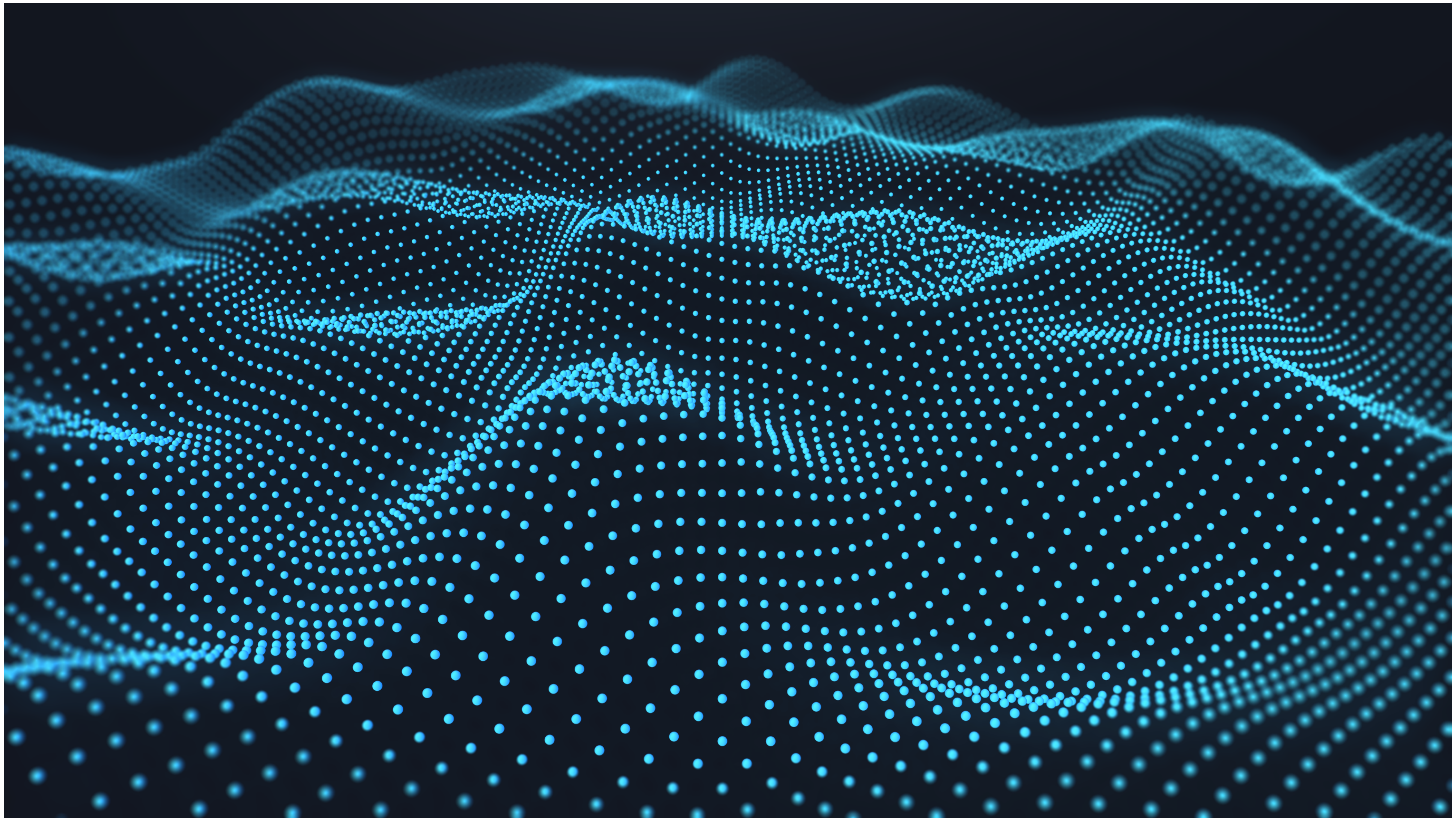
An overview of hydrodynamics for QCD



ECT* Trento, 18.9.2024

Utrecht University

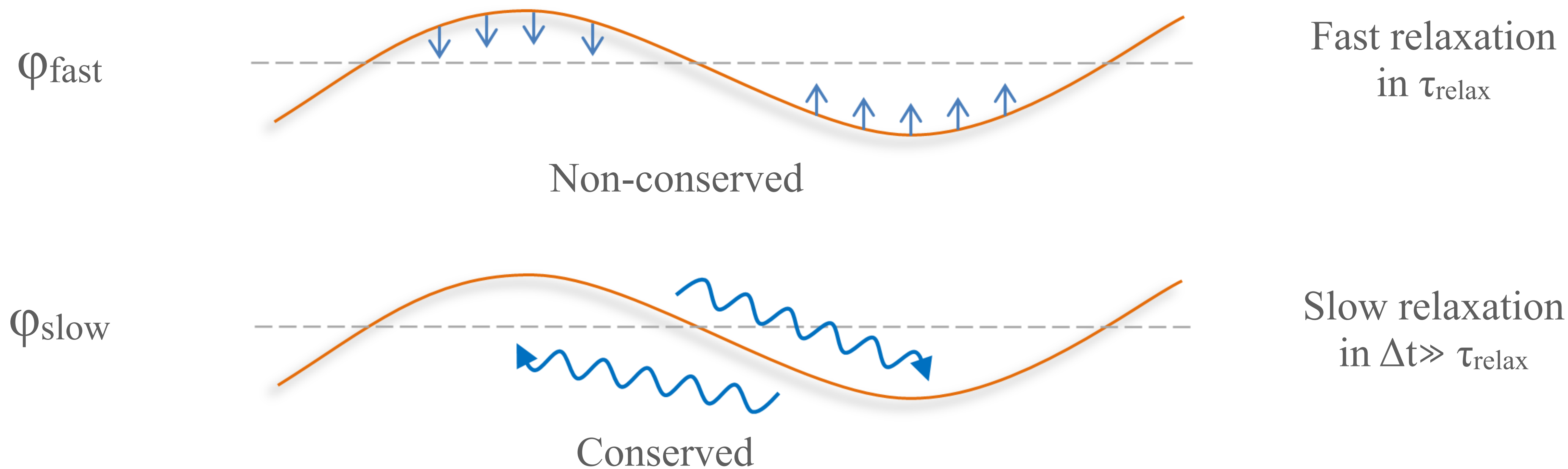
Umut Gürsoy

NWO

Outline

- Hydrodynamics for heavy ion collisions
- Bayesian analysis and characterization of transport
- Two gaps in theory: magnetic fields and spin flow
- Magneto-hydrodynamics and associated transport channels
- Spin-hydrodynamics

Hydrodynamics: theory of slow variables



Effective theory of conserved quantities,
organized in powers of derivatives $\partial_t \varphi_{slow}$, $\nabla \varphi_{slow}$

applicable e.g when $\tau T >> 1$

Relativistic hydrodynamics

Slow variables: energy-momentum current $T_{\mu\nu}$

Hydrodynamic equations $\nabla^{\mu} T_{\mu\nu} = 0$

Dynamical variables and constitutive relations: $u^{\mu} = dx^{\mu}/d\tau\,,\, T$

$$T^{\mu\nu} = E(T) u^{\mu} u^{\nu} + P(T)\left(u^{\mu} u^{\nu} + \eta^{\mu\nu}\right) + \mathcal{O}(\nabla)$$

→ Fluid velocity and temperature $u^{\mu}(x), \quad T(x)$

Relativistic hydrodynamics

- Initial conditions: temperature and velocity densities ?

- Microscopic data: EoS and transport coefficients

- Acausal propagation: numerical instabilities

Relativistic hydrodynamics

- Initial conditions: temperature and velocity densities ?

  Fluctuating Glauber model for incident nucleons;
  parametrisation of wounded nucleons $\Rightarrow$ parton densities

  Moreland et al. `15, `20

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Relativistic hydrodynamics

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  Lattice QCD for small baryon density;
  EFT, holographic models at intermediate density (neutron stars)

- Acausal propagation: numerical instabilities

Relativistic hydrodynamics

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  Lattice QCD for small baryon density;
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- Acausal propagation: numerical instabilities

  Second order hydrodynamics; Müller-Israel-Stewart model

  Denicol et al. `14

14 parameter MIS model

$$T^{\mu\nu} = E u^\mu u^\nu + (P + \Pi)\left(u^\mu u^\nu + \eta^{\mu\nu}\right) + \pi^{\mu\nu}$$

bulk viscous pressure

transverse traceless shear

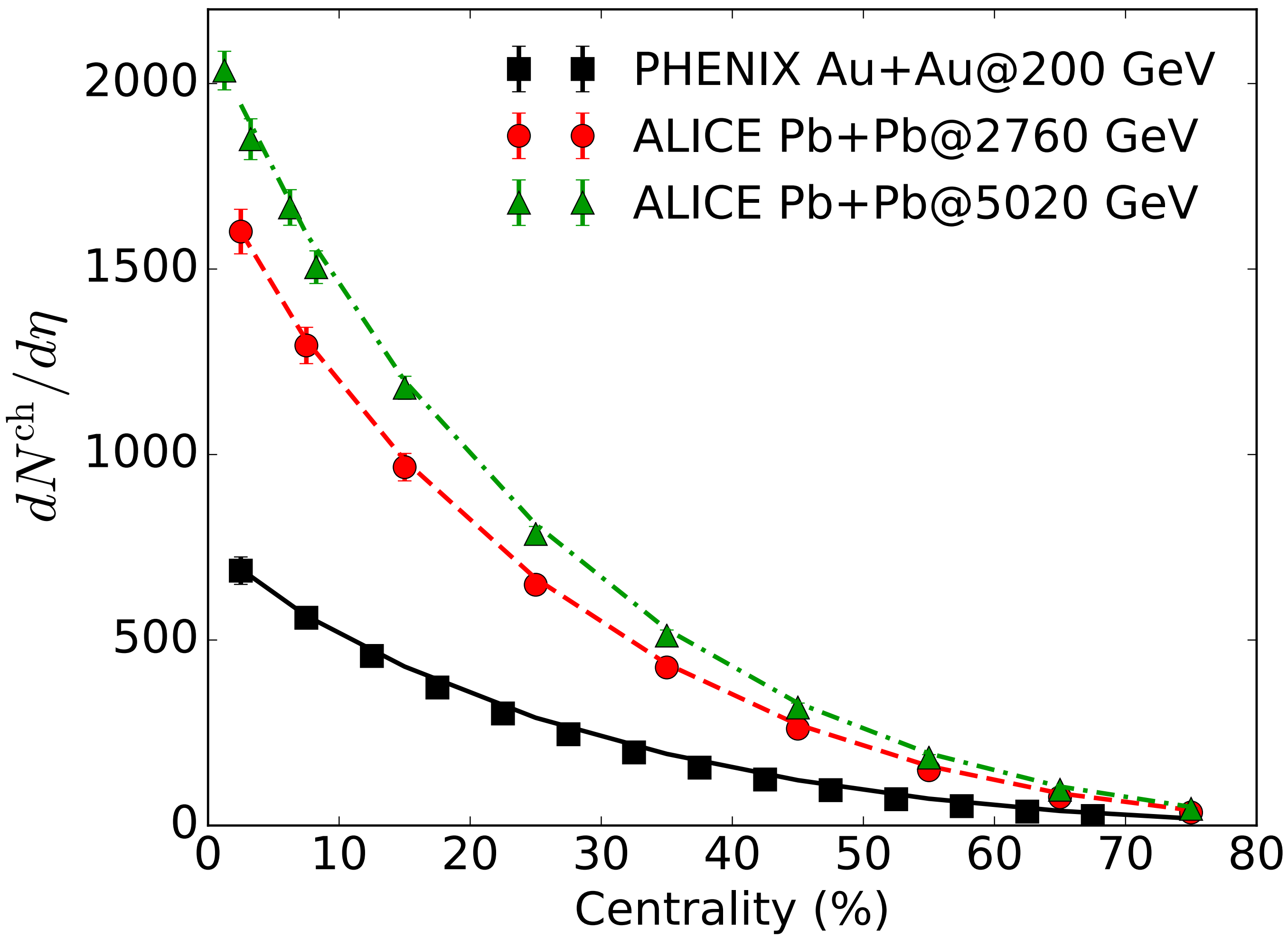
Instabilities avoided by finite relaxation times in dissipative terms

$$(u \cdot \nabla)\,\Pi = -\frac{1}{\tau_\Pi}\left[\Pi + \zeta \nabla \cdot u + \cdots\right],$$

$$\Delta^\mu_\alpha \Delta^\nu_\beta (u \cdot \nabla)\,\pi^{\alpha\beta} = -\frac{1}{\tau_\pi}\left[\pi^{\mu\nu} - 2\eta\sigma^{\mu\nu} + \cdots\right]$$

6 parameters for η(T) and ζ(T), and 8 second order parameters

Data vs hydro



Shen et al, `14

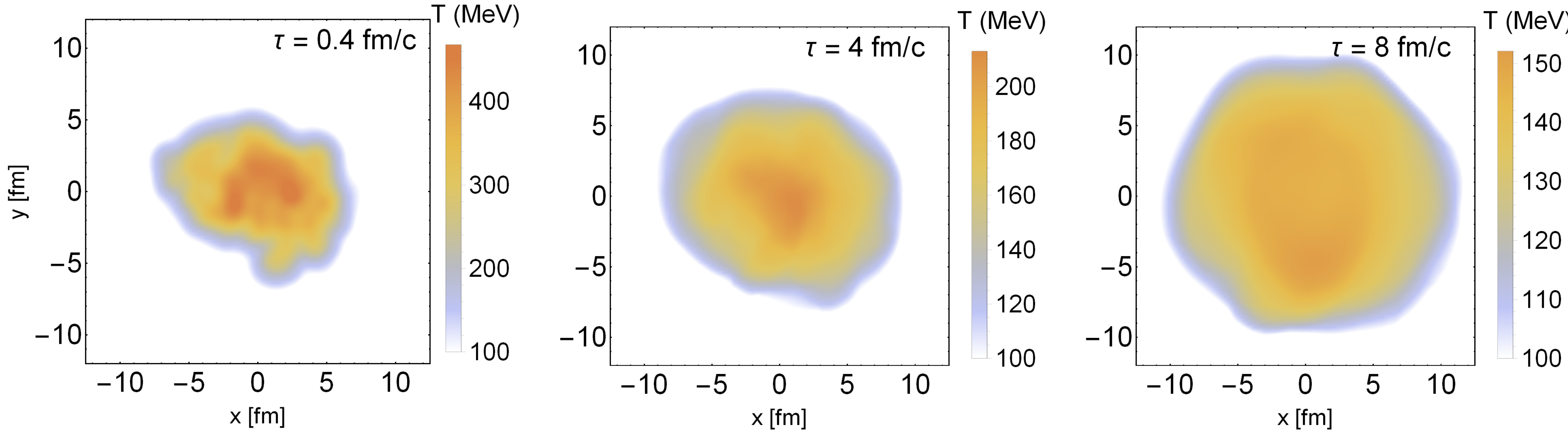
Marcus, Kharzeev, Rajagopal, Shen, UG '18

Boost invariant viscous hydro

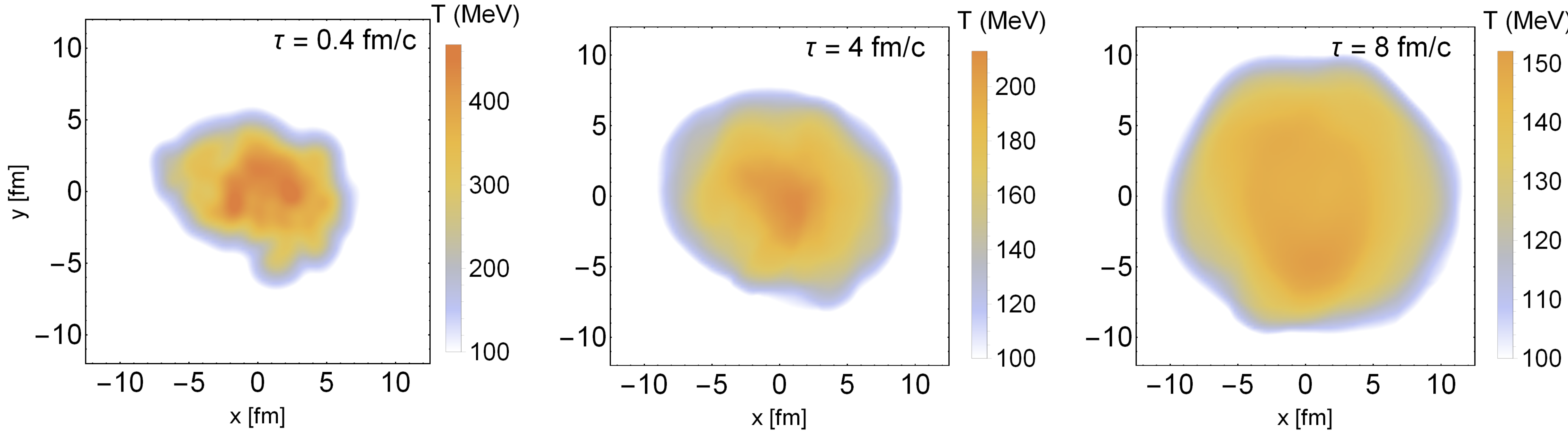
Lattice QCD ⇒ EoS

Data fit/kinetic theory ⇒ η, ζ, τ's

Advanced simulations



“Trajectum” framework: Nijs, van der Schee, Snellings, UG ’20



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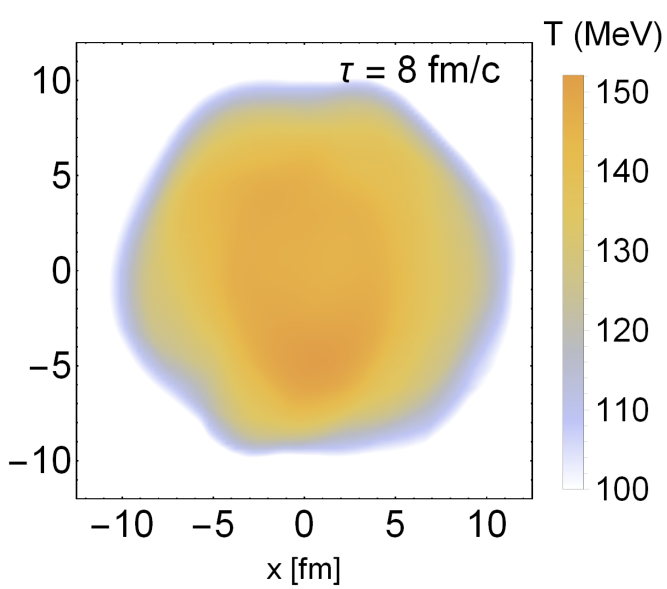
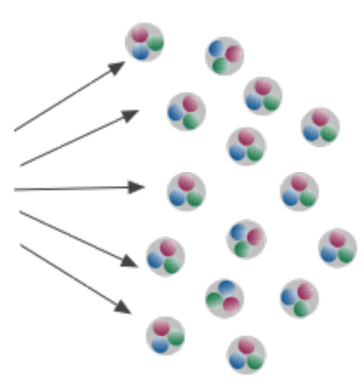
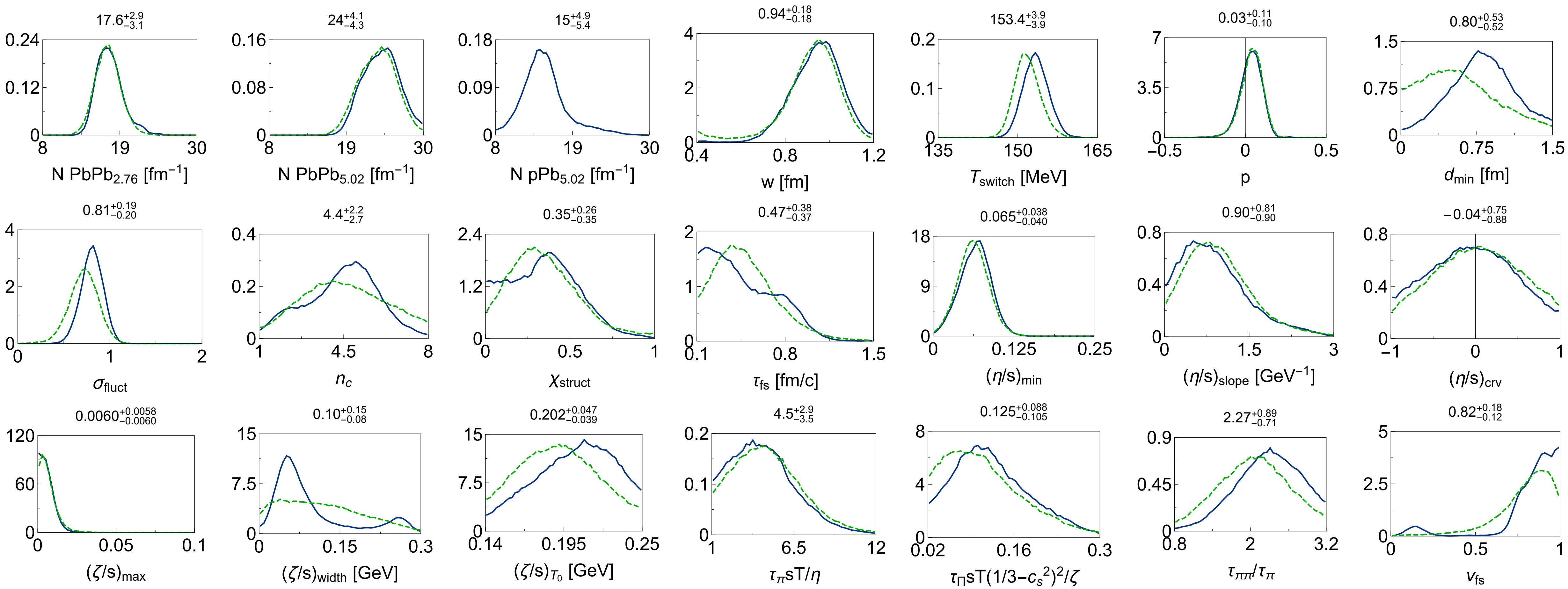


