Entropy production and dissipation in spin hydrodynamics

A relativistic quantum-statistical approach

A. Daher (IFJ PAN, Krakow, Poland)

In collaboration with: F. Becattini, X.L. Sheng, D. Wagner (INFN, Firenze, Italy)

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Narodowe Centrum Nauki

The Henryk Niewodniczański Institute of Nuclear Physics Polish Academy of Sciences

NAWA Narodowa Agencja Wymiany Akademickiej

Outline

1. **Motivations, remarks & goals**
2. **Quantum-statistical framework for relativistic fluid with spin current**
3. **Entropy current and entropy production rate**
4. **Dissipative currents: Method and results (Ongoing work)**
5. **Conclusions and outlooks**

Motivations, remarks & goals

Motivations

- **There is a growing interest in spin hydrodynamics,**

[1] W. Florkowski, B. Friman, A. Jaiswal, and E. Speranza, "Relativistic fluid dynamics with spin," *Phys. Rev. C* **97** no. 4, (2018) 041901, arXiv:1705.00587 [nucl-th].
[2] K. Hattori, M. Hongo, X.-G. Huang, M. Matsuo, and H. Taya, "Fate of spin polarization in a relativistic fluid: An entropy-current analysis," *Phys. Lett. B* **795** (2019) 100–106, arXiv:1901.06615 [hep-th].
[3] K. Fukushima and S. Pu, "Spin hydrodynamics and symmetric energy-momentum tensors – A current induced by the spin vorticity –," *Phys. Lett. B* **817** (2021) 136346, arXiv:2010.01608 [hep-th].
[4] M. Hongo, X.-G. Huang, M. Kaminski, M. Stephanov, and H.-U. Yee, "Relativistic spin hydrodynamics with torsion and linear response theory for spin relaxation," *JHEP* **11** (2021) 150, arXiv:2107.14231 [hep-th].
[5] D. She, A. Huang, D. Hou, and J. Liao, "Relativistic viscous hydrodynamics with angular momentum," *Sci. Bull.* **67** (2022) 2265–2268, arXiv:2105.04060 [nucl-th].
[6] A. D. Gallegos, U. Gürsoy, and A. Yarom, "Hydrodynamics of spin currents," *SciPost Phys.* **11** (2021) 041, arXiv:2101.04759 [hep-th].
[7] N. Weickgenannt, D. Wagner, E. Speranza, and D. H. Rischke, "Relativistic second-order dissipative spin hydrodynamics from the method of moments," *Phys. Rev. D* **106** no. 9, (2022) 096014, arXiv:2203.04766 [nucl-th].
[8] R. Biswas, A. Daher, A. Das, W. Florkowski, and R. Ryblewski, "Relativistic second-order spin hydrodynamics: An entropy-current analysis," *Phys. Rev. D* **108** no. 1, (2023) 014024, arXiv:2304.01009 [nucl-th].

- **Spin hydrodynamics includes spin current $< \widehat{S}^{\lambda\mu\nu} >$, and therefore quantum methods cannot be avoided.**

Remarks

- **Spin hydrodynamics suffers from the pseudo-gauge non-invariance,**

$$\widehat{T}'^{\mu\nu} = \widehat{T}^{\mu\nu} + \frac{1}{2}\nabla_\lambda \left(\widehat{\Phi}^{\lambda\mu\nu} - \widehat{\Phi}^{\mu\lambda\nu} - \widehat{\Phi}^{\nu\lambda\mu}\right),$$
$$\widehat{\mathcal{S}}'^{\mu\lambda\nu} = \widehat{\mathcal{S}}^{\mu\lambda\nu} - \widehat{\Phi}^{\mu\lambda\nu}.$$

As a specific example, in this talk, we will discuss pseudo-gauge transformation of the entropy production rate $\partial_\mu s^\mu$.

