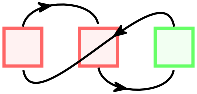
Second-Order Gradient Effects on Spin Alignment

Speaker: Zhong-Hua Zhang (张众华)

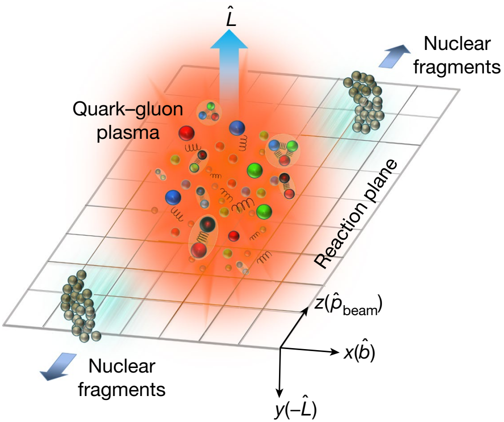
Collaborators: Xu-Guang Huang, Francesco Becattini, and Xin-Li Sheng

Fudan University, Shanghai

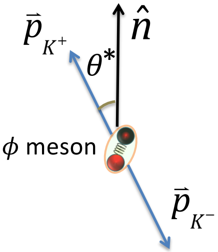
Sep. 19th @ ECT*



Spin Alignment



Heavy Ion Collision



$$\frac{1}{N}\frac{dN}{d\cos\theta^*} = \frac{1}{2} + \frac{3}{4}(3\cos^2\theta^* - 1)\underbrace{\boxed{\left(\Theta_{00} - \frac{1}{3}\right)}}_{\text{spin alignment}}$$

Liang, Wang, PLB 629, 20 (2005)

STAR, Nature 614, 244-248 (2023)

What is Tensor Polarization?

A massive vector boson's spin density matrix

$$\Theta_{rs} = \begin{pmatrix} \Theta_{1,1} & \Theta_{1,0} & \Theta_{1,-1} \\ \Theta_{0,1} & \Theta_{0,0} & \Theta_{0,-1} \\ \Theta_{-1,1} & \Theta_{-1,0} & \Theta_{-1,-1} \end{pmatrix}$$

$$\Theta = \frac{1}{3}\mathbb{1} + \frac{1}{2}\sum_{i=1}^{3} \boxed{P_i}\, S_i + \sum_{m=-2}^{2} (-1)^m \boxed{T_{2,m}}\, S_{2,-m}$$

$\langle S^i \rangle$: Vector polarization (3 DoFs) $\langle S_{2,m} \rangle$: Tensor polarization (5 DoFs)

with

$$S^1 = S_x = \frac{i}{\sqrt{2}} \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}, \quad S^2 = S_y = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad S^3 = S_z = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}.$$

And $S_{2,m} = \sum_{m1,m2} \langle 1, m1; 1, m2 | 2, m \rangle\, S_{1,m1} S_{1,m2}$ the rank-2 spherical irreducible tensor.

How to Detect Tensor Polarization?

- Strong decay, parity even $\Rightarrow T_{2,m}$ only:

$$\frac{1}{N}\frac{dN}{d\Omega}(\theta^*,\phi^*) = \frac{1}{4\pi} - \sqrt{\frac{3}{10\pi}}\sum_{m=-2}^{2}(-1)^m T_{2,m} Y_{2,-m}(\theta^*,\phi^*)$$

- For now, *spin alignment* only:

$$\frac{1}{N}\frac{dN}{d\cos\theta^*} = \frac{1}{2} + \frac{3}{4}(3\cos^2\theta^* - 1)\left(\Theta_{00} - \frac{1}{3}\right)$$

- $T_{2,\pm1}$, $T_{2,\pm2} \to \phi^*$ distribution (off-diagonal polarization)

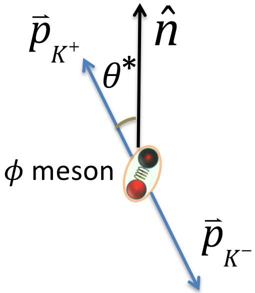
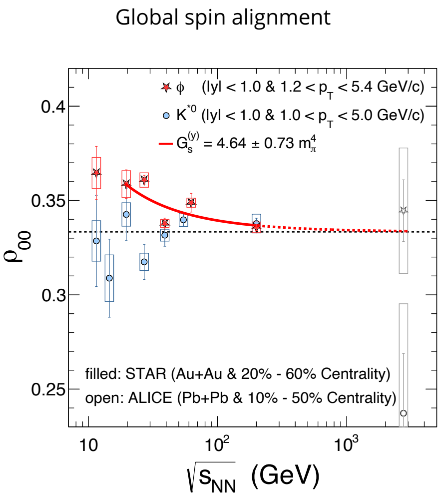


Figure: STAR, Nature.614.244-248 (2023)

Global Spin Alignment



- ϕ meson $\Theta_{00} > 1/3$ and too big

$$\Theta_{00} - \frac{1}{3} \sim P_\Lambda^2 \sim 10^{-4}$$

- K^{*0} different from ϕ

Figure: STAR, Nature 614, 244-248 (2023)

Physical Mechanisms

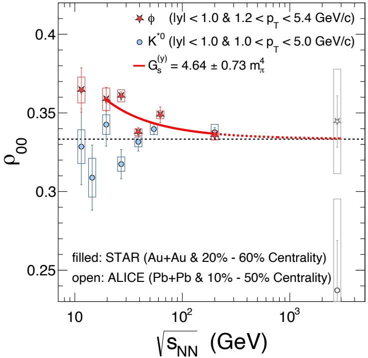
$$\phi \text{ meson: } \delta\Theta_{00} = \Theta_{00} - \frac{1}{3} \approx +c_\Lambda + c_B + c_s + c_F + c_L + \boxed{c_H} + c_\phi + c_g + \cdots$$

Physical mechanism	$\delta\Theta_{00}$
c_Λ: Quark coalescence + vorticity [1]	magnitude $\sim -10^{-4}$
c_B: Quark coalescence + EM-field [1]	magnitude $\sim 10^{-4}$
c_S: Spectrum splitting [2]	unclear
c_F: Quark fragmentation [3]	magnitude $\sim 10^{-5}$
c_L: Local spin alignment [4]	magnitude $\sim -10^{-2}$
c_H: Second-order hydro fields [5]	unclear
c_φ: Vector meson field [6]	> 0, fit to data
c_g: Glasma fields [7]	< 0, magnitude unclear

