On damping of spin waves and further developments in ideal-spin hydrodynamics

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Sep 17, 2024
Spin and quantum features of QCD plasma

Tiridates I (𐭕𐭉𐭓𐭉𐭃𐭕, Τιριδάτης)

Vologases I (𐭅𐭋𐭂𐭔)

Tiridates may have made it to Rome and even got Nero's royal treatment, kneeling and all, with a diadem and a kiss—but let's be honest, he never ventured to Trento.

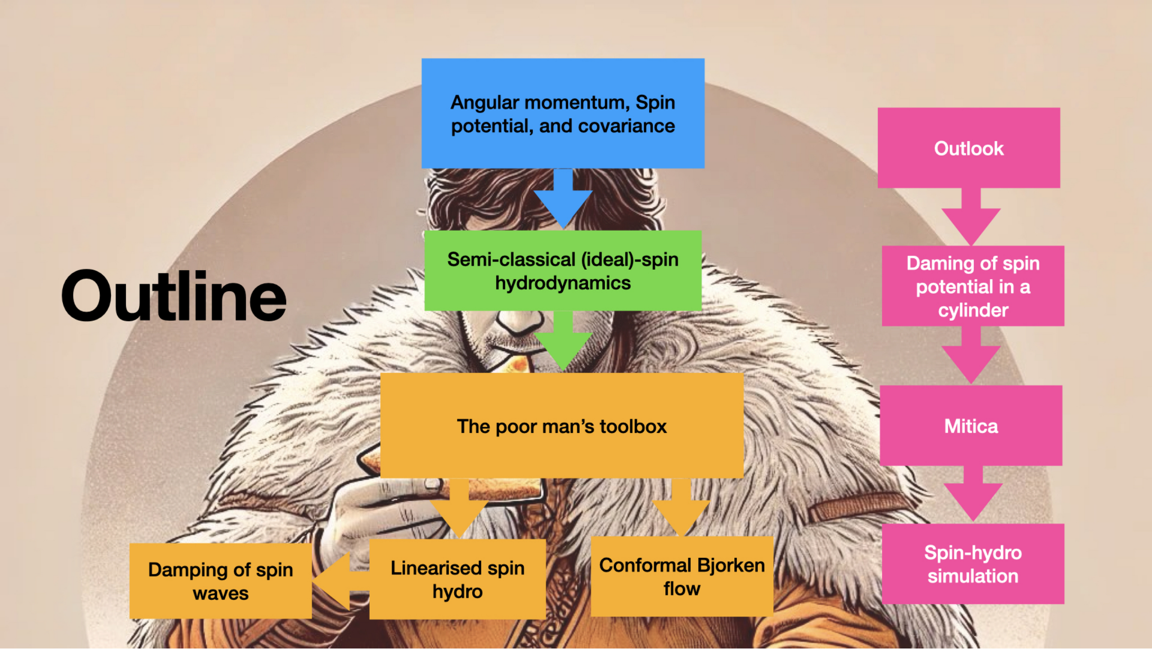
"It is far easier for you than for me to traverse so great a body of water. Therefore, if you will come to Asia, we can then arrange to meet each other."

"As the second Parthian to finally make the journey, I may not have knelt before an emperor, nor did I get a royal diadem, but I've made it here to Trento—pizza in hand, no less! Like Vologases, I knew the water was far easier to cross in this century. And while Tiridates earned his place in Rome, I'd say my journey to Trento has its own historic value."

In the last few months we were thinking about...

- Granted the success of the local-equilibrium (LEQ) assumption, what is the fate of the spin potential?
- Simplifying semi-classical spin hydrodynamics [Weickgenannt et al. (2022)] such that it can be solved with a minimal number of parameters
- Investigating the question of stability in spin hydrodynamics

Outline


Angular momentum, Spin potential, and covariance
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Angular momentum, Spin potential, and covariance

- A small body is inside a rigidly rotating environment
- The body reaches equilibrium with the environment when the entropy is maximized under the constraint of conserved charges $\{Q^I\}$ [Gavassino and Shokri (2023)]

$$\Phi = S + \alpha_I^\star Q^I \leq \log Z \qquad \alpha_I^\star = -\frac{\partial S_E}{\partial Q_E^I}$$

- The environment performs work to shift the system slightly out of equilibrium [Landau and Lifshitz (1980)]

$$\delta W_{\min} = \delta E - T_E \delta S + P_E \delta V - \boldsymbol{\Omega}_E \cdot \delta \mathbf{L} > 0$$

$$\underbrace{\left(\frac{1}{T_E} - \frac{\partial S}{\partial E}\right)\delta E + \left(\frac{P_E}{T_E} - \frac{\partial S}{\partial V}\right)\delta V - \left(\frac{\partial S}{\partial \mathbf{L}} + \frac{\boldsymbol{\Omega}_E}{T_E}\right)\cdot \mathbf{L}}_{\text{vanishes} \implies \text{Intensive parameters of the body relaxes to the ones of the environment}} + \underbrace{\mathcal{O}\left(\delta^2\right)}_{\text{Thermo inequalities}} > 0$$