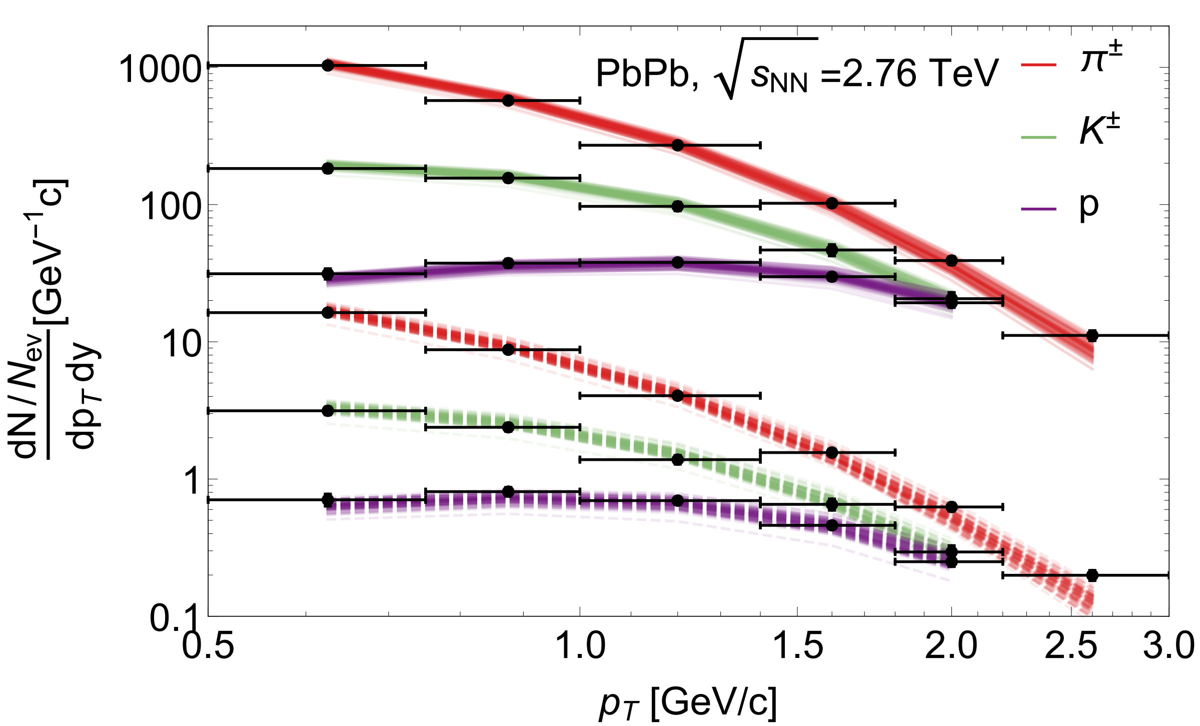


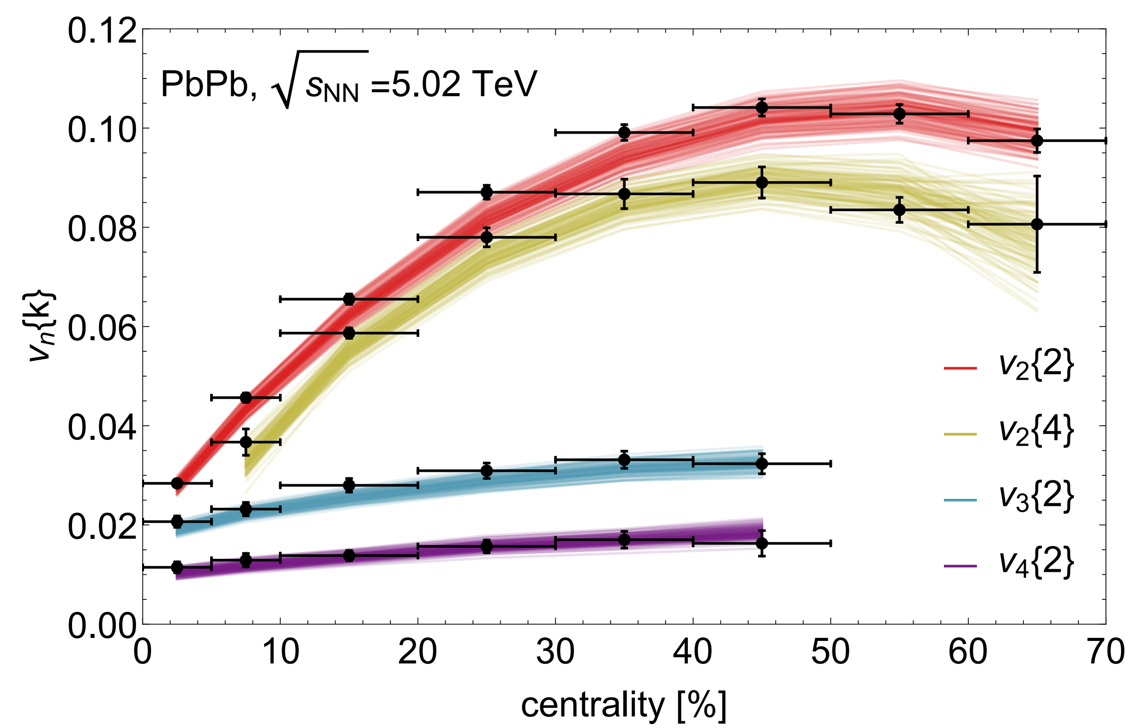
Flow parameters

$$\frac{d^2 N_i}{p_T dY dp_T d\phi_p} = \sum_n v_n(Y, p_T) \cos(n\phi_p)$$

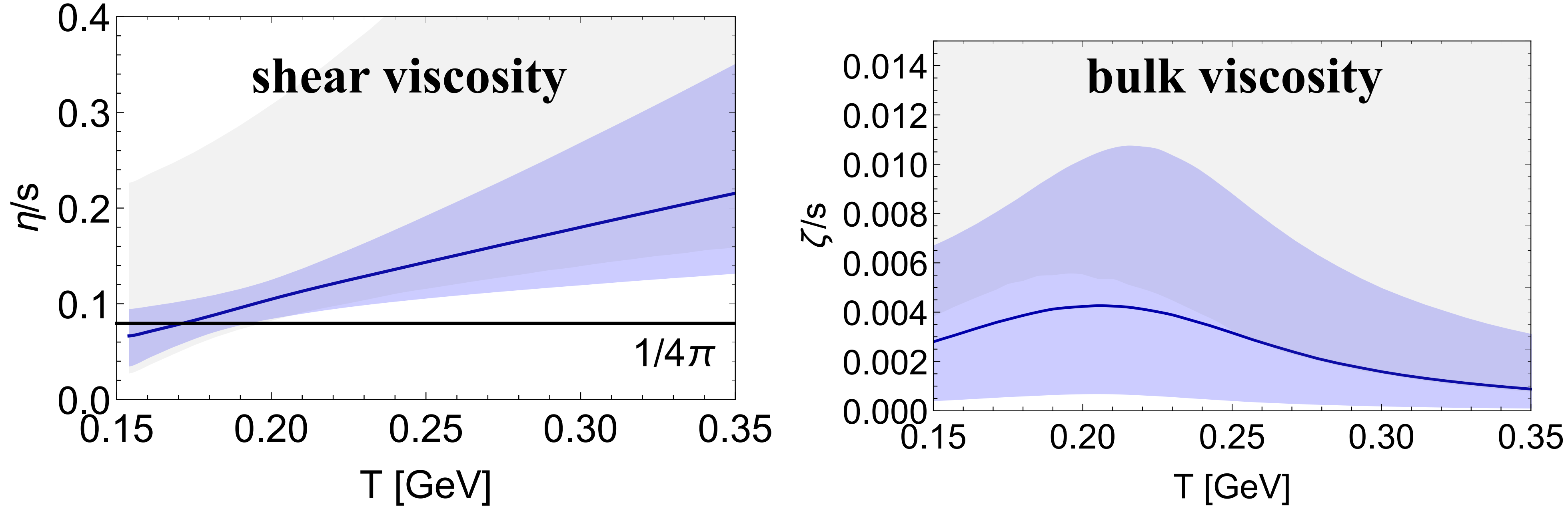
Bayesian analysis





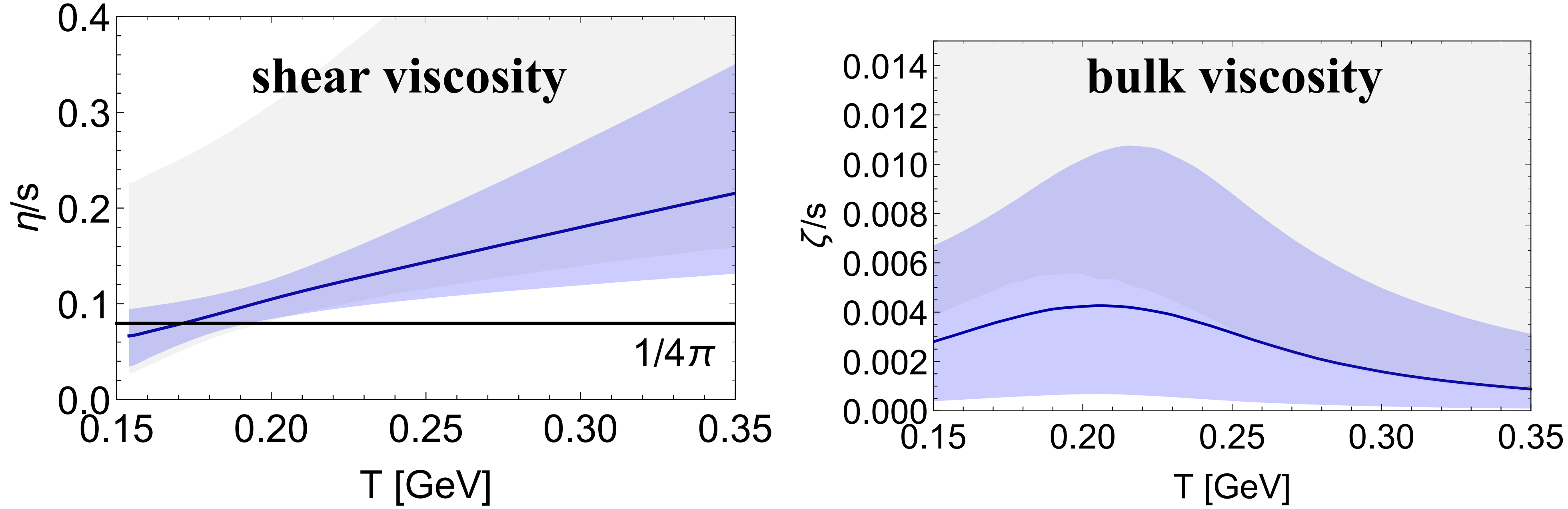


Transport properties



Nijs, van der Schee, Snellings, UG ’20

Transport properties



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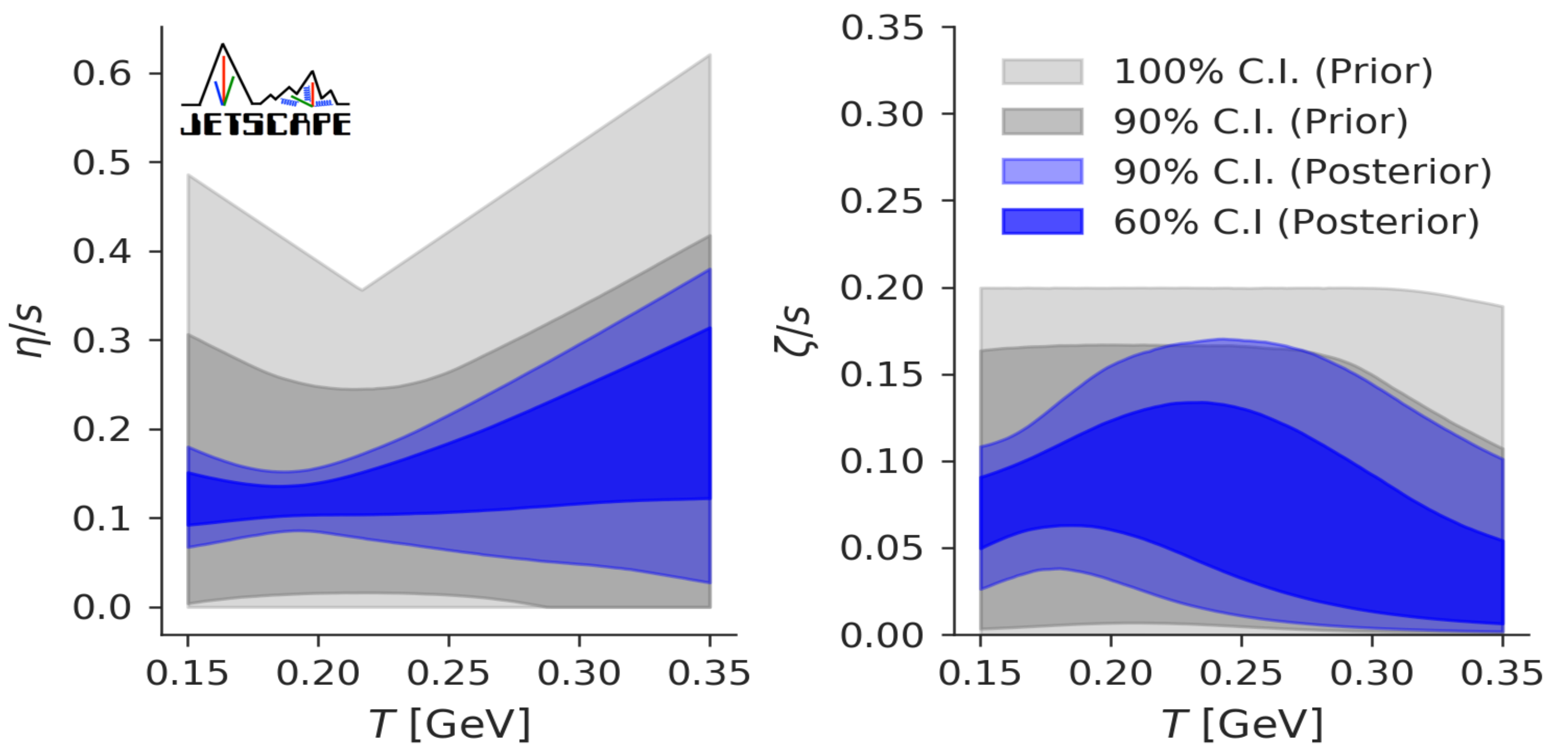


Jetscape collab. '20

How to reproduce T-dependence of η/s in holography?

Buchel, Liu, Starinets `04; Cremonini, Szepietowski, UG `12; Buchel `18, …

How to reproduce T-dependence of η/s in holography?

Buchel, Liu, Starinets `04; Cremonini, Szepietowski, UG `12; Buchel `18, …

Need higher derivative corrections coupled to functions of dilaton!

G(Φ) Riemann^2 suffices to fit Bayesian analysis result

T. Apostilidis, E. Preau, UG, ongoing

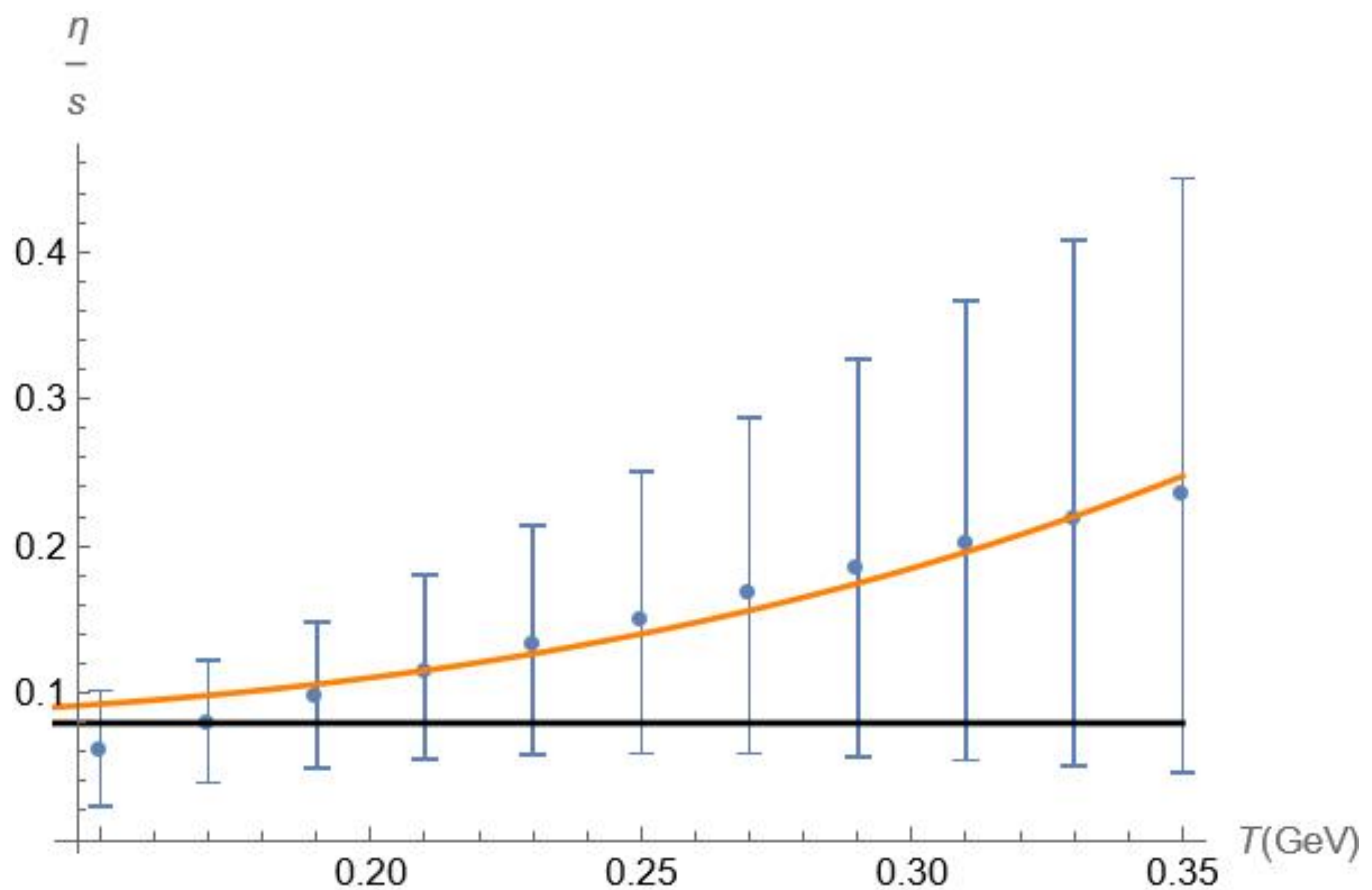
$$S = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g} \left[R - 2(\nabla\Phi)^2 + V(\Phi) + \ell^2 \beta G(\Phi) R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} \right]$$

η/s given by semi-analytic formula

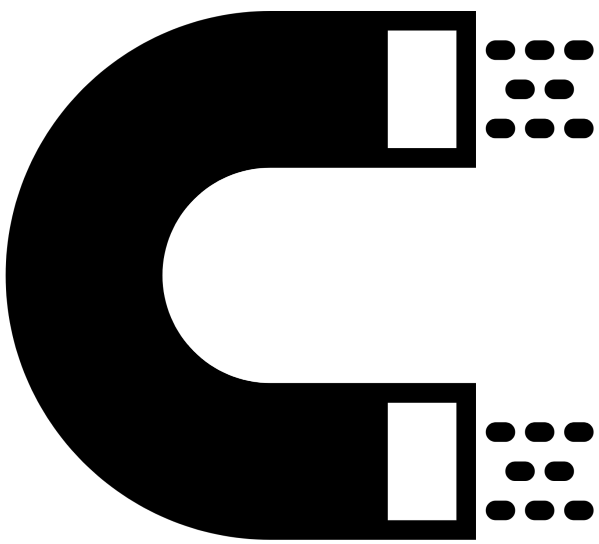
$$\frac{\eta}{s} = \frac{1}{4\pi} \left[1 + \frac{2}{3}\beta\ell^2 \left(-G(\Phi_h)V(\Phi_h) + \frac{3}{4} G'(\Phi_h) V'(\Phi_h) \right) \right]$$

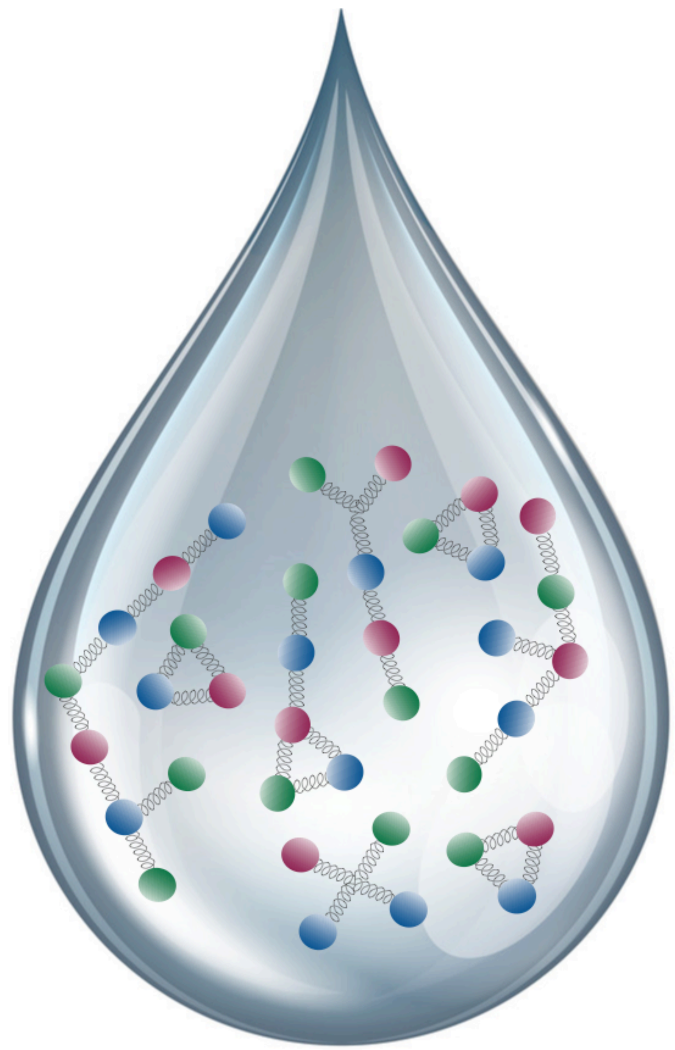
Solve for G(Φ) to fit data..