[1]. Liang, Wang, PLB 629, 20 (2005);
Yang *et al.*PRC 97, 034917 (2018);
Xia *et al.*PLB 817, 136325 (2021);
Becattini *et al.*PRC 88, 034905 (2013).
[2]. Liu, Li, arXiv: 2206.11890;
Sheng *et al.*, Eur.Phys.J.C 84, 299 (2024);
Wei, Huang, Chin.Phys.C 47,104015 (2023).
[3]. Liang, Wang PLB 629, 20 (2005);
[4]. Xia *et al.*PLB 817, 136325 (2021);
Gao, PRD 104, 076016 (2021).
[5]. Kumar, Yang, Gubler, PRD 109, 054038(2024);
Gao, Yang, Chin.Phys.C 48, 053114 (2024);
ZZ, Huang, Becattini, Sheng, 2024.
[6]. Sheng *et al.*, PRD 101, 096005 (2020);
Sheng *et al.*, PRD 102, 056013 (2020);
Sheng *et al.*, PRL 131, 042304 (2023).
[7]. Muller, Yang, PRD 105, L011901 (2022);
Kumar *et al.*, Phy. Rev. D108, 016020 (2023).

Why focus on Hydrodynamics?

Spin alignment

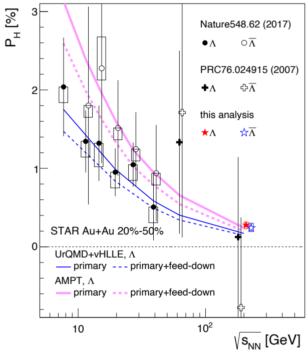


Graph: STAR, Nature.614.244 (2023)

ϕ mean field: X. Sheng *et al.*, PRD.101.096005 (2020)

Glasma: A. Kumar *et al.*, PRD.107.076025 (2023)

Λ's global polarization



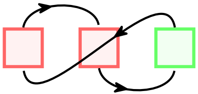
Graph: STAR, PRC.98.014910 (2018)

I. Karpenko, F. Becattini, EPJC.77.213 (2017)

H. Li *et al.*, PRC.96.054908 (2017)

Overview

- Diagram scheme



- Leading order of tensor polarization comes from $\mathcal{O}(\partial^2)$

$$(\partial\beta)(\partial\beta),\ (\partial\beta)\Omega,\ \Omega\Omega,\ \partial\partial\beta,\ \partial\Omega$$

thermal current $\beta_\mu(x) \sim \mathcal{O}(1)$ spin potential $\Omega_{\rho\sigma}(x) \sim \mathcal{O}(\partial)$

Spin Density Matrix and Wigner function

- Free Lagrangian for neutral vector bosons

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A_\mu A^\mu$$

- Future time-like (particle) Wigner function:

$$\widehat{W}_{+}^{\mu\nu}(x,k) = \frac{1}{2\pi}\int d^4s\, e^{ik\cdot s}\,\widehat{A}^{\nu}(x-\frac{s}{2})\widehat{A}^{\mu}(x+\frac{s}{2})\theta(k^2)\theta(k^0)$$

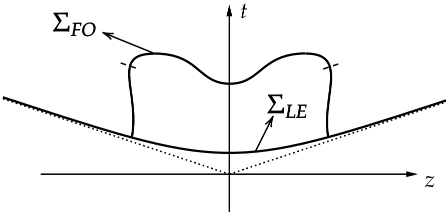
- Phase space distribution:

$$f(x,k) = \sum_r \epsilon_r^{\mu *}(k)\epsilon_r^{\nu}(k)W_{\mu\nu}(x,k)$$

$$\Theta_{rs}(x,k) = \epsilon_r^{\mu *}(k)\epsilon_s^{\nu}(k)W_{\mu\nu}(x,k)/f(x,k)$$

- Spin density matrix:

$$\Theta_{rs}(\boldsymbol{k}) \equiv \frac{\mathrm{Tr}\,(\widehat{\rho}\,\widehat{a}_{\boldsymbol{k}}^{s\dagger}\widehat{a}_{\boldsymbol{k}}^{r})}{\sum_r \mathrm{Tr}\,(\widehat{\rho}\,\widehat{a}_{\boldsymbol{k}}^{r\dagger}\widehat{a}_{\boldsymbol{k}}^{r})} = \frac{\int_{\Sigma_{\mathrm{FO}}} d\Sigma\cdot k\,\Theta_{rs}(x,k)f(x,k)}{\int_{\Sigma_{\mathrm{FO}}} d\Sigma\cdot k\, f(x,k)}$$



Freeze-out hypersurface

F. Becattini *et al.*, PLB 820, 136519 (2021)

LEDO

- Local equilibrium density operator (LEDO)

$$\widehat{\rho}_{\mathrm{LE}} = \frac{1}{Z_{\mathrm{LE}}} \exp\left\{ -\int_{\Sigma} d\Sigma_\mu(y) \left[\overset{\text{stress tensor}}{\widehat{T}^{\mu\nu}(y)} \underset{\text{thermal current } u_\nu/T \sim \mathcal{O}(1)}{\beta_\nu(y)} - \frac{1}{2} \overset{\text{spin tensor}}{\widehat{S}^{\mu\rho\sigma}(y)} \underset{\text{spin potential } \sim \mathcal{O}(\partial)}{\Omega_{\rho\sigma}(y)} \right] \right\}$$

Canonical currents: $T^{\mu\nu} = -F^{\mu\rho}\partial^\nu A_\rho - g^{\mu\nu}\mathcal{L}$, $S^{\mu\rho\sigma} = -F^{\mu\rho}A^\sigma + F^{\mu\sigma}A^\rho$

Integral measure: $d\Sigma_\mu(y) = d\Sigma(y) t_\mu(y)$, normal vector $t_\mu(y)$

- LEDO maximizes the entropy of $\widehat{\rho}_{\mathrm{LE}}$ on hypersurface Σ:

$$S[\widehat{\rho}_{\mathrm{LE}}] = -\,\mathrm{Tr}(\widehat{\rho}_{\mathrm{LE}} \ln \widehat{\rho}_{\mathrm{LE}}),$$

under matching conditions $t_\mu T^{\mu\nu}(x) = t_\mu \left\langle \widehat{T}^{\mu\nu}(x) \right\rangle_{\mathrm{LE}} [\beta, \Omega]$, $\quad t_\mu S^{\mu\rho\sigma}(x) = t_\mu \left\langle \widehat{S}^{\mu\rho\sigma}(x) \right\rangle_{\mathrm{LE}} [\beta, \Omega]$