- Angular momentum's intensive parameter (in equilibrium):

$$\frac{\partial S}{\partial \mathbf{J}} = -\frac{\mathbf{\Omega}_E}{T_E}$$

- In equilibrium $\alpha_I^\star = -\lambda_I$

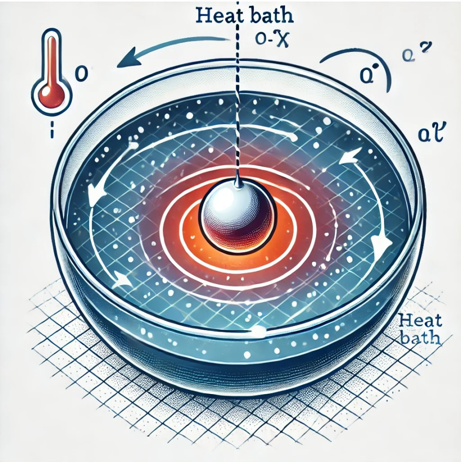
$$\hat{\rho} = \frac{1}{Z} e^{-\lambda_I \hat{Q}^I} \qquad Z = \mathrm{Tr}\left(e^{-\lambda_I \hat{Q}^I}\right)$$

- Covariantly charges are defined as the following integrals over arbitrary Cauchy hypersurfaces:

$$Q^I = \int \mathrm{d}\Sigma_\mu \, J^{\mu I} \quad \text{where} \quad D_\mu J^{\mu I} = 0$$

- We can then define

$$\Phi = \int \mathrm{d}\Sigma_\mu \, \phi^\mu \quad \text{where} \quad \phi^\mu = S^\mu + \alpha_I^\star J^{\mu I}$$



- Internal symmetries:

$$D_\mu N^\mu = 0 \to N = \int \mathrm{d}\Sigma_\mu \, N^\mu$$

- Conservation of energy-momentum and angular momentum

$$D_\mu T^{\mu\nu} = 0$$

- Perfect fluid ($J^{\lambda\mu\nu}$ is automatically conserved)

$$J^{\lambda\mu\nu} = L^{\lambda\mu\nu} + \mathcal{S}^{\lambda\mu\nu} \qquad L^{\lambda\mu\nu} \equiv 2T^{\lambda[\nu}x^{\mu]}$$

Notations and conventions:

$$\eta_{\mu\nu} = \mathrm{diag}(1,-1,-1,-1) \qquad A^{[\mu\nu]} \equiv \tfrac{1}{2}(A^{\mu\nu} - A^{\nu\mu})$$

“Those who violate the principle of general covariance, however swift their computations may be, cannot escape Mithra’s judgment; running through equations cannot save them, and manipulating coordinates will not hide the error. The formula that defies covariance collapses upon itself, and Mithra will find them, for the number of flawed transformations in their calculations will doom the entire result.”

Energy-Momentum Tensor and Spin Dynamics

- Non-symmetric energy-momentum tensor:

$$T^{\mu\nu} = T^{(\mu\nu)} + T^{[\mu\nu]}$$

- Spin dynamics postulate:

$$T^{[\mu\nu]} = \frac{1}{2}\nabla_\lambda \mathcal{S}^{\lambda\mu\nu}$$

- Fluxes of conserved charges with non-symmetric energy-momentum tensor:

$$\mathcal{J}^{\mu h} = T^{\mu\nu} K^h_\nu - \frac{1}{2}\mathcal{S}^{\mu\alpha\beta} D_{[\beta} K^h_{\alpha]}$$

- K^h are the independent Killing vectors $D_{(\mu} K_{\nu)} = 0$
- For generators of rotation $r = 1, 2, 3$ we find total angular momentum:

$$J^{\lambda r} = L^{\lambda r} + \mathcal{S}^{\lambda r} \qquad L^{\lambda r} \equiv T^{\lambda\nu} K^r_\nu \qquad \mathcal{S}^{\lambda r} \equiv \frac{1}{2}\mathcal{S}^{\lambda\mu\nu} D_{[\nu} K^r_{\mu]}$$

Happy now? Remember this is only in flat spacetime!