T. Apostilidis, E. Preau, UG, ongoing



Hydrodynamics

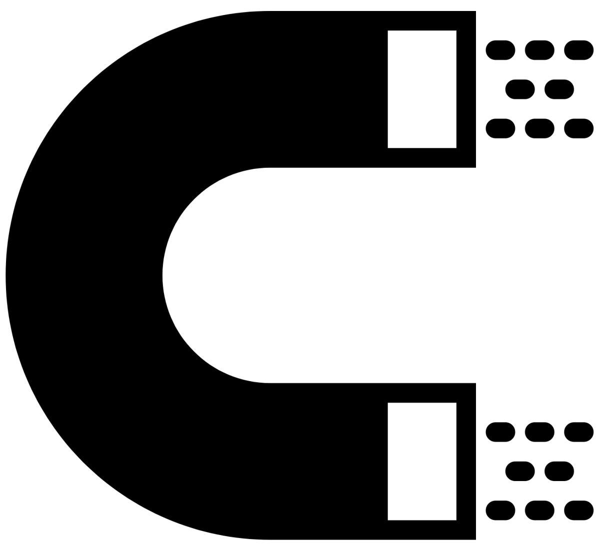




Magnetic field

$B \sim 10^{14}\, B_{MRI}$

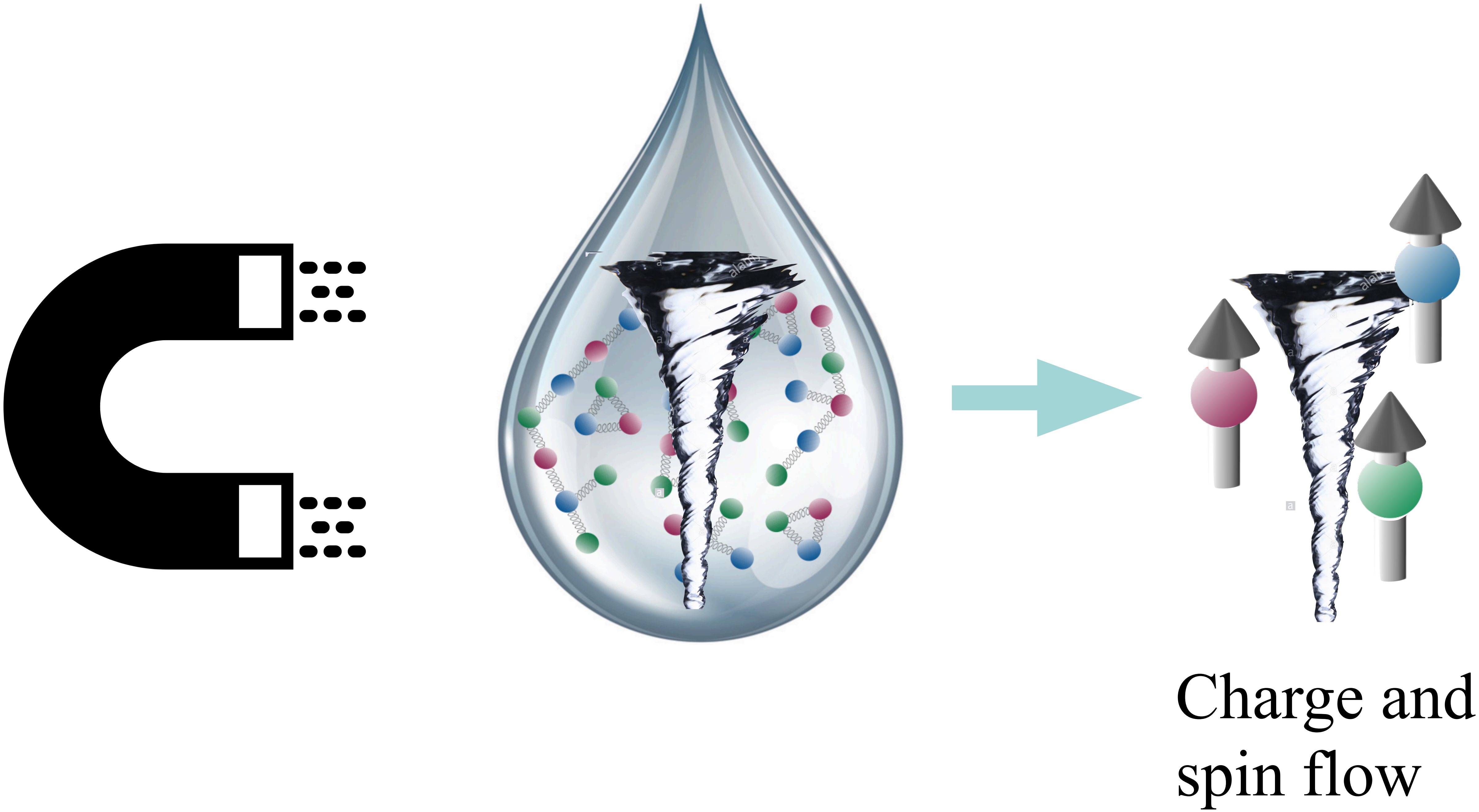
Hydrodynamics



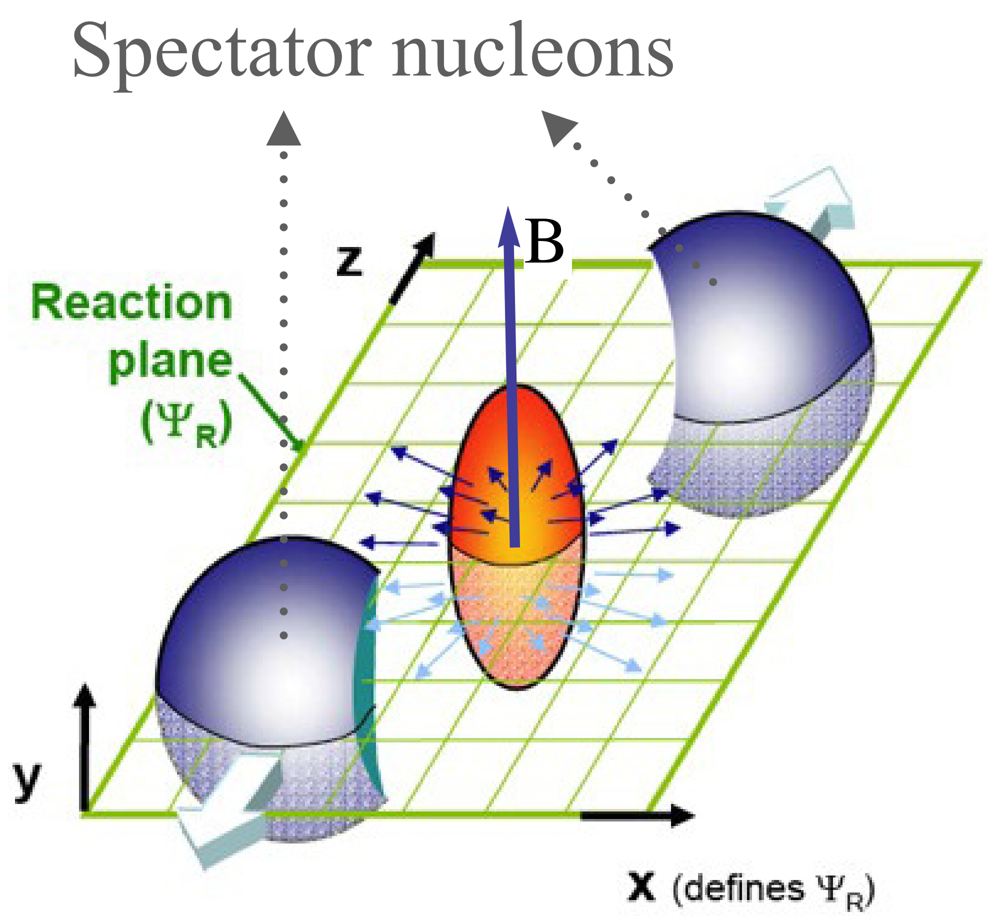
Strong vortical structure

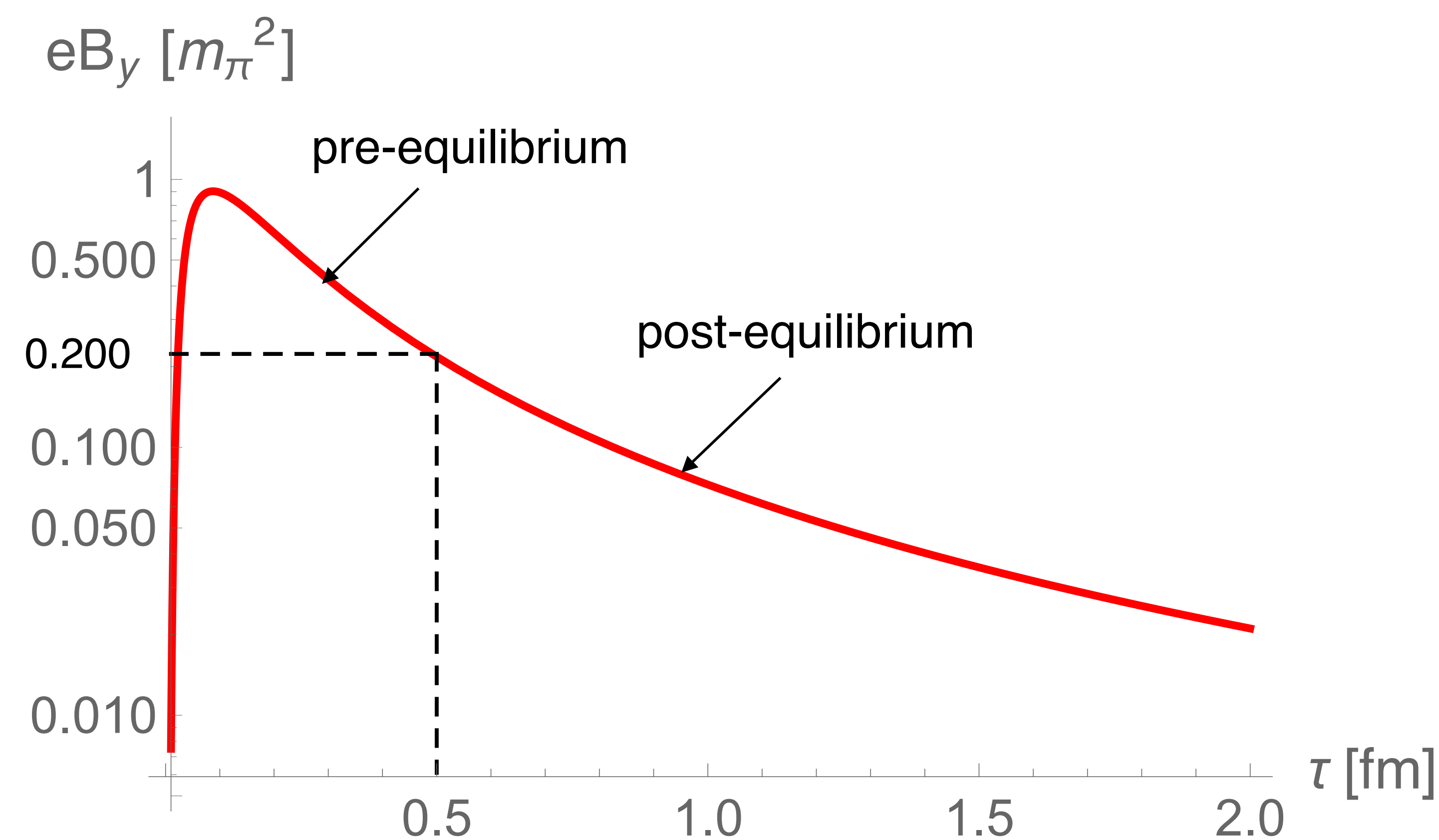
$\omega \sim 10^{22}\ s^{-1}$

Hydrodynamics



Magnetic field in heavy ion collisions





Skokov et al `09; Tuchin `10 `13;
Voronyuk et al `11
Mclerran, Skokov `13
Kharzeev, Rajagopal, UG `14
Inghrami, Becattini, Beraudo, del Zanna`12

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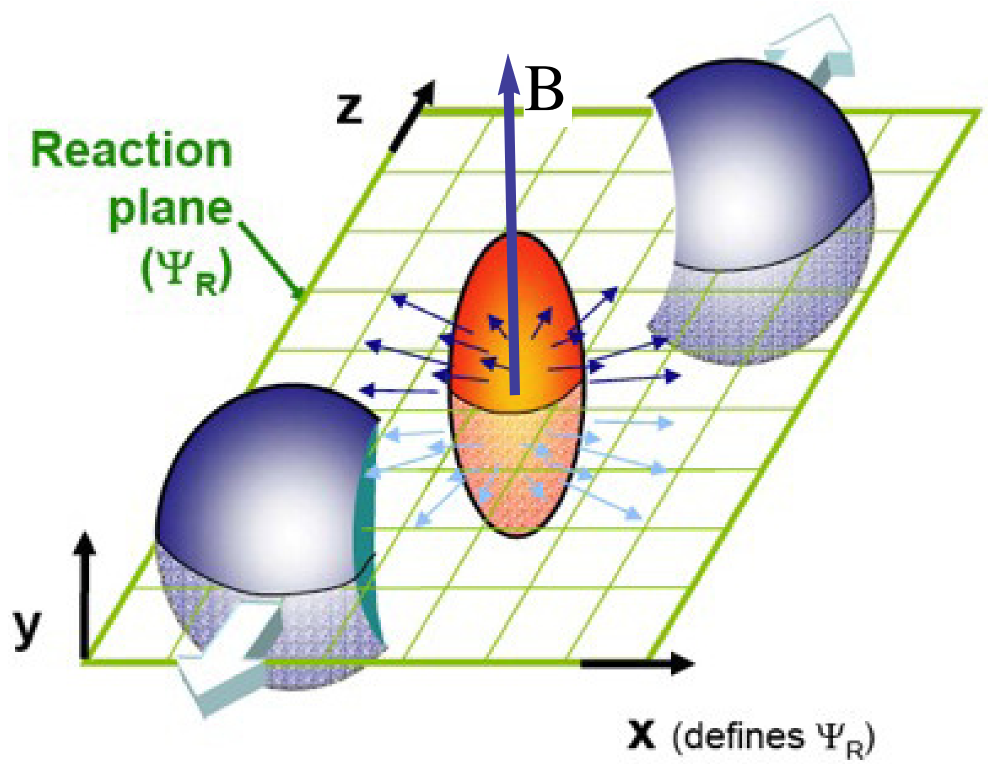

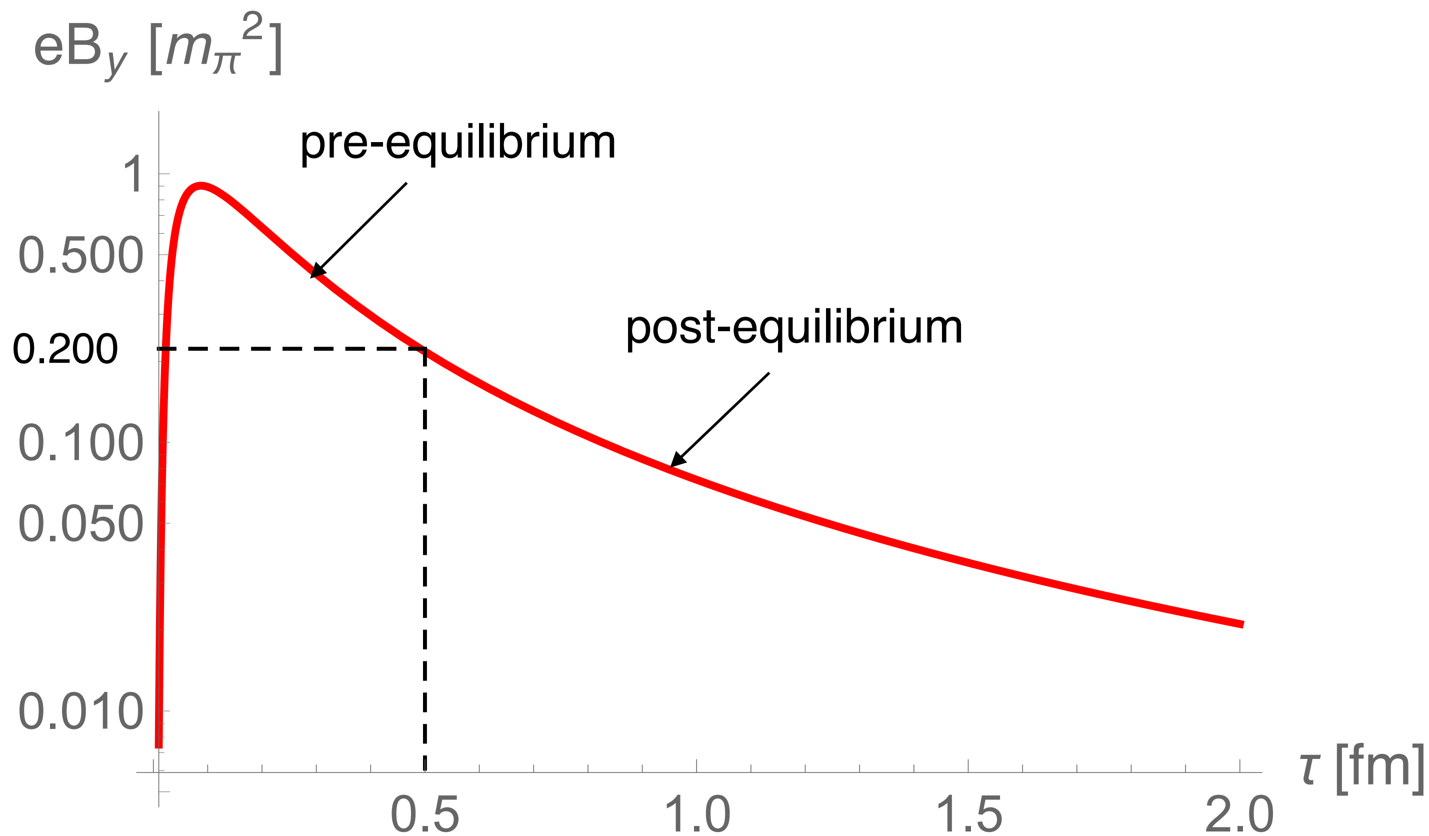

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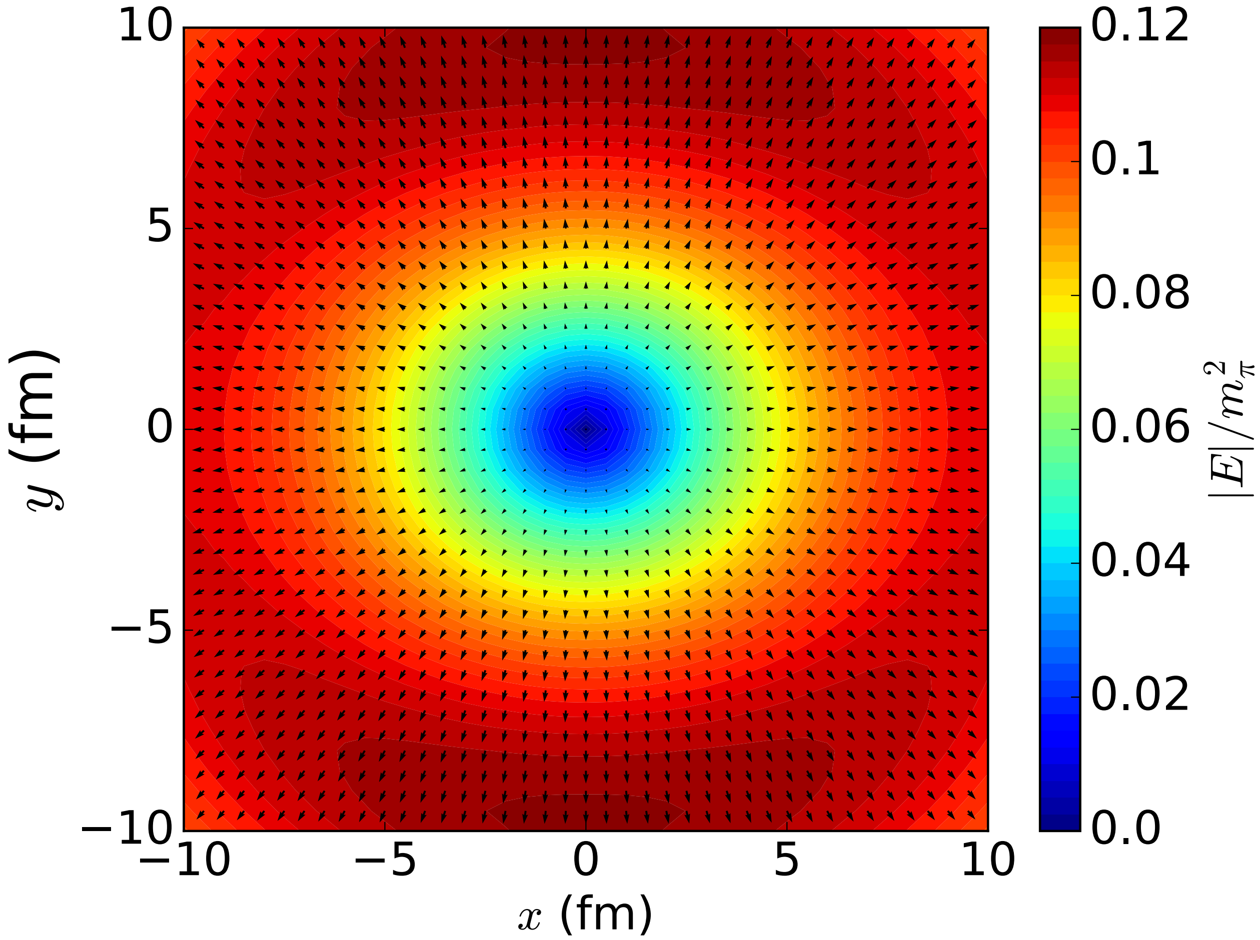
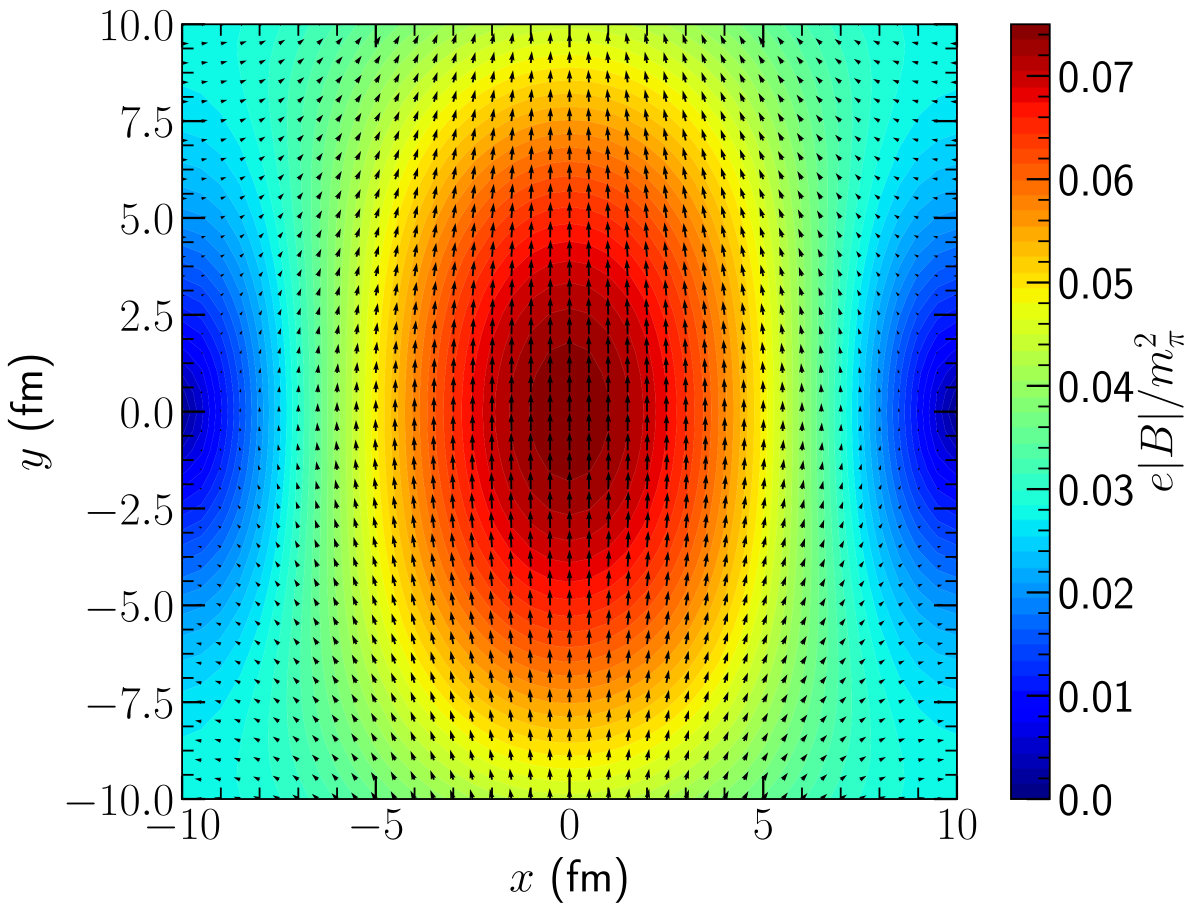
Cannot ignore *electric field*

