Emergent canonical spin tensor in the chiral symmetric hot QCD

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Spin and quantum features of QCD plasma
@ECT*

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In collaboration with A. Palermo

Based on: 2407.14345

Spin tensor

The spin tensor is the "spin" part of the total angular momentum density operator

$$\widehat{\mathcal{J}}^{\lambda,\mu\nu} = x^\mu \widehat{T}^{\lambda\nu} - x^\nu \widehat{T}^{\lambda\mu} + \widehat{\mathcal{S}}^{\lambda,\mu\nu}$$

Infinite ways to define the spin tensor through **pseudo-gauge transformations**

$$\widehat{T}'^{\mu\nu} = \widehat{T}^{\mu\nu} + \frac{1}{2}\nabla_\lambda \left(\widehat{\Phi}^{\lambda,\mu\nu} - \widehat{\Phi}^{\mu,\lambda\nu} - \widehat{\Phi}^{\nu,\lambda\mu} \right),$$

$$\widehat{\mathcal{S}}'^{\lambda,\mu\nu} = \widehat{\mathcal{S}}^{\lambda,\mu\nu} - \widehat{\Phi}^{\lambda,\mu\nu}, \quad \widehat{\Phi}^{\lambda,\mu\nu} = -\widehat{\Phi}^{\lambda,\nu\mu}$$

The total conserved charges are unaffected but the densities of energy, momentum and angular momentum differ by quantum terms.

Examples (Dirac field):

- The canoncial form is directly obtained from the Noether theorem

$$\widehat{\mathcal{S}}_{\rm C}^{\lambda,\mu\nu} = \frac{1}{2}\bar{\psi}\left\{\gamma^\lambda, \Sigma^{\mu\nu}\right\}\psi \quad \Sigma_{\mu\nu} = \frac{\rm i}{4}[\gamma_\mu, \gamma_\nu]$$

  It is dual to the axial current: $\widehat{\mathcal{S}}_{\rm C}^{\lambda,\mu\nu} = \epsilon^{\lambda\mu\nu\rho}\widehat{j}_{A\,\rho}$

- The Belinfante form has symmetric energy-momentum tensor and vanishing spin tensor

$$\widehat{\mathcal{S}}_{\rm B}^{\lambda,\mu\nu} = 0$$

Spin hydrodynamics

Spin hydrodynamics is necessary when the spin relaxation time scale is much longer than the time scale of e.g. kinetic equilibration

F. Becattini, W. Florkowski, E. Speranza, Phys. Lett. B 789 (2019) 419

Include the spin tensor in the hydro equations:

$$\partial_\mu T^{\mu\nu} = 0 \qquad \partial_\lambda \mathcal{S}^{\lambda,\mu\nu} = T^{\nu\mu} - T^{\mu\nu} \qquad \partial_\mu j^\mu = 0$$

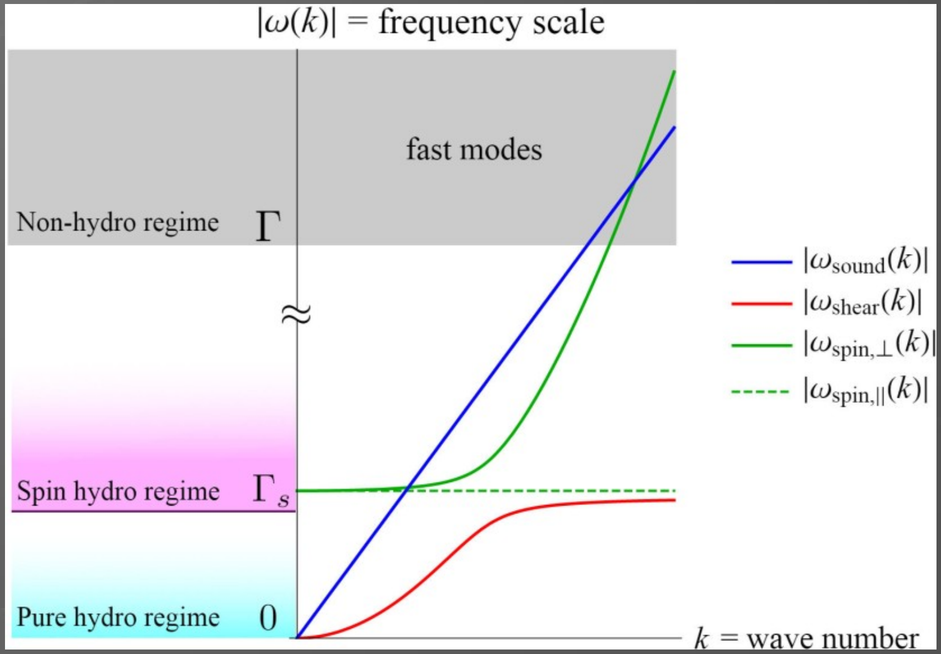
$$T^{\mu\nu} = T^{\mu\nu}(\beta,\zeta,\mathfrak{S}) \qquad \mathcal{S}^{\lambda,\mu\nu} = \mathcal{S}^{\lambda,\mu\nu}(\beta,\zeta,\mathfrak{S}) \qquad j^\mu = j^\mu(\beta,\zeta,\mathfrak{S})$$