Thermal Killing Vector

- We define the thermal Killing vector by combining $\{K_h\}$:

$$\beta^\star = -\alpha^\star_h K^h \qquad \beta^\star_\mu \beta^{\star\mu} > 0$$

- We can separate $h = 0$

$$\beta^\star = b^\star - \alpha^\star_m K^m \qquad b^\star = \frac{1}{T_0}\frac{\partial}{\partial t}$$

- This relation is the covariant form of the standard relation **[De Groot (1980)]**

$$\beta^\star_\mu = b^\star_\mu + \varpi^\star_{\mu\nu} x^\nu \ , \tag{1}$$

- The phi-vector:

$$\phi^\mu = S^\mu + \xi^\star N^\mu - T^{\mu\nu}\beta^\star_\nu + \frac{1}{2}\mathcal{S}^{\mu\alpha\beta}\varpi^\star_{\alpha\beta}$$

- Stationary point conditions:

$$\frac{u^\mu}{T} = \beta^{\star\mu} \qquad T^{[\mu\nu]} = 0 \qquad \frac{\mu}{T} = \xi^\star \qquad \Omega_{\alpha\beta} \equiv \varpi^\star_{\alpha\beta} = -D_{[\alpha}\beta_{\beta]}$$

- Spin potential:

$$\Omega_{\alpha\beta} = -\left.\frac{\partial s}{\partial \mathcal{S}^{\alpha\beta}}\right|_{n,\epsilon} \qquad \mathcal{S}^{\alpha\beta} = u_\mu \mathcal{S}^{\mu\alpha\beta}$$

- In LTE

$$\mathrm{d}s = \frac{1}{T}\,\mathrm{d}\varepsilon - \xi\,\mathrm{d}n - \Omega_{\alpha\beta}\,\mathrm{d}\mathcal{S}^{\alpha\beta}$$

What is semi-classical spin hydrodynamics?

“Then Ardvi Sura Anahita proceeded forth from the equations of hydrodynamics. Powerful were her arms, capable of controlling the most turbulent flows, as thick as the mightiest vortex, or still thicker; beautiful was her symmetry, and thus came she, strong, with the force of fluid momentum, thinking thus in her heart:”

“Who will praise me? Who will offer me a sacrifice of well-prepared simulations, with well-posed equations, and solutions cleanly resolved? To whom shall I cleave, who cleaves unto me, and honors conservation of charges, bestowing gifts of accurate results, and is of good will unto me?”

“For her brightness and glory, we offer a sacrifice of perfect solutions; we offer unto the holy Ardvi Sura Anahita, goddess of hydrodynamics, a good simulation, with flows fully resolved and no computational errors. Thus mayest thou guide us when appealed to!”

“Lo, we stand before thee to unveil the mysteries of relativistic spin hydrodynamics. Grant us thy favor, O Anahita, and bestow upon us thy power and glory in this noble pursuit!”

Aban Yasta, 133-137

In **global equilibrium**, the absence of dissipative terms is not determined by the microscopic nature of the fluid. Instead, it is governed by the geometry of the spacetime and the intensive parameters of the environment, which dictate the possible equilibrium states.

In **local thermal equilibrium**, thermodynamic quantities can be defined within macroscopically infinitesimal domains of the system, called **fluid cells**. The applicability of the local equilibrium approximation depends on the properties of the medium.

To reach global equilibrium, dissipation must occur. In a causal theory, this process does not happen instantaneously but takes time. Our question is: *what is the time scale required for the spin potential to relax to the thermal vorticity?*

Three assumptions of semiclassical spin hydro

- Macroscopic quantum corrections $\rightarrow$ semi-classical expansion in $\hbar$ (We truncate all the **equations** up to first order in $\hbar$) [?]
- **Assumption I**: spin tensor is small

$$\mathcal{S}^{\lambda\mu\nu} = \hbar S^{\lambda\mu\nu} \qquad \hbar D_\lambda S^{\lambda\mu\nu} = 2T^{[\nu\mu]}$$

- **Assumption II**: In global equilibrium

$$T^{[\nu\mu]} \overbrace{=}^{\text{geq}} 0 \qquad D_\lambda S^{\lambda\mu\nu} \overbrace{=}^{\text{geq}} 0 \qquad T^{(\mu\nu)} \overbrace{=}^{\text{geq}} T^{\mu\nu}_{(0)}$$

- **Assumption III**:

$$T^{(\mu\nu)} = \overbrace{\underset{\text{standard}}{T^{(\mu\nu)}}}^{\mathcal{O}\left(\hbar^0\right)} + \mathcal{O}\left(\hbar^2\right) \qquad T^{[\mu\nu]} = \mathcal{O}\left(\hbar^2\right) \rightarrow \overbrace{D_\mu T^{(\mu\nu)} = \mathcal{O}\left(\hbar^2\right)}^{\text{No back-reaction}}$$

The symmetric part of the energy-momentum tensor

$$T^{(\mu\nu)} = \mathcal{E} u^\mu u^\nu - \mathcal{P}\Delta^{\mu\nu} + \mathcal{Q}^\mu u^\nu + \mathcal{Q}^\nu u^\mu + \mathcal{T}^{\mu\nu}$$