$E \simeq E_0 \exp(-\sigma \tau) \sim E_0$

with $\sigma = 0.023$ fm^{-1} and $\tau \sim 10$ fm

Electromagnetic fields in QGP





z=0, τ=1 fm/c, Pb-Pb collision at 20-30% centrality, 2.76TeV

Marcus, Kharzeev, Rajagopal, Shen, UG '18

Magneto-hydrodynamics

Slow variables: energy-momentum, electromagnetic field

$$T_{\mu\nu} \qquad B^\mu \qquad E^\mu$$

- Magnetohydrodynamic equations

$$\nabla_\mu T^{\mu\nu} = F^{\rho\nu} J_{ext\,\rho}$$ **4 equations**

$$\nabla_\mu \left(F^{\mu\nu} - M^{\mu\nu} \right) = \rho u^\nu + J^\mu_{ext}$$ **4 equations**

$$\epsilon^{\mu\nu\alpha\beta} \nabla_\nu F_{\alpha\beta} = 0$$ **3 equations**

- Dynamical variables:

$u^\mu(x)$	$T(x)$	$\mu(x)$	$E^\mu(x)$	$B^\mu(x)$
3	**1**	**1**	**3**	**3**

with charge and electromagnetic polarization $\rho = \frac{\delta \mathcal{L}_m}{\delta \mu} \quad M^{\mu\nu} = \frac{\delta \mathcal{L}_m}{\delta F_{\mu\nu}} \quad \mu = u \cdot A$

Kovtun, Hernandez `17; Grozdanov, Hofman, Iqbal `17

Perturbative magneto-hydrodynamics

- Solve hydro without E, B $\Rightarrow$ fluid velocity $\vec{u}$

- Demand "no-force" in the rest frame

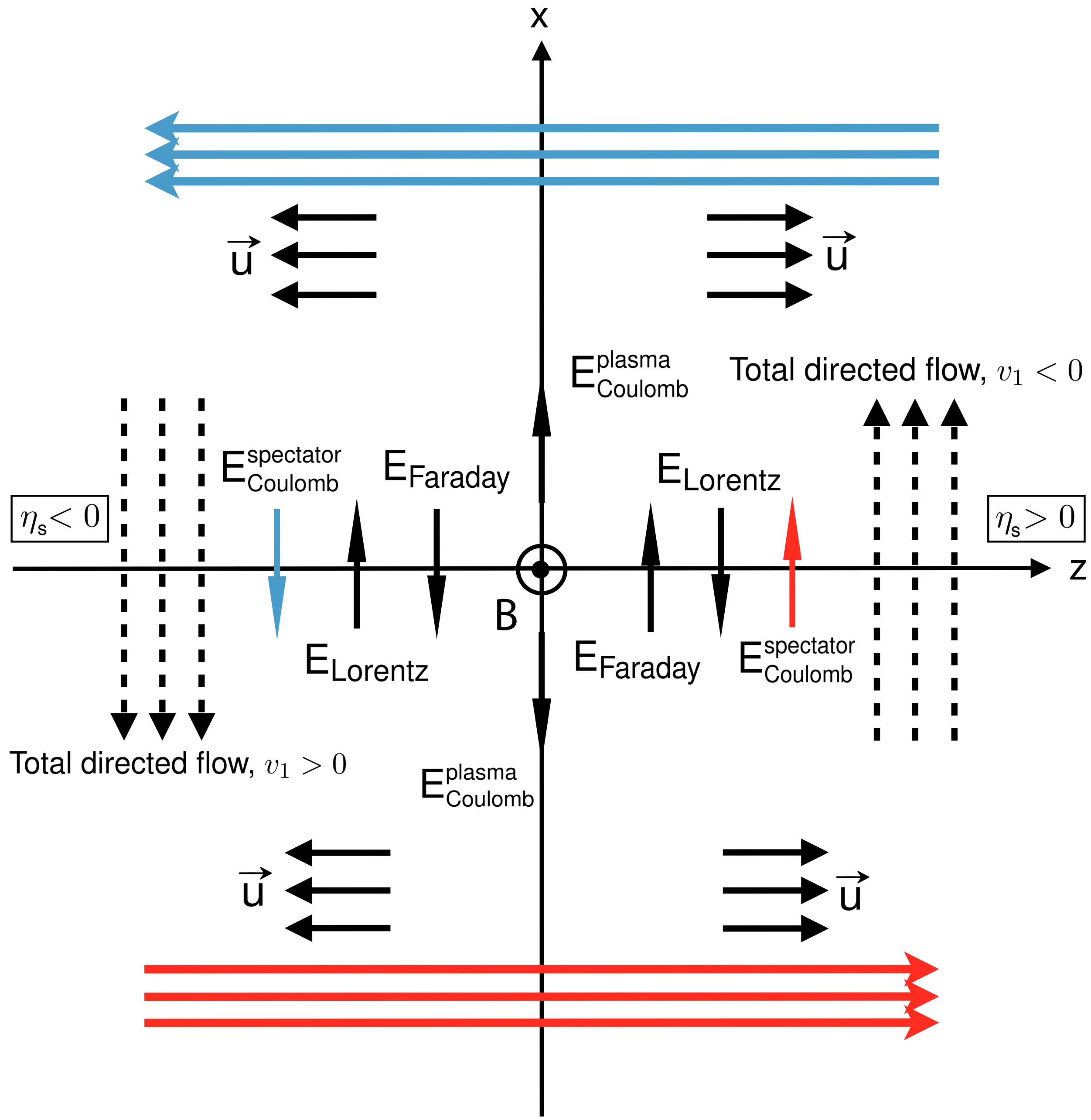
$$m\frac{d\vec{v}}{dt} = q\vec{v} \times \vec{B}' + q\vec{E}' - \mu m\vec{v} = 0$$

drag force

- Lorentz transform back to CM frame $\Rightarrow$ $\vec{u}[\vec{v}]$

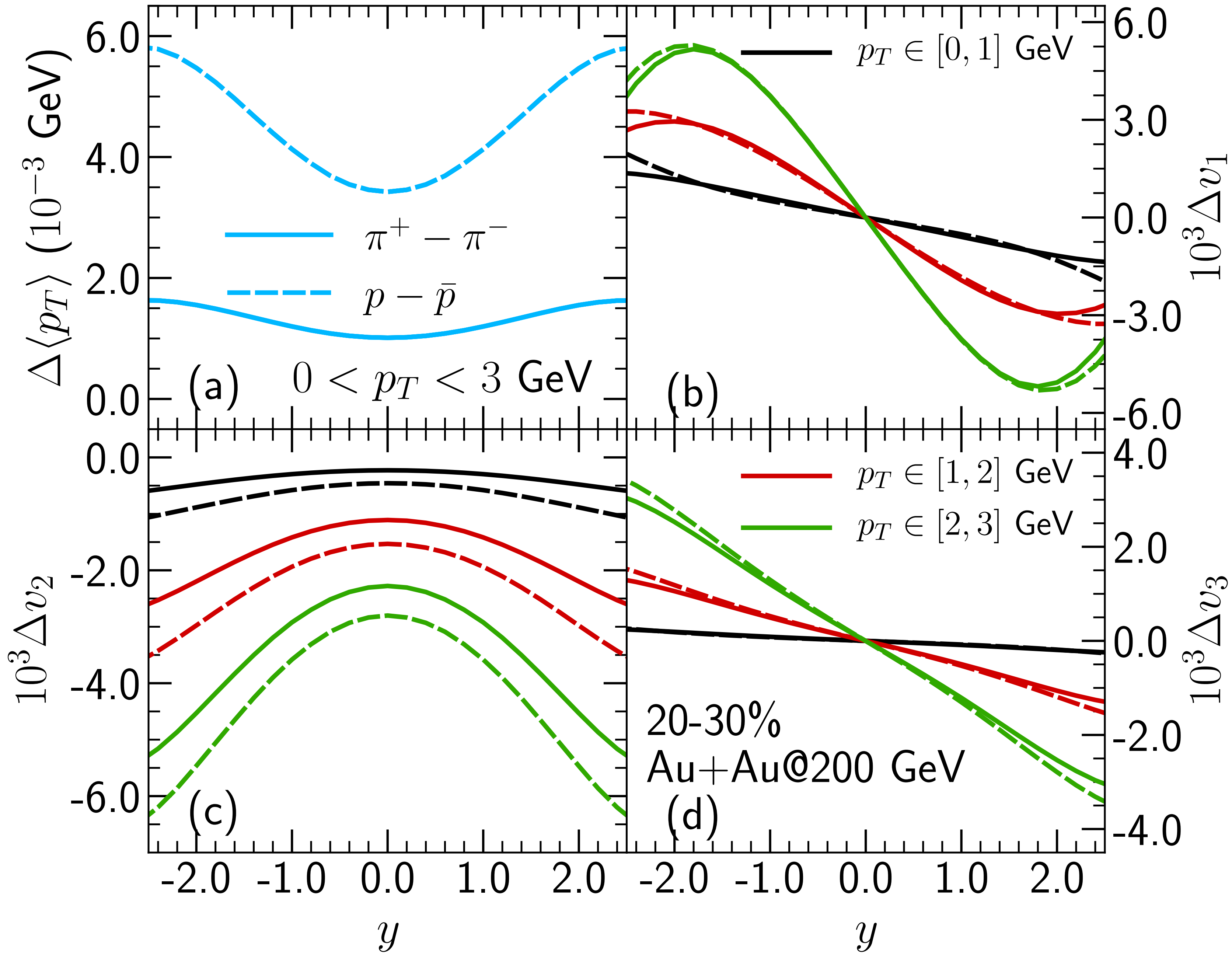
Kharzeev, Rajagopal, UG `14

Magnetically induced rapidity-odd flow

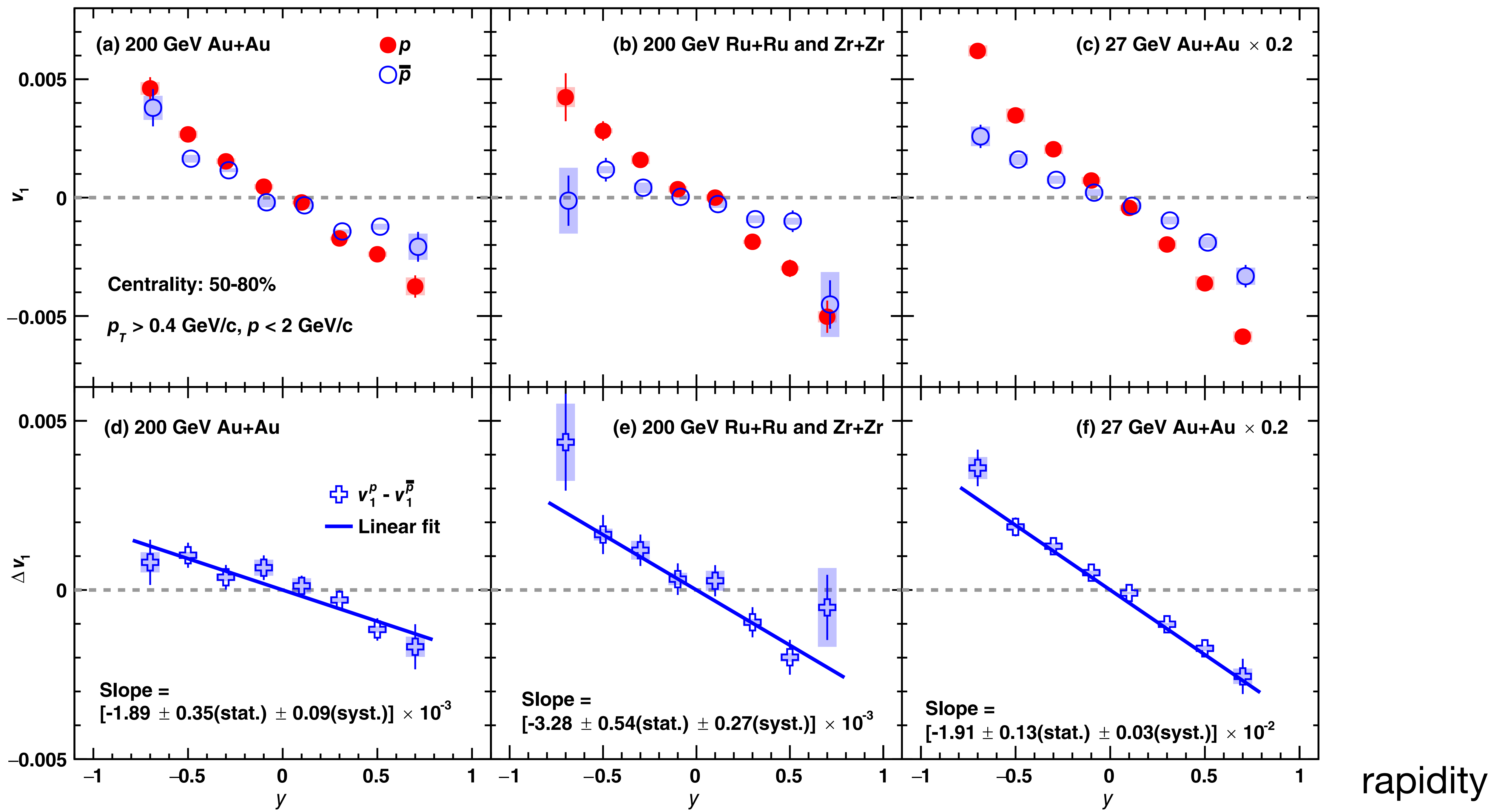


Results in ***rapidity-odd directed flow*** $\langle \cos\theta \rangle$

Magnetically induced currents in QGP

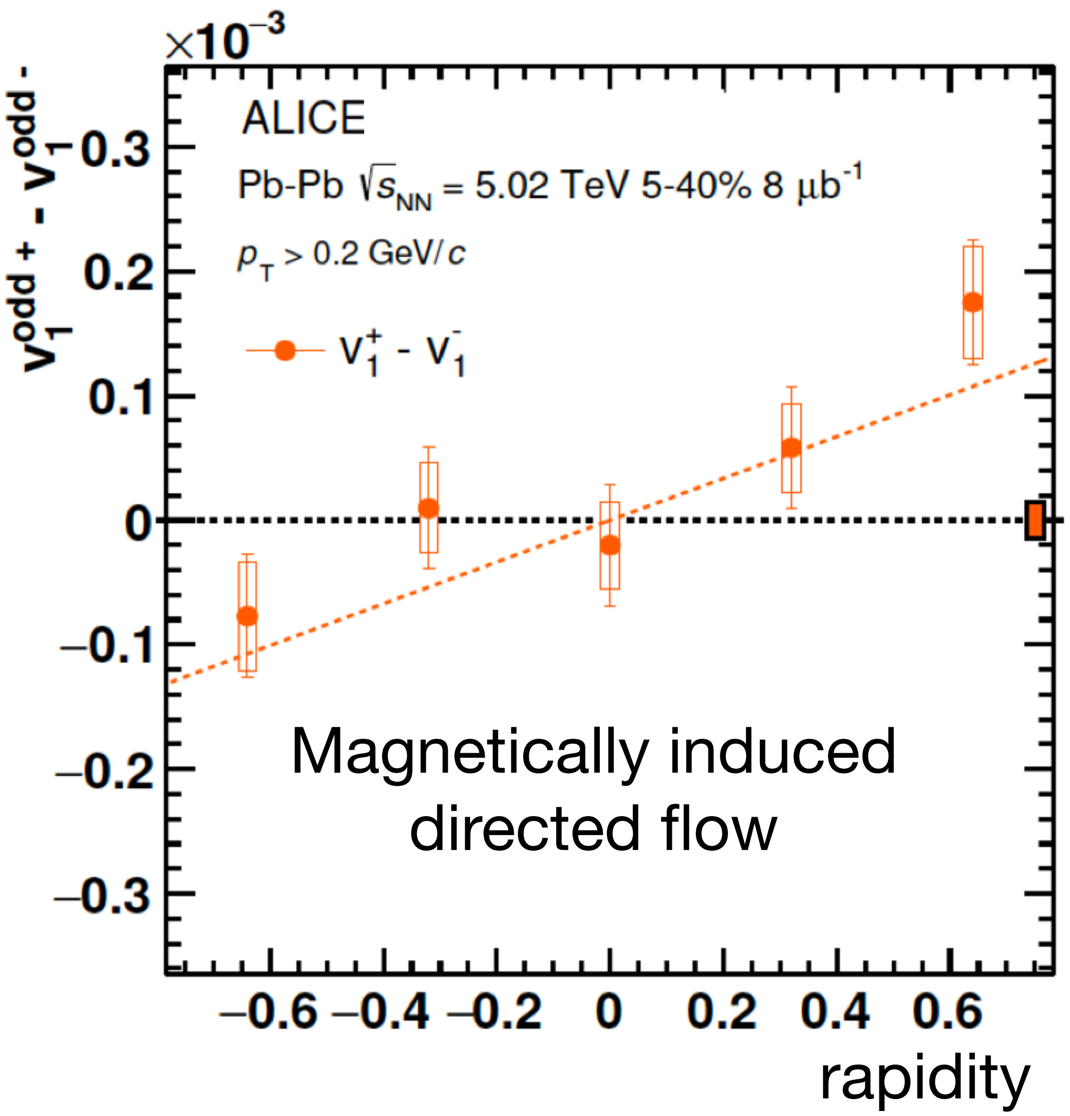


Kharzeev, Rajagopal, UG '14; Kharzeev, Rajagopal, Shen, Marcus, UG '18



STAR Collaboration, Phys. Rev. X. 14, 011028 (2024)

Effect recently observed at RHIC!



Effect also observed at LHC but with wrong sign…

ALICE Collaboration, Phys. Rev. Lett. 125, 022301 (2020)

Comparison to data:

Dubla, Snellings, UG ’20

- Non-perturbative backreaction of EM fields
- Electric and magnetic polarisation of medium
- Time dependence of transport coefficients
- Other transport, e.g. Hall conductivity, viscosities

Spin-hydrodynamics

Strong vortical structure

$\omega \sim 10^{22}\ s^{-1}$

Hydrodynamics with spin current

Slow variables: energy-momentum and spin current

$$T_{\mu\nu} \qquad S^{\lambda}_{\mu\nu}$$

Becattini et al ’08; Becattini, Piccinini ’08
Karabali, Nair ’14; Rischke et al ’14
Florkowski et al ’18 ’19; Hattori, X.-G. Huang et al ’19
Gallegos, UG ’19; Li, Stephanov, Yee ’20
Gallegos, Yarom, UG ’21; ’22

Spin effective action

Consider quantum field in a nontrivial Lorentz representation

$$e^{iW[e,\omega]} = \int D\Psi e^{iI[e,\omega,\Psi]}$$

Variations define the energy-momentum and spin current

$$T^{\mu\nu} = \frac{\delta W}{\delta e_\mu^a} e_a^\nu, \qquad S_{ab}^\lambda = \frac{\delta W}{\delta \omega_\lambda^{ab}}$$

Spin effective action

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Variations define the energy-momentum and spin current

$$T^{\mu\nu} = \frac{\delta W}{\delta e^a_\mu} e^\nu_a, \qquad S^\lambda_{ab} = \frac{\delta W}{\delta \omega^{ab}_\lambda}$$

Treat metric and spin connection independently in presence of torsion:

$$de^a + \omega^a_b e^b = T^a$$

$\Longrightarrow$ Keep T^a as external source, $T^a \to 0$ at the end.