Spin potential: $\mathfrak{S}$

Many recent developments

F. Becattini, S. Bhadury, J. Bhatt, A. Das, W. Florkowski, B. Friman, K. Fukushima, J.-H. Gao, C. Gale, K. Hattori, Y. Hidaka, M. Hongo, D. Hou, A. Huang, X.-G. Huang, A. Jaiswal, S. Jeon, M. Kaminski, A. Kumar, S. Li, Z.-T. Liang, J. Liao, Y.-C. Liu, K. Mameda, M. Matsuo, S. Pu, D. H. Rischke, R. Ryblewski, D. She, X.-L. Sheng, S. Shi, R. Singh, E. Speranza, M. Stephanov, H. Taya, Q. Wan, N. Weickgenannt, D.-L. Yang, H. U. Yee

Review: Becattini, Buzzegoli, Niida et al (2024)



M. Hongo, X. G. Huang, M. Kaminski, M. Stephanov and H. U. Yee, JHEP 11 (2021) 150

Statistical operator for Spin hydrodynamics

The statistical operator is obtained by maximizing the entropy $S = -\mathrm{tr}\left[\widehat{\rho}\log\widehat{\rho}\right]$ with the constraints of fixed energy-momentum and angular momentum-boost density

$$n_\lambda \mathrm{tr}\left[\widehat{\rho}\,\widehat{T}^{\lambda\nu}\right] = n_\lambda T^{\lambda\nu}$$

$$n_\lambda \mathrm{tr}\left[\widehat{\rho}\,\widehat{\mathcal{J}}^{\lambda,\mu\nu}\right] = n_\lambda \mathcal{J}^{\lambda,\mu\nu} \Rightarrow \quad n_\lambda \mathrm{tr}\left[\widehat{\rho}\,\widehat{\mathcal{S}}^{\lambda,\mu\nu}\right] = n_\lambda \mathcal{S}^{\lambda,\mu\nu}$$

General covariant local thermodynamic equilibrium density operator

$$\widehat{\rho} = \frac{1}{\mathcal{Z}}\exp\left\{-\int_\Sigma \mathrm{d}\Sigma\, n_\lambda\left[\widehat{T}^{\lambda\nu}\beta_\nu - \frac{1}{2}\mathfrak{S}_{\rho\sigma}\widehat{\mathcal{S}}^{\lambda,\rho\sigma}\right]\right\}$$

The density operator describing local thermal equilibrium is pseudo-gauge dependent

F. Becattini, W. Florkowski, and E. Speranza, Phys. Lett. B 789, 419 (2019)
W. Florkowski, A. Kumar, R. Ryblewski, Prog. Part. Nucl. Phys. 108 (2019) 103709
E. Speranza and N. Weickgenannt, Eur. Phys. J. A 57, 155 (2021)

Hydrodynamics is pseudo-gauge dependent

F. Becattini and L. Tinti, Phys. Rev. D 84, 025013 (2011) and Phys. Rev. D 87, 025029 (2013)
K. Fukushima and S. Pu, Phys. Lett. B 817, 136346 (2021)
A. Das, W. Florkowski, R. Ryblewski and R. Singh, Phys. Rev. D 103, L091502 (2021)

Pseudo-gauge dependence of Spin polarization

For instance, the relation between spin polarization and the properties of the fluid (thermal vorticity, thermal shear, etc) is pseudo-gauge dependent.