Zubarev, Prozorkevich, Smolyanskii, Theo. and Math. Phys. 40, 821 (1979)

van Weert, Ann. of Phys. 140, 133 (1982)

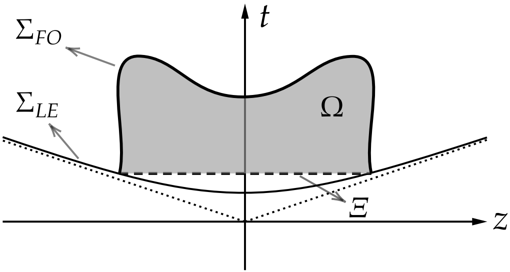
Becattini, Buzzegoli, Palermo, Particles 2, 197 (2019)

Basic Assumptions of Σ_{FO}

$$\widehat{\rho}_{\text{LE}} = \frac{1}{Z_{\text{LE}}} \exp\left\{ - \int_{\Sigma_{\text{FO}}} d\Sigma_\mu(y) \left[\widehat{T}^{\mu\nu}(y)\beta_\nu(y) - \frac{1}{2}\widehat{S}^{\mu\rho\sigma}(y)\Omega_{\rho\sigma}(y) \right] \right\}$$

- Large enough in the spatial direction
- Flat enough on the top
- Gauss's law:

$$\int_{\Sigma_{\text{FO}}} d\Sigma_\mu = \int_{\Xi} d\Sigma_\mu + \int_{\Omega} d^4x \frac{\partial}{\partial x^\mu}$$



Basic Assumptions of Σ_{FO}

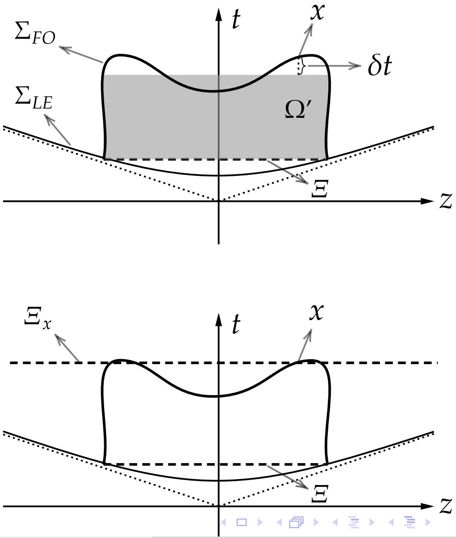
- Gauss's law:

$$\int_{\Sigma_{\text{FO}}} d\Sigma_\mu \approx \int_{\Xi} d\Sigma_\mu + \int_{\Omega'} d^4x \frac{\partial}{\partial x^\mu}$$

- Tensor polarization $@(x,k)$

$$\approx \mathcal{O}(\partial^2) + \delta t(x)\mathcal{O}(\partial^3)$$

Sheng, Becattini, Huang, ZZ, 2407.12130



Cumulant Expansion

- Density operator $\quad \widehat{\rho}_{\rm LE} = \frac{1}{Z_{\rm LE}} \exp\left\{\widehat{A} + \widehat{B}\right\}$, with $Z_{\rm LE} = \mathrm{Tr}\left(e^{\widehat{A}+\widehat{B}}\right)$

  "Gaussian" term $\quad \widehat{A}(x) = -\beta_\nu(x)\widehat{P}^\nu = -\beta_\nu(x)\int_\Sigma d\Sigma_\mu(y)\widehat{T}^{\mu\nu}(y)$

  "Perturbative" terms $\quad \widehat{B}(x) = -\int_\Sigma d\Sigma_\mu(y)\left[\widehat{T}^{\mu\nu}(y)(\beta_\nu(y) - \beta_\nu(x)) - \frac{1}{2}\widehat{S}^{\mu,\rho\sigma}(y)\Omega_{\rho\sigma}(y)\right]$

- Cumulant expansion: $e^{\widehat{A}+\widehat{B}} = e^{\widehat{A}}\sum_{n=0}^{\infty}\widehat{B}_n$, with $\widehat{B}_n \sim (\widehat{B})^n \sim \mathcal{O}(\partial^n)$

$$O(x) \equiv \mathrm{Tr}\left(\widehat{\rho}_{\rm LE}\widehat{O}(x)\right) = \frac{1}{Z_{\rm LE}}\mathrm{Tr}\left(e^{\widehat{A}(x)+\widehat{B}(x)}\widehat{O}(x)\right)$$

$$= \boxed{\sum_n \left\langle \widehat{B}_n\widehat{O}(x)\right\rangle_0 \Big/ \sum_n \left\langle \widehat{B}_n\right\rangle_0} = \boxed{\sum_n \left\langle \widehat{B}_n\widehat{O}(x)\right\rangle_{0,c}}$$

with $\langle\cdots\rangle_0 = \mathrm{Tr}\left(e^{\widehat{A}}\cdots\right)\Big/\mathrm{Tr}\left(e^{\widehat{A}}\right)$ the expectation value under the Gaussian-type distribution.

- Cumulant expansion

Noticing that $e^{s(\widehat{A}+\widehat{B})} = e^{s\widehat{A}} + \int_0^s d\lambda\, e^{(s-\lambda)\widehat{A}}\widehat{B}e^{\lambda(\widehat{A}+\widehat{B})}$

$$
\begin{aligned}
e^{\widehat{A}+\widehat{B}} =& e^{\widehat{A}} + \int_0^1 d\lambda\, e^{(1-\lambda)\widehat{A}}\widehat{B}e^{\lambda(\widehat{A}+\widehat{B})} \\
=& e^{\widehat{A}} + \int_0^1 d\lambda_1\, e^{(1-\lambda_1)\widehat{A}}\widehat{B}e^{\lambda_1\widehat{A}} + \int_0^1 dz_1 \int_0^{\lambda_1} dz_2\, e^{(1-\lambda_1)\widehat{A}}\widehat{B}e^{(\lambda_1-\lambda_2)\widehat{A}}\widehat{B}e^{\lambda_2(\widehat{A}+\widehat{B})} \\
=& \cdots
\end{aligned}
$$

$$
e^{\widehat{A}+\widehat{B}} = e^{\widehat{A}} \sum_{n=0}^{\infty} \widehat{B}_n,
$$

$$
\begin{aligned}
\widehat{B}_0 =& 1, \quad \widehat{B}_1 = \int_0^1 d\lambda_1 \widehat{B}(\lambda_1) \\
& \cdots \\
\widehat{B}_n =& \int_0^1 d\lambda_1 \int_0^{\lambda_1} d\lambda_2 \cdots \int_0^{\lambda_n - 1} d\lambda_n \widehat{B}(\lambda_1)\widehat{B}(\lambda_2)\cdots\widehat{B}(\lambda_n)
\end{aligned}
$$

with $\widehat{B}(\lambda_i) = e^{-\lambda_i\widehat{A}}\widehat{B}e^{\lambda_i\widehat{A}}$.