Gallegos,Yarom, UG `21 `22

Hydrodynamic degrees of freedom

$$\mathring{\nabla}_\mu T^{\mu\nu} = \frac{1}{2} R^{\rho\sigma\nu\lambda} S_{\rho\lambda\sigma} - T_{\rho\sigma} K^{\nu ab} e^\rho{}_a e^\sigma{}_b$$ 4 equations

$$\mathring{\nabla}_\lambda S^\lambda{}_{\mu\nu} = 2T_{[\mu\nu]} - 2S^\lambda{}_{\rho[\mu} e_{\nu]}{}^a e_\rho{}^b K_{\lambda ab}\,,$$ 6 equations

Hydrodynamic degrees of freedom

$$\mathring{\nabla}_\mu T^{\mu\nu} = \frac{1}{2} R^{\rho\sigma\nu\lambda} S_{\rho\lambda\sigma} - T_{\rho\sigma} K^{\nu ab} e^\rho{}_a e^\sigma{}_b \qquad \text{4 equations}$$

$$\mathring{\nabla}_\lambda S^\lambda{}_{\mu\nu} = 2T_{[\mu\nu]} - 2S^\lambda{}_{\rho[\mu} e_{\nu]}{}^a e_\rho{}^b K_{\lambda ab}\,, \qquad \text{6 equations}$$

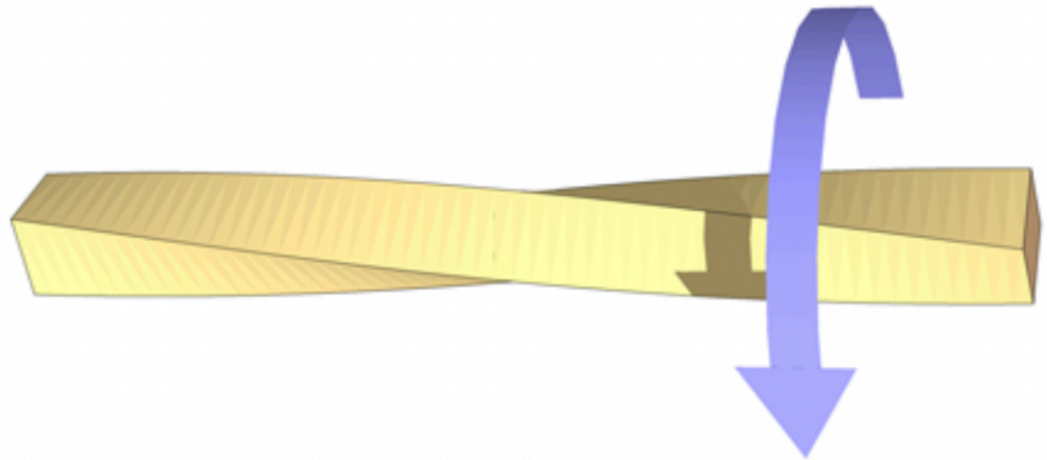
Hydrostatic action W[e,ω] invariant under ξ

10 dynamical variables:

$$T = \frac{T_0}{\sqrt{-\xi^2}}, \qquad u^a = \frac{e^a_\mu \xi^\mu}{\sqrt{-\xi^2}}, \qquad \mu^{ab} = \frac{\omega_\mu{}^{ab}\xi^\mu}{\sqrt{-\xi^2}}$$

Spin “chemical” potential

Solution to spin hydrodynamics



$$\underbrace{u^\mu K_\mu^{ab}}_{\text{torsion}} = \mu^{ab} - 2u^{[a}\underbrace{a^{b]}}_{\text{acceleration}} + \underbrace{\Omega^{ab}}_{\text{vorticity}}$$

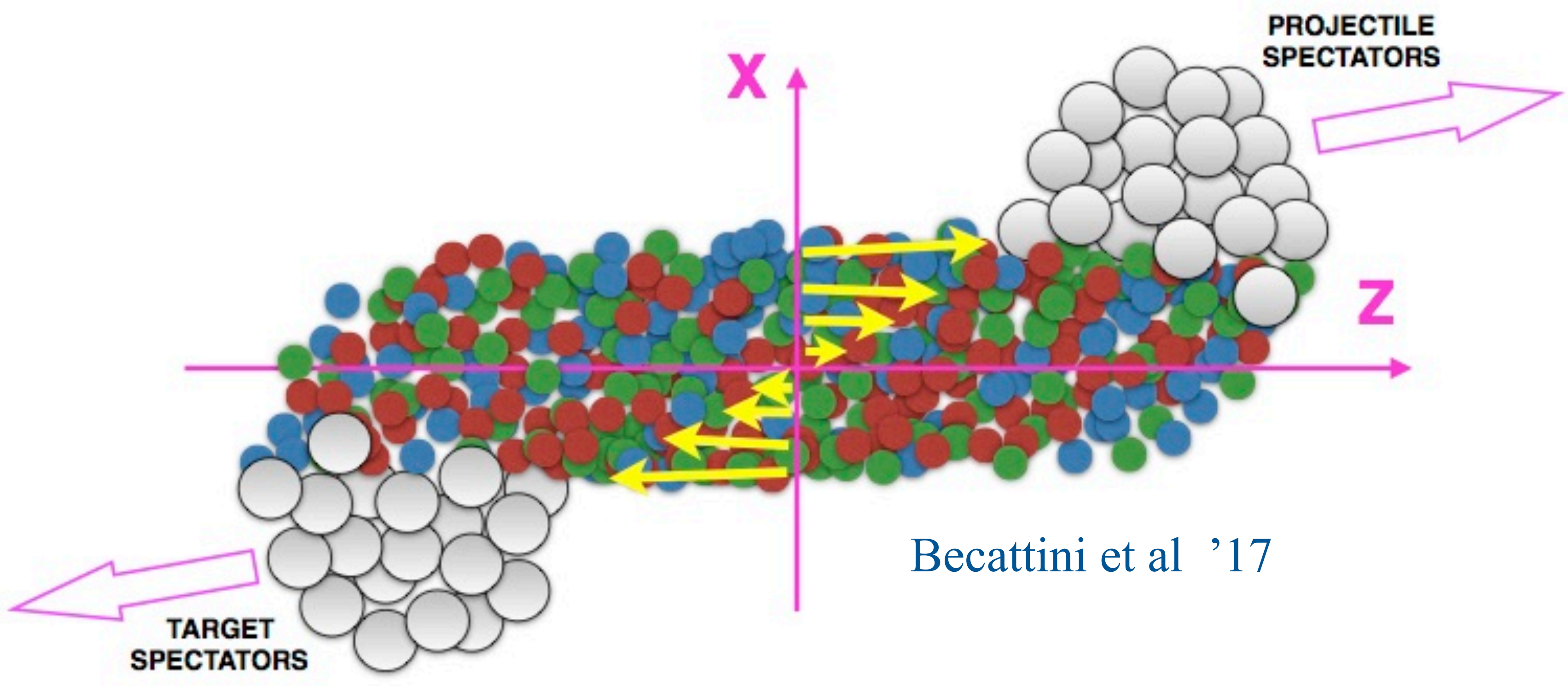
Holds beyond hydrostatics — **Gallegos, Yarom, UG '14**

Spin is "slave" to background flow:

$$\underbrace{\mu^{ab}}_{\text{spin potentials}} = -2u^{[a}\underbrace{a^{b]}}_{\text{acceleration}} + \underbrace{\Omega^{ab}}_{\text{vorticity}}$$

up to $O(\nabla^2)$

Application to HIC



Polarization of identified particle: Becattini et al. '13; Florkowski et al '19

$$\Pi_\mu(p) = -\frac{1}{4}\epsilon_{\mu\rho\sigma\beta}\frac{p^\beta}{m}\frac{\int d\Sigma_\lambda p^\lambda B(x,p)\mu^{\rho\sigma}}{2\int d\Sigma_\lambda p^\lambda n_F}$$

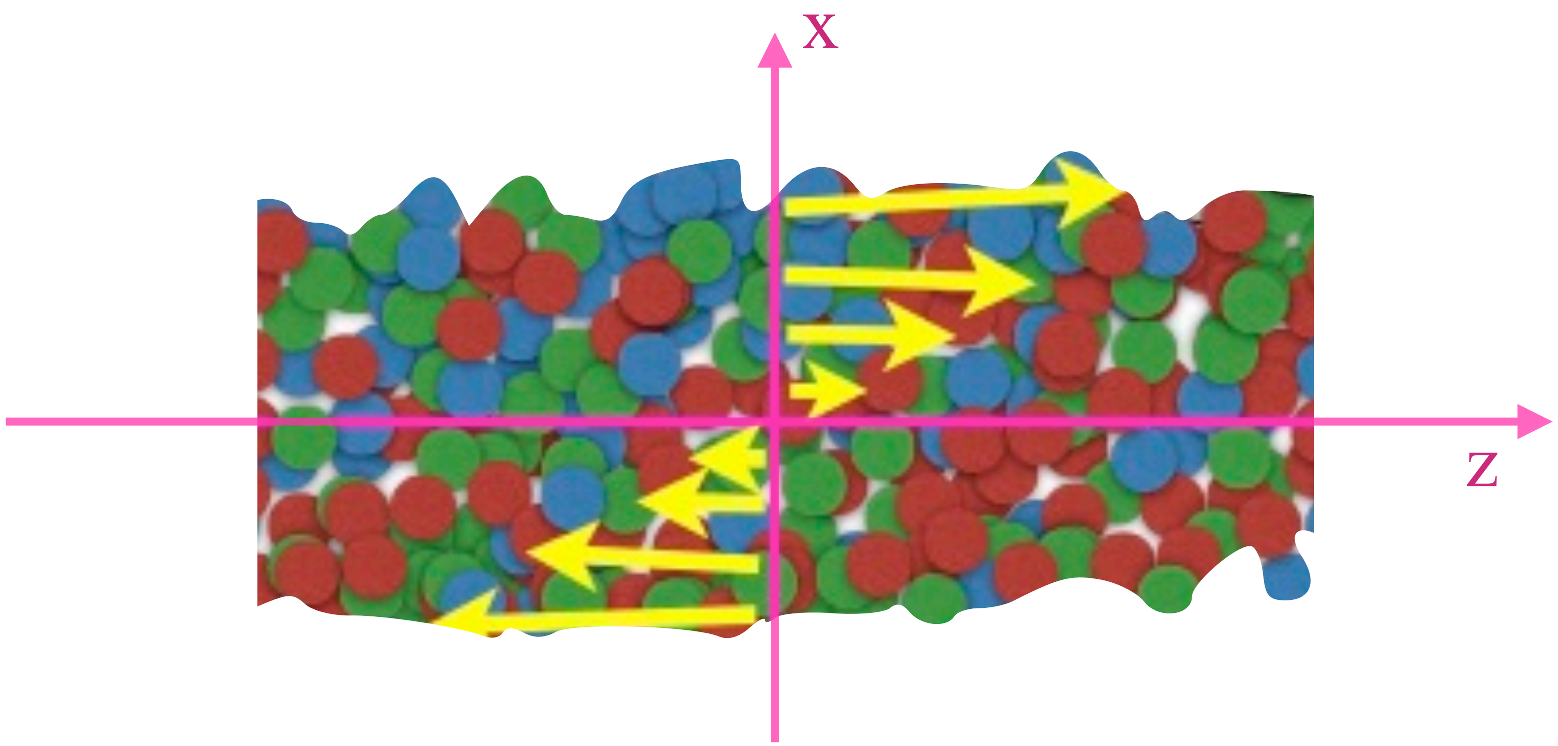
spin potential

freezout surface

Boltzmann type distribution

Spin hydrodynamics $\Longrightarrow$ spin potential

Bjorken flow with spin current



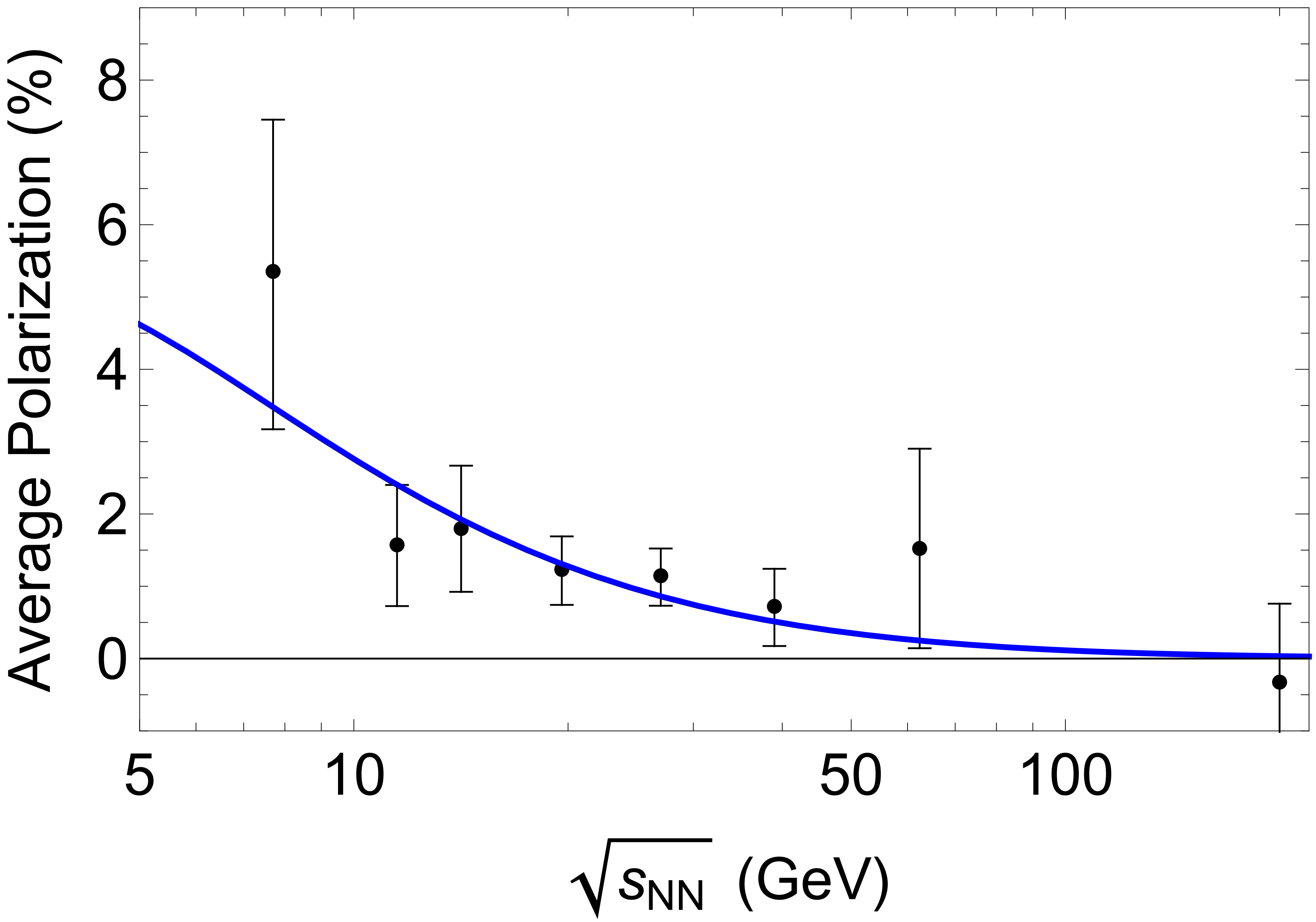
Nearly flat rapidity distribution $\Longrightarrow$ u, T, μ independent of η
Full symmetry of Bjorken flow: SO(1,1) x ISO(2) x Z_2

$$u^\tau = 1, \qquad T = T_0 \left(\frac{\tau_0}{\tau}\right)^{\frac{1}{3}} - \frac{\eta_0}{2\epsilon_0 \tau},$$

No global spin polarization $\Longrightarrow$ break symmetry by initial conditions

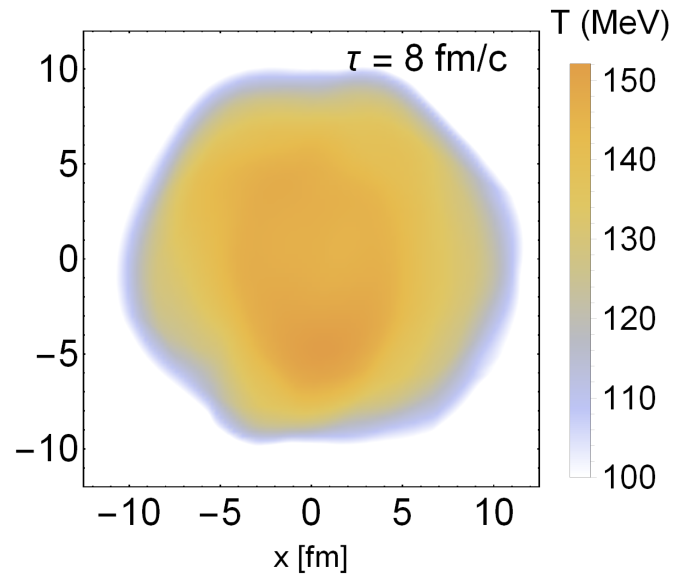
$$\delta u^\eta(\tau_0) \propto b\, q_x$$

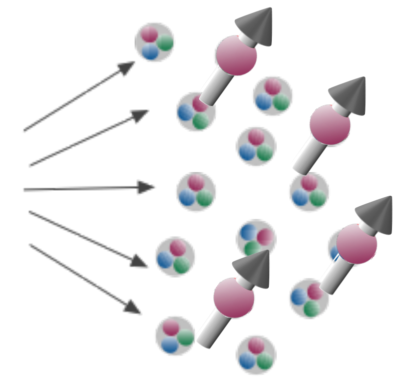
Comparison to data



Gallegos, Yarom, UG ’21

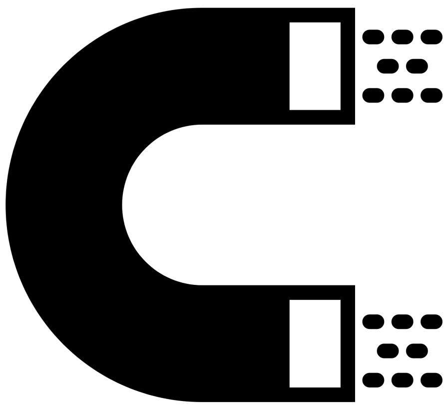
Loose ends

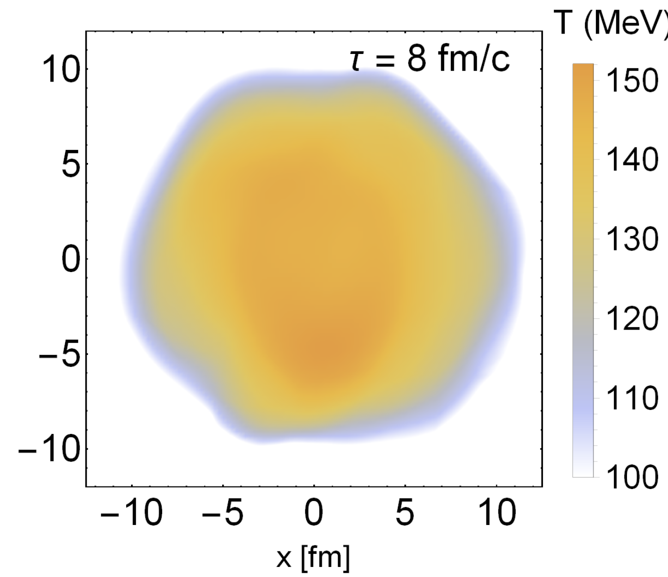


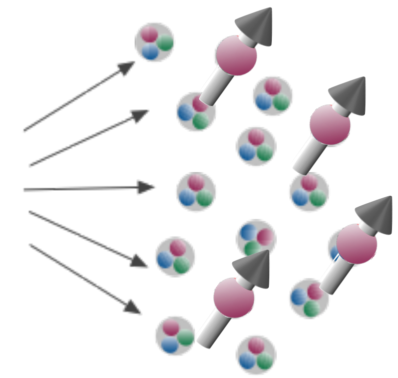


- Add vorticity to Trajectum ⟹ longitudinal polarization?

Loose ends







- Add vorticity to Trajectum $\Longrightarrow$ longitudinal polarization?
- Develop a theory of spin-magneto-hydro ?
- Interplay between chiral and spin transport?