MB, PRC 105 (2022) 4, 044907

- Belinfante

$$S^\mu_{\rm B}(k) \simeq S^\mu_\varpi(k) + S^\mu_\xi(k)$$

$$S^\mu_\varpi(k) = -\frac{1}{8m}\epsilon^{\mu\rho\sigma\tau}k_\tau \frac{\int_\Sigma {\rm d}\Sigma\cdot k\, n_{\rm F}\,(1-n_{\rm F})\,\varpi_{\rho\sigma}}{\int_\Sigma {\rm d}\Sigma\cdot k\, n_{\rm F}}$$

$$S^\mu_\xi(k) = -\frac{1}{4m}\epsilon^{\mu\lambda\sigma\tau}\frac{k_\tau k^\rho}{\varepsilon_k}\frac{\int_\Sigma {\rm d}\Sigma\cdot k\, n_{\rm F}\,(1-n_{\rm F})\,\hat t_\lambda \xi_{\rho\sigma}}{\int_\Sigma {\rm d}\Sigma\cdot k\, n_{\rm F}}$$

- Canonical

$$S^\mu_{\rm C}(k) \simeq S^\mu_{\rm B}(k) + \Delta^{\rm C}_\Theta S^\mu(k)$$

$$\Delta^{\rm C}_\Theta S^\mu(k) = \frac{\epsilon^{\lambda\rho\sigma\tau}\hat t_\lambda (k^\mu k_\tau - \eta^\mu_\tau m^2)}{m\varepsilon_k}\frac{\int_\Sigma \Sigma(x)\cdot k\, n_{\rm F}\,(1-n_{\rm F})\,(\varpi_{\rho\sigma}-\mathfrak{S}_{\rho\sigma})}{\int_\Sigma \Sigma\cdot k\, n_{\rm F}}$$

Also obtained in Y. C. Liu, X. G. Huang, Sci.China Phys.Mech.Astron. 65 (2022)

Note that the difference has linear independent terms in momentum and it is **impossible to reconcile** the two descriptions with a redefinition of thermodynamic fields.

MB, PRC 105 (2022) 4, 044907

Motivation and outline

- The spin tensor is a fundamental quantity to study out-of-equilibrium spin properties.
- Problem: pseudo-gauge symmetry allows many formulation of spin hydrodynamics with different physical predictions.

- **Understanding the problem**

  - Difference between a **Fundamental** and **Effective** spin tensor

  - Similar problem: Magnetization

- Solution: a specific form of the spin tensor emerges from the microscopic description of spin-spin interactions .

EM Magnetization

Also the electric current has a pseudo-gauge symmetry

$$\widehat{j}'^{\mu} = \widehat{j}^{\mu} + \underbrace{\nabla_\lambda \widehat{\mathcal{M}}^{\lambda\mu}}_{\text{bound current}} \ , \quad \text{Magnetization: } \widehat{\mathcal{M}}^{\lambda\mu} = -\widehat{\mathcal{M}}^{\mu\lambda}$$

Electric current definitions

- **Fundamental**: minimally coupled to gauge field
- **Effective**: one splits the bound and free currents to describe magnetized systems

EM Magnetization

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Electric current definitions

- **Fundamental**: minimally coupled to gauge field
- **Effective**: one splits the bound and free currents to describe magnetized systems

The magnetization tensor can also be induced by interactions.
Textbook example: Long range Ising model

$$\widehat{H} = -\frac{J}{2N}\sum_{i,j} S_i\, S_j \longrightarrow \widehat{\rho} = \sqrt{\frac{N\beta J}{2\pi}} \int_{-\infty}^{\infty} \mathrm{d}\mu \exp\left[-\frac{N\beta J}{2}\mu^2 + \beta J \mu \widehat{M}\right]$$

After an Hubbard-Stratonovich transformation

The following magnetization emerges:

$$\widehat{M} = \sum_i S_i$$

Fundamental definition of the spin tensor

In conventional gravitational physics the spin d.o.f. are not included: the spin connection is enslaved to the metric and the energy momentum tensor is symmetric.

In the Einstein-Cartan theory space-times with torsion are considered instead. This allows to define a spin tensor from the action varying the spin connection:

E. Speranza, N. Weickgenannt, Eur. Phys. J. A (2021)
M. Hongo, X.-G. Huang, M. Kaminski, M. Stephanov, H.-U. Yee, JHEP 11 (2021)
A. D. Gallegos, U. Gürsoy, and A. Yarom, SciPost Phys. 11, 041 (2021)
S. Floerchinger and E. Grossi, Phys. Rev. D 105, 085015 (2022)

$$\delta S = \int d^4x |e| \left(T^{\mu}{}_{a} \delta e^{a}{}_{\mu} + \frac{1}{2} S^{\lambda}{}_{ab} \delta \omega_{\lambda}{}^{ab} \right)$$