Semiclassical Landau frame

$$\mathcal{E} = u_\mu u_\nu T^{(\mu\nu)} = \varepsilon + \mathcal{O}\left(\hbar^2\right) \qquad \mathcal{P} = -\frac{1}{3}\Delta_{\alpha\beta}T^{\alpha\beta} = p + \Pi + \mathcal{O}\left(\hbar^2\right)$$

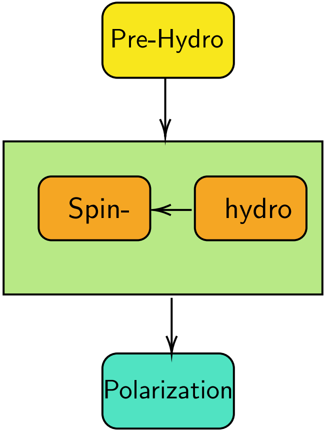
$$\mathcal{Q}^\mu = \Delta^{\mu\alpha} u^\beta T_{\alpha\beta} = \mathcal{O}\left(\hbar^2\right) \qquad \mathcal{T}^{\mu\nu} = \Delta^{\mu\nu}_{\alpha\beta} T^{(\alpha\beta)} = \pi^{\mu\nu} + \mathcal{O}\left(\hbar^2\right)$$

MIS-type EOM for dissipative fluxes

$$\tau_\Pi \dot{\Pi} + \Pi = -\zeta\theta + \cdots \qquad \tau_\pi \dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} + \cdots$$

Notations:

$$\dot{X} \equiv u^\mu D_\mu X \qquad \Delta^{\mu\nu}_{\alpha\beta} := \Delta^{(\mu}_\alpha \Delta^{\nu)}_\beta - \Delta^{\mu\nu}\Delta_{\alpha\beta}/3$$



- N_f fluid fields $\{\varphi^A\} = \{\varepsilon, u^\mu, \Pi, \pi^{\mu\nu}, \cdots\}$
- By solving standard dissipative hydro (DNMR in our case) we find $\{\varphi^A\}$
- Also N_s spin degrees $\{\psi^A\}$ of freedom (spin potential . . .)
- Knowing $\{\varphi^A\}$ we can solve $\hbar D_\lambda S^{\lambda\mu\nu} = 2T^{[\nu\mu]}$ and other required equations to find $\{\psi^A\}$
- Inserting the results into the formula for Π^μ on the FO surface we find the polarization

Ideal-spin hydrodynamics

- 24 components of $S^{\lambda\mu\nu}$ and 6 equations $\rightarrow$ further equations are needed (e.g., from the method of moments)
- In ideal-spin approximation d.o.f in $S^{\lambda\mu\nu}$ are only the 6 ones in $\Omega^{\mu\nu}$

$$S^{\lambda\mu\nu} = Au^\lambda \Omega^{\mu\nu} + Bu^\lambda u_\alpha \Omega^{\alpha[\mu} u^{\nu]} + Cu^\lambda \Omega^{\alpha[\mu} \Delta^{\nu]}{}_\alpha + Du_\alpha \Omega^{\alpha[\mu} \Delta^{\nu]\lambda} + E\Delta^\lambda{}_\alpha \Omega^{\alpha[\mu} u^{\nu]}$$

- Constraint from Assumption III: $B - C - D + T\frac{\partial E}{\partial T} = 0$

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* $\{A, B, C, D, E\}$ are functions of ε or, equivalently, T
* In quantum kinetic theory

$$A = \frac{\hbar T^2}{4m^2}\frac{\partial}{\partial T}(\varepsilon - 3P) \qquad B = \frac{\hbar T^2}{4m^2}\frac{\partial \varepsilon}{\partial T} \qquad C = D = E = -\frac{\hbar T^2}{4m^2}\frac{\partial P}{\partial T}$$

- The antisymmetric part of energy-momentum tensor

$$T^{[\mu\nu]} = -\hbar^2\Gamma^{(\kappa)} u^{[\mu} \left(\kappa^{\nu]} + \varpi^{\nu]\alpha} u_\alpha\right) + \frac{1}{2}\hbar^2\Gamma^{(\omega)}\epsilon^{\mu\nu\rho\sigma} u_\rho \left(\omega_\sigma + \beta\Omega_\sigma\right)$$
$$+ \hbar^2\Gamma^{(a)} u^{[\mu} \left(\beta a^{\nu]} + \nabla^{\nu]}\beta\right)$$

- Why "ideal": **spin** contributions to entropy production is of higher order in $\hbar$

$$\mathrm{d}s = \frac{1}{T}\,\mathrm{d}\varepsilon - \xi\,\mathrm{d}n + \mathcal{O}\left(\hbar^2\right)$$