- **It is indeed not trivial to generalize the local thermodynamic relations from "relativistic hydrodynamics" to "relativistic spin hydrodynamics",**

$$s^{\mu}_{LE} = \phi^{\mu} + T^{\mu\nu}_{\rm LE}\beta_{\nu} - \zeta j^{\mu}_{\rm LE} - \frac{1}{2}\Omega_{\lambda\nu}\mathcal{S}^{\mu\lambda\nu}_{\rm LE}\,.$$

$$\phi^{\mu}(x) = \int_0^{T(x)} \frac{\mathrm{d}T'}{T'^2}\left(T^{\mu\nu}_{\rm LE}(x)[T',\mu,\omega]u_{\nu}(x) - \mu(x)j^{\mu}_{\rm LE}(x)[T',\mu,\omega] - \frac{1}{2}\omega_{\lambda\nu}(x)\mathcal{S}^{\mu\lambda\nu}_{\rm LE}(x)[T',\mu,\omega]\right)$$

Yet, in some limit, it might converge to simple form (FUTURE WORK!!).

Goals

- **One of the main goals of spin hydro is to determine the dissipative currents:**

$$\delta T_S^{\mu\nu}\ ,\quad \delta j^{\mu}\ ,\quad \delta T_A^{\mu\nu}\ ,\quad \delta\mathcal{S}^{\lambda\mu\nu}$$

why?

1. **It will demonstrate how the spin current impacts the relativistic hydrodynamic dissipative currents $\Leftrightarrow$ Emergence of new transport coefficients?**
2. **To reveal the form of the spin dissipative currents and the corresponding spin transport coefficients.**
3. **To solve eventually the system's conservation laws:**

$$\partial_\mu T^{\mu\nu} = 0\ ,\quad \partial_\mu j^{\mu} = 0\ ,\quad \partial_\lambda \mathcal{S}^{\lambda\mu\nu} = T^{\nu\mu} - T^{\mu\nu}.$$

Quantum-statistical framework for relativistic fluid with spin current

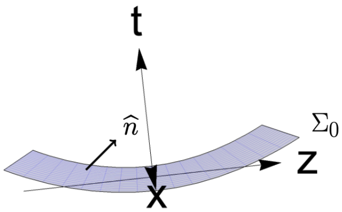
Local equilibrium is achieved at initial hypersurface Σ_0, where entropy is maximum provided that the mean vales of energy, momentum, particle number, and spin densities are their actual values:

$$S = -\operatorname{Tr}(\widehat{\rho}\log\widehat{\rho})$$

$$F[\widehat{\rho}] = -Tr[\widehat{\rho}\log\widehat{\rho}] - \int d\Sigma_0\, n_\mu \,(T^{\mu\nu}_{\rm LE} - T^{\mu\nu})\,\beta_\nu(x) - \int d\Sigma_0\, n_\mu\,(j^\mu_{\rm LE} - j^\mu)\,\zeta(x)$$
$$- \int d\Sigma_0\, n_\mu \left(S^{\mu\lambda\nu}_{\rm LE} - S^{\mu\lambda\nu}\right)\Omega_{\lambda\nu}(x)$$

$$T^{\mu\nu}_{\rm LE} \sim Tr\left[\widehat{\rho}\,\widehat{T}^{\mu\nu}\right]$$

$$T^{\mu\nu} \equiv \text{Actual Value}$$



Local equilibrium is achieved at initial hypersurface Σ_0, where entropy is maximum provided that the mean vales of energy, momentum, particle number, and spin densities are their actual values:

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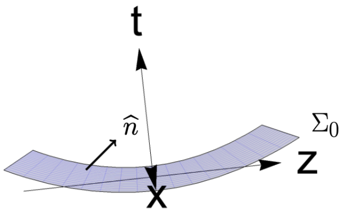