Ambiguity in spin current

Total angular momentum

$$J^{\lambda\mu\nu} = \underbrace{x^\mu T^{\lambda\nu} - x^\nu T^{\lambda\mu}}_{\text{orbital}} + \underbrace{S^{\lambda\mu\nu}}_{\text{spin}}$$

Conservation laws

$$\partial_\mu T^{\mu\nu} = 0\,, \qquad \partial_\lambda J^{\lambda\mu\nu} = 0$$

Preserved by

$$T'^{\mu\nu} = T^{\mu\nu} + \frac{1}{2}\partial_\lambda \left(\Phi^{\lambda\mu\nu} - \Phi^{\mu\lambda\nu} - \Phi^{\nu\lambda\mu}\right)$$

$$S'^{\lambda\mu\nu} = S^{\lambda\mu\nu} - \Phi^{\lambda\mu\nu}$$

Choice I: $$\nabla_\mu T^{\mu\nu} = 0\,, \qquad \nabla_\lambda S^{\lambda}_{ab} = T_{ab} - T_{ba}$$

Choice II: $$\nabla_\mu T^{'\,\mu\nu} = 0\,, \qquad T^{'\,\mu\nu} - T^{'\,\nu\mu} = 0$$

Satisfied only under the EoM

Choice III: $$T^{''\,\mu\nu} \equiv T^{'\,\nu\mu} - \frac{1}{2}\left(T^{'\,\nu\mu} - T^{'\,\mu\nu}\right)\,,$$

$$\nabla_\mu T^{''\,\mu\nu} = 0$$

No access to spin transport

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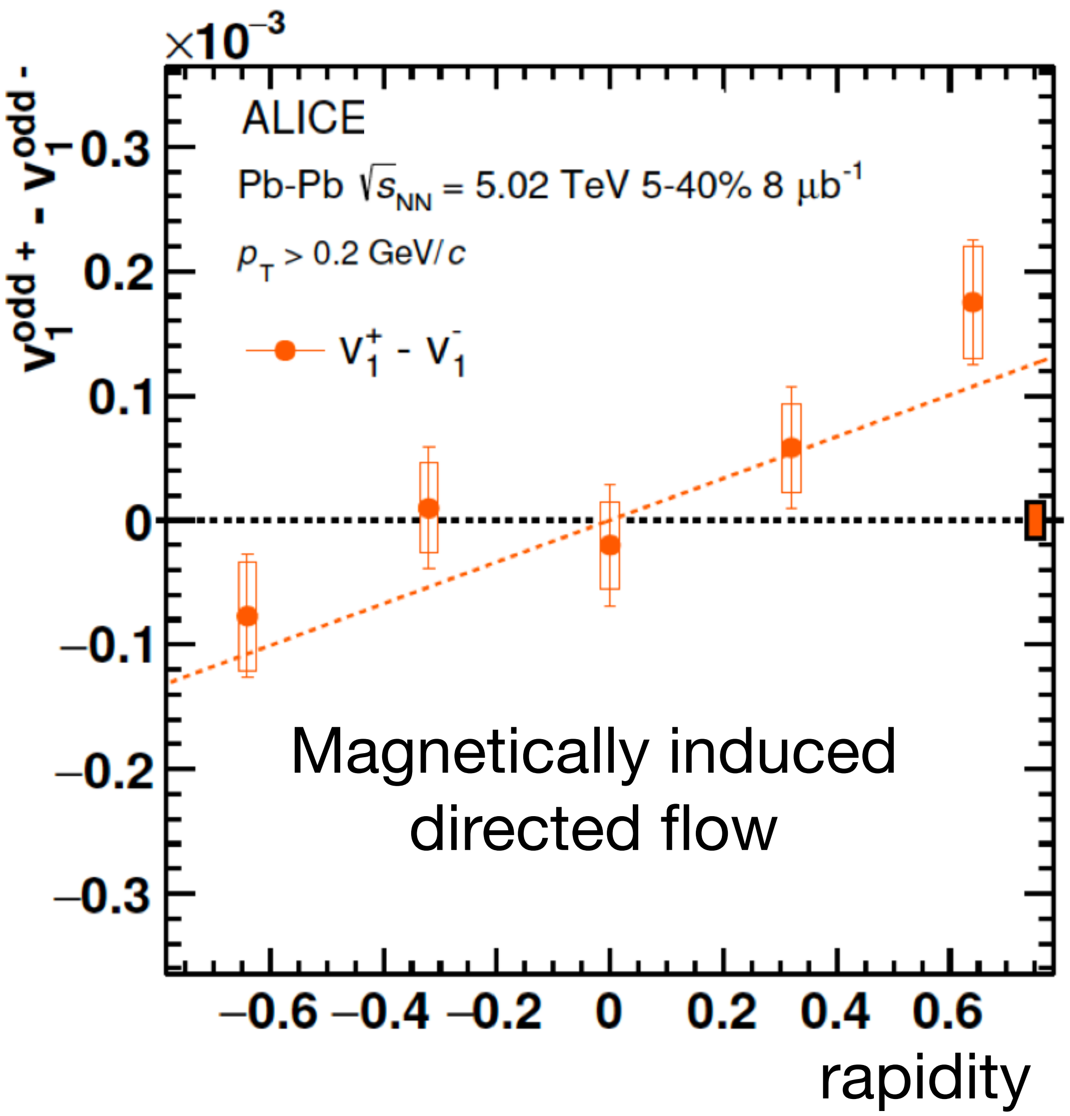
$\nabla_\mu T^{''\,\mu\nu} = 0$

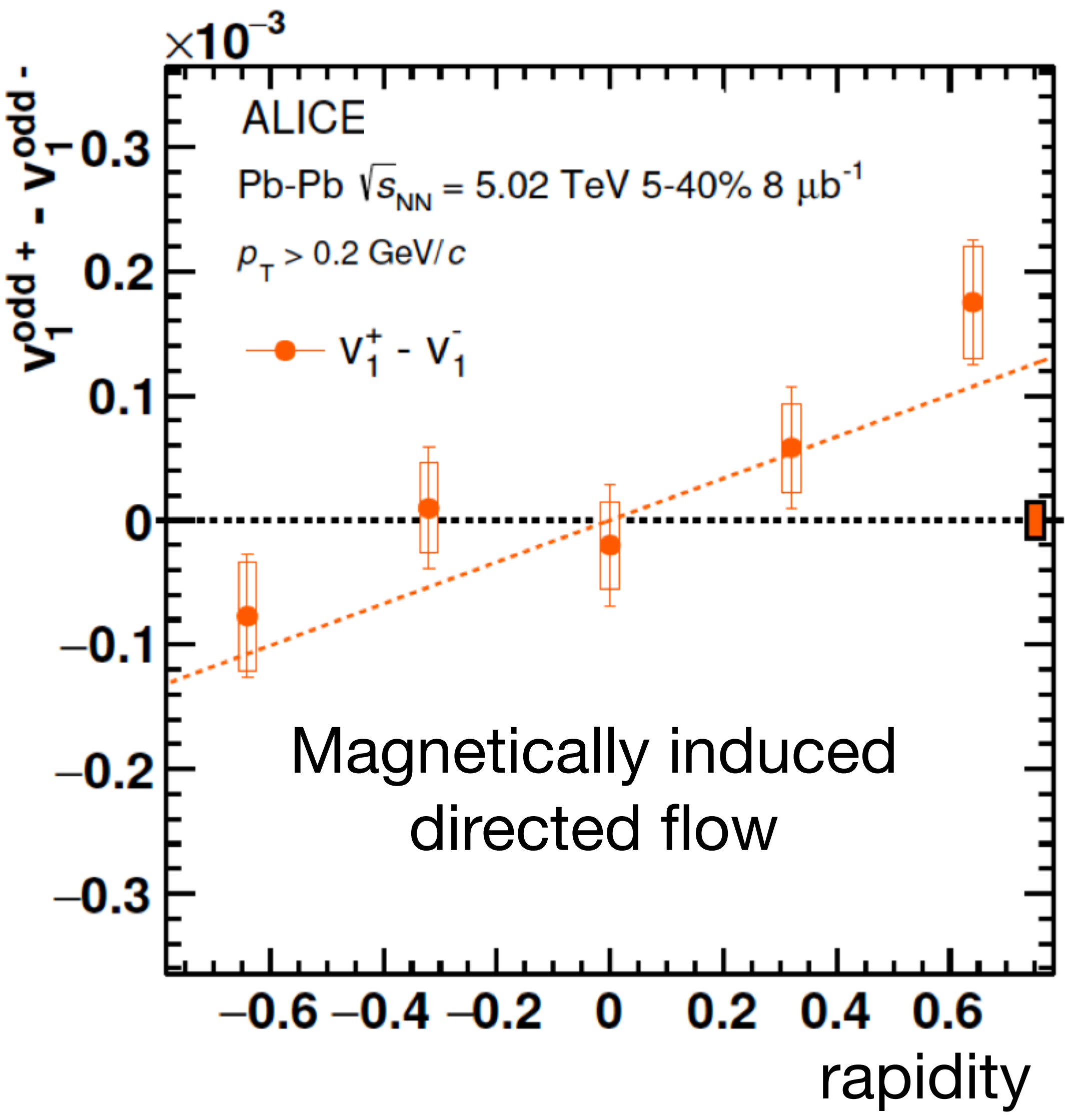
No access to spin transport

Consistency with choice III $\Longrightarrow$ spin transport determined by energy-momentum flow
External torsion fixes the ambiguity, organizes hydro expansion unambiguously



Effect recently observed at LHC!

ALICE Collaboration, Phys. Rev. Lett. 125, 022301 (2020)



Effect observed at LHC

ALICE Collaboration, Phys. Rev. Lett. 125, 022301 (2020)

Comparison to data:

Dubla, Snellings, UG '20

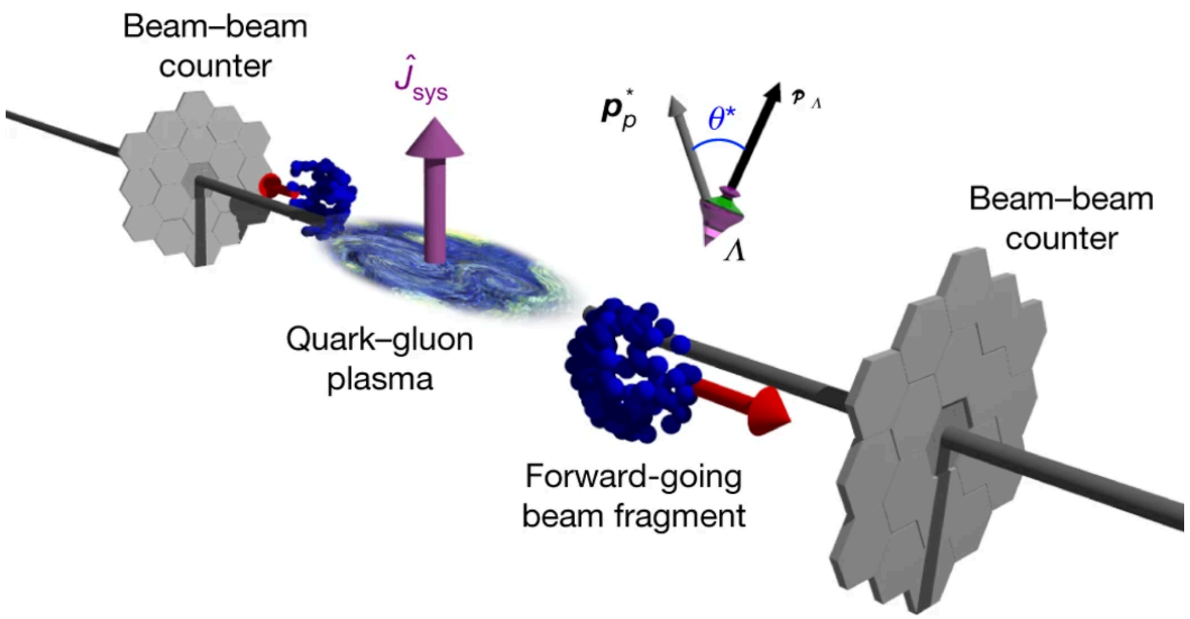
- Non-perturbative backreaction of EM fields
- Electric and magnetic polarisation of medium
- Time dependence of transport coefficients
- Other transport, e.g. Hall conductivity, viscosities

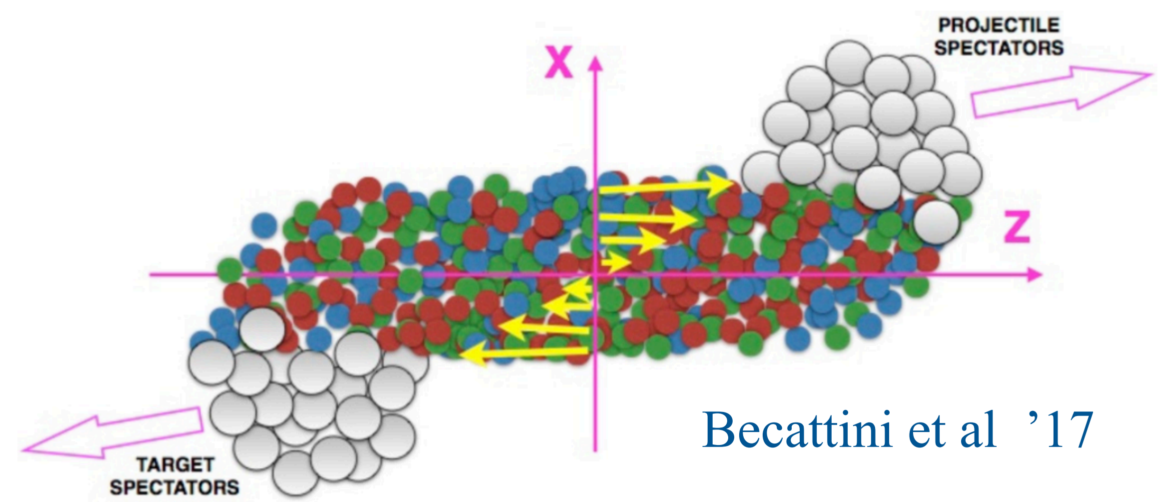
Spin-hydrodynamics

Strong vortical structure

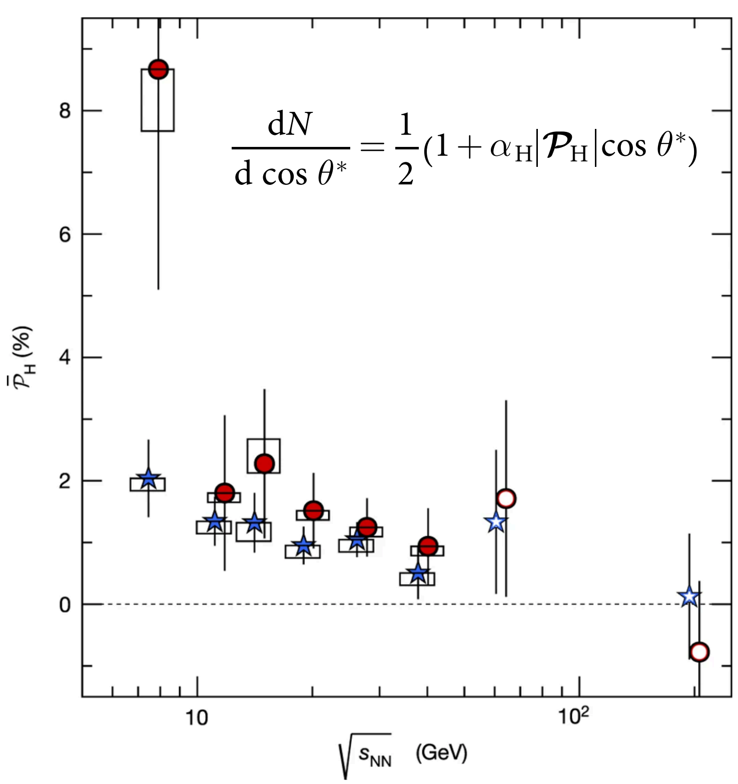
$\omega \sim 10^{22}\ s^{-1}$

Spin hydrodynamics





Becattini et al '17

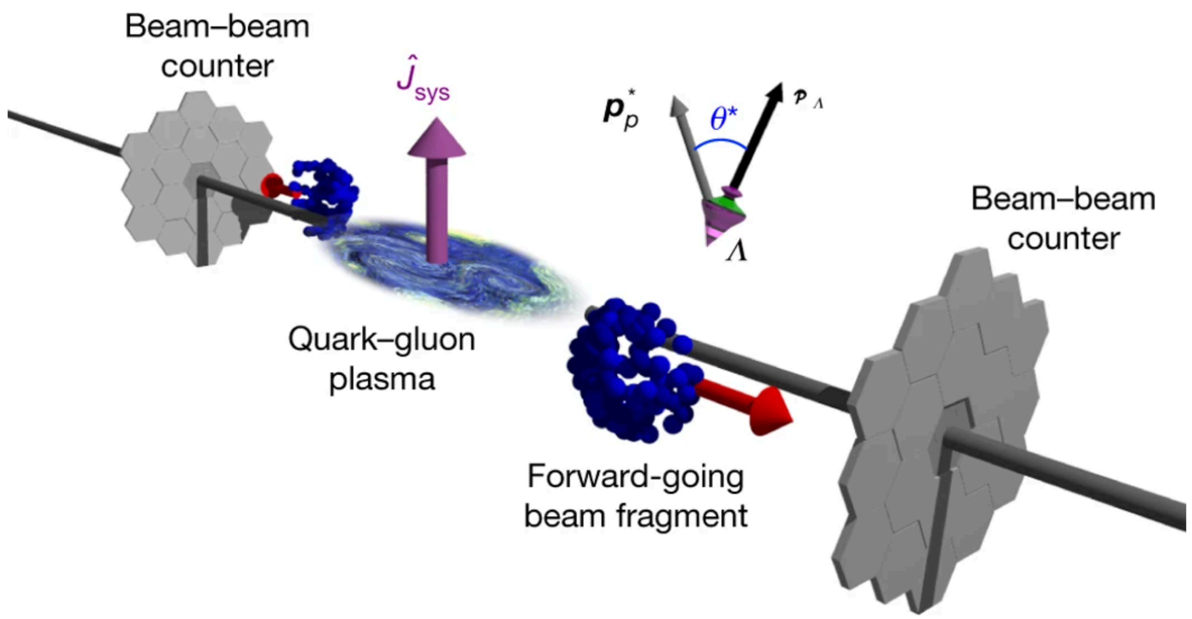


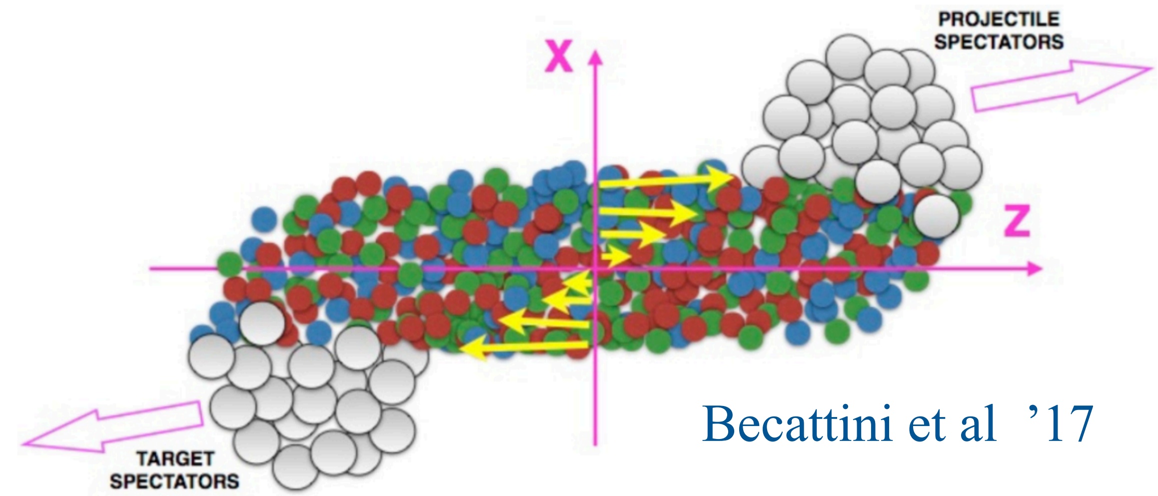
Global hyperon polarization at RHIC
by spin-orbit coupling $\vec{S} \cdot \vec{J}$

QGP: most vortical fluid: $\omega \sim 10^{22}\ s^{-1}$

STAR collaboration, RHIC `19

Spin hydrodynamics





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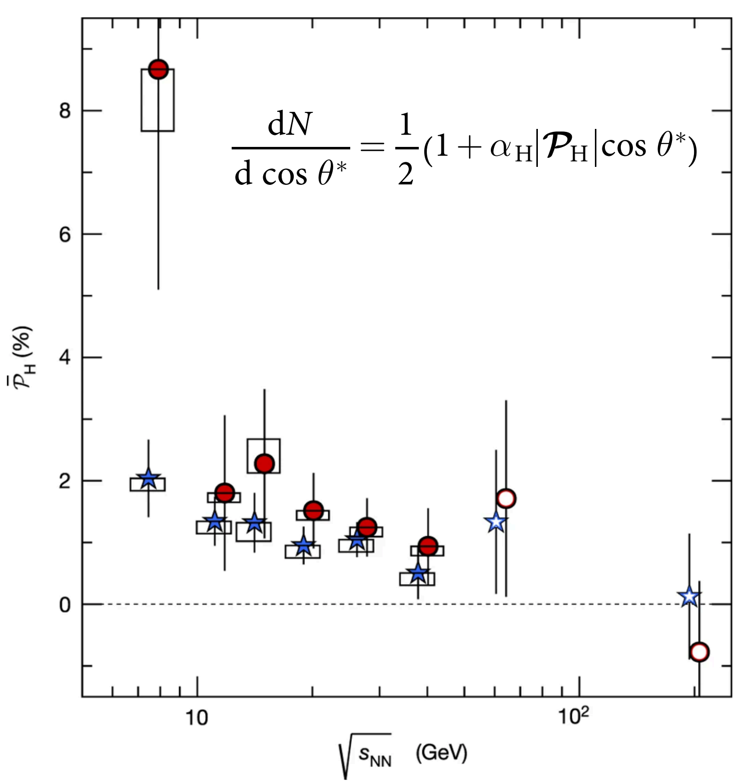

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STAR collaboration, RHIC `19

$\Longrightarrow$ hydrodynamic description?