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* $\{\Gamma^{(\kappa)}, \Gamma^{(\omega)}, \Gamma^{(a)}\}$ are functions of ε or, equivalently, T
* Notations

$$\Omega^\mu = -\tfrac{1}{2}\epsilon^{\mu\nu\alpha\beta}u_\nu D_\alpha u_\beta \qquad \kappa^\mu = -\Omega^{\mu\nu}u_\nu \qquad \omega^\mu = \tfrac{1}{2}\epsilon^{\mu\nu\alpha\beta}u_\nu\Omega_{\alpha\beta} \qquad a_\mu = u^\alpha D_\alpha u_\mu$$

Hang in there, we are almost at the fun part!

The poor man’s toolbox

We need a solution of hydrodynamics to feed into the spin equations

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We need a solution of hydrodynamics to feed into the spin equations

In a poor man's toolbox:

- **Damping of spin waves in a hydrostatics background** [D. Wagner, M.S, and D. H. Rischke arXiv:2405.00533] (Similar works: [Ambrus et al. (2022)] and [Singh et al. (2023)])
- **Linear spin hydro** [J. Sammet, M.S., D. Wagner, and D. H. Rischke, in preparation] (Similar works: [Ren et al. (2024)] and [Daher et al. (2024)])
- **Bjorken spin hydro** [A. Chiarini, M.S., D. Wagner, and D. H. Rischke work in progress] [(Kind of) similar work: [Singh et al. (2021)]]
- **Rigidly rotating fluid** [A. Chiarini, M.S., D. Wagner, A. Dash, and D. H. Rischke work in progress]

What can we possibly learn from each one?

Different contributions can be investigated in these three simple setups

$$\tau_\kappa \dot{\kappa}^{\langle\mu\rangle} = \overbrace{F(\theta, \dot{\beta})}^{\text{Bjorken}} \kappa^\mu + (\cdots) \left(\overbrace{\omega^{\mu\nu}}^{\text{Rigid rot.}} + \overbrace{\sigma^{\mu\nu}}^{\text{Bjorken}} \right) \kappa_\nu$$

$$+ \overbrace{\epsilon^{\mu\alpha\beta\sigma}\omega_\sigma G_{\alpha\beta}}^{\text{Linear and Rigid rot.}} + (\cdots) \overbrace{u_\mu T^{[\mu\nu]}}^{\text{Linear and Rigid rot.}}$$

Equations for both κ and ω are relaxation-type equations

$$\tau_\kappa = -\frac{A - B - C}{\hbar \Gamma^{(\kappa)}} \qquad \tau_\omega = -\frac{E}{\hbar \Gamma^{(\kappa)}}$$

In quantum kinetic theory

$$\tau_\kappa = \frac{T}{2m^2\Gamma^{(\kappa)}} (\varepsilon + P) \qquad \tau_\omega = \frac{T}{4m^2\Gamma^{(\omega)}} (\varepsilon + P) \left(1 - \frac{1}{v_s^2}\right)$$

Linear regime

Linearized ideal-spin hydrodynamics

Main idea: linearized equations of hydrodynamics in a homogeneous equilibrium configuration have linear wave solutions

- Independent degrees of freedom in $T^{(\mu\nu)}$ are $\varphi \in \{\beta = 1/T, u^\mu, \cdots\}$ and in $S^{\lambda\mu\nu}$ are $\psi \in \{\kappa^\mu, \omega^\mu, \cdots\}$
- For each $X \in \{\phi, \psi\}$: $X_0 \to X_0 + \delta X$ (X_0 is constant in a homogeneous equilibrium configuration)
- Fourier transform

$$\delta X(x) = \int \frac{\mathrm{d}^4 k}{(2\pi)^4} e^{ik\cdot x/\hbar} \delta X(k)$$

Linearized ideal-spin hydrodynamics

- Insert δX into the EOM (in Fourier space)

$$ik_\mu \delta T^{(\mu\nu)} = 0 \qquad ik_\lambda \delta S^{\lambda\mu\nu} = \delta T^{[\nu\mu]} \quad \text{and EOM for dissipative fluxes}$$

- EOM to the matrix form $\mathbf{M}\,\delta\vec{\mathbf{X}} = 0$
- This equation has solutions iif $\det(\mathbf{M}) = 0$
- $\Longrightarrow$ dispersion relations for eigenfrequencies $\omega = \omega(\mathbf{k})$ (if $k^\mu = (\omega, \mathbf{k})$)
- Performing this procedure for the example of ideal-spin hydrodynamics and DNMR (or MIS) theory with shear viscosity alone $\rightarrow$ spin and fluid waves are decoupled!

$$\omega^2 - i\hbar a\omega - v_{\mathfrak{s}}^2\mathbf{k}^2 - \hbar^2 b = 0 \qquad a = \frac{\tau_\kappa + \tau_\omega}{\tau_\kappa\tau_\omega} \quad b = \frac{1}{\tau_\kappa\tau_\omega} \quad v_{\mathfrak{s}}^2 = \frac{\Gamma^{(\kappa)}\tau_\kappa}{4\Gamma^{(\omega)}\tau_\omega}$$

Where did all fluid's contributions go?