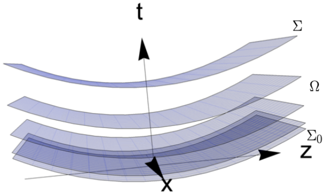
$$F[\widehat{\rho}] = -Tr[\widehat{\rho} \log \widehat{\rho}] - \int d\Sigma_0 \, n_\mu \, (T^{\mu\nu}_{\rm LE} - T^{\mu\nu}) \, \beta_\nu(x) - \int d\Sigma_0 \, n_\mu \, (j^\mu_{\rm LE} - j^\mu) \, \zeta(x)$$
$$- \int d\Sigma_0 \, n_\mu \left(S^{\mu\lambda\nu}_{\rm LE} - S^{\mu\lambda\nu} \right) \Omega_{\lambda\nu}(x)$$

$$T^{\mu\nu}_{\rm LE} \sim Tr\left[\widehat{\rho}\widehat{T}^{\mu\nu}\right]$$
$$T^{\mu\nu} \equiv \text{Actual Value}$$

$$\widehat{\rho}_{\rm LE} = \frac{1}{Z} \exp\left[- \int_{\Sigma_0} {\rm d}\,\Sigma_\mu \left(\widehat{T}^{\mu\nu} \beta_\nu - \zeta \widehat{j}^\mu - \frac{1}{2} \Omega_{\lambda\nu} \widehat{S}^{\mu\lambda\nu} \right) \right]$$

The Lagrange multipliers are obtained by solving the constraint equations at Σ_0. Their evolution is determined by solving the conservation equations:

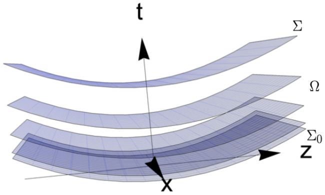
- $\beta^\mu \to u^\mu = \beta^\mu/\sqrt{\beta^2} \quad T = 1/\sqrt{\beta^2}$
- $\zeta = \mu/T$
- $\Omega_{\mu\nu} = \omega_{\mu\nu}/T$
- Thermal Shear: $\xi_{\mu\nu} = \frac{1}{2}\left(\nabla_\mu\beta_\nu + \nabla_\nu\beta_\mu\right)$ Thermal Vorticity: $\varpi_{\mu\nu} = \frac{1}{2}\left(\nabla_\nu\beta_\mu - \nabla_\mu\beta_\nu\right)$



Using Gauss theorem:

$$\widehat{\rho} = \frac{1}{Z} \exp \left[\underbrace{- \int_{\Sigma} \mathrm{d}\,\Sigma_\mu \left(\widehat{T}^{\mu\nu} \beta_\nu - \zeta \widehat{j}^\mu - \frac{1}{2} \Omega_{\lambda\nu} \widehat{\mathcal{S}}^{\mu\lambda\nu} \right)}_{\widehat{\rho}_{\mathrm{LE}}(t) \ at \ \Sigma} + \underbrace{\int_{\Omega} d\Omega \ \widehat{T}_S^{\mu\nu} \xi_{\mu\nu} + \widehat{T}_A^{\mu\nu} (\Omega_{\mu\nu} - \varpi_{\mu\nu}) - \frac{1}{2} \widehat{\mathcal{S}}^{\mu\lambda\nu} \nabla_\mu \Omega_{\lambda\nu}}_{\text{Dissipative Corrections}} \right]$$

Using Gauss theorem:



$$\widehat{\rho} = \frac{1}{Z}\exp\left[\underbrace{-\int_{\Sigma} \mathrm{d}\,\Sigma_\mu \left(\widehat{T}^{\mu\nu}\beta_\nu - \zeta\widehat{j}^\mu - \frac{1}{2}\Omega_{\lambda\nu}\widehat{\mathcal{S}}^{\mu\lambda\nu}\right)}_{\widehat{\rho}_{\mathrm{LE}}(t)\ at\ \Sigma} + \underbrace{\int_{\Omega} d\Omega\ \widehat{T}_S^{\mu\nu}\xi_{\mu\nu} + \widehat{T}_A^{\mu\nu}(\Omega_{\mu\nu} - \varpi_{\mu\nu}) - \frac{1}{2}\widehat{\mathcal{S}}^{\mu\lambda\nu}\nabla_\mu\Omega_{\lambda\nu}}_{\text{Dissipative Corrections}}\right]$$

