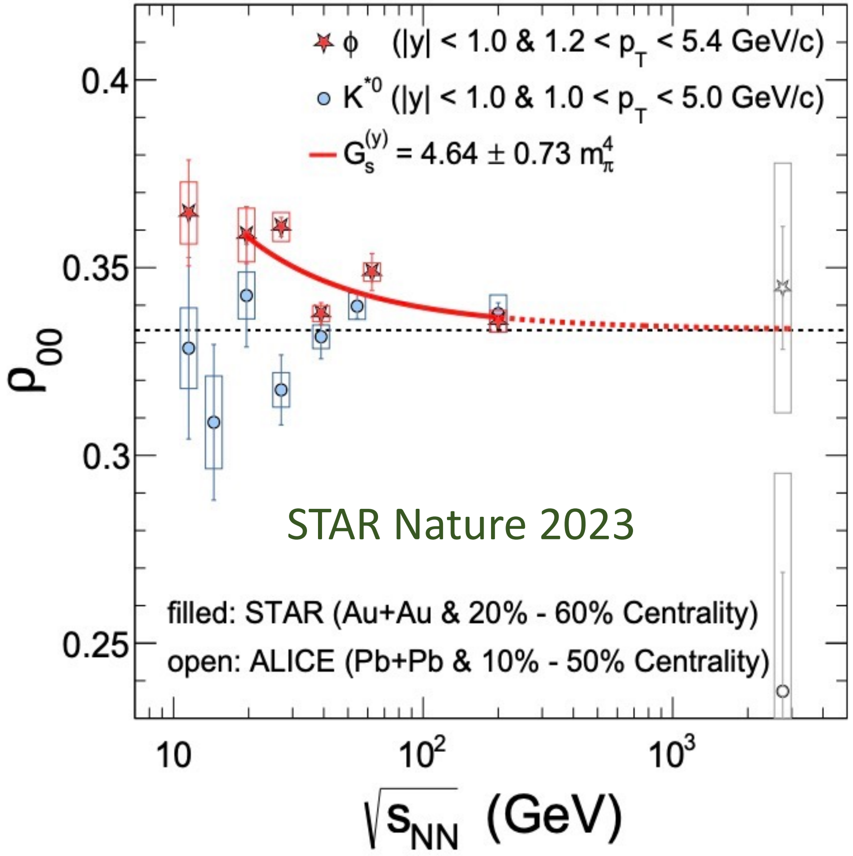
$$-n_0(1-n_0)\frac{1}{\varepsilon_0}\epsilon^{\mu\nu\alpha\rho}u_\nu p_\rho p^\lambda\partial_{(\alpha}(\beta u)_{\lambda)}$$

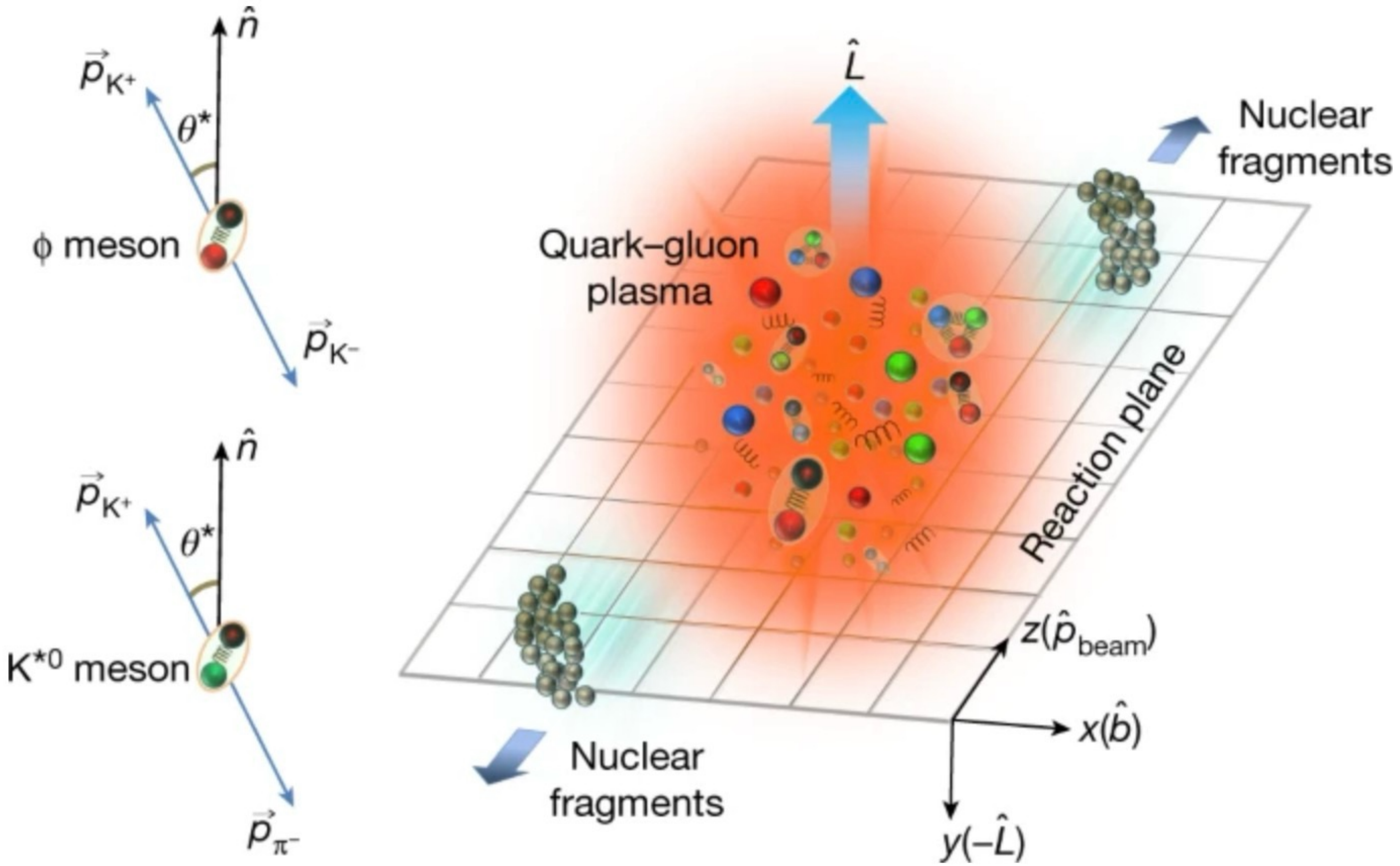
❖Right sign, large phenomenologically

Fu, Liu, Pang, Song, Yin, PRL, 2021;
Becatinni, Buzzegoli, Palermo, Inghirami,Karpenko PRL, 2021

The Spin Alignment in Heavy-ion Collision

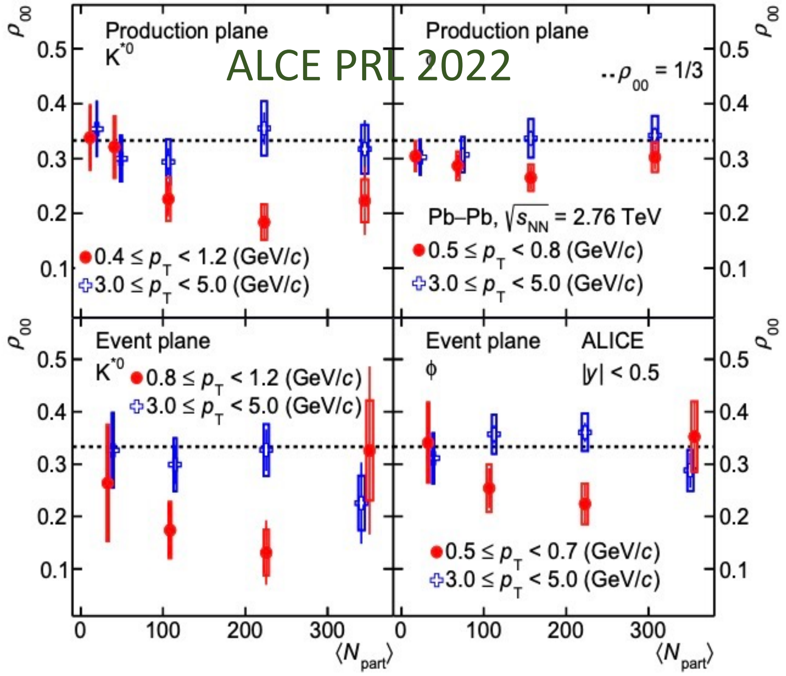
❖Experimental findings

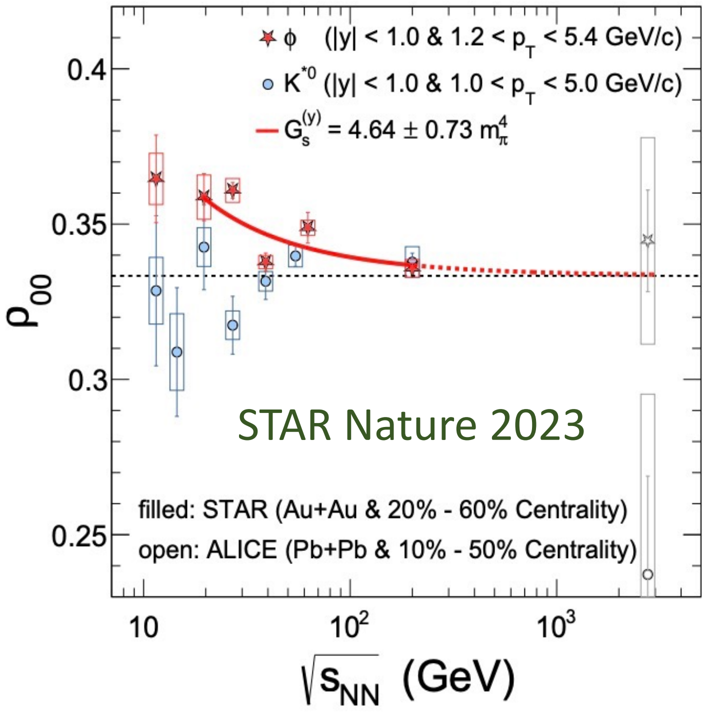




Challenges on Spin Alignment: 1) Large Magnitude

Experiments





1% level spin alignment, magnitude are large

Early Theories

❖ Coalescence picture — Liang & Wang PLB 2005

$$\frac{1+P_qP_{\bar{q}}}{3+P_qP_{\bar{q}}} \propto \left(\frac{\omega}{T}\right)^2$$

❖ Thermal picture — Becattini, Piccinini, Rizzo, PRC, 2008

$$\frac{1-(\hat{p}\cdot\widehat{\omega})^2}{18}\left(\frac{\omega}{T}\right)^2 \propto \left(\frac{\omega}{T}\right)^2$$

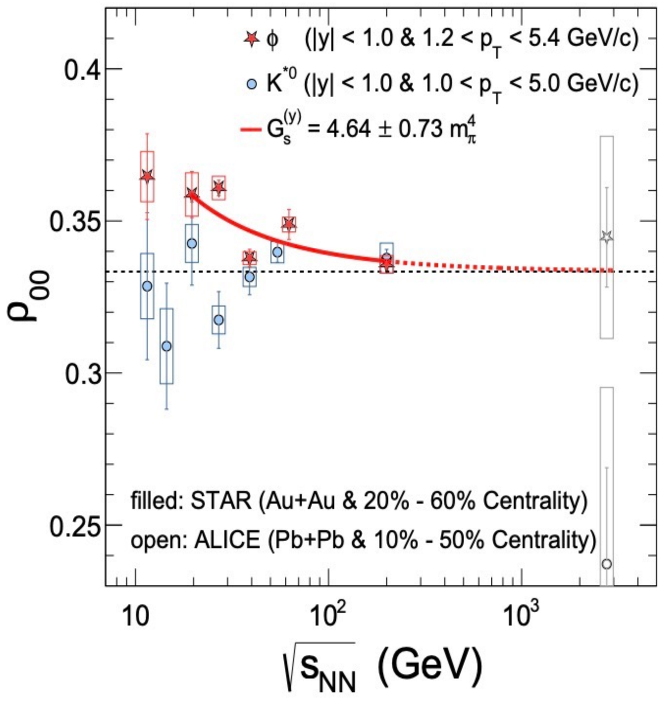
❖ $(\omega/T)^2 \sim 1/10^4$, 2nd order in gradient, small

❖ Problems：

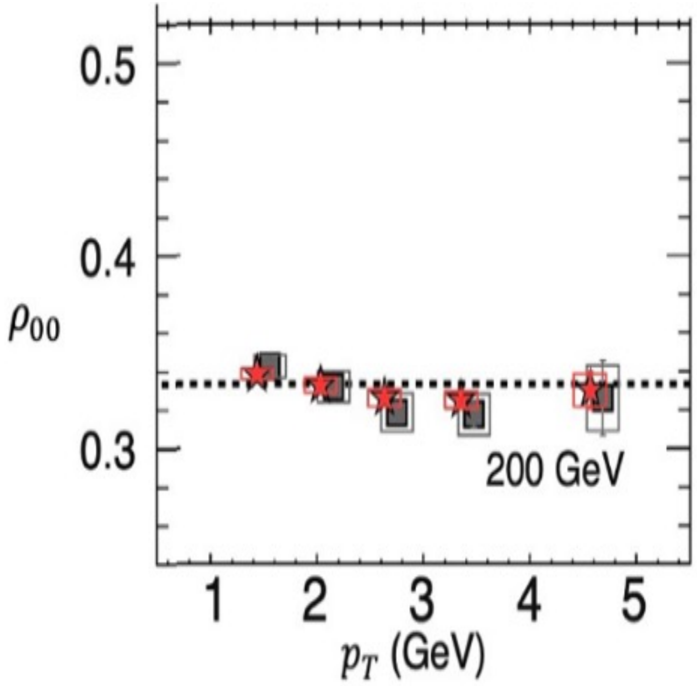
- Magnitude too small

Challenges on Spin Alignment: 2) Rich behaviors

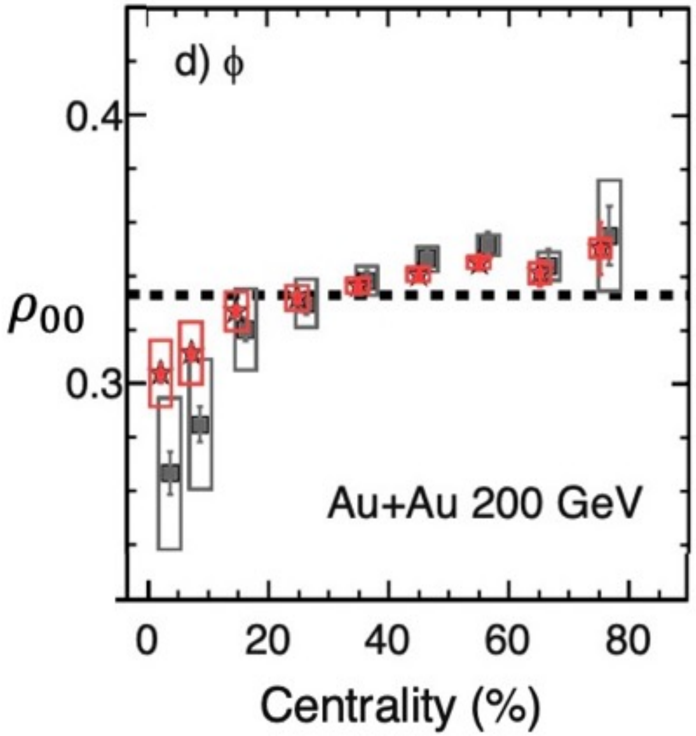
❖The Beam energy dependence



❖The p_T dependence the centrality dependence



❖The centrality dependence



Puzzling sign flip behaviors!