Hydrodynamics with spin current

Gallegos, Yarom, UG `21

Slow variables: energy-momentum and spin current

$$T_{\mu\nu} \qquad S^{\lambda}_{\mu\nu}$$

Salute to earlier work:

Becattini et al ’08; Becattini, Piccinini ’08
Karabali, Nair ’14
Florkowski et al ’18 ’19; Hattori, X.-G. Huang et al ’19
Gallegos, UG ’19; Li, Stephanov, Yee ’20

Ambiguity in spin current

Total angular momentum

$$J^{\lambda\mu\nu} = \underbrace{x^\mu T^{\lambda\nu} - x^\nu T^{\lambda\mu}}_{\text{orbital}} + \underbrace{S^{\lambda\mu\nu}}_{\text{spin}}$$

Conservation laws

$$\partial_\mu T^{\mu\nu} = 0\,, \qquad \partial_\lambda J^{\lambda\mu\nu} = 0$$

Preserved by

$$T'^{\mu\nu} = T^{\mu\nu} + \frac{1}{2}\partial_\lambda \left(\Phi^{\lambda\mu\nu} - \Phi^{\mu\lambda\nu} - \Phi^{\nu\lambda\mu}\right)$$

$$S'^{\lambda\mu\nu} = S^{\lambda\mu\nu} - \Phi^{\lambda\mu\nu}$$

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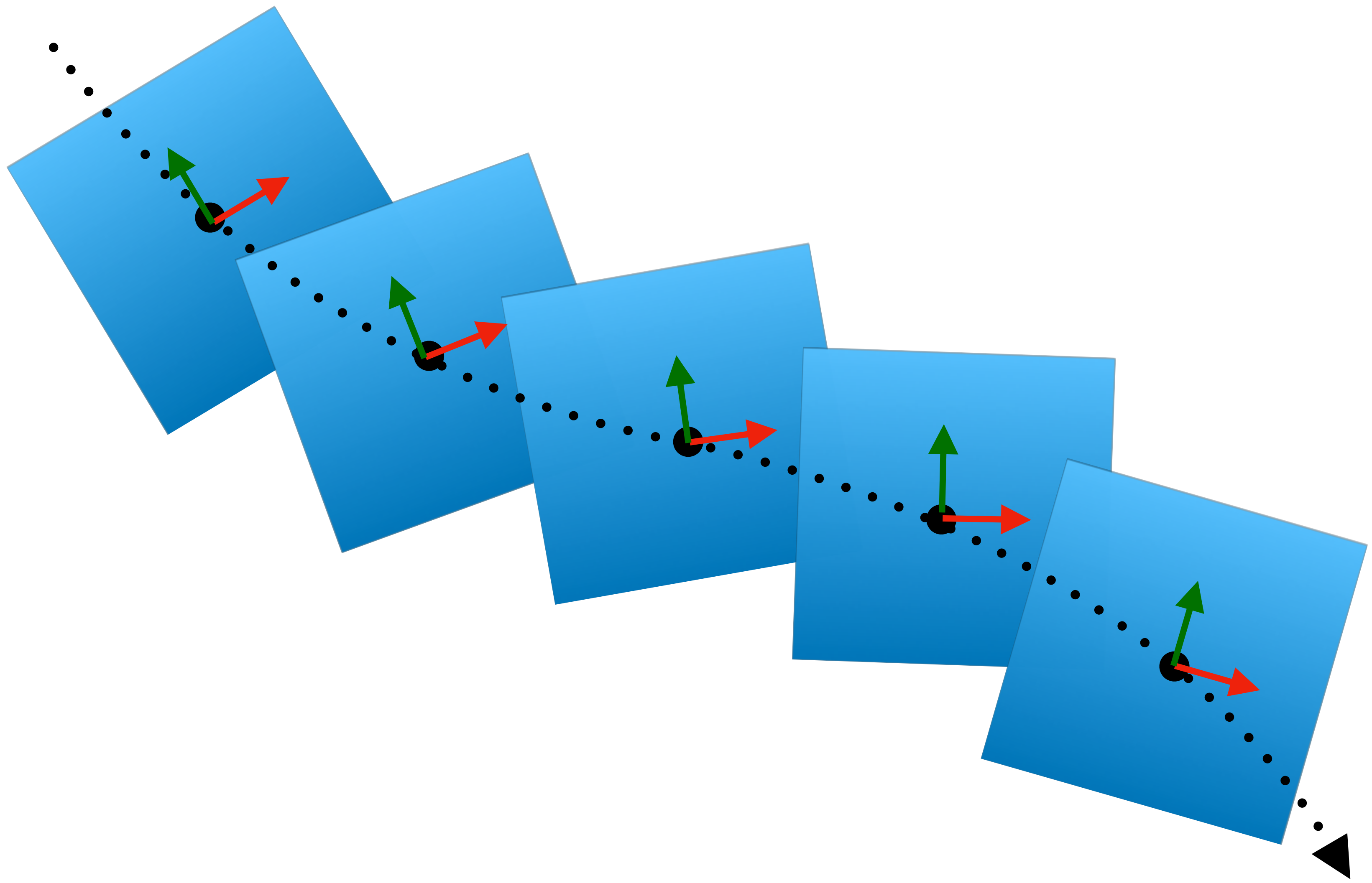
Torsion removes the ambiguity

Torsion



$$T^{\alpha}_{\mu\nu} = \Gamma^{\alpha}_{\mu\nu} - \Gamma^{\alpha}_{\nu\mu}$$

Asymmetric affine connection

Hydrodynamics with torsion

- More precisely, contorsion sources spin :

$$\omega_\mu^{ab} = \mathring{\omega}_\mu^{ab} + K_\mu^{ab}, \qquad \mathring{\omega} \sim \partial e$$

Hydrodynamics with torsion

- More precisely, contorsion sources spin :

$$\omega_\mu^{ab} = \mathring{\omega}_\mu^{ab} + K_\mu^{ab}, \qquad \mathring{\omega} \sim \partial e$$

- Hydrodynamics on a manifold with non-trivial torsion:

$$T^{\mu\nu} = \frac{\delta W}{\delta e_\mu^a} e_a^\nu, \qquad S_{ab}^\lambda = \frac{\delta W}{\delta \omega_\lambda^{ab}}$$

- Eventually K → 0, e.g. in QGP, non trivial spin current from O(K) terms in W[K]

Hydrodynamic equations

$$\mathring{\nabla}_\mu T^{\mu\nu} = \frac{1}{2} R^{\rho\sigma\nu\lambda} S_{\rho\lambda\sigma} - T_{\rho\sigma} K^{\nu ab} e^\rho{}_a e^\sigma{}_b$$ 4 equations

$$\mathring{\nabla}_\lambda S^\lambda{}_{\mu\nu} = 2T_{[\mu\nu]} - 2S^\lambda{}_{\rho[\mu} e_{\nu]}{}^a e_\rho{}^b K_{\lambda ab} \,,$$ 6 equations

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10 dynamical variables:

$$T \qquad u^\mu \qquad \mu^{ab} = \omega_\mu^{ab} u^\mu$$

Spin "chemical" potential

Analogous to electric potential

$$\mu_E = \frac{A_\mu \xi^\mu}{\sqrt{-\xi^2}}$$

Constitutive relations

Hydrodynamic action: most general scalar from T, u, e, K and derivatives

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$$\mu^{ab} = 2u^{[a}\underbrace{m^{b]}}_{\text{“electric”}} + \epsilon^{abcd}u_c\underbrace{\tilde{M}_d}_{\text{“magnetic”}}$$

“electric” 3 d.o.f.

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“electric” 3 d.o.f.

“magnetic” 3 d.o.f.

Hydrodynamic action:

$$W = P(T, m^2, \tilde{M}^2, m \cdot \tilde{M}) + \mathcal{O}(\partial u, \partial T, K_\perp, \partial m, \partial \tilde{M})$$

ideal fluid pressure

gradient corrections

Ideal spin fluid

Pressure of ideal spin fluid: $P(T, m^2, \tilde{M}^2, m \cdot \tilde{M})$

Constitutive relations:

$$T_i^{\alpha\beta} = \epsilon u^\alpha u^\beta + P\Delta^{\alpha\beta} - 2\left(\frac{\partial P}{\partial m^2} + 4\frac{\partial P}{\partial M^2}\right) u^\alpha M^{\beta\gamma} m_\gamma$$

$$S_i{}^{\lambda}{}_{\alpha\beta} = u^\lambda \rho_{\alpha\beta}\,,$$

spin density

susceptibilities

$m \times \tilde{M}$ "spin Poynting"

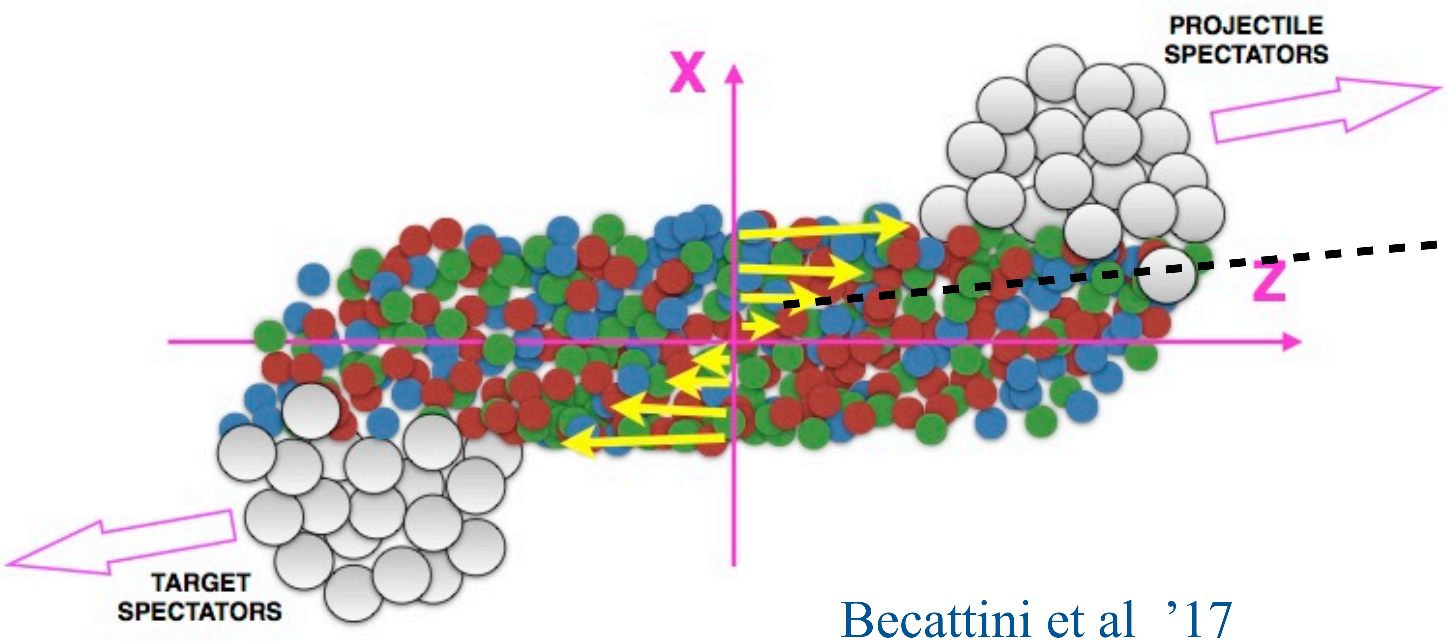
$$\left(M^{ab} \equiv \epsilon^{abcd} u_c \tilde{M}_d\,, \qquad \tilde{m}^{ab} \equiv \epsilon^{abcd} u_c m_d \right)$$

$$\epsilon = -P + \frac{\partial P}{\partial T} T + \frac{1}{2}\rho_{ab}\mu^{ab}\,,$$

$$\rho_{\alpha\beta} = 8\frac{\partial P}{\partial M^2} M_{\alpha\beta} + \frac{\partial P}{\partial m \cdot \tilde{M}}\left(4\tilde{m}_{\alpha\beta} - u_\alpha \tilde{M}_\beta + \tilde{M}_\alpha u_\beta\right) + 2\frac{\partial P}{\partial m^2}\left(u_\alpha m_\beta - m_\alpha u_\beta\right)$$

Gibbs-Duhem relations

Application to HIC



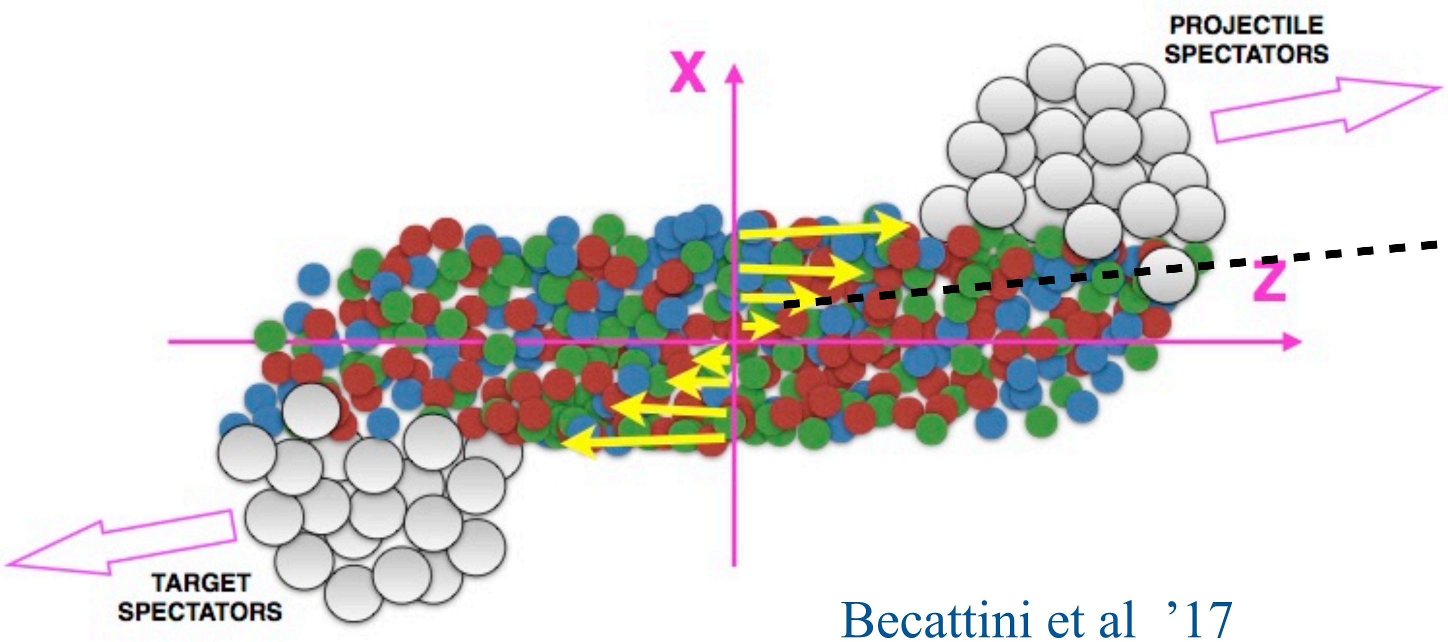
Becattini et al '17

General lore:

∇ x (u/T) = $\Omega_{thermal}$

spin-orbit $\Longrightarrow$ polarization

Application to HIC



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spin-orbit ⟹ polarization

Polarization of hyperon:

$$\Pi_\mu(p) = -\frac{1}{4}\epsilon_{\mu\rho\sigma\beta}\frac{p^\beta}{m}\frac{\int d\Sigma_\lambda p^\lambda B(x,p)\mu^{\rho\sigma}}{2\int d\Sigma_\lambda p^\lambda n_F}$$

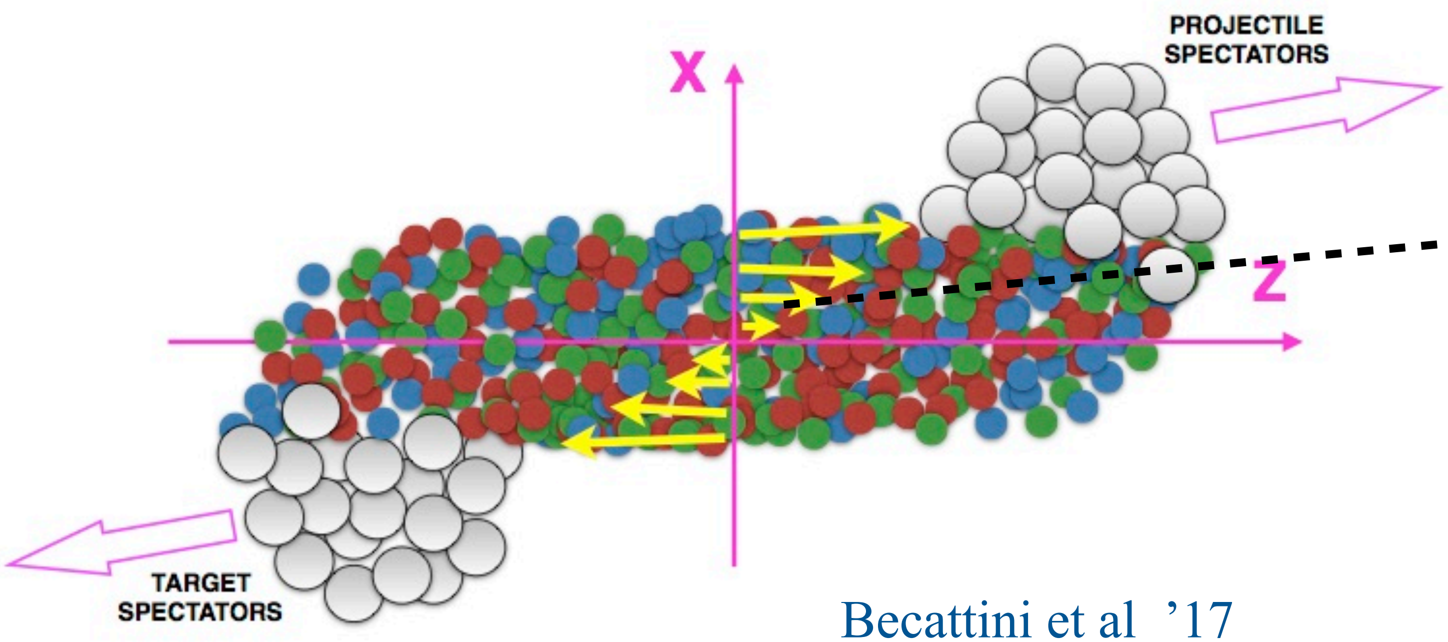
spin potential

freezout surface

Boltzmann type distribution

Becattini et al. '13; Florkowski et al '19 identified spin potential ⇔ Ω_{thermal}

Application to HIC



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Spin hydrodynamics $\Longrightarrow$ spin potential

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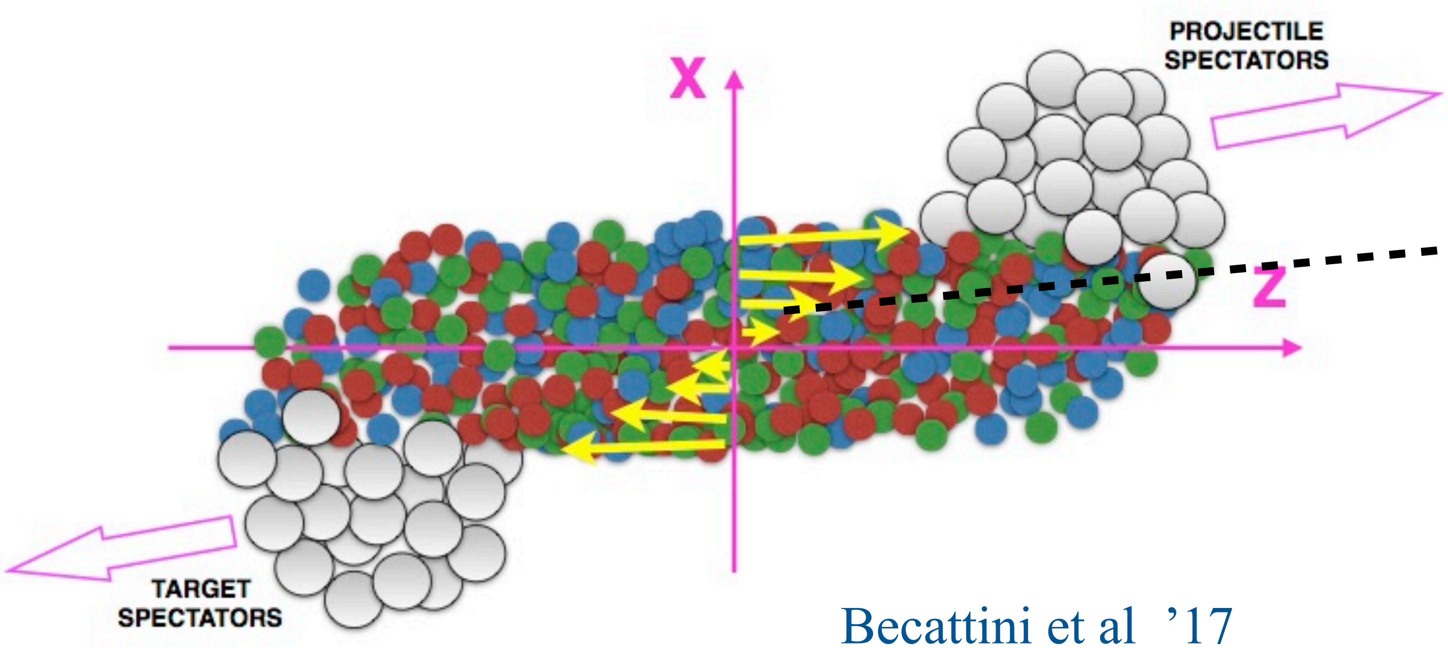

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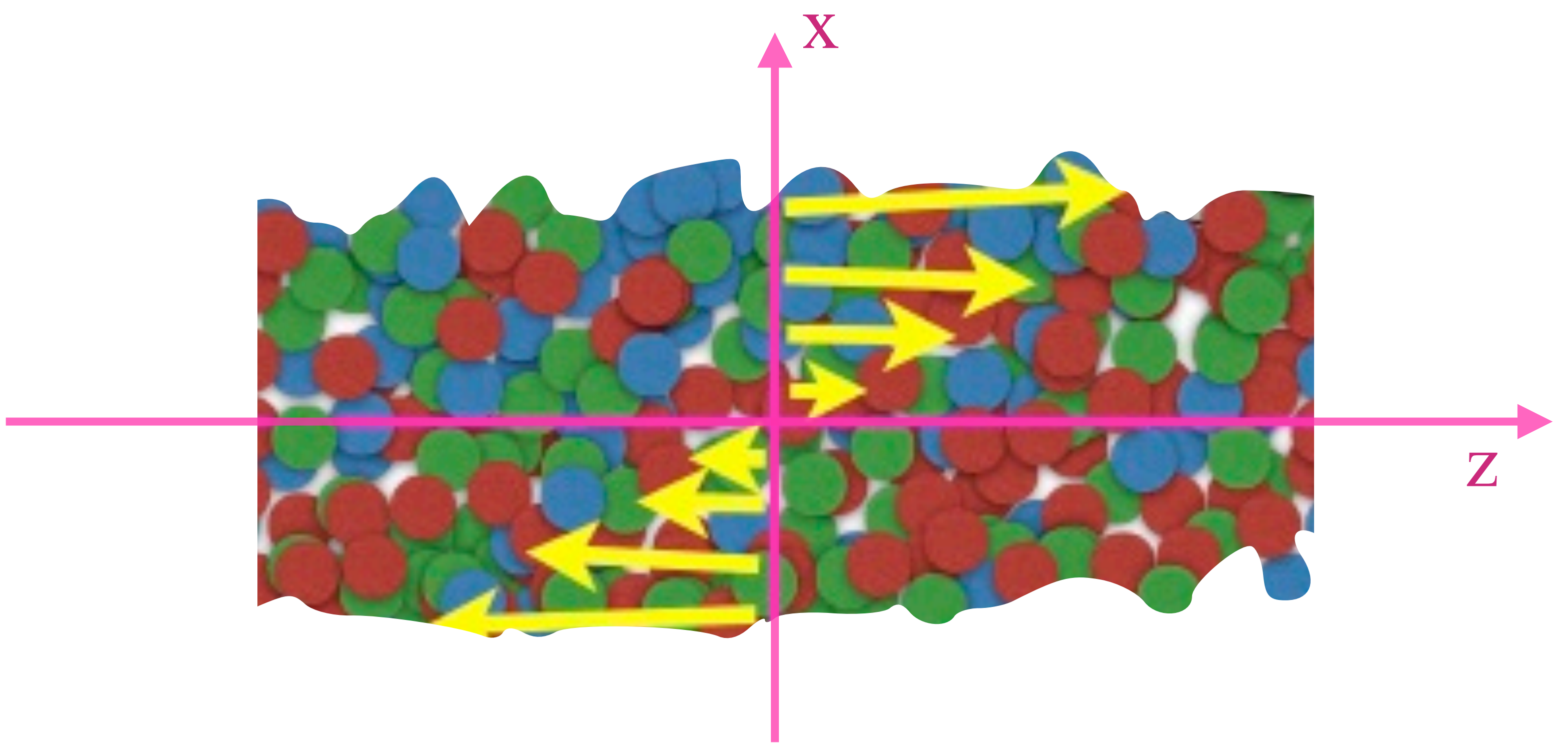
Spin hydrodynamics $\Longrightarrow$ spin potential $\Longrightarrow$ Ω_{thermal} in equilibrium