- Let's assume a fluid with N_f degrees of freedom: φ^A where $A = 1 \cdots N_f$ (all possible dissipative contributions, multiple charges etc)
- Such that all these equations in the linear order are written as

$$M_{\mathrm{f}}^{AB} \delta\varphi^B = \mathcal{O}\left(\hbar^2\right)$$

- And $S^{\lambda\mu\nu}$ with N_s degrees of freedom: ψ^A where $A = 1 \cdots N_s$ (ideal and dissipative)
- The N_s equations in linear order take the form

$$M_{\mathrm{s}}^{AB} \delta\psi^B + \underbrace{M_{\mathrm{fs}}^{AB} \delta\varphi^B}_{\text{source terms}} = 0$$

Where did all fluid's contributions go?

- All equations can be collectively written as

$$\begin{pmatrix} M_{\mathrm{f}} & \mathcal{O}(\hbar^2) \\ M_{\mathrm{fs}} & M_{\mathrm{s}} \end{pmatrix} \begin{pmatrix} \delta\varphi \\ \delta\psi \end{pmatrix} = \mathbf{0} \,.$$

- But

$$\begin{vmatrix} M_{\mathrm{f}} & 0 \\ M_{\mathrm{fs}} & M_{\mathrm{s}} \end{vmatrix} = \begin{vmatrix} M_{\mathrm{f}} & 0 \\ 0 & M_{\mathrm{s}} \end{vmatrix} = \det(M_{\mathrm{f}})\det(M_{\mathrm{s}})$$

In the absence of back-reaction from the spin to the fluid, the linear characteristic equation that determines the spin modes is decoupled from the fluid modes.

- This means that our theory is in some sense incomplete and stability criteria cannot be extracted
- The situation is not better with information current method:

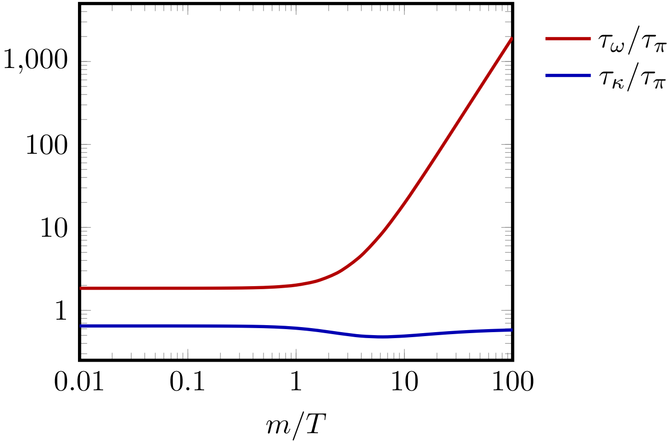
$$\phi^{\mu} = S^{\mu} + \xi^{\star} N^{\mu} - T^{(\mu\nu)} \beta^{\star}_{\nu} + \mathcal{O}\left(\hbar^{2}\right)$$

- There can be spin-induced contributions to $\mathcal{Q}^{\mu}$ and $\pi^{\mu\nu}$ and $\mathcal{P}$ (e.g., in **[Florkowski and Hontarenko (2024)]**)
- For example, in equilibrium

$$\mathcal{Q}^{\mu} \sim \frac{1}{2} T S^{\langle\mu\rangle\alpha\beta} \varpi^{\star}_{\alpha\beta}$$

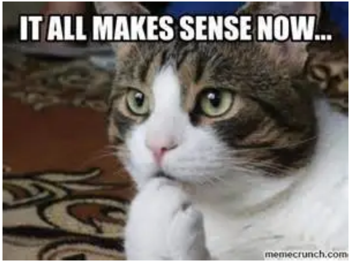
- However, we can still learn about the fate of spin potential

Timescales in spin hydrodynamics



Relaxation times simiar/larger than typical dissipative timescale τ_π for small/large m/T [Wagner et al. (2024)]

Timescales in spin hydrodynamics

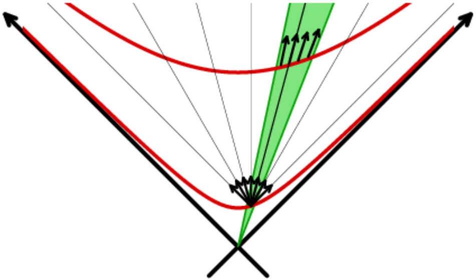


- At late times and small $\mathbf{k}$ the timescale t_d in damping factor $\exp\left(-t/t_d\right)$ is determined by τ_ω
- Spin degrees of freedom relax quite fast in high-energy collisions
- ... while these timescales for low-energy collisions might be even larger than the lifetime of the fireball
- A possible explanation of why the results of [Becattini et al. (2021)] are consistent with data

Bjorken flow

Conformal Bjorken flow

A theorist's favorite solution which unfortunatley does not have thermal vorticity.