How we progress?

New ideas

- ❖ New external field — Sheng, Oliva, Liang, Wang, Wang, 2206.05868
- ❖ Initial stage physics, such as Glasma — *Kumar, Müller, Yang, PRD* 2023
- ❖ Others attempts — Wei and Huang arxiv:2303.01897
- ❖ Study with thermal field theory carefully, discover missing effects, such as Shear Induced Tensor Polarization（SITP） — Li, Liu, 2022, arXiv:2206.11890

Attempt to convince you in this talk that these new effects are:

- **Natural**, appeared naturally once included more realistic physics in theory
- **Universal**, applying to all interacting medium with massive vector boson
- **Large and Rich**, magnitude can be large & containing rich physics

Similar physics also discussed later in Wagner, Weickgenannt, Speranza , PRR, 2022

Outline

1) Spin observables in heavy-ion collisions
2) Theory for tensor polarization and spin alignment
3) How to produce a large enough spin alignment?
4) How to generate the rich behavior of the spin alignment in a model?
5) Summary and outlooks

Structure of the density matrix

❖The spin-1 boson has (8 degrees of freedom)

$$\rho_{ss'} = \begin{pmatrix} \rho_{11} & \rho_{10} & \rho_{1-1} \\ \rho_{01} & \rho_{00} & \rho_{0-1} \\ \rho_{-11} & \rho_{-10} & \rho_{-1-1} \end{pmatrix} = \frac{1}{3}\delta_{ss'} + \frac{1}{2}\mathcal{P}_k (J_k)_{ss'} - \mathcal{T}_{ij}(J_{(i}J_{j)} - \frac{2}{3}\delta_{ij}\mathbf{1})_{ss'}$$

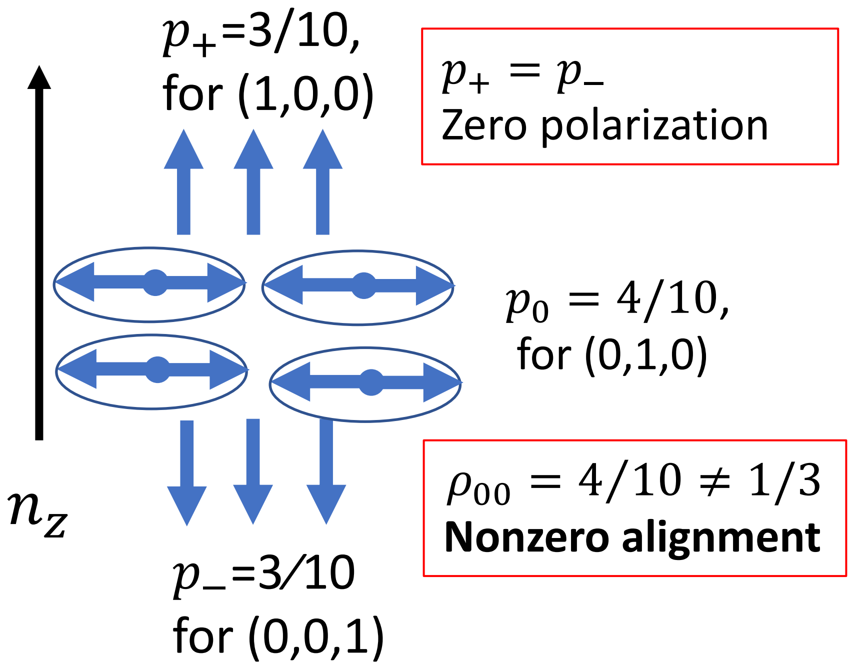
$$J_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, J_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}, J_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

Structure of the density matrix

❖The spin-1 boson has (8 degrees of freedom)

❖For a density matrix (quantize along z direction)

$$\rho = \begin{pmatrix} p_+ & & \\ & p_0 & \\ & & p_- \end{pmatrix}$$



❖Trivial case $p_+ = p_- = p_0 = 1/3$, no polarization

❖Vector Polarization exist when $\mathcal{P} = p_+ - p_- \neq 0$

❖Tensor Polarization & alignment exist if $p_0 \neq 1/3, T_{zz} = \rho_{00} - \frac{1}{3}$ (Even $\mathcal{P} = 0$, when $p_+ = p_-$,)

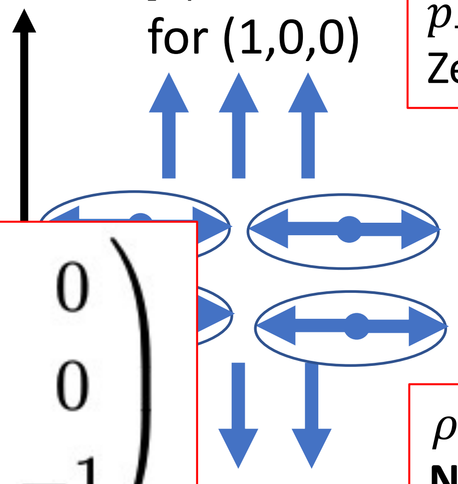
❖**Only** the tensor polarization part contribute to $\rho_{00} - 1/3$

Structure of the density matrix

❖The spin-1 boson has (8 degree of freedom)

$$\rho_{ss'} = \begin{pmatrix} \rho_{11} & \rho_{10} & \rho_{1-1} \\ \rho_{01} & \rho_{00} & \rho_{0-1} \\ \rho_{-11} & \rho_{-10} & \rho_{-1-1} \end{pmatrix} = \frac{1}{3}\delta_{ss'} + \frac{1}{2}\mathcal{P}_k (J_k)_{ss'} - \mathcal{T}_{ij}\left(J_{(i}J_{j)} - \frac{2}{3}\delta_{ij}\mathbf{1}\right)_{ss'}$$

$$J_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, J_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}, J_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$



$p_+ = p_-$
Zero polarization

$\rho_{00} = 4/10 \neq 1/3$
Nonzero alignment

❖Trivial case $p_+ = p_- = p_0 = 1/3$, no polarization

❖Vector Polarization exist when $\mathcal{P} = p_+ - p_- \neq 0$

❖Tensor Polarization & alignment exist if $p_0 \neq 1/3$, $T_{zz} = \rho_{00} - \frac{1}{3}$ (Even $\mathcal{P} = 0$, when $p_+ = p_-$,)

❖**Only** the tensor polarization part contribute to $\rho_{00} - 1/3$

Gradient Expansion and Symmetry analysis

❖Expansion up to 1st order gradient expansion

$$T^{\mu\nu} = \tilde{\Delta}^{\langle\mu}_{\lambda}\tilde{\Delta}^{\nu\rangle}_{\gamma}\Big[\kappa_0^u u^\lambda u^\gamma + \kappa_1^u u^\lambda u^\gamma + \kappa_{\rm sh}\sigma^{\lambda\gamma} + \kappa_T u^{(\lambda}\partial_\perp^{\gamma)}\beta + \kappa_{\rm su} u^{(\lambda}\sigma^{\gamma)\alpha}\tilde{p}_\alpha + \kappa_{\rm ou} u^{(\lambda}\Omega^{\gamma)\alpha}\tilde{p}_\alpha + \cdots\Big]$$

T-even (for $T^{\mu\nu}$)

T-even, 0th order (for $\kappa_0^u u^\lambda u^\gamma$)

T-odd, Shear Again! (for $\kappa_{\rm sh}\sigma^{\lambda\gamma}$)

Early theories include terms such as$(\omega/T)^2 \sim (1/100)^2$, 2nd order in gradient

Many Missing BUT Naturally Allowed Contribution at Lower Orders!

❖Why missed before?

- In-medium spectral properties/interactions required, **not been well studied before**
- Shear Induced Tensor Polarization(SITP) with κ_{sh} to be T-odd and indicate the nature of the dissipative physics, **not been studied before**

Could we find these terms in a concrete calculation?
Yes, see later

The Spin alignment from thermal field theory

❖The density matrix can be derived from Wigner function:

$$\mathcal{T}^{\mu\nu} \equiv \mathcal{W}^{\langle\mu\nu\rangle} \equiv \mathcal{W}^{(\mu\nu)} - \frac{1}{3}\tilde{\Delta}^{\mu\nu}\mathcal{S} = 2\tilde{\Delta}^{\langle\mu}_{\lambda}\tilde{\Delta}^{\nu\rangle}_{\gamma}W_{+}^{(\lambda\gamma)} \qquad \varrho_{ss'}(x,\boldsymbol{p}) = \epsilon^{\mu}_{s'}(\boldsymbol{p})\epsilon^{\nu*}_{s}(\boldsymbol{p})\mathcal{W}_{\mu\nu}(x,\boldsymbol{p})$$

❖The Wigner function can be related to correlation functions/Green functions that is well-defined in quantum field theory

$$W^{\mu\nu}(x,\boldsymbol{p}) \equiv \varepsilon_{\boldsymbol{p}} \int dp^0 \int d^4y e^{ip\cdot y}\langle V^{\mu}(x_-)V^{\nu}(x_+)\rangle$$

❖Either with a direct calculation or employ linear response theory(Dissipative term in Zubarev formalism)

$$W^{\mu\nu}_{+(1)} = \varepsilon_{\boldsymbol{p}} \lim_{\nu,q\to 0}\frac{\partial}{\partial\nu}[-\mathrm{Im}G^{\mu\nu\lambda\gamma}_{R+}(\nu,\boldsymbol{q},\boldsymbol{p})]\xi_{\lambda\gamma}$$

$$G^{\mu\nu\lambda\gamma}_{R}(\nu,\boldsymbol{q},\boldsymbol{p})$$

Wigner Trans

$$(-i)\Theta(t-t')\langle[V^{\mu}(t,\boldsymbol{x}^-)V^{\nu}(t,\boldsymbol{x}^+),T^{\lambda\gamma}(t',\boldsymbol{z})]\rangle,$$

❖We can obtain the coefficients of the general expansion in previous slide

$$\mathcal{T}^{\mu\nu} = \tilde{\Delta}^{\langle\mu}_{\lambda}\tilde{\Delta}^{\nu\rangle}_{\gamma}\Big[\kappa^{u}_{0}u^{\lambda}u^{\gamma} + \kappa^{u}_{1}u^{\lambda}u^{\gamma} + \kappa_{\mathrm{sh}}\sigma^{\lambda\gamma} + \kappa_{T}u^{(\lambda}\partial^{\gamma)}_{\perp}\beta + \kappa_{\mathrm{su}}u^{(\lambda}\sigma^{\gamma)\alpha}\tilde{p}_{\alpha} + \kappa_{\mathrm{ou}}u^{(\lambda}\Omega^{\gamma)\alpha}\tilde{p}_{\alpha} + \cdots\Big)$$

Total Theory Results

❖Full one loop results:

$$\mathcal{T}^{\mu\nu} = \tilde{\Delta}^{\langle\mu}_{\lambda}\tilde{\Delta}^{\nu\rangle}_{\gamma}\Big[\kappa^u_0 u^\lambda u^\gamma + \kappa^u_1 u^\lambda u^\gamma + \kappa_{\rm sh}\sigma^{\lambda\gamma} + \kappa_T u^{(\lambda}\partial^{\gamma)}_{\perp}\beta + \kappa_{\rm su} u^{(\lambda}\sigma^{\gamma)\alpha}\tilde{p}_\alpha + \kappa_{\rm ou} u^{(\lambda}\Omega^{\gamma)\alpha}\tilde{p}_\alpha + \cdots\Big]$$

$$\kappa^u_0 = \frac{\alpha_0}{-\tilde{v}^2}n_0,\ \kappa^u_1 = \left[\alpha_{\rm sh}\left(c_s^2 - \frac{1}{3}\right)\theta + \frac{\alpha_{\rm sp}\xi_p}{-\tilde{v}^2}\right]\beta n_0$$

$$\kappa_{\rm sh} = \alpha_{\rm sh}\beta n_0,\quad \kappa_T = 0,\quad \kappa_{\rm su} = 0,\quad \kappa_{\rm ou} = 0$$

$$\alpha_{\rm sh} = \frac{4\varepsilon_{\boldsymbol p}\pi}{\beta n(\varepsilon_{\boldsymbol p})}\int_0^\infty \frac{\partial n(\omega)}{\partial\omega}d\omega(\omega^2 - \varepsilon^2_{\boldsymbol p})A^2_{T/L}(\omega,\boldsymbol p)$$

$$\alpha_{\rm sp} = \frac{4\varepsilon_{\boldsymbol p}\pi}{\beta n(\varepsilon_{\boldsymbol p})}\int_0^\infty \frac{\partial n(\omega)}{\partial\omega}d\omega\varepsilon^2_{\boldsymbol p}(A^2_T(\omega,\boldsymbol p) - A^2_L(\omega,\boldsymbol p))$$

$$\alpha_0 = 2\varepsilon_{\boldsymbol p}\int_0^\infty d\omega\frac{n(\omega)}{n(\varepsilon_{\boldsymbol p})}\left[(A_L - A_T) - \frac{\Delta\omega^2\tilde{v}^2}{p^2}A_L\right]$$

❖Features of the result

- **Natural**, suggested by symmetry, verified in concrete thermal field theory calculation, all have been done is a more careful theory study with more realistic spectral functions
- **Universal**, SITP exist in all spin-1 particles including heavy quarkonium, in relativistic or non-relativistic (SITP has a coefficient have no mass suppression)

Total Theory Results

❖Full one loop results:

$$T^{\mu\nu} = \tilde{\Delta}^{\langle\mu}_{\lambda}\tilde{\Delta}^{\nu\rangle}_{\gamma}\left[\kappa^u_0 u^\lambda u^\gamma + \kappa^u_1 u^\lambda u^\gamma + \kappa_{\rm sh}\sigma^{\lambda\gamma} + \kappa_T u^{(\lambda}\partial^{\gamma)}_{\perp}\beta\right]$$

$$\rho_{00} = \epsilon^0_\lambda(p)\epsilon^0_\gamma(p)\left(\kappa^u_0 u^\lambda u^\gamma + \kappa^u_1 u^\lambda u^\gamma + \kappa_{sh}\sigma^{\lambda\gamma}\right)/\text{norm}$$

$$\kappa^u_0 = \frac{\alpha_0}{-\tilde{v}^2}n_0, \quad \kappa^u_1 = \left[\alpha_{\rm sh}\left(c_s^2 - \frac{1}{3}\right)\theta + \frac{\alpha_{\rm sp}\xi_p}{-\tilde{v}^2}\right]\beta n_0$$

$$\kappa_{\rm sh} = \alpha_{\rm sh}\beta n_0, \quad \kappa_T = 0, \quad \kappa_{\rm su} = 0, \quad \kappa_{\rm ou} = 0$$

$$\alpha_{\rm sh} \approx -\frac{2\Delta\varepsilon_{\boldsymbol p}}{\Gamma_{\boldsymbol p}} + 2\frac{\Delta\varepsilon_{\boldsymbol p}}{\Gamma_{\boldsymbol p}}\frac{\Delta\varepsilon_{\boldsymbol p}}{T} + \frac{\Gamma_{\boldsymbol p}}{2T} \sim \mathcal{O}(1)$$

$$\alpha_{\rm sp} \approx -\frac{\varepsilon_{\boldsymbol p}}{\Gamma_{\boldsymbol p}}\left(\frac{\Gamma^\Delta_{\boldsymbol p}}{\Gamma_{\boldsymbol p}} - \frac{\Delta\varepsilon_{\boldsymbol p}}{T}\frac{\Gamma^\Delta_{\boldsymbol p}}{\Gamma_{\boldsymbol p}} + \frac{\Gamma_{\boldsymbol p}}{T}\frac{\omega^\Delta_{\boldsymbol p}}{\Gamma_{\boldsymbol p}}\right) \sim \mathcal{O}(\delta^{-1}_{\rm qp}\delta_{\rm sp}).$$

$$\alpha_0 \approx (\omega^T_{\boldsymbol p} - \omega^L_{\boldsymbol p})/T$$

❖Features of the result

- **Natural**, suggested by symmetry, verified in concrete thermal field theory calculation, all have been done is a more careful theory study with more realistic spectral functions
- **Universal**, SITP exist in all spin-1 particles including heavy quarkonium, in relativistic or non-relativistic (SITP has a coefficient have no mass suppression)

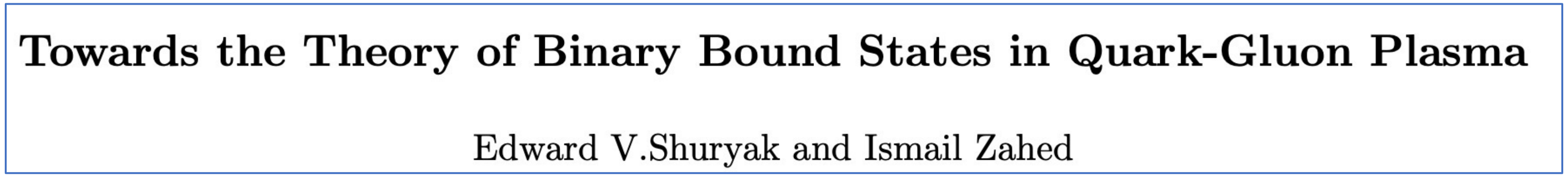
Outline

1) Spin observables in heavy-ion collisions
2) Theory for tensor polarization and spin alignment
3) How to produce a large enough spin alignment?
4) How to generate the rich behavior of the spin alignment in a model?
5) Summary and outlooks

Spectral properties of in-medium degree of freedom

❖Where the mesonic resonance forms?