Gradient Expansion

- Wigner function: $$W_{\mu\nu}(x,k) = \sum_{n=0}^{\infty} W_{\mu\nu}^{(n)}(x,k)$$

$$W_{\mu\nu}^{(n)}(x,k) = \left\langle \widehat{B}_n \widehat{W}_{\mu\nu}(x,k) \right\rangle_{0,c}$$

Zeroth-Order Result

- Cumulant expansion at 0th order: $W^{(0)}_{\mu\nu}(x,k) = \left\langle \widehat{W}_{\mu\nu}(x,k) \right\rangle_0$
- "Free" distribution: $\left\langle \widehat{a}^{s\dagger}_{\boldsymbol{k}} \widehat{a}^{r}_{\boldsymbol{q}} \right\rangle_0 = (2\pi)^3 \delta^{rs} \delta^{(3)}(\boldsymbol{k}-\boldsymbol{q})(2E_{\boldsymbol{k}})\, n_B(\beta(x)\cdot k)$

  Bose-Einstein distribution: $n_B(x) = 1/(e^x - 1)$

- Zeroth-order Wigner fn.: nothing polarized.

$$
\begin{aligned}
W^{(0)}_{\mu\nu}(x,k) = &\int \frac{d^3\boldsymbol{p}_1}{(2\pi)^3} \frac{d^3\boldsymbol{p}_2}{(2\pi)^3} (2\pi)^3 \frac{1}{2E_{\boldsymbol{p}_1}} \frac{1}{2E_{\boldsymbol{p}_2}} e^{-i(p_1-p_2)\cdot x} \\
&\times \sum_{a_1,a_2} \delta^{(4)}\left(k - \frac{p_1}{2} - \frac{p_2}{2}\right) \left\langle \widehat{a}^{a_2\dagger}_{\boldsymbol{p}_2} \widehat{a}^{a_1}_{\boldsymbol{p}_1} \right\rangle_0 \epsilon^{\mu}_{a_1}(p_1) \epsilon^{\nu *}_{a_2}(p_2). \\
= &-\delta(k^2 - m^2)\theta(k^0)\, \Delta^{(k)}_{\mu\nu}\, n_B(\beta(x)\cdot k)
\end{aligned}
$$

projection $\perp$ to k

$$\Delta^{(k)}_{\mu\nu} = \eta_{\mu\nu} - k_\mu k_\nu / m^2$$

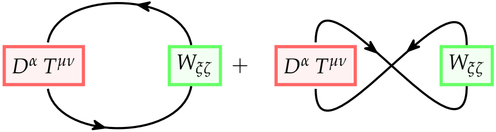
- Recall that $\Theta_{rs}(x,k) \propto \epsilon^{\mu *}_r(k) \epsilon^{\nu}_s(k) W_{\mu\nu}(x,k)$, which leads to $\Theta^{(0)}_{rs} = \delta_{rs}$

First-Order Gradient Expansion

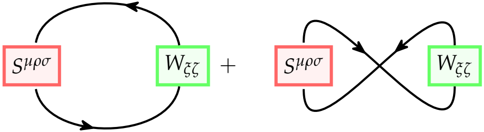
Cumulant expansion: $W^{(1)}_{\mu\nu}(x,k) = \left\langle \widehat{B}_1 \widehat{W}_{\mu\nu}(x,k) \right\rangle_{0,c} = \left\langle \widehat{B}_1 \widehat{W}_{\mu\nu}(x,k) \right\rangle_0 - \left\langle \widehat{B}_1 \right\rangle_0 \left\langle \widehat{W}_{\mu\nu}(x,k) \right\rangle_0$

$$\widehat{B}_1 = -n_\mu \partial_\alpha \beta_\nu(x) \int_0^1 d\lambda \int_P d^3\boldsymbol{y}\, (y-x)^\alpha \widehat{T}^{\mu\nu}(y - i\lambda\beta(x)) + \frac{1}{2} n_\mu \Omega_{\rho\sigma}(x) \int_0^1 d\lambda \int_P d^3\boldsymbol{y}\, \widehat{S}^{\mu\rho\sigma}(y - i\lambda\beta(x))$$

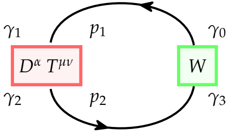
$$\int_0^1 d\lambda \int_P d^3\boldsymbol{y}\, (y-x)^\alpha \left\langle \widehat{T}^{\mu\nu}(y - i\lambda\beta(x)) \widehat{W}_{\xi\zeta}(x,k) \right\rangle_{0,c} =$$



$$\int_0^1 d\lambda \int_P d^3\boldsymbol{y} \left\langle \widehat{S}^{\mu\rho\sigma}(y - i\lambda\beta(x)) \widehat{W}_{\xi\zeta}(x,k) \right\rangle_{0,c} =$$