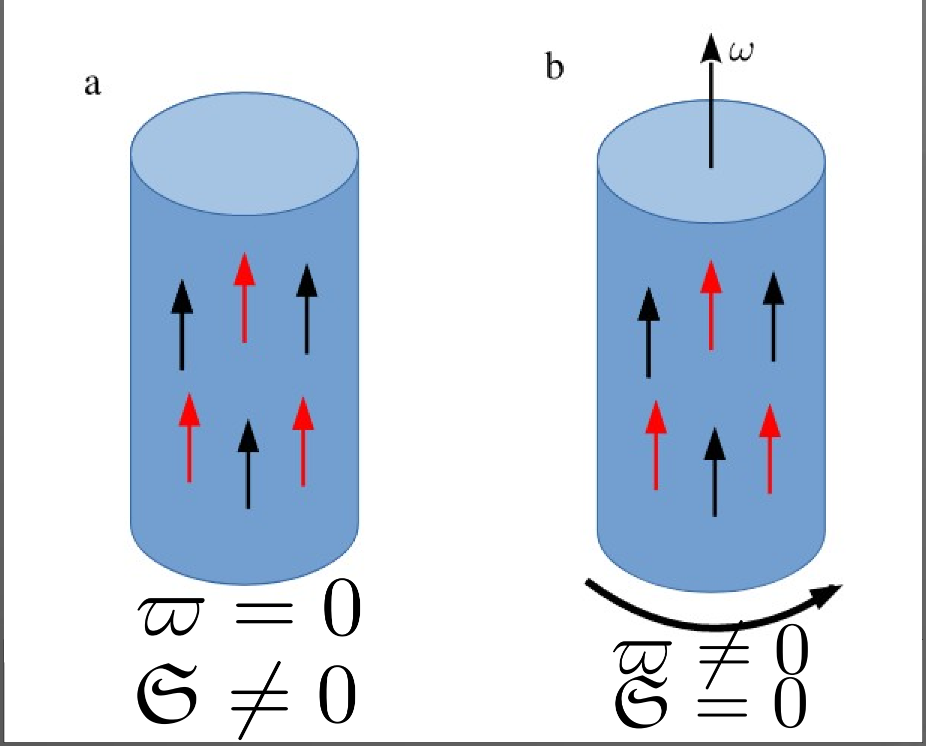
The canonical spin tensor is obtained in a minimal coupling with gravity.

Effective definition of the spin tensor

The spin potential emerges when imposing the constraint on the angular momentum of the system

$$n_\lambda \mathrm{tr}\left[\widehat{\rho}\,\widehat{\mathcal{S}}^{\lambda,\mu\nu}\right] = n_\lambda \mathcal{S}^{\lambda,\mu\nu}$$

Effective definition: choose the spin tensor that fits the phenomenological description of the system.



a) Spin polarization with no rotation is impossible to describe without a spin tensor (Belinfante)

b) Spin polarization and rotation possible without spin tensor

F. Becattini, W. Florkowski, and E. Speranza, Phys. Lett. B 789, 419 (2019)

Effective definition of the spin tensor

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Effective definition: choose the spin tensor that fits the phenomenological description of the system.

Phenomenological Landau theory

The Landau function is a free energy obtained constraining the magnetization.

$$\mathrm{e}^{-\beta L\{M_\Lambda(\boldsymbol{x})\}} = \mathrm{tr}\left\{\mathrm{e}^{-\beta H\{S_i\}}\delta\left[\sum_{i\in\boldsymbol{x}} S_i - M_\Lambda(\boldsymbol{x})\right]\right\}$$

Also spin hydrodynamics free energy is obtained constraining the spin tensor.

What spin tensor should we use?

Interactions can make the choice.

MB, A. Palermo, 2407.14345

$$Z\big|_{\mathrm{NJL}+\varpi} = Z\big|_{\text{Spin hydro with canonical spin}}$$

The partition function of the finite temperature NJL model

$$\mathcal{L}_{\mathrm{NJL}} = \bar{q}\left(\frac{i}{2}\overleftrightarrow{\not{D}} - \hat{m}\right)q + G_A\left(\bar{q}\gamma_\mu\gamma^5 q\right)\left(\bar{q}\gamma^\mu\gamma^5 q\right) + \cdots$$

with a mean field pseudo vector current f^σ generated by rotation and/or magnetic field

$\equiv$

The partition function of non-dissipative spin hydrodynamics of free Dirac field with a **canonical** spin tensor

$$f^\sigma = \frac{1}{2}\epsilon^{\sigma\mu\nu\rho}(\varpi_{\mu\nu} - \mathfrak{S}_{\mu\nu})n_\rho$$