- Chemical freezeout
- Late stage of QGP?

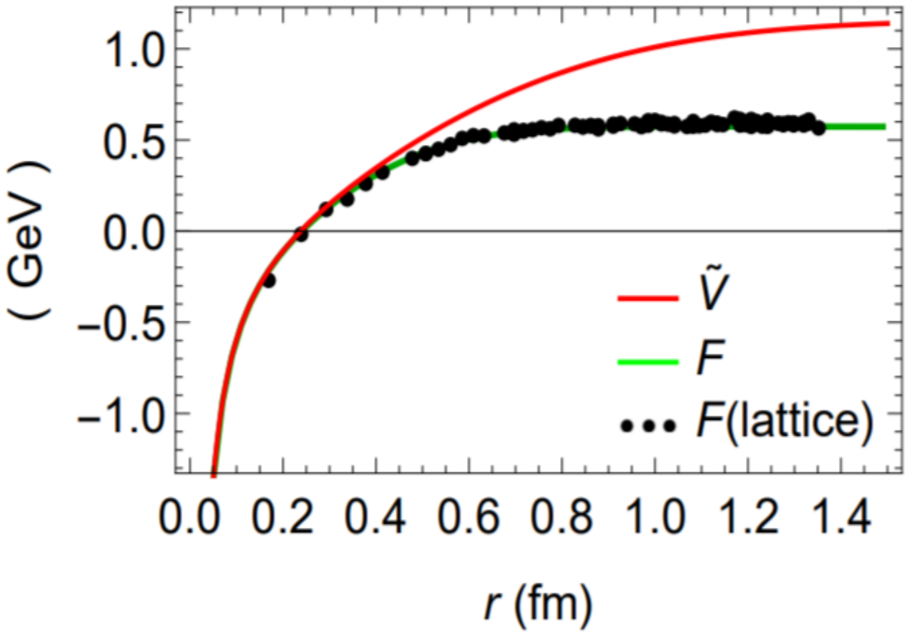


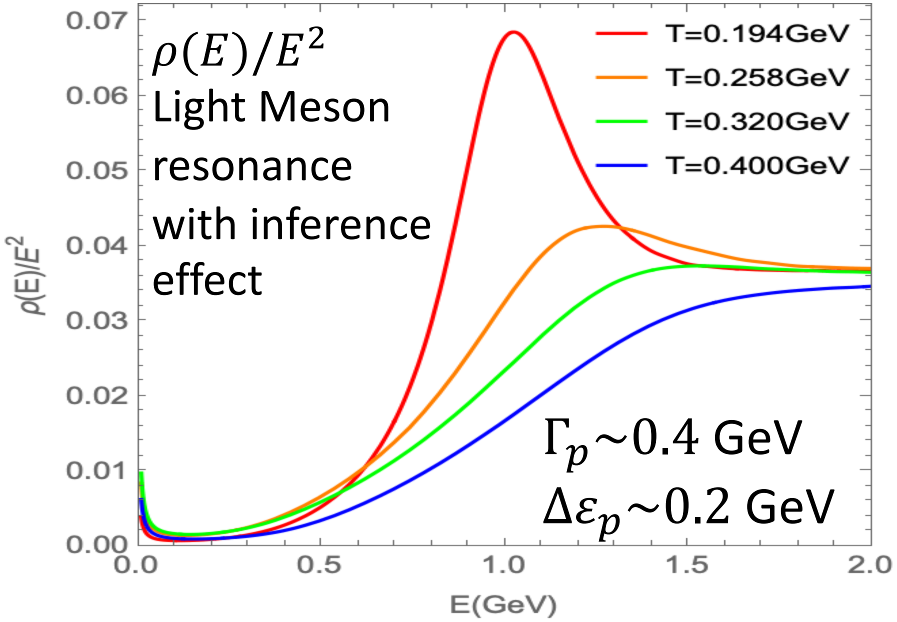
Towards the Theory of Binary Bound States in Quark-Gluon Plasma

Edward V.Shuryak and Ismail Zahed

Shuryak, Zahed, PRD 2004

❖Large confining potential ⟺ mesonic resonance at $T \sim 0.2$ GeV





Liu, Rapp, PRC 2018

Spectral properties of in-medium degree of freedom

❖ Where the mesons of freedom forms?

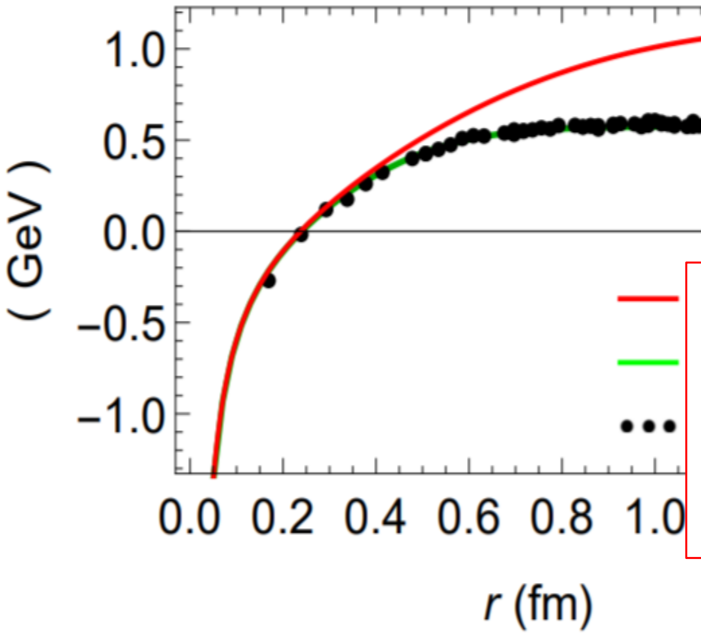
- Chemical freezeout
- Late stage of QGP?

Towards the Theory of Binary Bound States in Quark-Gluon Plasma

Edward V.Shuryak and Ismail Zahed

Shuryak, Zahed, PRD 2004

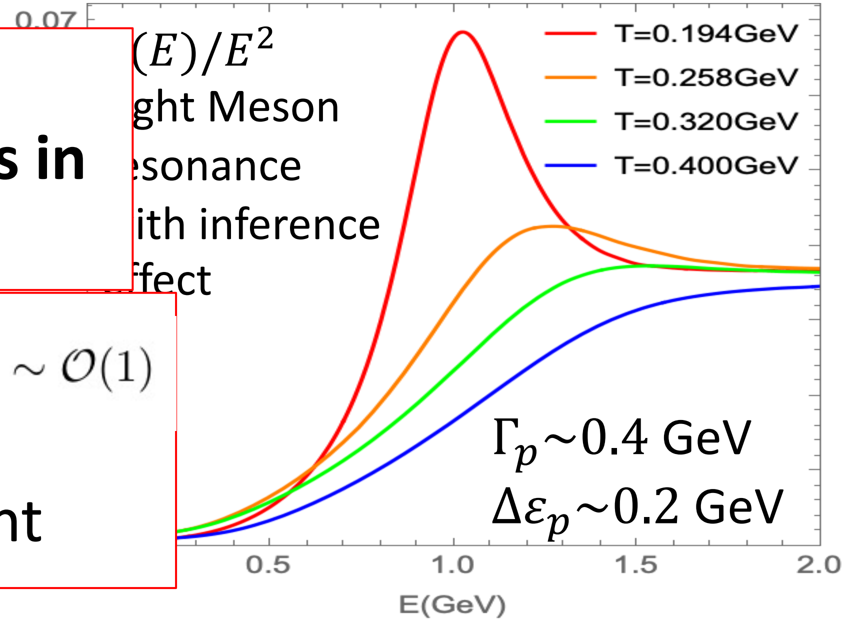
❖ Large confining potential ⟺ mesonic resonance at $T \sim 0.2$ GeV



Large width can be generated by collisions in strong coupling

$$\alpha_{\rm sh} \approx -\frac{2\Delta\varepsilon_p}{\Gamma_p} + 2\frac{\Delta\varepsilon_p}{\Gamma_p}\frac{\Delta\varepsilon_p}{T} + \frac{\Gamma_p}{2T} \sim \mathcal{O}(1)$$

$\alpha_{\rm sh} \sim 1$, Large Coefficient

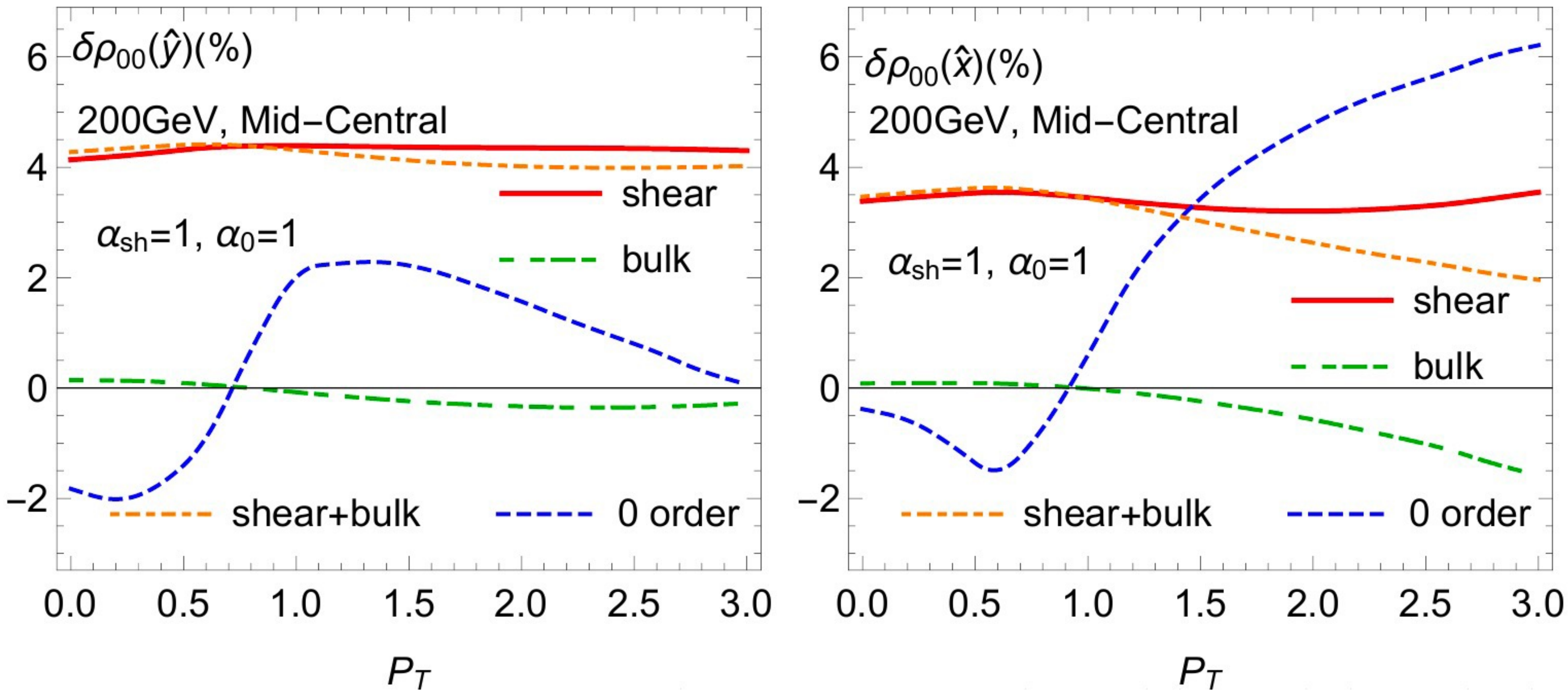


Liu, Rapp, PRC 2018

Phenomenology implication

❖The connection to $\delta\rho_{00} \equiv \rho_{00} - 1/3$

$$\delta\rho_{00}(\hat{n}_{\rm pr}, \boldsymbol{p}) = \frac{\int d\Sigma^\lambda p_\lambda \mathcal{T}^{\mu\nu}(x,\boldsymbol{p})\hat{n}_\mu(\boldsymbol{p})\hat{n}_\nu(\boldsymbol{p})}{d\Sigma^\lambda p_\lambda \mathcal{S}(x,\boldsymbol{p})}$$



α_{sh}~1, estimate from spectral of light heavy meson in previous slide **(no p, T dependence)**

α_0~1, just a value for convenience (need better spectral function to estimate)

❖ **Large** phenomenologically, especially **SITP** can generate ~1% level spin alignment at the relatively late stage of QGP phase

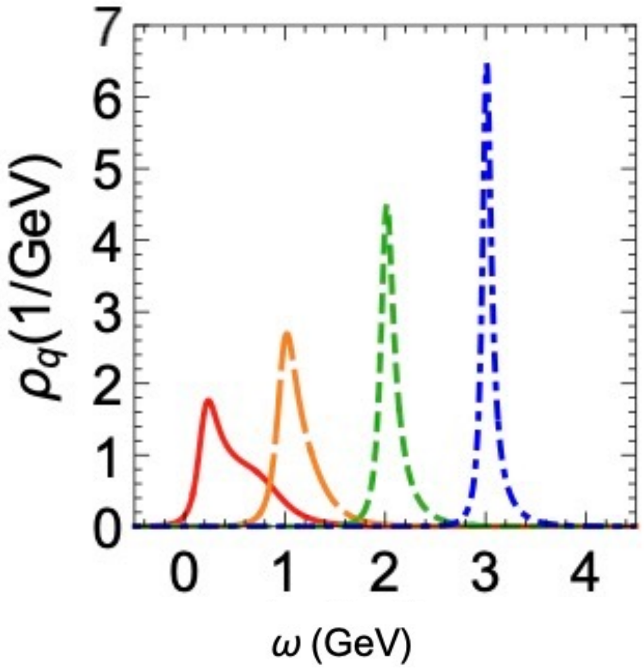
❖Could this mechanism explain the rich behavior such as p_T and centrality dependence of spin alignment?