First-Order Gradient Expansion: Diagram Rules



$$= \int_0^1 d\lambda \int_P d^3\boldsymbol{y}(y-x)^\alpha \prod_{i=0}^{3}\left(\sum_{a_i}\int \frac{d^3\boldsymbol{p}_i}{(2\pi)^3}\frac{1}{2E_{\boldsymbol{p}_i}}\right)(2\pi)^3\delta^{(4)}\left(k-\frac{p_1}{2}-\frac{p_2}{2}\right)\Bigg[t^{\mu\nu}{}_{\gamma_1\gamma_2}(p_1,-p_2)e^{i(p_0-p_3)\cdot x}$$

$$\times \left\langle \hat{a}^{a_1}_{\boldsymbol{p}_1}\hat{a}^{a_0\dagger}_{\boldsymbol{p}_0}\right\rangle_0 \left\langle \hat{a}^{a_2\dagger}_{\boldsymbol{p}_2}\hat{a}^{a_3}_{\boldsymbol{p}_3}\right\rangle_0 \epsilon^{a_0*}_{\gamma_0}(p_0)\epsilon^{a_1}_{\gamma_1}(p_1)\epsilon^{a_2*}_{\gamma_2}(p_2)\epsilon^{a_3}_{\gamma_3}(p_3)e^{-i(p_1-p_2)\cdot(y_1-i\lambda_1\beta(x))}\Bigg]$$

$$= \delta(k^0-E_{\boldsymbol{k}})\int_0^1 d\lambda\, \underbrace{D^\alpha t^{\mu\nu}{}_{\gamma_1\gamma_2}(p_1,-p_2)}_{\text{vertex}}\, \overbrace{(2n\cdot p_1)(n_B(\beta\cdot p_1)+1)\quad (2n\cdot p_2)n_B(\beta\cdot p_2)}^{\text{Bose-Einstein distributions from lines}}\, \underbrace{\left(-\Delta^{\gamma_0\gamma_1}_{(p_1)}\right)\quad\left(-\Delta^{\gamma_2\gamma_3}_{(p_2)}\right)}_{\text{spins sums from lines}}\Bigg|_{\overset{\text{momentum conservation}}{p_1=p_2=k}}$$

$$t^{\mu\nu}{}_{\gamma_1\gamma_2}(p_1,p_2) = \frac{e^{-\lambda(p_1+p_2)\cdot\beta}}{(2n\cdot p_1)(2n\cdot p_2)}\left[p_1^\mu p_2^\nu \eta_{\gamma_1\gamma_2} - p_{2,\gamma_2}p_2^\nu \eta^\mu{}_{\gamma_1} - \frac{1}{2}(p_1\cdot p_2)\eta^{\mu\nu}\eta_{\gamma_1\gamma_2} + \frac{1}{2}p_{1,\gamma_2}p_{2,\gamma_1}\eta^{\mu\nu} - \frac{1}{2}m^2\eta^{\mu\nu}\eta_{\gamma_1\gamma_2}\right]$$

$$D^\alpha = \left(-\frac{i}{2}\right)\left(\eta^{\alpha\zeta}-n^\alpha n^\zeta\right)\left(\frac{\partial}{\partial p_1^\zeta}-\frac{\partial}{\partial p_2^\zeta}\right)$$

ZZ, Huang, Becattini, Sheng, to appear

- Adaptability (higher-order results, additional operators in LEDO, fermions)

First-Order Gradient Expansion

- Recall that: $W^{(1)}_{\mu\nu}(x,k) = -n_\xi \partial_\alpha \beta_\lambda(x) \int_0^1 d\lambda \int_P d^3\boldsymbol{y}(y-x)^\alpha \left\langle \widehat{T}^{\xi\lambda}(y - i\lambda\beta(x))\widehat{W}_{\mu\nu}(x,k)\right\rangle_{0,c}$ + spin potential contribution

$$W^{(1)}_{\perp,\mu\nu}(x,k) = -i\delta(k^2-m^2)\theta(k^0)n_B(1+n_B)\Delta^{(k)}_{\mu\rho}\Delta^{(k)}_{\nu\sigma}\Big[\underbrace{\varpi^{\rho\sigma}}_{\text{thermal vorticity}} - \Xi_\alpha{}^{[\rho}\Big(\underbrace{\xi^{\sigma]\alpha}}_{\text{thermal shear}} + \underbrace{\delta\Omega^{\sigma]\alpha}}_{\text{net spin potential: } \Omega-\varpi}\Big)\Big]$$

with: $\Xi^{\mu\nu} = \eta^{\mu\nu} - \dfrac{k^\mu n^\nu}{(n\cdot k)}$

- Spin alignment at leading order:

$$\delta\Theta_{00}(x,k) \approx \frac{1}{3n_B\delta(k^2-m^2)\theta(k^0)}\left(\epsilon^\mu_y(k)\epsilon^\nu_y(k) + \frac{1}{3}\Delta^{\mu\nu}_{(k)}\right)W_{\mu\nu}(x,k)$$

- Space-time reversal odd: $\boxed{W^{(1)}_{\mu\nu} = -W^{(1)}_{\nu\mu} \;\Rightarrow\; \delta\Theta_{00} = 0 + \mathcal{O}(\partial^2)}$

Space-time Reversal Property

- Space-time reversal w.r.t. x, $PT:\ y \to y' = 2x - y$

$$\widehat{\rho} = \frac{1}{Z}\exp\left\{-\int_{\Sigma} d\Sigma_\mu(y)\left[\widehat{T}^{\mu\nu}(y)\,\beta_\nu(y) - \frac{1}{2}\widehat{S}^{\mu\rho\sigma}(y)\,\Omega_{\rho\sigma}(y)\right]\right\}$$

$$\Downarrow\ (PT)\widehat{T}^{\mu\nu}(y)(PT)^{-1} = \widehat{T}^{\mu\nu}(y'),\quad (PT)\widehat{S}^{\mu\rho\sigma}(y)(PT)^{-1} = -\widehat{S}^{\mu\rho\sigma}(y')$$

$$(PT)\widehat{\rho}(PT)^{-1} = \frac{1}{Z}\exp\left\{-\int_{\Sigma'} d\Sigma_\mu(y')\left[\widehat{T}^{\mu\nu}(y')\,\beta'_\nu(y') - \frac{1}{2}\widehat{S}^{\mu\rho\sigma}(y')\,\Omega'_{\rho\sigma}(y')\right]\right\}$$

$\Sigma \to \Sigma'$ $\qquad \beta_\nu(y) \qquad -\Omega_{\rho\sigma}(y)$

- Wigner function: $W_{\mu\nu}(x,k) = \mathrm{Tr}\left(\widehat{\rho}\widehat{W}_{\mu\nu}(x,k)\right) = \mathrm{Tr}\left((PT)\widehat{\rho}(PT)^{-1}(PT)\widehat{W}_{\mu\nu}(x,k)(PT)^{-1}\right)$
  $(PT)\widehat{W}_{\mu\nu}(x,k)(PT)^{-1} = \widehat{W}_{\nu\mu}(x,k)$
- For a hyperplane, $\Sigma' = \Sigma$: $W_{\mu\nu}(x,k)[\beta_\nu, \Omega_{\rho\sigma}] = W_{\nu\mu}(x,k)[\beta'_\nu, \Omega'_{\rho\sigma}]$
- Recall the power counting rules: $\beta_\nu \sim \mathcal{O}(1)$, $\Omega_{\rho\sigma} \sim \mathcal{O}(\partial) \Rightarrow \boxed{W^{(n)}_{\mu\nu}(x,k) = (-1)^n W^{(n)}_{\nu\mu}(x,k)} \sim \mathcal{O}(\partial^n)$