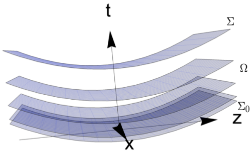
This implies that dissipation in spin hydrodynamics occurs when:

$$\xi \neq 0 \quad \Omega \neq \varpi \quad \nabla\Omega \neq 0$$

Entropy current and entropy production rate

Near local equilibrium at the hypersuface Σ, the entropy is defined as:

$$
\begin{aligned}
S &= -\operatorname{Tr}\left[\widehat{\rho}_{\mathrm{LE}}(t)\log\widehat{\rho}_{\mathrm{LE}}(t)\right] \\
&= \log Z_{\mathrm{LE}} + \int_\Sigma \mathrm{d}\Sigma_\mu \left[\operatorname{Tr}\left(\widehat{\rho}_{\mathrm{LE}}\widehat{T}^{\mu\nu}\right)\beta_\nu - \zeta \operatorname{Tr}\left(\widehat{\rho}_{\mathrm{LE}}\widehat{j}^{\mu}\right) - \frac{1}{2}\Omega_{\lambda\nu}\operatorname{Tr}\left(\widehat{\rho}_{\mathrm{LE}}\widehat{\mathcal{S}}^{\mu\lambda\nu}\right)\right]
\end{aligned}
$$



Can we define an entropy current out of S ? In other words, is it possible to show that $\log Z_{\mathrm{LE}}$ is an extensive quantity?

$$\log Z_{\mathrm{LE}} = \int_{\Sigma} \mathrm{d}\Sigma_{\mu} \phi^{\mu}$$

[F. Becattini, D. Rindori PhysRevD.99.125011]

where ϕ^{μ} is defined as thermodynamic potential vector field:

$$\phi^{\mu}(x) = \int_{0}^{T(x)} \frac{\mathrm{d}T'}{T'^{2}} \left(T^{\mu\nu}_{\mathrm{LE}}(x)[T', \mu, \omega] u_{\nu}(x) - \mu(x) j^{\mu}_{\mathrm{LE}}(x)[T', \mu, \omega] - \frac{1}{2} \omega_{\lambda\nu}(x) \mathcal{S}^{\mu\lambda\nu}_{\mathrm{LE}}(x)[T', \mu, \omega] \right)$$

For a fluid at global equilibrium with vanishing thermal vorticity $\varpi_{\mu\nu} = 0$:

$$\phi^{\mu} = p\, \beta^{\mu}$$

where "p" is the hydrostatic pressure.

Therefore, entropy current exists:

$$S = \int_\Sigma \mathrm{d}\Sigma_\mu \, \phi^\mu + T^{\mu\nu}_{\mathrm{LE}}\beta_\nu - \zeta j^\mu_{\mathrm{LE}} - \frac{1}{2}\Omega_{\lambda\nu}\mathcal{S}^{\mu\lambda\nu}_{\mathrm{LE}}$$

$$s^\mu_{LE} = \phi^\mu + T^{\mu\nu}_{\mathrm{LE}}\beta_\nu - \zeta j^\mu_{\mathrm{LE}} - \frac{1}{2}\Omega_{\lambda\nu}\mathcal{S}^{\mu\lambda\nu}_{\mathrm{LE}} .$$

In quantum theory, we only have the total entropy, not the entropy current. We need to construct an entropy current through an integral. However, this introduces ambiguities, as several fields can lead to the same integral. However if $s^\mu - s^\mu_{LE} \perp n^\mu$**,**

$$s^\mu = \phi^\mu + T^{\mu\nu}\beta_\nu - \zeta j^\mu - \frac{1}{2}\Omega_{\lambda\nu}\mathcal{S}^{\mu\lambda\nu} \qquad \phi^\mu = \int_0^T \frac{\mathrm{d}T'}{T'^2}\left(T^{\mu\nu}[T']u_\nu - \mu j^\mu[T'] - \frac{1}{2}\omega_{\lambda\nu}\mathcal{S}^{\mu\lambda\nu}[T']\right)$$