$\varpi_{\mu\nu}$: Thermal vorticity

$\mathfrak{S}_{\mu\nu}$: Canonical spin potential

Partition function in Canonical ideal spin hydrodynamics

$$\widehat{\rho} = \frac{1}{\mathcal{Z}} \exp\left\{ -\int_\Sigma \mathrm{d}\Sigma\, n_\lambda \left[\widehat{T}_B^{\lambda\nu}\beta_\nu + \frac{1}{2}(\varpi - \mathfrak{S})_{\rho\sigma}\widehat{\mathcal{S}}_{\mathrm{C}}^{\lambda,\rho\sigma} \right] \right\}$$

The partition function of local thermal equilibrium is obtained as a path integral in a auxiliary curved space-time constructed with the thermodynamic fields

T. Hayata, Y. Hidaka, T. Noumi and M. Hongo, PRD 92 (2015)
M. Hongo, Annals Phys. 383 (2017)

$\tilde{x}$: new thermal coordinates
Using imaginary time (Euclidean):

$$\mathcal{Z} = \int \mathcal{D}\psi\mathcal{D}\bar{\psi}\, \mathrm{e}^{-S_E - S_{\mathfrak{S}}}\,, \quad S_E = \int_0^{|\beta|} \mathrm{d}\tau \mathrm{d}^3\tilde{\boldsymbol{x}}\sqrt{\tilde{g}_E}\left[\bar{\psi}\left(\frac{\overrightarrow{\not{D}} - \overleftarrow{\not{D}}}{2} + m \right)\psi + V(x) \right]$$

$$\overrightarrow{D}_{\tilde{0}} = \partial_{\tilde{0}} + \frac{1}{2}\varpi_{\tilde{0}}{}^{ab}\Sigma_{ab}, \quad \overleftarrow{D}_{\tilde{0}} = \partial_{\tilde{0}} - \frac{1}{2}\varpi_{\tilde{0}}{}^{ab}\Sigma_{ab}, \quad \overrightarrow{D}_{\tilde{i}} = \overleftarrow{D}_{\tilde{i}} = \partial_{\tilde{i}}$$

The spin connection $\varpi_{\tilde{\lambda}}{}^{ab}$ is linked to the thermal vorticity: $\varpi_{\tilde{0}\tilde{i}\tilde{j}} = -\frac{1}{2|\beta|}\left(\partial_{\tilde{i}}\beta_{\tilde{j}} - \partial_{\tilde{j}}\beta_{\tilde{i}} \right)$

$$S_{\mathfrak{S}} = \frac{1}{2}\int_0^{|\beta|} \mathrm{d}\tau \mathrm{d}^3\tilde{\boldsymbol{x}}\sqrt{\tilde{g}_E}\; n_{\tilde{\lambda}}(\varpi_{\tilde{\rho}\tilde{\sigma}} - \mathfrak{S}_{\tilde{\rho}\tilde{\sigma}})\widehat{\mathcal{S}}_{\mathrm{C}}^{\tilde{\lambda},\tilde{\rho}\tilde{\sigma}}$$

Spin potential action

Geometrical interpretation

$$S_{\mathfrak{S}} = \frac{1}{2}\int_0^{|\beta|} \mathrm{d}\tau \mathrm{d}^3\tilde{\boldsymbol{x}}\,\sqrt{\tilde{g}_E}\,\bar{\psi}\left[e_a^{\tilde{\mu}}\gamma^a\,\overrightarrow{\mathfrak{s}}_{\tilde{\mu}} - \overleftarrow{\mathfrak{s}}_{\tilde{\mu}} e_a^{\tilde{\mu}}\gamma^a\right]\psi$$

$$\overrightarrow{\mathfrak{s}}_{\tilde{\mu}} = \frac{1}{2}\omega_{\tilde{\mu}ab}\Sigma^{ab}$$

$$\omega_{\tilde{\lambda}}{}^{bc} = \frac{1}{2}e^b_{\tilde{\mu}}e^c_{\tilde{\nu}}n_{\tilde{\lambda}}(\varpi^{\tilde{\mu}\tilde{\nu}} - \mathfrak{S}^{\tilde{\mu}\tilde{\nu}})$$

The spin potential can be included as a contribution to spin connection that is independent of the metric, i.e. like a space-time with torsion

$$D_{\tilde{\mu}} \to D_{\tilde{\mu}} + \mathfrak{s}_{\tilde{\mu}} \qquad \omega' = \varpi + (\varpi - \mathfrak{S}) = 2\varpi - \mathfrak{S}$$

A fundamental (proper) definition of the canonical spin tensor is the minimal coupling with torsion!