Second-Order Gradient Expansion

- Based on the cumulant expansion:

$$\begin{aligned}
\widehat{\mathcal{B}}_2 \equiv & \int_0^1 d\lambda_1 \int_0^{\lambda_1} d\lambda_2 \int_{\Sigma_{\mathrm{FO}}} d\Sigma_{\mu_1}(y_1) d\Sigma_{\mu_2}(y_2) \\
& \times \Big[\partial_{\alpha_1}\beta_{\nu_1}(x)\partial_{\alpha_2}\beta_{\nu_2}(x)(y_1-x)^{\alpha_1}(y_2-x)^{\alpha_2}\widehat{T}^{\mu_1\nu_1}(y_1^{(\beta)})\widehat{T}^{\mu_2\nu_2}(y_2^{(\beta)}) \\
& - \frac{1}{2}\partial_{\alpha_1}\beta_{\nu_1}(x)\Omega_{\rho_2\sigma_2}(x)(y_1-x)^{\alpha_1}\widehat{T}^{\mu_1\nu_1}(y_1^{(\beta)})\widehat{S}^{\mu_2,\rho_2\sigma_2}(y_2^{(\beta)}) \\
& - \frac{1}{2}\Omega_{\rho_1\sigma_1}(x)\partial_{\alpha_2}\beta_{\nu_2}(x)(y_2-x)^{\alpha_2}\widehat{S}^{\mu_1,\rho_1\sigma_1}(y_1^{(\beta)})\widehat{T}^{\mu_2\nu_2}(y_2^{(\beta)}) \\
& + \frac{1}{4}\Omega_{\rho_1\sigma_1}(x)\Omega_{\rho_2\sigma_2}(x)\widehat{S}^{\mu_1,\rho_1\sigma_1}(y_1^{(\beta)})\widehat{S}^{\mu_2,\rho_2\sigma_2}(y_2^{(\beta)}) \Big] \\
& + \int_0^1 d\lambda_1 \int_{\Sigma_{\mathrm{FO}}} d\Sigma_{\mu_1}(y_1) \Big[-\partial_{\alpha_1}\partial_{\alpha_2}\beta_{\nu_1}(x)\frac{1}{2}(y_1-x)^{\alpha_1}(y-x)^{\alpha_2}\widehat{T}^{\mu_1\nu_1}(y_1^{(\beta)}) \\
& + \frac{1}{2}\partial_{\alpha_1}\Omega_{\rho_1\sigma_1}(x)(y_1-x)^{\alpha_1}\widehat{S}^{\mu_1,\rho_1\sigma_1}(y_1^{(\beta)}) \Big]
\end{aligned}$$

- Wigner function induced by second-order hydro fields:

$$W^{(2)}_{\mu\nu}(x,k) = \left\langle \widehat{\mathcal{B}}_2 \widehat{W}_{\mu\nu}(x,k) \right\rangle_{0,c}$$

$$W^{(2)}_{\gamma_0\gamma_5}|_{TT}(x,k) = \hat{n}_{\mu_1}\hat{n}_{\mu_2}\partial_{\alpha_1}\beta_{\nu_1}(x)\partial_{\alpha_2}\beta_{\nu_2}(x)\left\{\boxed{D^{\alpha_1}_{(1)}\,T^{\mu_1\nu_1}_{(1)}}\quad\boxed{D^{\alpha_2}_{(2)}\,T^{\mu_2\nu_2}_{(2)}}\quad\boxed{W_{\gamma_0\gamma_5}} + \text{other 15 diagrams}\right\}$$

$$W^{(2)}_{\gamma_0\gamma_5}|_{TS}(x,k) = \hat{n}_{\mu_1}\hat{n}_{\mu_2}\partial_{\alpha_1}\beta_{\nu_1}(x)\Omega_{\rho_2\sigma_2}(x)\left\{\boxed{D^{\alpha_1}_{(1)}\,T^{\mu_1\nu_1}_{(1)}}\quad\boxed{S^{\mu_2\rho_2\sigma_2}_{(2)}}\quad\boxed{W_{\gamma_0\gamma_5}} + \text{other 15 diagrams}\right\}$$

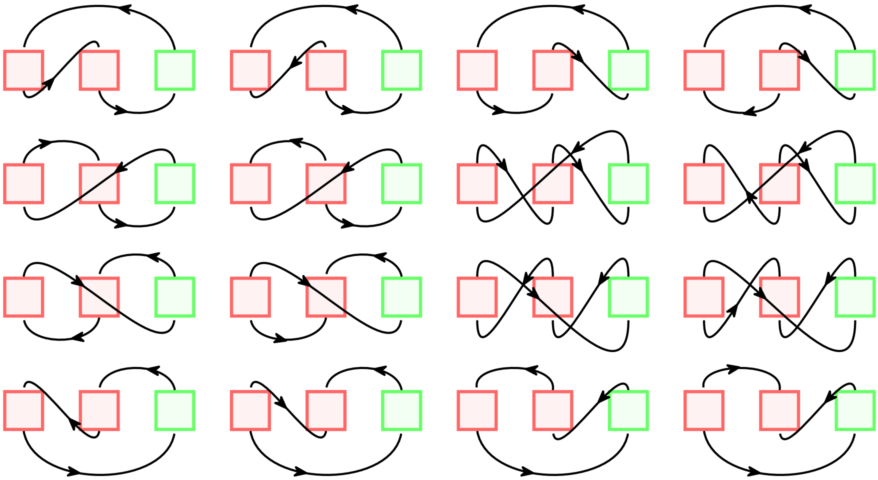
$$W^{(2)}_{\gamma_0\gamma_5}|_{ST}(x,k) = \hat{n}_{\mu_1}\hat{n}_{\mu_2}\Omega_{\rho_1\sigma_1}(x)\partial_{\alpha_2}\beta_{\nu_2}(x)\left\{\boxed{S^{\mu_1\rho_1\sigma_1}_{(1)}}\quad\boxed{D^{\alpha_2}_{(2)}\,T^{\mu_2\nu_2}_{(2)}}\quad\boxed{W_{\gamma_0\gamma_5}} + \text{other 15 diagrams}\right\}$$