Conformal Bjorken flow

- There is a clever parameterization found by Heller and Spalinski (2015):

$$w = T\tau \qquad f(w) = 1 + \frac{\tau}{T}\frac{\mathrm{d}T}{\mathrm{d}\tau} \qquad \mathcal{A} = 18\left(f(w) - \frac{2}{3}\right)$$

- Slow-roll approximation captures the MIS **attractor** at early and late times

$$f(w) = \frac{2}{3} - \frac{w}{8C_{\tau\pi}} + \frac{\sqrt{64C_\eta C_{\tau\pi} + 9w^2}}{24C_{\tau\pi}} \qquad C_{\tau\pi} = T\tau_\pi \qquad C_\eta = \frac{\eta}{s}$$

A quick solution

- Using the slow-roll approximation we find at late times

$$\mathbf{x}_\perp \propto \exp\left(-\frac{C_{\tau\mathbf{x}}}{C_{\tau\pi}^2} w\right) w^{C_{\tau\mathbf{x}}/\rho_\mathbf{x}} \qquad \mathbf{x}_\parallel \propto \exp\left(-\frac{C_{\tau\mathbf{x}}}{C_{\tau\pi}^2} w\right)$$

- Numerical inspection with the assumption of a very small initial value of $\mathbf{x} \in \{\kappa, \omega\}$ shows that:
  - τ_π has a much more important effect than τ_ω and τ_κ
  - A large value of ρ_κ (ρ_ω) can amplify a very small initial spin potential to a very large one

Final words

- Numerically solving spin EOM on top of an uncharged fluid in global equilibrium with a non-vanishing thermal vorticity
- ... to understand better the equilibration timescale of spin degrees of freedom [A. Chiarini, M.S., D. Wagner, A. Dash, and D. H. Rischke, work in progress]s
- **Mitica** - Flexible Hadron Polarization Analysis Code [N. Saß, M.S, A. Palermo, David Wagner, H. Elfner, and Dirk H. Rischke, work in progress]: specialized code designed to implement and compare various hadron polarization formulas, including a novel approach based on quantum kinetic theory. This code builds upon and extensively adapts algorithms from existing particlization frameworks, maintaining the use of OpenMP for optimized performance and faster execution.

Thank you for your attention!

Backup

Local Λ polarization

Solving standard dissipative hydrodynamics and feeding the results into polarization formula of [?] on the freezeout surface: preliminary results signal that the standard shear tensor contributes in the right direction [N. Saß, M.S, A. Palermo, David Wagner, H. Elfner, and Dirk H. Rischke, work in progress]

$$S^{\mu}(p)_{\mathrm{NS}} = \int d\Sigma \cdot p \, \frac{f_{0p}}{2\mathcal{N}} \Bigg\{ - \frac{\hbar}{2m} \tilde{\Omega}^{\mu\nu} p_{\nu} + \left(g^{\mu}_{\nu} - \frac{u^{\mu} p_{\langle\nu\rangle}}{E_p} \right) \times \left[\mathfrak{e} \chi_{\mathfrak{p}} \left(\tilde{\Omega}^{\nu\rho} - \tilde{\varpi}^{\nu\rho} \right) u_{\rho} - \chi_{\mathfrak{q}} \mathfrak{d} \beta_0 \sigma_{\rho}{}^{\langle\alpha} \epsilon^{\beta\rangle\nu\sigma\rho} u_{\sigma} p_{\langle\alpha} p_{\beta\rangle} \right] \Bigg\}$$

Using Q^I_{tot} as constants of motion, we expand the entropy of the environment:

$$S_E(Q^I_{\text{tot}} - Q^I) = S_E(Q^I_{\text{tot}}) + \alpha^\star_I Q^I + \cdots \tag{2}$$

Where:

$$\alpha^\star_I = -\frac{\partial S_E(Q^J_{\text{tot}})}{\partial Q^I_E} \tag{3}$$

The total entropy up to first order in Q^I is:

$$S_{\text{tot}} \approx S_E(Q^I_{\text{tot}}) + \Phi \tag{4}$$

Where:

$$\Phi = S + \alpha^\star_I Q^I \tag{5}$$

Φ must be non-decreasing in time as S_{tot} is non-decreasing.