At global equilibrium the spin potential is equal to the thermal vorticity, thus no extra terms at global equilibrium.

Spin potential action

Geometrical interpretation

$$S_{\mathfrak{S}} = \frac{1}{2}\int_0^{|\beta|} \mathrm{d}\tau \mathrm{d}^3\tilde{\boldsymbol{x}}\, \sqrt{\tilde{g}_E}\, \bar{\psi}\left[e^{\tilde{\mu}}_a \gamma^a \overrightarrow{\mathfrak{s}_{\tilde{\mu}}} - \overleftarrow{\mathfrak{s}_{\tilde{\mu}}} e^{\tilde{\mu}}_a \gamma^a\right]\psi$$

$$\overrightarrow{\mathfrak{s}_{\tilde{\mu}}} = \frac{1}{2}\omega_{\tilde{\mu}ab}\Sigma^{ab}$$

$$\omega_{\tilde{\lambda}}{}^{bc} = \frac{1}{2} e^b_{\tilde{\mu}} e^c_{\tilde{\nu}} n_{\tilde{\lambda}} (\varpi^{\tilde{\mu}\tilde{\nu}} - \mathfrak{S}^{\tilde{\mu}\tilde{\nu}})$$

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A fundamental (proper) definition of the canonical spin tensor is the minimal coupling with torsion!

The Canonical spin tensor is dual to the axial current

$$S_{\mathfrak{S}} = -\int_0^{|\beta|} \mathrm{d}\tau \mathrm{d}^3\tilde{\boldsymbol{x}} \sqrt{\tilde{g}_E}\, j^{\tilde{\mu}}_A f_{\tilde{\mu}}$$

where

$$f^{\tilde{\sigma}} \equiv \frac{1}{2}\epsilon^{\tilde{\sigma}\tilde{\mu}\tilde{\nu}\tilde{\rho}}(\varpi_{\tilde{\mu}\tilde{\nu}} - \mathfrak{S}_{\tilde{\mu}\tilde{\nu}}) n_{\tilde{\rho}}$$

Partition function of thermal NJL model

We start from the zero temperature NJL model, including spin-spin interactions

$$\mathcal{L}_{\rm NJL} = \bar{q}\left(\frac{i}{2}\overleftrightarrow{\not\partial} - \hat{m}\right)q + G_A\left(\bar{q}\gamma_\mu\gamma^5 q\right)\left(\bar{q}\gamma^\mu\gamma^5 q\right) + \cdots$$

At **global equilibrium with thermal vorticity** the partition function is written as an Euclidean path integral in the emergent thermal space-time

$$\mathcal{Z}_{\rm NJL} = \int \mathcal{D}q\mathcal{D}\bar{q}\,\mathrm{e}^{-S_{\rm NJL}[q,\,\bar{q};\,e_a^{\tilde{\mu}}]} = \int \mathcal{D}q\mathcal{D}\bar{q}\mathcal{D}f\,\mathrm{e}^{-S_f[q,\,\bar{q},\,f_{\tilde{\mu}};\,e_a^{\tilde{\mu}}]}$$

Hubbard-Stratonovich transformation

$$S_f = \int_0^{|\beta|}\mathrm{d}\tau\int\mathrm{d}^3\tilde{\boldsymbol{x}}\,\sqrt{\tilde{g}_E}\left[\mathcal{L}_0 + \frac{1}{4G_A}f_{\tilde{\mu}}f^{\tilde{\mu}} - j_{\mathrm{A}\tilde{\sigma}}f^{\tilde{\sigma}}\right]$$

The resulting partition function from the pseudo-axial field f is equivalent to the partition function of spin hydrodynamics in the mean field limit identifying

$$f^\sigma = \frac{1}{2}\epsilon^{\sigma\mu\nu\rho}(\varpi_{\mu\nu} - \mathfrak{S}_{\mu\nu})n_\rho$$

Mean pseudo-vector: CSE and AVE

Non-vanishing f^σ = mean axial current $\langle\, \widehat{j}_{\rm A}^{\mu} \,\rangle$
= mean canonical spin tensor $\langle\, \widehat{\mathcal{S}}_{\rm C}^{\lambda,\mu\nu} \,\rangle = \epsilon^{\lambda\mu\nu\rho} \langle\, \widehat{j}_{{\rm A}\,\rho} \,\rangle$

In order to have a non-vanishing mean pseudo vector f^σ we need something that provides:

- Rotational symmetry breaking
- Parity breaking

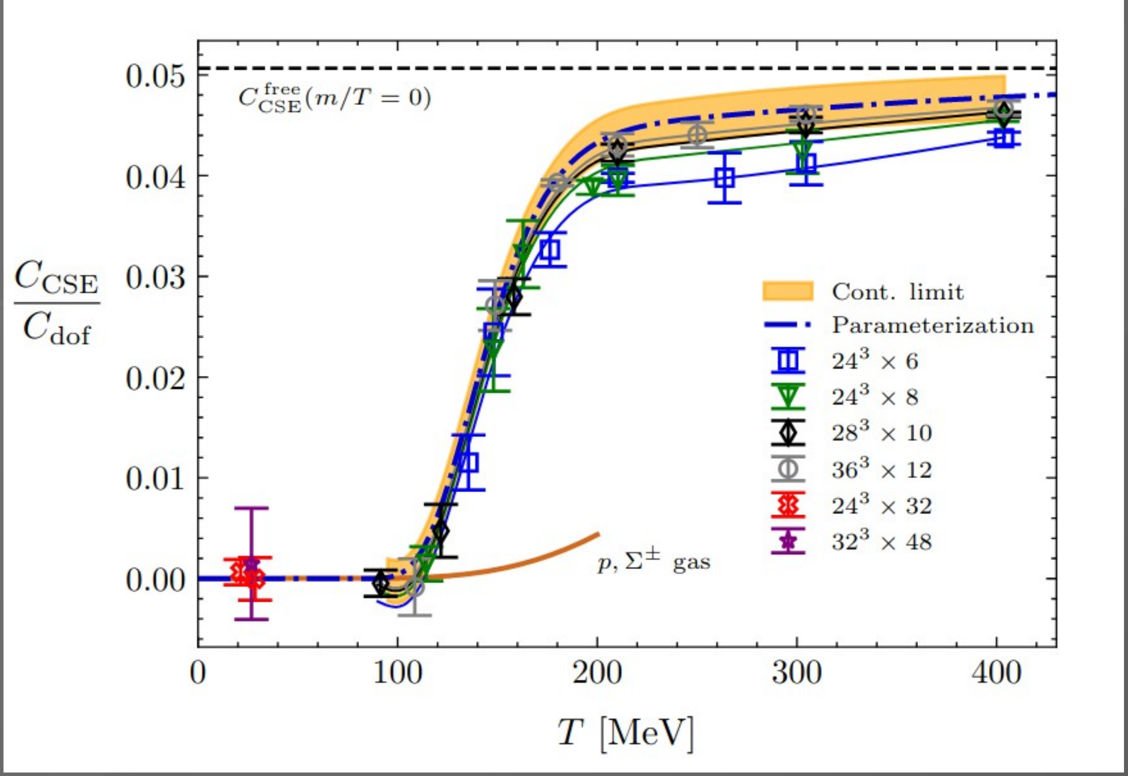
This can be done by the magnetic field $\boldsymbol{B}$ and the rotation $\boldsymbol{\Omega}$ through the Chiral Separation Effect (CSE) and the Axial Vortial Effect (AVE)

For a massless fermion:

$$\langle\, \widehat{\boldsymbol{j}}_{\rm A} \,\rangle = \frac{\mu}{2\pi^2}\boldsymbol{B} + \left(\frac{T^2}{6} + \frac{\mu^2}{2\pi^2}\right)\boldsymbol{\Omega}$$

QGP

In the chiral symmetric phase of hot QCD, lattice calculations at the physical point obtained a CSE conductivity approaches the one calculated for free massless fermions.



B. B. Brandt, G. Endrodi, E. Garnacho-Velasco and G. Marko, JHEP 02 (2024), 142

We expect the same behavior for the AVE (with possible radiative corrections)

Hot QCD + B, Ω → **AVE, CSE** → **Mean canonical spin tensor** → Spin-Spin interactions → **Spin fluid and canonical spin potential**

Summary and outlook

- The spin tensor is a fundamental quantity to study out-of-equilibrium spin properties.
- Problem: pseudo-gauge symmetry allows many formulation of spin hydrodynamics with different physical predictions.
- Solution: a specific form of the spin tensor emerges from the microscopic description of spin-spin interactions .
- A rotating system with NJL interactions is described by the non-dissipative spin hydrodynamic in the canonical pseudo-gauge.

$$Z|_{\text{NJL}+\varpi} = Z|_{\text{Spin hydro with canonical spin}}$$

- Indication that the QGP has out-of-equilibrium macroscopic spin properties described by a canonical spin tensor and spin potential:

  New direction to study spin properties of heavy-ion collisions

Thank you!