Using the entropy current $s^\mu = \phi^\mu + T^{\mu\nu}\beta_\nu - \frac{1}{2}\Omega_{\lambda\nu}S^{\mu\lambda\nu}$, we obtain:

$$\partial_\mu s^\mu = \left(T_S^{\mu\nu} - T_{S(\mathrm{LE})}^{\mu\nu}\right)\xi_{\mu\nu} - \left(j^\mu - j^\mu_{\mathrm{LE}}\right)\partial_\mu\zeta + \left(T_A^{\mu\nu} - T_{A(\mathrm{LE})}^{\mu\nu}\right)\left(\Omega_{\mu\nu} - \varpi_{\mu\nu}\right) - \frac{1}{2}\left(\mathcal{S}^{\mu\lambda\nu} - \mathcal{S}^{\mu\lambda\nu}_{\mathrm{LE}}\right)\partial_\mu\Omega_{\lambda\nu}$$

$\varpi_{\mu\nu}$: is the thermal vorticity

This formula is a generalization of what was obtained ***C. Van Weert*** **without spin:**

C. van Weert, "Maximum entropy principle and relativistic hydrodynamics," *Annals of Physics, Volume 140, Issue 1,1982* .

Applying pseudo-gauge transformation to the entropy current (for $\Omega = \varpi$), we get

$$
\begin{aligned}
\phi'^{\mu} &= \phi^{\mu} + \int_0^T \frac{\mathrm{d}T'}{T'} \left[\nabla_\lambda A^{\lambda\mu} - \Phi^{\lambda\mu\nu}\xi_{\lambda\nu}\right], \\
s'^{\mu} &= s^{\mu} + \int_0^T \frac{\mathrm{d}T'}{T'} \left[\nabla_\lambda A^{\lambda\mu} - \Phi^{\lambda\mu\nu}\xi_{\lambda\nu}\right] + \nabla_\lambda A^{\lambda\mu} - \Phi^{\lambda\mu\nu}\xi_{\lambda\nu}.
\end{aligned}
$$

where $A^{\lambda\mu} = \beta_\nu/2\left(\Phi^{\lambda\mu\nu} - \Phi^{\nu\lambda\mu} + \Phi^{\mu\nu\lambda}\right)$ is an anti-symmetric 2-tensor. The last term on the right-hand side cannot be written as a total derivative of an anti-symmetric tensor like in the case of entropy-gauge.

Therefore, the divergence of the entropy current is, in general, not invariant under a pseudo-gauge transformation.

Dissipative currents: Method and results (Ongoing work)

$$\delta T_S^{\mu\nu} = H^{\mu\nu\rho\sigma}\xi_{\rho\sigma} + K^{\mu\nu\rho}\partial_\rho\zeta + L^{\mu\nu\rho\sigma}\left(\Omega_{\rho\sigma} - \varpi_{\rho\sigma}\right) + M^{\mu\nu\rho\sigma\tau}\partial_\rho\Omega_{\sigma\tau},$$

$$\delta T_A^{\mu\nu} = N^{\mu\nu\rho\sigma}\xi_{\rho\sigma} + P^{\mu\nu\rho}\partial_\rho\zeta + Q^{\mu\nu\rho\sigma}\left(\Omega_{\rho\sigma} - \varpi_{\rho\sigma}\right) + R^{\mu\nu\rho\sigma\tau}\partial_\rho\Omega_{\sigma\tau},$$

$$\delta j^\mu = G^{\mu\rho\sigma}\xi_{\rho\sigma} + I^{\mu\rho}\partial_\rho\zeta + O^{\mu\rho\sigma}(\Omega_{\rho\sigma} - \varpi_{\rho\sigma}) + F^{\mu\rho\sigma\tau}\partial_\rho\Omega_{\sigma\tau},$$

$$\delta S^{\mu\lambda\nu} = T^{\mu\lambda\nu\rho\sigma}\xi_{\rho\sigma} + U^{\mu\lambda\nu\rho}\partial_\rho\zeta + V^{\mu\lambda\nu\rho\sigma}\left(\Omega_{\rho\sigma} - \varpi_{\rho\sigma}\right) + W^{\mu\lambda\nu\rho\sigma\tau}\partial_\rho\Omega_{\sigma\tau}.$$