Outline

1) Spin observables in heavy-ion collisions
2) Theory for tensor polarization and spin alignment
3) How to produce a large enough spin alignment?
4) How to generate the rich behavior of the spin alignment in a model?
5) Summary and outlooks

Rich physics in spectral functions

❖Spectral function (quark) as a function of momentum

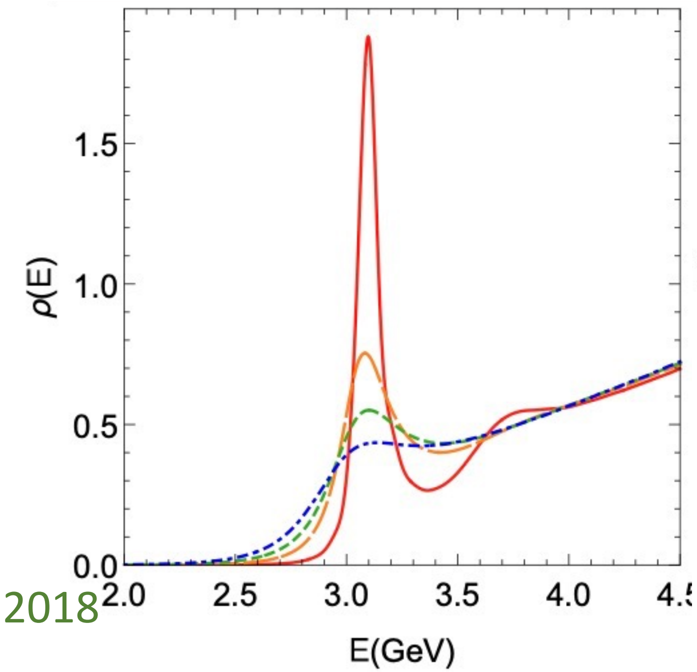


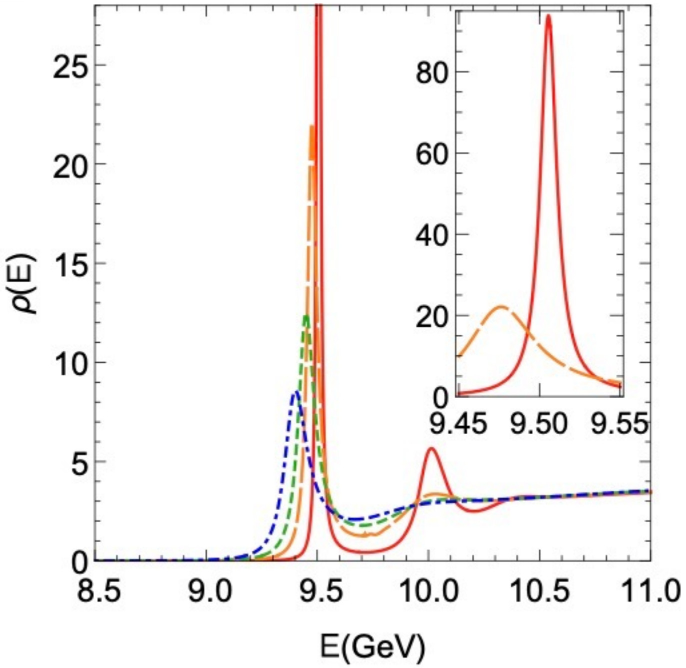
p=0 GeV
p=1 GeV
p=2 GeV
p=3 GeV

Bound states' p dependence is much hard, but doable

Liu, Rapp, PRC 2018

❖Different particles, different formation T, different mass shift





Spectral properties are manifestations of in-medium QCD interactions

Rich QCD interactions → Rich spectral properties → Rich behavior in spin alignment

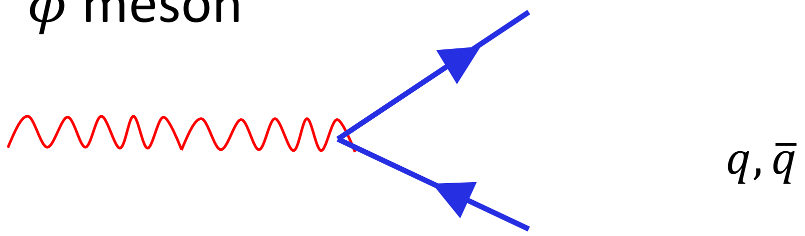
Above spectral functions are using T-matrix approach, pretty challenging for spin-alignment calculation. We will illustrate some of the physics using a toy model in following

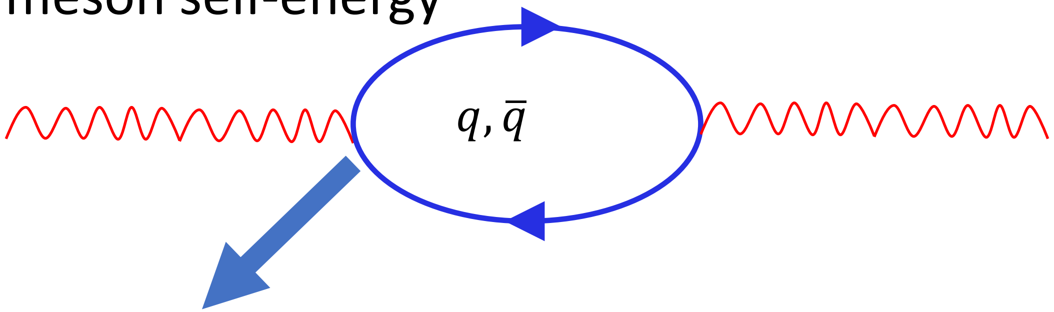
A toy quark meson model

❖The toy quark meson Lagrangian,

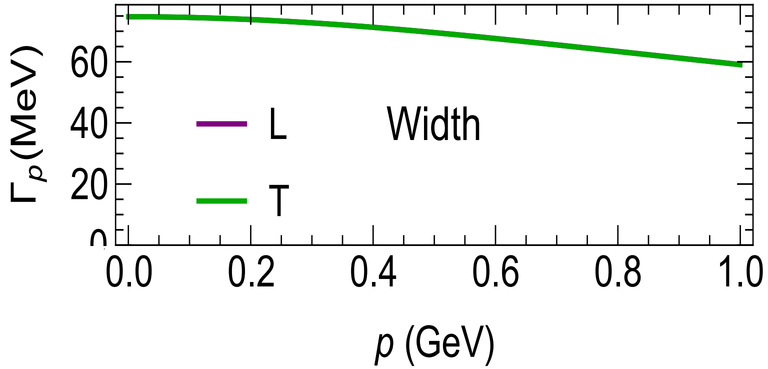
$$L = \bar{\psi}\gamma^{\mu} i(\partial_{\mu} + igA_{\mu})\psi - m_s\,\bar{\psi}\psi - \frac{1}{4}F^2 + m^2A^2$$

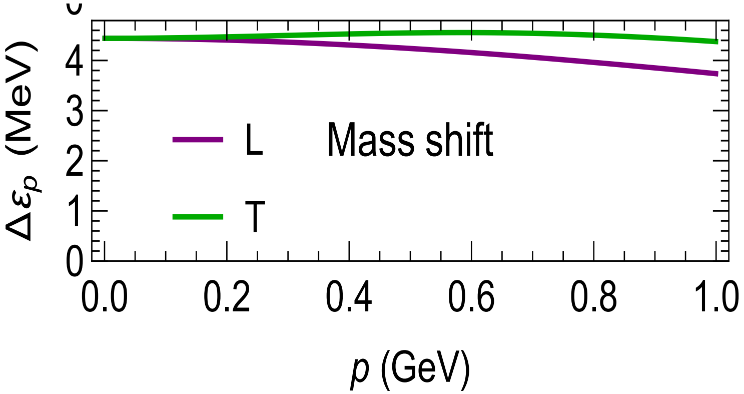
❖ The interaction is $g\ \bar{\psi}\gamma^{\mu}A_{\mu}\psi$ can generate interaction illustrated as





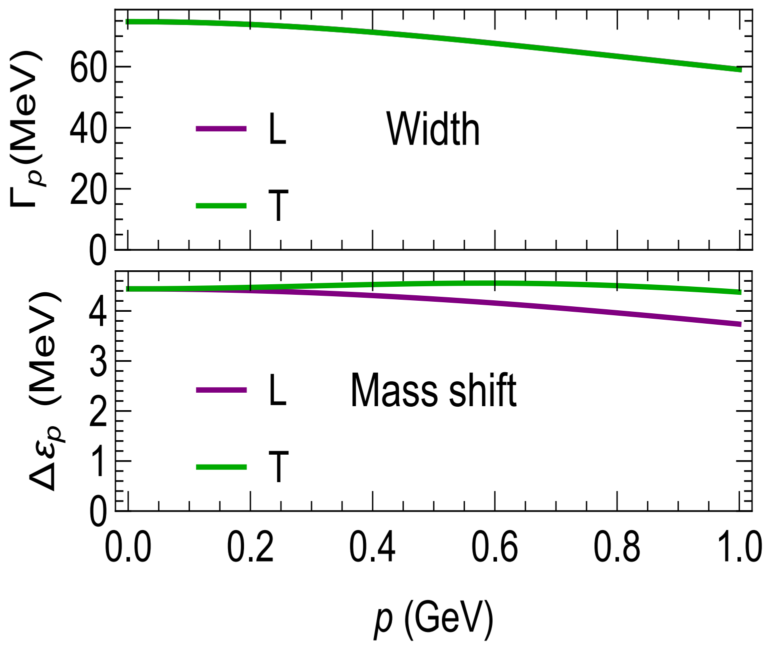
❖This self energy can generate **width** and **mass shift** in spectral function **with more information such as p dependence**

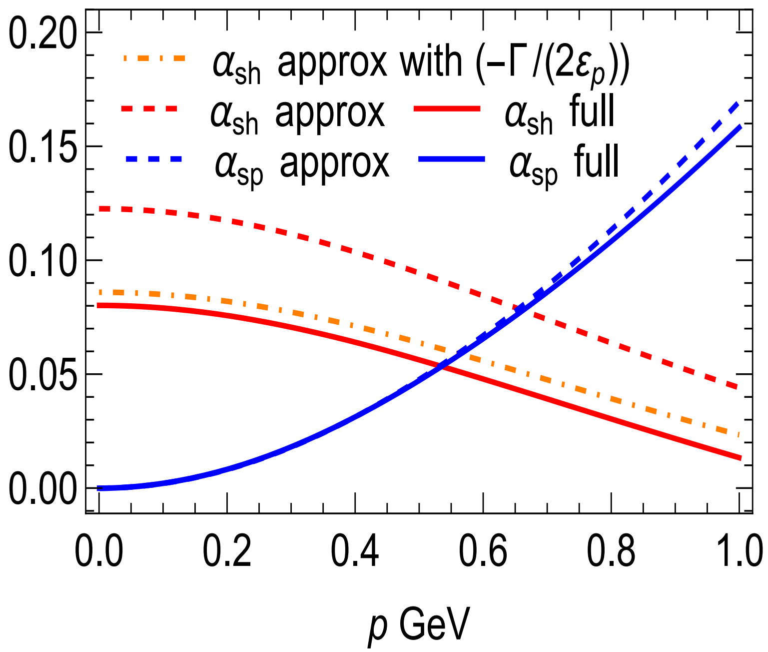




The mass shift and width of QM model

❖The rich behavior of the mass shift and the width of this interaction can lead to rich behavior of the coefficient





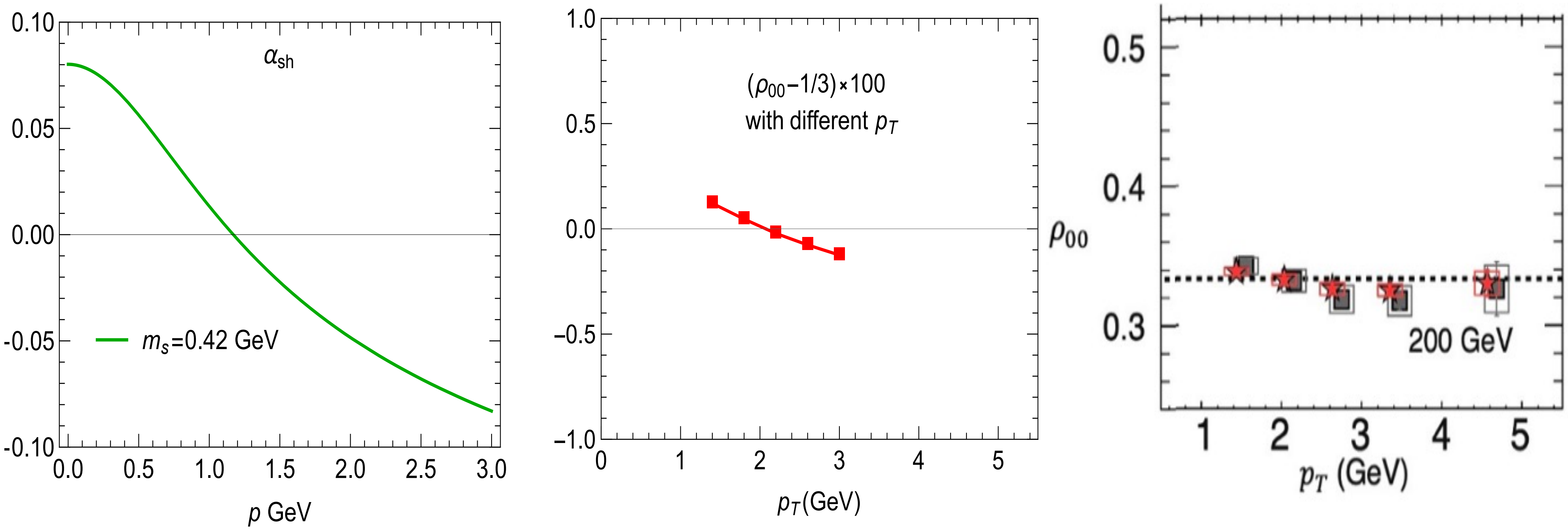
$$\alpha_{\rm sh} \approx -\frac{2\Delta\varepsilon_{\boldsymbol{p}}}{\Gamma_{\boldsymbol{p}}} + 2\frac{\Delta\varepsilon_{\boldsymbol{p}}}{\Gamma_{\boldsymbol{p}}}\frac{\Delta\varepsilon_{\boldsymbol{p}}}{T} + \frac{\Gamma_{\boldsymbol{p}}}{2T} \sim \mathcal{O}(1)$$

$$\alpha_{\rm sp} \approx -\frac{\varepsilon_{\boldsymbol{p}}}{\Gamma_{\boldsymbol{p}}}\left(\frac{\Gamma^{\Delta}_{\boldsymbol{p}}}{\Gamma_{\boldsymbol{p}}} - \frac{\Delta\varepsilon_{\boldsymbol{p}}}{T}\frac{\Gamma^{\Delta}_{\boldsymbol{p}}}{\Gamma_{\boldsymbol{p}}} + \frac{\Gamma_{\boldsymbol{p}}}{T}\frac{\omega^{\Delta}_{\boldsymbol{p}}}{\Gamma_{\boldsymbol{p}}}\right) \sim \mathcal{O}(\delta_{\rm qp}^{-1}\delta_{\rm sp}).$$