Generalization to Larger Systems

- Even if the body's charges are not smaller than the environment's, the equilibrium conditions still hold.
- Perturbing the body: $\hat{\rho}' = \hat{\rho} + \delta\hat{\rho}$
- Using Bogoliubov inequality:

$$S[\hat{\rho}'] + \alpha_I^\star \operatorname{Tr}\left(\hat{\rho}'\hat{Q}^I\right) < \ln Z \tag{6}$$

- Hence, $\Phi \leq \ln Z$, maximized in equilibrium.

Physical Meaning of Φ

For a small body with volume V, energy E, and entropy S:

$$-T_E\Phi = E - T_E S \tag{7}$$

- The environment performs work to shift the system slightly out of equilibrium.

$$\delta W_{\min} = \delta E - T_E\delta S + P_E\delta V - \boldsymbol{\Omega}_E\cdot\delta\mathbf{L} > 0\ . \tag{8}$$

- The minimum work required:

$$\delta W_{\min} = P_E\delta V - T_E\delta\Phi \tag{9}$$

Thermodynamic Stability

Expanding δE in terms of δV and δS:

- First-order derivatives cancel: $\delta\Phi = 0$ at first order.
- Second-order terms yield standard thermodynamic inequalities:

$$C_v > 0 \quad , \quad \left.\frac{\partial P}{\partial V}\right|_T < 0 \quad , \quad C_p > 0 \tag{10}$$

This is the essence of the Gibbs stability criterion.

Conformal Bjorken flow

- Let's assume conformal symmetry: $\varepsilon = 3P \sim T^4 \quad \tau_\pi = C_{\tau\pi}/T \quad \eta = C_\eta \frac{\varepsilon+P}{T}$
- Reasons to love Bjorken flow (as a theorist):
  1. A coordinate system (Milne) in which $u^\mu = (1, \mathbf{0})$:

$$\mathrm{d}s^2 = \mathrm{d}\tau^2 - \mathrm{d}x^2 - \mathrm{d}y^2 - \tau^2\,\mathrm{d}\eta_s{}^2 \qquad \tau^2 = t^2 - z^2 \qquad \tanh\eta_s = \frac{z}{t}$$

  2. All quantities are functions of τ only $\implies$ EOM become ODEs
  3. Energy-momentum tensor is diagonal $T^\mu_\nu = \mathrm{diag}(\varepsilon, P_\perp, P_\perp, P_\parallel)$
- The pressure anisotropy is due to the shear-stress tensor (which has one degree of freedom)

$$\mathcal{A} \equiv \frac{P_\parallel - P_\perp}{P_{\mathrm{EQ}}}$$

- There is a clever parameterization found by Heller and Spalinski (2015):

$$w = T\tau \qquad f(w) = 1 + \frac{\tau}{T}\frac{\mathrm{d}T}{\mathrm{d}\tau} \qquad \mathcal{A} = 18\left(f(w) - \frac{2}{3}\right)$$

Bjorken ideal-spin conformal hydrodynamics

- The coefficients from conformal symmetry

$$\{A, B, C, D, E\} \to \{A, B, C, D, E\} T^3 \qquad \{\Gamma^{(\kappa)}, \Gamma^{(\omega)}\} \to \{\Gamma^{(\kappa)}, \Gamma^{(\omega)}\} T^4$$

- Constraint becomes algebraic

$$3E + B - C - D = 0$$

- Using rotational symmetry

$$\kappa^\mu = \left(0, \kappa_\perp(\tau), 0, \frac{\kappa_\parallel(\tau)}{\tau}\right) \qquad \omega^\mu = \left(0, \omega_\perp(\tau), 0, \frac{\omega_\parallel(\tau)}{\tau}\right)$$

Spin equations of motion

- The same equations are found for $\mathbf{x} \in \{\omega, \kappa\}$:

$$C_{\tau\mathbf{x}} \left(w \frac{\mathrm{d}}{\mathrm{d}w} + \frac{\mathcal{A}}{6} \right) \mathbf{x}_\perp + (w - \rho_\mathbf{x})\, \mathbf{x}_\perp = 0$$
$$C_{\tau\mathbf{x}} \left(w \frac{\mathrm{d}}{\mathrm{d}w} + \frac{\mathcal{A}}{6} \right) \mathbf{x}_\parallel + w\mathbf{x}_\parallel = 0$$

- The timescales are redefined as $C_{\tau\mathbf{x}} = T\tau_\mathbf{x}$
- Couplings to the shear tensor: $T\rho_\kappa = D/(\hbar\Gamma^\kappa)$ and $T\rho_\omega = E/(\hbar\Gamma^\omega)$

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