$$W^{(2)}_{\gamma_0\gamma_5}|_{SS}(x,k) = \hat{n}_{\mu_1}\hat{n}_{\mu_2}\Omega_{\rho_1\sigma_1}(x)\Omega_{\rho_2\sigma_2}(x)\left\{\boxed{S^{\mu_1\rho_1\sigma_1}_{(1)}}\quad\boxed{S^{\mu_2\rho_2\sigma_2}_{(2)}}\quad\boxed{W_{\gamma_0\gamma_5}} + \text{other 15 diagrams}\right\}$$

$$W^{(2)}_{\gamma_0\gamma_5}|_{T}(x,k) = \hat{n}_{\mu_1}\partial_{\alpha_1}\partial_{\alpha_2}\beta_{\nu_1}(x)\left\{\boxed{D^{\alpha_1}_{(1)}D^{\alpha_2}_{(1)}\,T^{\mu_1\nu_1}_{(1)}}\quad\boxed{W_{\gamma_0\gamma_5}} + \text{the other diagram}\right\}$$

$$W^{(2)}_{\gamma_0\gamma_5}|_{S}(x,k) = \hat{n}_{\mu_1}\partial_{\alpha_1}\Omega_{\rho_1\sigma_1}(x)\left\{\boxed{D^{\alpha_1}_{(1)}\,S^{\mu_1\rho_1\sigma_1}_{(1)}}\quad\boxed{W_{\gamma_0\gamma_5}} + \text{the other diagram}\right\}$$

16 Diagrams



Second-Order Gradient Expansion

- Following the cumulant expansion and the diagram rules:

projected Wigner fn.

$$\boxed{W^{(2)}_{\perp,\mu\nu}}|_{\varpi^2} = \frac{1}{2}\delta(k^2-m^2)\theta(k^0)n_B(1+n_B)(1+2n_B)\left(\eta_{\rho\sigma}-\frac{k_\rho k_\sigma}{2m^2}\right)\ \varpi_{\mu\rho}(x)\ \ \varpi_{\sigma\nu}(x)\ +\Delta^{(k)}_{\mu\nu}\cdots$$

$$\boxed{W^{(2)}_{\perp,\mu\nu}}|_{\partial\varpi} = \delta(k^2-m^2)\theta(k^0)n_B(1+n_B)(1+2n_B)\frac{1}{2(n\cdot k)}\ \partial_\alpha\varpi_{\rho\sigma}(x)$$

$$\times\left\{\eta^{\alpha\rho}\hat{k}^\sigma n_\mu n_\nu - n^\rho\hat{k}^\sigma\eta^\alpha{}_{(\mu}n_{\nu)} - 2\eta^{\alpha\rho}\eta^\sigma{}_{(\mu}n_{\nu)} + 2n^\rho\eta^\sigma{}_{(\mu}\eta_{\nu)}{}^\alpha\right\}+\Delta^{(k)}_{\mu\nu}\cdots$$

$$\vdots$$

- PT even: $W^{(2)}_{\mu\nu}(x,k) = W^{(2)}_{\nu\mu}(x,k)$

- Spin alignment: $\delta\Theta_{00}(x,k) \approx \frac{1}{3n_B\delta(k^2-m^2)\theta(k^0)}\left(\epsilon^\mu_y(k)\epsilon^\nu_y(k)+\frac{1}{3}\Delta^{\mu\nu}_{(k)}\right)W_{\mu\nu}(x,k)$

Spin Alignment: Quadratic Terms

- Decompose thermal vorticity $\varpi_{\rho\sigma} = \underbrace{a_{[\rho}}_{\text{acceleration in lab frame}} \hat{n}_{\sigma]} + \epsilon_{\rho\sigma\alpha\beta} \underbrace{\omega^{\alpha}}_{\text{rotation in lab frame}} \hat{n}^{\beta}$
- Spin alignment induced by squared vorticity

$$\delta\Theta_{00}|_{\varpi^2}(x,k) = -\frac{1}{6}(1+n_B)(1+2n_B)\left[(\underbrace{\boldsymbol{\omega}_{\boldsymbol{k}}}_{\text{rotation in rest frame}} \cdot \boldsymbol{e}_y)^2 - \frac{1}{3}\boldsymbol{\omega}_{\boldsymbol{k}}^2\right]$$
$$-\frac{1}{12}(1+n_B)(1+2n_B)\left[(\underbrace{\boldsymbol{a}_{\boldsymbol{k}}}_{\text{acceleration in rest frame}} \cdot \boldsymbol{e}_y)^2 - \frac{1}{3}\boldsymbol{a}_{\boldsymbol{k}}^2\right]$$

- Acceleration and rotation in the rest frame of the vector meson

$$\boldsymbol{\omega}_{\boldsymbol{k}} = \frac{E_{\boldsymbol{k}}}{m}\boldsymbol{\omega} - \frac{\boldsymbol{a}\times\boldsymbol{k}}{m} - \frac{(\boldsymbol{\omega}\cdot\boldsymbol{k})\boldsymbol{k}}{m(E_{\boldsymbol{k}}+m)}, \quad \boldsymbol{a}_{\boldsymbol{k}} = \frac{E_{\boldsymbol{k}}}{m}\boldsymbol{a} + \frac{\boldsymbol{\omega}\times\boldsymbol{k}}{m} - \frac{(\boldsymbol{a}\cdot\boldsymbol{k})\boldsymbol{k}}{m(E_{\boldsymbol{k}}+m)}$$

their transformation is similar to that of the EM field.