The p_T behavior

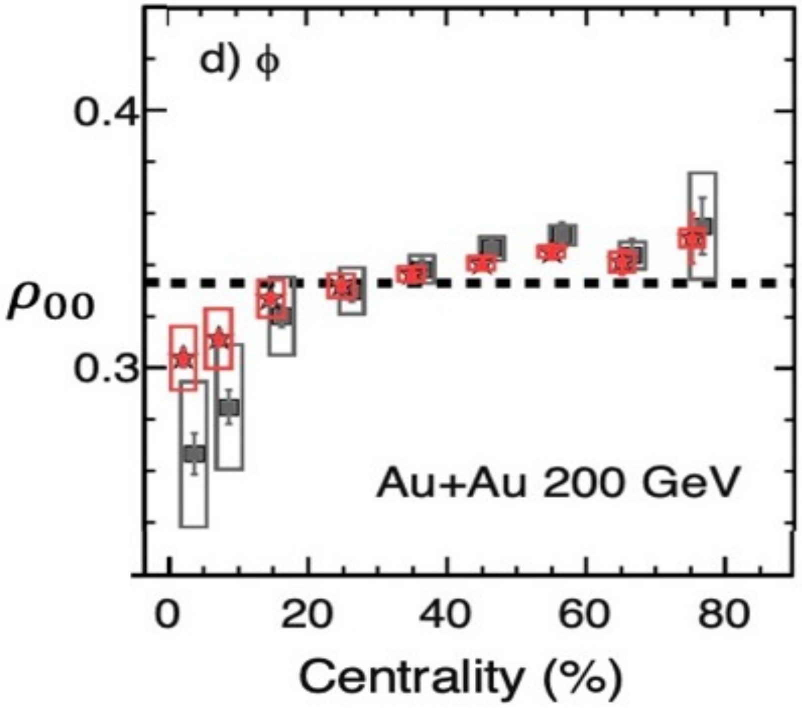
❖The coefficient will flip sign due to competing of this two term

$$\alpha_{\rm sh} \approx -\frac{2\Delta\varepsilon_{\boldsymbol{p}}}{\Gamma_{\boldsymbol{p}}} + 2\frac{\Delta\varepsilon_{\boldsymbol{p}}}{\Gamma_{\boldsymbol{p}}}\frac{\Delta\varepsilon_{\boldsymbol{p}}}{T} + \frac{\Gamma_{\boldsymbol{p}}}{2T} \sim \mathcal{O}(1)$$



❖The competing between mass-shift/width term and width/T will flip the sign of coefficient leading to spin alignment flip sign with increasing p_T

Could we understand centrality dependence

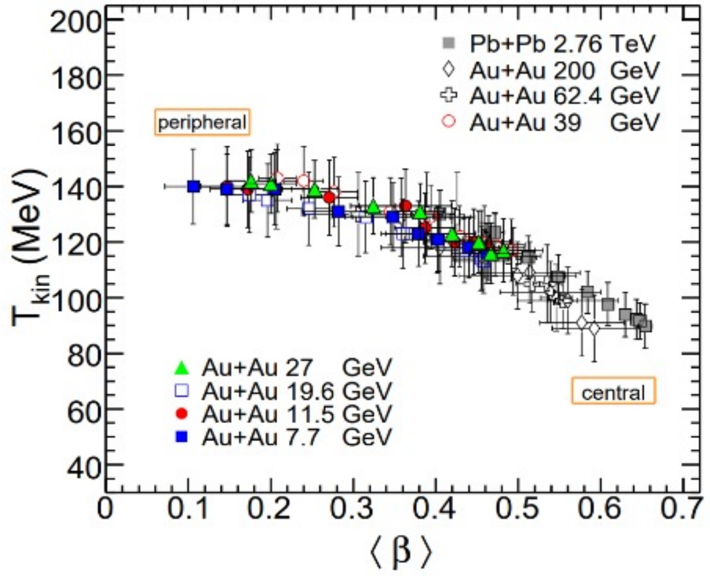


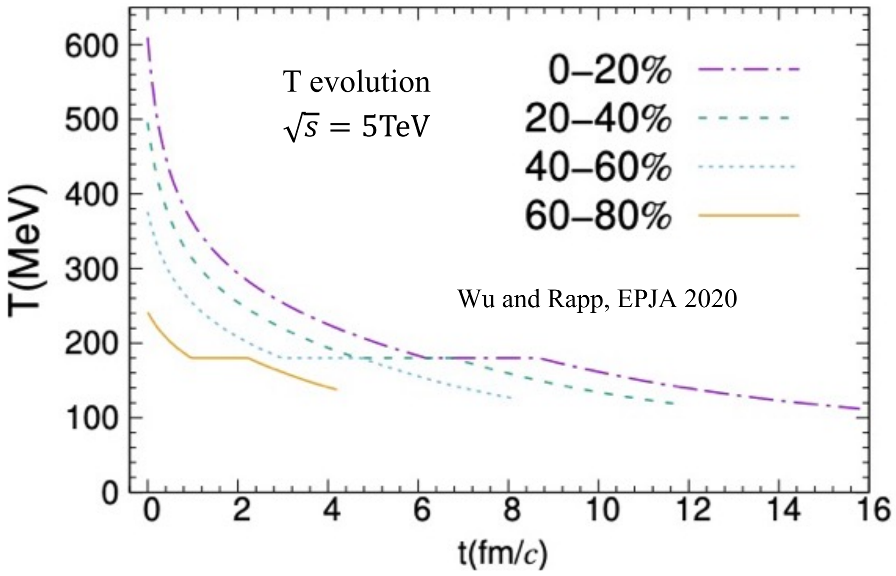
Not as straightforward as p_T, but we can still do it with some reasonable assumptions

Toward centrality dependence

❖Some Background knowledge:

- freezeout temperature as a function of centrality

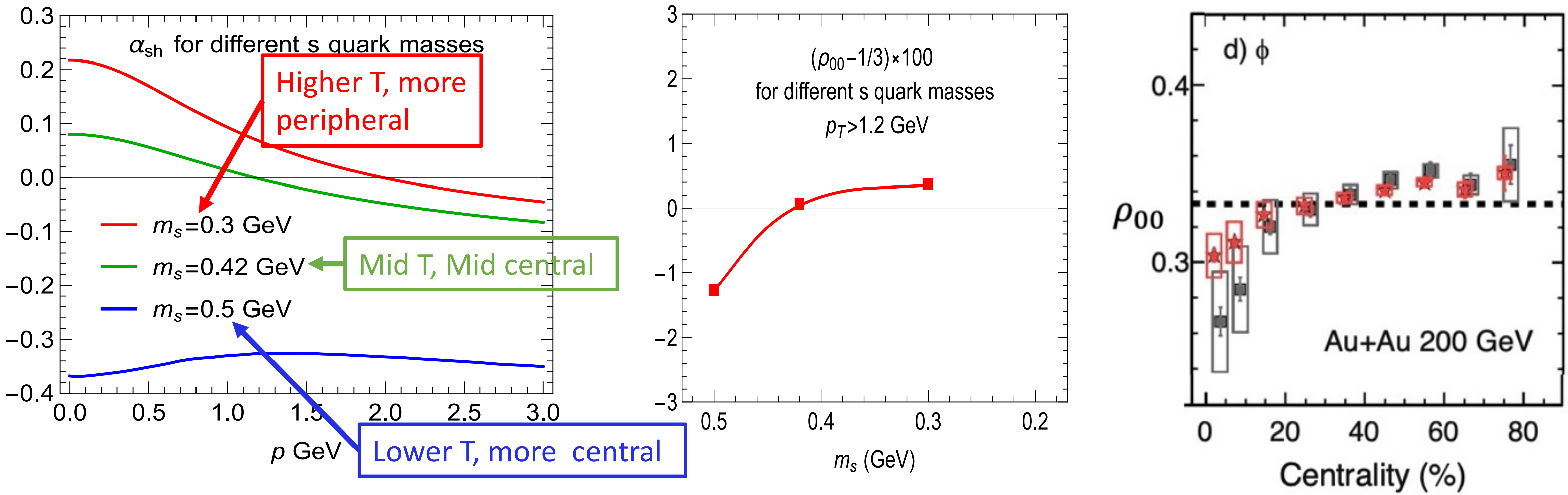




- More central, large system, long lifetime, low kinetic freezeout temperature
- More central↔lower freezeout temperature; more peripheral↔higher freezeout temperature
- Spin freezeout is more like kinetic freezeout, we consider it as "spin kinetic freezeout"
- At different temperature, some parameter of the model such as "effective strange mass" can change drastically around the phase boundary in a small temperature window.

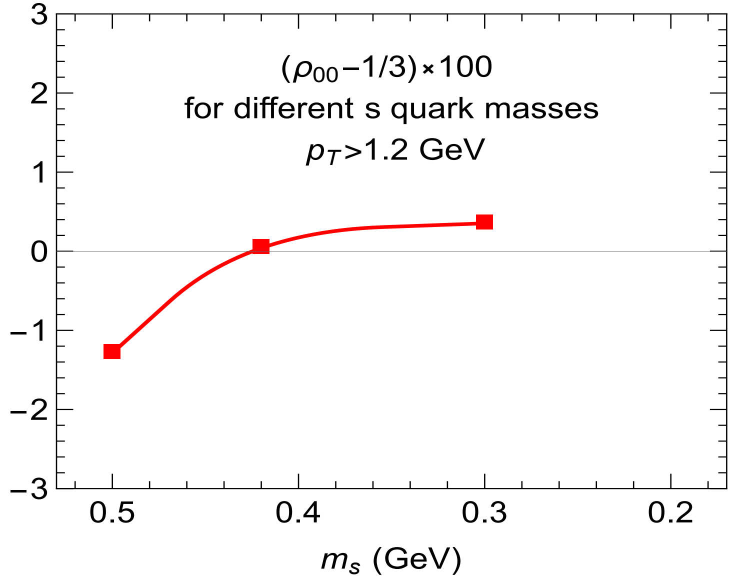
The mass dependence of the spin alignment

❖ Mass dependence

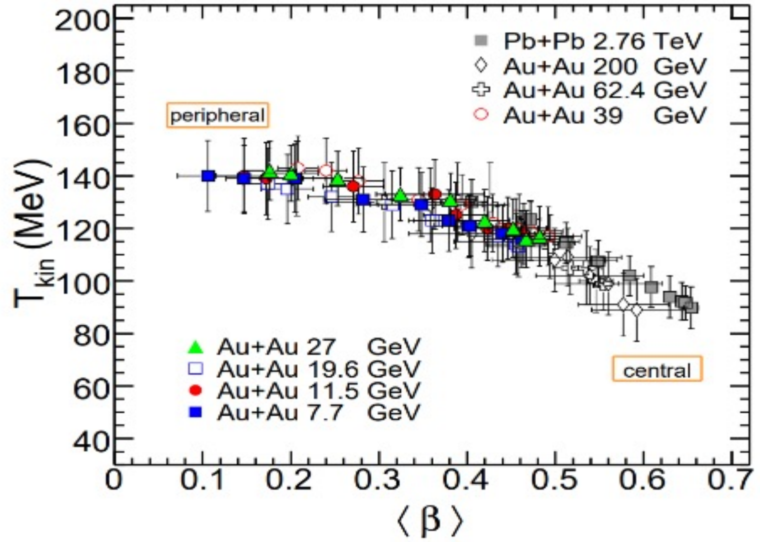


Relate mass depemdence to centrality depdence?

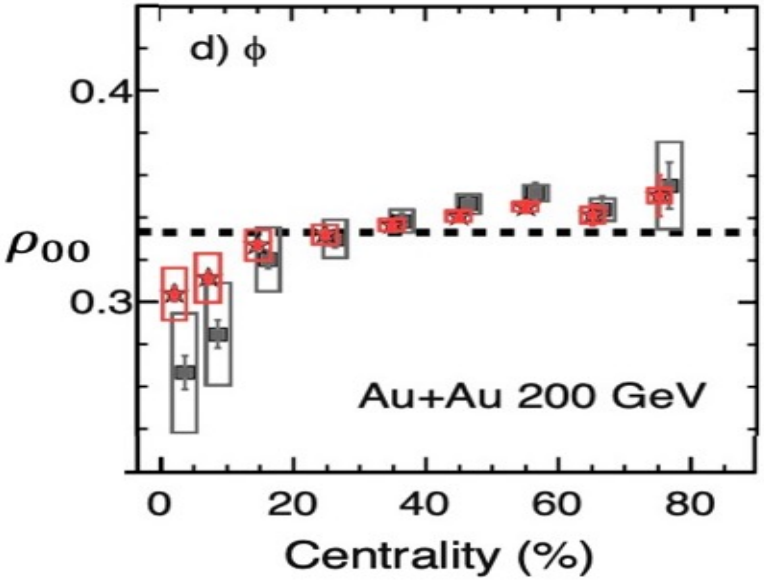
❖ "mass increase as T decrease" + "freezeout T decrease at centrality increase"



+



=

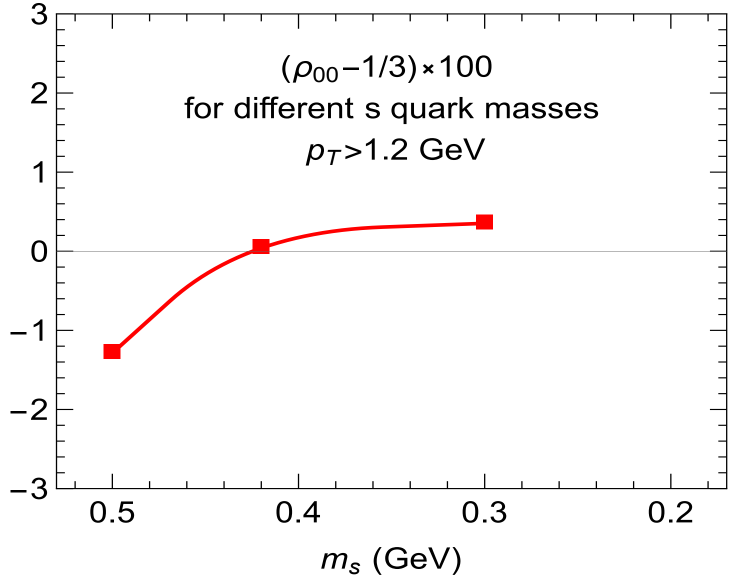


Physics that melt the quark mass at higher T:

1) Collective modes of quarks at low frequency
2) Chiral symmetry restoration
3) …

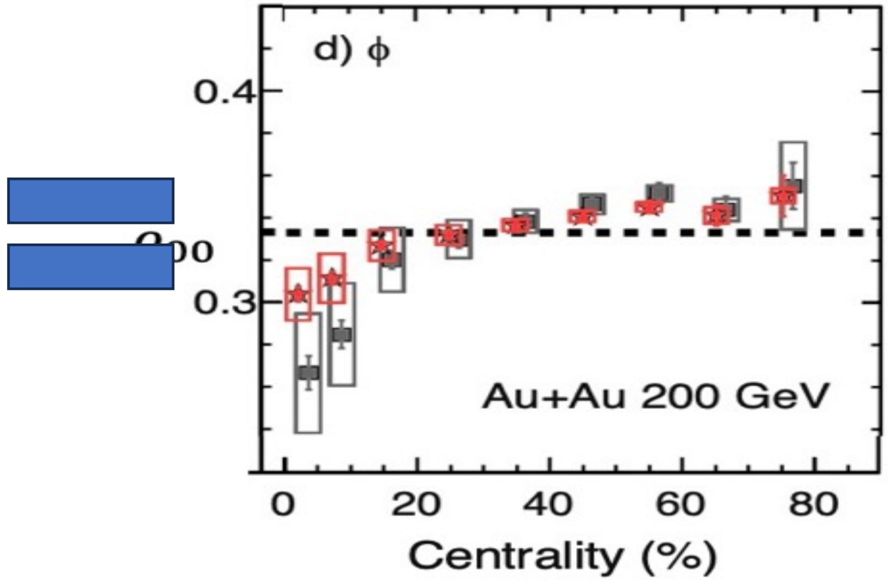
Relate mass depemdence to centrality depdence?