Hence the goal reduces to determining the coefficient tensors:

$$H^{\mu\nu,\rho,\sigma}\,,\quad K^{\mu\nu\rho}\,,\quad L^{\mu\nu\rho\sigma}\,,\quad M^{\mu\nu\rho\sigma\tau}$$

$$N^{\mu\nu\rho\sigma}\,,\quad P^{\mu\nu\rho}\,,\quad Q^{\mu\nu\rho\sigma}\,,\quad R^{\mu\nu\rho\sigma\tau}$$

$$G^{\mu\rho\sigma}\,,\quad I^{\mu\rho}\,,\quad O^{\mu\rho\sigma}\,,\quad F^{\mu\rho\sigma\tau}$$

$$T^{\mu\lambda\nu\rho\sigma}\,,\quad U^{\mu\lambda\nu\rho}\,,\quad V^{\mu\lambda\nu\rho\sigma}\,,\quad W^{\mu\lambda\nu\rho\sigma\tau}$$

Using irreducible representation of *SO(3)* group,

$$\text{Vector:}\quad V^{\mu} = (0 \oplus 1)$$

$$\text{Symmetric 2-tensor:}\quad B^{\mu\nu} = (0 \oplus 0 \oplus 1 \oplus 2)$$

$$\text{Antisymmetric 2-tensor:}\quad A^{\mu\nu} = (1 \oplus 1)$$

Our hydrodynamics "tools" existing at global equilibrium,

$$u^{\mu}\,,\quad \Delta^{\mu\nu}\,,\quad \epsilon^{\mu\nu\alpha\beta}$$

Therefore, the irreducible representation, interms of our hydrodynamic variables:

$$\text{Vector:}\quad V^{\mu} = (u^{\mu} \oplus \Delta^{\mu}_{\alpha})$$

$$\text{Symmetric 2-tensor:}\quad B^{\mu\nu} = (u^{\mu}u^{\nu} \oplus \Delta^{\mu\nu} \oplus u^{\mu}\Delta^{\nu}_{\alpha} + u^{\nu}\Delta^{\mu}_{\alpha} \oplus \Delta^{\mu}_{\alpha}\Delta^{\nu}_{\beta} + \Delta^{\nu}_{\alpha}\Delta^{\mu}_{\beta}),$$

$$\text{Antisymmetric 2-tensor:}\quad A^{\mu\nu} = (u^{\mu}\Delta^{\nu}_{\alpha} - u^{\nu}\Delta^{\mu}_{\alpha} \oplus \epsilon^{\mu\nu\tau\alpha}u_{\tau}).$$

Hence we can decompose any tensor interms of its *SO(3)* irreducible components, by using V^{μ} , $B^{\mu\nu}$, $A^{\mu\nu}$**:**