Spin Alignment: Linear Terms

- Spin alignment induced by curl of $\boldsymbol{\omega}$:

$$\delta\Theta_{00}\big|_{\partial\boldsymbol{\omega}}(x,k) = -\frac{1}{9}(1+n_B)\left\{\left[\frac{3\overbrace{\gamma_k}^{E_{\boldsymbol{k}}/m}(\gamma_k-1)}{2m(\gamma_k+1)}(v^y)^2 - \frac{\gamma_k{}^2+1}{2m\gamma_k}\right]\right.$$

$$\left.\times \underbrace{\boldsymbol{v}}_{\text{velocity of particle}}\cdot(\nabla\times\boldsymbol{\omega}) + \frac{3}{m}(v^y)(\nabla\times\boldsymbol{\omega})^y\right\},$$

- Spin alignment induced by divergence and gradient of $\boldsymbol{a}$:

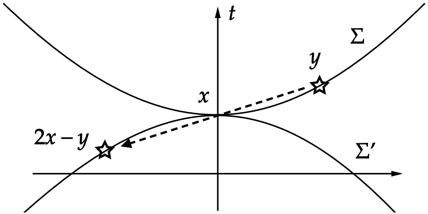
$$\begin{aligned}\delta\Theta_{00}\big|_{\partial\boldsymbol{a}}(x,k) =& \frac{1}{9}(1+n_B)\left\{\left[\frac{\gamma_k{}^2-3}{2m\gamma_k} - \frac{3\gamma_k}{2m}(v^y)^2\right]\nabla\cdot\boldsymbol{a}\right.\\ &+\left[\frac{3\gamma_k{}^2(\gamma_k-1)}{2m(\gamma_k+1)^2}(v^y)^2 - \frac{\gamma_k}{2m}\right](\boldsymbol{v}\cdot\nabla)(\boldsymbol{a}\cdot\boldsymbol{v}) + \frac{3}{m\gamma_k}\partial_y a^y\\ &\left.+\frac{3\gamma_k}{m(\gamma_k+1)}(\boldsymbol{v}\cdot\nabla)(a^y v^y) + \frac{3(\gamma_k-1)}{2m(\gamma_k+1)}(v^y\partial_y)(\boldsymbol{a}\cdot\boldsymbol{v})\right\}\end{aligned}$$

Break PT Symmetry of the System

How to break $W^{(1)}_{\mu\nu} = -W^{(1)}_{\nu\mu}$ to get $\delta\Theta_{00} \propto \partial_\alpha\beta_\nu,\, \Omega_{\rho\sigma}$?

- Dissipative process $\rightarrow$ Time reversal symmetry broken W. Dong *et al.*, PRD 109, 056025 (2024); ...
- Chiral symmetry breaking $\rightarrow$ Parity symmetry broken not clear
- Freeze-out hypersurface curved talk at Chirality 2024

Recall that $W^{(1)}_{\mu\nu} = -W^{(1)}_{\nu\mu}$ is based on $\Sigma = \Sigma'$



Curved Hypersurface: Results

- New terms induced by local curvature of the hypersurface:

local curvature

$$\boxed{W_{\perp}^{(1)\mu\nu}|_B(x,k)} = \delta(k^2-m^2)\theta(k^0)n_B(1+n_B)\frac{\boxed{B_{\alpha\beta}(x)}}{2(n\cdot k)}\left\{\boxed{\xi_{\rho\sigma}(x)}\left(\eta^{\rho\sigma}+\frac{k^\rho k^\sigma}{2m^2}\right)\Xi^{\alpha(\mu}\Xi^{\beta\nu)}\right.$$

$$\left. + \left(\boxed{\delta\Omega_{\rho\sigma}(x)} - \boxed{\xi_{\rho\sigma}(x)}\right)\Xi^{\alpha\rho}\Xi^{\beta(\mu}\Delta_{(k)}^{\nu)\sigma} + \boxed{(\Delta_{(k)}^{\mu\nu}\cdots)}\right\}$$

projected Wigner fn.

net spin potential: $\Omega - \omega$

thermal shear

no spin polarization

- Recall that:

$$\Xi^{\mu\nu} = \left(\eta^{\mu\nu} - \frac{k^\mu n^\nu}{(n\cdot k)}\right)$$

$$\delta\Theta_{00}(x,k) \approx \frac{1}{3n_B\delta(k^2-m^2)\theta(k^0)}\left(\epsilon_y^\mu(k)\epsilon_y^\nu(k) + \frac{1}{3}\Delta_{(k)}^{\mu\nu}\right)W_{\mu\nu}(x,k)$$

- Spin alignment at $\mathcal{O}(\partial)$!

Conclusion and Outlook

- Flat freezeout hypersurface $\Rightarrow$ spin alignment $\propto \mathcal{O}(\partial^2)$

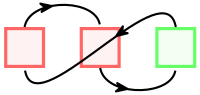
$$(\partial\beta)(\partial\beta),\ (\partial\beta)\Omega,\ \Omega\Omega,\ \partial\partial\beta,\ \partial\Omega$$

$\partial_\mu \beta_\nu =$ thermal vorticity + therma shear, $\quad \Omega_{\rho\sigma}$ – spin potential

- Curved freezeout hypersurface $\Rightarrow$ spin alignment $\propto$ thermal shear ...

- Spin alignment also at the first order of gradient:
  Dissipation;
  Chiral symmetry breaking...

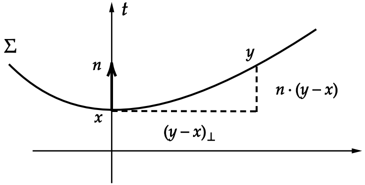
Thank you!



Geometry of Curved Hypersurface

- Assume Σ is a space-like 3D hypersurface:
  $F(x) = 0, \ \forall x \in \Sigma$
  Choose a proper F, s.t. $v_0 > 0$, with $v_\mu = \partial F(x)/\partial x^\mu$
  Normal vector of the hypersurface: $n_\mu = v_\mu / \sqrt{v \cdot v}$
- Taylor expansion around x upto second order:

$$n \cdot (y - x) = \frac{1}{2} B_{\mu\nu} (y-x)^\mu (y-x)^\nu + \mathcal{O}\left(((y-x)_\perp)^3 \right)$$



- Curvature tensor:

projection $\perp$ to n

$$B_{\mu\nu}(x) \equiv -\frac{1}{\sqrt{v \cdot v}} \Delta^{(n)}_{\mu\rho} \Delta^{(n)}_{\nu\sigma} \frac{\partial^2 F(x)}{\partial x_\rho \partial x_\sigma}$$

curvature of curve C $\qquad$ $\Delta^{(n)}_{\mu\nu} T^\mu T^\nu$

$$B_{\mu\nu} T^\mu T^\nu = \kappa_C \left(-T_\perp^2 \right)$$