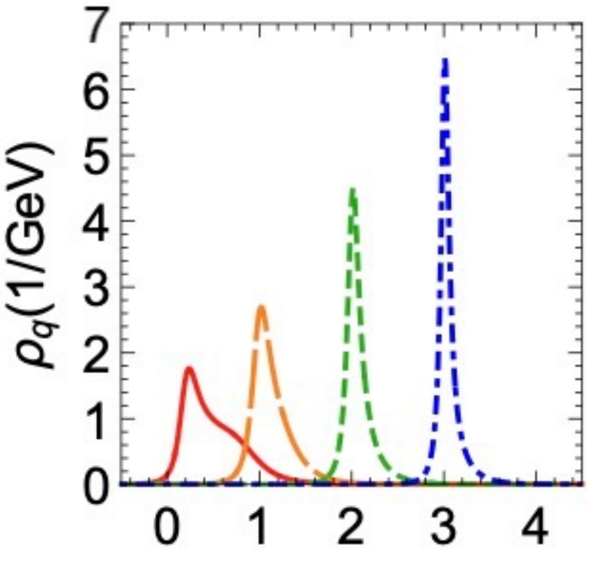
❖ "mass increase as T decrease" + "freezeout T decrease at more central"



Physics that melt the quark mass at higher T:

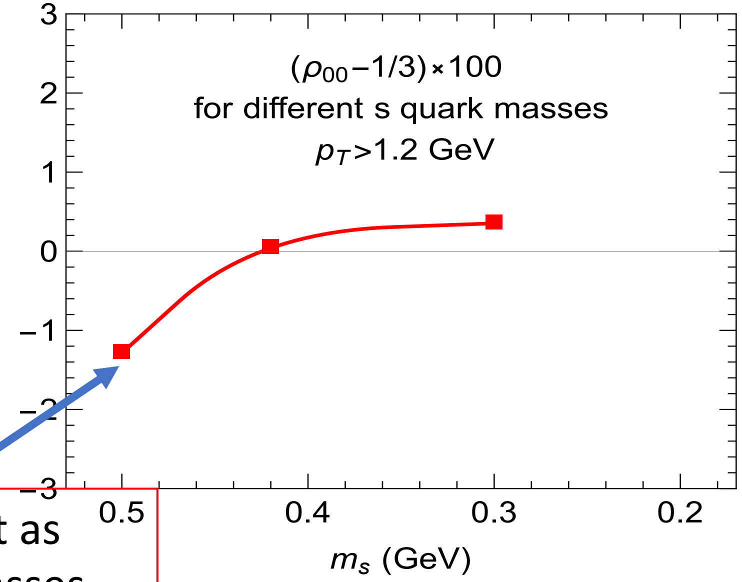
1) Collective modes of quarks at low frequency
2) Chiral symmetry restoration
3) …





Relate mass depemdence to centrality depdence?

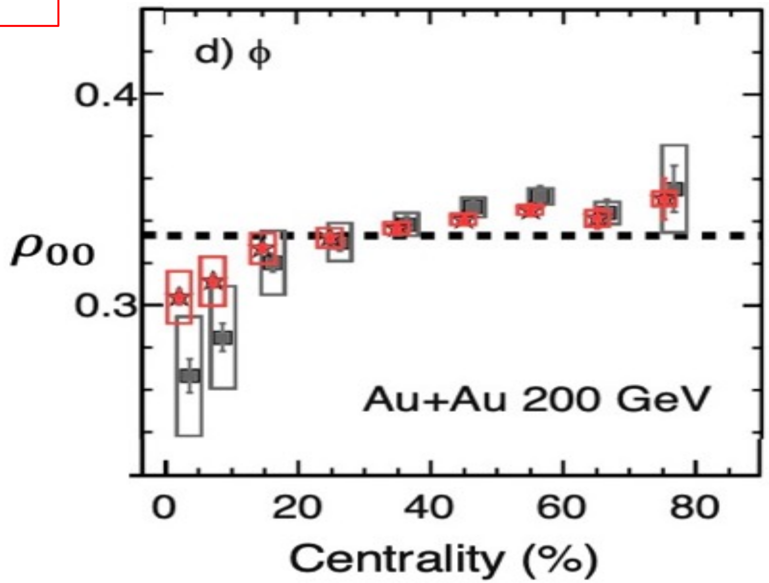
- “mass increase as T decrease” + “freezeout T decrease at more central”

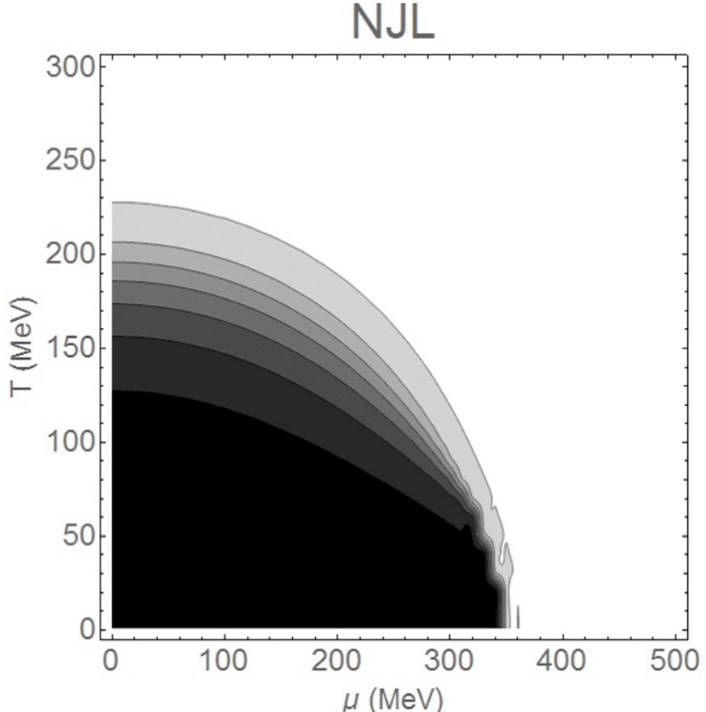


Regard it as Kaon masses

Physics that melt the quark mass at higher T:

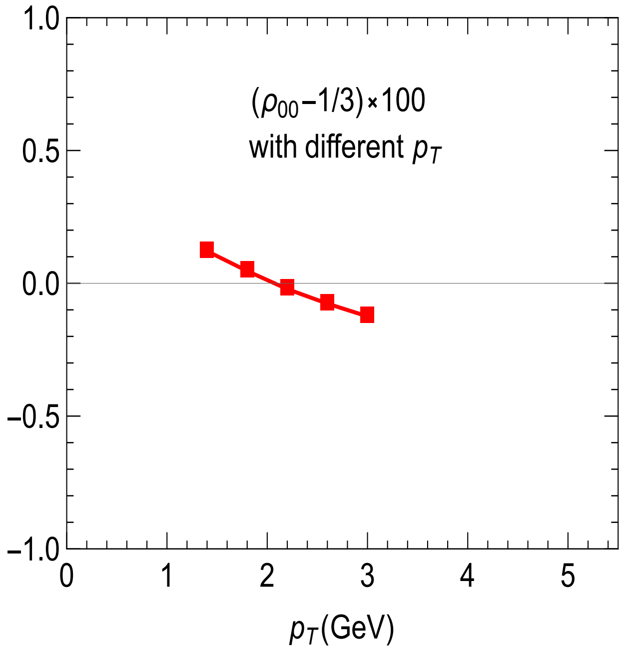
1) Collective modes of quarks at low frequency
2) Chiral symmetry restoration
3) …

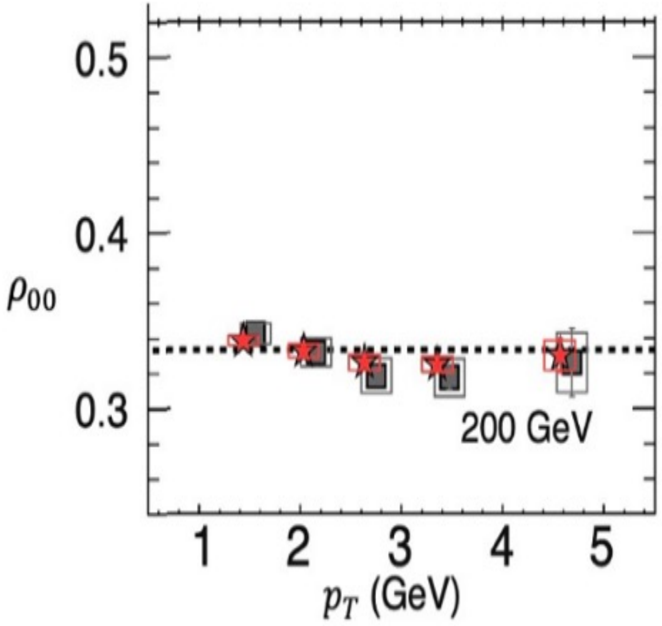


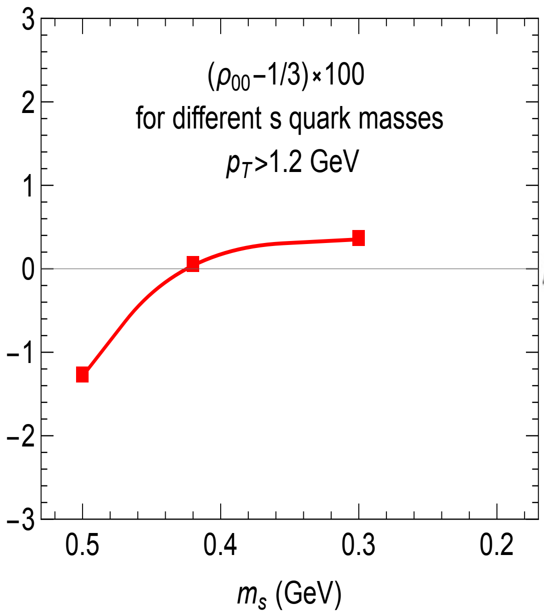


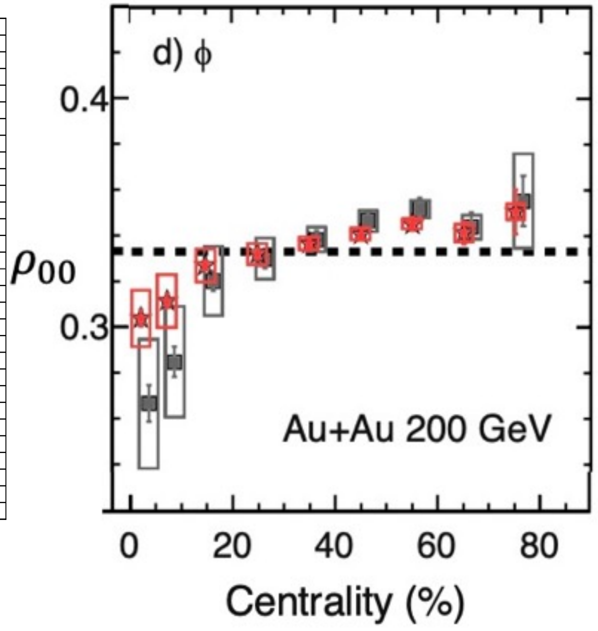
Remarks for rich behaviors of spin alignment

- ❖Using toy model spectral functions with p, T, mass (many others) species dependence, can include **Rich** physics that may help in understand rich structures observed in experiment
- ❖More realistic model more can leads to even richer physics and behaviors that is promising for fully describe the observations quantitatively.