1. **Any rank**
2. **Any symmetry**

$$W^{\mu\lambda\nu\rho\sigma\tau}: \begin{cases}
w_1\, u^\mu u^{[\lambda} u^\rho u^{[\sigma} \Delta^{\nu]\tau]} \\
w_2\, u^\mu u^{[\lambda} u^\rho u_\beta \epsilon^{\sigma\tau\beta\nu]} \\
w_3\, u^\mu u^\rho \Delta^{[\lambda[\tau} \Delta^{\nu]\sigma]} \Leftrightarrow u^\mu u_\beta u^\rho u_\gamma \epsilon^{\lambda\nu\beta\alpha} \epsilon^{\sigma\tau\gamma}{}_\alpha \\
w_4\, u^\mu u_\beta u^\rho u^{[\sigma} \epsilon^{\lambda\nu\beta\tau]} \\
w_5\, u^{[\lambda} u^{[\sigma} \Delta^{\mu\nu]} \Delta^{\tau]\rho} \\
w_6\, u^{[\lambda} u_\beta \Delta^{\mu\nu]} \epsilon^{\sigma\tau\beta\rho} \\
w_7\, u_\beta u^{[\sigma} \Delta^{\tau]\rho} \epsilon^{\lambda\nu\beta\mu} \\
w_8 (\Delta^{[\lambda\rho} \Delta^{\mu[\sigma} \Delta^{\nu]\tau]} + \frac{1}{2} \Delta^{[\lambda[\tau} \Delta^{\nu]\sigma]} \Delta^{\mu\rho}) \Leftrightarrow u_\beta u_\gamma \epsilon^{\lambda\nu\beta\mu} \epsilon^{\sigma\tau\gamma\rho} \\
w_9\, u^\rho u^{[\sigma} u^{[\lambda} u_\alpha \epsilon^{\mu\nu]\alpha\tau]} \\
w_{10}\, u^\rho u^{[\sigma} \Delta^{[\lambda\tau]} \Delta^{\mu\nu]} \Leftrightarrow u^\rho u^\sigma u_a u_c \epsilon^{\mu\beta\tau a} \epsilon^{\lambda\nu c}{}_\beta - u^\rho u^\tau u_a u_c \epsilon^{\mu\beta\sigma a} \epsilon^{\lambda\nu c}{}_\beta \\
w_{11}\, u^\rho \Delta^{\mu[\tau} \epsilon^{\lambda\nu\sigma]\alpha} u_\alpha \Leftrightarrow u^\rho u_z u_a u_c \epsilon^{\sigma\tau zy} \epsilon^{\mu\beta}{}_y{}^a \epsilon^{\lambda\nu c}{}_\beta \\
w_{12}\, u^{[\lambda} u^\rho \Delta^{\mu[\tau} \Delta^{\nu]\sigma]} \Leftrightarrow u^\rho u^\lambda u_a u_z \epsilon^{\sigma\tau zy} \epsilon^{\mu\nu a}{}_y - u^\rho u^\nu u_a u_z \epsilon^{\sigma\tau zy} \epsilon^{\mu\lambda a}{}_y \\
w_{13}\, u^{[\sigma} u_w u^\mu u^{[\lambda} \epsilon^{\rho\tau]w\nu]} \\
w_{14}\, u^\mu u^{[\sigma} \Delta^{[\lambda\tau]} \Delta^{\nu]\rho} \Leftrightarrow u^\sigma u_w u^\mu \epsilon^{\rho\tau w\beta} \epsilon^{\lambda\nu c}{}_\beta u_c - u^\tau u_w u^\mu \epsilon^{\rho\sigma w\beta} \epsilon^{\lambda\nu c}{}_\beta u_c \\
w_{15}\, (u^{[\lambda} u^{[\sigma} \Delta^{\mu\tau]} \Delta^{\nu]\rho} - u^{[\lambda} u^{[\sigma} \Delta^{\nu]\tau]} \Delta^{\mu\rho}) \Leftrightarrow u^\sigma u_w u^\lambda u_a \epsilon^{\rho\tau wv} \epsilon^{\mu\nu a}{}_v - u^\sigma u_w u^\nu u_a \epsilon^{\rho\tau wv} \epsilon^{\mu\lambda a}{}_v - u^\tau u_w \epsilon^{\rho\sigma wv} \epsilon^{\mu\nu a}{}_v u_a u^\lambda + \imath \\
w_{16}\, (u^{[\sigma} \Delta^{\mu\tau]} \epsilon^{\lambda\nu\rho\alpha} u_\alpha - u^{[\sigma} \epsilon^{\lambda\nu\tau]\alpha} u_\alpha \Delta^{\mu\rho}) \Leftrightarrow u^\sigma u_w u_a u_c \epsilon^{\rho\tau wb} \epsilon^{\mu\beta}{}_b{}^a \epsilon^{\lambda\nu c}{}_\beta - u^\tau u_w u_a u_c \epsilon^{\rho\sigma wb} \epsilon^{\mu\beta}{}_b{}^a \epsilon^{\lambda\nu c}{}_\beta \\
w_{17}\, u^\mu u^{[\nu} \Delta^{\lambda][\tau} \Delta^{\rho\sigma]} \Leftrightarrow u_w u_z u^\mu u^\lambda \epsilon^{\rho y\nu w} \epsilon^{\sigma\tau z}{}_y - u_w u_z u^\mu u^\nu \epsilon^{\rho y\lambda w} \epsilon^{\sigma\tau z}{}_y \\
w_{18}\, u^\mu \Delta^{\rho[\sigma} \epsilon^{\lambda\nu\tau]\alpha} u_\alpha\; u_w u_z u_c u^\mu \epsilon^{\rho y\beta w} \epsilon^{\sigma\tau z}{}_y \epsilon^{\lambda\nu c}{}_\beta \\
w_{19}\, (u^{[\lambda} \Delta^{\nu]\rho} \epsilon^{\mu\sigma\tau\alpha} u_\alpha - u^{[\lambda} \Delta^{\mu\rho} \epsilon^{\nu]\sigma\tau\alpha} u_\alpha) \Leftrightarrow u_w u_z u^\lambda u_a \epsilon^{\rho ybw} \epsilon^{\sigma\tau z}{}_y \epsilon^{\mu\nu a}{}_b - u_w u_z u^\nu u_a \epsilon^{\rho ybw} \epsilon^{\sigma\tau z}{}_y \epsilon^{\mu\lambda a}{}_b \\
w_{20}\, \Delta^{[\lambda\mu} \Delta^{\nu][\tau} \Delta^{\rho\sigma]} \Leftrightarrow u_w u_z u_a u_c \epsilon^{\rho ybw} \epsilon^{\sigma\tau z}{}_y \epsilon^{\lambda\nu c}{}_\beta \epsilon^{\mu\beta}{}_b{}^a \\
w_{21}\, u^{[\sigma} u^{[\lambda} \Delta^{\tau]\nu]} \Delta^{\rho\mu} \\
w_{22}\, u^{[\sigma} u_\alpha \epsilon^{\lambda\nu\alpha\tau]} \Delta^{\mu\rho} \\
w_{23}\, u^{[\lambda} \epsilon^{\sigma\tau\alpha\nu]} u_\alpha \Delta^{\rho\mu} \\
w_{24}\, \Delta^{\lambda[\tau} \Delta^{\nu\sigma]} \Delta^{\rho\mu} \Leftrightarrow \epsilon^{\sigma\tau zy} u_z \epsilon^{\lambda\nu c}{}_y u_c \Delta^{\rho\mu} + \epsilon^{\sigma\tau zx} u_z \Delta^{\rho\mu} u_c \epsilon^{\lambda\nu c}{}_x
\end{cases}$$