Bjorken flow with spin current



Nearly flat rapidity distribution $\Longrightarrow$ u, T, μ independent of η
Full symmetry of Bjorken flow: SO(1,1) x ISO(2) x Z_2

$$u^\tau = 1, \qquad T = T_0 \left(\frac{\tau_0}{\tau}\right)^{\frac{1}{3}} - \frac{\eta_0}{2\epsilon_0 \tau},$$

No global spin polarization $\Longrightarrow$ break symmetry by initial conditions

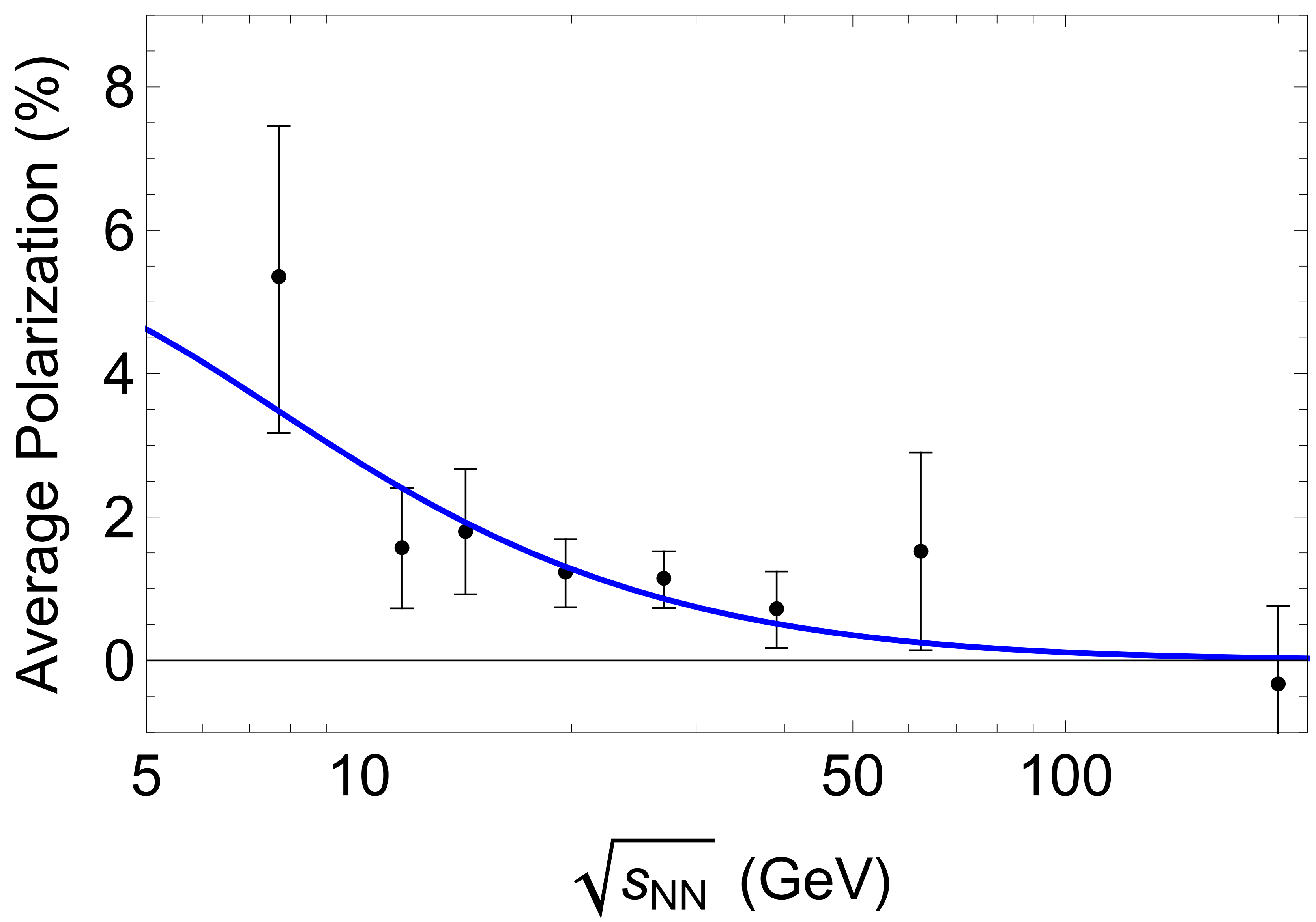
$$\delta u^\eta(\tau_0) \propto b\, q_x$$

Comparison to data

Hydrodynamic solution, for small “kinematic viscosity”/time $\frac{3\eta_0}{4\epsilon_0}\frac{1}{T\tau} \ll 1$

Floerschinger, Wiedemann ’11

$$\delta m^x(\tau) \propto \tau^{-\frac{8}{3}} e^{-\frac{9q^2\eta_0\tau_0}{16T_0\epsilon_0}\left(\frac{\tau}{\tau_0}\right)^{\frac{4}{3}}}, \qquad \delta M^{x\eta}(\tau) \propto q^2\,\tau^{-\frac{5}{3}} e^{-\frac{9q^2\eta_0\tau_0}{16T_0\epsilon_0}\left(\frac{\tau}{\tau_0}\right)^{\frac{4}{3}}}$$



Bayes' theorem

$$P(A|B) = P(B|A)\frac{P(A)}{P(B)}$$

Probability of A

Probability that A happens if B happened

Likelihood of A given B

Probability of B

Anomalous transport

Chiral magnetic and vortical effects:

$$\vec{J}_f = \frac{1}{2\pi^2}\mu_5 \left[3\, q_f(e\vec{B}) + 2(\mu\vec{\omega})\right]$$

Axial anomaly
(e.g. sphaleron decay)

Vorticity $\nabla \times \vec{u}$

Anomalous transport

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Not yet discovered in heavy ion collisions

Isobar run, STAR collab. 2021

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→ get B and ω by different means

Hydrodynamics in action formalism

Jensen et al ’12; Banerjee et al ’12

Hydrodynamics in action formalism

Jensen et al '12; Banerjee et al '12

- Example: charged fluid in presence of external sources

$$g_{\mu\nu}(x) \qquad A_\mu(x)$$

- Most general scalar $\quad S_{hydro} = \int d^4x \sqrt{g}\, W[g, A]$

- Diffeomorphism and gauge invariance: hydro equations

$$\partial_\mu T^{\mu\nu} = F^{\mu\nu} J_\mu \qquad \partial_\mu J^\mu = 0$$

- Thermal equilibrium: timelike Killing vector ξ

$$|\xi| = 1/T \qquad \xi/|\xi| = u \qquad u \cdot A = \mu_E$$

- Expand W in T, u, μ_E and derivatives: constitutive relations

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- Expand W in T, u, μ_E and derivatives: constitutive relations

$\Longrightarrow$ Spin hydrodynamics from W[e,ω]

→ vielbein (e)

→ spin connection (ω)

Spin effective action

Consider quantum field in a nontrivial Lorentz representation

$$e^{iW[e,\omega]} = \int D\Psi e^{iI[e,\omega,\Psi]}$$

Variations define the energy-momentum and spin current

$$T^{\mu\nu} = \frac{\delta W}{\delta e^a_\mu} e^\nu_a, \qquad S^\lambda_{ab} = \frac{\delta W}{\delta \omega^{ab}_\lambda}$$

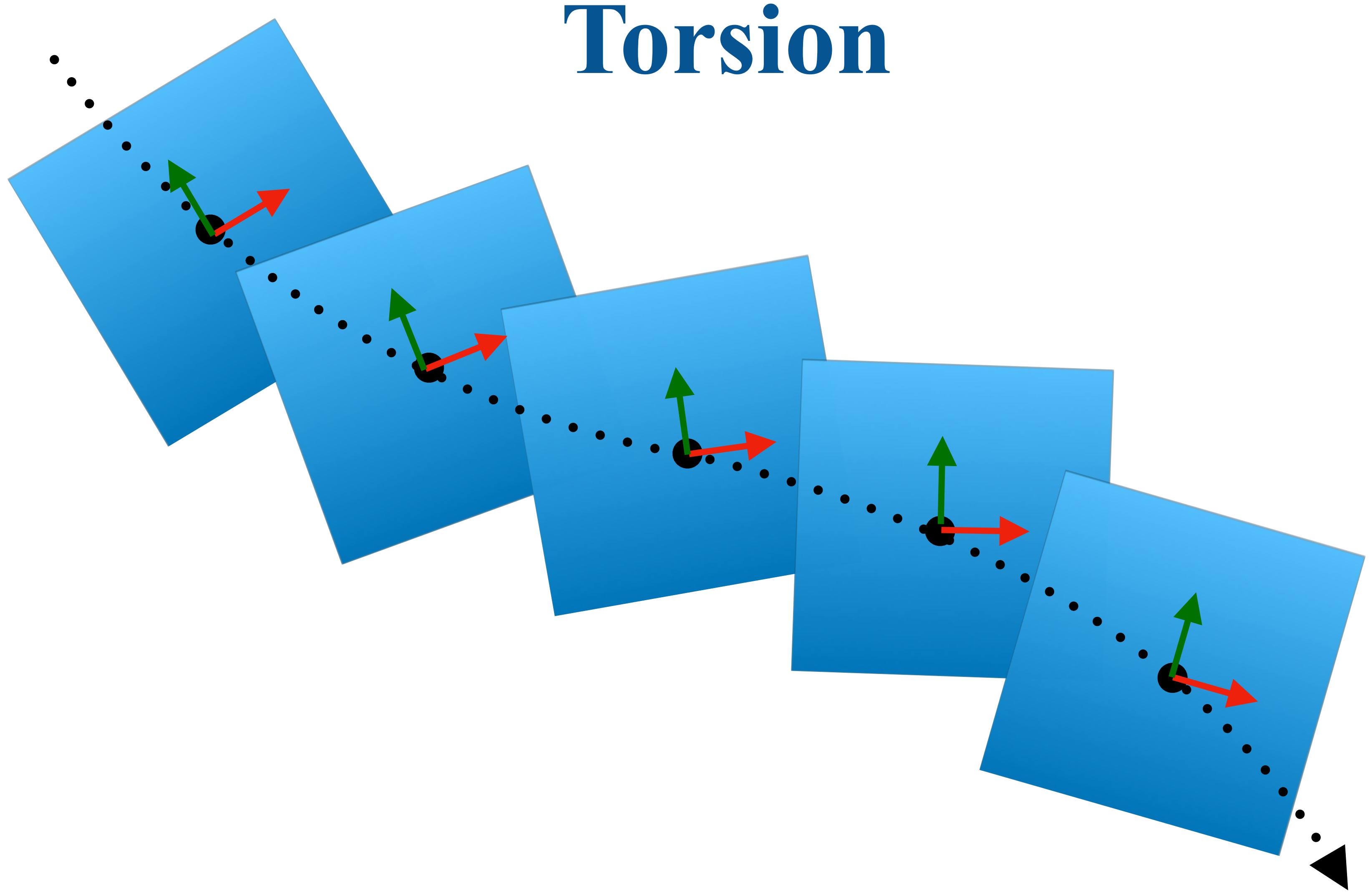
Metric and spin connection are independent in presence of torsion:

$$de^a + \omega^a_b e^b = T^a$$

Belinfante-Rosenberg ambiguity is torsion $\Phi^{\lambda\mu\nu} \Leftrightarrow T^a_{\mu\nu}$

$\Longrightarrow$ Keep T^a as external source, $T^a \to 0$ at the end.

Torsion



$$T^{\alpha}_{\mu\nu} = \Gamma^{\alpha}_{\mu\nu} - \Gamma^{\alpha}_{\nu\mu}$$

Asymmetric affine connection

$$T^a = de^a + \omega^a_b \wedge e^b$$

Covariant derivative of vierbein

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Metric and spin connection are dependent:

$$de^a + \omega^a_b e^b = 0$$

$$\left.\frac{\delta W}{\delta e}\right|_{constraint} \quad \Rightarrow \quad T'^{\mu\nu} = T^{\mu\nu} + \frac{1}{2}\partial_\lambda \left(S^{\lambda\mu\nu} - S^{\mu\lambda\nu} - S^{\nu\lambda\mu}\right)$$

Hydrodynamic equations

Require invariance of W[e,ω] under

1. Diffeomorphisms

$$\delta_\xi e^a = \mathcal{L}_\xi e^a \,, \qquad \delta_\xi \omega^{ab} = \mathcal{L}_\xi \omega^{ab}$$

2. Local Lorentz transformations

$$\delta_\lambda e^a = -\lambda^a{}_b e^b \,, \qquad \delta_\lambda \omega^{ab} = D\lambda^{ab}$$

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Relativistic hydrodynamics with spin current

$$\mathring{\nabla}_\mu T^{\mu\nu} = \frac{1}{2} R^{\rho\sigma\nu\lambda} S_{\rho\lambda\sigma} - T_{\rho\sigma} K^{\nu ab} e^\rho{}_a e^\sigma{}_b$$ analogous to EM work $F^{\mu\nu} J_\mu$

$$\mathring{\nabla}_\lambda S^\lambda{}_{\mu\nu} = 2T_{[\mu\nu]} - 2S^\lambda{}_{\rho[\mu} e_{\nu]}{}^a e_\rho{}^b K_{\lambda ab} \,,$$ antisymm. stress generates S

Hydrostatic equilibrium

Thermal equilibrium in presence of time-independent sources

$$\pounds_\xi e^a{}_\mu = 0 \qquad\qquad \pounds_\xi \omega_\mu^{ab} = 0$$

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Thermal equilibrium in presence of time-independent sources

$$\pounds_\xi e^a{}_\mu = 0$$

$$\pounds_\xi \omega_\mu^{ab} = 0$$

$$a_\mu = -\frac{\mathring{\nabla}_\mu T}{T}$$

acceleration

$$\mu^{ab} = u^\mu \underbrace{K_\mu{}^{ab}}_{\text{torsion}} + 2u^{[a}\underbrace{a^{b]}}_{\text{acceleration}} - \underbrace{\Omega^{ab}}_{\text{vorticity}}$$

(thermal vorticity)

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(thermal vorticity)

- Spin potential $\Longrightarrow$ thermal vorticity at equilibrium
- Torsion and thermal vorticity $\Longrightarrow$ spin density
- μ^{ab} are independent variables out of equilibrium

Hydrostatic equilibrium

Thermal equilibrium in presence of time-independent sources

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- Torsion and thermal vorticity $\Longrightarrow$ spin density
- μ^{ab} are independent variables out of equilibrium

$\Longrightarrow$ To be determined by hydrodynamic equations + constitutive relations

Summary and outlook

- Bayesian analysis + hydro $\Rightarrow$ transport coefficients of QGP

  Collisions with different nuclei e.g. Oxygen Nijs, van der Schee '21

  Extension to include magnetic fields and rotation

- Magneto-hydrodynamics and a perturbative scheme

  Full magnetohydro needed to explain data $\Rightarrow$ CME, CVE

- Systematic study of spin transport in relativistic hydrodynamics

  Realistic hydro simulations, resolve open puzzles:
  sign in longitudinal polarization Becattini and Karpenko '16; Bhadury et al '21

- Holographic description of magnetic QGP and the spin flow

First order hydrostatics

In this talk: Conformal and parity invariant fluid

Weyl invariance: $\delta S = 0\,, \qquad e^a{}_\mu \to e^\phi e^a{}_\mu$

$\Longrightarrow$ Conformal Ward identity of spin fluid: $\quad T^\mu{}_\mu = \mathring{\nabla}_\mu S_\lambda{}^{\lambda\mu}$

$$\epsilon = \epsilon_0 T^4 + 3\rho_0 M^2 T^2 + \cdots\,, \quad P = \frac{1}{3}\epsilon_0 T^4 + \rho_0 M^2 T^2 + \cdots\,, \quad \rho_{ab} = 8\rho_0 T^2 M_{ab} + \cdots$$

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Most general correction to ideal fluid:

$$W_h = \int d^4x |e| \left(\chi^{(1)} T^3 \kappa + 2\chi_1^{(2)} T^2 \kappa_A^{\mu\nu} M_{\mu\nu} + 2\chi_2^{(2)} T^2 K^{\mu\nu} M_{\mu\nu} \right)$$

linear in torsion

$$\Longrightarrow\quad S^\lambda_{ab} = u^\lambda \rho_{ab} + 2T^3\chi^{(1)}\Delta^\lambda{}_{[a}u_{b]} - 4T^2\chi_1^{(2)} M^\lambda{}_{[a}u_{b]} + 4T^2\chi_2^{(2)} u^\lambda M_{ab}$$

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$$\Longrightarrow\quad S^\lambda_{ab} = u^\lambda \rho_{ab} + 2T^3\chi^{(1)}\Delta^\lambda{}_{[a}u_{b]} - 4T^2\chi_1^{(2)} M^\lambda{}_{[a}u_{b]} + 4T^2\chi_2^{(2)} u^\lambda M_{ab}$$

universal
"unsourced" component

Non-equilibrium corrections

All conformal and parity invariant contributions that vanish at equilibrium:

$$\delta S^{\lambda}{}_{ab} = 2\sigma_1 \underbrace{\sigma^{\lambda}{}_{[a} u_{b]}}_{\text{shear induced spin current}} + 2\sigma_2 \hat{M}^{\lambda}{}_{[a} u_{b]} + 2\sigma_3 \Delta^{\lambda}{}_{[a} \hat{m}_{b]} + 2\sigma_4 u^{\lambda} u_{[a} \hat{m}_{b]} + 2\sigma_5 u^{\lambda} \hat{M}_{ab}$$

$$\hat{M} = M + \Omega \qquad \hat{m} = m - a$$

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shear induced
spin current

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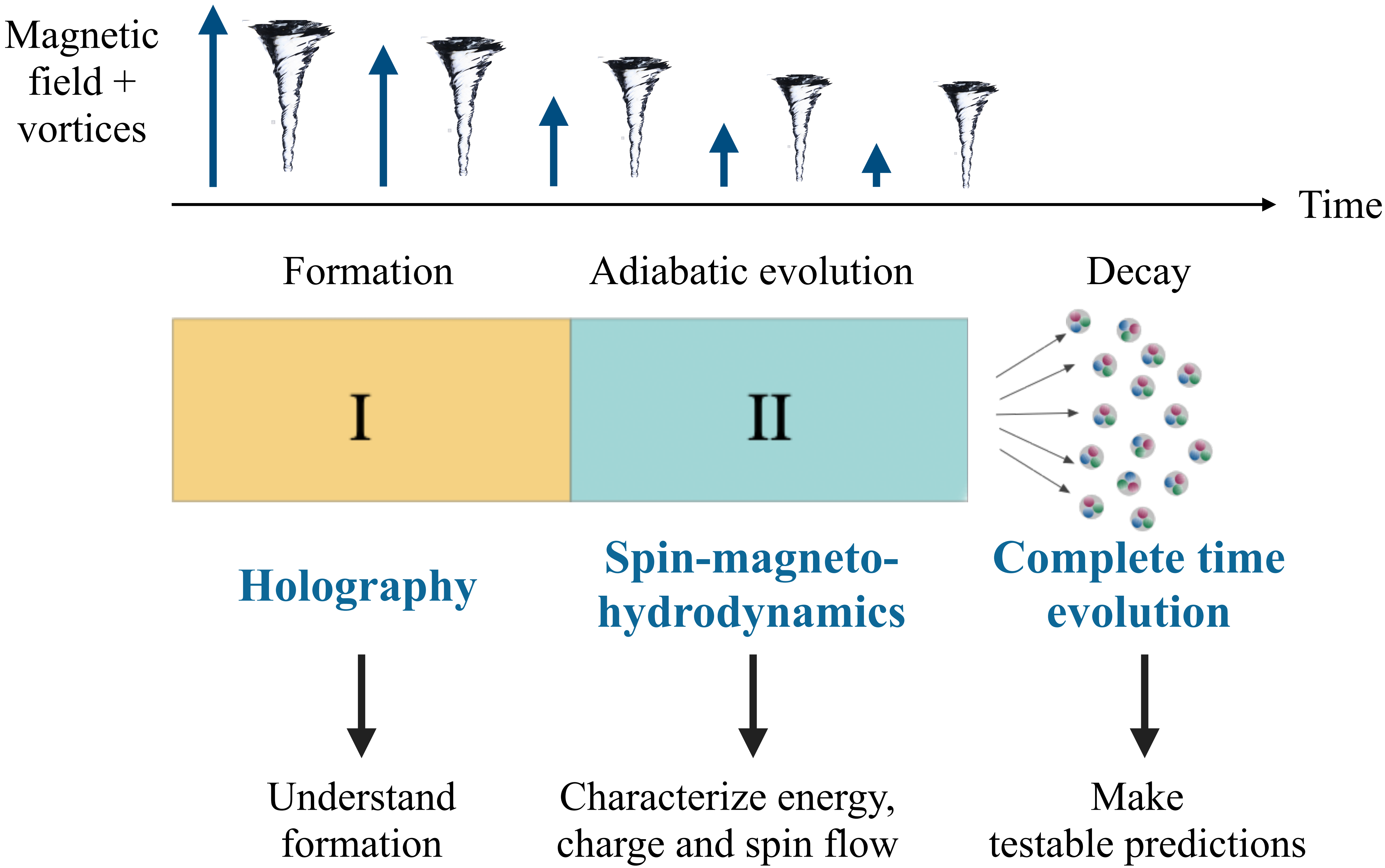
Constraints on transport coefficients: $\mathring{\nabla}_{\mu} J^{\mu}_{S} \geq 0$

+ Onsager relations, CPT etc.



“Hydro-holographic” theory of strongly interacting plasmas