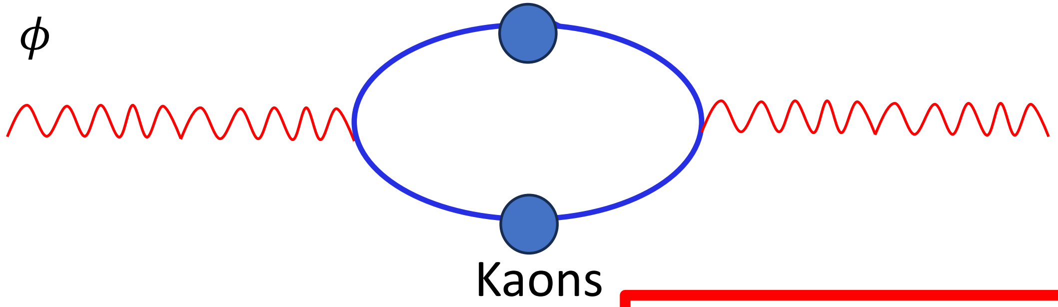


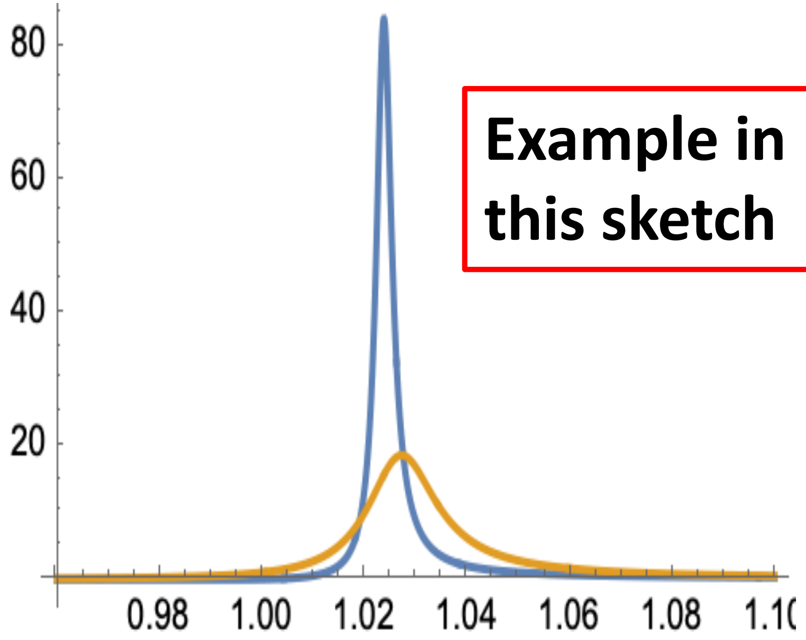




On Going: Chiral perturbation calculation ϕ meson's Spectra

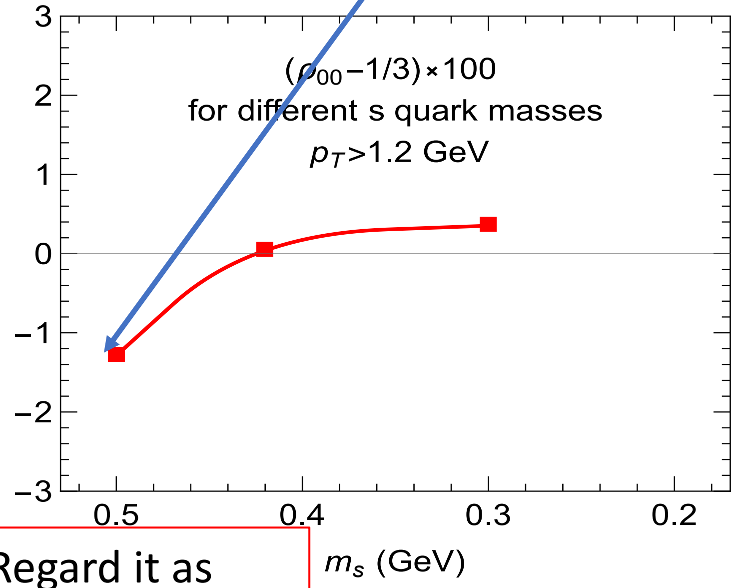
❖ The dressed kaons



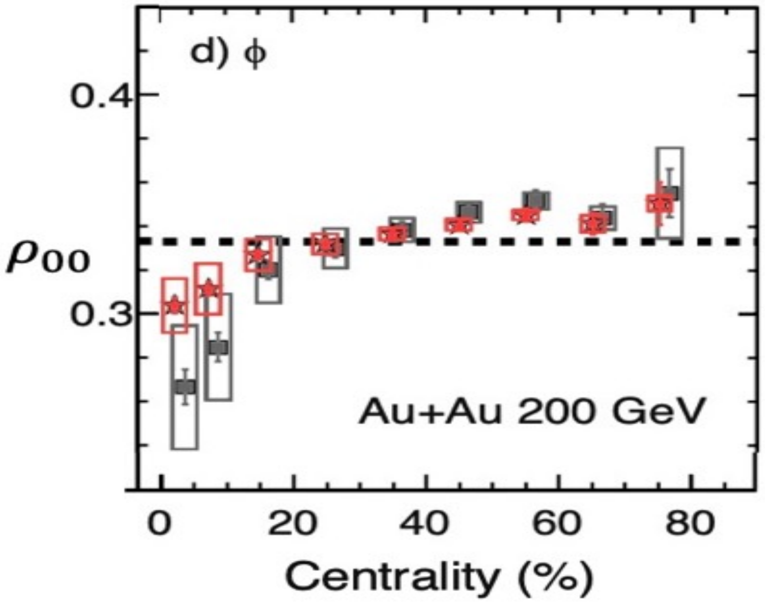


Example in this sketch

Negative α_{sh}
Negative spin alignment



Regard it as Kaon masses



Summary

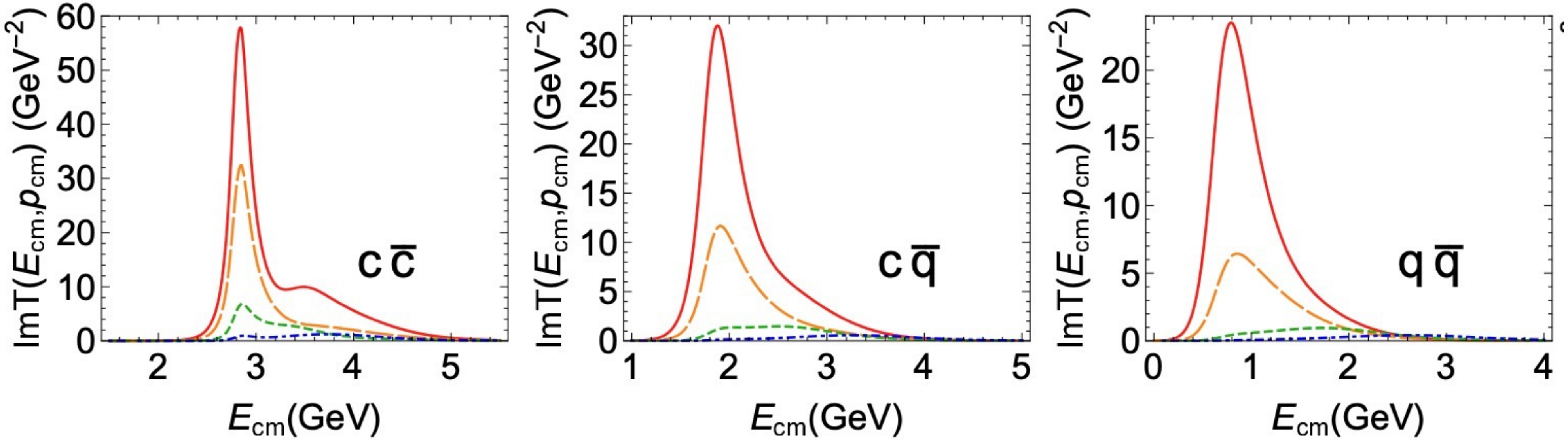
- ❖Discovered a **Shear-Induced Tensor Polarization(SITP)**, together with other new 0^{th} and 1^{st} order effects
- ❖**Natural**, allowed by the symmetry and verified in calculation
- ❖**Universal**, SITP exist in all interacting many-body system with spin-1 particle, in relativistic/non-relativistic scenarios.
- ❖**Large and Rich,** effects especially SITP can contribute to spin alignment at the order of ~1% level, could generate "right" and rich behavior even in a simple model.

Standard many-body interactions (such as collisions) can lead to large spin alignment with the discovery of the missing new effects, such as SITP!

Many works need to be done to make quantitative predictions, due to the rich physics and complexity in strongly coupled many-body system

Backup slides

❖T-matrix resonances without interference effects



Total Theory Results

❖Total results:

$$T^{\mu\nu} = \tilde{\Delta}^{\langle\mu}_{\lambda}\tilde{\Delta}^{\nu\rangle}_{\gamma}\Big[\kappa_0^u u^\lambda u^\gamma + \kappa_1^u u^\lambda u^\gamma + \kappa_{\rm sh}\sigma^{\lambda\gamma} + \kappa_T u^{(\lambda}\partial_\perp^{\gamma)}\beta$$
$$+\kappa_{\rm su} u^{(\lambda}\sigma^{\gamma)\alpha}\tilde{p}_\alpha + \kappa_{\rm ou} u^{(\lambda}\Omega^{\gamma)\alpha}\tilde{p}_\alpha + \cdots\Big]$$

$$\kappa_0^u = \frac{\alpha_0}{-\tilde{v}^2}n_0,\ \kappa_1^u = \left[\alpha_{\rm sh}\left(c_s^2 - \frac{1}{3}\right)\theta + \frac{\alpha_{\rm sp}\xi_p}{-\tilde{v}^2}\right]\beta n_0$$
$$\kappa_{\rm sh} = \alpha_{\rm sh}\beta n_0,\quad \kappa_T = 0,\quad \kappa_{\rm su} = 0,\quad \kappa_{\rm ou} = 0$$

$$\alpha_{\rm sh} \approx -\frac{2\Delta\varepsilon_p}{\Gamma_p} + 2\frac{\Delta\varepsilon_p}{\Gamma_p}\frac{\Delta\varepsilon_p}{T} + \frac{\Gamma_p}{2T} \sim \mathcal{O}(1)$$
$$\alpha_{\rm sp} \approx -\frac{\varepsilon_p}{\Gamma_p}\left(\frac{\Gamma_p^\Delta}{\Gamma_p} - \frac{\Delta\varepsilon_p}{T}\frac{\Gamma_p^\Delta}{\Gamma_p} + \frac{\Gamma_p}{T}\frac{\omega_p^\Delta}{\Gamma_p}\right) \sim \mathcal{O}(\delta_{\rm qp}^{-1}\delta_{\rm sp}).$$
$$\alpha_0 \approx (\omega_p^T - \omega_p^L)/T$$

Nonanalytical energy-shift more subtle

❖Features of the result

- **Natural**, suggested by symmetry, verified in concrete thermal field theory calculation, all have been done is a more careful theory study with more realistic spectral functions
- **Universal**, SITP exist in all spin-1 particles including heavy quarkonium, in relativistic or non-relativistic (SITP has a coefficient have no mass suppression)

Relation to Wigner function

❖Full Wigner function

$$W^{\mu\nu}(x,\boldsymbol{p}) \equiv \varepsilon_{\boldsymbol{p}} \int dp^0 \int d^4y e^{ip\cdot y} \langle V^\mu(x_-) V^\nu(x_+) \rangle \;= W_+^{\mu\nu}(x,\boldsymbol{p}) + W_-^{\mu\nu}(x,\boldsymbol{p})$$

❖Positive mode $W_+^{\mu\nu}$, with projections, and normalization

$$\mathcal{W}^{\mu\nu}(x,\boldsymbol{p}) \equiv 2\tilde{\Delta}^\mu_\alpha \tilde{\Delta}^\nu_\beta W_+^{\alpha\beta}(x,\boldsymbol{p})$$

$\tilde{\Delta} = -\eta^{\mu\nu} + \tilde{p}^\mu \tilde{p}^\nu / \tilde{p}^2$

$\tilde{p}$ is on-shell 4 momentum

❖The density matrices related to it as

$$\varrho_{ss'}(x,\boldsymbol{p}) = \epsilon^\mu_{s'}(\boldsymbol{p}) \epsilon^{\nu *}_s(\boldsymbol{p}) \mathcal{W}_{\mu\nu}(x,\boldsymbol{p})$$

❖Decomposition of $\mathcal{W}^{\mu\nu}$

$$\mathcal{W}^{\mu\nu} = \frac{1}{3}\tilde{\Delta}^{\mu\nu}\mathcal{S} + \mathcal{W}^{[\mu\nu]} + \mathcal{T}^{\mu\nu}$$

$$\mathcal{T}^{\mu\nu} \equiv \mathcal{W}^{\langle\mu\nu\rangle} \equiv \mathcal{W}^{(\mu\nu)} - \frac{1}{3}\tilde{\Delta}^{\mu\nu}\mathcal{S} = 2\tilde{\Delta}^{\langle\mu}_{\lambda}\tilde{\Delta}^{\nu\rangle}_{\gamma} W_+^{(\lambda\gamma)}$$

Related to Wigner function

❖Full Wigner function

$$W^{\mu\nu}(x,\mathbf{p}) \equiv \varepsilon_{\mathbf{p}} \int dp^0 \int d^4y e^{ip\cdot y} \langle V^\mu(x_-) V^\nu(x_+)\rangle \ = W_+^{\mu\nu}(x,\mathbf{p}) + W_-^{\mu\nu}(x,\mathbf{p})$$

❖Positive mode $W_+^{\mu\nu}$, with projections, and normalization

$$\mathcal{W}^{\mu\nu}(x,\mathbf{p}) \equiv 2\tilde{\Delta}^\mu_\alpha \tilde{\Delta}^\nu_\beta W_+^{\alpha\beta}(x,\mathbf{p})$$

$\tilde{\Delta} = -\eta^{\mu\nu} + \tilde{p}^\mu \tilde{p}^\nu / \tilde{p}^2$

$\tilde{p}$ is on-shell 4 momentum

❖The density matrices related to it as

$$\varrho_{ss'}(x,\mathbf{p}) = \epsilon^\mu_{s'}(\mathbf{p}) \epsilon^{\nu *}_s(\mathbf{p}) \mathcal{W}_{\mu\nu}(x,\mathbf{p})$$

❖Decomposition of $\mathcal{W}^{\mu\nu}$

$$\mathcal{W}^{\mu\nu} = \frac{1}{3}\tilde{\Delta}^{\mu\nu}\mathcal{S} + \mathcal{W}^{[\mu\nu]} + \mathcal{T}^{\mu\nu} \quad \Longleftrightarrow \quad = \frac{1}{3}\delta_{ss'} + \frac{1}{2}\mathcal{P}_k (J_k)_{ss'} - \mathcal{T}_{ij}(J_{(i}J_{j)} - \frac{2}{3}\delta_{ij}\mathbf{1})_{ss'}$$

❖Tensor polarization

$$\mathcal{T}^{\mu\nu} \equiv \mathcal{W}^{\langle\mu\nu\rangle} \equiv \mathcal{W}^{(\mu\nu)} - \frac{1}{3}\tilde{\Delta}^{\mu\nu}\mathcal{S} = 2\tilde{\Delta}^{\langle\mu}_{\lambda}\tilde{\Delta}^{\nu\rangle}_{\gamma} W_+^{(\lambda\gamma)}$$

Gradient Expansion and Symmetry analysis

- Expansion up to 1st order gradient expansion

$$T^{\mu\nu} = \tilde{\Delta}^{\langle\mu}_{\lambda}\tilde{\Delta}^{\nu\rangle}_{\gamma}\Big[\kappa_0^u u^\lambda u^\gamma + \kappa_1^u u^\lambda u^\gamma + \kappa_{\rm sh}\sigma^{\lambda\gamma} + \kappa_T u^{(\lambda}\partial_\perp^{\gamma)}\beta + \kappa_{\rm su} u^{(\lambda}\sigma^{\gamma)\alpha}\tilde{p}_\alpha + \kappa_{\rm ou} u^{(\lambda}\Omega^{\gamma)\alpha}\tilde{p}_\alpha + \cdots\Big]$$

T-even

T-even, 0th order

T-odd, Shear Again!

Early theories include terms such as $(\omega/T)^2 \sim (1/100)^2$, 2nd order in gradient

Many Missing BUT Naturally Allowed Contribution at Lower Orders!

- Why missed before?
  - In-medium spectral properties/interactions required, **not been well studied before**
  - Shear Induced Tensor Polarization(SITP) with κ_{sh} to be T-odd and indicate the nature of the dissipative physics, **not been studied before**

Could we find these terms in a concrete calculation?

Yes, see later

0^{th} order—a compact non-perturbative result

❖The tensor polarization related to spectral function as

$$\mathcal{T}^{\mu\nu}_{(0)} = 2\tilde{\Delta}^{\langle\mu}_{\alpha}\tilde{\Delta}^{\nu\rangle}_{\beta}\int_0^\infty dp^0 \int d^4y e^{ip\cdot y}\langle V^\alpha(x_-)V^\beta(x_+)\rangle = 2\tilde{\Delta}^{\langle\mu}_{\alpha}\tilde{\Delta}^{\nu\rangle}_{\beta}\int_0^\infty dp^0 n(p^0)A^{\alpha\beta}(p)$$

❖The spectral function $\Delta^{\mu\nu}_L = v^\mu v^\nu/(-v^2)$, $\Delta^{\mu\nu}_T = \Delta^{\mu\nu} - \Delta^{\mu\nu}_L$, $\tilde{v}^\mu = \tilde{\Delta}^{\mu\nu}u_\nu$

$$A^{\mu\nu} = \sum_{a=L,T}\Delta^{\mu\nu}_a A_a,\; A_a = \frac{1}{\pi}\mathrm{Im}\,\frac{-1}{p^2 - m^2 - \Pi_a}.$$

❖The result

$$\mathcal{T}^{\mu\nu}_{(0)} = \alpha_0 n(\varepsilon_{\boldsymbol{p}})\tilde{\Delta}^{\langle\mu\nu\rangle}_L = \frac{\alpha_0}{-\tilde{v}^2}n(\varepsilon_{\boldsymbol{p}})\tilde{\Delta}^{\langle\mu}_{\lambda}\tilde{\Delta}^{\nu\rangle}_{\gamma}u^\lambda u^\gamma,$$

$$\alpha_0 = 2\varepsilon_{\boldsymbol{p}}\int_0^\infty d\omega\,\frac{n(\omega)}{n(\varepsilon_{\boldsymbol{p}})}\left[(A_L - A_T) - \frac{\Delta\omega^2\tilde{v}^2}{p^2}A_L\right]$$

$\propto \dfrac{\varepsilon^T_{\boldsymbol{p}} - \varepsilon^L_{\boldsymbol{p}}}{T}$ (T-even) $\quad\quad \propto \Gamma^2$