- **Matching conditions**

$$n_\mu(\delta T_S^{\mu\nu} + \delta T_A^{\mu\nu}) = 0, \quad n_\mu \delta j^\mu = 0, \quad n_\mu \delta S^{\mu\lambda\nu} = 0.$$

- $\partial_\mu s^\mu$ **is *SO(3)* invariant, and is a true scalar** $\longrightarrow$ **parity invariant**

- $\partial_\mu s^\mu \geq 0$

This allows us to cancel out all the non-physical coefficients.

Results

1. **We have retrieved the standard expressions of $\delta T_S^{\mu\nu}$ and δj^μ in relativistic hydrodynamics.**

2. **The expressions for $\delta T_S^{\mu\nu}$, $\delta T_A^{\mu\nu}$, and δj^μ include contributions proportional to $(\Omega_{\mu\nu} - \varpi_{\mu\nu})$ and $\partial_\lambda \Omega_{\mu\nu}$. Such new contributions are not corrections or only valid in a specific limit!**

3. **The expression for $\delta S^{\lambda\mu\nu}$.**

The results are almost ready, so please stay tuned !

Conclusions and outlooks

Conclusions

1. **We used a first-principle quantum-statistical method to derive the entropy current and the entropy production rate.**

2. **We studied the pseudo-gauge transformation for the entropy production.**

3. **We developed a new method based on the *SO(3)* irreducible representation that allows for the decomposition of any rank-tensor given any symmetry.**

4. **We showed that $\delta T_S^{\mu\nu}$, $\delta T_A^{\mu\nu}$, δj^μ admit contributions proportional to $(\Omega_{\mu\nu} - \varpi_{\mu\nu})$ and $\partial_\lambda \Omega_{\mu\nu}$, and obtained the expression of $\delta S^{\lambda\mu\nu}$ (to be finalized)**

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Outlooks

1. **Develop a second-order theory based on the results of our method's form of dissipative currents.**

2. **Perform a physical analysis of spin transport and relaxation times.**

3. **Numerically solve the conservation laws.**

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