EM Magnetization

What about other terms in the NJL model?

$$\mathcal{L}_\sigma = \bar{\psi}\left(\frac{i}{2}\overleftrightarrow{\partial\!\!\!/} - \hat{m}\right)\psi + g_\sigma\left(\bar{\psi}\sigma^{\mu\nu}\psi\right)^2, \quad \sigma^{\mu\nu} = \frac{\mathrm{i}}{2}[\gamma^\mu, \gamma^\nu]$$

The partition function in rotation after the HS transformation:

$$\mathcal{Z}_\sigma = \int \mathcal{D}\psi\mathcal{D}\bar{\psi}\mathcal{D}\mathcal{P}\, \mathrm{e}^{-S_\sigma[\psi, \bar{\psi}, \mathcal{P}_{\tilde{\mu}\tilde{\nu}}; e_a^{\tilde{\mu}}]}$$

$$S_\sigma = \int_0^{|\beta|} \mathrm{d}\tau \int \mathrm{d}^3\tilde{\boldsymbol{x}}\, \sqrt{\tilde{g}_E}\left[\mathcal{L}_0 + \frac{1}{4g_\sigma}\mathcal{P}_{\tilde{\mu}\tilde{\nu}}\mathcal{P}^{\tilde{\mu}\tilde{\nu}} - \bar{\psi}\sigma^{ab}\psi e_a^{\tilde{\mu}} e_b^{\tilde{\nu}}\mathcal{P}^{\tilde{\mu}\tilde{\nu}}\right]$$

This corresponds to local equilibrium with electromagnetic polarization

$$\mathcal{P}_{\mu\nu} = \frac{1}{2}\left(n_\mu \mathcal{P}_\nu - n_\mu \mathcal{P}_\mu\right)$$

$$\widehat{\rho} = \frac{1}{\mathcal{Z}}\exp\left\{-\int_\Sigma \mathrm{d}\Sigma\, n_\lambda\left[\widehat{T}_B^{\lambda\nu}\beta_\nu - \widehat{j}^\lambda\zeta - \widehat{\mathcal{M}}^{\lambda\nu}\mathcal{P}_\nu\right]\right\}, \quad \widehat{\mathcal{M}}^{\lambda\nu} = \bar{\psi}\sigma^{\lambda\nu}\psi$$

Linear independent terms in spin polarization

Consider two different set of thermodynamic fields, one in the Belinfante PG and one in the Canonical: $\{\beta_{\rm B} : \varpi_{\rm B}, \xi_{\rm B}\}\,, \quad \{\beta_{\rm C} : \varpi_{\rm C}, \xi_{\rm C}; \mathfrak{S}_{\rm C}\}$

Respectively, the spin polarizations are

$$S^\mu_{\rm B}[\beta^{\rm B}](k) = -\frac{1}{8m}\epsilon^{\mu\rho\sigma\tau}k_\tau I_{\rho\sigma}(\varpi^{\rm B}) - \frac{1}{4m}\epsilon^{\mu\lambda\sigma\tau}\frac{k_\tau k^\rho}{\varepsilon_k}\hat{t}_\lambda I_{\rho\sigma}(\xi^{\rm B}),$$

$$S^\mu_{\rm C}[\beta^{\rm C}, \Omega^{\rm C}](k) = -\frac{1}{8m}\epsilon^{\mu\rho\sigma\tau}k_\tau I_{\rho\sigma}(\varpi^{\rm C}) - \frac{1}{4m}\epsilon^{\mu\lambda\sigma\tau}\frac{k_\tau k^\rho}{\varepsilon_k}\hat{t}_\lambda I_{\rho\sigma}(\xi^{\rm C})$$
$$+\frac{\epsilon^{\lambda\rho\sigma\tau}\hat{t}_\lambda(k^\mu k_\tau - \eta^\mu_\tau m^2)}{8m\varepsilon_k}I_{\rho\sigma}(\varpi^{\rm C} - \mathfrak{S}^{\rm C}).$$

Where, $I_{\rho\sigma}(\Lambda) = \dfrac{\int_\Sigma {\rm d}\Sigma\cdot k\, n_{\rm F}\,(1-n_{\rm F})\,\Lambda_{\rho\sigma}}{\int_\Sigma {\rm d}\Sigma\cdot k\, n_{\rm F}}$

The only way the two polarizations are equal for every value of momentum is if

$$\varpi^{\rm B} = \varpi^{\rm C}, \quad \xi^{\rm B} = \xi^{\rm C}, \quad \mathfrak{S}^{\rm C} = \varpi^{\rm C}$$

Which correspond to global equilibrium conditions!