1st order–linear response theorem

$$\xi_{\lambda\gamma} \equiv \beta^{-1}\partial_{(\lambda}(\beta u)_{\gamma)} \approx \sigma_{\lambda\gamma} + \left[\frac{1}{3}\bar{\Delta}_{\lambda\gamma} + c_s^2 u_\lambda u_\gamma\right]\theta$$

❖Linear response (like those for calculate η/s)

$$W^{\mu\nu}_{+(1)} = \varepsilon_{\boldsymbol{p}} \lim_{\nu,q\to 0} \frac{\partial}{\partial\nu}[-\mathrm{Im}G^{\mu\nu\lambda\gamma}_{R+}(\nu,\boldsymbol{q},\boldsymbol{p})]\xi_{\lambda\gamma}$$

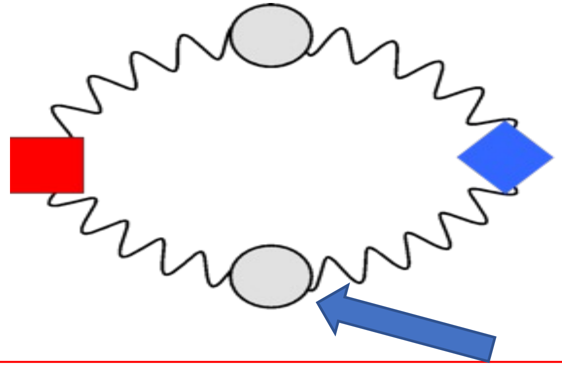
❖The green function is connected to energy momentum tensor

$G^{\mu\nu\lambda\gamma}_{R}(\nu,\boldsymbol{q},\boldsymbol{p})$ ⟷ Wigner Trans ⟷ $(-i)\Theta(t-t')\langle[V^\mu(t,\boldsymbol{x}^-)V^\nu(t,\boldsymbol{x}^+),T^{\lambda\gamma}(t',\boldsymbol{z})]\rangle,$

$$T^{\mu\nu} \equiv -F^{\mu}{}_{\alpha}F^{\nu\alpha} + m^2V^\mu V^\nu - \eta^{\mu\nu}\left(-F^2/4 + m^2V^2/2\right)$$

❖One skeleton/dressed loop calculation with spectral functions

$$G^{\mu\nu\lambda\gamma}_{R+}(\nu,\boldsymbol{q},\boldsymbol{p}) = -\int_0^\infty dk_0 \int_0^\infty dk_0' \frac{n(k_0')-n(k_0)}{\nu+k_0'-k_0+i0^+} \times \sum_{a,b=L,T} A_a(k)A_b(k')I^{\mu\nu\lambda\gamma}_{ab}(k,k').$$



In-medium interactions are implicitly included in self-energies!

1^{st} order–linear response theorem

$$\xi_{\lambda\gamma} \equiv \beta^{-1}\partial_{(\lambda}(\beta u)_{\gamma)} \approx \sigma_{\lambda\gamma} + \left[\frac{1}{3}\bar{\Delta}_{\lambda\gamma} + c_s^2 u_\lambda u_\gamma\right]\theta$$

❖ Linear response (like those for calculate η/s)

$$W^{\mu\nu}_{+(1)} = \varepsilon_{\boldsymbol{p}} \lim_{\nu,q\to 0} \frac{\partial}{\partial\nu}[-\mathrm{Im}G^{\mu\nu\lambda\gamma}_{R+}(\nu,\boldsymbol{q},\boldsymbol{p})]\xi_{\lambda\gamma}$$

❖ The green function is connected to energy momentum tensor

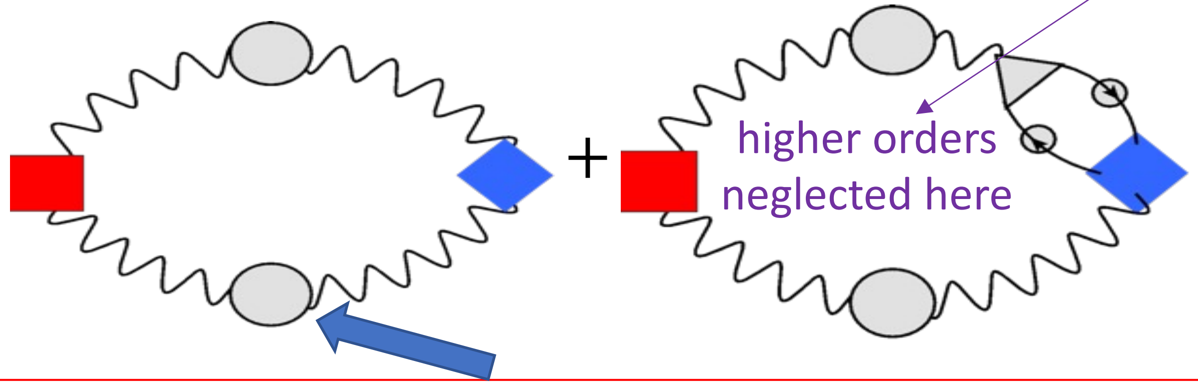
$$G_R^{\mu\nu\lambda\gamma}(\nu,\boldsymbol{q},\boldsymbol{p}) \quad \xleftrightarrow{\text{Wigner Trans}} \quad (-i)\Theta(t-t')\langle[V^\mu(t,\boldsymbol{x}^-)V^\nu(t,\boldsymbol{x}^+), T^{\lambda\gamma}(t',\boldsymbol{z})]\rangle,$$

$$T^{\mu\nu} \equiv -F^\mu{}_\alpha F^{\nu\alpha} + m^2V^\mu V^\nu - \eta^{\mu\nu}\left(-F^2/4 + m^2V^2/2\right)$$

+ higher order terms

❖ One skeleton/dressed loop calculation with spectral functions

$$G^{\mu\nu\lambda\gamma}_{R+}(\nu,\boldsymbol{q},\boldsymbol{p}) = -\int_0^\infty dk_0 \int_0^\infty dk'_0 \frac{n(k'_0)-n(k_0)}{\nu+k'_0-k_0+i0^+} \times \sum_{a,b=L,T} A_a(k)A_b(k')I^{\mu\nu\lambda\gamma}_{ab}(k,k').$$



In-medium interactions are implicitly included in self-energies!

1^{st} order–linear response theorem

$$\xi_{\lambda\gamma} \equiv \beta^{-1}\partial_{(\lambda}(\beta u)_{\gamma)}$$
$$\approx \sigma_{\lambda\gamma} + \left[\frac{1}{3}\bar{\Delta}_{\lambda\gamma} + c_s^2 u_\lambda u_\gamma\right]\theta$$

❖Linear response (like those for calculate η/s)

$$W^{\mu\nu}_{+(1)} = \varepsilon_{\boldsymbol{p}} \lim_{\nu,q\to 0} \frac{\partial}{\partial\nu}[-\mathrm{Im}G^{\mu\nu\lambda\gamma}_{R+}(\nu,\boldsymbol{q},\boldsymbol{p})]\xi_{\lambda\gamma}$$

❖The green function is connected to energy momentum tensor

$$G^{\mu\nu\lambda\gamma}_{R}(\nu,\boldsymbol{q},\boldsymbol{p}) \overset{\text{Wigner Trans}}{\longleftrightarrow} (-i)\Theta(t-t')\langle[V^\mu(t,\boldsymbol{x}^-)V^\nu(t,\boldsymbol{x}^+),T^{\lambda\gamma}(t',\boldsymbol{z})]\rangle,$$

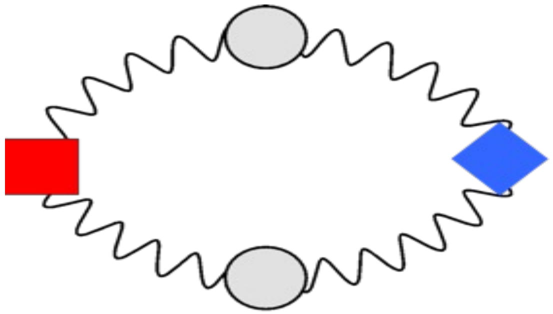
$$T^{\mu\nu} \equiv -F^{\mu}{}_{\alpha}F^{\nu\alpha} + m^2V^\mu V^\nu - \eta^{\mu\nu}\left(-F^2/4 + m^2V^2/2\right)$$

❖One skeleton/dressed loop calculation with spectral functions

$$G^{\mu\nu\lambda\gamma}_{R+}(\nu,\boldsymbol{q},\boldsymbol{p}) = -\int_0^\infty dk_0 \int_0^\infty dk_0' \frac{n(k_0')-n(k_0)}{\nu+k_0'-k_0+i0^+} \times \sum_{a,b=L,T} A_a(k)A_b(k')I^{\mu\nu\lambda\gamma}_{ab}(k,k')\,.$$

❖With some calculation:

$$\begin{aligned} I^{\mu\nu\lambda\gamma}_{ab}(k,k') &= [k^\lambda k'^\gamma + k^\gamma k'^\lambda]\Delta^{\nu\alpha}_a(k)\Delta^{\mu}_{b,\alpha}(k') \\ &- [k_\alpha k'^\gamma \Delta^{\nu\lambda}_a(k)\Delta^{\mu\alpha}_b(k') + k^\gamma k'_\alpha \Delta^{\nu\alpha}_a(k)\Delta^{\mu\lambda}_b(k')] \\ &- [k^\lambda k'_\alpha \Delta^{\nu\alpha}_a(k)\Delta^{\mu\gamma}_b(k') + k_\alpha k'^\lambda \Delta^{\nu\gamma}_a(k)\Delta^{\mu\alpha}_b(k')] \\ &+ (k_\alpha k'^\alpha - m^2)[\Delta^{\nu\lambda}_a(k)\Delta^{\mu\gamma}_b(k') + \Delta^{\nu\gamma}_a(k)\Delta^{\mu\lambda}_b(k')] \\ &- \eta^{\gamma\lambda}[(k^\zeta k'_\zeta - m^2)\eta_{\alpha\beta} - k_\beta k'_\alpha]\Delta^{\nu\alpha}_a(k)\Delta^{\mu\beta}_b(k')] \end{aligned}$$



$$k = (k_0, \boldsymbol{p}+\boldsymbol{q}/2),$$
$$k' = (k_0', \boldsymbol{p}-\boldsymbol{q}/2).$$

Compact results for 1st order

- A one-line formula for tensor polarization

$$\mathcal{T}^{\mu\nu}_{(1)} = \beta n(\varepsilon_{\boldsymbol{p}})\tilde{\Delta}^{\langle\mu}_{\lambda}\tilde{\Delta}^{\nu\rangle}_{\gamma}\left[\alpha_{\rm sh}\xi^{\gamma\lambda} + \alpha_{\rm sp}\xi_p\frac{u^\lambda u^\gamma}{-\tilde{v}^2}\right]$$

with coefficient

$$\alpha_{\rm sh} = \frac{4\varepsilon_{\boldsymbol{p}}\pi}{\beta n(\varepsilon_{\boldsymbol{p}})}\int_0^\infty \frac{\partial n(\omega)}{\partial\omega}d\omega(\omega^2-\varepsilon_{\boldsymbol{p}}^2)A^2_{T/L}(\omega,\boldsymbol{p})$$

$$\alpha_{\rm sp} = \frac{4\varepsilon_{\boldsymbol{p}}\pi}{\beta n(\varepsilon_{\boldsymbol{p}})}\int_0^\infty \frac{\partial n(\omega)}{\partial\omega}d\omega\varepsilon_{\boldsymbol{p}}^2(A^2_T(\omega,\boldsymbol{p})-A^2_L(\omega,\boldsymbol{p}))$$

in quasi-particle spectral function $A_a(\omega,\boldsymbol{p}) \approx \frac{1}{2\varepsilon_{\boldsymbol{p}}}\frac{1}{\pi}\mathrm{Im}\frac{-1}{\omega-\omega^a_{\boldsymbol{p}}+i\Gamma^a_{\boldsymbol{p}}/2}$

$$\alpha_{\rm sh} \approx -\frac{2\Delta\varepsilon_{\boldsymbol{p}}}{\Gamma_{\boldsymbol{p}}} + 2\frac{\Delta\varepsilon_{\boldsymbol{p}}}{\Gamma_{\boldsymbol{p}}}\frac{\Delta\varepsilon_{\boldsymbol{p}}}{T} + \frac{\Gamma_{\boldsymbol{p}}}{2T} \sim \mathcal{O}(1)$$

$$\alpha_{\rm sp} \approx -\frac{\varepsilon_{\boldsymbol{p}}}{\Gamma_{\boldsymbol{p}}}\left(\frac{\Gamma^\Delta_{\boldsymbol{p}}}{\Gamma_{\boldsymbol{p}}} - \frac{\Delta\varepsilon_{\boldsymbol{p}}}{T}\frac{\Gamma^\Delta_{\boldsymbol{p}}}{\Gamma_{\boldsymbol{p}}} + \frac{\Gamma_{\boldsymbol{p}}}{T}\frac{\omega^\Delta_{\boldsymbol{p}}}{\Gamma_{\boldsymbol{p}}}\right) \sim \mathcal{O}(\delta^{-1}_{\rm qp}\delta_{\rm sp}).$$

T-odd, dissipative

- Width Γ_p, energy/mass-shift $\Delta\varepsilon_p$, split of width Γ^Δ_p and energy $\omega^\Delta_{\boldsymbol{p}}$

Well-defined old players in thermal field theory, no extra new players are required

Total Theory Results

❖Total results:

$$T^{\mu\nu} = \tilde{\Delta}^{\langle\mu}_{\lambda}\tilde{\Delta}^{\nu\rangle}_{\gamma}\Big[\kappa_0^u u^\lambda u^\gamma + \kappa_1^u u^\lambda u^\gamma + \kappa_{\rm sh}\sigma^{\lambda\gamma} + \kappa_T u^{(\lambda}\partial_\perp^{\gamma)}\beta + \kappa_{\rm su} u^{(\lambda}\sigma^{\gamma)\alpha}\tilde{p}_\alpha + \kappa_{\rm ou} u^{(\lambda}\Omega^{\gamma)\alpha}\tilde{p}_\alpha + \cdots\Big]$$

$$\kappa_0^u = \frac{\alpha_0}{-\tilde{v}^2}n_0,\quad \kappa_1^u = \left[\alpha_{\rm sh}\left(c_s^2 - \frac{1}{3}\right)\theta + \frac{\alpha_{\rm sp}\xi_p}{-\tilde{v}^2}\right]\beta n_0$$

$$\kappa_{\rm sh} = \alpha_{\rm sh}\beta n_0,\quad \kappa_T = 0,\quad \kappa_{\rm su} = 0,\quad \kappa_{\rm ou} = 0$$

$$\alpha_{\rm sh} = \frac{4\varepsilon_{\mathbf p}\pi}{\beta n(\varepsilon_{\mathbf p})}\int_0^\infty \frac{\partial n(\omega)}{\partial\omega}d\omega(\omega^2 - \varepsilon_{\mathbf p}^2)A^2_{T/L}(\omega,\mathbf{p})$$

$$\alpha_{\rm sp} = \frac{4\varepsilon_{\mathbf p}\pi}{\beta n(\varepsilon_{\mathbf p})}\int_0^\infty \frac{\partial n(\omega)}{\partial\omega}d\omega\varepsilon_{\mathbf p}^2(A_T^2(\omega,\mathbf{p}) - A_L^2(\omega,\mathbf{p}))$$

more subtle

❖Features of the result

- **Natural**, suggested by symmetry, verified in concrete thermal field theory calculation, all have been done is a more careful theory study with more realistic spectral functions
- **Universal**, SITP exist in all spin-1 particles including heavy quarkonium, in relativistic or non-relativistic (SITP has a coefficient have no mass suppression)